Multiplication Clouds Have Turtled Linings

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Multiplication Clouds Have Turtled Linings
by Judi Harris

Does it seem that April showers may have made your students' multiplication facts a bit rusty? Believe it or not, here is a way for them to practice the sevens table:

In drawing this figure, they can review the eights facts:

And this design can help them to memorize the sixes table:

Aligned for Review

G. H. Hardy told us that "a mathematician, like a painter or a poet, is a maker of patterns." Turtled designs make mathematical patterns viewable in graphic form. Since it is easier to remember a series of numbers that fit an overall pattern than it is to memorize seemingly random numbers, it makes good pedagogic sense to help children to recognize multiplication table patterns by drawing them with the turtle.

Consider, for example, the nines table. It is often the most difficult for students to memorize.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9</td>
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<tr>
<td>2</td>
<td>18</td>
<td>36</td>
<td>54</td>
<td>72</td>
<td>90</td>
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<td></td>
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<td>18</td>
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<tr>
<td>3</td>
<td>27</td>
<td>54</td>
<td>81</td>
<td>108</td>
<td>135</td>
<td>162</td>
<td>189</td>
<td>216</td>
<td>243</td>
<td>27</td>
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<tr>
<td>4</td>
<td>36</td>
<td>72</td>
<td>108</td>
<td>144</td>
<td>180</td>
<td>216</td>
<td>252</td>
<td>288</td>
<td>324</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>90</td>
<td>135</td>
<td>180</td>
<td>225</td>
<td>270</td>
<td>315</td>
<td>360</td>
<td>405</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>54</td>
<td>108</td>
<td>162</td>
<td>216</td>
<td>270</td>
<td>324</td>
<td>378</td>
<td>432</td>
<td>486</td>
<td>54</td>
</tr>
<tr>
<td>7</td>
<td>63</td>
<td>126</td>
<td>189</td>
<td>252</td>
<td>306</td>
<td>369</td>
<td>422</td>
<td>485</td>
<td>538</td>
<td>63</td>
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<tr>
<td>8</td>
<td>72</td>
<td>144</td>
<td>216</td>
<td>288</td>
<td>360</td>
<td>432</td>
<td>504</td>
<td>576</td>
<td>648</td>
<td>72</td>
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<td>9</td>
<td>81</td>
<td>162</td>
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<td>567</td>
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<td>729</td>
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<tr>
<td>10</td>
<td>90</td>
<td>180</td>
<td>270</td>
<td>360</td>
<td>450</td>
<td>540</td>
<td>630</td>
<td>720</td>
<td>810</td>
<td>90</td>
</tr>
</tbody>
</table>

You may have already asked your students to notice the order of the digits in the ones column:

9 8 7 6 5 4 3 2 1 0

The sequence of numerals in the tens place is similar:

0 1 2 3 4 5 6 7 8 9

And if you add each pair of digits, what is the result in each case?

Attending to patterns such as these make drill more meaningful and (hopefully) more interesting. Let us now look at multiplication facts in more graphic detail.

Digital Lineage

If we arrange the digits 0 - 9 at equidistant intervals around the circumference of a circle,
then draw a line connecting the ones’ place digits for the nines facts, in order,

the result is a decagon.

But if we connect the ones’ place digits for the fours table, 4 8 2 6 0 4 8 2 6 0

the resulting graphic is a bit more surprising.

Sketch the ones’ digit design for the sixes facts here, and you will receive a surprise of a different sort.

Why do you think some of these patterns are the same?

Fall in Line

Turtling these patterns can begin by using Logo to measure 10 evenly-spaced positions around the circumference of a 360-step circle. We know that the circumference of any circle is approximately equal to pi (~3.14) multiplied by twice the circle’s radius. Therefore, to center the circle on the screen, we can reset the turtle’s position to one radius to the left of HOME, 360 / 3.14 / 2 ~ 57, so we tell the turtle to

PENUP
SETPOS [-57 0]
PENDOWN

A 360-step circle can be drawn by entering the command

REPEAT 360 [ FORWARD 1 RIGHT 1 ]

but we would like to stop at 10 equal intervals along the way, and ask the turtle what its screen position is each time.

REPEAT 10 [ PRINT POS REPEAT 36 [ FORWARD 1 RIGHT 1 ] ]

Round these numbers to the nearest integers, and construct ten subprocedures that output the ten screen positions as lists.

TO PLACE0
OUTPUT LIST -57 0
END

TO PLACE1
OUTPUT LIST -46 34
END

TO PLACE2
OUTPUT LIST -18 55
END

TO PLACE3
OUTPUT LIST 18 55
END

TO PLACE4
OUTPUT LIST 46 35
END

TO PLACE5
OUTPUT LIST 58 1
END

TO PLACE6
OUTPUT LIST 47 -33
END

TO PLACE7
OUTPUT LIST 18 -54
END

TO PLACE8
OUTPUT LIST -17 -54
END

TO PLACE9
OUTPUT LIST -46 -34
END

You may want to make your circle demarcations on a larger (i.e., 720-step) or smaller circle.

Single File

The superprocedure should be constructed so that students can enter a list of digits that represent the ones’ place of a multiplication table to make the computer draw the corresponding pattern. If a user’s list begins with 3 6 9..., the turtle should move from PLACE3 to PLACE6 to PLACE9, leaving a turtled trail as it goes.
Let us assume that the user’s digit list is stored in a variable named DIGITS. The procedure MAKE.LINE tells the turtle to reset its screen position according to the first elements of :DIGITS.

```
TO MAKE.LINE :POINT
SETPOS RUN PLACE :POINT
END

MAKE.LINE (FIRST :DIGITS)
```

MAKE.LINE calls a subprocedure (PLACE) that translates the digit from the list into a PLACE0-PLACE9 subprocedure name.

```
TO PLACE :NUMERAL
OUTPUT LIST WORD "PLACE :NUMERAL"
END
```

The recursive procedure DRAW tells the turtle to proceed from point to point around the circle, according to the order specified by the user’s digit list.

```
TO DRAW :DIGIT.LIST
IF EMPTYP :DIGIT.LIST [SETPOS
RUN PLACE :FINAL.POINT STOP]
MAKE.LINE (FIRST :DIGITS)
DRAW BUTFIRST :DIGIT.LIST
END
```

DRAW is called by the superprocedure DESIGN.

```
TO DESIGN
CLEARSCREEN
PENUP
SETPOS [-57 0]
PENDOWN
CLEARTEXT
PRINT [PLEASE TYPE THE DIGIT LIST HERE:]
MAKE "DIGITS READLIST
MAKE "FINAL.POINT (FIRST :DIGITS)
PENUP MAKE.LINE (FIRST :DIGITS)
PENDOWN
DRAW :DIGITS
END
```

DESIGN PLEASE TYPE THE DIGIT LIST HERE:

```
3 6 9 2 5 8 1 4 7 0
```

Drop a Line

Like rabbits in this spring season, patterns beget patterns.

```
WHAT IF THESE SAME TOOLS WERE USED TO EXAMINE SEQUENCES OF ONES’ PLACE NUMERALS WHEN MULTIPLICATION FACT ANSWER DIGITS ARE SUMMED?

1 x 5 = 5 Sum: 5
2 x 5 = 10 Sum: 1
3 x 5 = 15 Sum: 6
4 x 5 = 20 Sum: 2
5 x 5 = 25 Sum: 7
6 x 5 = 30 Sum: 3
7 x 5 = 35 Sum: 8
8 x 5 = 40 Sum: 4
9 x 5 = 45 Sum: 9
10 x 5 = 50 Sum: 5
```

What patterns might emerge if answer digits were subtracted? Multiplied? Why are some patterns identical? Why are others asymmetrical? Would similar graphics emerge using addition facts?

Perhaps this spring’s out-of-line(s) classroom behavior should be multiplied!

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About the Cover

Cory Dahlquist drew this picture when he was a 6th grader in Karen Thimmesch's class at Galtier Magnet School in St. Paul. Paul Krocheski, computer teacher at Galtier, writes that Cory’s picture was the result of a project "to use squares, rectangles, circles and triangles in a master procedure to produce a human and/or animal representation." He notes that this project helps students become better at understanding the use of subprocedures in a master program.