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## NUMERICAL RANGES OF THE PRODUCT OF OPERATORS

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## NUMERICAL RANGES OF THE PRODUCT OF OPERATORS

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*Abstract.* We study containment regions of the numerical range of the product of operators  $A$  and  $B$  such that  $W(A)$  and  $W(B)$  are line segments. It is shown that the containment region is equal to the convex hull of elliptical disks determined by the spectrum of  $AB$ , and conditions on  $A$  and  $B$  for the set equality holding are obtained. The results cover the case when  $A$  and  $B$  are self-adjoint operators extending the previous results on the numerical range of the product of two orthogonal projections.

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