2012

Preservers of Matrix Pairs with a Fixed Inner Product Value

Chi-Kwong Li
William & Mary, ckli@wm.edu

Lucijan Plevnik

Peter Semrl

Recommended Citation

Li, C. K., Plevnik, L., & Šemrl, P. (2012). Preservers of matrix pairs with a fixed inner product value. Oper. Matrices, 6(3), 433-464.

Operators and Matrices
Volume 6, Number 3 (2012), 433–464
doi:10.7153/oam-06-29

PRESERVERS OF MATRIX PAIRS WITH A FIXED INNER PRODUCT VALUE

CHI-KWONG LI, LUCIJAN PLEVNIK AND PETER ŠEMRL

Abstract. Let $\mathscr{V}$ be the set of $n\times n$ hermitian matrices, the set of $n\times n$ symmetric matrices, the set of all effects, or the set of all projections of rank one. Let c be a real number. We characterize bijective maps $\phi:\mathscr{V}\to\mathscr{V}$ satisfying $\operatorname{tr}(AB)=c \iff \operatorname{tr}(\phi(A)\phi(B))=c$ with some additional restrictions on c, depending on the underlying set of matrices.

Mathematics subject classification (2010): 15A86, 15B57.
Keywords and phrases: hermitian matrix; real symmetric matrix; projection; effect; trace; preserver.

REFERENCES

[1] J. T. CHAN, C. K. LI, AND N. K. SZE, *Mappings on matrices: Invariance of functional values of matrix products*, J. Aust. Math. Soc. **81** (2006), 165–184.
[2] J. T. CHAN, C. K. LI, AND N. K. SZE, *Mappings preserving spectra of products of matrices*, Proc. Amer. Math. Soc. **135** (2007), 977–986.
[3] S. CLARK, C. K. LI, AND L. RODMAN, *Spectral radius preservers of products of nonnegative matrices*, Banach J. Math. Anal. **2** (2008), 107–120.
[4] M. DOBOVIŠEK, B. KUZMA, G. LEŠNJAK, C. K. LI, AND T. PETEK, *Mappings that preserve pairs of operators with zero triple Jordan product*, Linear Algebra Appl. **426** (2007), 255–279.
[5] V. FORSTALL, A. HERMAN, C. K. LI, N. S. SZE, AND V. YANNELLO, *Preservers of eigenvalue inclusion sets of matrix products*, Linear Algebra Appl. **434** (2011), 285–293.
[6] J. HARTMAN, A. HERMAN, AND C. K. LI, *Preservers of eigenvalue inclusion sets*, Linear Algebra Appl. **433** (2010), 1038–1051.
[7] J. C. HOU, C. K. LI, AND N. C. WONG, *Jordan isomorphisms and maps preserving spectra of certain operator products*, Studia Math. **184** (2008), 31–47.
[8] J. C. HOU, C. K. LI, AND N. C. WONG, *Maps preserving the spectrum of generalized Jordan product of operators*, Linear Algebra Appl. **432** (2010), 1049–1069.
[9] C. K. LI, E. POON, AND N. S. SZE, *Preservers for norms of Lie product*, Operators and Matrices **3** (2009), 187–203.
[10] C. K. LI AND L. RODMAN, *Preservers of spectral radius, numerical radius, or spectral norm of the sum on nonnegative matrices*, Linear Algebra Appl. **430** (2009), 1739–1761.
[11] C. K. LI, P. ŠEMRL, AND N. K. SZE, *Maps preserving the nilpotency of products of operators*, Linear Algebra Appl. **424** (2007), 222–239.
[12] L. MOLNÁR, *Selected preserver problems on algebraic structures of linear operators and on function spaces*, Springer-Verlag, Berlin, Heidelberg, 2007.
[13] U. UHLHORN, *Representation of symmetry transformations in quantum mechanics*, Ark. Fysik **23** (1963), 307–340.
[14] Z.-X. WAN, *Geometry of matrices*, World Scientific, Singapore, New Jersey, London, Hong Kong, 1996.

Paper OaM-06-29