Hydrography and Hydrodynamics of Virginia Estuaries II: Studies of the Distribution of Salinity and Dissolved Oxygen in the Upper York System

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HYDROGRAPHY AND HYDRODYNAMICS OF VIRGINIA ESTUARIES

II. Studies of the Distribution of Salinity and Dissolved Oxygen in the Upper York System

Special Report in Applied Marine Science and Ocean Engineering

NUMBER 13

VIRGINIA INSTITUTE OF MARINE SCIENCE
GLOUCESTER POINT, VIRGINIA 23062
II. STUDIES OF THE DISTRIBUTION OF SALINITY AND DISSOLVED OXYGEN
IN THE UPPER YORK SYSTEM

by

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and
W. J. Hargis, Jr.

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Dr. William J. Hargis, Jr.
Director

August, 1971
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Tables</td>
<td>iv</td>
</tr>
<tr>
<td>List of Figures</td>
<td>v</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>vii</td>
</tr>
<tr>
<td>Abstract</td>
<td>ix</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Description of the Study Area</td>
<td>4</td>
</tr>
<tr>
<td>Hydrographic Survey</td>
<td>6</td>
</tr>
<tr>
<td>Field Operation</td>
<td>6</td>
</tr>
<tr>
<td>Analysis of Experimental Data</td>
<td>12</td>
</tr>
<tr>
<td>Results</td>
<td>13</td>
</tr>
<tr>
<td>Mathematical Analysis</td>
<td>23</td>
</tr>
<tr>
<td>Basic Concepts</td>
<td>23</td>
</tr>
<tr>
<td>Computer Models</td>
<td>28</td>
</tr>
<tr>
<td>Evaluation of Parameters</td>
<td>31</td>
</tr>
<tr>
<td>Description of the Mathematical Model</td>
<td>54</td>
</tr>
<tr>
<td>Computational Method</td>
<td>54</td>
</tr>
<tr>
<td>Description of the Computer Program</td>
<td>56</td>
</tr>
<tr>
<td>Salinity Model Demonstration</td>
<td>59</td>
</tr>
<tr>
<td>Dissolved Oxygen Model Demonstration</td>
<td>67</td>
</tr>
<tr>
<td>Conclusions</td>
<td>76</td>
</tr>
<tr>
<td>Recommendations</td>
<td>78</td>
</tr>
<tr>
<td>List of References</td>
<td>79</td>
</tr>
<tr>
<td>Appendices</td>
<td></td>
</tr>
<tr>
<td>A. Results of Slack Water Runs</td>
<td>80</td>
</tr>
<tr>
<td>B. Profiles of the Cross Sections</td>
<td>88</td>
</tr>
<tr>
<td>C. Tidal Observations</td>
<td>102</td>
</tr>
<tr>
<td>D. Listing of Input Data and Output Data from Salinity and DO Verification for Time-dependent Model</td>
<td>107</td>
</tr>
<tr>
<td>E. Graphical Summary of Data Collected During Operation York River, October, 1969</td>
<td>132</td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Geometrical Data for the Station Locations</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>Local Inflow Drainage Area - Mattaponi and York</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>Local Inflow Drainage Area - Pamunkey</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>Discharge Record at Hanover, Virginia</td>
<td>34</td>
</tr>
<tr>
<td>5</td>
<td>Wetland Acreage</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>Values of Dispersion Coefficient for each Reach</td>
<td>45</td>
</tr>
<tr>
<td>7</td>
<td>Computed Tidal Flow for Stations P20 Through P15</td>
<td>47</td>
</tr>
<tr>
<td>8</td>
<td>Computed Tidal Flow for Stations P14 Through P09</td>
<td>48</td>
</tr>
<tr>
<td>9</td>
<td>Computed Tidal Flow for Stations P08 Through P03</td>
<td>49</td>
</tr>
<tr>
<td>10</td>
<td>Computed Tidal Flow for Stations P02, P01, Y01, M02, M01</td>
<td>50</td>
</tr>
<tr>
<td>11</td>
<td>Computed Tidal Flow for Stations M14 Through M10</td>
<td>51</td>
</tr>
<tr>
<td>12</td>
<td>Computed Tidal Flow for Stations M09 Through M04</td>
<td>52</td>
</tr>
<tr>
<td>13</td>
<td>Computed Tidal Flow for Station M03</td>
<td>53</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Map of the York, Pamunkey and Mattaponi river watersheds.</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Map of stations Mattaponi and Pamunkey showing stations occupied in Operation York River.</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>Schematic diagram of Pamunkey stations occupied in Operation York River.</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>Schematic diagram of Mattaponi stations occupied in Operation York River.</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>Longitudinal salinity distributions for Mattaponi.</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>Longitudinal salinity distributions for Pamunkey.</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>Distributions of maximum and minimum water temperatures for the Mattaponi.</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>Distributions of maximum and minimum water temperatures for the Pamunkey.</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>Longitudinal distributions of maximum and minimum dissolved oxygen concentration for the Mattaponi.</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>Longitudinal distributions of maximum and minimum dissolved oxygen concentration for the Pamunkey.</td>
<td>19</td>
</tr>
<tr>
<td>11</td>
<td>Longitudinal profiles of mean depth for Mattaponi.</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>Longitudinal profiles of mean depth for Pamunkey.</td>
<td>21</td>
</tr>
<tr>
<td>13a</td>
<td>Processes of advection and tidal mixing redistributing contaminant from a discharge.</td>
<td>25</td>
</tr>
<tr>
<td>13b</td>
<td>Concentration profile of contaminant.</td>
<td>25</td>
</tr>
<tr>
<td>14</td>
<td>Mass balance over volume element AΔX.</td>
<td>26</td>
</tr>
<tr>
<td>15</td>
<td>Isopleths of DO on a salinity versus temperature plot.</td>
<td>42</td>
</tr>
<tr>
<td>16</td>
<td>Flow diagram for organization of subroutines in time-dependent model.</td>
<td>60</td>
</tr>
<tr>
<td>17</td>
<td>Salinity verification - calculated and observed Pamunkey salinity distributions for August 14, 1970.</td>
<td>63</td>
</tr>
<tr>
<td>18</td>
<td>Salinity verification - calculated and observed Pamunkey salinity distributions for September 29, 1970.</td>
<td>64</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>19</td>
<td>Salinity verification - salinity observations and calculated location of $5%$ isohaline in the Pamunkey</td>
<td>65</td>
</tr>
<tr>
<td>20</td>
<td>Salinity verification - observed and calculated salinity versus time at Chelsea Farm</td>
<td>66</td>
</tr>
<tr>
<td>21</td>
<td>DO verification - Pamunkey River high-water slack observed and calculated results</td>
<td>69</td>
</tr>
<tr>
<td>22</td>
<td>DO verification - Mattaponi River high-water slack observed and calculated results</td>
<td>70</td>
</tr>
<tr>
<td>23</td>
<td>DO verification - Pamunkey River low-water slack observed and calculated results</td>
<td>71</td>
</tr>
<tr>
<td>24</td>
<td>DO verification - Mattaponi River low-water slack observed and calculated results</td>
<td>72</td>
</tr>
<tr>
<td>25</td>
<td>DO verification - observations at P01 and P02 and calculated results at reach 18 versus time</td>
<td>73</td>
</tr>
<tr>
<td>26</td>
<td>DO verification - observations at M01 and M02 and calculated results at reach 32 versus time</td>
<td>74</td>
</tr>
<tr>
<td>27</td>
<td>DO verification - observations at P03 and P04 and calculated results at reach 16 versus time</td>
<td>75</td>
</tr>
</tbody>
</table>
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ABSTRACT

Mathematical models have been developed for predicting salinity and dissolved oxygen distributions in the upper York River system, including the tidal portions of the Mattaponi and Pamunkey. The study area and field project are described. Summaries are given of the field data and the method of evaluation of parameters.

Two programs are described: a time-dependent dissolved oxygen model and a time-dependent salinity model. Basic equations and program descriptions are given and verification results are presented.
INTRODUCTION

Marine scientists and coastal engineers have sought mathematical models which properly predict changes in the concentration and distribution of conservative and non-conservative substances in estuaries. The substances most frequently investigated are salinity and dissolved oxygen because they are of prime importance to estuarine organisms. This study deals with concentration changes of salinity and dissolved oxygen resulting from three main factors: changes in stream discharge, fluctuations in the volume rate of flow and the addition of oxygen-consuming organic matter along the course of an estuary. The first plan for a dissolved oxygen model was published by Streeter and Phelps in 1925. The basic equation expresses the rate of change of dissolved oxygen and includes terms for advection, deoxygenation and re-aeration. Camp (1963) has modified the Streeter-Phelps equation and introduced other parameters such as photosynthesis, bottom deposits, dispersion coefficients and mean tidal velocity to develop the most popular steady state oxygen sag equations. The digital computer proved to be a valuable aid in the solution of Camp's equations (Harrison & Fang, 1970).

Thomann (1963) put the unsteady state diffusion equation in a finite-difference form, i.e., in a form dealing with the average concentrations in a finite number of segments. The equation in this form can be programmed in a straightforward manner for solution by a digital computer. The computer approach has been applied to several river systems, e.g. the Delaware (Jeglic, 1967) and Potomac (Hetling, 1969).

There has been some recent work concerning the hydrographic characteristics of the upper York system. The Chesapeake Bay Institute of The Johns

The present study develops computer models for the prediction of dissolved oxygen concentration in the Mattaponi, Pamunkey and upper York rivers. Two models are presented, one for oxygen, the other for salinity; both are based on the finite difference method of volume integration between reaches suggested by Thomann (1963). The work was done in cooperation with the Water Control Board and the Division of Water Resources, Department of Conservation and Development of the Commonwealth of Virginia. These models will serve as a guide for planning and management of the river system.
Figure 1. Map of the York, Pamunkey and Mattaponi river watersheds.
DESCRIPTION OF THE STUDY AREA

The upper estuary of the York River includes the Pamunkey and Mattaponi rivers (Fig. 1). The streams of the York River basin flow in a southeasterly direction from their source in the foothills of the Blue Ridge Mountains in Virginia to the Chesapeake Bay near Yorktown, Virginia. The York River is formed by the confluence of the Mattaponi and Pamunkey rivers at West Point and is an estuary for its entire course (about 35 miles*). The Pamunkey and Mattaponi channels are meandering and are tidal for 56 and 41 miles, respectively. The tidal distances, however, are subject to variation due to changes in the fluvial flow.

The mean range of tide at the mouth of the York River is two feet, increasing to 2.9 feet at West Point. At Walkerton on the Mattaponi, 29 miles upstream from West Point, the mean tidal range is 3.9 feet and at New Castle on the Pamunkey, 52 miles upstream from West Point, it is two feet.

The drainage basin comprises 2663 square miles, of which 909 and 1477 square miles belong to the Mattaponi and Pamunkey, respectively. Bottom slopes are about 10 feet per mile immediately upstream from the fall line and reduce to about 1.5 feet per mile in the coastal plain below the fall line.

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*Miles referred to herein are statute miles. When the notation "mile number" is used to designate a particular feature, the distances are statute miles upstream from the mouth of the York River. For this paper, both the Lord Delaware Bridge over the Pamunkey and the Eltham Bridge over the Mattaponi at West Point were taken to be at mile number 35.
The area is characterized by mild winters and hot summers, with an average precipitation of 41 inches per year. Snowfall averages 14 inches per year. Summer is the wettest season, while autumn is the driest (U.S. ESSA, 1958-1967).
HYDROGRAPHIC SURVEY

Field Operation

In order to construct correct mathematical models for predicting changes in the spatial and temporal distribution of salinity and dissolved oxygen in the Mattaponi and Pamunkey rivers, field data were needed to provide values for the various coefficients and parameters used in modeling as well as for verification purposes. Required information consisted of the following: basin geometry, fresh water discharge, mean cross-sectional velocities within successive reaches, mean discharge through cross sections for at least one complete tidal cycle, tidally-induced fluctuations in mean salinity and dissolved oxygen at each cross section, water level fluctuations, spatial distributions of salinity and dissolved oxygen for various flow conditions, and longitudinal changes in biochemical oxygen demand (BOD) for various flow conditions.

To satisfy these requirements, two types of field survey were planned and executed. The first survey measured temperature, salinity, dissolved oxygen, currents and water levels in the tidal portions of the Mattaponi and Pamunkey rivers. The surveyed area extended from two miles below West Point in the York River to 38 and 56 miles above West Point in the Mattaponi and Pamunkey rivers, respectively. Figure 2 is a map of the stations occupied during this field operation. Figures 3 and 4 show the stations schematically, together with landmarks. This area was surveyed in as short a time as could be achieved with the available equipment and personnel. Hence the measurements were as close to simultaneous as was practicable. A total of 37 transects were occupied during the survey. Each transect had between one and four stations, depending on river width. Distances between transects averaged three miles with closer spacing in the vicinity of West
Figure 2. Map of stations Mattaponi and Pamunkey showing stations occupied in Operation York River.
Figure 3. Schematic diagram of Pamunkey stations occupied in Operation York River.
Figure 4. Schematic diagram of Mattaponi stations occupied in Operation York River.
Point where the dissolved oxygen was expected to be lowest. Transects located near sharp bends were positioned at least four river widths from the bend to insure representative measurements of currents for the reach.

Salinity samples and temperature-velocity measurements were obtained at hourly intervals for twenty-five consecutive hours at each station. Sampling depths were at six-foot increments from surface to bottom (inclusive). Water samples for dissolved oxygen analysis were taken at hourly intervals, six feet below the surface at the central station on each transect except in the vicinity of West Point where DO sampling coincided with the temperature, salinity-velocity measuring scheme.

Water samples for salinity and dissolved oxygen analysis were obtained with Frautschy bottles, placed in 4-oz. sample bottles and subjected to laboratory analysis. Dissolved oxygen was analyzed using the Azide modification of the Winkler method (reagents up to and including H₂SO₄ were added in the field), while salinities were determined with a laboratory model inductance salinometer (Beckman Model RS7-A). Velocities were measured with Savonius rotor type current meters (Hydro Products Model 400 direction sensor and 460 current speed sensor and Marine Advisors Model B5 direction sensor and B1 speed sensor). Temperatures were sensed with thermistors (Applied Research Austin Model ET-100) attached to the current meter housings. Values for current speed and direction as well as water temperature were read from deck readout meters. Water samplers were lowered on a hand line and activated by a messenger. The current meter-thermistor combination was weighted with a 50-lb lead weight and lowered by a hand-cranked winch. All sampling was accomplished by two-man crews in small (17-22 ft) outboard boats.
Limitations of equipment and personnel precluded sampling at more than six transects at a time. (One boat and crew sampled all stations on a given transect once each hour). The entire survey was completed during a ten-day period in October 1969. A total of seven 25-hour sampling sessions were required, starting upstream in the Pamunkey and working towards West Point, and repeating the procedure in the Mattaponi. Stations in the vicinity of West Point were occupied twice.

Water level fluctuations (tides) were measured at West Point, Elsing Green and Newcastle Bridge on the Pamunkey and at Aylett on the Mattaponi. The West Point and Elsing Green facilities were established and operated by the Division of Water Resources. The other gauges were of a temporary type. However all gauges were surveyed for vertical control.

Bathymetry of each transect was obtained with a recording sonic depth sounder (Raytheon Model DE-719).

The second type of survey, a slack water survey, was begin in August 1970 and is continuing. Temperature is measured, and salinity, dissolved oxygen and BOD samples are taken monthly at local slack water before flood tide (low slack) or slack water before ebb tide (high slack) at a series of stations up the Mattaponi and Pamunkey rivers. If weather permits both slack waters are sampled. Stations generally coincide with those of the October 1969 survey, with sampling starting at West Point and proceeding up each river to follow the progressive wave of the tides. One station on each transect is sampled with measurements made six feet below the surface and six feet above the bottom.
Analysis of Experimental Data

**Data processing.** - Data collected in the field and the results of laboratory analyses were permanently recorded on a magnetic disk. This medium greatly speeded and facilitated the editing and processing of the data.

**Data reduction.** - From the data stored on the disk, various calculations were made. Section averages of the salinity, dissolved oxygen and temperature were calculated to provide input values for the models. Vertical integrals of the longitudinal component of velocity were calculated. These were used to compute tidal exchange fluxes.

Channel widths were determined from U. S. Geological Survey 7.5-minute quadrangles. Cross-sectional areas were determined by planimetry of the bottom profile data collected by VIMS personnel in January 1970. Section lengths were determined from C&GS navigation charts. The volume of a section was taken to be the mean of the end cross-sectional areas times the section length.

Tidal exchange fluxes were calculated from the vertical integrals of the longitudinal components of velocity. These were averaged over a cross-section and multiplied by the mean areas as determined from the bottom profile measurements. This approach is a simplification of Harlacher's method (Troskolsanski, 1967).

Measurements from the strip-chart records of the temporary tide gauges were corrected for the elevation of the staff with respect to sea level (1929 datum), corrected for variations in the paper feed rate and replotted.
Results

Figures 5 and 6 show the longitudinal salinity distributions for the Mattaponi and Pamunkey, respectively. Results are shown for high-water slack and low-water slack. Figures 7 and 8 show the distribution of maximum and minimum water temperatures at the time of the study for the Mattaponi and Pamunkey, respectively. Figures 9 and 10 show the longitudinal distribution of maximum and minimum dissolved oxygen concentration for the Mattaponi and Pamunkey, respectively.

Appendix B shows the profiles of the cross sections, with local mean low water as the datum (U.S.C. & G.S., 1971). Figures 11 and 12 show the longitudinal profiles of hydraulic depth for the Mattaponi and Pamunkey, respectively. Table 1 summarizes the geometric data for the system, showing the cross-sectional areas, widths and hydraulic depths at mean tidal height (U.S.C. & G.S., 1971).

The results of the tidal observations are shown in the figures in Appendix C. For the permanent tide gauges, copies of the actual tide records are shown. For the temporary tide gauges, the plots shown were made after correction for sea level and variations in the rate of the clock-work mechanism. The heights shown are referred to mean sea level (1929 Datum Plain).

Cross-sectional averages of the field measurements are displayed in graphical form in Appendix E.
Figure 5. Longitudinal salinity distributions for Mattaponi.
Figure 6. Longitudinal salinity distributions for Pamunkey.
Figure 7. Distributions of maximum and minimum water temperatures for the Mattaponi.
Figure 8. Distributions of maximum and minimum water temperatures for the Pamunkey.
Figure 9. Longitudinal distributions of maximum and minimum dissolved oxygen concentration for the Mattaponi.
Figure 10. Longitudinal distributions of maximum and minimum dissolved oxygen concentration for the Pamunkey.
Figure 11. Longitudinal profiles of mean depth for Mattaponi.
Figure 12. Longitudinal profiles of mean depth for Pamunkey.
<table>
<thead>
<tr>
<th>Transect</th>
<th>Area (ft²)</th>
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</tr>
</thead>
<tbody>
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MATHEMATICAL ANALYSIS

Basic Concepts

Hetling (1969) uses a checking account as an elementary example of a mathematical model. The numbers added to or subtracted from the total are not money, but they represent money, and (with careful accounting) they tell the true state of a real situation. Similarly, a mathematical model can be constructed to analyze the process of self-purification of a stream.

A river is not only a physical-chemical system but is also a vital, living community containing many forms of life in vast numbers. When organic material is deposited into a body of water, the river community tends to maintain itself in dynamic equilibrium, making natural and rapid adjustments to changes in the food supply, the availability of sufficient oxygen, and the rate of dissipation of its waste products. Bacteria begin to attack and alter the material; during this alteration dissolved oxygen is consumed. Often, this results in a noticeable decrease in the dissolved oxygen content in a stream below a source of waste, and is followed by an increasing oxygen concentration still farther downstream. This organic matter serves as a sink for dissolved oxygen. The losses incurred through such a sink are generally counteracted by sources of oxygen; the sources of oxygen in a stream are photosynthesis and re-aeration, i.e. the entrapment of oxygen from the overlying air. The systems tends to be self-purifying in the sense that the oxygen used up in consuming the organic matter is eventually replaced from the atmosphere.

Dissolved oxygen and organic matter tend to redistribute themselves spatially. In a river, the longitudinal distribution of oxygen and organic matter is much more variable than the lateral and vertical distributions.
Two mechanisms are at work in distributing contaminants and dissolved oxygen: advection and dispersion. Advection is the process whereby a contaminant is carried along by the stream; Figure 13a illustrates this phenomenon. Alternating ebb and flood flow past a contaminant discharge tend to dilute the contaminant without removing it entirely while freshwater discharge tends to carry it away.

Dispersion is the process whereby a contaminant is transferred by mixing. Turbulence causes a mass of water with a high concentration of contaminant to mix with another water mass of lower concentration. In this way the contaminant itself is spread out.

Going downstream from an outfall, advection and dispersion act in concert, both tending to carry the contaminant away from its source. Going upstream, however, they tend to oppose one another, advection tending to move the contaminant downstream, and dispersion tending to force it upstream. The result is as shown in Figure 13b; the concentration profile falls off gradually going downstream, but drops off quickly going upstream from a source.

The fate of a contaminant in a short reach $\Delta x$ is shown in Figure 14. Advection and dispersion tend to carry the contaminant away horizontally. At the same time, it is decaying within the reach. There may, in addition, be a source of pollutant. The picture is much the same for dissolved oxygen: advection, dispersion, decay and a source (re-aeration).

The foregoing physical basis for analysis leads to a partial differential equation containing terms representing advection, dispersion, decay, re-aeration and sources. This equation (a refinement of the continuity equation) forms the mathematical basis for analysis.
Figure 13a. Processes of advection and tidal mixing redistributing contaminant from a discharge.

Figure 13b. Concentration profile of contaminant.
Figure 14. Mass balance over volume element $A\Delta x$. 
Salinity modeling proceeds in the same way, except that there are no source of sink terms, since salt is a conservative substance.

The formula expressing the processes described above is called the mass transfer equation. This general equation for the conservation of mass of a substance in a turbulent fluid may be written in the following form:

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = \frac{\partial}{\partial x} \left( E_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( E_y \frac{\partial C}{\partial y} \right) \\
+ \frac{\partial}{\partial z} \left( E_z \frac{\partial C}{\partial z} \right) + S_0 - S_1
\]

Where \( C \) equals the concentration of the diffusing substance.

\( x, y, z \) = coordinate directions \( x \) (length), \( y \) (width), \( z \) (depth).

\( u, v, w \) = velocity components corresponding to the \( x, y \) and \( z \) directions.

\( E_x, E_y, E_z \) = turbulent diffusion coefficients corresponding to the coordinate directions.

\( S_0 \) = source; net nonadvective rate of addition of the substance across the boundaries of the system.

\( S_1 \) = sink; the net rate of removal of substance from the system.

In dealing with the distribution of a contaminant, it is frequently convenient to use the one-dimensional approximation in which the instantaneous concentration is considered to be uniform over each cross section, and all spatial variations are in the longitudinal direction of the estuary.

The one-dimensional form of equation (1) is obtained by ignoring \( \frac{\partial C}{\partial y} \) and \( \frac{\partial C}{\partial z} \) in comparison with \( \frac{\partial C}{\partial x} \). The resulting equation is:

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = \frac{\partial}{\partial x} \left( E_x \frac{\partial C}{\partial x} \right) + S_0 - S_1
\]
Methods of solution of this equation are based upon adding the re-aeration, photosynthesis and other sources of oxygen, and subtracting the biochemical oxygen demand, sludge deposits and other uses of oxygen with respect to time. Once the stream parameters are known for existing conditions, and the mathematical model is solved, certain parameters can be altered to reflect a new set of conditions, such as increased waste loads or the installation of sewage treatment plants.

Computer Models

Specific form must be given to the generalized source and sink terms in equation (2) in order to obtain equations useful for DO analysis. The following assumptions are therefore made:

1. the rate of atmospheric re-aeration is proportional to the difference between the dissolved oxygen concentration and the saturation concentration;
2. the rate of decay of organic material is proportional to the concentration of organic material;
3. the rate of settling out of organic matter is proportional to the concentration of organic matter present;
4. bottom deposits may exert an immediate demand on the oxygen in the overlying water.

In order to solve them using a digital computer, these equations must be put into a finite-difference form so that only a finite number of reaches need be considered. One such reach is shown in Figure 14. Because of advective and dispersive exchange across the ends of a reach, the concentration in one reach will depend on the concentrations in the two adjacent reaches. Since fixed boundary conditions are used, there result 2N equations
for N reaches. For reach i, the pair of equations for dissolved oxygen and BOD is:

\[
\frac{\partial C_i}{\partial t} = - k_{2i} (C_{si} - C_i) + k_{1i} L_i - S_i V_i^{-1} \\
- \frac{2A_{i+1}E_{i+1} (C_{i+1} - C_i) V_i^{-1}}{D_i} + \frac{Q_i}{V_i} (C_{i-1} - C_i) (1 - \phi_i) \quad \text{(3)}
\]

\[
+ \frac{2A_{i+1}E_{i+1} (C_{i+1} - C_i) V_i^{-1}}{D_{i+1}} - \frac{Q_{i+1}}{V_i} (C_i - C_{i+1}) \phi_i, \quad \text{and}
\]

\[
\frac{\partial L_i}{\partial t} + k_{1i} L_i = \frac{J_i}{V_i} \quad \text{(4)}
\]

In these equations, \(A_i\) is the area at the upstream boundary of the ith reach, \(E_i\) is the dispersion coefficient across the boundary, and \(Q_i\) is the flow rate of water across the boundary. The parameter \(C_{si}\) is the saturation concentration of oxygen in the ith reach. The decay coefficient is \(k_{1i}\), and \(k_{2i}\) is the re-aeration coefficient. The volume of the ith reach is \(V_i\), and \(D_i\) is its length. The source term for dissolved oxygen is \(S_i\), while \(J_i\) is the source term for BOD. The form of the interpolation factor \(\phi_i\) is

\[
\phi_i = 0.5 \left( 1 - \frac{1}{3\pi} \tan^{-1} \frac{Q_i}{20E_i A_i} (D_i + D_{i+1}) \right)
\]
The equation used for salinity is similar to the equations above except for different advective terms as follows:

\[
\frac{\partial S_i}{\partial t} \sim \frac{q_i}{v_i} \left[ S_{i-1} (1 - \varphi_i) + S_i \varphi_i \right] + \frac{q_{i+1}}{v_i} \left[ S_i (1 - \varphi_{i+1}) + S_{i+1} \varphi_i \right]
\]  

(5)

The salinity equation has no decay term but can have a term representing loading. The interpolation factor \( \varphi_i \) in the salinity equation is calculated in such a way as to prevent numerical instability in the case of predominance of advection over dispersion (Pence, Jeglic and Thomann, 1968).
EVALUATION OF PARAMETERS

Stream flow.- The biological oxidation of organic matter is a time-dependent process. The oxygen demands exerted by such matter occur at a distance downstream from the point of entry, this distance depending on the rate at which stream flow sweeps away the organic material. Experience has shown that several days are required before changes in river flow are reflected by changes in the biological characteristics of a river.

The flow measurements used in making the necessary analysis were obtained from U. S. Geological Survey for the gauging stations at Hanover, Virginia and Beulahville, Virginia. The Hanover gauging station on the Pamunkey River is located two miles east of Hanover at the bridge on State Highway 614. The drainage area above Hanover is 1072 square miles. The Beulahville station on the Mattaponi River is located 0.4 miles upstream from the bridge on State Highway 628, 2.8 miles north of Beulahville. The drainage area above the gauging station near Beulahville is 619 square miles. The drainage areas and distances between transects are presented in Tables 2 and 3. The Hanover gauging station provisional flow record is shown in Table 4.

Biochemical oxygen demand (BOD).- The concentration of biodegradable organic matter present in waste water in a polluted stream is conveniently measured by the BOD test. The standard test is the five-day 20 C BOD test in which the amount of oxygen required for the decomposition of the organic matter during a five-day period at 20 C, is determined. The limiting pollutional load on a stream may be expressed in terms of the pounds of BOD which may safely be disposed of in the stream each day. Samples taken directly from natural waters usually contain sufficient bacteria to carry on the oxidation, and seeding is not required. Likewise buffering and
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<th>Distance (Miles)</th>
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<th>Drainage area (sq.mi.)</th>
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Table 4. Discharge Record at Hanover, Va.

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SUM = 12323  
AVG = 410.8  
MAX = 910  
MIN = 269  

SUM = 8159  
AVG = 263.2  
MAX = 447  
MIN = 205
dilution are not required in cases of low BOD. In addition, estuarine water is self-buffered by its carbonate concentration.

The oxidation process is generally carried out in two stages: carbonaceous and nitrogenous. The first stage is accomplished by saprophytic organisms, those which derive their energy from the breakdown of organic substances. The second stage comes about by the oxidation of the nitrogenous portion of pollutants in the waste water. This aspect of biological degradation is accomplished by a group of bacteria referred to as nitrifiers. These have a slower reproductive rate than saprophytic organisms, so their effects are not noted in the 5-day test. Each stage is characterized by two steps: cell synthesis and respiration. In the carbonaceous stage, the energy required for synthesis is obtained from the transformation of the complex organic compounds into carbon dioxide and water. After the organic matter has been converted to bacterial cells, the endogenous respiration of the organisms occurs, also yielding carbon dioxide and water, and usually ammonia. If insufficient amounts of the nutrients nitrogen and phosphorous are present, they must be added initially in order that the process may proceed at a reasonable rate. In laboratory long-term BOD battle studies of raw sewage or industrial wastes, the nitrogenuous BOD becomes appreciable only after an incubation period of about 10 days. In the stream, the two stages frequently proceed simultaneously, although there may be lags in the nitrification stage in highly polluted streams or those with low dissolved oxygen.

There is no large population center or livestock industry on the York River. A stream such as the York River has a low population of nitrifiers, even with a relatively long history of introduction of paper
mill waste water. Hence the effect of the second stage BOD is insignificant.

The usual practice for measuring BOD is to store the sample at a fixed temperature of 20 °C for a period of five days and then to determine the drop in dissolved oxygen that has occurred. The following equation can be used for computing the ultimate BOD at 20 °C or other temperatures given the 5-day, 20 °C BOD:

\[
L = \frac{y (1.02)^{T-20}}{1 - 10^{-0.5}} + B_N^{0.5}
\]

where

- \( y \) = drop in DO occurring in incubated sample (mg/liter)
- \( T \) = temperature (°C)
- \( B_N \) = nitrogenous BOD

The first stage deoxygenation coefficient \( k_1 \) and \( k_3 \). The BOD is a measure of the concentration of decomposable organic matter in the polluted water. In the examination of stream water quality, the BOD concentration involves not only the amount of organic material which is decomposable by bacteria but also the time rate at which it will decompose aerobically. The BOD test may be used as a laboratory model of the deoxygenation process in the receiving waters. If laboratory results are to be applied to actual situations, estimates must be in keeping with: (1) the nature of the stream channel, flow, and flow variations; (2) the progress in time of the waste matter and the stream water itself; and (3) the transfer of pollutional load to the stream bottom, where it undergoes benthal decomposition.

The rate at which oxygen is utilized in the biochemical oxidation of organic material has been found to be proportional to the amount of organic matter present. This relationship is expressed mathematically as

\[
\frac{dL}{dt} = -k_1L
\]
where \( L \) = the ultimate oxygen demand

\[ k_1 = \text{rate constant (day}^{-1}\text{)} \]

\[ t = \text{time in days} \]

The sewage effluents and industrial wastes in water courses may cause the accumulation of sludge over relatively long periods of time. Settleable solids are deposited on the stream bottom by sedimentation. This is particularly true in estuaries, since wastes discharged into a salty water may undergo chemical coagulation. This causes the finely divided colloidal particles to aggregate and subsequently to settle out as bottom sediments. Moreover, due to bacterial activities, part of the contributed BOD is removed by biological flocculation from the flowing water in the stream. In the mathematical equations the rate of loss of BOD from the stream due to settling is given by

\[
\frac{dL}{dt} = -k_3L
\]

where \( k_3 \) = rate constant for settling of BOD (day\(^{-1}\)).

Re-aeration coefficient \( k_2 \).- There is a continuous interchange of oxygen at the air-water interface. When the concentration of dissolved oxygen in the water falls below the saturation concentration, there will be a net mass transfer of oxygen from the atmosphere to the water, referred to as re-aeration. The difference between the saturation concentration and actual concentration of dissolved oxygen in the water is called the oxygen deficit. The rate of oxygen transfer depends on the properties of the air-water interface, the gradient of the deficit, the intensity of turbulence of the stream flow, and temperature. The rate at which the oxygen content of the stream is replenished from the atmosphere may be estimated. The rate constant for replenishment is called the \( k_2 \) value. The \( k_2 \) values cannot be determined in the laboratory but must be computed from the results
of analyses of the water in the stream at a number of different stations. The computation of \( k_2 \) values is a much less precise procedure because of the complexity of the problem and due to the uncontrollable factors encountered in a natural stream. Expressed mathematically

\[
\frac{dD}{dt} = -k_2 D
\]

where \( D = \) dissolved oxygen deficit (mg/liter)

\( k_2 = \) re-aeration coefficient, days\(^{-1}\)

\( t = \) time, days

The re-aeration rate \( k_2 \) can be determined using the O'Connor-Dobbins formula:

\[
k_2 = \frac{24(D_C U)^{1/2}}{H^{3/2}}
\]

where:

\( D_C = \) molecular diffusivity of oxygen in water

\( 0.000081 \text{ ft}^2/\text{hr} @ 20 \text{ C} \)

\( H = \) water mean depth (feet)

\( U = \) mean tidal velocity (feet per hour)

For different temperatures it can be corrected by the following equation:

\[
k_T = k_{20} (\theta)^{T-20}, \text{ where}
\]

\( \theta = 1.015 \text{ to } 1.047 \)

**Estimation of benthal oxygen demand.** - There is not enough dissolved oxygen to maintain aerobic conditions in the bottom sediments which are relatively fine in texture but have a very high oxygen demand. Some of the anaerobic decomposition products diffuse upward through the pore water of the sediments to the overlying water and exert BOD in the flowing water. Therefore, the oxygen demand of the bottom sediments upon the overlying water becomes a very important part of the oxygen-balance studies.
The oxygen demand of the benthal deposits in this report are designated P and have the dimension mg/liter/day.

One method of estimating the benthal oxygen demand is based on measuring the volatile content of river bottom sludge samples and computing the oxygen demand. Another method consists of placing samples of bottom deposits in bottles and measuring the oxygen uptake over various time intervals.

Values of P in this report were assigned to those reaches of the river where the profile indicated settling might occur. The values used were selected by the best judgment of the design engineer. The benthal demand in such unpredictable factors would require that the data be constantly updated to reflect the changing river conditions.

**Photosynthesis.** - There are 15,000 acres of marsh or swamp draining into the estuary of the Pamunkey River and 8,238 acres into the Mattaponi River (see Table 5). The marsh vegetation, algae and aquatic plants in a stream share the common characteristic of all chlorophyll-containing plants, of being able to convert carbon dioxide into organic plant substances and at the same time release molecular oxygen to the environment. The amount of oxygen produced during photosynthesis varies with the intensity of sunlight and the density of the plant population. In addition, the same plants require oxygen for respiration at night and on cloudy days. In the stream pollution studies, careful consideration should be given as to the proper role played by plant photosynthesis in the overall oxygen balance of the stream. For use in the present method the overall net production of DO by photosynthesis is expressed in the mathematical equations as $\alpha$ and has the dimensions of mg/liter/day. The values of $\alpha$ can be derived from light and dark bottle tests. Since we did not test $\alpha$ values during the course of the
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sanitary survey, we are using values based on our best judgment from the 1969 hydrographic survey.

A number of other formulas have been suggested for estimating the rate constant for atmospheric re-aeration. They are useful mainly for relatively clean waters.

**Oxygen saturation.** - $C_s$ is the maximum quantity of oxygen which can dissolve in equilibrium with the partial pressure of oxygen in the atmosphere. Partial pressure is determined by the volume percentage of oxygen in the atmosphere in contact with water (approximately 21%) and depends on the temperature, barometric pressure and degree of water vapor saturation. Furthermore, the saturation value of DO is affected by the chemical characteristics of the water, such as the concentration of dissolved salts. $C_s$ decreases with an increase in temperature, water vapor pressure and concentration of solids and increases with barometric pressure. As salinity increases the solubility of oxygen decreases, being only about 82% as great for sea water as for fresh water (Camp, 1963). In oxygen balance studies of tidal estuaries, the solubility of oxygen should be adjusted for the average salt content in each reach. One of the reasons the stream quality becomes critical in warmer months is that less DO is available.

For this study an empirical equation was derived based on temperature and salt content (D. E. Carritt and E. J. Green, 1967).

$$C_{SAT} = 14.6244 - 0.367134 T + 0.0044972 T^2 - 0.0966 S + 0.00205 ST + 0.0002739 S^2$$

where $T = \text{temp. in } \degree C$

$S = \text{salinity in } \%_o (\text{parts per thousand})$

$C_{SAT} = \text{saturation concentration of DO in mg/liter.}$
Figure 15. Isopleths of DO on a salinity versus temperature plot.
Figure 15 shows isopleths of DO in mg/liter as a function of salinity and temperature.

**Temperature.** - In the relationship of BOD stabilization and DO concentration, temperature plays an important role. An increase in temperature has two effects: (1) the organic material is stabilized at a faster rate and, therefore, the DO is utilized at a higher rate and (2) the saturation value for dissolved oxygen is reduced, thereby decreasing the amount of oxygen that a stream can dissolve. Low stream flows reduce the waste removal rate and increase the waste concentration. For the Pamunkey River, low flow and high temperature both occur at the end of summer. From the viewpoint of stream pollution abatement, this represents the worst combination of these two significant factors. Measurements of water temperature during the summer months indicate that the average water temperature in July and August is about 25 C with occasional peaks as high as 28 C.

**Temperature correction for k₁ and k₂.** - The values of k₁ and k₂ in the input data are assumed to be for 20 C. The values of k₁ and k₂ at temperature T, used in the model, can be computed by following equations respectively:

\[
(k₁)_T = (k₁)₂₀ \cdot e^{0.095 (T-20)} \quad (13)
\]

\[
(k₂)_T = (k₂)₂₀ \cdot (1.038(T-20)) \quad (14)
\]

**Dispersion.** - The dispersion coefficient (or longitudinal mixing coefficient), E, for a sectionally homogeneous estuary may be evaluated for a particular stream flow from the average salinity, provided the velocity, U, is not negligible and the salt concentration at the upstream station is larger than one percent of the normal value at the lower end of
the estuary. This approach is based on the continuity equation applied to salinity (Harleman, Lee and Hall, 1967). For this evaluation

\[ S_a = S_b - 0.434 \frac{Ux}{E} \]  

where \( S_b \) = the average salinity concentration in \( \% \) at the downstream point and \( x \) = the distance upstream to a station where the concentration is \( S_a \). The \( S_a \) and \( S_b \) values were taken from the high-water distribution of salinity observed during the October, 1969, hydrographic survey. This equation is differentiated with respect to distance to obtain an explicit expression for dispersion coefficient. The result was used for the saline portion of the streams.

When the salinity in a reach of an estuary is less than one percent of the downstream boundary condition of the estuary, this method cannot be used because the mixing mechanism is quite different (Harleman, Lee and Hall, 1967). Instead the Modified Taylor's Equation is used to determine the turbulent diffusion coefficient:

\[ E = 77 (n) \frac{Q}{A} (R)^{5/6} \]  

where \( n \) = Manning's roughness coefficient, \( Q \) = average flow over several tidal cycles, \( A \) = the average cross-sectional area, and \( R \) = the average hydraulic radius in the reach. The values used in this report are presented in Table 6.

For the short-term prediction of dissolved oxygen and BOD, it is inappropriate to use the dispersion coefficients calculated for use in the long-term salinity prediction model. The reason for this conclusion is that tidal flushing and shear flow turbulence cause the longitudinal dispersion observed over a long period, while for time scales smaller than a tidal period, tidal velocity is included in the advection term and shear
Table 6. Calculated Dispersion Coefficients

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flow turbulence along is acting as a dispersion mechanism. It has been found in operating the dissolved oxygen model that reducing the dispersion coefficients to about five percent of the values used in the salinity model gives adequate representation of the dissolved oxygen sag regions. However, computer results obtained using the higher dispersion coefficients tended to diminish the difference in concentration between one reach and the next, hence, the region near the sag was not adequately modeled.

**Tidal exchange.**—The velocity measurements made during the field survey were used to compute the tidal exchange volume flux for the time-dependent model. For each station for each hour, the vertical integral of the streamwise component of velocity was computed. These were averaged over the stations or a transect to obtain an average for the transect. These results were multiplied by the cross-sectional areas computed from the results of the bathymetry survey to obtain volume flow rates through each cross section for each hour. This technique is a form of Harlacher's method (Troskolanski, 1960). The results of these computations are shown in Tables 7 through 13. According to the sign convection used, flood tides are positive and ebb tides negative.
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Table 12. Computed Tidal Flows

(ft$^3$/sec$^{-1} \times 10^4$)

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Table 13. Computed Tidal Flows

(ft$^3$/sec$^{-1}$ x $10^4$)

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DESCRIPTION OF THE MATHEMATICAL MODEL

Computational Method

The time-dependent equations are solved by integration using an explicit scheme. Using the distribution of dissolved oxygen and BOD plus all the geometric, hydraulic and waste-load information at a given time as initial conditions, the equations are integrated numerically over a given time interval. The final answers are printed out and a new time interval is established. The geometric, hydraulic or load conditions may be changed at the beginning of any interval. After the program finishes integrating over the final time interval specified, it reinitializes to begin a new run.

For computational purposes, the upper York system has been divided into a number of reaches. Computations are performed using sectional average values of the system parameters and dependent variables. Some parameters are defined at the center of a reach, others are defined between reaches.

The numbering system used for the salinity model is shown below. Reach volume and inter-reach area are given as examples of each of the two types of variables. Inter-reach variables are needed for the "fictitious" sections up-and downstream of the system modeled. The number 151 is arbitrarily assigned to the extra numbers needed for the upstream end of the second branch.
Below is the numbering system used for the sectioning of the oxygen model. The two reaches farthest upstream have been eliminated, in order to apply actual measurements as boundary conditions. The reach of the York farthest upstream has been broken into two halves, one associated with each tributary.
Description of the Computer Program

The program has been modified from DECS-III (Jeglic, 1967). Additional information concerning input and output can be found in that report.

"Namelist" inputs.- For reading data into the program, the "Fortran namelist" system is used. The first column of each card must be left blank. The beginning of a namelist, say CONTRL, would be announced by a card punched with "&CONTRL" beginning in column 2. The namelist must end with the characters "&END". It is not necessary to specify all the variables belonging to a particular namelist.

Besides fixed and floating point variables, it is possible to read in logical and hollerith variables.

Data arrangement.- The input data deck begins with three cards specifying the date of the computer run, the user's name and the problem title. Next come a series of four namelists; CONTRL, MODEL, INITL, and TIMDEP. Described below are changes in, and additions to, the data input as described in the DECS-III report (Jeglic, 1967).

A. Namelist CONTRL

Two fixed-point variables have been added to this namelist. IBRAN is the number identifying the first section on the right-hand tributary. For the DO model, IBRAN = 20. IFORK identifies the section where the two branches meet. IFORK = 38 is the appropriate input for the DO model. NSECTS and SINGLE are not allowed as alternate mnemonics.

B. Namelist MODEL

One must add the following variables because of the two-fork system. The entries below pertain to the upstream end of the right branch.
LENGTH (150), analagous to LUPPER
ZCO, ZC (I), analagous to XCO & XC(I)
and ZLO, ZL(I), analagous to XLO & XL(I)
The entries CSO and CS(I) have been changed so that saturation concentration can be computed as a function of salinity and temperature.
The entries CQO, CSO, XCO, ZCO, YCO, XLO, ZLO, and YLO must all end in zero.

C. Namelist INITL

The following entries have been added for the upstream boundary of the Mattaponi branch:

K(l51) is the eddy diffusion coefficient for the interface upstream of the first reach;

Q(l51) is the flow entering the first reach;

AREA (151) is the area at the upstream end of the first reach;

CRUPP is the analogue of CUPP;

RLUPP is the analogue of LUPP.

D. Namelist TIMDEP

Q(l51), K(l51) and AREA(l51) have the same function as in the namelist INITL.

RECYCL = T causes the program to start over using a second set of data. In the namelist CONTRL and MODEL, one need enter only the variables which have been changed from the previous run. The namelist INITL and TIMDEP must be filled out completely, even if nothing has been changed from the preceding run.

Organization of subroutines.- The so-called main program (MAIN) of the program does very little. It reads and prints the three title cards and then transfers control to a subroutine called PRELIM. As the name
suggests, this subroutine carries out the preliminary setting up needed for the program. It reads the namelists CONTRL, MODEL, and INITL and prints out the information from them, as well as setting up the prescribed options. Each time a namelist is read, a subroutine named CHECK is called to make sure that the necessary information is indeed given. If it is not, a subroutine named TROUBL is called and this subroutine prints a diagnostic message and transfers control back to MAIN, which starts a new run.

From PRELIM, control is next transferred to entry PREWRT in subroutine CYCLE. This subroutine is the central one of the program. It is CYCLE which reads the namelist TIMDEP and from the information received sets up the limits of a given time interval and, by appropriate calls, gets the time integration going. Namelist INITL differs from namelist TIMDEP in the manner of the printout only.

Initial conditions having been established and an integration interval designated; a sequence of subroutines is called to perform the integration. First, subroutine STPSIZ calls subroutine INTEGR to (i) integrate over the time step and (ii) integrate over the two half-steps. The two results are compared, using subroutine ERRORM, and if the agreement is close enough, STPSIZ proceeds to the next time step. If it is not, the time step is cut in half and another attempt made.

Subroutine RUNKUT, when called by INTEGR to perform the integration uses, as the name implies, a Runge-Kutta algorithm for computational stability and accuracy. This subroutine also uses the explicit time derivatives computed by the subroutines DCDTA and DLDTB for dissolved oxygen and BOD respectively.
The subroutine SETHDR prints page headings and headings for data
sets. The subroutine SETCAL computes time intervals for subroutine CYCLE,
using the day, month and year of the beginning and end of the time interval.

Figure 16 shows the flow diagram taken from the report on DECS-III
(Jeglic, 1967).

Salinity modeling. - The program for salinity works much as described
above however, several differences need to be mentioned. Most importantly,
the subroutine DCDTA has been eliminated, since there is only one dependent
variable. Only fixed boundary conditions have been allowed in the salinity
model, although these may be changed at the start of every new time interval.
The constants dealing with re-aeration and decay rate have been eliminated,
and only dummy temperatures and values of P need be specified. Values of
J (in pounds per day) may be input to signify either a saline waste discharge
or a quantity of salt introduced with a lateral inflow.

Salinity Model Demonstration

Sample input. - The salinity model was verified using:

1. Slack-water runs made by the Division of Oceanography beginning
   in August, 1970;

2. Other VIMS data, where suitable for the purpose; and

3. Data gathered from August to October 1968 by the Chesapeake
   Corporation.

Since the most important practical problem is salinity intrusion over a
time period of several months, highwater slack data were used. Appendix
D contains input data and computer printout for the 1970 verification.
The computer simulation is started 3.5 months in advance of the first
slack-water run being verified, in order to remove transients caused
by the arbitrariness of the initial conditions.
Figure 16. Flow diagram for organization of subroutines in time-dependent model.
Sample output.- The program output is divided into several sections. The first of these is a list of program controls (see Appendix D). The volume parameters in the next section are for expressing volume as a function of discharge and are not used in the program. Next comes a printout of the geometric data used in the program, viz the inter-reach, cross-sectional area and the volume, hydraulic depth and length of each reach. Next comes a printout of the required hydraulic and loading information. The quantity $U$ is the magnitude of the tidal current. The quantity $J$ represents salinity loading in pounds per day, while $P$ has no significance in the salinity program. The quantity $K$ is the dispersion coefficient, specified at the interface between reaches. The discharge, $Q$, is likewise specified at the interface between two reaches.

The list of initial conditions concludes the printout of the information needed to start the integration process. Integration is carried out over specified time intervals, and one can have several outputs printed out after a specified number of intervals (note the entry PRINTOUT FREQUENCY). The first two of these geometric inputs and hydraulic and load inputs have already been described. One may wish them printed out occasionally because the entries in them may be changed at the beginning of any interval. The printout of the integration history is used mainly for diagnostic purposes. The printout of Computed System Parameters may be desired occasionally to check the results of changes in the geometric, hydraulic or load inputs.

The final printout for an integration interval is the salinity distribution at the end of that interval. The boundary conditions are printed out after the salinity distribution; first the left branch, then the right branch, then the downstream end.
Verification. - The period from the beginning of July, 1970 to
the middle of October in that year, was a time of gradually decreasing
flow and of increasing salinity intrusion in the Mattaponi and Pamunkey.
Therefore, it provides a suitable test for the salinity model. The computer
model was integrated from May 1, 1970 until September 29, 1970 using
boundary conditions obtained from prevailing seasonal salinity conditions
in the upper York system. The river discharge used in the computer program
for a given day was the median discharge over the month preceding that
day. Figures 17 and 18 show comparisons of field data with model results
for the Pamunkey River. These graphs show the slack water run results for
August 14 and September 29, 1970, together with the calculated results.

The boundary conditions used in the salinity are prevailing seasonal
values at Bell's Rock at high-water slack, based on accumulated VIMS data.
For the winter and spring months, 12‰ is specified while 15‰ is used for
summer and 16‰ for autumn.

During 1968 there were several low-water slack runs made by VIMS
personnel. In these runs, particular isohalines were sampled, rather than
particular fixed locations. By adding 5.7 miles to the location of the 5‰
isohaline at low-water slack, its location at high-water slack may be
approximated. Figure 19 shows the observed location of the 5‰ isohaline
and the calculated results. Interpolation between reach midpoints was
used to determine the model result for the location of the 5‰ isohaline.

The Chesapeake Corporation also conducted slack-water runs in 1968.
Figure 20 shows the comparison between the high-water slack observation
at Chelsea Farm, on the Mattaponi and the calculated results for the same
point. The calculated results were obtained by interpolating between the
model results for reaches 29 and 30, which bracket Chelsea Farm.
Figure 17. Salinity verification - calculated and observed Pamunkey salinity distributions for August 14, 1970.
Figure 18. Salinity verification - calculated and observed Pamunkey salinity distributions for September 29, 1970.
Figure 19. Salinity verification - salinity observations and calculated location of 5% isohaline in the Pamunkey.
Figure 20. Salinity verification - observed and calculated salinity versus time at Chelsea Farm.
Dissolved Oxygen Model Demonstration

Initial conditions.- The time-dependent model of dissolved oxygen is intended for short-range prediction (one or two days) on an hourly basis, including the daily temperature cycle, the tidal contribution to discharge and the daily photosynthesis cycle. Effects not observable in a steady-state model can be detected in this way. For example, one can determine the effect of tidal motion in spreading a pollutant, night-time oxygen minima or immediate effects of a large BOD "spill".

This sort of prediction fits well with the field data that were collected, which are quasi-synoptic measurements of the variables for a period of two tidal cycles. The quasi-synoptic measurements had to be synthesized into a truly synoptic record. This was accomplished for tidal currents by shifting the records for the various "days" in time so that the times for low-water slack coincide. The same was done for salinity and oxygen. Temperature records were shifted in time by a whole number of days. The result is a single record starting at 1200 on October 17 (approximately low-water slack). The BOD distribution was determined from data collected on later slack-water runs. The rate of input of loading was determined partly from (1) the reported outfall from the pulp mill at West Point, (2) estimated outfall from the West Point sewage treatment plant based on standard values of the contribution per person per day, and (3) estimates of the marsh contribution needed to maintain steady values of BOD. These estimates were made by a succession of trial runs of the model for ten full tidal cycles. For each run, the loading was adjusted to counteract the drift in BOD observed in the previous run. The same procedure was used for primary or benthal oxygen demand (P).

The boundary conditions express dissolved oxygen as a function of temperature. The upstream conditions define DO to be 80% of saturation
in fresh water. The formula is

\[
\text{CUPP} = \text{CRUPP} = 11.7 - 0.294T + 0.0036T^2
\]  

(17)

The downstream condition (based on field data) is that DO be 75% of saturation at 14\% salinity. The formula is

\[
\text{CLOW} = 0.32 - 0.237T + 0.0032T^2
\]  

(18)

In both formulae, T is temperature in C. The BOD boundary conditions were fixed at values gathered from VIMS slack-water runs in August, 1970.

**Time-dependent inputs.** - The tidal contribution to the flow is computed internally, using the magnitude of the tidal current and assuming that the initial time corresponds to a low-water slack. The manner of arriving at the temperature, tidal current and salinity inputs has already been explained. There were, in addition, hourly inputs of estimated primary oxygen demand and photosynthesis.

**Verification results.** - The model was run with the above initial conditions and varying parameters for a period of ten tidal cycles of model time. Figures 21 and 22 show the oxygen concentration at the first high-water slack for the two rivers. Figure 25 shows the calculated dissolved oxygen concentration versus time at reach 18, with the observed results at stations PO1 and PO2. Figure 26 shows the DO concentration at reach 32, together with the observed values at stations MO1 and MO2. Figure 27 shows the DO concentration at reach 16, with the observed values at stations PO3 and PO4. In the last three figures, "percent of saturation" refers to saturation concentration at 19.0 C and \(0\%\). Hence values in excess of 100\% saturation are not supersaturated compared to the local saturation concentration.
Figure 21. DO verification - Pamunkey River high-water slack observed and calculated results.
Figure 22. DO verification - Mattaponi River high-water slack observed and calculated results.
Figure 23. DO verification - Pamunkey River low-water slack observed and calculated results.
Figure 24. DO verification - Mattaponi River low-water slack observed and calculated results.
Figure 25. DO verification - observations at PO1 and PO2 and calculated results at reach 18 versus time.
Figure 26. DO verification - observations at M01 and M02 and calculated results at reach 32 versus time.
Figure 27. DO verification - observations at P03 and P04 and calculated results at reach 16 versus time.
CONCLUSIONS

A time-dependent model was developed from Thomann's (1963) equations. This model is capable of predicting the dissolved oxygen and BOD levels occurring in the upper York River and its tributaries under conditions of tidal motion and time-varying flow rates and pollutional loadings. The model has been verified according to data collected during October, 1969.

A time-dependent model has also been developed for predicting salinity variation over time periods of several months, but without tidal motion. With suitable inputs, other contaminants could also be modeled.

An extensive field survey was undertaken to gather background data for constructing the models. A series of slack-water sampling runs has been started to provide year-round verification data for the models.

Parameters which required precise description in this model include flow, dispersion coefficients, unit rates of BOD decay, atmospheric re-aeration, tidal exchange, waterway geometry and waste loading.

The dissolved oxygen concentration appears to be out of phase with the salinity upstream from West Point on the Pamunkey, suggesting that low oxygen water is being carried upstream on the flood tide. One of the values of the time-dependent model is that it enables one to study such localized, short-term variations of dissolved oxygen.

The Mattaponi and Pamunkey rivers have relatively strong tidal flows and, therefore, high re-aeration rates. Any man-made change in the mechanical properties of the rivers would have to be examined for its effect on the rate of re-aeration.

For certain thermal effluent studies (e.g. Pritchard-Carpenter, 1967) the freshwater discharge is replaced by a quantity called net new water,
derived by assuming a two-layer system in which salt balance is maintained by means of an upstream flow in the bottom layer and a return flow in the top layer. The return flow plus the fresh water discharge gives the net new water. The net new water concept has not been used in this study because BOD is distributed throughout the water column, and so will not be flushed by a (supposedly) high flow in the top layer.

The model results are only as good as the data employed; results are dependent on the rate parameters used for computation. Model development requires extensive analysis of data describing the physical, chemical and biological characteristics of the river system. The original collection and analysis of data for a medium-sized river may require anywhere from three to five years of effort. Its estuary may take longer since its characteristics are more complex.

Time and funding limitations precluded further field and laboratory work necessary to adequately describe these parameters. Estimates of their values were made, based on close examination of available data from this study and other river and estuarine studies. Such estimates, although used, are undesirable since the river is a dynamic entity. A more correct procedure would require a continuing series of field surveys to re-evaluate these parameters.

More sophisticated modeling techniques await future development. Present methods do clearly describe, within board limits, existing and projected future conditions. The accuracy of the methods described will be increased as better data become available. At the present time, Thomann's equations are widely accepted, mathematical relationships used for estimating the effects of pollutions on a stream. They are among the best tools available for engineering decisions. The natural conditions in the streams and therefore, the parameters in the equations may, in the future, differ substantially from values estimated at this time.
RECOMMENDATIONS

It is recommended that a study of the thickness, extent and biochemical nature of the benthal deposits be made for the Mattaponi and Pamunkey rivers. Furthermore, the nature of the oxygen exchange between the rivers and their marshes and swamps needs to be studied. It would also be useful to know the nature of the BOD exchange between rivers and wetlands.

Consideration should be given to a model which predicts the hydraulic and pollutional aspects of a river simultaneously, i.e. that can predict the tidal height and dissolved oxygen level simultaneously.

A continuing program of measurements needs to be maintained to enable updating of the parameters used in the equations. Thus the effects of changes in river flow, or climate, or state of health of the wetlands, may be detected and included in the model.
REFERENCES


APPENDIX A

RESULTS OF SLACK WATER RUNS
PAMUNKEY RIVER
29 IX 70
HWS

△ SALINITY  ○ TEMPERATURE
● D.O.

D.O. (mg/liter⁻¹)

MILES FROM RIVER MOUTH

SALINITY %
PAMUNKEY RIVER
12 XI 70
HWS

△ SALINITY  ○ TEMPERATURE
● D.O.

D.O. (mg/liter⁻¹)

SALINITY %

TEMP. °C

MILES FROM RIVER MOUTH
MATTAPONI RIVER
20 VIII 70
SBF
△ SALINITY ○ TEMPERATURE ● D.O.

MILES FROM RIVER MOUTH

D.O. (mg/liter⁻¹)

SALINITY %

TEMP. °C

2 4 6 8 10

12 14

30 35 40 45 50
APPENDIX B

PROFILES OF THE CROSS SECTIONS
Y 02
MILE 31.0

Y 01
MILE 32.3
P 17
MILE 79.2

P 16
MILE 74.6

P 15
MILE 70.9

DEPTH (FT.)

DISTANCE (FT.)
APPENDIX C

TIDAL OBSERVATIONS
PAMUNKEY RIVER - ELSING GREEN (69 MILES)
TYPE OF GAUGE: PERMANENT
REFERENCE: 1929 DATUM PLAIN

OCTOBER, 1969
MATTAPONI RIVER - AYLETT LANDING (71.0 MILES)
TYPE OF GAUGE: BUBBLER
REFERENCE: 1929 DATUM PLAIN

OCTOBER, 1969
PAMUNKEY RIVER — NEW CASTLE BRIDGE (85.8 MILES)
TYPE OF GAUGE: BUBBLER
REFERENCE: 1929 DATUM PLAIN

OCTOBER, 1969
ELTHAM BRIDGE (35 MILES)
TYPE OF GAUGE: PERMANENT
REFERENCE: 1929 DATUM PLAIN

OCTOBER, 1969
APPENDIX D

LISTING OF INPUT DATA AND OUTPUT DATA FROM SALINITY AND DO VERIFICATION FOR TIME-DEPENDENT MODEL
TIMEDEP DISSOLVED OXYGEN MODEL BASED ON DECS-III PROGRAM
IBM 360/50 COMPUTER

PROBLEM TITLE: OD MODEL - DEMONSTRATION
DATE OF RUN: SEPT 17 71
SUBMITTED BY: P. HYER
NO. OF ESTUARY SECTIONS - 38.
RIGHT BRANCH - 20.
CROTCH - 34.
NO. OF INITIAL INTEGRATION STEPS - 50.
INTEGRATION TYPE - CONTROLLED STEP SIZE.
DOUBLE VARIABLE INTEGRATION.

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**Date:** October 17, 1969  
**Time:** 1200

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**October 17, 1969**  
**Time:** 1200

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CROTCH - 34.
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SINGLE VARIABLE INTEGRATION.
INTEGRATION TOLERANCES -
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3. 1.0000E 00

TEMPERATURE INPUT IN DEGREES CENTIGRADE.

CONVERSION FACTORS USED ON INPUT VARIABLE -

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PRINTOUT FREQUENCY -

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2. HYDRAULIC AND LOAD INPUTS * EVERY 11 TIME INTERVAL(S)
3. INTEGRATION HISTORY * EVERY 10 TIME INTERVAL(S)
4. COMPUTED SYSTEM PARAMETERS * EVERY 10 TIME INTERVAL(S)
5. FINAL ANSWERS * EVERY 1 TIME INTERVAL(S)
PROBLEM CONSTANTS

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A & = 0.0 \\
B & = 0.0 \\
PK & = 0.0 \\
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\end{align*}
\]

COEFFICIENTS FOR \( C(S, T) \) QUADRATIC

\[
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C(2) & = -0.367130E 00 \\
C(3) & = 0.273900E-03 \\
C(4) & = 0.205000E-02 \\
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\end{align*}
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BOUNDARY COEFFICIENTS

INPUT TIME VARYING CONSTANTS
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**May 1, 1970**

**Time: 1059**

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**DAY 0.458**  
**MAY 1, 1970**  
**TIME 1059**

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<tr>
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<td>11.800</td>
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**BOUNDARY VALUES**  
**SALINITY, 1.0000E-02, 1.0000E-02, 1.1000E 00**
SEPT 17 71
DO MODEL - DEMONSTRATION
P. HYSER
&CONTROL
NSEC=38, IFORK=34, IBRAN=20,
PFREQ=25, 25, 5, 25.1,
ERCALL=.005, .02, .1, ERLALL=.005, .02, .1,
FIXED=F, NSTEPS=50, CFLGTH=1000., CFAREA=1000., CFVOL=1.0E6, DEXPOL=1,
CFK=50.,
CFU=-1.,
CFJ=1000.,
CFP=1000.,
QDELQ=2,
SINGLE=F
BOUND=1,
&END
&MODEL
LENGTH= 14., 24., 20., 11., 27., 17., 19., 9.9, 10.8, 12.5, 17.4, 8.2, 14.7, 13.1, 13.2,
13.1, 13.1, 4.5, 6.56, 22., 15., 21., 16.6, 16.4, 16.4, 13.1, 18.7, 22.7, 10.5, 10.5, 9.82, 3.28,
6.56, 6.54, 4*6.54,
LUPPER=21., LLOWER=16., LENGTH(150)=5.,
CSO=14.6244, CS=-.0966, -.3671, .0002739, .00205, .004497,
XC0=11.7, ZC0=11.7, YC0=9.3,
XC=-.294, .0036, ZC=-.294, .0036, YC=-.237, .0032,
XLO=0.4, YLO=3.3, ZLO=0.4,
THETA=1.0238, D=2.25E-8, A=.23, B=0., PK=1.0,
NU=1.099, &END
&INITL
DATE=10, 17, 69
MILTIM=1200
19.7, 19.6, 19.2, 18.9, 19.0,
14.3, 15.3, 15.5, 16.4, 17.3, 18.0, 18.2, 18.3, 18.5, 18.6, 18.7,
18.8, 18.8, 19.0, 19.0, 4*19.0,
VOL= 20.5, 41.4, 51.9, 47.6, 18.5, 0.284, 0.445, 0.190, .205, .275, .466, .200, .294, .257,
.243, .270, .383, .200, .310, .245, .410, .111, .130, .207, .308, .265,
340., 363., 178., 195., 177., 83.3, 140., 5*420.,
H= 10.0, 9.0, 10.3, 10.6, 11.4, 14.6, 15.7, 18.0, 19.8, 17.2, 15.8, 15.2, 16.9, 15.2,
13.0, 14.2, 17.4, 18.4, 18.5, 6.6, 6.7, 8.4, 12.2, 15.0, 15.5, 13.8, 18.4, 20.0, 13.7, 12.8,
14.6, 11.7, 10.0, 8.5, 4*8.5,
U=.45, .095, .095, .095, .065, .055, .08, .13, .15, .15, .15, .15, .15, .15, 2.05,
2.05, 2.10, 2.30, 1.80, 1.90, 1.50,
0.49, .050, .035, .105, .102, .139, .130, .165, .165, .260, .175, .207, .125,
1.25, 1.41, 4*1.7,
P=0, .072, 2.4, 2.3, 7.3, 10.8, 2.8, 4.3, 5.7, 6.8, 9.8, 3.0, 11.5,
10.5, 9.3, 6.8, 14.2, 6.6, 10.6,
0.18, 2.5, 5.9, 5.1, 5.5, 9.2, 7.9, 6.3, 7.6, 4.4, 5.3, 4.4, 3.9, 4.8,
7.2, 8.2, 8.8, 7.5, 10.2,
4*10.2,
J=0, 0, 18., 15., 58., 1.19, 1.93, .83, 1.51, 2.6, 4.1, 1.0, 2.0, 1.8, 1.7, 1.0, 1.4,
75.0, .48,
.06, .51, .14, .32, .95, 1.1, 2.6, 4.0, 3.1, 1.8, .72, .44, .59, .54, 1.0,
1.0, 1,,
Q=419.5, Q(151)=243.9,
DELQ=4.4, 8.1, 6.6, 3.5, 8.8, 5.5, 6.6, 3.3, 3.5, 4.0, 5.1, 2.6, 4.8, 4.4, 4.4,
4.4, 4.4, 1.1, 20.3,
7.6, 5.3, 7.4, 5.7, 5.7,
5.7, 4.7, 5.9, 7.2, 3.8, 3.6, 3.6, 1.1, 1.2, 4, 2.5, 4*0.,
AREA = 1.563, 1.875, 2.250, 3.188, 5.720, 10.000, 26.750, 20.125, 36.0, 37.500, 46.9, 47.0, 2.313, 4.500, 8.250, 9.875, 17.750, 23.500, 29.250, 16.500, 24.250, 20.500, 30.0, 27.0, 24.000, 5*64.3,
AREA(151) = .750,
K = .025, .025, .02, .01, .01, .02, .06, .085, .29, .22, .31, .35, .44, .60, .65, 1.09, 1.17, 1.14, 1.01, 1.29, .01, .02, .035, .03, .04, .34, .15, .34, .39, .33, .52, .53, .70, 1.29, 1.49, 4*1.49,
K(151) = .01,
SALT = 0.06, 0.06, 0.06, 0.06, 0.06, 0.06, 0.06, 0.06, 0.07, 0.16, 0.32, 0.77, 1.51, 2.96, 5.14, 7.94, 9.49, 9.7,
       0.06, 0.06, 0.06, 0.06, 0.06, 0.06, 0.06, 0.06, 0.06, 0.09, 0.55, 2.09, 5.00, 8.63, 11.32, 12.40, 12*4.4,
L = 0.4, 0.3, 0.3, 0.3, 0.4, 0.4, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.1, 1.2, 1.8, 2.2, 3.4, 3.9, 0.1, 0.1, 0.2, 0.3, 0.4, 0.6, 0.7, 0.8, 1.0, 1.1, 1.1, 1.4, 0.9, 2.6, 2.1, 1.9, 1.8, 1.8, 1.4,
       C = 8.2, 7.8, 6.4, 6.2, 5.9, 5.9, 6.6, 6.4, 6.0, 6.3, 6.5, 6.7, 6.8, 6.6, 7.0, 6.0, 4.8, 3.8, 4.0, 8.4, 7.2, 6.7, 6.8, 6.7, 6.8, 6.8, 6.7, 6.7, 6.5, 6.6, 6.3, 6.0, 5.6, 5.7, 4*5.7,
&END
&TIMDEP
DATE=10, 17, 69
MILTIM=1813
&END
&TIMDEP
DATE=10, 18, 69
MILTIM=0025
&END
&TIMDEP
MILTIM=0638
&END
&TIMDEP
MILTIM=1903
&END
&TIMDEP
DATE=10, 19, 69
MILTIM=0116
&END
&TIMDEP
MILTIM=0728
&END
&TIMDEP
MILTIM=1341
&END
&TIMDEP
MILTIM=1953
&END
&TIMDEP
DATE=10, 20, 69, MILTIM=0206,
&END
&TIMDEP
MILTIM=0819,
&END
&TIMDEP
MILTIM=1431,
&END
&TIMDEP
MILTIM=2044,
&END
&TIMDEP
DATE=10,21,69, MILTIM=0256,
&END
&TIMDEP
MILTIM=0909,
&END
&TIMDEP
MILTIM=1522,
&END
&TIMDEP
MILTIM=2134,
&END
&TIMDEP
DATE=10,22,69, MILTIM=0347,
&END
&TIMDEP
MILTIM=0959,
&END
&TIMDEP
MILTIM=1612,
&END
&TIMDEP RECYCL=T
SEPT  17  71
70 VERIFICATION E(S)
'AUL  V.  HYER
&CONTRL
NSECT=35, IFORK=34, IBRAN=20,
PFRQ=10, 11, 10, 10, 1,
ERCALL=.05, .1, .1, ERLALL=.05, .1, 1.
FIXED=F, NSTEPS=250, CFLGTH=1000., CFAREA=1000., CFVOL=1.0E6, DEXPOL=1,
CFU=0.,
CFK=1000.,
QUELQ=2,
SINGL=T, LNAME=12H SALINTY
BOUND=2,
&END
&MODEL
LENGTH=21.,
13.1, 13.1, 3.28,
5., 22., 15., 21., 16.6, 16.4,
16.4, 13.1, 18.7, 22.7, 10.5, 10.9, 9.82, 3.28,
6.56, 6.54,
LUPPER=5., LLOWER=16., LENGTH(150)=5.,
CSO=0.,
THETA=0., D=0., A=0., B=0., PK=0., NU=0.,
&END
&INITL
DATE=5, 1, 70,
MILTIM=1100,
L=9*0.1, 0.2, 0.3, 0.8, 1.6, 2.9, 5.3, 7.6, 9.0, 10.1, 10.8,
9*0.1, 3.1, 6.0, 7.8, 9.5, 10.5, 11.1, 11.8,
LUPP=0.1, RLPUPP=0.1,
LLOW=11.0,
H=8.4, 10.0, 9.0, 10.3, 10.6, 11.4, 14.6, 15.7, 18.0, 19.8, 17.2, 15.8, 15.2, 16.9, 15.2,
13.0, 14.2, 17.4, 18.4, 3.2, 5.6, 6.7, 8.4, 12.2, 15.0, 15.5, 13.8, 18.4, 20.0, 13.7, 12.8,
14.6, 11.7, 10.0, 8.5,
U=0.01, 0.02, 0.09, 0.2, 2.0, 3.1, 2.9, 1.0, 1.1, 1.3, 1.1, 0.9, 1.0, 1.7, 1.9, 1.6,
2.0, 1.9, 1.9, 1.7,
0.01, 1.3, 1.5, 1.4, 1.9, 1.3, 1.0, 1.3, 1.7, 2.0, 1.5, 1.7, 1.0, 1.4, 1.1,
J=35*0., P=35*0.,
AREA=837.1, 1.563, 1.875, 2.250, 3.188, 5.750, 10.000, 26.750, 19.750, 17.500, 20.250,
22.700, 29.000, 18.700, 16.000, 20.750, 20.125, 29.250, 37.500, 46.900,
750.2, 313.4, 500.8, 250.9, 8.75, 17.750, 23.500, 19.250, 16.500, 16.500,
24.250, 20.500, 30.500, 27.000, 71.250, 64.287,
AREA(151)=.317,
VOL=22.3, 20.5, 41.4, 4.0, 5.6, 9.4, 7.6, 185.0, 284.0, 445.0, 190.0, 205.0, 275.0, 466.0, 200.0,
294.0, 257.0, 243.0, 270.0, 383.0, 129.0, 1.56, 24.5, 41.0, 111.0, 130.0, 207.0, 308.0, 265.0,
340.0, 363.0, 178.0, 195.0, 177.0, 83.3, 485.0, 386.0,
Q= 483., Q(151)=244.0,
DELP= 8.6, 4.3, 8.6, 8.6, 4.3, 12.9, 4.3, 8.6,
7*4.3, 8.6, 2* 4.3,
2.1, 7.2, 5.3, 7.2, 3*5.3, 4.7, 6.4, 7.9, 3.8, 2*3.6, 1.1, 2*4.3, 0.02,
.02, .025, .025, .02, .02, .01, .01, .02, .06, .085, .29, .22, .31, .35,
.44, .60, .65, 1.09, 1.17, 1.14, 1.01,
.01, .01, .02, .035, .03, .04, .34, .15, .34, .39, .33, .52, .53,
0.70, 1.29, 1.49,
K(151)=.01,
TEMP=35*20.,
&END
&TIMDEP
DATE=6, 15, 70,
Q= 483.
&END
&TIMDEP
DATE=7,15,70,
LLow=15.0,
Q=183.0,
DELQ=3.3, 1.6, 2*3.3, 1.6, 4.9, 1.6, 3.3, 7*1.6, 3.3, 2*1.6,
2.1, 7.2, 5.3, 7.2, 3*5.3, 4.7, 6.4, 7.9, 3.8, 2*3.6, 1.1, 2*4.3,
&END
&TIMDEP
DATE=8,14,70,
Q=261.0,Q(151)=179.0,
DELQ=4.7, 2.3, 2*4.7, 2.3, 7.0, 2.3, 4.7, 7*2.3, 4.7, 2*2.3,
1.5, 5.6, 3.9, 5.5, 3*4.2, 3.4, 4.7, 5.8, 2.8, 2*2.7, 0.8, 2*2.3,
&END
&TIMDEP
DATE=9,29,70,
LLow=17.6,
Q=39.0,Q(151)=41.0,
DELQ=0.7, 0.4, 2*0.7, 0.4, 1.1, 0.4, 0.7, 7*0.4, 0.7, 2*0.4,
0.4, 1.3, 6.7, 1.3, 3*1.0, 0.8, 1.1, 1.3, 3*0.6, 0.2, 2*0.4,
&END
&TIMDEP RECYCL=T
&END
APPENDIX E

GRAPHICAL SUMMARY OF DATA COLLECTED DURING

OPERATION YORK RIVER, OCTOBER, 1969
M 04
39.4 MILES
20 X 1969 - 21 X 1969

D. O. (mg/liter)

35
30
25
20
15
10
5

TEMPERATURE

39
34
30
25
20
15
10
5

DISSOLVED
OXYGEN

SALINITY (%)

TEMPERATURE (°C)

SALINITY

TIME (HRS.)
07 12 17 22 03 08
Dissolved Oxygen (mg/liter)

Temperature (°C)

Salinity (%)

Time (Hrs.)

M I I
63.0 MILES

18 X 1969 - 19 X 1969

Dissolved Oxygen

Temperature

Salinity

18 23 04 09

18 19
M 13  18 X 1969 - 19 X 1969
70.0 MILES

D.O. (mg/liter)

TEMPERATURE

Dissolved Oxygen

TIME (HRS.)
D.O. (mg/liter)

TIME (HRS.)

TEMPERATURE

DISSOLVED OXYGEN

M 14  18 X 1969 - 19 X 1969
71.0 MILES

19 0 5 10 15 20
M 15
72.0 MILES

18 X 1969 - 19 X 1969

D.O. (mg/liter)

Temperature (°C)

TIME (HRS.)

Dissolved Oxygen

Temperature
Y01  17 X 1969-18 X 1969

Dissolved Oxygen

Temperature

Salinity

Time (Hrs.)
P O 2
35.4 MILES

17 X 1969 - 18 X 1969

Dissolved Oxygen

Temperature

Salinity (%)
D.O. (mg/liter) vs. Time (Hrs.)

- Dissolved Oxygen (D.O.)
- Temperature
- Salinity

P 04
38.5 MILES

16 X 1969 - 17 X 1969

Temperature (°C)

Salinity (%)

Time (Hrs.)

08 13 18 23 04 09
Dissolved Oxygen
Temperature
Salinity

P 05
41.0 MILES

16 X 1969 - 17 X 1969

TIME (HRS.)

TEMPERATURE (°C)

SALINITY (%)

D.O. (mg/liter)
P 06
43.5 MILES

16 X 1969 - 17 X 1969

DISSOLVED OXYGEN

TEMPERATURE

SALINITY

D.O. (mg/liter)

SALINITY (%)
Dissolved Oxygen (mg/liter) vs. Time (Hrs)

Salinity (%) vs. Time (Hrs)

Temperature (°C) vs. Time (Hrs)

P 07  16 X 1969 - 17 X 1969
47.3 MILES

- 30
- 25
- 20
- 15
- 10
- 5
0

- 5
- 10
- 15
- 20
- 25
- 30

08 13 18 23 04 09 09

DISSOLVED OXYGEN
TEMPERATURE
SALINITY

TIME (HRS)
P 09  14 X 1969 - 15 X 1969
52.8 MILES

D.O. (mg/liter)

TEMPERATURE (°C)

SALINITY (‰)

TIME (HRS.)
P 10
54.8 MILES
14 X 1969 - 15 X 1969

DISSOLVED OXYGEN

TEMPERATURE

SALINITY

TIME (HRS.)

D.O. (mg/liter)

SALINITY (‰)

TEMPERATURE (°C)

23 04 09 14 19 0
P 14  14 X 1969 - 15 X 1969
68.7 MILES

D. O. (mg/liter)

TIME (HRS.)
Dissolved Oxygen (mg/liter) and Temperature (°C) over 74.6 miles from 13th to 14th of October 1969.
P19  13 X 1969 - 14 X 1969
85.8 MILES

D.O. (mg/liter)

TEMPERATURE

DISSOLVED OXYGEN

TIME (HRS.)

TEMPERATURE (°C)
Dissolved Oxygen

Temperature

P20
89.0 MILES

13 X 1969 - 14 X 1969

Dissolved Oxygen

Temperature

Time (Hrs.)

D.O. (mg/liter)

Temperature (°C)