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Weighted power counting and perturbative unitarity

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We consider the relationship between renormalizability and unitarity at a Lifshitz point in d dimensions. We test tree unitarity for theories containing only scalars and fermions, and for pure gauge theory. In both cases, we find the requirement of weighted power-counting renormalizability is equivalent to that of tree unitarity.

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I. INTRODUCTION

Lorentz violating (LV) field theories have been studied extensively, with constraints being placed on LV operators of the standard model (see, for example, Refs. [1,2]). The idea of breaking Lorentz invariance by imposing Lifshitz-point scaling opened the doors to rendering previously nonrenormalizable theories renormalizable [3]. The main incentive is to obtain an UV (albeit LV) completion of a nonrenormalizable field theory which becomes Lorentz invariant in the infrared. There are some advantages to invoking Lorentz violation. From the point of view of eliminating unwanted ultraviolet divergences, there are many regularization techniques available. In each technique, the regularization is usually removed in some manner, but Lorentz violation provides a physical cutoff [4]. Also, by imposing Lifshitz-point scaling, one can make virtually any theory power-counting renormalizable. Unfortunately, this is not a panacea as, for instance, there is no *a priori* equivalence between power-counting renormalizability and unitarity. For example, the standard model is power-counting renormalizable, but one can derive perturbative unitarity bounds on the Higgs mass. The preceding remarks are from the point of view of a Wilsonian quantum field theory. It is interesting to note, as recently proposed by Dvali *et al.* [5,6], it may be possible to have nonrenormalizable, strongly coupled theories which self-unitarize by formation of extended, classical field configurations. Thus, the indication of strong coupling does not necessarily imply new physics, but the theory may begin to obstruct short distance measurements in analogy to the formation of black holes in two-to-two scattering at trans-Planckian energy.

Most recently, Lifshitz-point field theories have gained popularity because of the prospect of producing a consistent, renormalizable quantum theory of gravity [3,7]. A Lifshitz point is a conformal fixed point invariant under anisotropic rescalings of space and time, with suitable scaling dimensions for fields. The anisotropic scaling leads to a modification of power-counting arguments for renormalizability, and also changes the relativistic phase space factor thereby altering the condition for perturbative unitarity. Many theories can be constructed in which Lifshitz-point scaling restores renormalizability; for example, consider gauge theories in higher dimensions [8–12]. Unfortunately, Lifshitz type UV-completions of these theories are not necessarily without problems. The proposed UV completion of five-dimensional QED exhibits a fine-tuning problem [12], and in the case of Hořava-Lifshitz gravity there has been concern over the consistency of various versions of the theory [13–18]. Some of these versions become strongly coupled at a certain scale and there is a breakdown of the perturbative expansion (for recent reviews related to this problem see [19,20]). One way to see this breakdown is to check the bound for perturbative unitarity [13,21]. Since making a theory power-counting renormalizable does not guarantee the absence of strong coupling, it is interesting to ask what happens to perturbative unitarity for an arbitrary theory at a Lifshitz point.

The purpose of this paper is to present, in a simple setting, the manner in which making a theory renormalizable affects perturbative unitarity– in particular, perturbative unitarity at tree level [21]. We will quickly review some necessary background material for a theory containing scalars and fermions, and then for a pure gauge theory. Then, we will derive the condition for tree unitarity in tree-level scattering processes, and apply it in these two settings.

II. BACKGROUND

We attempt to succinctly present the relevant material on scalars, fermions, and gauge fields at a Lifshitz point. A more complete story of scalars and fermions can be found in Ref. [3], and for a more detailed discussion of gauge fields, see Refs. [8,9]. We will, for the most part, follow the notation of [9], where we consider a spacetime manifold of dimension d to be split as the product $\mathbb{R} \times \mathcal{M}_{\bar{d}}$. The spatial manifold $\mathcal{M}_{\bar{d}}$ is of dimension $\bar{d}$ and the symmetry group considered as $O(\bar{d})$. In general, we can consider the spacetime manifold to be split into two sets of coordinates. If we assume time and some spatial coordinates to be in the first set, then the second set contains only spatial coordinates. When appropriate, we will use a hat to denote the set of coordinates containing time and a bar to denote the remaining spatial coordinates. So, for instance, the dimension of the spacetime is $d = \hat{d} + \bar{d}$. As evidenced above, we will work in the special case of $\hat{d} = 1$, where time is split from the spatial coordinates. The case $\hat{d} = 1$ is important because it is contained in a set of sufficient conditions for the absence of spurious subdivergences, as described in section III. Also, $\hat{d} = 1$ is the case considered for Hořava-Lifshitz gravity.

Scaling at a Lifshitz point by the parameter λ results in the transformation

$$\hat{x} \longrightarrow \lambda\hat{x}, \qquad \bar{x} \longrightarrow \lambda^{1/z}\bar{x}, \tag{2.1}$$

where z is a positive nonzero integer representing the severity of the difference in scaling. For this to be a symmetry of the action the fields must scale accordingly.

A. Scalars and fermions

For the purposes of this paper, the free part of the Lagrangian for a scalar and fermion can be written as

$$\mathcal{L}_{\text{free}} = \frac{1}{2}(\hat{\partial}\phi)^2 + \frac{1}{2\Lambda_L^{2z-2}}(\bar{\partial}^z\phi)^2 + \bar{\psi}i\hat{\not{\partial}}\psi + \frac{1}{\Lambda_L^{z-1}}\bar{\psi}(i\bar{\not{\partial}})^z\psi. \tag{2.2}$$

We have made use of some shorthand notation, which can be written out explicitly as

$$(\hat{\partial}\phi)^2 = \sum_{i,j}^{\hat{d}}(\hat{\partial}_i\phi)(\hat{\partial}_j\phi)\eta^{ij}, \quad \text{and}$$

$$(\bar{\partial}^z\phi)^2 = \sum_{\substack{(i_1,\cdots,i_z)\\(j_1,\cdots,j_z)}}^{d}(\bar{\partial}_{i_1}\cdots\bar{\partial}_{i_z}\phi)(\bar{\partial}_{j_1}\cdots\bar{\partial}_{j_z}\phi)\eta^{i_1j_1}\cdots\eta^{i_zj_z},$$

where the indices of the first sum start from one and the indices of the second sum all start from $\hat{d}+1$. The tensor η is the d-dimensional Minkowski metric with components $\eta_{11} = 1$, $\eta_{ii} = -1$ for $i > 1$, and the rest are zero. Shorthand notation was also used to write the fermion part of the Lagrangian, with contractions between partial derivatives and gamma matrices, but we omit the explicit form as it is clear from the above scalar example. Finally, the parameter Λ_L dictates the energy at which the anisotropic scaling is important. If we assign the weighted dimensions

$$[\hat{\partial}] = 1, \qquad [\bar{\partial}] = \frac{1}{z}, \tag{2.3}$$

we see that the weighted dimension of the spacetime volume element $[d^dx] = [d\hat{x}d^{\bar{d}}\bar{x}] = -1 - \bar{d}/z \equiv -đ$. Thus, the weighted dimension of the Lagrangian is $đ$. By comparison, we also find the following assignments:

$$[\phi] = \tfrac{1}{2}(đ - 2), \qquad [\psi] = \tfrac{1}{2}(đ - 1). \tag{2.4}$$

The propagator for the scalar field will take the following form:

$$i\Delta_F(p) = \frac{i}{\hat{p}^2 - \bar{p}^{2z}/\Lambda_L^{2z-2}}, \tag{2.5}$$

and we see, as $[1/\hat{p}^2] = -2$, the weighted dimension (or weight) of the propagator is minus two. Analogously, the weight of the fermion propagator is minus one.

B. Gauge fields

If we decompose the gauge field as $A = (\hat{A}, \bar{A})$ and the covariant derivative as $D = (\hat{D}, \bar{D}) = (\hat{\partial} - ig\hat{A}, \bar{\partial} - ig\bar{A})$, where g is the gauge coupling, we have the following weighted dimensions:

$$[g\hat{A}] = [\hat{D}] = 1, \qquad [g\bar{A}] = [\bar{D}] = \frac{1}{z}. \tag{2.6}$$

We can also separate the field strength by its components, and make the following shorthand definitions:

$$\hat{F} \equiv F_{\hat{\mu}\,\hat{\nu}}, \qquad \tilde{F} \equiv F_{\hat{\mu}\,\bar{\nu}}, \qquad \bar{F} \equiv F_{\bar{\mu}\,\bar{\nu}}. \tag{2.7}$$

For the case where $\hat{d} = 1$, we have that $\hat{F}$ is identically zero, but we will temporarily assume the case of general $\hat{d}$ to determine the weight assignments. If we consider the term $(\hat{\partial}\hat{A})^2$ to be of weight $đ$, then we can determine the weight of the gauge coupling $[g] = 2 - đ/2$, and the weights of the gauge fields and field strength components:

$$[\hat{A}] = \frac{đ}{2} - 1, \qquad [\bar{A}] = \frac{đ}{2} - 2 + \frac{1}{z}, \qquad [\hat{F}] = \frac{đ}{2},$$
$$[\tilde{F}] = \frac{đ}{2} - 1 + \frac{1}{z}, \qquad [\bar{F}] = \frac{đ}{2} - 2 + \frac{2}{z}. \tag{2.8}$$

Also, for later calculations, the weights of the propagators are [8]

$$\begin{aligned} \hat{P} &\equiv [\langle\hat{A}\,\hat{A}\rangle] = -2, \\ \tilde{P} &\equiv [\langle\hat{A}\,\bar{A}\rangle] = -3 + \frac{1}{z}, \\ \bar{P} &\equiv [\langle\bar{A}\,\bar{A}\rangle] = -4 + \frac{2}{z}. \end{aligned} \tag{2.9}$$

We will only be concerned with cases where the couplings appearing in interactions, λ_i, have positive weight. In particular, we wish to investigate the class of theories which have all $[\lambda_i] \geq \chi$, where χ is some non-negative, minimal weight and the Lagrangian is written as

$$\mathcal{L} = \frac{1}{\bar{g}^2}\mathcal{L}_r(\bar{g}A, \bar{g}\,\bar{C}, \bar{g}C), \tag{2.10}$$

where C and $\bar{C}$ denote the ghosts and antighosts. The coupling $\bar{g}$ (not necessarily the gauge coupling) is a factor of the interaction couplings, $\lambda_i = \bar{\lambda}_i\bar{g}^{n_i-2}$, and has weight

$$[\bar{g}] = \min_i \frac{[\lambda_i]}{n_i - 2}, \tag{2.11}$$

where n_i corresponds to the i-th vertex with n external legs ($n > 2$). The weight of $\bar{g}$ satisfies the relations

$$[\lambda_i] \geq (n_i - 2)[\bar{g}], \quad \text{and} \quad 0 \leq [\bar{g}] \leq [g], \tag{2.12}$$

such that $[\bar{\lambda}_i] \geq 0$. Weighted power-counting renormalizable Lagrangians of the form in (2.10) have been proven to be renormalizable (see Ref. [9]).

We note that the hat component of the gauge field has the same weight as the scalar field, while the bar component has lower weighted dimension. In some instances, the weight of $\bar{F}$ can even be negative. If we write the vertices as products of $\bar{g}F$ and covariant derivatives, $\bar{F}$ may have negative weight while preserving polynomiality of the Lagrangian. In order to have a finite number of interaction terms, we require $[\bar{g}\bar{F}] > 0$, as this covers the other components of F as well. Thus, $[\bar{g}]$ is bounded above and below:

$$-[\bar{F}] < [\bar{g}] \leq [g]. \tag{2.13}$$

Of course, if $[\bar{F}]$ is positive the lower bound is zero. The range of possible values for the weight of $\bar{g}$ will dictate the set of allowed interactions; consequently, $[\bar{g}] = [g]$ is the most restrictive.

C. Power counting

We will now quickly review the method of weighted power counting for a single field, as in Ref. [3]. Consider a diagram with E external legs, I internal lines, L loops, and V vertices. In general, the diagram will involve an integral of the form

$$\prod_i^L \left(\int d\hat{q}_i d^{\bar{d}} \bar{q}_i \right) \prod_j^I P_j \prod_k^V V_k, \tag{2.14}$$

where P_i are the propagators on the internal lines and V_k are the vertices in the diagram. If a vertex contains n hat derivatives and m bar derivatives, we define the weighted degree of divergence of an N-point vertex of type α as $\delta_N^{(\alpha)} = n + m/z$. We also define the number of vertices, $v_N^{(\alpha)}$, corresponding to an N-point interaction of type α. The weighted superficial degree of divergence (ω) can be written as the sum of the contribution from the loop measure, propagators, and (α, N)-type vertices carrying momentum factors of weight $\delta_N^{(\alpha)}$.

$$\omega = L\text{đ} + PI + \sum_{(\alpha,N)} \delta_N^{(\alpha)} v_N^{(\alpha)}, \tag{2.15}$$

where the weight of the propagator is P and the final term is the sum over all the vertices in the diagram. Using the topological relations $L = I - V + 1$ and $E + 2I = \sum N v_N^{(\alpha)}$, we arrive at the expression

$$\omega = L\text{đ} - \frac{E}{2}(\text{đ} + P) + \sum v_N^{(\alpha)}(\delta_N^{(\alpha)} - D(N)), \tag{2.16}$$

where $D(N) \equiv \text{đ}(1 - \frac{N}{2}) - P\frac{N}{2} = \text{đ} - \frac{N}{2}(\text{đ} + P)$. Now, the condition for weighted power-counting renormalizability is $\delta_N^{(\alpha)} \leq D(N)$. This relation implies there are no couplings of negative weighted dimension. Likewise, it implies there are no operators of weighted dimension greater than đ. Since we will deal in some detail with $D(N)$, for two types of theories, we show its resulting expression in each case. Note, for all fields (f) the dimensions of the fields may be written as $[f] = \frac{1}{2}(\text{đ} + P_f)$, where P_f is the weight of the propagator of f. For theories only containing scalars and fermions, the result for $D(N)$ may be written as

$$D(N_B + N_F) = \text{đ} - N_B[\phi] - N_F[\psi], \tag{2.17}$$

where N_B is the number of bosons and N_F is the number of fermions. A similar expression is obtained for pure gauge theories:

$$D(\hat{N} + \bar{N} + N_{gh}) = \text{đ} - \hat{N}[\hat{A}] - \bar{N}[\bar{A}] - N_{gh}[C], \tag{2.18}$$

where $\hat{N}$ is the number of hat-component gauge fields, $\bar{N}$ is the number of bar-component gauge fields, and N_{gh} is the number of ghosts and antighosts. Since we will only be concerned with tree-level diagrams, we set $N_{gh} = 0$.

III. PERTURBATIVE UNITARITY CONDITION

In order to determine the condition for perturbative unitarity, we proceed by developing the formalism in analogy to the more familiar discussion in four dimensions maintaining Lorentz invariance (a similar derivation, scattering scalars in four dimensions at a Lifshitz point, was found in [18]). We may start with the expression of the generalized optical theorem for forward scattering [22]:

$$2\,\mathrm{Im}[\mathcal{M}(k_1k_2 \to k_1k_2)] = \sum_n \int d\Pi_n |\mathcal{M}(k_1k_2 \to \{q_n\})|^2. \tag{3.1}$$

Labelling the initial state as "a" and separating out the elastic portion, we have

$$2\,\mathrm{Im}[\mathcal{M}(a \to a)] - \int \frac{d^{\bar{d}}\bar{q}_1 d^{\bar{d}}\bar{q}_2}{(2\pi)^{2\bar{d}} E_1 E_2} |\mathcal{M}(a \to q_1q_2)|^2 (2\pi)^d \times \delta^{(d)}(k_1 + k_2 - q_1 - q_2) > 0. \tag{3.2}$$

To proceed with the derivation, we presume the scattering takes place in the center-of-mass frame. Assuming we have a dispersion relation that looks like $E = \sqrt{f(\bar{q}) + m^2}$, where $f(\bar{q})$ is a positive, monotonic function of the magnitude of the spatial momenta, we can perform most of the integrals to get

$$\frac{\bar{q}_1^{\bar{d}-1}}{(2\pi)^{\bar{d}-1} 4E_{\mathrm{cm}} f'(\bar{q}_1)} \int d\Omega_{\bar{d}-1} |\mathcal{M}|^2 \sim \frac{1}{4(2\pi)^{\bar{d}-1}} E_{\mathrm{cm}}^{\bar{d}/z-3} \int d\Omega_{\bar{d}-1} |\mathcal{M}|^2, \tag{3.3}$$

where we have taken $f'(\bar{q}) \sim \bar{q}^{2z-1}$, at high energy. In general, for two-to-two scattering $\lambda_1\lambda_2 \rightarrow \lambda_3\lambda_4$, $\mathcal{M}$ is the helicity amplitude $\mathcal{M}_{\lambda_1\lambda_2,\lambda_3\lambda_4}$, where λ_i corresponds to the helicity of the i-th particle. The scattering takes place in a plane, and the amplitude is a function of $E_{\rm cm}$ and the angle θ between incoming and outgoing particles. The helicity amplitude can then be expanded in terms of Wigner d functions: $d^j_{\Lambda\Lambda'}(\theta)$, with $\Lambda = \lambda_1 - \lambda_2$ and $\Lambda' = \lambda_3 - \lambda_4$. In the following, we assume specific helicity configurations such that $\Lambda = \Lambda' = 0$, where the d functions become the Legendre polynomials: $d^j_{00}(\theta) = P_j(\cos(\theta))$. This is done for clarity of presentation, but it should be possible to generalize the result to arbitrary helicity considerations. Now, we expand the invariant scattering amplitude in terms of Legendre polynomials:

$$\mathcal{M}(E_{\rm cm}, \cos(\theta)) = 16\pi\sum_j (2j+1)a_j^{el}P_j(\cos(\theta)). \quad (3.4)$$

Plugging this into (3.2), we get the following expression:

$$\begin{aligned} &32\pi\sum_j (2j+1)\,\mathrm{Im}(a_j^{el})P_j(\cos(\theta)) \\ &\quad - C(\bar{d})E_{\rm cm}^{\bar{d}/z-3}\sum_j (2j+1)|a_j^{el}|^2 > 0, \end{aligned} \quad (3.5)$$

where $C(\bar{d})$ is a constant, which depends on $\bar{d}$, resulting from the various integrations. Since the scattering matrix for elastic scattering is diagonal in j, Eq. (3.5) constrains each partial-wave amplitude a_j^{el} independently. After some rearranging, we arrive at the following:

$$\mathrm{Re}(a_j^{el})^2 + \left[\mathrm{Im}(a_j^{el}) - \frac{16\pi}{C(\bar{d})}E_{\rm cm}^{-(\bar{d}/z-3)}\right]^2 < \left[\frac{16\pi}{C(\bar{d})}E_{\rm cm}^{-(\bar{d}/z-3)}\right]^2. \quad (3.6)$$

The above inequality defines the unitarity circle; as long as we are within the circle, perturbative unitarity holds. This translates into a bound on the energy growth of the right-hand side of Eq. (3.1):

$$\begin{aligned} \frac{1}{2}\sum_n \int d\Pi_n |\mathcal{M}(a \rightarrow \{q_n\})|^2 &= \mathrm{Im}[\mathcal{M}(a \rightarrow a)] \\ &\lesssim (\mathrm{const})E^{-(\not{d}-4)}. \end{aligned} \quad (3.7)$$

The condition for tree unitarity then follows from some dimensional analysis. Using

$$[d\Pi_n] = n(\not{d} - 2) - \not{d}, \quad \text{and assuming} \quad \mathcal{M} \sim E^\beta, \quad (3.8)$$

we get, from the energy bound (3.7),

$$\beta \leq 2 - \frac{n}{2}(\not{d} - 2) = \not{d} - \frac{N}{2}(\not{d} - 2), \quad (3.9)$$

where, in the final equality, we substituted $n = N - 2$.

A. Application to scalars and fermions

In order to check the condition of tree unitarity for scalars and fermions, it is useful to rewrite the unitarity condition as

$$\beta \leq D(N) + \sum_i^N \delta(f_i), \quad (3.10)$$

where the sum is over external lines, and $\delta(f_i)$ is the highest power of energy the field f_i, when contracted with an external state, can contribute to the scattering amplitude. A scalar external line contributes an energy of E^0, while a fermion external line contributes, at most, $E^{1/2}$. For example, the four-point interaction $\mathcal{L} \supset \phi\phi\bar{\psi}\psi$ has $\sum \delta(f) = 2\delta(\phi) + 2\delta(\psi) = 2(0) + 2(1/2) = 1$. Thus, the tree-level scattering amplitude grows at most like E. Now, consider the general interaction term written schematically as

$$k\hat{\partial}^s\bar{\partial}^t\phi^{N_B}(\bar{\psi}\psi)^{N_F/2}, \quad (3.11)$$

where N_B counts the number of scalars, N_F is the number of fermions, $t + s$ is the number of derivatives, and k is a constant of dimensionality κ. We should note, perturbative unitarity can also be violated if the propagator contains more than two time derivatives. For the argument that there are no more than two time derivatives, and, in particular, no time derivatives in interactions with $N > 2$, the reader may check Ref. [3]. The weighted degree of divergence of the $(N_B + N_F)$-point interaction in Eq. (3.11) is $\delta_{N_B+N_F} = s + t/z$. The contribution from the external lines can be represented as $\sum \delta(f)$, as defined before. Substituting $\beta = \delta_{N_B+N_F} + \sum \delta(f)$ into Eq. (3.10) we get

$$\delta_{N_B+N_F} \leq D(N_B + N_F), \quad (3.12)$$

which is the condition for weighted renormalizability from before. So, for an N-point interaction, the unitarity condition is equivalent to the renormalizability condition.

To check tree unitarity for a tree-level diagram containing a propagator, Eq. (3.10) is again the most convenient. This condition, for a vertex with N_1 lines connected to a vertex with N_2 lines by the field $f_{\rm prop}$ with a propagator of weight P, is

$$\begin{aligned} &\delta_{N_1} + \delta_{N_2} + P + \sum_i^{N_1}\delta(f_i) + \sum_i^{N_2}\delta(f_i) - 2\delta(f_{\rm prop}) \\ &\quad \leq D(N_1 + N_2 - 2) + \sum_i^{N_1+N_2-2}\delta(f_i), \end{aligned} \quad (3.13)$$

where $\delta(f_{\rm prop})$ is the energy factor the field $f_{\rm prop}$ would contribute were it an external line. We can expand $D(N_1 + N_2 - 2) = D(N_1) + D(N_2) + D(-2) - 2\not{d}$, and use the fact that $D(-2) = 2\not{d} + P$ to arrive at the condition:

$$\delta_{N_1} + \delta_{N_2} \leq D(N_1) + D(N_2), \quad (3.14)$$

which always holds since the individual vertices are renormalizable. The result (3.14), along with the result of (3.12), implies that, for scalars and fermions in tree-level scattering processes, the tree unitarity condition is equivalent to the condition of weighted power-counting renormalizability.

B. Application to gauge fields

The treatment of gauge theory is arguably more interesting than that of scalars and fermions. For instance, in a four-dimensional Lorentz invariant theory, we cannot simply add a mass term for a gauge field, as the resulting theory would violate unitarity. After witnessing the troubles present in the original version of Hořava-Lifshitz gravity, it is natural to wonder, from the perspective of obtaining an UV-complete, higher dimensional gauge theory, what happens to perturbativity.

For gauge fields at tree level, the analysis is analogous to the treatment for scalars and fermions above, so we will briefly reiterate the arguments. The condition of tree unitarity can again be written in the form of Eq. (3.10), where the external line contributions are $\delta(\hat{A}) = 0$ and $\delta(\bar{A}) = -1 + \frac{1}{z}$. We consider the following schematic N-point vertex:

$$\bar{\lambda}_i \bar{g}^{N-2} \hat{\partial}^s \bar{\partial}^t \hat{A}^{\hat{N}} \bar{A}^{\bar{N}}, \tag{3.15}$$

where $\hat{N}$ is the number of $\hat{A}$s and $\bar{N}$ is the number of $\bar{A}$s. We may write $\beta = \delta_{\hat{N}+\bar{N}} + \sum \delta(f)$, and from Eq. (3.10) we obtain

$$\delta_{\hat{N}+\bar{N}} \leq D(\hat{N} + \bar{N}), \tag{3.16}$$

which is the condition for power-counting renormalizability. Similarly for N_1-point and N_2-point vertices connected by a field with propagator of weight P, we arrive at the following expression:

$$\delta_{N_1} + \delta_{N_2} + P \leq D(N_1 + N_2 - 2). \tag{3.17}$$

Since the propagator could be $\langle \hat{A} \bar{A} \rangle$, we make use of the relation $\tilde{P} = \frac{1}{2}(\hat{P} + \bar{P})$. The result is the same as for the scalar and fermion case:

$$\delta_{N_1} + \delta_{N_2} \leq D(N_1) + D(N_2), \tag{3.18}$$

which holds if we assume each vertex is power-counting renormalizable. So, at tree level, weighted power-counting renormalizable pure gauge theories satisfy perturbative unitarity.

IV. CONCLUSIONS

We have seen that while imposing Lifshitz-point scaling can render a theory renormalizable, it also modifies the relativistic phase space factor and thereby the condition for perturbative unitarity. For the theories considered, the tree unitarity condition holds if and only if the Lagrangian is weighted power-counting renormalizable.

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