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A gravitational-wave standard siren measurement of the Hubble constant


On 17 August 2017, the Advanced LIGO1 and Virgo2 detectors observed the gravitational-wave event GW170817—a strong signal from the merger of a binary neutron-star system3. Less than two seconds after the merger, a γ-ray burst (GRB 170817A) was detected within a region of the sky consistent with the LIGO–Virgo-derived location of the gravitational-wave source4–6. This sky region was subsequently observed by optical astronomy facilities7, resulting in the identification of an optical transient signal within about ten arcseconds of the galaxy NGC 4993. This detection of GW170817 in both gravitational waves and electromagnetic waves represents the first 'multi-messenger' astronomical observation. Such observations enable GW170817 to be used as a 'standard siren' (meaning that the absolute distance to the source can be determined directly from the gravitational-wave measurements) to measure the Hubble constant. This quantity represents the local expansion rate of the Universe, sets the overall scale of the Universe and is of fundamental importance to cosmology. Here we report a measurement of the Hubble constant that combines the distance to the source inferred purely from the gravitational-wave signal with the recession velocity inferred from measurements of the redshift using the electromagnetic data. In contrast to previous measurements, ours does not require the use of a cosmic 'distance ladder': the gravitational-wave analysis can be used to estimate the luminosity distance out to cosmological scales directly, without the use of intermediate astronomical distance measurements. We determine the Hubble constant to be about 70 kilometres per second per megaparsec. This value is consistent with existing measurements18,19, while being completely independent of them. Additional standard siren measurements from future gravitational-wave sources will enable the Hubble constant to be constrained to high precision.

The Hubble constant \(H_0\) measures the mean expansion rate of the Universe. At nearby distances (less than about 50 Mpc) it is well approximated by the expression

\[
v_H = H_0 d
\]

where \(v_H\) is the local 'Hubble flow' velocity of a source and \(d\) is the distance to the source. At such distances all cosmological distance measures (such as luminosity distance and comoving distance) differ at the order of \(v_H/c\), where \(c\) is the speed of light. Because \(v_H/c \approx 1\%\) for GW170817, the differences between the different distance measures are much smaller than the overall errors in distance. Our measurement of \(H_0\) is similarly insensitive to the values of other cosmological parameters, such as the matter density \(\Omega_m\) and the dark-energy density \(\Omega_L\).

To obtain the Hubble flow velocity at the position of GW170817, we use the optical identification of the host galaxy NGC 4993. This identification is based solely on the two-dimensional projected offset and is independent of any assumed value of \(H_0\). The position and redshift of this galaxy allow us to estimate the appropriate value of the Hubble flow velocity. Because the source is relatively nearby, the random relative motions of galaxies, known as peculiar velocities, need to be taken into account. The peculiar velocity is about 10% of the measured recessional velocity (see Methods).

The original standard siren proposal14 did not rely on the unique identification of a host galaxy. By combining information from around 100 independent gravitational-wave detections, each with a set of potential host galaxies, an estimate of \(H_0\) accurate to 5% can be obtained even without the detection of any transient optical counterparts22. This is particularly relevant, because gravitational-wave networks will detect many binary black-hole mergers over the coming years23 and these are not expected to be accompanied by electromagnetic counterparts. Alternatively, if an electromagnetic counterpart has been identified but the host galaxy is unknown, then the same statistical method can be applied but using only those galaxies in a narrow beam around the location of the optical counterpart. However, such statistical analyses are sensitive to several complicating effects, such as the incompleteness of current galaxy catalogues or the need for dedicated follow-up surveys, and to a range of selection effects24. Here we use the identification of NGC 4993 as the host galaxy of GW170817 to perform a standard siren measurement of the Hubble constant15–18.

Analysis of the gravitational-wave data associated with GW170817 produces estimates for the parameters of the source, under the assumption that general relativity is the correct model of gravity. We are most interested in the joint posterior distribution on the luminosity distance and binary orbital inclination angle. For the analysis we fix the location of the gravitational-wave source on the sky to the identified location of the counterpart (see Methods for details).

An analysis of the gravitational-wave data alone finds that GW170817 occurred at a distance \(d = 43.8^{+2.5}_{-6.8}\) Mpc (all values are quoted as the maximum posterior value with the minimal-width 68.3% credible interval). The distance quoted here differs from that in other studies3, because here we assume that the optical counterpart represents the true sky location of the gravitational-wave source instead of marginalizing over a range of potential sky locations. The uncertainty of approximately 15% is due to a combination of statistical measurement error from the noise in the detectors, instrumental calibration uncertainties3 and a geometrical factor that depends on the correlation of distance with inclination angle. The gravitational-wave measurement is consistent with the distance to NGC 4993 measured using the Tully–Fisher relation\(^{9,25}\), \(d_{TF} = 41.1 \pm 5.8\) Mpc.

The measurement of the gravitational-wave polarization is crucial for inferring the binary inclination. This inclination, \(\iota\), is defined as the angle between the line-of-sight vector from the source to the detector and the orbital-angular-momentum vector of the binary system. For electromagnetic phenomena it is typically not possible to tell whether a system is orbiting clockwise or anticlockwise (or, equivalently, face-on or face-off), and sources are therefore usually characterized.
by a viewing angle defined as min(\(\eta\), 180° − \(\eta\)), with \(\eta\) in the range [0°, 180°]. By contrast, gravitational-wave measurements can identify the sense of the rotation, and so \(\eta\) ranges from 0° (anticlockwise) to 180° (clockwise). Previous gravitational-wave detections by the Laser Interferometer Gravitational-wave Observatory (LIGO) had large uncertainties in luminosity distance and inclination\(^{23}\) because the two LIGO detectors that were involved are nearly co-aligned, preventing a precise polarization measurement. In the present case, owing to the addition of the Virgo detector, the cosine of the inclination can be constrained at 68.3% (1\(\sigma\)) confidence to the range \([-1.00, −0.81]\), corresponding to inclination angles in the range [144°, 180°]. This inclination range implies that the plane of the binary orbit is almost, but not quite, perpendicular to our line of sight to the source (\(\eta\approx 180°\)), which is consistent with the observation of a coincident \(\gamma\)-ray burst\(^{4–6}\). We report inferences on \(\cos \eta\) because our prior for it is flat, so the posterior is proportional to the marginal likelihood for it from the gravitational-wave observations.

Electromagnetic follow-up observations of the gravitational-wave sky-localization region\(^7\) discovered an optical transient\(^8–11\) in close proximity to the galaxy NGC 4993. The location of the transient was previously observed by the Distance Less Than 40 Mpc (DLT40) survey on 27.99 July 2017 universal time (UT) and no sources were found\(^{10}\). We estimate the probability of a random chance association between the optical counterpart and NGC 4993 to be 0.004% (Methods). In what follows we assume that the optical counterpart is associated with GW170817, and that this source resides in NGC 4993.

To compute \(H_0\) we need to estimate the background Hubble flow velocity at the position of NGC 4993. In the traditional electromagnetic calibration of the cosmic ‘distance ladder’\(^{19}\), this step is commonly carried out using secondary distance indicator information, such as the Tully–Fisher relation\(^{25}\), which enables the background Hubble flow velocity in the local Universe to be inferred by scaling back from more distant secondary indicators calibrated in quiet Hubble flow. We do not adopt this approach here, however, to preserve more fully the independence of our results from the electromagnetic distance ladder. Instead we estimate the Hubble flow velocity at the position of NGC 4993 by correcting for local peculiar motions.

NGC 4993 is part of a collection of galaxies, ESO 508, which has a center-of-mass recession velocity relative to the frame of the cosmic microwave background (CMB)\(^{26}\) of 3.327 ± 72 km s\(^{-1}\). We correct the group velocity by 310 km s\(^{-1}\) owing to the coherent bulk flow\(^{28,29}\) towards the Great Attractor (Methods). The standard error on our estimate of the peculiar velocity is 69 km s\(^{-1}\), but recognizing that this value may be sensitive to details of the bulk flow motion that have been imperfectly modelled, in our subsequent analysis we adopt a more conservative estimate\(^{29}\) of 150 km s\(^{-1}\) for the uncertainty on the peculiar velocity at the location of NGC 4993 and fold this into our estimate of the uncertainty on \(\eta\). From this, we obtain a Hubble velocity \(v_H = 3.017 ± 0.166\) km s\(^{-1}\).

Once the distance and Hubble-velocity distributions have been determined from the gravitational-wave and electromagnetic data, respectively, we can constrain the value of the Hubble constant. The measurement of the distance is strongly correlated with the measurement of the inclination of the orbital plane of the binary. The analysis of the gravitational-wave data also depends on other parameters describing the source, such as the masses of the components\(^23\). Here we treat the uncertainty in these other variables by marginalizing over the posterior distribution on system parameters\(^5\), with the exception of the position of the system on the sky, which is taken to be fixed at the location of the optical counterpart.

We carry out a Bayesian analysis to infer a posterior distribution on \(H_0\) and inclination, marginalized over uncertainties in the recessional and peculiar velocities (Methods). In Fig. 1 we show the marginal posterior for \(H_0\). The maximum a posteriori value with the minimal 68.3% credible interval is \(H_0 = 70.0^{+12.9}_{−8.0}\) km s\(^{-1}\) Mpc\(^{-1}\). Our estimate agrees well with state-of-the-art determinations of this quantity, including CMB measurements from Planck\(^{20}\) (67.74 ± 0.46 km s\(^{-1}\) Mpc\(^{-1}\), ‘TT, TE, EE + lowP + lensing + ext’) and type Ia supernova measurements from SHoES\(^{21}\) (73.24 ± 1.74 km s\(^{-1}\) Mpc\(^{-1}\)), and with baryon acoustic oscillations measurements from SDSS\(^{39}\), strong lensing measurements from H0LiCOW\(^{31}\), high-angular-multipole CMB measurements from SPT\(^{32}\) and Cepheid measurements from the Hubble Space Telescope key project\(^{19}\). Our measurement is an independent determination of \(H_0\). The close agreement indicates that, although each method may be affected by different systematic uncertainties, we see no evidence at present for a systematic difference between gravitational-wave–based estimates and established electromagnetic–based estimates. As has been much remarked on, the Planck and SHoES results are inconsistent at a level greater than about 3\(\sigma\). Our measurement does not resolve this inconsistency, being broadly consistent with both.

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**Figure 1** | GW170817 measurement of \(H_0\). The marginalized posterior density for \(H_0\) \(p(H_0\mid GW170817)\), is shown by the blue curve. Constraints at 1\(\sigma\) (darker shading) and 2\(\sigma\) (lighter shading) from Planck\(^{20}\) and SHoES\(^{23}\) are shown in green and orange, respectively. The maximum a posteriori value and minimal 68.3% credible interval from this posterior density function is \(H_0 = 70.0^{+12.9}_{−8.0}\) km s\(^{-1}\) Mpc\(^{-1}\). The 68.3% (1\(\sigma\)) and 95.4% (2\(\sigma\)) minimal credible intervals are indicated by dashed and dotted lines, respectively.

**Figure 2** | Inference on \(H_0\) and inclination. The posterior density of \(H_0\) and \(\cos \eta\) from the joint gravitational-wave–electromagnetic analysis are shown as blue contours. Shading levels are drawn at every 5% credible level, with the 68.3% (1\(\sigma\); solid) and 95.4% (2\(\sigma\); dashed) contours in black. Values of \(H_0\) and 1\(\sigma\) and 2\(\sigma\) error bands are also displayed from Planck\(^{20}\) and SHoES\(^{21}\). Inclination angles near 180° (\(\cos \eta = −1\)) indicate that the orbital angular momentum is antiparallel to the direction from the source to the detector.
The posterior density on $\cos(\theta)$ ($p(\cos(\theta))$) is shown for various assumptions about the prior distribution of $H_\Omega$. The analysis of the joint gravitational-wave and electromagnetic data with a 1/$H_\Omega$ prior density gives the blue curve; using values of $H_\Omega$ from Planck and SH0ES as a prior on $H_\Omega$ gives the green and red curves, respectively. Choosing a narrow prior on $H_\Omega$ converts the precise Hubble velocity measurements for the group containing NGC 4993 to a precise distance measurement, breaking the distance inclination degeneracy and leading to strong constraints on the inclination. Minimal 68.3% (1\,\sigma) credible intervals are indicated by dashed lines. Because our prior on inclination is flat on $\cos(\theta)$, the densities in this plot are proportional to the marginalized likelihood for $\cos(\theta)$.

One of the main sources of uncertainty in our measurement of $H_\Omega$ is due to the degeneracy between distance and inclination in the gravitational-wave measurements. A face-on or face-off binary far away has a similar gravitational-wave amplitude to that of an edge-on binary closer in. This relationship is captured in Fig. 2, which shows posterior contours in the $H_\Omega$-$\cos(\theta)$ parameter space.

The posterior in Fig. 1 results from the vertical projection of Fig. 2, marginalizing out uncertainties in $\cos(\theta)$ to derive constraints on $H_\Omega$. Alternatively, it is possible to project horizontally, and thereby marginalize out $H_\Omega$ to derive constraints on $\cos(\theta)$. If instead of deriving $H_\Omega$ we take the existing constraints on $\cos(\theta)$ from SH0ES as priors, we are able to improve our constraints on $H_\Omega$, as shown in Fig. 3. Assuming the Planck value for $H_\Omega$, the minimal 68.3% credible interval for $\cos(\theta)$ is $[−1.00, −0.92]$ (corresponding to an inclination angle in the range $[157^\circ, 177^\circ]$). Assuming the SH0ES value of $H_\Omega$, it is $[−0.97, −0.85]$ (corresponding to an inclination angle in the range $[148^\circ, 166^\circ]$). We note that the face-off $i = 180^\circ$ orientation for the SH0ES result is just outside the 90% confidence range. It will be particularly interesting to compare these constraints to those from modelling of the short $\gamma$-ray burst, afterglow and optical counterpart associated with GW170817.

We have presented a standard siren determination of the Hubble constant, using a combination of a distance estimate from gravitational-wave observations and a Hubble velocity estimate from electromagnetic observations. Our measurement does not use a ‘distance ladder’ and makes no prior assumptions about $H_\Omega$. We find $H_\Omega = 70.0^{+12.9}_{−12.0} \text{km s}^{-1} \text{Mpc}^{-1}$, which is consistent with existing measurements. This first gravitational-wave–electromagnetic multi-messenger event demonstrates the potential for cosmological inference from gravitational-wave standard sirens. We expect that additional multi-messenger binary neutron-star events will be detected in the coming years, and combining subsequent independent measurements of $H_\Omega$ from these future standard sirens will lead to an era of precision gravitational-wave cosmology.
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METHODS
Probability of optical counterpart association with NGC 4993. We calculate the probability that an NGC 4993-like galaxy (or brighter) is misidentified as the host by asking how often the centre of one or more such galaxies falls by random chance within a given angular radius θ of the counterpart. Assuming Poisson counting statistics this probability is given by \( P = 1 - \exp\left(-\pi \theta^2 S \left( m < m^c \right) \right) \) where \( S \left( m \right) \) is the surface density of galaxies with apparent magnitude equal to or brighter than \( m \). From the local galaxy sample distribution in the infrared (-K-band) apparent magnitudes, we obtain \( S(-K) = 0.68 \times 10^{4.04 M_B - 10.7} \) per square degree. As suggested previously\(^{24}\), we set \( \theta \) equal to twice the half-light radius of the galaxy, for which we use diameter of NGC 4993 of about 1.1 arcmin, as measured in the near-infrared band (the predominant emission band for early-type galaxies).

NGC 4993 resides in a group of galaxies whose center-of-mass recession velocity relative to the CMB frame\(^{25}\) is \( 3,327 \pm 72 \) km s\(^{-1}\). We assume that all of the galaxies in the group are at the same distance and therefore have the same Hubble flow velocity, which we assign to be the Hubble velocity of GW170817. This assumption is accurate to within 1\% given that the radius of the group is approximately 0.4 Mpc. To calculate the Hubble flow velocity of the group, we correct its measured recessional velocity by the peculiar velocity caused by the local gravitational field. This is a large correction\(^{25,26}\); typical peculiar velocities are 300 km s\(^{-1}\) equivalent to about 10\% of the total recessional velocity at a distance of 40 Mpc.

We use the 6dF galaxy redshift survey peculiar velocity map\(^{26,27}\), which used more than 8,000 Fundamental Plane galaxies to map the peculiar velocity field in the southern hemisphere out to redshift \( z \leq 0.055 \). We weight the peculiar velocity corrections from this catalogue with a Gaussian kernel centered on the sky position of NGC 4993 and with a width of 8\% Mpc; the kernel width is independent of \( H_0 \) and is equivalent to a width of 800 km s\(^{-1}\) in velocity space, typical of the widths used in the catalogue itself. There are ten galaxies in the 6dF peculiar velocity catalogue within one kernel width of NGC 4993. In the CMB frame\(^{26}\), the weighted radial component of the peculiar velocity and associated uncertainty is \( \langle v_p \rangle = 310 \pm 69 \) km s\(^{-1}\).

We verified the robustness of this peculiar velocity correction by comparing it with the velocity field reconstructed from the 2MASS redshift survey\(^{25,27}\). This exploits the linear relationship between the peculiar velocity and mass density fields smoothed on scales larger than about 8h\(^{-1}\) Mpc, and the constant of proportionality can be determined by comparison with radial peculiar velocities of individual galaxies estimated from, for example, Tully–Fisher and type Ia supernovae distances. Using these reconstructed peculiar velocities, which have a larger associated uncertainty\(^{29}\) of 150 km s\(^{-1}\), at the position of NGC 4993 we find a Hubble velocity in the CMB frame of \( 3,047 \pm 31 \) km s\(^{-1}\). In excellent agreement with the result derived using 6dF. We adopt this larger uncertainty on the peculiar velocity correction in recognition that the peculiar velocity estimated from the 6dF data may represent an imperfect model of the true bulk flow at the location of NGC 4993. For our inference of the Hubble constant we therefore use a Hubble velocity \( v_H = 3,017 \pm 166 \) km s\(^{-1}\) with 68.3\% uncertainty.

Finally, we emphasize again the independence of our Hubble–constant inference from the electromagnetic distance scale, but note the consistency of our gravitational-wave distance estimate to NGC 4993 with the Tully–Fisher distance estimate derived by scaling back the Tully–Fisher relation calibrated with more distant galaxies in quiet Hubble flow\(^{25}\). This consistency also strongly supports the robustness of our estimate for the Hubble velocity of NGC 4993.

Summary of the model. Given observed data from a set of gravitational-wave detectors, \( x_{GW} \) parameter estimation is used to generate a posterior on the parameters that determine the waveform of the gravitational-wave signal. Parameters are inferred within a Bayesian framework\(^{28}\) by comparing strain measurements in the two LIGO detectors and in the Virgo detector with the gravitational waveforms expected from the inspiral of two point masses\(^{29}\) under general relativity. We use algorithms for removing short-lived detector noise artefacts\(^{30}\) and use approximate point-particle waveform models.\(^{39,41,42}\) We have verified that the systematic changes in the results presented here from incorporating non-point-mass tidal effects\(^{43,44}\) and from different data processing methods are much smaller than the statistical uncertainties in the measurement of \( H_0 \) and the orbital inclination angle of the binary.

From this analysis we can obtain the parameter estimation likelihood of the observed gravitational-wave data, marginalized over all parameters that characterize the gravitational-wave signal except \( d \) and \( \cos(\theta) \).

\[
p(x_{GW}(d, \cos(\theta))) = \int p(x_{GW}(d, \cos(\lambda))) \, p(\lambda) \, d\lambda
\]

The other waveform parameters are denoted by \( \lambda \), with \( p(\lambda) \) denoting the corresponding prior.

Given perfect knowledge of the Hubble flow velocity of the gravitational-wave source \( v_H \), this posterior distribution can be readily converted into a posterior on \( d \) and \( \cos(\theta) \) as:

\[
p(H_0, \cos(v_H)) \propto \langle \cos(\theta) \rangle \, p(x_{GW}(d, \cos(\theta))) \, p(v_H) \, p(\lambda) d\lambda
\]

where \( p(d) \) and \( p(\cos(\theta)) \) are the prior distributions on distance and inclination. For the Hubble velocity \( v_H = 3,017 \) km s\(^{-1}\), the maximum a posteriori distance from the gravitational-wave measurement of 43.8 Mpc corresponds to \( H_0 = 68.9 \) km s\(^{-1}\) Mpc\(^{-1}\), so this procedure would be expected to generate a posterior on \( H_0 \) that peaks close to that value.

Although the above analysis is conceptually straightforward, it makes several assumptions. In practice, the Hubble flow velocity cannot be determined exactly and must be corrected for uncertain peculiar velocities. This correction does not explicitly set a prior on \( H_0 \), but instead inherits a 1/\( H_0^2 \) prior from the usual \( p(d) \propto d^3 \) prior used in gravitational-wave parameter estimation. In addition, the log likelihood in this model is that a redshift has been obtained first and the distance is then measured using gravitational waves. Because gravitational-wave detectors cannot be pointed, we cannot target particular galaxies or redshifts for gravitational-wave sources. In practice, we wait for a gravitational-wave event to trigger the analysis and this introduces potential selection effects that we must consider. We see below that the simple analysis described above does give results that are consistent with a more careful analysis for this first detection. However, the simple analysis cannot be readily extended to include second and subsequent detections, so we now describe a more general framework that does not suffer from these limitations.

We suppose that we have observed a gravitational-wave event, which generated data \( x_{GW} \) in our detectors, and that we have also measured a recessional velocity for the host \( v_H \) and the peculiar velocity field \( v_p \) in the vicinity of the host. These observations are statistically independent and so the combined likelihood is

\[
p(x_{GW}, v_H, v_p) = N(\langle v_{GW} \rangle - \langle H_0 \rangle v_H, \sigma_{GW}^2) \times \exp(-\sigma^2/2)
\]

where \( N(\langle v \rangle, \sigma^2) \) is the normal (Gaussian) probability density with mean \( \langle v \rangle \) and standard deviation \( \sigma \) evaluated at \( x \). The measured recessional velocity \( v_{GW} = 3,323 \) km s\(^{-1}\), with uncertainty \( \sigma_{GW} = 72 \) km s\(^{-1}\), is the mean velocity and standard error for the members of the group hosting NGC 4993 taken from 2MASS\(^{27}\), corrected to the CMB frame\(^{26}\) We take a similar Gaussian likelihood for the measured peculiar velocity \( v_p = 310 \) km s\(^{-1}\), with uncertainty \( \sigma_p = 150 \) km s\(^{-1}\).
signals that generate a response of sufficiently high amplitude. The decision about whether to include an event in the analysis is a property of the data only, in this case \(x_{GW}, v_\text{r}, v_\text{p}\), but the fact that we condition our analysis on a signal being detected, that is, the data exceeding these thresholds, means that the likelihood must be renormalized to become the likelihood for detected events. This is the role of \(N_\text{d} = \int p(d|x_{GW}, cos, \lambda) p(v_\text{r}, v_\text{p}, H_0)p(p(v_\text{r})|v_\text{p}) dp\)

\[
\int p(d|x_{GW}, cos, \lambda) p(v_\text{r}, v_\text{p}, H_0)p(p(v_\text{r})|v_\text{p}) dp
\]

where the integral is over the full prior ranges of the parameters \(|d, v_\text{r}, v_\text{p}, cos, \lambda|\) and over datasets that would be selected for inclusion in the analysis (that is, that exceed the specified thresholds). If the integral was over all datasets then it would evaluate to 1, but because the range is restricted there can be a non-trivial dependence on parameters characterizing the population of sources, in this case \(H_0\).

In our analysis, there are in principle selection effects in both the gravitational-wave data and the electromagnetic data. However, around the time of detection of GW170817, the LIGO–Virgo detector network had a detection horizon of approximately 190 Mpc for binary neutron-star events, within which electromagnetic measurements are largely complete. For example, the counterpart associated with GW170817 had a brightness of about 17 mag in the I band at 40 Mpc (refs 8–13); this source would be about 22 mag at 400 Mpc, and therefore still detectable by survey telescopes such as DECam well beyond the gravitational-wave horizon. Even the dimmest theoretical light curves for kilonovae are expected to peak at about 22.5 mag at the LIGO–Virgo horizon.

We therefore expect that gravitational-wave selection effects are dominant and ignore electromagnetic selection effects. The fact that the fraction of binary neutron-star events that will have observed kilonova counterparts is presently unknown does not modify these conclusions, because we can restrict our analysis to only gravitational-wave events with kilonova counterparts.

For the gravitational-wave data, the decision about whether or not to analyse an event is determined largely by the signal-to-noise ratio \(\rho\) of the event. A reasonable model for the selection process is a cut in signal-to-noise ratio; that is, \(d > d_{\text{threshold}}\) for some threshold \(d_{\text{threshold}}\). However, if the redshift prior is volumetric, \(p(z) \propto d^2\), then the selection-effect term is proportional to \(H_0^2\), which cancels a similar correction to the likelihood and gives a posterior on \(H_0\) that is identical to the canonical analysis.

For a single event, any choice of prior can be mapped to our canonical analysis with a different prior on \(H_0\). For any reasonable prior choices on \(d, \rho, z\), we would expect to gradually lose sensitivity to the particular prior choice as further observed events are added to the analysis. However, to illustrate the uncertainty that comes from the prior choice for this first event, we compare in Extended Data Fig. 2 and Extended Data Table 1 the results from the canonical prior choice \(p(d) \propto d^2\) to those from two other choices: using a flat prior on \(z\) and assuming a velocity correction due to the peculiar velocity of NGC 4993 that is a Gaussian with width 250 km s\(^{-1}\). To do the first of these, the posterior samples from gravitational-wave parameter estimation have to be re-weighted, because they are generated with the \(d^2\) prior used in the canonical analysis. We first ‘undo’ the default prior before applying the desired new prior.

The choice of a flat prior on \(z\) is motivated by the simple model described above, in which we imagine first making a redshift measurement for the host and then using that as a prior for analysing the gravitational-wave data. Setting priors on distance and redshift, the simple analysis gives the same result as the canonical analysis, but now we set a prior on redshift and \(H_0\) and obtain a different result. This is to be expected because we are making different assumptions about the underlying population, and it arises for similar reasons as the different biases in peculiar velocity measurements based on redshift-selected or distance-selected samples.

As can be seen in Extended Data Table 1, the results change by less than 1\(\sigma\), as measured by the statistical error of the canonical analysis.

By increasing the uncertainty in the peculiar velocity prior, we test the assumptions in our canonical analysis that (1) NGC 4993 is a member of the nearby group of galaxies, and (2) that this group has a center-of-mass velocity close to the Hubble flow. The results in Extended Data Table 1 summarize changes in the values of \(H_0\) and in the error bars.

We conclude that the effect of a reasonable change to the prior is small relative to the statistical uncertainties for this event.

### Incorporating additional constraints on \(H_0\)

By including previous measurements of \(H_0\) we can constrain the orbital inclination more precisely. We do this by setting the \(H_0\) prior in equation (3) to \(p(H_0|H_0\sigma) = N(H_0\sigma)^2\) for ShoES11 and 22: \(H_1 = 73.24 \pm 1.04 \text{ km s}^{-1}\text{Mpc}^{-1}\) and \(\sigma_{1,1} = 1.74 \text{ km s}^{-1}\text{Mpc}^{-1}\), and for Planck 20: \(H_0 = 67.74 \pm 1.91 \text{ km s}^{-1}\text{Mpc}^{-1}\) and \(\sigma_0 = 0.46 \text{ km s}^{-1}\text{Mpc}^{-1}\).

### Data and code availability

The publicly available codes and data can be found at the LIGO Open Science Center (https://losc.ligo.org).
Extended Data Figure 1 | Graphical model illustrating the statistical relationships between the data and parameters. Open circles indicate parameters that require a prior; filled circles describe measured data, which are conditioned on in the analysis. Here we assume that we have measurements of the gravitational-wave data $x_{GW}$, a recessional velocity (that is, redshift) $v_r$, and the mean peculiar velocity in the neighborhood of NGC 4993 $(v_p)$. Arrows flowing into a node indicate that the conditional probability density for the node depends on the source parameters; for example, the conditional distribution for the observed gravitational-wave data $p(x_{GW} | d, \cos \iota)$ depends on the distance and inclination of the source (and additional parameters, here marginalized out).
Extended Data Figure 2 | Using different assumptions compared to our canonical analysis. The posterior distribution on $H_0$ discussed in the main text is shown in black, the alternative flat prior on $z$ (discussed in Methods) gives the distribution shown in blue, and the increased uncertainty (250 km s$^{-1}$) applied to our peculiar velocity measurement (also discussed in Methods) is shown in pink. Minimal 68.3% (1σ) credible intervals are shown by dashed lines.
Extended Data Table 1 | Summary of constraints on the Hubble constant, binary inclination and distance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>68.3% Symm.</th>
<th>68.3% MAP</th>
<th>90% Symm.</th>
<th>90% MAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0/(\text{km s}^{-1}\text{ Mpc}^{-1})$</td>
<td>74.0$^{+16.0}_{-8.0}$</td>
<td>70.0$^{+12.0}_{-8.0}$</td>
<td>74.0$^{+33}_{-12}$</td>
<td>70.0$^{+28}_{-11}$</td>
</tr>
<tr>
<td>$H_0/(\text{km s}^{-1}\text{ Mpc}^{-1})$ (flat in z prior)</td>
<td>81$^{+27}_{-13}$</td>
<td>71.0$^{+23.0}_{-9.0}$</td>
<td>81$^{+50}_{-17}$</td>
<td>71.0$^{+48}_{-11}$</td>
</tr>
<tr>
<td>$H_0/(\text{km s}^{-1}\text{ Mpc}^{-1})$ (250 km s$^{-1}$ $\sigma_{v_r}$)</td>
<td>74.0$^{+16.0}_{-9.0}$</td>
<td>70.0$^{+14.0}_{-9.0}$</td>
<td>74.0$^{+33}_{-14}$</td>
<td>70.0$^{+29}_{-14}$</td>
</tr>
<tr>
<td>$\cos \iota$ (GW only)</td>
<td>$-0.88^{+0.18}_{-0.09}$</td>
<td>$-0.974^{+0.164}_{-0.026}$</td>
<td>$-0.88^{+0.32}_{-0.11}$</td>
<td>$-0.974^{+0.332}_{-0.026}$</td>
</tr>
<tr>
<td>$\cos \iota$ (SHoES)</td>
<td>$-0.901^{+0.065}_{-0.057}$</td>
<td>$-0.912^{+0.061}_{-0.059}$</td>
<td>$-0.901^{+0.106}_{-0.083}$</td>
<td>$-0.912^{+0.095}_{-0.086}$</td>
</tr>
<tr>
<td>$\cos \iota$ (Planck)</td>
<td>$-0.948^{+0.052}_{-0.036}$</td>
<td>$-0.982^{+0.060}_{-0.016}$</td>
<td>$-0.948^{+0.091}_{-0.046}$</td>
<td>$-0.982^{+0.104}_{-0.018}$</td>
</tr>
<tr>
<td>$\iota$/deg (GW only)</td>
<td>152$^{+14}_{-17}$</td>
<td>167$^{+13}_{-23}$</td>
<td>152$^{+20}_{-27}$</td>
<td>167$^{+13}_{-37}$</td>
</tr>
<tr>
<td>$\iota$/deg (SHoES)</td>
<td>154.0$^{+9.0}_{-8.0}$</td>
<td>156.0$^{+10.0}_{-7.0}$</td>
<td>154.0$^{+15}_{-12}$</td>
<td>156.0$^{+21}_{-11}$</td>
</tr>
<tr>
<td>$\iota$/deg (Planck)</td>
<td>161.0$^{+8.0}_{-8.0}$</td>
<td>169.0$^{+8.0}_{-12.0}$</td>
<td>161.0$^{+12}_{-12}$</td>
<td>169.0$^{+11}_{-18}$</td>
</tr>
<tr>
<td>$d$/ (Mpc)</td>
<td>41.1$^{+4.0}_{-7.3}$</td>
<td>43.8$^{+2.9}_{-6.9}$</td>
<td>41.1$^{+5.6}_{-12.6}$</td>
<td>43.8$^{+5.6}_{-13.1}$</td>
</tr>
</tbody>
</table>

We give both 1σ (68.3%) and 90% credible intervals for each quantity. ‘Symm.’ refers to a symmetric interval (for example, median and 5%–95% range); ‘MAP’ refers to maximum a posteriori intervals (for example, MAP value and smallest range enclosing 90% of the posterior). Values given for $\iota$ are derived from arccosine-transforming the corresponding values for $\cos \iota$, so the ‘MAP’ values differ from those that would be derived from the posterior on $\cos \iota$. © 2017 Macmillan Publishers Limited, part of Springer Nature. All rights reserved.