Combined Analysis of $\nu(\mu)$ Disappearance and $\nu(\mu) \rightarrow \nu(e)$ Appearance in MINOS Using Accelerator and Atmospheric Neutrinos

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Combined Analysis of $\nu_\mu$ Disappearance and $\nu_\mu \rightarrow \nu_e$ Appearance in MINOS Using Accelerator and Atmospheric Neutrinos


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We report on a new analysis of neutrino oscillations in MINOS using the complete set of accelerator and atmospheric data. The analysis combines the $\nu_\mu$ disappearance and $\nu_e$ appearance data using the three-flavor

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The study of neutrino oscillations has entered a precision era in which the experimental data can be used to probe the three-flavor framework of mixing between the neutrino flavor eigenstates (ν_e, ν_μ, ν_τ) and mass eigenstates (ν_1, ν_2, ν_3). In the standard theory, neutrino mixing is described by the unitary PMNS matrix [1], parametrized by three angles θ_{12}, θ_{23}, θ_{13}, and a phase δ_{CP}. The oscillation probabilities additionally depend on the two mass-squared differences Δm^2_{23} and Δm^2_{13}, where Δm^2_{ij} = m_i^2 - m_j^2. The current generation of experiments has measured all three mixing angles and the mass-squared differences using accelerator, atmospheric, reactor, and solar neutrinos [2]. Most recently, the smallest mixing angle, θ_{13}, has been measured precisely by reactor neutrino experiments [3–5]. However, the picture is not yet complete. The value of δ_{CP}, which determines the level of CP violation in the lepton sector, has not yet been measured. It is also not known whether the neutrino mass hierarchy is normal (Δm^2_{32} > 0) or inverted (Δm^2_{32} < 0), whether sin^2 2θ_{23} is maximal, or if not, whether the mixing angle θ_{23} lies in the lower (θ_{23} < π/4) or higher (θ_{23} > π/4) octant. These unknowns, which are essential to a complete understanding of neutrino mass and mixing, can be probed by long-baseline neutrino experiments.

The MINOS long-baseline experiment [6] has published measurements of oscillations using accelerator and atmospheric neutrinos and antineutrinos. The oscillations observed by MINOS are driven by the larger mass-squared difference Δm^2_{32}; hence, many features of the data can be described by an effective two-flavor model with a single mass-squared difference Δm^2 and mixing angle θ. In this approximation, the ν_μ and ν_τ survival probabilities are

\[ P(ν_μ \rightarrow ν_μ) \approx 1 - \sin^2 2θ \sin^2 \left( \frac{Δm^2 L}{2E_ν} \right), \]

where L_ν is the neutrino propagation distance and E_ν is the neutrino energy. A previous two-flavor analysis of ν_μ and ν_τ disappearance using the combined accelerator and atmospheric data from MINOS yielded |Δm^2| = 2.41^{+0.09}_{-0.10} \times 10^{-3} eV^2 and sin^2 2θ = 0.950^{+0.035}_{-0.036} [7]. The statistical weight of the data now enables MINOS to constrain the full three-flavor model of ν_μ and ν_τ disappearance. The uncertainty on Δm^2 is approaching the size of the smaller mass-squared difference, Δm^2_{13}, which is neglected in the two-flavor model. Moreover, the precise knowledge of θ_{13} enables an analysis of the data based on the full set of mixing parameters. In this Letter we present the three-flavor analysis of the combined MINOS data.

In the three-flavor framework, the oscillations are driven by two mass-squared differences Δm^2_{32} and Δm^2_{31}, where Δm^2_{32} = Δm^2_{3j} + Δm^2_{2j}. The interference between the resulting two oscillation frequencies leads to terms in the oscillation probabilities that depend on all the mixing parameters. The leading-order ν_μ and ν_τ survival probabilities in vacuum take the same form as the two-flavor approximation in Eq. (1), with the effective parameters given by [8]:

\[ \sin^2 2θ = 4\sin^2 θ_{23}\cos^2 θ_{13}(1 - \sin^2 θ_{23}\cos^2 θ_{13}), \]

\[ Δm^2 = Δm^2_{32} + Δm^2_{23}\sin^2 θ_{12} \]

\[ + Δm^2_{12} \cos δ_{CP} \sin θ_{12} \tan θ_{23} \sin 2θ_{12}. \]  

The exact symmetries of the two-flavor model under θ → π/2 − θ and Δm^2 → −Δm^2 lead to approximate degeneracies in the octant of θ_{23} and mass hierarchy in the three-flavor formalism.

For neutrinos traveling through matter, the propagation eigenstates are modified by the MSW effect [9]. In this case, the mixing angle θ_{13} is replaced by a modified version, θ_M, given by [10]

\[ \sin^2 2θ_M = \frac{\sin^2 2θ_{13}}{\sin^2 2θ_{13} + (A - \cos 2θ_{13})^2}. \]  

The size of the matter effect is determined by the parameter A = ±2\sqrt{2}G_F n_e E_ν/Δm^2_{31}, where G_F is the Fermi weak coupling constant, n_e is the density of electrons, and the sign of A is positive (negative) for neutrinos (antineutrinos). Equation (3) shows that sin^2 2θ_M is maximal at A = cos 2θ_{13}. This condition leads to the resonant enhancement of ν_μ ↔ ν_τ oscillations, which can significantly alter the magnitude of ν_μ disappearance. The effect is present for neutrinos in the normal hierarchy and for antineutrinos in the inverted hierarchy. An MSW resonance is predicted to occur in multi-GeV, upward-going atmospheric neutrinos, which travel through Earth’s mantle [11]. MINOS is the first experiment to probe this resonance by measuring ν_μ and ν_τ interactions separately with atmospheric neutrinos, yielding sensitivity to the mass hierarchy and θ_{23} octant.

MINOS [12] has previously reported measurements of ν_e and ν_τ appearance in accelerator ν_μ and ν_τ beams. Measurements of ν_μ → ν_e appearance in accelerator neutrinos have also been published by T2K [13]. Both results are based on three-flavor analyses. For accelerator neutrinos, the ν_μ → ν_e appearance probability in matter, expanded to second order in α = Δm^2_{31}/Δm^2_{32} (≈ 0.03), is given by [14]:

formalism. We measure |Δm^2_{32}| = [2.28–2.46] \times 10^{-3} eV^2 (68% C.L.) and sin^2θ_{23} = 0.35–0.65 (90% C.L.) in the normal hierarchy, and |Δm^2_{32}| = [2.32–2.53] \times 10^{-3} eV^2 (68% C.L.) and sin^2θ_{23} = 0.34–0.67 (90% C.L.) in the inverted hierarchy. The data also constrain δ_{CP}, the θ_{23} octant degeneracy and the mass hierarchy; we disfavor 36% (11%) of this three-parameter space at 68% (90%) C.L.
The complete MINOS produced by the NuMI facility [15], located at the Fermi Accelerator Laboratory. The first term in Eq. (4) is proportional to \( \sin^2 \theta_{23} \) and breaks the \( \theta_{23} \) octant degeneracy. In addition, the dependence on \( A \) is sensitive to the mass hierarchy and the second term in the expansion is sensitive to CP violation. In this Letter, we strengthen the constraints on \( \delta_{CP} \), the \( \theta_{23} \) octant, and the mass hierarchy obtained from the MINOS disappearance data [12] by combining the complete MINOS disappearance and appearance data and by exploiting the improved precision on \( \theta_{13} \) from reactor experiments.

In the MINOS experiment, the accelerator neutrinos are produced by the NuMI facility [15], located at the Fermi National Accelerator Laboratory. The complete MINOS accelerator neutrino data set comprises exposures of 10.71 \( \times \) 10\(^{20} \) protons on target using a \( \nu_\mu \)-dominated beam and 3.36 \( \times \) 10\(^{20} \) protons on target using a \( \bar{\nu}_\mu \)-enhanced beam [7]. These data were acquired in the “low energy” NuMI beam configuration [15], where the neutrino event energy peaks at 3 GeV. The spectrum and composition of the beam are measured using two steel-scintillator tracking detectors with toroidal magnetic fields. The Near and Far detectors are located 1.04 and 735 km downstream of the production target, respectively. The 5.4 kton Far Detector is installed 705 m (2070 m water equivalent) underground in the Soudan Underground Laboratory and is equipped with a scintillator veto shield for rejection of cosmic-ray muons. These features have enabled MINOS to collect 37.88 kton-y of atmospheric neutrino data [16].

The oscillation analysis uses charged-current (CC) interactions of both muon and electron neutrinos. These events are distinguished from neutral-current (NC) backgrounds by the presence of a muon track or electromagnetic shower, respectively. The events also typically contain shower activity from the hadronic recoil system. The selection of accelerator \( \nu_\mu \) CC and \( \bar{\nu}_\mu \) CC events is based on a multivariate \( k \)-nearest-neighbor classification algorithm using a set of input variables characterizing the topology and energy deposition of muon tracks [17]. The selected events are separated into contained-vertex neutrinos, with reconstructed interaction positions inside the fiducial volume of the detectors, and non-fiducial muons, in which the neutrino interactions occur outside the fiducial volume or in the surrounding rock. The contained-vertex events are further divided into candidate \( \nu_\mu \) and \( \bar{\nu}_\mu \) interactions based on the curvature of their muon tracks. In the oscillation fit, the events are binned as a function of reconstructed neutrino energy. For contained-vertex events, this is taken as the sum of the muon and hadronic shower energy measurements; for nonfiducial muons, the muon energy alone is used as the neutrino energy estimator. To improve the sensitivity to oscillations, the contained-vertex \( \nu_\mu \) events from the \( \nu_\mu \)-dominated beam are also binned according to their calculated energy resolution [18–20]. The predicted energy spectra in the Far Detector are derived from the observed data in the Near Detector using a beam transfer matrix [21].

The selection of accelerator \( \nu_e \) CC and \( \bar{\nu}_e \) CC events is based on a library-event-matching algorithm that performs hit-by-hit comparisons of contained-vertex shower-like events with a large library of simulated neutrino interactions [22–24]. The events are required to have reconstructed energies in the range 1–8 GeV, where most of the \( \nu_e \) and \( \bar{\nu}_e \) appearance is predicted to occur. The 50 best-matching events from the library are used to calculate a set of classification variables that are combined into a single discriminant using an artificial neural network. The selection does not discriminate between \( \nu_e \) and \( \bar{\nu}_e \) interactions. The selected events are binned as a function of the reconstructed energy and library-event-matching discriminant. The background contributions from NC, \( \nu_\mu \) CC, and \( \bar{\nu}_\mu \) CC interactions, and intrinsic \( \nu_e \) CC and \( \bar{\nu}_e \) CC interactions from the beam, are determined using samples of Near Detector data collected in different beam configurations. The backgrounds in the Far Detector are calculated from these Near Detector components [25]. The rates of appearance in the Far Detector are derived from the \( \nu_\mu \) CC and \( \bar{\nu}_\mu \) CC spectra measured in the Near Detector [12].

Atmospheric neutrinos are separated from the cosmic-ray muon background using selection criteria that identify either a contained-vertex interaction or an upward-going or horizontal muon track [26,27]. For contained-vertex events, the background is further reduced by checking for associated energy deposits in the veto shield. The event selection yields samples of contained-vertex and nonfiducial muons, which are each separated into candidate \( \nu_\mu \) CC and \( \bar{\nu}_\mu \) CC interactions. These samples of muons are binned as a function of \( \log_{10}(E) \) and \( \cos \theta_\mu \), where \( E \) is the reconstructed energy of the event in GeV and \( \theta_\mu \) is the zenith angle of the muon track. This two-dimensional binning scheme enhances the sensitivity to the MSW resonance. The results remain in close agreement with the two-flavor analysis of \( \nu_\mu \) and \( \bar{\nu}_\mu \) disappearance, in which these data were binned as a function of \( \log_{10}(L/E) \) [7]. A sample of contained-vertex showers is also selected from the data, composed mainly of NC, \( \nu_e \) CC, and \( \bar{\nu}_e \) CC interactions. These events are grouped into a single bin, since they have negligible sensitivity to oscillations but constrain the overall flux normalization. The predicted event rates in each selected sample are calculated from a Monte Carlo simulation of atmospheric neutrino interactions in the Far Detector [16,28]. The cosmic-ray muon backgrounds are obtained from the observed data by reweighting the events tagged by the veto shield according to the measured shield inefficiency [26].
For all the data samples, the predicted event spectra in the Far Detector are reweighted to account for oscillations, and the backgrounds from $\nu_e$ and $\bar{\nu}_e$ appearance are included. The oscillation probabilities are calculated directly from the PMNS matrix using algorithms optimized for computational efficiency [29]. The probabilities account for the propagation of neutrinos through Earth. For atmospheric neutrinos, a constant electron density of 1.36 mol cm$^{-3}$ is assumed. For atmospheric neutrinos, Earth is modeled by four layers of constant electron density: an inner core ($r < 1220$ km, $n_e = 6.05$ mol cm$^{-3}$); an outer core ($1220 < r < 3470$ km, $n_e = 5.15$ mol cm$^{-3}$); the mantle ($3470 < r < 6336$ km, $n_e = 2.25$ mol cm$^{-3}$); and the crust ($r > 6336$ km, $n_e = 1.45$ mol cm$^{-3}$). This four-layer approximation reflects the principal structures of the PREM model [30]. Comparisons with a more detailed 42-layer model yield similar oscillation results.

The oscillation parameters are determined by applying a maximum likelihood fit to the data. The parameters $\Delta m^2_{32}$, $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$ and $\delta_{CP}$ are varied in the fit. The mixing angle $\theta_{13}$ is subject to an external constraint of $\sin^2 \theta_{13} = 0.0242 \pm 0.0025$, based on a weighted average of the published results from the Daya Bay [31], RENO [4], and Double Chooz [5] reactor experiments. This constraint is incorporated into the fit by adding a Gaussian penalty term to the likelihood function. The fit uses fixed values of $\Delta m^2_{21} = 7.54 \times 10^{-5}$ eV$^2$ and $\sin^2 \theta_{12} = 0.307$ [32].

The impact of these two parameters is evaluated by shifting them in the fit according to their uncertainties; the resulting shifts in the fitted values of $\Delta m^2_{32}$ and $\sin^2 \theta_{23}$ are found to be negligibly small. The likelihood function contains 32 nuisance parameters, with accompanying penalty terms, that account for the major systematic uncertainties in the simulation of the data [16,23,33]. The fit proceeds by summing the separate likelihood contributions from the $\nu_\mu$ disappearance [7] and $\nu_e$ appearance [12] data sets, taking their systematic parameters to be uncorrelated.

Figure 1 shows the 2D confidence limits on $\Delta m^2_{32}$ and $\sin^2 \theta_{23}$, obtained by maximizing the likelihood function at each point in this parameter space with respect to $\sin^2 \theta_{13}$.

![Figure 1](https://example.com/figure1.png)

**FIG. 1.** The left panels show the 68% and 90% confidence limits on $\Delta m^2_{32}$ and $\sin^2 \theta_{23}$ for the normal hierarchy (top) and inverted hierarchy (bottom). At each point in this parameter space, the likelihood function is maximized with respect to $\sin^2 \theta_{13}$, $\delta_{CP}$ and all of the systematic parameters. The $-2\Delta \log(L)$ surface is calculated relative to the overall best fit, which is indicated by the star. The right panels show the 1D likelihood profiles as a function of $\Delta m^2_{32}$ and $\sin^2 \theta_{23}$ for each hierarchy. The horizontal dotted lines indicate the 68% and 90% C.L.

<table>
<thead>
<tr>
<th>Mass hierarchy</th>
<th>$\theta_{23}$ octant</th>
<th>$\Delta m^2_{32}/10^{-3}$ eV$^2$</th>
<th>$\sin^2 \theta_{23}$</th>
<th>$\sin^2 \theta_{13}$</th>
<th>$\delta_{CP}/\pi$</th>
<th>$-2\Delta \log(L)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m^2_{32} &lt; 0$</td>
<td>$\theta_{23} &lt; \pi/4$</td>
<td>$-2.41$</td>
<td>$0.41$</td>
<td>$0.243$</td>
<td>$0.62$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\Delta m^2_{32} &lt; 0$</td>
<td>$\theta_{23} &gt; \pi/4$</td>
<td>$-2.41$</td>
<td>$0.61$</td>
<td>$0.241$</td>
<td>$0.37$</td>
<td>$0.09$</td>
</tr>
<tr>
<td>$\Delta m^2_{32} &gt; 0$</td>
<td>$\theta_{23} &lt; \pi/4$</td>
<td>$+2.37$</td>
<td>$0.41$</td>
<td>$0.242$</td>
<td>$0.44$</td>
<td>$0.23$</td>
</tr>
<tr>
<td>$\Delta m^2_{32} &gt; 0$</td>
<td>$\theta_{23} &gt; \pi/4$</td>
<td>$+2.35$</td>
<td>$0.61$</td>
<td>$0.238$</td>
<td>$0.62$</td>
<td>$1.74$</td>
</tr>
</tbody>
</table>

**TABLE I.** The best-fit oscillation parameters obtained from this analysis for each combination of mass hierarchy and $\theta_{23}$ octant. Also listed are the $-2\Delta \log(L)$ values for each of the four combinations, calculated relative to the overall best-fit point.
$\delta_{CP}$, and all of the systematic parameters. Also shown are the corresponding 1D likelihood profiles as a function of $\Delta m_{32}^2$ and $\sin^2 \theta_{23}$. The 68% (90%) confidence limits on these parameters are calculated by taking the range of negative log-likelihood values with $-2\Delta \log(L) < 1.00$ (2.71) relative to the overall best fit. This yields $|\Delta m_{32}^2| = [2.28 \pm 2.46] \times 10^{-3} \text{ eV}^2$ (68% C.L.) and $\sin^2 \theta_{23} = 0.35 \pm 0.65$ (90% C.L.) in the normal hierarchy, and $|\Delta m_{32}^2| = [2.32 \pm 2.53] \times 10^{-3} \text{ eV}^2$ (68% C.L.) and $\sin^2 \theta_{23} = 0.34 \pm 0.67$ (90% C.L.) in the inverted hierarchy. The data disfavor maximal mixing ($\theta_{23} = \pi/4$) by $-2\Delta \log(L) = 1.54$. The measurements of $\Delta m_{32}^2$ are the most precise that have been reported to date.

The data also constrain $\delta_{CP}$, the $\theta_{23}$ octant degeneracy and the mass hierarchy. Table I lists the best-fit oscillation parameters for each combination of octant and mass hierarchy, and the differences in negative log-likelihood relative to the overall best fit. Assuming $\theta_{23} > \pi/4$ ($\theta_{23} < \pi/4$), the data prefer the inverted hierarchy by $-2\Delta \log(L) = 1.65$ (0.23). The combination of normal hierarchy and higher octant is disfavored by 1.74 units of $-2\Delta \log(L)$, strengthening the previous constraints from $\nu_e$ and $\nu_x$ appearance [12]. Figure 2 shows the 1D likelihood profile as a function of $\delta_{CP}$ for each of the four possible combinations. The data disfavor 36% (11%) of the parameter space defined by $\delta_{CP}$, the $\theta_{23}$ octant, and the mass hierarchy at 68% (90%) C.L.

In summary, we have presented the first combined analysis of $\nu_\mu$ disappearance and $\nu_e$ appearance data by a long-baseline neutrino experiment. The results are based on the complete set of MINOS accelerator and atmospheric neutrino data. A combined analysis of these data sets yields precision measurements of $\Delta m_{32}^2$ and $\sin^2 \theta_{23}$, along with new constraints on the three-parameter space defined by $\delta_{CP}$, the $\theta_{23}$ octant, and the mass hierarchy.

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