Massive Photons: An Infrared Regularization Scheme for Lattice QCD plus QED

Michael G. Endres
MIT, Ctr Theoret Phys, Cambridge, MA 02139 USA;

Andrea Shindler
Forschungszentrum Julich, IAS, IKP, D-52428 Julich, Germany;

Andrea Shindler
Forschungszentrum Julich, JCHP, D-52428 Julich, Germany;

Andre Walker-Loud
Coll William & Mary, Dept Phys, Williamsburg, VA 23187 USA

Follow this and additional works at: https://scholarworks.wm.edu/aspubs

Recommended Citation

This Article is brought to you for free and open access by the Arts and Sciences at W&M ScholarWorks. It has been accepted for inclusion in Arts & Sciences Articles by an authorized administrator of W&M ScholarWorks. For more information, please contact scholarworks@wm.edu.
Massive photons: an infrared regularization scheme for lattice QCD+QED

Michael G. Endres,1 Andrea Shindler,2‡ Brian C. Tiburzi,3,4,5 § and André Walker-Loud6,7,8,†

1Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
2IAS, IKP and JCHP, Forschungszentrum Jülich, 52428 Jülich, Germany
3Department of Physics, The City College of New York, New York, NY 10031, USA
4Graduate School and University Center, The City University of New York, New York, NY 10016, USA
5RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA
6Department of Physics, College of William and Mary, Williamsburg, VA 23187-8795, USA
7Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, VA 23606, USA
8Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

PACS numbers: 11.15.Ha, 12.38.-t, 12.38.Gc

Introduction – Approximately 95% of the visible mass of the universe arises from the binding of quarks into nucleons by the strong interactions of quantum chromodynamics (QCD). The relative mass difference between the proton and neutron is approximately 0.07%, and is attributed to two sources of isospin symmetry breaking in the Standard Model, namely, differences in the down and up quark masses and their electromagnetic charges. Although these breaking effects are minute, they play an essential role in our understanding of the universe. For example, the primordial abundance of light nuclear elements in the early universe is exquisitely sensitive to the excess mass of the neutron compared to the proton [1, 2].

Lattice QCD (LQCD) provides a first principles approach for determining isospin breaking effects in hadronic and nuclear processes. There are a handful of LQCD calculations of the strong contribution to the nucleon mass splitting [2, 3] and a comparable number that determine the electromagnetic corrections [4, 5]. One impressive calculation includes both sources of isospin breaking simultaneously and yields, among other quantities, a postdiction for the nucleon isospin splitting with \( \approx 5\sigma \) statistical significance [8]. There exists an alternate means for determining the electromagnetic self-energy of the nucleon, from the Cottingham Formula [17–20], which makes use of experimental cross sections as input to dispersion integrals. However, the uncertainty attained with this method [21, 22] is not yet competitive with the LQCD calculations.

Although inclusion of electromagnetism in LQCD is theoretically straight-forward [24, 25], it presents practical challenges due to the long-range nature of the electromagnetic (QED) interactions. Specifically, such interactions give rise to power-law finite volume (FV) corrections, and their removal via extrapolation requires computationally demanding simulations performed at multiple volumes. An analytic understanding of the power-law FV effects within such setups [8, 26–28] have enabled reliable FV extrapolations of the single hadron spectrum.

Despite the successful application of present techniques, there are a number of reasons for considering new methods. Control over FV modifications to light nuclear binding energies seem to require particularly large volumes [26]. There are quantities in addition to the spectrum for which precise knowledge of the QED modifications are needed, for example, corrections to hadronic matrix elements [29] and charged particle scattering [30], both of which suffer from infrared (IR) challenges. LQCD calculations are performed with multiple ultraviolet (UV) regulators, providing valuable cross-checks on the continuum extrapolation of many important quantities [31]. Multiple IR regulators can do the same for LQCD calculations that include QED, but to date, only a few other formulations have been considered [32, 33]. Of those, only one is constructed with a local quantum field theory (QFT) [33, 34]. Finally, computationally efficient means of accounting for IR effects are always desirable, not just for lattice QCD+QED, but anywhere long-range Coulomb interactions are present (see, e.g., [35]).

Motivated by these considerations, we demonstrate the viability of an alternative IR regulator for lattice QCD+QED simulations: namely, the introduction of a photon mass \( m_\gamma \). Although a photon mass term manifestly violates gauge-invariance, it maintains locality and its effects on hadronic quantities can be reliably quantified and accounted for within an effective field theory (EFT) framework. The introduction of a new scale, \( m_\gamma \),
implies an additional extrapolation within our approach. With the aid of analytic formulas, however, we demonstrate that for the spectrum, such extrapolations can be performed at a single volume and yield results that are consistent with conventional approaches. In the remaining sections, we present the salient features of our calculation.

**Analytic Considerations**—In continuum Euclidean spacetime, the $R\xi$ gauge fixed action for the massive photon is given by

$$\mathcal{L}_\gamma = \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2\xi} (\partial_\mu A_\nu)^2 + \frac{1}{2} m_r^2 A_\mu^2 \quad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; throughout this study, we work in Landau gauge, corresponding to the limit $\xi \rightarrow 0$. An Abelian theory, such as QED, with a massive vector gauge field is still perturbatively renormalizable. This well known result follows from the fact that it is possible to find a BRST transformation that leaves the Lagrangian invariant up to a total divergence [38]. The BRST symmetry is preserved if one uses a gauge invariant UV cutoff [37], such as a spacetime lattice, thus the renormalizability follows from the power-counting theorems for a lattice regularization [38].

We consider three forms of corrections to correlators and hadron mass differences at leading order in the fine-structure constant $\alpha = e^2/(4\pi)$. These corrections arise from either the zero mode contribution to the partition function, the presence of a finite photon mass, or FV effects. The analytic forms of these corrections are determined from an EFT for hadrons of mass $M$ ($M = m_\gamma$, $m_\pi$, $m_K^+$, and $m_K^0$) and charge $Q$; the naive expansion is in $m_r/M$ (i.e. $\Lambda_{UV} = M$) [29]. The EFT is a generalization of nonrelativistic QED (NRQED) [40] for hadrons that includes a photon mass term, and additional operators that are unconstrained by gauge invariance.

1. **Zero mode**: For sufficiently small $m_r$, the zero mode of the temporal photon field appearing in Eq. [1] must be treated nonperturbatively [41]. In this regime, the two-point function for single hadrons has the form

$$C(\tau) = Ze^{-\gamma M \tau - \alpha x^2}, \quad (2)$$

where $Z$ is an overlap factor; the zero-mode contribution appears as $x = (4\pi Q^2)/(2m_r^2 L^3 T)$, and vanishes as $T \rightarrow \infty$ at fixed $L$ and $m_r$.

2. **Photon mass**: The hadrons electromagnetic mass shift can be determined as a function of photon mass, order-by-order in an expansion in powers of $m_r/M$. With the electromagnetic mass written as $M(\alpha, m_r)$, we define the mass shift $\Delta_\gamma M(\alpha, m_r) = M(\alpha, m_r) - M(\alpha, 0)$, which is UV finite. These IR shifts are given by

$$\Delta_\gamma M^{LO} = -\frac{\alpha}{2} Q^2 m_r^2, \quad \Delta_\gamma M^{NLO} = \left( C \alpha - \frac{\alpha}{4\pi} Q^2 \right) \frac{m_r^2}{M}. \quad (3)$$

The leading-order (LO) expression is non-analytic in the squared photon mass, whereas the next-to-leading order (NLO) expression is analytic but arises from both loops and local contributions [12]: the $N^2 LO$ correction is of order $\Delta_\gamma M^{N^2LO} = \mathcal{O}(m_r^3/M^2)$. The latter two orders are accompanied by coefficients not fixed by the hadron charge.

3. **Finite volume**: The effects of FV can similarly be calculated using an NRQED approach. This is a finite photon mass generalization of that pursued by [26,27]. The FV corrections to the electromagnetic mass are written as $\delta_L M(\alpha, m_\gamma, L) = M(\alpha, m_\gamma, L) - M(\alpha, m_\gamma, \infty)$, and for charged hadrons, are given up to NLO by

$$\delta_L M^{LO} = 2\pi \alpha Q^2 m_\gamma \left[ I_1(m_\gamma, L) - \frac{1}{(m_\gamma L)^3} \right], \quad \delta_L M^{NLO} = \pi \alpha Q^2 \frac{m_\gamma^2}{M} \left[ 2I_{1/2}(m_\gamma, L) + I_{3/2}(m_\gamma, L) \right]. \quad (4)$$
TABLE I. QED$_{TL}$ induced mass splittings, extrapolated to $L \to \infty$ ($m_s = 0$, $K_s = 0$).

<table>
<thead>
<tr>
<th>splitting</th>
<th>$K_L$</th>
<th>$\chi^2$/dof</th>
<th>$\Delta M/M \times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p - n$</td>
<td>1</td>
<td>0.07/2</td>
<td>0.73(05)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.03/1</td>
<td>0.70(13)</td>
</tr>
<tr>
<td>$K^+ - K^0$</td>
<td>1</td>
<td>0.29/2</td>
<td>3.71(06)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.17/1</td>
<td>3.68(20)</td>
</tr>
</tbody>
</table>

TABLE II. QED$_M$ induced mass splittings, extrapolated to $m_s = 0$ ($L/a = 24$, $K_L = 1$).

<table>
<thead>
<tr>
<th>splitting</th>
<th>$m_{\gamma}/m_\pi$ range</th>
<th>$K_s$</th>
<th>$\chi^2$/dof</th>
<th>$\Delta M/M \times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p - n$</td>
<td>1/14 - 1</td>
<td>2</td>
<td>0.09/5</td>
<td>0.79(06)</td>
</tr>
<tr>
<td></td>
<td>1/4 - 1/2</td>
<td>1</td>
<td>0.06/2</td>
<td>0.81(08)</td>
</tr>
<tr>
<td>$K^+ - K^0$</td>
<td>1/14 - 1</td>
<td>2</td>
<td>0.42/5</td>
<td>3.77(06)</td>
</tr>
<tr>
<td></td>
<td>1/4 - 1/2</td>
<td>1</td>
<td>0.12/2</td>
<td>3.79(06)</td>
</tr>
</tbody>
</table>

where

$$I_n(z) = \frac{1}{2^n + \pi n} \prod_{n \neq 0} K_{2-n} \left( \frac{z}{|\nu|} \right)$$

and $\nu \in \mathbb{Z}^3$. By contrast, the leading nonvanishing correction for neutral baryons (mesons) appears at $N^2$LO ($N^3$LO). Because the zero mode of the temporal photon is treated exactly in Eq. [2], the FV corrections are calculated with this mode removed—a manifestation of which is the subtracted term appearing at LO.

**Lattice Parameters and Ensembles** — Electroweak numerical calculations of the hadron spectrum were performed using a modified version of the Chroma software suite [43]. Studies were performed using dynamical SU(3) flavor symmetric isotropic QCD gauge field configurations generated using a tadpole-improved L"uscher-Weisz gauge action and clover fermion action. The configurations correspond to a single lattice spacing $a = 0.1453(16)$ fm, three spatial extents: $L \sim 3.48$ fm, 4.64 fm and 6.96 fm, and temporal extents $T > L$. The pion (kaon) and nucleon masses in physical units are $m_\pi = m_K = 807.0(9.1)$ MeV and $m_n = 1.634(18)$ GeV, respectively. This choice of masses ensures that the only appreciable FV corrections to hadron masses are those arising from QED effects. The QCD ensembles used in this work comprise 956 ($L/a = 24, T/a = 48$), 515 ($L/a = 32, T/a = 48$) and 342 ($L/a = 48, T/a = 64$) configurations and are a subset of those described in [44]; further details regarding the ensembles, lattice action and parameters can be found there.

Uncorrelated photon field configurations $A_\mu$ were generated using two different lattice actions: a conventional massless Coulomb gauge-fixed action with the zero-mode removed [11] [24] [25] (QED$_{TL}$) [43], and a naive lattice discretized form of Eq. [1] (QED$_M$), where derivatives are replaced by finite differences. Note that in Euclidean space, Landau gauge is a complete gauge-fixing condition, and therefore in the latter case, the path integration over nonzero-modes is well defined in the $m_s \to 0$ limit. The photon mass values considered in this work are obtained by $m_\gamma/m_\pi \in [1/4, 1/7, 1/4, 1/3, 5/12, 1/2, 7/12, 1]$. In both cases, results were obtained by computing correlation functions on QCD+QED gauge configurations generated by post-multiplying each QCD configurations by a single $e^{ie\phi A_\mu}$, where $Q_\mu = 2/3$, $Q_d = Q_s = -1/3$. In the electroquenched approximation with SU(3) flavor symmetry, isospin splittings have missing contributions that are $\mathcal{O}(a^2)$, and therefore negligible for this study.

In the electroquenched theory, the fine structure coupling does not renormalize and therefore we take it to be equal to its experimental value, $\alpha^{-1} = 137.036 \ldots$, measured in the Thomson limit. The presence of electromagnetic interactions demands renormalization of the valence bare quark masses $m_q$, however. Since our lattice regulator breaks chiral symmetry, this leads to an additive shift in the quark mass. We tune the valence quark masses so that, in the presence of electromagnetic interactions, the neutral $q\bar{q}$ meson mass $m_{qq}$ obtained from the connected part of the $q\bar{q}$ correlation function is sufficiently close to the pion (kaon) mass $m_\pi$. For our electroquenched calculation, this choice of renormalization is robust but the quark mass renormalization in the full QCD+QED does not allow for a unique separation of the QED and QCD effects [46]. All measurements were performed using valence quark masses $am_\mu = -0.25501$ and $am_\pi = -0.24750$ (the QCD bare quark mass is $am_q = -0.2450$); the resulting mistuning for the charge neutral mesons was $\Delta m_{qq}/m_\pi \lesssim 0.1\%$ for all values of $m_\pi/m_s \lesssim 1$, where $\Delta m_{qq} = m_{qq} - m_\pi$.

The mistuning from strong isospin-breaking can be estimated using chiral symmetry. For the kaon, one finds

$$\frac{\Delta m_{K^+ - K^0}}{m_K} \approx \frac{1}{2} \frac{\Delta m_{uu} - \Delta m_{dd}}{m_K} \lesssim 0.0004,$$

while the nucleon correction is given by

$$\frac{\Delta m_{n-p}}{m_n} \approx \frac{\alpha_{d-u}}{4\pi f_\pi m_n} \frac{2(\Delta m_{dd} - \Delta m_{uu})}{m_n^2} \lesssim 0.0002.$$  

We can estimate the parameter $\alpha_{d-u}$ from the LQCD determination of the $m_q - m_u$ contribution to the nucleon mass splitting [28] and find $\Delta m_{p-n}/m_n \lesssim 0.0002$. In both cases, mistuning is a potentially sizable correction to our results, which affects both the QED$_{TL}$ and QED$_M$ determinations. Although a precise quark mass tuning is required for practical applications, it is not needed in the present proof-of-principle study [47].

**Analysis and Results** — Shell-shell and shell-point correlation functions were estimated using a single measurement per configuration, with a randomly chosen spacetime source location. Following [43], we average observables over $+e$ and $-e$ on each configuration in order
to exactly cancel off the $O(\epsilon)$ contributions to statistical noise. Mass differences due to electromagnetic effects can be determined from the late-time dependence of single hadron correlation functions $C^A(\tau)$ and $C^B(\tau)$, by studying the plateau region of an effective mass difference $\Delta M_{AB}^\tau(\tau) = M_{AB}^\tau(\tau) - M_{AB}^\tau(0)$. By exploiting the correlations between $A$ and $B$, we are able to extract a clear signal for the mass difference. For the nucleons, we consider a generalized effective mass formula of the form:

$$M_{\text{eff,exp}}(\tau) = -\frac{1}{a} \log \frac{C(\tau + a)}{C(\tau)} + 2\tau + xa ,$$

(8)

which neglects the backward propagation of states on a lattice of finite temporal extent $T$. For mesons, we account for the backward propagating state by considering a generalized effective mass formula of the form:

$$M_{\text{eff,cosh}}(\tau) = \frac{1}{a} \cosh^{-1} \left[ \frac{e^{h(\tau,a)} + e^{h(\tau,-a)}}{2} \right] - xT ,$$

(9)

where $h(\tau,a) = xa(a - T + 2\tau) + \log[C(\tau + a)/C(\tau)]$. Both formulas treat the zero mode of the temporal photon field appearing in Eq. 2 non-perturbatively (for neutral hadrons $x = 0$ and these expressions reduce to their conventional forms). Although this contribution is negligible compared to the hadron masses, for the lattice parameters considered it can be comparable in magnitude to the mass differences we wish to extract. Fig. 1 provides an explicit example of the behavior of $\Delta M_{\text{eff}}(\tau)$ for the kaon mass splitting, computed both with and without the zero-mode contribution accounted for.

Mass differences were determined for all volumes and photon masses via a correlated constant least-squares fit to $\Delta M_{\text{eff}}$ in the plateau region, as demonstrated in Fig. 1. An analogous determination from exponential fits to a ratio of correlation functions yielded consistent results. Systematic uncertainties were estimated by varying the region over which fits were performed, and all uncertainties were added in quadrature. Extracted mass shifts were subsequently extrapolated to vanishing photon mass and/or the infinite volume limit using the fit formula:

$$\Delta M(\alpha, L, m_\gamma) = \Delta M(\alpha) + \sum_{k=0}^{K_\gamma} \Delta_\gamma M^{N^k\text{LO}}(\alpha, m_\gamma)$$

$$+ \sum_{k=0}^{K_L} \delta_\gamma M^{N^k\text{LO}}(\alpha, m_\gamma, L),$$

(10)

where $K_\gamma$ and $K_L$ indicate the order of each extrapolation. In the case of mass splittings, an appropriate linear combination of mass shift formulas were used. Note that for the $QED_{TL}$ extrapolations, $K_\gamma = 0$; the appropriate FV formulas for $\delta_\gamma M^{N^k\text{LO}}$ retain $T$-dependence, and may be found in §3.

We carry out two independent analyses to test the viability of our proposal: 1) an infinite volume extrapolation of $QED_{TL}$ induced mass differences, as is conventionally performed, and 2) an $m_\gamma \rightarrow 0$ extrapolation of $QED_M$ induced mass differences using data at a single FV, but after having first removed the lowest order FV contributions, $\delta_\gamma M$. Both types of extrapolation were performed using Eq. 10 noting that many of the lowest-order contributions are fixed by theory. Results for the first analysis, using all three volumes, are provided in Table 1 and representative fits are shown in Fig. 2. Results for the second analysis on the smallest volume are provided in Table II for comparison, and shown in Fig. 3. Analogous $QED_M$ extrapolations, performed at each of the three volumes, are summarized in Fig. 4 and are consistent not only with each other, but also the $QED_{TL}$ extrapolations. In all cases, we find that the numerical and theoretical mass corrections are in excellent agreement down to at least $m_\gamma L \sim 1$.

The most computationally demanding part of our calculation involves multiple inversions of the Dirac operator. Assuming, conservatively, a linear scaling with spacetime volume, the total inversion cost for $L/a = 32$ is $515/956 \times (32/24)^3 \sim 1.3$ times greater than that of $L/a = 24$. By comparison, the $L/a = 48$ inversion cost
FIG. 4. $QED_M$ extrapolations (points) performed independently at each volume. Systematic errors are estimated by considering fits over multiple ranges of $m_q$ and orders $K_\gamma \leq 3$ (summarized by the histograms). Summary of $QED_{TL}$ extrapolations are indicated by gray horizontal bands.

is $\sim 3.8$ times greater. The $L/a = 24$ extrapolations using $m_q/\bar{m}_\pi \in [1/4, 1/2]$ data, provided in Table I, are consistent with those using all values of $m_q$. The results are also consistent with the $QED_{TL}$ extrapolation using three volumes, provided in Table II but required only 4/5 the computational cost. We therefore conclude that for the same precision and accuracy, the numerical cost of our $QED_M$ calculation of the mass splittings is comparable to or less than that of $QED_{TL}$.

Conclusion – This work demonstrates that it is possible to reliably estimate infinite volume hadron mass differences induced by electromagnetism on a single lattice volume with $QED_M$. Conservatively, the pion-less EFT employed in this work is valid for $m_q \ll 2m_\pi$ and $m_q L \gtrsim 4$. Provided these inequalities are satisfied, the analytic expressions obtained for the mass shift are valid up to $O(m_\gamma^2/M^3, \alpha^2)$, and are independent of the pion mass; from our numerics, it appears that this order is sufficient to obtain reliable extrapolations of the mass shifts in the regime $m_q/\bar{m}_\pi \lesssim 1$ and $m_q L \gtrsim 1$.

On pre-existing lattice configurations, and for equal computational cost, we obtain an equally precise uncertainty in extrapolated differences as compared to the traditional method. This cost comparison does not account for the significant overhead of generating the configurations in the first place. The results of our analysis pave the way for a more complete treatment of QED corrections using this approach. When considering more involved LQCD calculations, such as charged-particle scattering [30], our method provides a mass gap to produce a photon, thus increasing the range of energy for which the standard Liitscher method [48, 49] for obtaining the scattering phase shift can be employed. It will be interesting to explore these types of calculations, and also to use our method with chiral fermions, which do not suffer from additive quark mass renormalization. Finally, it would be interesting to see if our method of screened interactions coupled with analytic extrapolation techniques is of benefit to quantum many-body calculations.

We would like to thank W. Detmold, R. Edwards, B. Joó, D. Richards and K. Orginos for the use of the JLab/W&M QCD gauge field configurations and D. B. Kaplan, T. C. Luu, and M. J. Savage for useful conversations and correspondences. Additionally, we would like to thank A. Patella and N. Tantalo for stimulating discussions during the Lattice 2015 conference. We acknowledge the hospitality of the International Institute of Physics at the Federal University of Rio Grande de Norte and the Institute for Nuclear Theory at the University of Washington (Nuclear Reactions Workshop [50]), where portions of this work were completed. Computations for this study were carried out on facilities of the USQCD Collaboration, which are funded by the Office of Science of the U.S. Department of Energy. M. G. E was supported by the the U. S. Department of Energy Early Career Research Award DE-SC0010495, and monies from the Dean of Science Office at MIT. B. C. T. was supported in part by a joint City College of New York-RIKEN/Brookhaven Research Center fellowship, a grant from the Professional Staff Congress of the CUNY, and by the U.S. National Science Foundation, under Grant No. PHY15-15738. A. W-L. was supported in part by the U.S. Department of Energy (DOE) contract DE-AC05-06OR23177, under which Jefferson Science Associates, LLC, manages and operates the Jefferson Lab and by the U.S. DOE Early Career Award contract DE-SC0012180.
\[ \text{[10]} \text{ S. Basak et al. (MILC), PoS LATTICE2008, 127 (2008), arXiv:0812.4486 [hep-lat]} \]
\[ \text{[11]} \text{ A. Portelli et al. (Budapest-Marseille-Wuppertal), PoS LATTICE2010, 121 (2010), arXiv:1011.4189 [hep-lat]} \]

\[ \text{[20]} \text{ J. C. Collins, Nucl. Phys. B149, 90 (1979)} \]
\[ \text{[21]} \text{ J. Gasser and H. Leutwyler, Phys. Rept. 87, 77 (1982)} \]
\[ \text{[26]} \text{ Z. Davoudi and M. J. Savage, Phys. Rev. D90, 054503 (2014), arXiv:1402.6741 [hep-lat]} \]
\[ \text{[33]} \text{ B. Lucini, A. Patella, A. Ramos, and N. Tantalo, (2015), to appear in PoS LATTICE2015} \]
\[ \text{[34]} \text{ B. Lucini, A. Patella, A. Ramos, and N. Tantalo, JHEP 02, 076 (2016), arXiv:1509.01636 [hep-th]} \]
\[ \text{[35]} \text{ G. Makov and M. C. Payne, Phys. Rev. B 51, 4014 (1995)} \]
\[ \text{[36]} \text{ J. C. Collins, Renormalization (Cambridge Univ. Press, 1984)} \]
\[ \text{[37]} \text{ M. Luscher, Conf.Proc. C880628, 451 (1988)} \]
\[ \text{[40]} \text{ W. Caswell and G. Lepage, Phys. Lett. B167, 437 (1986)} \]
\[ \text{[41]} \text{ The local operator contributing at NLO is} \phi^+\phi\psi^+\psi, \text{where the scalar} \phi \text{picks up a vacuum expectation value, thereby Higgsing the photon. Whereas the operator} m_A^2 A_{\mu} A_{\nu} \psi^+\psi \text{contributes to masses at N^2LO, it is a prime example of a gauge non-invariant operator.} \]
\[ \text{[43]} \text{ S. Beane et al. (NPLQCD), Phys. Rev. D87, 034506 (2013), arXiv:1206.5219 [hep-lat]} \]
\[ \text{[44]} \text{ Note that this formulation of QED violates reflection positivity, leading to the ill-behavior of charged particle propagators for } T \to \infty \text{ at fixed } L \text{ [8]. Our QCD_{T=1} computation uses } T/L \sim 1, \text{ for which the FV effect is mild.} \]
\[ \text{[46]} \text{ An additional mistuning effect arises because of the } T \text{-dependence of quark masses in } QCD_{T=1}. \text{ Addressing this is required for practical applications, or could be eliminated by using the } QED_{T=1} \text{ formulation.} \]
\[ \text{[47]} \text{ M. Luscher, Commun. Math. Phys. 105, 153 (1986)} \]
\[ \text{[48]} \text{ M. Luscher, Nucl. Phys. B354, 531 (1991)} \]