APPENDICES

CHESAPEAKE BAY SHORELINE STUDY:
Headland Breakwaters and Pocket Beaches For Shoreline
Erosion Control

FINAL REPORT

By

C. S. Hardaway
G. R. Thomas
J.-H. Li

Virginia Institute of Marine Science
The College of William and Mary
Gloucester Point, Virginia 23062

A Final Report Obtained Under Contract With
The United States Army Corps of Engineers, Norfolk District
With The Virginia Department of Conservation and Recreation
via the

JOINT COMMONWEALTH PROGRAMS ADDRESSING
SHORE EROSION IN VIRGINIA

Special Report in Applied Marine Science and Ocean Engineering
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APPENDIX A

Profile Data

- Chippokes State Park
- Parkway Breakwaters
- Hog Island Breakwaters
- Drummonds Field
- Waltrip
- Hog Island Headlands
- Yorktown Bays
- Summerille
Chippokes State Park

Profiles 1-33

Jul 28, 1987
Nov 12, 1987
Feb 23, 1988
Sep 21, 1988
Dec 14, 1988
Mar 28, 1989
Jun 08, 1989
Oct 03, 1989
Apr 17, 1990
CHIPPOKES STATE PARK
PROFILE NO. 31

CHIPPOKES STATE PARK
PROFILE NO. 31
Parkway Breakwaters

Profiles 1-27

Jul 29, 1987
Nov 16, 1987
Feb 26, 1988
Sep 14, 1988
Dec 06, 1988
Mar 17, 1989
May 24, 1989
Oct 12, 1989
Apr 03, 1990
PARKWAY BREAKWATERS
PROFILE NO. 34

PARKWAY BREAKWATERS
PROFILE NO. 04
PARKWAY BREAKWATERS
PROFILE NO. 99

--- DEC0688
--- SEP1488
--- FEB2688
--- NOV1687
--- JUL2987

FEET (MSL)

0 50 100 150 200 250

PARKWAY BREAKWATERS
PROFILE NO. 99

--- APR0390
--- OCT1289
--- MAY2489
--- MAR1789
--- DEC0688

FEET

0 50 100 150 200 250
Hog Island Breakwaters

Profiles 1-61

May 18, 1987
Jun 22, 1987
Sep 01, 1987
Feb 09, 1988
Sep 07, 1988
Nov 22, 1988
Mar 16, 1989
Oct 04, 1989
May 01, 1990
HOG ISLAND BREAKWATERS
PROFILE NO. 09

- SEPO788
- FEB0988
- SEPO187
- JUN2287
- MAY1887

FEET

- UHW
- MSL
- MLW

FEET

- MAY0190
- OCT0489
- MAR1689
- NOV2288
- SEPO788

FEET
HOG ISLAND BREAKWATERS
PROFILE NO. 55

--- SEP0788
--- FEB0988
--- SEP0187
--- JUN2287
--- MAY1887

FEET

--- MHW
--- MSL
--- MLW

--- MAY0190
--- OCT0489
--- MAR1689
--- NOV2288
--- SEP0788

FEET (MSL)

--- MHW
--- MSL
--- MLW

FEET

0 25 50 75 100 125 150
Drummonds Field

Profiles 1-35

Sep 09, 1987
Dec 01, 1987
Feb 24, 1988
Sep 13, 1988
Dec 08, 1988
Mar 30, 1989
Jun 15, 1989
Oct 10, 1989
May 11, 1990
Waltrip

Profiles 1-17

Sep 27, 1988
Dec 20, 1988
Mar 20, 1989
Oct 11, 1989
Apr 13, 1990
Hog Island Headlands

Profiles 1-30

Aug 12, 1987
Oct 20, 1987
Feb 10, 1988
Sep 06, 1988
Dec 15, 1988
Mar 01, 1989
Jun 07, 1989
Oct 05, 1989
May 02, 1990
Yorktown Bays

Profiles 1-7

Aug 07, 1987
Nov 17, 1987
Mar 28, 1988
Sep 28, 1988
Nov 30, 1988
Mar 15, 1989
May 23, 1989
Oct 13, 1989
Apr 04, 1990
Summerville

Profiles 1-12

Sep 18, 1987
Dec 11, 1987
Mar 08, 1988
Sep 02, 1988
Dec 05, 1988
Mar 22, 1989
Jun 08, 1989
Oct 06, 1989
Apr 30, 1990
APPENDIX B

Model Tombolos
Dr. K.D. Suh
With Appendices
A Numerical Model for Predicting Shoreline Change Due to Offshore Breakwaters

by
Kyung Duck Suh
Virginia Institute of Marine Science
Gloucester Point, VA 23062

1 Introduction

A numerical model is a valuable tool for assessing beach changes in the vicinity of coastal structures. The advanced knowledge on waves and currents, and their interaction and the resulting sediment transport, associated with the increased capacities of large computers and the improved numerical modeling algorithms, has made it possible to apply the complex models such as a multi-line model (Perlin & Dean, 1985) or general 3-D topographical change models (Wang et al., 1975; Watanabe, 1982) to the numerical modeling of shoreline problems. Due to the relatively accurate prediction of the shoreline change with less computing efforts, however, the one-line model has been widely used by many coastal engineers. The GENESIS model (Hanson & Kraus, 1989) may be one of the most sophisticated one-line models to exist, though its public access is yet limited. In the present paper, a one-line numerical model is developed that simulates the shoreline change in the lee of offshore breakwaters.

Most one-line models reported to date use the Cartesian coordinate system, one of the axes of which is taken to be approximately parallel to the shoreline. The longshore sediment transport and the shoreline movement are assumed to be parallel and perpendicular, respectively, to this axis. If the shoreline makes a large angle with this axis as on a tombolo behind an offshore breakwater, however, this assumption is drastically violated. To overcome this circumstance, models using an orthogonal curvilinear coordinate system have been developed, under the assumption that any point on the shoreline moves perpendicular to the instantaneous shoreline and the longshore sediment transport is parallel to the local shoreline, see LeBlond (1972), Suh (1985), Kobayashi & Dalrymple (1986). The present model also uses the orthogonal curvilinear coordinate system. Another feature of the present model is that it uses the longshore sediment transport coefficients that vary depending on the breaking wave heights. The inclusion of this feature seems to be crucial in the situation in which the longshore variation of breaking wave height is large as in the coastal area protected by offshore breakwaters. The present model is also able to handle the formation of a tombolo in which the shoreline attaches the breakwater.

The developed model is applied to the simulation of the shoreline response during the first eight months after the construction of the Chippokes State Park Breakwaters, Virginia (Hardaway et al., 1988).
2 Numerical Model

2.1 Shoreline model

The orthogonal curvilinear coordinate system used in the present model is shown in Fig. 1 along with some other notations. $s$ is the coordinate following the shoreline whose positive direction is chosen such that the beach is located on the right-hand side of the coordinate $s$. $(x_s, y_s)$ is the location of an arbitrary point on the curved shoreline in terms of the Cartesian coordinate system.

$$\vec{n} = \left( \frac{\partial x_s}{\partial s}, \frac{\partial y_s}{\partial s} \right) = (\cos \theta, \sin \theta)$$  \hspace{1cm} (2.1)

is the unit tangential vector to the shoreline in the direction of increasing $s$,

$$\vec{\tilde{n}} = \left( -\frac{\partial y_s}{\partial s}, \frac{\partial x_s}{\partial s} \right) = (-\sin \theta, \cos \theta)$$  \hspace{1cm} (2.2)

is the seaward unit normal vector to the shoreline, and $\theta$ is the angle between $\vec{n}$ and the $x$-axis which is measured counterclockwise from the positive $x$-direction. $Q$ is the volumetric longshore sediment transport rate and $q$ is the volumetric offshore sediment transport rate per unit shoreline length. $\alpha_b$ is the breaking wave angle between the wave crest and the $x$-axis which is measured counterclockwise from the positive $x$-direction.

Assume that the point, $(x_s, y_s)$, moves perpendicular to the instantaneous shoreline, so that

$$\left( \frac{\partial x_s}{\partial t}, \frac{\partial y_s}{\partial t} \right) = e \vec{n},$$  \hspace{1cm} (2.3)

in which $e$ is the rate of shore-normal movement of shoreline. Using the notation of complex variables,

$$z_s = x_s + iy_s,$$  \hspace{1cm} (2.4)

in which $i = \sqrt{-1}$, and expressing $e$ as

$$e = \frac{1}{D} \left( \frac{\partial Q}{\partial s} + q \right),$$  \hspace{1cm} (2.5)

Eq. (2.3) can be written as

$$\frac{\partial z_s}{\partial t} = \frac{1}{D} \left( \frac{\partial Q}{\partial s} + q \right) \exp \left\{ i \left( \theta - \frac{\pi}{2} \right) \right\},$$  \hspace{1cm} (2.6)
in which \( D \) is the depth of profile closure at which no measurable change in bottom elevation occurs. If \( \theta \) is very small, the real and imaginary parts of the above equation give

\[
\frac{\partial x_s}{\partial t} \simeq 0,
\]
\[
\frac{\partial y_s}{\partial t} \simeq -\frac{1}{D} \left( \frac{\partial Q}{\partial x_s} + q \right), \quad (s \simeq x_s \text{ for } \theta \simeq 0),
\]

respectively. Note that Eq. (2.8) is the sediment continuity equation of traditional one-line models using the Cartesian coordinate system.

A widely-used expression for the longshore sediment transport rate in places where diffraction dominates is that proposed by Ozasa & Brampton (1980):

\[
Q = \Gamma H_b^{3/2} \left\{ K_1 \sin(2\delta_b) - K_2 \frac{\partial H_b}{\partial s} \cot \beta \cos \delta_b \right\},
\]

where

\[
\Gamma = \frac{g^{1/2}}{16(s_* - 1)(1 - p) \kappa^{1/2}},
\]
\[
\delta_b = \alpha_b - \theta.
\]

is the breaking wave angle relative to the shoreline under the assumption that the breakerline and the shoreline are locally parallel, and \( g \) = gravitational acceleration; \( s_* \) = specific gravity of sediment relative to fluid; \( p \) = porosity of sediment; \( \kappa \) = ratio of wave height to water depth at breaking; \( H_b \) = breaking wave height; \( \tan \beta = \) beach slope; \( K_1, K_2 \) = empirical longshore sediment transport coefficients. The first term in Eq. (2.9) describes the sediment transport rate due to obliquely incident waves, whereas the second term describes the transport due to longshore gradient of breaking wave heights, which has been found to be of great importance in cases where diffraction dominates. The beach slope can be calculated by the empirical formula proposed by Sunamura (1984):

\[
\tan \beta = \begin{cases} 
0.013 \left( \frac{H_b}{s_*^{1/2} d_*^{1/2}} \right)^{1/2} + 0.15 & \text{for laboratory,} \\
0.12 \left( \frac{H_b}{s_*^{1/2} d_*^{1/2}} \right)^{1/2} & \text{for field,}
\end{cases}
\]

in which \( d_* \) = sediment grain size; \( T \) = wave period.

A number of studies has been performed to determine the coefficient \( K_1 \). A value of \( K_1 = 0.77 \) was originally determined by Komar & Inman (1970) using their field experiment data, and a decrease from 0.77 to 0.58 was recommended by Kraus et al. (1982) based on their own sand tracer experiments. Including laboratory data as well as field data, Das (1972) obtained a coefficient 0.35. This reduction may be due to the laboratory transport conditions that are not fully developed to
initiate sediment movement or support the suspended sediment particles. It is expected that the longshore transport coefficient would fall below the value of 0.77 or 0.58 even in the field if the transport condition is not fully developed as in the lee of an offshore breakwater, for example.

Dean (1973) proposed an expression for the longshore sediment transport rate:

\[
Q = \frac{5\pi}{16} K_f \frac{H_b^2 \kappa \tan \beta C_b C_{gb}}{16 c(s_a - 1)w_s} \sin 2\delta_b, \tag{2.13}
\]

in which \( K_f \) = a constant representing the fraction of wave energy consumed by falling sediment particles; \( C_b, C_{gb} \) = phase and group velocities at wave breaking; \( c \) = bottom friction coefficient; and \( w_s \) = fall velocity of sediment particles. The comparison of the first term in Eq. (2.9) and the above equation gives

\[
K_1 = \frac{5\pi}{16} K_f \frac{g^{1/2} \kappa^{1/2}(1 - p) \tan \beta}{cw_s} H_b^{1/2}. \tag{2.14}
\]

According to Longuet-Higgins (1972), \( c/\tan \beta = \) constant, that is, the bottom friction coefficient is proportional to the beach slope. Assuming that the breaking wave height is the only variable which changes in Eq. (2.14), it would seem to follow that \( K_1 \propto H_b^{1/2} \). Since wave diffraction is the most dominant factor for changing the breaking wave height behind offshore breakwaters, and the coefficient, \( K_1 \), is a dimensionless value, the following expression for the longshore sediment transport rate is proposed:

\[
Q = \Gamma H_b^{5/2} K_D^{-1/2} \left( K_1 \sin(2\delta_b) - K_2 \frac{\partial H_b}{\partial \delta} \cot \beta \cos \delta_b \right), \tag{2.15}
\]

in which \( K_D \) = diffraction coefficient. This equation reduces to Eq. (2.9) in the open region where \( K_D = 1 \), and gives a smaller transport rate than Eq. (2.9) in the lee of a breakwater where the transport condition is not fully developed.

The second transport coefficient, \( K_2 \), has not received great attention of researchers. Ozasa & Brampton (1980) used \( K_2 = 3.24 K_1 \). Hanson & Kraus (1989), however, recommended that the value of \( K_2 \) is typically 0.5 to 1.0 times that of \( K_1 \).

The onshore transport rate, \( q \), in Eq. (2.6) represents sink or source of sediment from/to offshore or backshore area. Typical sources include sediment discharge from rivers, bank or dune erosion due to precipitation, beach nourishment and transport due to offshore wind, whereas typical sinks include permanent offshore transport of sediment due to severe storms or long-term sea level rise, sand mining and transport due to onshore wind. These quantities depend on the particular situation of the model area and vary with both time and longshore distance. The present model assumes zero onshore transport of sediment.

One of the basic assumptions of one-line models is that the beach has a constant depth of profile closure throughout the model area, within which erosion or accretion of beach occurs. The
expression proposed by Hallermeier (1983) is adopted with a deep water wave height, $H_o$, in place of the extreme wave height for an annual seaward limit of profile change:

$$D = 2.9H_o(s_* - 1)^{-1/2}. \tag{2.16}$$

The deep water wave height is calculated by

$$H_o = H_B \sqrt{\frac{C_{gB}}{C_{go}}}, \tag{2.17}$$

in which $H_B$ = wave height at the location of breakwaters; $C_{gB}, C_{go}$ = wave group velocities in deep water and at the location of breakwaters.

### 2.2 Breaking wave model

The sediment continuity equation, (2.6), can be solved for the shoreline position, $z_*$, if the wave heights and angles along the breakerline are given. In the present model, the wave condition (height, period and angle of incidence) at the location of the breakwaters is given as input data and the wave deformation in the lee of the breakwaters is computed. It is assumed that the wave condition is constant spatially at the location of the breakwaters and gaps. This assumption will be appropriate if the offshore bottom topography does not deviate drastically from straight and parallel contours, the offshore distances of the breakwaters are almost constant, and the longshore distance of the model area is not too long.

The three major phenomena which alter the wave in the lee of the breakwaters are refraction, diffraction and shoaling. There are some means to compute the combined wave refraction-diffraction, e.g., Larsen (1976) or Southgate (1985), if the bathymetry between the breakwaters and the shoreline is known. But that is not the case for a one-line model. Thus, in the present model, it is assumed that all these wave phenomena occur independently, so that the breaking wave height, $H_b$, can be calculated by

$$H_b = K_D K_R K_S H_B, \tag{2.18}$$

in which $H_B$ = wave height at the location of breakwaters, and $K_D, K_R, K_S$ = coefficients of diffraction, refraction, and shoaling. The remainder of this section is devoted to the computation of these coefficients. The procedure for computing the breaking wave angle will be explained where appropriate.

For illustration, let us consider two offshore breakwaters as shown in Fig. 2. The shoreline on the left salient will be affected by both the wave diffracted around the left tip of the left breakwater and the wave diffracted through the gap. Perlin (1979) assumed that each wave behaves
independently and the resulting sediment transport can be calculated by adding vectorially the sediment transported by each wave. Kraus (1983) used the highest wave of the two waves to calculate the sediment transport. The Kraus’ approach adopted in this study looks more reasonable as the salient grows or a tombolo is formed. The entire shoreline shown in Fig. 2, then, can be divided into three diffraction regions: the left one from the left end of the shoreline to the middle of the left salient (diffraction around the left tip of the left breakwater), central one from the middle of the left salient to the middle of the right salient (diffraction through the gap), and the right one from the middle of the right salient to the right end of the shoreline (diffraction around the right tip of the right breakwater).

The diffraction analysis used in this model is based on the theory of Penney & Price (1952). The assumptions in their theory include a semi-infinitely long breakwater with an infinitesimal thickness, linear theory and a constant depth. In the left diffraction region in Fig. 2, the breakwater is assumed to be infinitely long to the right direction and the constant water depth is taken as the depth at the breakwater tip, \( h_B \). Similar assumptions are made for the right diffraction region. In the central diffraction region behind the gap, each breakwater is assumed to be infinitely long to the directions away from the gap, and the solutions from each breakwater are superimposed to calculate the actual diffraction coefficient, \( K_D \):

\[
K_D = K_D^L K_D^R,
\]

(2.19)

in which \( K_D^L, K_D^R \) = diffraction coefficients calculated independently for the left and right breakwaters by Penney & Price method.

The diffraction coefficient at the breakline position, \( B \), in Fig. 2, is assumed to be the same as that calculated for the corresponding shoreline position, \( S \). The diffraction coefficient, \( K_D \), is expressed by

\[
K_D = \text{mod}\{F(r, \theta_D)\} = \{F_r(r, \theta_D)^2 + F_i(r, \theta_D)^2\}^{1/2},
\]

(2.20)

where

\[
F(r, \theta_D) = F_r(r, \theta_D) + iF_i(r, \theta_D)
= [\cos\{k_B r \cos(\theta_D - \theta_o)\} - i \sin\{k_B r \cos(\theta_D - \theta_o)\}] \cdot \{f_r(\sigma) + if_i(\sigma)\}
+ [\cos\{k_B r \cos(\theta_D + \theta_o)\} - i \sin\{k_B r \cos(\theta_D + \theta_o)\}] \cdot \{f_r(\sigma') + if_i(\sigma')\}.
\]

(2.21)

\( r \) is the distance from the breakwater tip to the shoreline, and the angles, \( \theta_D \) and \( \theta_o \), are as shown in Fig. 2. The wave number, \( k_B \), at the location of the breakwater is related to the water depth, \( h_B \), and the wave angular frequency, \( \omega (= 2\pi / T) \), by the dispersion relationship

\[
\omega^2 = g k_B \tanh k_B h_B,
\]

(2.22)

and
\[ f(\sigma) = f_r(\sigma) + i f_i(\sigma) = \frac{1}{2} \{ 1 + C(\sigma) + S(\sigma) \} + \frac{i}{2} \{ C(\sigma) - S(\sigma) \}, \quad (2.23) \]

in which
\[
\sigma = 2 \left( \frac{k_B r}{\pi} \right)^{1/2} \sin \left( \frac{\theta_D - \theta_o}{2} \right),
\]
\[
\sigma' = -2 \left( \frac{k_B r}{\pi} \right)^{1/2} \sin \left( \frac{\theta_D + \theta_o}{2} \right).
\]

The Fresnel integrals, \( C(\sigma) \) and \( S(\sigma) \), are expressed by
\[
C(\sigma) = \int_0^\sigma \cos \left( \frac{\pi}{2} u^2 \right) du,
\]
\[
S(\sigma) = \int_0^\sigma \sin \left( \frac{\pi}{2} u^2 \right) du,
\]
which can be computed using the computer programs given in IBM (1970).

The method of determining the breaking wave angle closely follows that of Kraus (1983). First, for each point of the shoreline, the offshore bottom contours from the shoreline to the location of the breakwater tip are assumed to be straight and parallel to the local orientation of the shoreline. The approximate location of wave breaking, \( (x_b, y_b) \), corresponding to the shoreline position, \( (x_s, y_s) \), is calculated by
\[
x_b = x_s - w_b \sin \theta,
\]
\[
y_b = y_s + w_b \cos \theta,
\]
in which
\[
w_b = \frac{h_b}{\tan \beta} = \frac{K_D H_B}{\kappa \tan \beta}
\]
is the surf zone width. For the breaking points located in Zone I, III, or V in Fig. 2, the breaking wave angle, \( \alpha_b \), and the coefficients of refraction and shoaling are computed by
\[
\alpha_b = \theta + \sin^{-1} \left\{ \frac{k_B}{k_b} \sin (\alpha_B - \theta) \right\},
\]
\[
K_R = \left\{ \frac{\cos (\alpha_B - \theta)}{\cos (\alpha_b - \theta)} \right\}^{1/2},
\]
\[
K_S = \left( \frac{C_{TB}}{C_{Sb}} \right),
\]
in which the subscripts \( B \) and \( b \) refer to the values at the location of breakwater and wave breaking, respectively.

In Zone II and IV in Fig. 2, the diffracted wave crests near the tips of the breakwaters are assumed to be circular in planform. The diffracted wave is then refracted shoreward. Fig. 3 is
the enlargement of the left half of Fig. 2. The wave reached at the left tip of the breakwater with an angle \( \alpha_B \) is diffracted and leaves the tip with an angle \( \theta_R \) to refract to the breaking point, \( B_1 \). The unknown angle, \( \theta_R \), is measured counterclockwise from the line which is perpendicular to the assumed straight and parallel bottom contours. It is convenient to use local shoreline-oriented coordinates \((x', y')\) as in Fig. 3, which are related to the computational coordinates \((x, y)\) by

\[
x' = (x - x_b) \cos \theta + (y - y_b) \sin \theta, \\
y' = -(x - x_b) \sin \theta + (y - y_b) \cos \theta,
\]

The origin \((x', y') = (0, 0)\) corresponds to the breaking point \(B_1(x_b, y_b)\). Considering the wave ray refracted from the left tip of the breakwater, \((x'_1, y'_1)\), to the breaking point, \(B_1\), in Fig. 3, we have

\[
\tan \theta' = -\frac{dx'}{dy'},
\]

and the ray path equation between these two points becomes

\[
0 - x'_1 = \int_{y'_1}^{0} dx' = \int_{0}^{y'_1} \tan \theta' dy'.
\]

On the other hand, Snell's law gives

\[
k_B \sin \theta_R = k \sin \theta'
\]

on the ray. Using shallow water approximation and assuming plane beach,

\[
k_B = \frac{2\pi}{g^{1/2}h_B^{1/2}T},
\]

\[
k = \frac{2\pi}{g^{1/2}h(y')^{1/2}T},
\]

\[
h(y') = \frac{(h_B - h_b)}{y'_1} y' + h_b.
\]

Substitution into Eq. (2.38) gives

\[
\sin \theta' = \frac{\sin \theta_R}{h_B^{1/2}} \left\{ \left( (h_B - h_b) \frac{y'}{y'_1} + h_b \right) \right\}^{1/2}
\]

Assuming \(h_b(\ll h_B) = 0\),

\[
\sin \theta' = C y'^{1/2},
\]
in which \( G := \sin \theta_R / y'^{1/2} \). Accordingly,

\[
\tan \theta' := \frac{G y'^{1/2}}{(1 - G^2 y')}^{1/2}.
\]  

Substituting the above equation into Eq. (2.37) and integrating using the substitution of \( t = (1 - G^2 y')^{1/2} \) give

\[
-x' = -\frac{y'}{\sin^2 \theta_R} \left\{ \sin \theta_R \cos \theta_R + \sin^{-1}(\cos \theta_R) - \sin^{-1}(1) \right\}.
\]  

Similarly, for the wave diffracted around the right tip of the breakwater in Fig. 3,

\[
x'_r = \frac{y'_r}{\sin^2 \theta_R} \left\{ \sin \theta_R \cos \theta_R + \sin^{-1}(\cos \theta_R) - \sin^{-1}(1) \right\},
\]  

in which \( \theta_R \) is measured clockwise from the line which is perpendicular to the assumed straight and parallel bottom contours as shown in Fig. 3, and the origin of the shoreline-oriented coordinates \((x', y')\) is located at the breaking point, \( B_2 \).

Eqs. (2.45) and (2.46) can be solved for \( \theta_R \) by the Newton-Raphson method. The breaking wave angle, \( \alpha_b' \), relative to the \( x' \)-axis is calculated from

\[
k_b \sin \theta_R = k_b \sin \alpha_b',
\]  

and the breaking angle, \( \alpha_b \), relative to the \( z \)-axis is then given by

\[
\alpha_b = \begin{cases} 
\alpha_b' + \theta & \text{for wave diffracted at the left tip}, \\
-\alpha_b' + \theta & \text{for wave diffracted at the right tip}.
\end{cases}
\]  

The corresponding refraction coefficient is computed by

\[
K_R = \left( \frac{\cos \theta_R}{\cos \alpha_b'} \right)^{1/2},
\]

and the shoaling coefficient is computed by Eq. (2.33). As the salient grows, in the region near the apex of the salient, \( \theta_R \) bigger than \( \theta_{R_{\text{max}}} \) can be calculated, but this is impossible due to the presence of the breakwater. The breaking wave angles in this region are obtained by the interpolation (proportional to the longshore distance) from those at the ends of the region, and the refraction coefficients are assumed to be 1.0.
2.3 Finite-difference equations and boundary conditions

An explicit finite-difference method is used to solve Eqs. (2.6), (2.15) and (2.11) numerically for the wave condition computed along the shoreline. The location and orientation of the shoreline and the breaking wave angle are defined at the nodal points, and the longshore transport rate is defined between the nodal points, as shown in Fig. 4. The sediment continuity equation, (2.6), at the ith point with \( q = 0 \) can be expressed as the following finite-difference form:

\[
z'_{s_i} = z_{s_i} + \frac{\Delta t}{D} \left( \frac{Q_i - Q_{i-1}}{\Delta s_i + \Delta s_{i-1}} \right) \exp \left\{ i \left( \frac{\theta_i + \theta_{i-1}}{2} - \frac{\pi}{2} \right) \right\},
\]

in which \( \Delta t \) is the time step. The prime denotes the quantity being solved for the next time step and the unprimed quantities are known quantities at the present time step. The quantities, \( \Delta s_i \), \( \theta_i \), and \( Q_i \), are computed by

\[
\Delta s_i = \left\{ (x_{s_{i+1}} - x_{s_i})^2 + (y_{s_{i+1}} - y_{s_i})^2 \right\}^{1/2},
\]

\[
\theta_i = \tan^{-1} \left( \frac{y_{s_{i+1}} - y_{s_i}}{x_{s_{i+1}} - x_{s_i}} \right),
\]

and

\[
Q_i = \Gamma \left( \frac{H_{b_i} + H_{b_{i+1}}}{2} \right)^{5/2} \left( \frac{K_{D_i} + K_{D_{i+1}}}{2} \right)^{1/2} \left\{ K_1 \sin(2\delta_{b_i}) - K_2 \frac{H_{b_{i+1}} - H_{b_i}}{\Delta s_i} \left( \frac{\cot \beta_i + \cot \beta_{i+1}}{2} \right) \cos \delta_{b_i} \right\},
\]

in which

\[
\delta_{b_i} = \frac{\alpha_{b_i} + \alpha_{b_{i+1}}}{2} - \theta_i.
\]

The explicit finite-difference scheme gives unstable solution for a large time step \( \Delta t \). A useful guideline for choosing \( \Delta t \) for stable solution can be obtained by linearizing Eq. (2.8) with respect to \( y_s \). The linearization is made by assuming that \( \delta_b \ll 1 \). Then Eq. (2.11) becomes \( \delta_b \simeq \alpha_b - \partial y_s / \partial x_s \), and Eq. (2.9) gives

\[
\frac{\partial Q}{\partial x_s} \simeq -2K_1 \Gamma H_b^{5/2} \partial^2 y_s / \partial x_s^2
\]

with \( \partial H_b / \partial s = 0 \). Substitution of Eq. (2.55) into Eq. (2.8) with \( q = 0 \) gives

\[
\frac{\partial y_s}{\partial t} \simeq \epsilon \frac{\partial^2 y_s}{\partial x_s^2},
\]

with \( \epsilon = \partial^2 y_s / \partial x_s^2 \).
in which $\epsilon = 2K_1 \Gamma H_b^{5/2}/D$. Since Eq. (2.56) is a diffusion equation, the following stability criterion holds:

$$\Delta t \leq \frac{\Delta x^2}{2 \epsilon}.$$  \hspace{1cm} (2.57)

This stability criterion can give the first approximation of the time step for stable solution. If the computed shoreline shows saw-tooth instability, a smaller time step will be needed.

There might be several types of possible boundary conditions depending on the situation at the extremeties of the shoreline stretch to be modeled. The first is a fixed beach at the boundaries, which implies that the sediment transport rate remains constant near the boundaries so that the beach retains an equilibrium state there. The fixed boundary condition is applicable when the length of the beach is large enough so that the sediment transport at the boundaries does not affect the region where the offshore breakwaters are simulated. The second is a floating boundary condition which allows the shoreline to change at the boundaries by assuming linear variation of longshore sediment transport rate near the boundaries, for example, $Q_{IB+1} = 2Q_{IB} - Q_{IB-1}$, in which $IB$ indicates model boundary. The third is a forced boundary condition, which is applicable when the movement of the boundary is a priori known. In the present model, the final position of the boundary is given as input data and it is assumed that the shoreline at the boundary moves in proportion to the elapsed time from the initial position to the final position. The last one is a no-flux boundary condition which is applicable in the case in which there is an impermeable terminal groin at the boundary.

### 2.4 Treatment of tombolo formation

When the shoreline attaches an impermeable offshore breakwater, its offshore movement should stop there. But the numerical model that does not appreciate the presence of the breakwater may predict shoreline positions passed over the breakwater. Thus, at each time step we have to examine if any point of the shoreline has passed over the breakwater and pull it back to the location of the breakwater if any. Using the discretized shoreline position as shown in Fig. 5, there are three cases we can consider. In each figure, the dashed line denotes the shoreline position at the previous time step, and the solid line and the dash-dotted line denote the uncorrected and corrected shoreline positions, respectively, at the present time step. In the following analysis, the primed quantities represent the corrected shoreline position.

**Case 1 (Fig. 5 (a))**: Only the $i$th point passed over the breakwater to reach the point $C(x_i, y_i)$. We want to pull it back to the point $C'(x'_i, y'_i)$, which is the intersection of the breakwater and the line passing the points $C$ and $C'$. The equation of the breakwater whose end points are $(x_1, y_1)$ and $(x_r, y_r)$ is

$$y = \tan \theta_B (x - x_1) + y_1,$$  \hspace{1cm} (2.58)

in which
\[ \theta_B = \tan^{-1}\left( \frac{y_r - y_l}{x_r - x_l}\right) \]  

(2.59)

is the angle between the breakwater and the x-axis which is measured counterclockwise from the positive x-direction. The equation of the line passing the points C and \(C'\) is

\[ y = \cot \frac{\theta_{i-1} + \theta_i}{2} (x_{s_i} - x) + y_{s_i}, \]  

(2.60)

in which \(\theta_i\) and \(\theta_{i-1}\) are as defined in Fig. 4, and \(-\cot\{(\theta_i + \theta_{i-1})/2\}\) represents the slope of the line passing C and \(C'\). The intersection of Eqs. (2.58) and (2.60) is the point \(C'(x'_{s_i}, y'_{s_i})\) whose coordinates are

\[ x'_{s_i} = \frac{x_{s_i} \cot \frac{\theta_{i-1} + \theta_i}{2} + x_l \tan \theta_B + y_{s_i} - y_l}{\cot \frac{\theta_{i-1} + \theta_i}{2} + \tan \theta_B}, \]  

(2.61)

\[ y'_{s_i} = \tan \theta_B (x'_{s_i} - x_l) + y_l. \]  

(2.62)

In order to satisfy the continuity of sediment, the points \(B(x_{s_{i-1}}, y_{s_{i-1}})\) and \(D(x_{s_{i+1}}, y_{s_{i+1}})\) should move to the points \(B'(x'_{s_{i-1}}, y'_{s_{i-1}})\) and \(D'(x'_{s_{i+1}}, y'_{s_{i+1}})\), respectively, at the expense of pulling back the point C to \(C'\). In other words, \(\Delta BCC' = \Delta ABB' + \Delta BB'C'\) and \(\Delta CCC'D = \Delta C'DD' + \Delta DD'E\), which in turn can be expressed as

\[
\frac{1}{2} \epsilon_i \Delta s_{i-1} \cos \left\{ \tan^{-1}\left( \frac{y_{s_{i-1}} - y_{s_{i-2}}}{x_{s_{i-1}} - x_{s_{i-2}}} \right) - \frac{\theta_{i-1} + \theta_i}{2} \right\}
\]

\[ = \frac{1}{2} \epsilon_{i-1} \Delta s_{i-2} \cos \left\{ \frac{\theta_{i-2} + \theta_{i-1}}{2} - \tan^{-1}\left( \frac{y_{s_{i-2}} - y_{s_{i-1}}}{x_{s_{i-2}} - x_{s_{i-1}}} \right) \right\}
\]

\[ + \frac{1}{2} \epsilon_{i-1} \Delta s'_{i-1} \cos \left\{ \frac{\theta_{i-2} + \theta_{i-1}}{2} - \tan^{-1}\left( \frac{y'_{s_{i-1}} - y_{s_{i-1}}}{x'_{s_{i-1}} - x_{s_{i-1}}} \right) \right\}, \]  

(2.63)

and

\[
\frac{1}{2} \epsilon_i \Delta s_i \cos \left\{ -\tan^{-1}\left( \frac{y_{s_{i+1}} - y_{s_i}}{x_{s_{i+1}} - x_{s_i}} \right) + \frac{\theta_i + \theta_{i-1}}{2} \right\}
\]

\[ = \frac{1}{2} \epsilon_{i+1} \Delta s'_{i} \cos \left\{ -\frac{\theta_i + \theta_{i+1}}{2} + \tan^{-1}\left( \frac{y'_{s_i} - y_{s_i}}{x'_{s_i} - x_{s_i}} \right) \right\}
\]

\[ + \frac{1}{2} \epsilon_{i+1} \Delta s_{i+1} \cos \left\{ -\frac{\theta_i + \theta_{i+1}}{2} - \tan^{-1}\left( \frac{y_{s_{i+1}} - y'_{s_i}}{x_{s_{i+1}} - x'_{s_i}} \right) \right\}, \]  

(2.64)

in which \(\epsilon_i = \{(x_{s_i} - x'_{s_i})^2 + (y_{s_i} - y'_{s_i})^2\}^{1/2}\) is the length of \(CC'\), \(\epsilon_{i-1}\) and \(\epsilon_{i+1}\) are the lengths of \(BB'\) and \(DD'\), respectively, that are unknown, and

\[ \Delta s'_{i-1} = \{(x'_{s_{i-1}} - x_{s_{i-1}})^2 + (y'_{s_{i-1}} - y_{s_{i-1}})^2\}^{1/2}, \]  

(2.65)

\[ \Delta s'_i = \{(x_{s_{i+1}} - x'_{s_i})^2 + (y_{s_{i+1}} - y'_{s_i})^2\}^{1/2} \]  

(2.66)
are the lengths of $BC'$ and $CD'$, respectively. From the values $\epsilon_{i-1}$ and $\epsilon_{i+1}$ calculated by Eqs. (2.63) and (2.64), respectively, the corrected shoreline positions, $B'(x'_{s_{i-1}}, y'_{s_{i-1}})$ and $D'(x'_{s_{i+1}}, y'_{s_{i+1}})$, are obtained as
\[
x'_{s_{i-1}} = x_{s_{i-1}} - \epsilon_{i-1} \sin \frac{\theta_{i-2} + \theta_{i-1}}{2},
\]
\[
y'_{s_{i-1}} = y_{s_{i-1}} + \epsilon_{i-1} \cos \frac{\theta_{i-2} + \theta_{i-1}}{2},
\]
and
\[
x'_{s_{i+1}} = x_{s_{i+1}} - \epsilon_{i+1} \sin \frac{\theta_{i} + \theta_{i+1}}{2},
\]
\[
y'_{s_{i+1}} = y_{s_{i+1}} + \epsilon_{i+1} \cos \frac{\theta_{i} + \theta_{i+1}}{2}.
\]
If the corrected shoreline position passes over the breakwater, it is pulled back to the location of the breakwater by
\[
x''_{s_{i-1}} = \frac{x'_{s_{i-1}} \cot \frac{\theta_{i-2} + \theta_{i-1}}{2} + x_i \tan \theta_B + y'_{s_{i-1}} - y_i}{\cot \frac{\theta_{i-2} + \theta_{i-1}}{2} + \tan \theta_B},
\]
\[
y''_{s_{i-1}} = \tan \theta_B (x''_{s_{i-1}} - x_i) + y_i,
\]
or
\[
x''_{s_{i+1}} = \frac{x'_{s_{i+1}} \cot \frac{\theta_{i} + \theta_{i+1}}{2} + x_i \tan \theta_B + y'_{s_{i+1}} - y_i}{\cot \frac{\theta_{i} + \theta_{i+1}}{2} + \tan \theta_B},
\]
\[
y''_{s_{i+1}} = \tan \theta_B (x''_{s_{i+1}} - x_i) + y_i,
\]
as in Eqs. (2.61) and (2.62). The double-primes denote the shoreline positions pulled back to the location of the breakwater.

**Case 2 (Fig. 5 (b)):** The $(i-1)$th and $i$th points passed over the breakwater to reach the points $C(x_{s_{i-1}}, y_{s_{i-1}})$ and $D(x_{s_{i}}, y_{s_{i}})$, respectively. These points are pulled back to the points $C'(x'_{s_{i-1}}, y'_{s_{i-1}})$ and $D'(x'_{s_{i}}, y'_{s_{i}})$, respectively, whose coordinates can be obtained, using the same method for determining the position of the point $C'$ in Fig. 5 (a), as
\[
x'_{s_{i-1}} = \frac{x_{s_{i-1}} \cot \frac{\theta_{i-1} + \theta_{i-1}}{2} + x_i \tan \theta_B + y_{s_{i-1}} - y_i}{\cot \frac{\theta_{i-1} + \theta_{i-1}}{2} + \tan \theta_B},
\]
\[
y'_{s_{i-1}} = \tan \theta_B (x'_{s_{i-1}} - x_i) + y_i,
\]
and
\[
x'_{s_{i}} = \frac{x_{s_{i}} \cot \frac{\theta_{i} + \theta_{i}}{2} + x_i \tan \theta_B + y_{s_{i}} - y_i}{\cot \frac{\theta_{i} + \theta_{i}}{2} + \tan \theta_B},
\]
\[
y'_{s_{i}} = \tan \theta_B (x'_{s_{i}} - x_i) + y_i.
\]
The continuity of sediment requires the relations, $\Delta CC'D' + \Delta BCC' = \Delta BB'C' + \Delta ABB'$ and $\Delta CDD' + \Delta DDE' = \Delta D'E'E + \Delta EE'F$, which can be rewritten as
\[
\frac{1}{2} \epsilon_{i-1} \Delta s'_{i-1} \cos \left\{ -\tan^{-1} \left( \frac{y'_{s_{i}} - y_{s_{i-1}}}{x'_{s_{i}} - x_{s_{i-1}}} \right) + \frac{\theta_{i-2} + \theta_{i-1}}{2} \right\}
\]
$$
\begin{align}
&\frac{1}{2} \epsilon_{i-1} \Delta s_{i-2} \cos \left\{ \tan^{-1} \left( \frac{y_{s_{i-1}} - y_{s_{i-2}}}{x_{s_{i-1}} - x_{s_{i-2}}} \right) - \frac{\theta_{i-2} + \theta_{i-1}}{2} \right\} \\
&= \frac{1}{2} \epsilon_{i-2} \Delta s'_{i-2} \cos \left\{ \frac{\theta_{i-3} + \theta_{i-2}}{2} - \tan^{-1} \left( \frac{y'_{s_{i-1}} - y_{s_{i-2}}}{x'_{s_{i-1}} - x_{s_{i-2}}} \right) \right\} \\
&+ \frac{1}{2} \epsilon_{i-2} \Delta s_{i-3} \cos \left\{ \frac{\theta_{i-3} + \theta_{i-2}}{2} - \tan^{-1} \left( \frac{y_{s_{i-3}} - y'_{s_{i-3}}}{x_{s_{i-3}} - x_{s_{i-2}}} \right) \right\} \\
&= \frac{1}{2} \epsilon_{i} \Delta s_{i-1} \cos \left\{ \tan^{-1} \left( \frac{y_{s_{i}} - y_{s_{i-1}}}{x_{s_{i}} - x_{s_{i-1}}} \right) - \frac{\theta_{i-1} + \theta_{i}}{2} \right\} \\
&+ \frac{1}{2} \epsilon_{i} \Delta s_{i} \cos \left\{ - \tan^{-1} \left( \frac{y_{s_{i+1}} - y_{s_{i}}}{x_{s_{i+1}} - x_{s_{i}}} \right) + \frac{\theta_{i-1} + \theta_{i}}{2} \right\} \\
&= \frac{1}{2} \epsilon_{i+1} \Delta s'_{i} \cos \left\{ - \frac{\theta_{i} + \theta_{i+1}}{2} + \tan^{-1} \left( \frac{y_{s_{i+1}} - y_{s_{i}}}{x_{s_{i+1}} - x_{s_{i}}} \right) \right\} \\
&+ \frac{1}{2} \epsilon_{i+1} \Delta s_{i+1} \cos \left\{ \frac{\theta_{i} + \theta_{i+1}}{2} - \tan^{-1} \left( \frac{y_{s_{i+2}} - y_{s_{i+1}}}{x_{s_{i+2}} - x_{s_{i+1}}} \right) \right\},
\end{align}
$$

(2.79)

and

$$
\begin{align}
&\frac{1}{2} \epsilon_{i} \Delta s_{i} \cos \left\{ \tan^{-1} \left( \frac{y_{s_{i}} - y_{s_{i-1}}}{x_{s_{i}} - x_{s_{i-1}}} \right) - \frac{\theta_{i-1} + \theta_{i}}{2} \right\} \\
&= \frac{1}{2} \epsilon_{i} \Delta s_{i} \cos \left\{ - \tan^{-1} \left( \frac{y_{s_{i+1}} - y_{s_{i}}}{x_{s_{i+1}} - x_{s_{i}}} \right) + \frac{\theta_{i-1} + \theta_{i}}{2} \right\} \\
&= \frac{1}{2} \epsilon_{i+1} \Delta s'_{i} \cos \left\{ - \frac{\theta_{i} + \theta_{i+1}}{2} + \tan^{-1} \left( \frac{y_{s_{i+1}} - y_{s_{i}}}{x_{s_{i+1}} - x_{s_{i}}} \right) \right\} \\
&+ \frac{1}{2} \epsilon_{i+1} \Delta s_{i+1} \cos \left\{ \frac{\theta_{i} + \theta_{i+1}}{2} - \tan^{-1} \left( \frac{y_{s_{i+2}} - y_{s_{i+1}}}{x_{s_{i+2}} - x_{s_{i+1}}} \right) \right\},
\end{align}
$$

(2.80)

in which \(\epsilon_{i-1} = \{(x_{s_{i-1}} - x'_{s_{i-1}})^2 + (y_{s_{i-1}} - y'_{s_{i-1}})^2\}^{1/2}\) and \(\epsilon_{i} = \{(x_{s_{i}} - x'_{s_{i}})^2 + (y_{s_{i}} - y'_{s_{i}})^2\}^{1/2}\) are the lengths of \(CD'\) and \(DD'\), \(\epsilon_{i-2}\) and \(\epsilon_{i+1}\) are the unknown lengths of \(BB'\) and \(EE'\), and \(\Delta s'_{i-1}, \Delta s'_{i-2}\) and \(\Delta s'_{i}\) are the lengths of \(CD'\), \(BC'\) and \(DE'\), respectively. Eqs. (2.79) and (2.80) can be solved for \(\epsilon_{i-2}\) and \(\epsilon_{i+1}\), respectively, from which the corrected shoreline positions, \(B'(x_{s_{i-2}}, y_{s_{i-2}})\) and \(E'(x'_{s_{i+1}}, y'_{s_{i+1}})\), can be calculated as

$$
\begin{align}
x'_{s_{i-2}} &= x_{s_{i-2}} - \epsilon_{i-2} \sin \left( \frac{\theta_{i-3} + \theta_{i-2}}{2} \right), \\
y'_{s_{i-2}} &= y_{s_{i-2}} + \epsilon_{i-2} \cos \left( \frac{\theta_{i-3} + \theta_{i-2}}{2} \right),
\end{align}
$$

(2.81)

and

$$
\begin{align}
x'_{s_{i+1}} &= x_{s_{i+1}} - \epsilon_{i+1} \sin \left( \frac{\theta_{i} + \theta_{i+1}}{2} \right), \\
y'_{s_{i+1}} &= y_{s_{i+1}} + \epsilon_{i+1} \cos \left( \frac{\theta_{i} + \theta_{i+1}}{2} \right).
\end{align}
$$

(2.82)

(2.83)

(2.84)

If the corrected shoreline position passes over the breakwater, it is pulled back to the location of the breakwater by the manner similar to that used for Case 1 as in Eqs. (2.71) to (2.74).

Case 3 (Fig. 5 (c)): In the present model, if any two adjacent points touch the breakwater, the longshore sediment transport rate between the points is set to zero. Thus, if \(N (\geq 3)\) consecutive points touch the breakwater at a certain time step, the model predicts no movement of the inner \((N - 2)\) points and only the two outermost points can move for the next time step. Fig. 5 (c) is such a case with \(N = 3\). Since the longshore sediment transport rates between the points \(C'\) and \(D\) and between \(D\) and \(E'\) are zero, the point \(D\) did not move. But the two end points, \(C'\) and \(E'\), moved to the points, \(C\) and \(E\), respectively, which should be pulled back to \(C'\) and \(E'\),
respectively, as the breakwater blocks the offshore movement of the shoreline. In order to find the position of the points $B'$ and $F'$, we need to solve the relationships for continuity of sediment, $\Delta C'C + \Delta BCC' = \Delta B'B'C' + \Delta ABB'$ and $\Delta DED' + \Delta EE'F = \Delta E'F'F + \Delta F'F'G$, which can be rewritten as

\[
\begin{align*}
\frac{1}{2} \epsilon_{i-1} \Delta s_{i-1} \cos \left\{ -\tan^{-1} \left( \frac{y_{s_{i-1}} - y_{s_{i-1}}}{x_{s_{i-1}} - x_{s_{i-1}}} \right) + \frac{\theta_{i-2} + \theta_{i-1}}{2} \right\} \\
+ \frac{1}{2} \epsilon_{i-2} \Delta s_{i-2} \cos \left\{ \tan^{-1} \left( \frac{y_{s_{i-2}} - x_{s_{i-2}}}{x_{s_{i-2}} - x_{s_{i-2}}} \right) - \frac{\theta_{i-2} + \theta_{i-1}}{2} \right\} \\
= \frac{1}{2} \epsilon_{i-2} \Delta s_{i-2} \cos \left\{ \frac{\theta_{i-3} + \theta_{i-2}}{2} - \tan^{-1} \left( \frac{y_{s_{i-2}} - y_{s_{i-2}}}{x_{s_{i-2}} - x_{s_{i-2}}} \right) \right\} \\
+ \frac{1}{2} \epsilon_{i-2} \Delta s_{i-3} \cos \left\{ \frac{\theta_{i-3} + \theta_{i-2}}{2} - \tan^{-1} \left( \frac{y_{s_{i-3}} - y_{s_{i-3}}}{x_{s_{i-3}} - x_{s_{i-3}}} \right) \right\}
\end{align*}
\]

(2.85)

and

\[
\begin{align*}
\frac{1}{2} \epsilon_{i+1} \Delta s_{i+1} \cos \left\{ \tan^{-1} \left( \frac{y_{s_{i+1}} - y_{s_{i}}}{x_{s_{i+1}} - x_{s_{i}}} \right) - \frac{\theta_{i} + \theta_{i+1}}{2} \right\} \\
+ \frac{1}{2} \epsilon_{i+1} \Delta s_{i+1} \cos \left\{ -\tan^{-1} \left( \frac{y_{s_{i+2}} - y_{s_{i+1}}}{x_{s_{i+2}} - x_{s_{i+1}}} \right) + \frac{\theta_{i} + \theta_{i+1}}{2} \right\} \\
= \frac{1}{2} \epsilon_{i+1} \Delta s_{i+1} \cos \left\{ \frac{\theta_{i+1} + \theta_{i+2}}{2} + \tan^{-1} \left( \frac{y_{s_{i+2}} - y_{s_{i+1}}}{x_{s_{i+2}} - x_{i+1}} \right) \right\} \\
+ \frac{1}{2} \epsilon_{i+2} \Delta s_{i+2} \cos \left\{ \frac{\theta_{i+1} + \theta_{i+2}}{2} - \tan^{-1} \left( \frac{y_{s_{i+3}} - y_{s_{i+2}}}{x_{s_{i+3}} - x_{s_{i+2}}} \right) \right\}
\end{align*}
\]

(2.86)

in which $\epsilon_{i-1}$, $\epsilon_{i-2}$, $\epsilon_{i+1}$ and $\epsilon_{i+2}$ are the lengths of $CC'$, $BB'$, $EE'$ and $FF'$, respectively, and $\Delta s_{i-2}$ and $\Delta s_{i+1}$ are the lengths of $BC'$ and $EF'$, respectively. The above two equations can be solved for $\epsilon_{i-2}$ and $\epsilon_{i+2}$, respectively. Then, the corrected shoreline position, $B'(x_{s_{i-2}}, y_{s_{i-2}})$, can be calculated by the same expressions as those given in Eqs. (2.81) and (2.82), and similarly the point $F'(x_{s_{i+2}}, y_{s_{i+2}})$ is computed by

\[
\begin{align*}
x_{s_{i+2}}' &= x_{s_{i+2}} - \epsilon_{i+2} \sin \frac{\theta_{i+2} + \theta_{i+1}}{2}, \\
y_{s_{i+2}}' &= y_{s_{i+2}} + \epsilon_{i+2} \cos \frac{\theta_{i+2} + \theta_{i+1}}{2}
\end{align*}
\]

(2.87)

(2.88)

Again, if the corrected shoreline position passes over the breakwater, it is pulled back to the location of the breakwater as before.

3 Application

The developed numerical model was applied to the simulation of the shoreline change near the Chippokes State Park Breakwaters, James River, Surry County, Virginia (Fig. 6) for the first eight
months after construction. The Chippokes State Park Breakwaters consisted of six shore-parallel offshore breakwaters of 15.24 m crest length, 22.86 m gap and 9.14 m offshore distance (from the initial MHW line to the centerline of the breakwaters) was constructed during June 1987 and the shoreline change observed in September 1987 and February 1988 has been reported by Hardaway et al. (1988).

3.1 Wave hindcast

There are no available wave data near the project site for the simulation period. So the input wave data at the location of the breakwaters were hindcasted from the wind data at Norfolk, Virginia, which is located about 50 km southeast from the project site. The wind data include speed and direction at every three hours. Assuming that the wave direction corresponds to the wind direction, as seen in Fig. 6, the project site is affected by the wind blowing within the directional window fanning from 320° to 20° of direction measured clockwise from true north. It is also assumed that in order to generate the wave field that affects the shoreline change, wind should blow for more than nine hours (i.e., more than three observations) within the directional window. The average wind speed and direction for the period are calculated by vector-averaging the observations given every three hours. A constant wave field corresponding to the averaged wind speed and direction is assumed for that period.

The significant wave height and period at the location of the breakwaters are computed using the method of Kiley (1989), which is essentially a shallow water estuarine version of the quasi-empirical/quasi-theoretical wind wave prediction model developed by Bretschneider (1966) and modified by Camfield (1977). The method includes the variation in water depth, the effect of the surrounding land forms on the computation of the effective fetch, wave growth due to wind stress and wave decay due to bottom friction and percolation. The effect of refraction is not included in the computation of the wave height since the method assumes shore-normal wind direction. The wave angle at the location of the breakwaters is determined by Snell’s law assuming that the offshore bottom contours are straight and parallel to the z-axis and the deep water wave direction (at the center of the river) is same as the wind direction. The z-axis of the shoreline prediction model that corresponds to the baseline of the beach profile measurement in the report of Hardaway et al. (1988) is tilted by 9° counterclockwise from the E-W axis. The computed wave data are listed in Appendix A.

3.2 Input data

The input data file used for the Chippokes State Park Breakwaters is listed at the end of Appendix B. Some of the data need supplementary explanations as follows:

1. SI unit is used for all the numeric data.

2. The wave conditions listed on Lines 3 and 4 correspond to the wave data listed in the table in Appendix A, which were hindcasted using the wind data at Norfolk, Virginia.
3. $K_1 = 0.58$ was used as suggested by Kraus et al. (1982). $K_2 = 0.5K_1$ was used tentatively. The purpose of this application of the model is just to check if the model works properly for a typical field condition. Calibration or sensitivity test of the model by using various combinations of $K_1$ and $K_2$ may be needed if the model is used for the purpose of designing offshore breakwaters.

4. $\Delta t = 120$ sec used is somewhat larger than that computed by Eq. (2.57). But the computed shoreline position did not show any evidence of instability.

5. Seven print outputs at the end of every month from July 1987 till February 1988 (except September 1987 during which there was no measurable wave) were made in the output file. Only two of them (August 1987 and February 1988), however, were used for the comparison with the field measurements reported in Hardaway et al. (1988).

6. All the water depths and shoreline position in the model are with respect to MHW level, since the shoreline position in the report of Hardaway et al. is presented in terms of MHW line.

7. The coordinates of the breakwater tips listed on Line 11 correspond to the shoreward corners of the bases of the breakwaters. This choice becomes more reasonable than using the coordinates corresponding to the centerlines of the breakwaters as the breakwater slope becomes steeper and the breakwater width becomes larger.

8. Forced boundary conditions were used at both boundaries since the shoreline change in the areas far from the breakwaters was not available.

9. Diffraction coefficients were computed every 30 time steps (i.e., every 1 hour).

3.3 Result

Fig. 7 shows the measured (Fig. 7 (a)) and computed (Figs. 7 (b) and (c)) shoreline changes. Fig. 7 (b) is the result with varying longshore sediment transport coefficients (cf. Eq. (2.15)), whereas Fig. 7 (c) is that with constant coefficients (cf. Eq. (2.9)). In Figs. 7 (b) and (c), the shoreline positions in June 1987, September 1987 and February 1988 are indicated by solid line, dashed line and long-dashed line, respectively.

In September 1987 the field measurement shows the formation of double salients behind each breakwater and in February 1988 the double salients coalesced into a single tombolo. As can be seen in Fig. 7 (b), the formation of double salients in September 1987 is predicted by the model using varying longshore transport coefficients, even though it is not so clear as in the measurement. This feature is hard to be observed in Fig. 7 (c) which is the result using constant coefficients.

The final shoreline positions in February 1988 are almost identical between the two model results. The model predicted smaller tombolos and more prominent erosion behind the gaps compared with those in the measurement. This may be due to the addition of sediment to the system by runoff and bank erosion as reported in Hardaway et al. (1988). Note that the on-offshore sediment transport was assumed to be zero in the present model.
4 Conclusion and Suggestion

A one-line numerical model for predicting shoreline change in the vicinity of offshore breakwaters was developed and compared with the field data at Chippokes State Park Breakwaters, Virginia. The use of longshore sediment transport coefficients that vary in proportion to square root of diffraction coefficients could predict the formation of double salients during the initial stage of shoreline change after the construction of the breakwaters. The model also could handle the formation of tombolos as well as the growth of salients.

In many cases beach nourishment is used in combination with offshore breakwaters for beach protection. The beach nourishment could be incorporated in the model by advancing the initial shoreline position by the amount corresponding to the volume of beach nourishment. Assuming that the characteristic of the borrowed sediment is same as that of native sediment, the amount of initial shoreline advance may be computed as \( V/\ell D_t \), in which \( V \) = volume of beach nourishment, \( \ell \) = the length of the shoreline stretch along which beach nourishment is applied, and \( D_t \) = depth of annual seaward limit of profile change which may be obtained by the expression proposed by Hallermeier (1983).

References


Appendix A: Wind and Wave Data

The wind and wave data used for the simulation of the shoreline change at the Chippokes State Park Breakwaters, Virginia from June 1987 till February 1988 are listed in the following table. In the table, the wind direction, $\alpha_w$, is measured clockwise from true north, and the wave angles, $\alpha_o$ and $\alpha_B$, are as defined in Fig. 1. $U$ is wind speed, and $H_B$ and $T_B$ are the significant wave height and period at the location of the breakwaters.
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Appendix B: Input Data File

At the beginning of the program, it requires the names of input and output files, which should be typed in on the computer terminal. The input data file is an existing file and the output file is created by the program. The input data file an example of which is included at the end of this appendix contains requests for information in a series of lines. Lines of text (the request portion) should be neither added nor deleted from the input file, as the program will skip over these request lines to read the input values. A free format is used for the input data except the initial shoreline positions on Line 18. If several values are required in a line, they may be separated by a space or by a comma, or both.

1. The first line of the input file requests a project identification, which may be up to 80 characters long.

2. The second line requests mean diameter (in millimeter), specific gravity and porosity of sediment.

3. The input wave condition may change with time. Assuming that for a certain period the input wave condition is constant, the number of changing wave conditions in each run of the model is entered here.

4. The significant wave height (meter), wave period (second), wave angle (degree) and duration (hour) of each wave condition at the location of breakwaters are read in line by line. The number of data lines should match the number of wave conditions entered on Line 3. Between each neighboring wave conditions there may be a period in which the sea state is calm or the wave direction is away from the project site due to offshore wind. These periods are not included, so the sum of the duration of each wave condition may be smaller than the real simulation time.

5. Values of the longshore sediment transport coefficients, $K_1$ and $K_2$, are entered here. In the project site where the magnitudes of these coefficients are not well determined, they may be used as model calibration parameters.

6. A guideline for choosing a proper time step $DT$ (called $\Delta t$ in the main text of this paper) is discussed in §2.3. The larger the grid size $\Delta x$ is, the larger time step $\Delta t$ can be chosen. A smaller time step increases the computational run time, whereas a larger time step degrades the accuracy of shoreline prediction. If a time step larger than that suggested in Eq. (2.57), a warning message is written in the output file. If the predicted shoreline shows unrealistic saw-toothed shape, it may be cured by reducing $\Delta t$ in most cases. The total number of time steps is computed by summing up the durations (converted to seconds) on Line 4 and dividing by $DT$.

7. In many situations it is informative to study the time evolution of the computed shoreline change. The value entered here specifies the total number of simulated times when the computed shoreline position should be printed in the output file. The output of the initial shoreline position does not have to be included, since it is a default output. The output of the final shoreline position, however, should be included if it is wanted.
8. The print time steps should be specified by the number of time steps. The number of print time steps should match the number entered on Line 7.

9. Enter the number of offshore breakwaters.

10. Enter the water depths (meter) at different diffraction regions. The first water depth is that near the left tip of the leftmost breakwater, the second one is that at the first gap, the third one is that at the second gap, and so on, and the last one is the water depth near the right tip of the rightmost breakwater. The number of the water depths specified should be larger than the number of breakwaters entered on Line 9 by one.

11. Enter the coordinates \((x, y)\) of the left and right tips of each breakwater starting from the leftmost breakwater. The number of data lines should match the number of breakwaters entered on Line 9.

12. The flags LBC and RBC inform the program which boundary condition is applied at the left and right boundaries, respectively. The choice of boundary conditions is discussed in §2.3.

13. If a forced boundary condition is applied, the \(y\)-coordinate of the boundary at the final time step should be specified. It is assumed that the \(z\)-coordinate does not change. If any other boundary condition is applied, zero \((0, 0)\) should be entered.

14. The beach slope is computed by Eq. (2.12) depending on the flag entered here. 0 is for laboratory tests and 1 is for field tests.

15. In the present model, it is proposed that the longshore sediment transport coefficients can vary in proportion to square root of the diffraction coefficient as in Eq. (2.15). The flag 1 includes this concept in the model so that the longshore sediment transport rate, \(Q\), is computed by Eq. (2.15), whereas with the flag 0 the model uses constant transport coefficients so that \(Q\) is computed by Eq. (2.9).

16. A large portion of the computing time is consumed for the computation of diffraction coefficients, which are not sensitive to the slow evolution of shoreline. Thus, by computing the diffraction coefficients at an integer multiple of the time step a large amount of computing time can be saved without sensible degradation of model result. If we enter the value of 10, for example, the diffraction coefficients computed for the initial shoreline position are used for the next ten time steps until the new diffraction coefficients are computed for the shoreline position at the eleventh time step, and the new diffraction coefficients are used for the next ten time steps, and so forth.

17. Enter the total number of points of shoreline including the boundary points.

18. The coordinates \((x, y)\) of the initial shoreline position are entered in FORMAT(6F10.3). Since two values \((x\ and \ y)\) are needed for each shoreline point, one data line contains three shoreline points.
1. Project identification:
   Chippokes State Park Breakwaters, Virginia
2. Mean diameter(mm), Specific gravity, and Porosity of sediment:
   0.44 2.65 0.4
3. Number of wave conditions changing in a sequence:
   43
4. Wave height(m), Period(sec), Angle(deg) and Duration(hr):
   0.180 1.85  -4.56  18.
   0.180 1.85  -7.33  12.
   0.180 1.85  -8.41  12.
   0.180 1.85  -2.59  15.
   0.186 1.89   2.54   9.
   0.216 2.04  -9.43  45.
   0.201 1.97  -16.30  9.
   0.180 1.85  -14.91  9.
   0.162 1.74  -7.90  12.
   0.171 1.80  -12.41 15.
   0.171 1.80  -13.83 12.
   0.116 1.36  -24.19  9.
   0.137 1.51   14.84  9.
   0.152 1.68   3.30   24.
   0.186 1.89   8.99   12.
   0.186 1.88  18.71  9.
   0.146 1.51  24.04  9.
   0.171 1.80  -2.57  15.
   0.152 1.68  -6.59  9.
   0.186 1.89  11.81  9.
   0.140 1.60  11.56  15.
   0.223 2.07  10.71  9.
   0.201 1.97  12.16  12.
   0.180 1.85  -16.92 24.
   0.180 1.85  -3.13  9.
   0.152 1.68  12.81  9.
   0.152 1.68  2.27  15.
   0.186 1.89  10.04  9.
   0.171 1.80  -8.93  12.
   0.180 1.85  -13.27 21.
   0.162 1.74  -14.00 15.
   0.207 2.00  -3.45  27.
   0.152 1.68  -10.07 15.
   0.171 1.80  -12.07 15.
   0.201 1.97  -10.85 15.
   0.186 1.89   5.21  15.
   0.186 1.89  -0.74  21.
   0.186 1.89  -1.14  15.
   0.152 1.68  -14.42 12.
   0.201 1.97  13.41  9.
   0.171 1.80  -15.64 9.
   0.180 1.85  -0.82  12.
   0.195 1.93  -4.14  27.
5. K1 and K2:
   0.58 0.29
6. DT(sec) and Total number of time steps:
   120.0 18450
7. Total number of print outputs:
   7
8. Print time steps:
   1260
   1980
   5580
9. Number of breakwaters: 6

10. Water depths (m) at different regions:
- 0.77 0.80 0.86 0.84 0.83 0.83 0.77

11. \((X_l, Y_l)\) and \((X_r, Y_r)\) of each breakwater:
- 38.93 28.22 60.25 26.19
- 77.39 25.93 99.06 25.60
- 115.61 25.08 137.16 24.41
- 154.90 24.84 176.33 25.36
- 193.95 25.20 215.62 25.00
- 233.72 25.00 255.87 25.00

12. Flags for LBC & RBC (1=Fixed; 2=Floating; 3=Forced; 4=No-flux):
- 3, 3

13. Final y-coords. of left & right boundaries (0.0 if not forced boundary):
- 26.40 17.50

14. Flag for type of test (0=laboratory; 1=field):
- 1

15. Flag for longshore transport coefficients (0=constant; 1=varying):
- 1

16. Time step for calculation of diffraction coefficients:
- 30

17. Total number of points of shoreline location:
- 195

18. Initial shoreline positions \((X(I), Y(I)), I=1,NP\) in FORMAT(6F10.3):

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Captions of figures

1. Orthogonal curvilinear coordinate system and definition of model variables.
2. Variables used for computation of diffraction coefficients and zoning for computation of breaking wave angle and the coefficients of refraction and shoaling.
3. Refraction of the wave diffracted at the breakwater tip.
4. Finite-difference representation of shoreline and associated transport around ith point.
5. Illustrations for tombolo formation.
6. Location map of Chippokes State Park Breakwaters, Virginia.
7. Comparison between measurement and computation of shoreline change near Chippokes State Park Breakwaters, Virginia.
Figure 1
Figure 5