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# Higher-derivative Lee–Wick unification

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## ABSTRACT

We consider gauge coupling unification in Lee–Wick extensions of the Standard Model that include higher-derivative quadratic terms beyond the minimally required set. We determine how the beta functions are modified when some Standard Model particles have two Lee–Wick partners. We show that gauge coupling unification can be achieved in such models without requiring the introduction of additional fields in the higher-derivative theory and we comment on possible ultraviolet completions.

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## 1. Introduction

The Lee–Wick Standard Model (LWSM) has been proposed as a possible solution to the hierarchy problem [1], motivated by the ideas of Ref. [2]. For each Standard Model particle, higher-derivative quadratic terms are introduced so that propagators fall off more quickly with momentum. Although gauge invariance implies that higher-derivative interaction terms must also be present, power-counting arguments indicate that the ultraviolet divergences in loop diagrams are no greater than logarithmic, even when the usually problematic Higgs sector is taken into account [1].

The presence of higher-derivative quadratic terms leads to additional poles in the two-point function of each Standard Model field. The higher-derivative theory can be recast using an auxiliary field approach as a dimension-four Lagrangian, with additional fields corresponding to the new Lee–Wick partner states [1]. In the original LWSM proposal, each Standard Model particle has a single LW partner which, in the dimension-four form of the Lagrangian, has wrong-sign kinetic and mass terms. Due in part to this sign difference, the LW partners play the role of Pauli–Villars regulators in loop diagrams, so that the cancellation of quadratic divergences found in the equivalent higher-derivative theory is reproduced. Unlike Pauli–Villars regulators, however, Lee–Wick particles are taken to be physical. It has been argued that Lee–Wick field theories preserve macroscopic causality as long as the LW partners can decay [3], and that gauge boson scattering remains unitary despite the presence of massive LW vector meson states [4]. The evidence in favor of the consistency of LW theories [3–5] and the simple mechanism that they provide for solving the hierarchy problem has motivated a number of recent studies of the formal properties and phenomenology of the LWSM and related theories [6].

It has been pointed out that LW theories with more than a single LW partner field can be constructed if higher-derivative quadratic terms beyond the minimally required set are included [7]. Letting  $N$  refer to the number of poles in the two-point function of each field in the higher-derivative form of the theory, the LWSM most frequently discussed in the literature corresponds to  $N = 2$ ; Ref. [7] showed how one may construct the  $N = 3$  generalization of the LWSM, and provided the mappings between the Lagrangian in its higher-derivative (HD), auxiliary field (AF) and Lee–Wick (LW) forms, where the latter refers to the theory with quadratic terms that are canonical aside from their overall signs. Clearly, generalization to LW theories with  $N > 3$  is possible. However, one might ask whether anything useful is gained in constructing such theories, aside from intellectual exercise. In Ref. [7], it was pointed out that the heavier LW partner of each Standard Model field in the  $N = 3$  theory is an ordinary particle (corresponding to a state with positive norm), and therefore might be distinguishable at colliders from the lightest LW partner. In this Letter, we point out another, potentially useful feature of theories in which some ordinary particles have more than a single LW partner: gauge coupling unification can be achieved at the one-loop level without requiring the introduction of new particles that remain light in the limit that the LW scale is taken to infinity.

This Letter is organized as follows. In the next section we show how the computation of beta functions in the  $N = 2$  theory, as considered by Grinstein and O’Connell [8], is modified in the  $N = 3$  case. Notably, the doubling of the number of the massive LW gauge bosons does not lead to a doubling of their contribution to the beta functions, so that one cannot naively extrapolate the answer from that of the  $N = 2$  theory. In Section 3 we study one-loop unification assuming that each Standard Model particle has either one or two LW partners. In Section 4 we suggest possible extra-dimensional ultraviolet completions for some models of this type and we summarize our conclusions.

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## 2. Beta functions

We employ the background field method, where gauge fields are expanded about a classical background  $B^\mu$ ,

$$A^\mu \rightarrow B^\mu + A^\mu, \quad (2.1)$$

where we use the notation  $A_\mu \equiv A_\mu^a T^a$ , etc., with the gauge group generators normalized  $\text{Tr} T^a T^b = \frac{1}{2} \delta^{ab}$ . The gauge fixing term is given by

$$\mathcal{L}_{\text{gf}} = -\frac{1}{2g^2} \text{Tr}(D^\mu A_\mu)^2, \quad (2.2)$$

where the covariant derivative is with respect to the background field

$$D^\mu = \partial^\mu - iB^\mu. \quad (2.3)$$

The gauge-fixed Lagrangian is invariant under a residual gauge symmetry in which  $B^\mu$  transforms as a gauge field and  $A^\mu$  as a matter field in the adjoint representation. Working to quadratic order in the fluctuating field  $A_\mu$  and performing its functional integral, one obtains the one-loop effective action for the background field, including the kinetic term

$$-\frac{1}{2} c_B \text{Tr} B_{\mu\nu} B^{\mu\nu}. \quad (2.4)$$

The beta function for the gauge coupling can be extracted from the coefficient  $c_B$ . This construction is well known and discussed in textbooks; we refer the reader to Ref. [9] for a detailed review, and Ref. [8] for a discussion of subtleties that can arise in LW theories.

Grinstein and O'Connell demonstrated in the  $N=2$  LWSM that the same beta functions are obtained whether one works in the HD or the LW form of the theory [8]. We expect the same to hold true for theories with  $N > 2$ , though in these cases the HD form of the theory is more cumbersome for Feynman diagram calculations. As a consistency check, we will do one example in an  $N=3$  theory where it is tractable to compute beta functions in both the HD and LW forms of the theory: we consider the contribution to the  $SU(N)$  beta function from a complex scalar in the fundamental representation. For the remaining beta function calculations that we need, we work with the simpler LW form of the Lagrangian.

The  $N=3$  Lagrangian for a complex scalar in the fundamental representation of  $SU(N)$  is given by [7]

$$\begin{aligned} \mathcal{L}_{\text{HD}} = & \hat{D}_\mu \hat{H}^\dagger \hat{D}^\mu \hat{H} - m_H^2 \hat{H}^\dagger \hat{H} - \frac{1}{M_1^2} \hat{H}^\dagger (\hat{D}_\mu \hat{D}^\mu)^2 \hat{H} \\ & - \frac{1}{M_2^4} \hat{H}^\dagger (\hat{D}_\mu \hat{D}^\mu)^3 \hat{H} + \mathcal{L}_{\text{int}}(\hat{H}), \end{aligned} \quad (2.5)$$

where  $\hat{D}_\mu = \partial_\mu - iA_\mu - iB_\mu$ , and the  $M_i$  determine the masses of the LW partners. (We assume that the  $M_i$  are comparable and not far above the weak scale.) The logarithmically divergent part of  $c_B$  determines the beta function. Equivalently, one can find the beta function by computing the wave-function renormalization  $Z$  of the fluctuating field  $A$  in background field gauge. Rescaling the fields so that the gauge coupling appears in the covariant derivatives and writing  $Z = 1 + a/\epsilon + \dots$  in dimensional regularization with  $\epsilon = 4 - d$ , the  $\beta$  function is given by [8]

$$\beta = -\frac{1}{4} g^2 \frac{\partial a}{\partial g} \equiv \frac{bg^3}{16\pi^2}. \quad (2.6)$$

In the present example, the necessary vertices can be extracted from Eq. (2.5) and are shown in Fig. 1. The three-point coupling shown in Fig. 1(a) has the Feynman rule

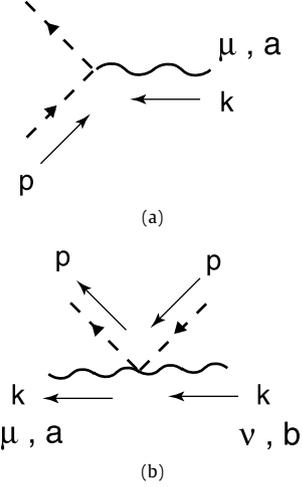


Fig. 1. Higgs-gauge boson vertices relevant to the calculation of the gauge boson two-point function, with momenta restricted accordingly.

$$\begin{aligned} i\Gamma_\mu^{(3)a}(p, k) & \equiv ig(2p+k)_\mu T^a \left\{ 1 - \frac{1}{M_1^2} [p^2 + (p+k)^2] \right. \\ & \left. + \frac{1}{M_2^4} [p^4 + p^2(p+k)^2 + (p+k)^4] \right\}, \end{aligned} \quad (2.7)$$

with the momenta and indices shown in the diagram. The four-point coupling shown in Fig. 1(b) has the Feynman rule

$$\begin{aligned} i\Gamma_{\mu\nu}^{(4)ab}(p, k) & \equiv ig^2 T^a T^b \left\{ g_{\mu\nu} + \frac{1}{M_1^2} [-2p^2 g_{\mu\nu} - (2p+k)_\mu (2p+k)_\nu] \right. \\ & \left. + \frac{1}{M_2^4} [3p^4 g_{\mu\nu} + (2p^2 + (p+k)^2)(2p+k)_\mu (2p+k)_\nu] \right\} \\ & + (a \leftrightarrow b, k \rightarrow -k), \end{aligned} \quad (2.8)$$

in the simplified case where the momenta are chosen as shown in the diagram (the more general result will not be required). Finally, the  $\hat{H}$  propagator is given by

$$\tilde{D}(p) = \frac{i}{p^2 - m_H^2 - p^4/M_1^2 + p^6/M_2^4}. \quad (2.9)$$

The one-loop contributions to the gauge boson two-point function are given by

$$i\Pi_{1\mu\nu}^{ab}(k) = \int \frac{d^4 p}{(2\pi)^4} \text{Tr}[i\Gamma_{\mu\nu}^{(4)ab}(p, k)\tilde{D}(p)], \quad (2.10)$$

for the diagram obtained from Fig. 1(b) by closing the scalar line, and

$$\begin{aligned} i\Pi_{2\mu\nu}^{ab} = & \int \frac{d^4 p}{(2\pi)^4} \text{Tr}[i\Gamma_\mu^{(3)a}(p, k)\tilde{D}(p) \\ & \times i\Gamma_\nu^{(3)b}(p+k, -k)\tilde{D}(p+k)], \end{aligned} \quad (2.11)$$

for the diagram that is second order in Fig. 1(a) vertex. Using the expressions given in Eqs. (2.7), (2.8) and (2.9), the logarithmically divergent parts of  $\Pi_{1\mu\nu}^{ab}$  and  $\Pi_{2\mu\nu}^{ab}$  may be extracted by expanding the integrands in powers of  $p^{-1}$ . One finds the  $\epsilon$  poles

$$\begin{aligned}
i\Pi_{1\mu\nu}^{ab}(k) &\rightarrow -\frac{ig^2}{16\pi^2} \left(\frac{2}{\epsilon}\right) \left[3\frac{M_2^4}{M_1^2} g_{\mu\nu} + 5k_\mu k_\nu + k^2 g_{\mu\nu}\right] \delta^{ab}, \\
i\Pi_{2\mu\nu}^{ab}(k) &\rightarrow \frac{ig^2}{16\pi^2} \left(\frac{2}{\epsilon}\right) \left[3\frac{M_2^4}{M_1^2} g_{\mu\nu} + \frac{11}{2}k_\mu k_\nu + \frac{1}{2}k^2 g_{\mu\nu}\right] \delta^{ab}
\end{aligned} \tag{2.12}$$

which combine to give the desired transverse form

$$i\Pi_{\mu\nu}^{ab} \rightarrow -\frac{1}{2} \frac{ig^2}{16\pi^2} \left(\frac{2}{\epsilon}\right) (k^2 g_{\mu\nu} - k_\mu k_\nu) \delta^{ab}. \tag{2.13}$$

From Eq. (2.6), it then follows that the contribution to the  $SU(N)$  beta function is given by  $\Delta b = 1/2$ . By comparison, the contribution to the  $SU(2)$  beta function due to the Higgs doublet in the Standard Model is  $\Delta b = 1/6$ . In the LW form of the  $N = 3$  theory, this result is enhanced by a factor of three due to the contribution of the LW partners. (One can check that the LW sign changes in vertices and propagators occur in each diagram an even number of times.) Thus, the result for  $\Delta b$  computed in the HD form of the  $N = 3$  theory reproduces the result of the LW form, as one would expect.

One might draw the incorrect conclusion from this example that the contribution to the Standard Model beta functions from the bosonic LW states is simply enhanced by a factor of  $3/2$  in going from the  $N = 2$  to the  $N = 3$  theory. While this is true for the LW Higgs fields (which couple to the gauge fields like their Standard Model counterpart, up to signs), it is not true in the LW gauge sector. The gauge-boson self-interactions in the LW form of an  $N = 3$  theory were found in Ref. [7]; the couplings of the two LW partners,  $A_2$  and  $A_3$ , to the massless gauge field,  $A_1$ , in an  $SU(N)$  gauge theory are given by [7]

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2} \text{Tr}(D_\mu A_{2\nu} - D_\nu A_{2\mu})^2 - \frac{1}{2} \text{Tr}(D_\mu A_{3\nu} - D_\nu A_{3\mu})^2 \\
&\quad - \frac{ig}{(m_3^2 - m_2^2)} \text{Tr}(F_{1\mu\nu} [m_3 A_2^\mu - m_2 A_3^\mu, m_3 A_2^\nu - m_2 A_3^\nu]),
\end{aligned} \tag{2.14}$$

where  $m_2$  and  $m_3$  are the mass eigenvalues of the LW partners and the covariant derivative here is given by  $D^\mu A_j^\nu = \partial^\mu A_j^\nu - ig[A_1^\mu, A_j^\nu]$  for  $j = 2, 3$ . We can write these interactions in a form that more easily allows us to compare the result in the  $N = 2$  and  $N = 3$  theories. Let  $\mathbf{A} \equiv [A_2, A_3]^T$  and define

$$\begin{aligned}
\eta &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad M^2 = \begin{pmatrix} m_2^2 & 0 \\ 0 & m_3^2 \end{pmatrix}, \\
C &= \frac{1}{m_3^2 - m_2^2} \begin{pmatrix} m_3^2 & -m_2 m_3 \\ -m_2 m_3 & m_2^2 \end{pmatrix}.
\end{aligned} \tag{2.15}$$

Eq. (2.14) is contained in

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2} \text{Tr}[(D_\mu \mathbf{A}_\nu - D_\nu \mathbf{A}_\mu)^T \eta (D^\mu \mathbf{A}^\nu - D^\nu \mathbf{A}^\mu)] \\
&\quad - \text{Tr}[\mathbf{A}_\mu^T \eta M^2 \mathbf{A}^\mu] - 2ig \text{Tr}[F_1^{\mu\nu} \mathbf{A}_\mu^T C \mathbf{A}_\nu],
\end{aligned} \tag{2.16}$$

where we have also included the gauge boson mass terms. The Feynman rule for the  $A_1 \mathbf{A}^2$  vertex shown in Fig. 2, is then

$$\begin{aligned}
i\Gamma_{\alpha\beta\mu}^{(3)abc}(p, k) &= -gf^{abc} \{ \eta [(2p+k)_\mu g_{\alpha\beta} - p_\alpha g_{\mu\beta} - (p+k)_\beta g_{\mu\alpha}] \\
&\quad + C[-k_\beta g_{\alpha\mu} + k_\alpha g_{\beta\mu}] \},
\end{aligned} \tag{2.17}$$

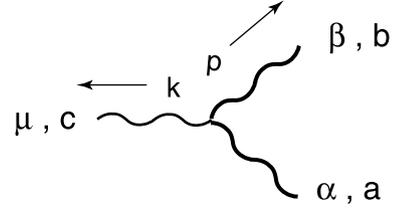


Fig. 2. Gauge-LW gauge boson vertex from Eq. (2.16). The heavy line represents the LW gauge boson field  $\mathbf{A} \equiv [A_2, A_3]^T$ .

with the matrices  $\eta$  and  $C$  as previously defined. The propagator for the column vector of LW gauge field partners is given in matrix form by

$$\tilde{D}(p)_{\mu\nu}^{ab} = i\eta [g_{\mu\nu} - M^{-2} p_\mu p_\nu] [p^2 - M^2]^{-1} \delta^{ab}. \tag{2.18}$$

It is now straightforward to evaluate the  $A_1$  two-point function and extract the logarithmically divergent part, as in our previous scalar example. We find

$$\Pi_{\mu\nu}^{ab} \rightarrow -i \frac{\Delta b g^2}{16\pi^2} \left(\frac{2}{\epsilon}\right) (k^2 g_{\mu\nu} - k_\mu k_\nu) \delta^{ab}, \tag{2.19}$$

where

$$\Delta b = -\frac{1}{2} \text{Tr}[\mathbb{1} + 6C\eta + C\eta C\eta - C\eta M^{-2} C\eta M^2] C_2 \tag{2.20}$$

and where  $C_2$  is the quadratic Casimir for the adjoint representation,  $f^{acd} f^{bcd} = C_2 \delta^{ab}$ . Substituting the matrices from Eq. (2.15), one obtains  $\Delta b = -9/2C_2$ . As a check, we note that in the  $N = 2$  theory, where there is a single LW partner with mass  $m_2$ , the appropriate Lagrangian is obtained via the substitutions  $\mathbb{1} \rightarrow 1$ ,  $C = \eta = 1$  and  $M = m_2^2$ . In this case, Eq. (2.20) yields  $\Delta b = -7/2C_2$ , in agreement with the result quoted in Ref. [8]. The remaining pure gauge contribution from the light field  $A_1$  and ghosts yields  $\Delta b = -11/3C_2$  as in the  $N = 2$  theory. We note that the contribution of the massive LW states in the  $N = 3$  theory is not twice the  $N = 2$  result due to the third term of Eq. (2.14), which leads to a loop diagram in which both the fields  $A_2$  and  $A_3$  propagate.

### 3. One-loop gauge unification

We may now apply the results of the previous section to evaluate one-loop gauge coupling unification. For the pure gauge contributions to the beta functions  $b_i$  of the gauge group factor  $G_i$ , the Standard Model results are  $-11/3C_2(G_i)$ ; these are modified to either  $-43/6C_2(G_i)$  or  $-49/6C_2(G_i)$  in the  $N = 2$  and  $N = 3$  LW extensions, respectively, following the discussion in the previous section. The contribution to the  $b_i$  for each chiral matter field is multiplied by either 3 or 5, since there are one or two Dirac partners in the  $N = 2$  and  $N = 3$  LW extension, respectively. Finally, the Higgs field contribution is multiplied by either 2 or 3, since each LW partner is also complex scalar. Ref. [8] notes that the LWSM does not unify at one loop, unless multiple Higgs doublets are included. As Table 1 indicates, we find this is the case if 8 Higgs doublets are included in the  $N = 2$  theory, or 6 in  $N = 3$ . However, we can now consider models in which each field has at least one LW partner, with some having two. These models solve the hierarchy problem since they are at least as convergent as the  $N = 2$  theory. This provides a wide range of possibilities for achieving more accurate unification. In Table 1 we give some of the simpler successful models, with the SM and MSSM one-loop results provided for comparison. The experimental central values of  $\alpha_1^{-1}(m_Z) = 59.00$  and  $\alpha_2^{-1}(m_Z) = 29.57$  [10] are taken as inputs, unification is assumed and  $\alpha_3^{-1}(m_Z)$  is then predicted. Of

**Table 1**

Predictions for  $\alpha_3^{-1}(m_Z)$  assuming one-loop unification. The experimental value is  $8.2169 \pm 0.1148$  [10]. The GUT scale is defined by  $\alpha_1^{-1}(M_{\text{GUT}}) = \alpha_2^{-1}(M_{\text{GUT}})$ . The abbreviations used are as follows: H = Higgs doublets, gen. = generation, LH = left handed.

Model	$N = 3$ fields	$(b_3, b_2, b_1)$	$M_{\text{GUT}}$ (GeV)	$\alpha_3^{-1}(m_Z)$	Error
SM	–	$(-7, -19/6, 41/10)$	$1 \times 10^{13}$	14.04	$+50.8\sigma$
MSSM	–	$(-3, 1, 33/5)$	$2 \times 10^{16}$	8.55	$+2.9\sigma$
$N = 2$ 1H LWSM	none	$(-19/2, -2, 61/5)$	$4 \times 10^7$	14.03	$+50.6\sigma$
$N = 3$ 1H LWSM	all	$(-9/2, 25/6, 203/10)$	$9 \times 10^6$	13.76	$+48.3\sigma$
$N = 2$ 8H LWSM	none	$(-19/2, 1/3, 68/5)$	$1 \times 10^8$	7.76	$-4.01\sigma$
$N = 3$ 6H LWSM	all	$(-9/2, 20/3, 109/5)$	$2 \times 10^7$	7.85	$-3.16\sigma$
$N = 2$ 1H LWSM	gluons	$(-25/2, -2, 61/5)$	$4 \times 10^7$	7.81	$-3.55\sigma$
$N = 2$ 1H LWSM	gluons, 1 gen. quarks	$(-59/6, 0, 41/3)$	$7 \times 10^7$	8.40	$+1.55\sigma$
$N = 2$ 1H LWSM	1 gen. LH fields	$(-49/6, 2/3, 191/15)$	$4 \times 10^8$	8.03	$-1.66\sigma$
$N = 2$ 2H LWSM	LH leptons	$(-19/2, 1/3, 68/5)$	$1 \times 10^8$	7.76	$-4.01\sigma$
$N = 2$ 2H LWSM	gluons, quarks, 1H	$(-9/2, 9/2, 169/10)$	$3 \times 10^8$	8.21	$-0.06\sigma$

course, Table 1 does not represent an exhaustive list of the possible variations on the LWSM. It illustrates that models with improved gauge coupling unification at the one-loop level can be achieved in the higher-derivative LW theories of Ref. [7] by choosing an appropriate set of higher-derivative terms, beyond the minimally required set, without adding additional fields in the HD theory. It should be noted that the results in Table 1 will be altered by two-loop corrections to the running of the gauge couplings, which have not been computed in any version of the LWSM. In addition, specific models will have threshold corrections that will modify these results. It should be understood that the deviations from the experimental value of  $\alpha_3^{-1}(m_Z)$  shown in the table are subject to these uncertainties.

#### 4. Completions and conclusions

Although we will not attempt to construct explicit ultraviolet completions that are consistent with the LW theories listed in Table 1, a number of points are worth noting. First, the unification shown assumes the GUT normalization of hypercharge, the choice that leads to unification in conventional SU(5) or trinified gauge theories. Nevertheless, it is possible in strongly coupled string theories for the string and unification scales to coincide, so that one may never realize a grand unified field theory at any intermediate point. If one were interested in conventional grand unification, then two issues become relevant. First, the LW theories in Table 1 unify at a scale much lower than in the Standard Model, with the GUT scale ranging from  $4 \times 10^7$  to  $4 \times 10^8$  GeV in the more successful models. Ref. [8] points out that the low unification scale in their multi-Higgs LWSM is not consistent with semi-simple unification, due to the constraint from proton decay. However, this assessment may be overly pessimistic. Higher-dimensional SU(5) GUTs can avoid the problem of proton decay from GUT gauge boson exchange by placing fermions at orbifold fixed points where the wave functions of the offending bosons vanish (see, for example, the discussion in [11]). There does not seem to be any reason why the same approach couldn't be adapted here. The compactification scale in theories where GUT symmetry is broken by orbifold projection can be taken at or near the grand unification scale (as in Ref. [12]), so the effective theory at lower energies is four-dimensional; the beta functions shown in Table 1 therefore apply, as does the accounting of divergences in four-dimensional LW theories. (Unification in theories with a lower compactification scale would require a different analysis since the gauge coupling running is affected significantly by Kaluza–Klein thresholds.) The advantage of higher-dimensional GUTs is that one can place incomplete multiplets of matter fields at orbifold fixed points where the GUT symmetry is broken. At such fixed points, it is consistent with gauge invariance to write down different higher-derivative kinetic

terms for what would otherwise be different components of a single GUT multiplet in a 4D theory. This approach makes it feasible, for example, to have an  $N = 2$  LW unified theory where only the left-handed fermions of one generation have  $N = 3$  partners (i.e., the third from last example in Table 1). Finally, one may pursue trinification, as advocated in Ref. [8], so that there is no gauge-boson-induced proton decay. In this case, the extra-dimensional construction has similar benefits. In the  $N = 3$  six-Higgs doublet model, for example, one does not need to introduce six complete 27-plets if the GUT group is broken by extra-dimensional boundary conditions on an interval, an approach discussed in Refs. [13]. It is also worth noting that in trinified theories where the equality of SU(3) gauge couplings at the unification scale is a consequence of string boundary conditions rather than a discrete cyclic symmetry of the field theory [14], the presence of  $N = 3$  gluons would be consistent with the SU(3)<sup>3</sup> gauge symmetry and would allow unification without a large multiplicity of Higgs doublets.

In summary, we have shown that the particle content needed to fix one-loop gauge unification in the LWSM can be introduced in a more restricted way than previously considered, by extending the non-generic set of HD interactions that are consistent with the LW construction to higher order for some Standard Model fields. Computation of the pure gauge contributions to the beta functions requires a computation that does not seem to generalize trivially to theories with arbitrary  $N$ , and was computed here for the next-to-minimal case of  $N = 3$ . Explicit unified field theories that correspond to some of the solutions discussed in the previous section seem plausible in the framework of orbifold GUTs, where matter fields may be placed at fixed points with reduced gauge symmetry so that HD kinetic terms may differ between fields that would otherwise live within the same 4D GUT multiplet. The construction of explicit unified theories of this type seems worthy of further investigation.

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