Euler And The Turtle

Judi Harris

College of William and Mary

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By Judi Harris

Let's face it, facilitators. Most of us have a Peter Pan or Wendy hiding behind our wise, adult facades, who reign in full glory whenever we sniff Logo in the classroom air! If "growing up" means losing the joyful, experimentative, problem-solving flexibility that is innately childlike, then we Logo-ers must reserve a special corner of our personalities that will never "mature." Chances are you have already done that, and your love for Logo stems partly from the opportunity that it affords to exercise the "never-neverland" option.

**Modes of Mischief**

Spring is in the air as I write at my desk, and memories flood back of how that special vernal aroma would incite even the most studious of my childhood friends to mischief! We perfected the technique (we thought) of appearing to listen in class, while really concentrating on something quite different.

I remember specifically one puzzle "fad" that mesmerized us for several weeks. The challenge was to draw this figure without crossing a line already drawn, or lifting the pencil from the paper. Can you figure out how to do it given those constraints?

We finally did solve the problem, but the "magic" of it remained; some pictures could be drawn this way, and others couldn't. I doubt that any of us realized the inherent mathematical or problem-solving value of such an activity. We also would have been surprised to find out that one of the world's most prolific mathematicians pondered a similar puzzle in the early 18th century. He discovered the reason behind the "magic," also.

**Euler and the Opus**

The Swiss mathematician Leonhard Euler (pronounced "oiler") had a Peter Pan in his personality long before Barrie conceived the character. Euler reportedly had 13 grandchildren, and is said to have created mathematical theorems with a baby on his lap and children playing at his feet. He wrote about 800 pages a year of good quality mathematical manuscripts, and is credited with concocting the original ideas of topology.

Enter the 20th century turtle, and the 18th century Euler line drawings like the one at left get a burst of Logo power! Here are some other designs which can be drawn without the turtle's pen going up or its crossing or retracing a line that it has already drawn.
Euler continued

Designs such as these below cannot be produced according to those process specifications.

![Euler Line Designs](image)

So what’s the “magic?” What do the two groups each have in common within their individual attribute boundaries that is dissimilar across groups? Logo students could pose, consider, discuss, revise, prove, and disprove theories after sufficient experimentation with “turtled” Euler lines. Why don’t YOU stop reading for now, and call out your Peter or Wendy to help you play with this?

Euler’s Observations

Euler examined the line segments extending from the vertices of polygons and classified the intersections as “even vertices” and “odd vertices.”

Even Vertices

Odd Vertices

He observed that the number of odd vertices in a figure is always even, and then went on to say:

- If there are no odd vertices in a polygon, then it can be drawn using any vertex as the starting point.
- If there are two odd vertices in a polygon, it can be drawn with Euler lines by starting at one odd vertex and finishing at the other.
- Otherwise, a polygon cannot be drawn without crossing or retracing lines, or lifting the pencil.

What powerful ideas! Do you want to play again, now, before you read on? Or did you already solve the Euler Logo puzzles?

See the Magic

Wise teachers capitalize upon children’s natural interests and preferred activities. Why not make a set of polygons that can be drawn according to Euler’s specifications? Challenge your students not only to solve the puzzles with the turtle, but to try to see the pattern to the “magic.” Most importantly, they could make new Euler line puzzles for their classmates to solve!

Oh - by the way - if your Peter or Wendy wants to check how mine solved the Euler puzzles mentioned earlier in the article, help them to send a stamped self-addressed long envelope to:

Judi Harris
104 Woodside Road, B-306
Haverford, PA 19041

Judi Harris is an elementary school use facilitator in Haverford, PA, and a graduate education instructor for a number of institutes of higher learning. She is also an educational computing consultant whose clients include the Ministry of Education of Kuwait. Her CompuServe number is 75116,1207.

From The Editor

by Tom Lough

Educators are enjoined to encourage students to take risks. Taking risks is an important part of the Logo learning environment.

All too recently, we were reminded that the price to pay for taking a risk can be high. In spite of the shuttle tragedy, however, the quest for knowledge continues.

This space is dedicated to the memory of our professional colleague

Christa McAuliffe

and her fellow travelers,

Francis Scoobee,

Michael Smith,

Judith Resnick,

Ronald McNair,

Ellison Onizuka,

and

Gregory Jarvis.

May their willingness to take risks be an inspiration to us all.

FD 104!