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Judi Harris
College of William and Mary

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Euler and the Turtle
by Judi Harris

Let's face it, facilitators. Most of us have a Peter Pan or Wendy hiding behind our wise, adult facades, who reign in full glory whenever we sniff Logo in the classroom air. If "growing up" means losing the joyful, experimentative, problem-solving flexibility that is innately childlike, then we must reserve a special corner of our personalities that will never "mature." Chances are that you have already done that, and your fondness for Logo stems partly from the opportunity it affords to exercise the "never-never land option."

Modes of Mischief

Soon spring will be in the air, and in anticipation, memories flood back of how that special vernal aroma would incite even the most studious of my childhood friends to mischief. We perfected the technique (we thought) of appearing to listen in class, while really concentrating on something quite different.

I remember specifically one puzzle "fad" that mesmerized us for several weeks. The challenge was to draw this figure without crossing a line already drawn, or lifting the pencil from the paper. How would you do that, given those constraints?

We finally did solve the problem, but the "magic" of it remained; some pictures could be drawn this way, and others couldn't. I doubt that any of us recognized the inherent instructional value of such an activity. We also would have been surprised to discover that one of the world's most prolific mathematicians pondered a similar puzzle in the early 18th century. He also identified the pattern behind the "magic."

Euler and the Opus

The Swiss mathematician Leonhard Euler (pronounced "oiler") had a Peter Pan in his personality long before Barrie conceived the character. Euler reportedly had 13 grandchildren and is said to have created mathematical theorems with a baby on his lap and children playing at his feet. He wrote about 800 pages per year of good quality mathematical manuscripts, and is credited with concocting the original ideas of topology.

Enter the 20th century Logo turtle, and 18th century Euler line drawings like the one that intrigued us in elementary school and get a burst of Logo power. Here are some other designs which can be drawn without lifting the turtle's pen, crossing or retracing a line.
Designs such as these cannot be produced according to those procedural specifications.

So what’s the “magic”? What attributes are similar within these two groups of figures that are also dissimilar between groups? Logo students could pose, consider, discuss, revise, prove, and disprove theories after sufficient experimentation with “turtled” Euler lines. Why don’t YOU stop reading for now, and call out your Peter or Wendy to help you to play with this?

Euler’s Observations

Euler examined line segments extending from the vertices of polygons, and classified the intersections as “even vertices” or “odd vertices.”

He observed that the number of odd vertices in any figure is always even, and then went on to say:

• If there are no odd vertices in a polygon, then it can be drawn using any vertex as the starting point.

• If there are two odd vertices in a polygon, it can be drawn by starting at one odd vertex and finishing at the other without lifting the pen, crossing, or retracing a line.

• Otherwise, a polygon cannot be drawn without crossing or retracing lines, or lifting the pen.

What powerful ideas! Do you want to play again before you read on? Or did you already solve these Euler Logo puzzles?

See the Magic

Wise teachers capitalize upon children’s natural interests and preferred activities. Why not make a set of polygons that can be drawn according to Euler’s specifications? Challenge your students not only to solve the puzzles with the turtle, but to try to see the pattern to the “magic.” Most importantly, encourage them to create Euler puzzles of their own for their classmates to solve.

Oh – by the way – if your Peter or Wendy wants to check how mine solved the Euler puzzles mentioned earlier in this article, help them to send a stamped, self-addressed envelope to me at the address at the end of the article or send an electronic request on either BitNet (JudiH@Virginia.bitnet), or CompuServe (75116,1207).

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Judi Harris
621F Madison Avenue
Charlottesville, VA 22903
CIS: 75116,1207
BitNet: JudiH@Virginia