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A Mathematical Model of Chincoteague Bay, Virginia

John Joseph Vaccaro

College of William and Mary - Virginia Institute of Marine Science

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A MATHEMATICAL MODEL OF CHINCOTEAGUE BAY, VIRGINIA

A Thesis
Presented to
The Faculty of the School of Marine Science
The College of William and Mary in Virginia

In Partial Fulfillment
Of the Requirements for the Degree of
Master of Arts

by
John Joseph Vaccaro
1976
APPROVAL SHEET

This thesis is submitted in partial fulfillment of the requirements for the degree of Master of Arts in Marine Science

John Joseph Vaccaro

Approved, August 1976

Bruce J. Neilson, Ph.D.

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Robert Byrne, Ph.D.

Paul Hyer, Ph.D.

Albert Kuo, Ph.D.

Richard Wetzel, Ph.D.
Dedication

This thesis is dedicated to Jane.
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A MATHEMATICAl MODEL OF CHINCOTEAGUE BAY, VIRGINIA
I. INTRODUCTION

In the past years, several mathematical models of circulation and dispersion have been developed. These numerical tidal models have been developed due to the increased importance of the ecology of the estuarine systems. Accurate time varying predictive models for studies on circulation and water quality are now important for planning future land development and waste load allocations in the proximity of an estuarine system.

An estuarine system can be represented by a mathematical formulation consisting of interacting variables. The system responds to natural and manmade external inputs by spatial and temporal arrangement of pertinent parameters. The essence of the mathematical formulation of the system is partial differential equations with variable coefficients. With known or estimated initial data and boundary conditions, the equations comprise an initial value problem whose solution will yield spatial and temporal arrangement of the unknown parameters. This solution will comprise a realistic mathematical hydrodynamic model of an estuary.

The model of Leendertse (1967) has been chosen as the hydrodynamic model for Chincoteague Bay, Virginia.
(Figure 1.). This model incorporates the time-dependent, two-dimensional, vertically-integrated equations of motion and continuity. The parabolic mass-balance equations for conservative constituents are solved for the transport of dissolved constituents by the method of Peaceman and Rachford (1955) as presented by Hess et al (1975).

The sections that follow will deal with the formulation of the mathematics of the problem and the solution method. The model use is then described with pertinent tests to determine effects on model prediction of the tidal dynamics of the Bay. The final part correlates the verification of the model with field data collected by the VIMS Department of Physical Oceanography and Maryland Department of Natural Resources during the period from August 18, 1975 to August 28, 1975.
Figure 1. Map of Chincoteague Bay, Virginia
II. MATHEMATICS OF THE MODEL

2.0 Introduction

In this section the working time-dependent hydrodynamic and convective-diffusive equations for unsteady motion will be developed. The derivation is a vertical integration of the equations that express the conservation of mass and momentum, and mass-balance. It will be shown what assumptions are involved and which terms are neglected in order to arrive at simplified solvable equations. The equations will be applied in the Eulerian form with a right-handed coordinate system such that the X- and Y- axes, $X_1$ and $X_2$, are in the horizontal plane and the Z- axis, $X_3$, is directed vertically upward. (Figure 2.). The actual derivation of the momentum equations will not be done since this can be found in many standard texts, e.g. Lamb (1932), and Schlichting (1968). The assumptions made in deriving the Navier-Stokes equations, conservation of mass, and mass-balance, along with the simplifying assumptions are:

1. The fluid is incompressible.
2. The fluid is a continuum.
3. The fluid is isotropic.
4. The stress components are symmetric.
5. The Coriolis terms involving the vertical velocity are small and can be neglected.
Figure 2. Definition Sketch of Variables
6. There are no density currents.
7. The variation of stress in the horizontal directions is negligible.
8. The tidal wave is much longer than the depth.
9. Barometric and wind effects on atmospheric pressure are small.
10. Fickian type diffusion.
11. Estuary is vertically homogenous.

2.1 Conservation of Mass

The conservation of mass states that the mass of a parcel of fluid remains constant. Mathematically stated,

\[ \frac{d \rho}{dt} = 0 \]  

2.1

This can be expanded to the form

\[ \frac{\partial \rho}{\partial t} + \sum_{i=1}^{3} (u_i \rho) = 0 \]  

2.2

where the first term is the temporal change of mass, the second term, convective term, is the change experienced by the fluid due to movement into a different location in space, and where

- \( x_i = \) a point in space, where \( x_1 = x, x_2 = y, x_3 = z \)
- \( t = \) time
- \( u_i = \) the instantaneous velocity at \( x_i \)
- \( \rho = \) the instantaneous density

By the assumption of incompressibility (2.2) can be reduced to

\[ \frac{\partial u_i}{\partial x_i} = 0 \]  

2.3

Now the instantaneous velocity can be written as
the sum of the average velocity over some period of time and the difference between the instantaneous and average velocity, the fluctuating velocity, whose mean is equal to zero, therefore

\[ u_i = \bar{u}_i + u'_i \]  \hspace{1cm} (2.4)

where \( \bar{u}_i \) = the average velocity over time, tidal and non-tidal component
\( u'_i \) = the fluctuating velocity, departure from the mean

A similar expression can be written for density as

\[ \rho = \bar{\rho} + \rho' \]  \hspace{1cm} (2.5)

where \( \bar{\rho} \) = average density at point \( x_i \)
\( \rho' \) = fluctuating density at point \( x_i \)

Substituting equations 2.4 and 2.5 into equation 2.3 and neglecting terms where the density and velocity fluctuations are multiplied together since they are small, Pritchard (1971) and Fofonoff (1962), results in

\[ \frac{\partial \bar{u}_i}{\partial x_i} = 0 \]  \hspace{1cm} (2.6)

Expanding equation 2.6 in rectilinear coordinates results in

\[ \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \]  \hspace{1cm} (2.7)

The average velocity can be written as the sum of the vertical average value of the velocity over the water depth and a term that represents the deviation of the
vertically averaged velocity from the mean velocity.

Therefore,

\[ \bar{u} = u + u' \]
\[ \bar{v} = v + v' \]
\[ \bar{w} = w + w' \]  \hspace{1cm} 2.8

where the vertically averaged value of the deviation term over the water depth is equal to zero and

\[ \bar{u} = \int_{h}^{b} \bar{u} \, dz \]  \hspace{1cm} 2.9

Neglecting the deviation term in equation 2.8 because of the shallow depths, substitution of equation 2.8 into eq. 2.7, and vertically integrating through the use of Leibnitz' Rule which states,

\[ \frac{d}{dt} \int_{a}^{b} f(x,t) \, dx = \int_{a}^{b} \frac{df}{dt} \, dx + f(b) \frac{db}{dt} - f(a) \frac{da}{dt} \]  \hspace{1cm} 2.10

results in the expression

\[ \frac{\partial}{\partial x} \int_{h}^{b} u \, dz + u(h) \frac{dh}{dt} - u(-h) \frac{dh}{dt} + \frac{\partial}{\partial y} \int_{h}^{b} v \, dz + v(h) \frac{dh}{dt} - v(-h) \frac{dh}{dt} = 0 \]  \hspace{1cm} 2.11

The kinematic boundary conditions are

\[ \bar{w} \big|_{h} = \frac{\partial}{\partial x} \frac{2}{2} + \frac{\partial}{\partial y} \frac{2}{2} + \frac{\partial}{\partial z} \frac{2}{2} \big|_{x=h} \]  \hspace{1cm} 2.12

\[ \bar{w} \big|_{-h} = \frac{\partial}{\partial x} \frac{2}{2} + \frac{\partial}{\partial y} \frac{2}{2} \big|_{x=-h} \]

Applying the kinematic boundary conditions to equation 2.11 and then dividing by \( H = h + \bar{h} \) yeilds the working equation for the conservation of mass

\[ \frac{\partial H \bar{u}}{\partial x} + \frac{\partial \bar{u} \bar{v}}{\partial y} + \frac{\partial \bar{v}}{\partial t} = 0 \]  \hspace{1cm} 2.13
2.2 Conservation of Momentum

The conservation of momentum is an expression of Newton's Second Law and states that the time rate change of momentum of a fluid parcel is equal to the sum of the forces acting on it. Using the assumptions from Section 2.0, the momentum equations are

\[ \rho \frac{\partial}{\partial t} (u_i) + \rho u_j \frac{\partial u_i}{\partial x_j} = \frac{\partial p}{\partial x_i} - 2 \epsilon_{ijk} \Omega_j \rho u_k - \rho \mu \frac{\partial^2 u_i}{\partial x_j^2} \]

where

- \( \epsilon_{ijk} \) = the cyclic tensor
- \( \delta_{ij} \) = the Kroenker delta, unity when \( i=3 \), otherwise zero
- \( \Omega_j \) = components of the angular velocity of the Earth
- \( \mu \) = molecular viscosity
- \( p \) = pressure

Substituting equations 2.4 and 2.5 into equation 2.14, dividing through by \( p \), obeying the assumptions, letting \( f = 2 \Omega \sin(\phi) \) (where \( \phi \) is the latitude) and neglecting all vertical acceleration terms results in the equations

\[ \frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} - f u_i + \mu \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \]

\[ \frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} - f u_i + \mu \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_i} \frac{\partial u_j}{\partial x_j} \]

\[ \frac{\partial p}{\partial x_3} - g \]

Equation 2.17, the hydrostatic equation, is vertically integrated from some depth \( x_3 \) to the surface \( x_3 = l \) where
the pressure is equal to \( P_a \), the atmospheric pressure,
\[
\frac{\partial \tilde{P}}{\partial x_j} = -g \int_{x_j}^{x_j+h} \rho \, dx_j, \quad \text{or} \quad \tilde{P}_a - \tilde{P} = -g \rho \int_{x_j}^{x_j+h} \, dx_j
\]  
\[2.18\]

Applying equation 2.18 to the pressure term in equation 2.15, Leibnitz' Rule to the vertical integral of the density, and using a Boussinesq type approximation on density, results in
\[
\frac{1}{\rho} \frac{\partial \tilde{P}}{\partial x_i} = \frac{1}{\rho} \frac{\partial \tilde{P}_a}{\partial x_i} - g \frac{\partial \tilde{\rho}}{\partial x_i} - \frac{g \tilde{\rho}}{\rho} \int_{x_j}^{x_j+h} \frac{\partial \tilde{P}}{\partial x_j} \, dx_j
\]  
\[2.19\]

Similarly, the process can be done for the pressure term in the \( x_2 \)-component, resulting in the expressions for conservation of momentum in rectilinear coordinates, neglecting the molecular viscous terms and bars representing average values,
\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= -g \frac{\partial \rho}{\partial x} - \frac{\partial}{\partial x} \left( \rho \frac{\partial \rho}{\partial x} \right) \\
&+ f v - \frac{\partial}{\partial y} (u'v') - \frac{\partial}{\partial z} (u'w') \\
&- f u - \frac{\partial}{\partial x} (u'u) - \frac{\partial}{\partial y} (u'v') - \frac{\partial}{\partial z} (u'w')
\end{align*}
\]  
\[2.20\]

Introducing equation 2.8 into the above equations, vertically integrating from the bottom, \( z=-h \), to the surface \( z=\ell \) neglecting the convective (field) accelerations that contain the vertical velocities since they are small compared to the other terms, and noting from continuity that, \( \frac{\partial u}{\partial x} = 0 \) yields
Due to the assumption 9. in Section 2.0, the atmospheric pressure can be approximated by a constant and, therefore, is neglected in equations 2.22 and 2.23. Similarly, the sixth term in each equation, the slope of the isobaric surfaces due to density gradients, is considered small. This can be shown by a scale analysis with the method of Abbott (1960). Of second order importance in the equations of motion are the convective-inertia terms and the density gradient. In some cases the density gradient is the important term, as in the James River (Pritchard, 1953). Letting

\[ g \left( \eta - h \right) \frac{\partial \rho}{\partial h} \sim C^2 \left| \frac{\partial \rho}{\partial x} \right|, \rho \frac{\partial \nu}{\partial x} \sim \frac{\rho \omega \nu_0^2}{2c} \]

where \( \tau \eta = 12.42 \) hours, tide period \( 2.24 \)

\( C = \sqrt{gh} \), 13.9 ft/sec

\( \nu_0 = \) characteristic velocity, 1 ft/sec

\( h = \) mean depth, 6 feet

then the ratio

\[ D = \frac{C^3 \left| \frac{\partial \rho}{\partial x} \right|}{\omega \nu_0^2 \rho} \]

2.25

gives an approximation of the importance of the convective...
terms to the density gradients. Using appropriate values for each variable and letting density be approximated by salinity yields

\[ D = 3.0 \times 10^{-6} \]

\[ \text{if } \rho = 29.9 \% \]

\[ \frac{\partial \rho}{\partial x} = 2.5 \text{ m.} \]

The scale analysis shows that the slope term can be neglected. This result is compatible with Pritchard's (1966) analysis of the internal and longitudinal homogeneity of Chincoteague Bay. The eight and ninth terms in each equation are of the same form as Reynold's stresses but are different in character and, therefore, cannot be represented by similar relationships as the Reynold's stresses with coefficients of eddy viscosity. These terms represent side friction and are neglected for computational purposes.

The bottom stress terms can be represented by the quadratic friction relationships

\[ \frac{1}{\mu_p} \tau_{b,x} = g \frac{U (U' + \bar{U})}{H C} \]

\[ \frac{1}{\mu_p} \tau_{b,y} = g \frac{V (U' + \bar{U})}{H C} \]

where

\[ C = \text{Chezy coefficient, which is related to Manning's } n \text{ by the expression, } C = \frac{1.49 (H)^{1/6}}{n} \]
2.2.1 Formulation of the Surface Shear Stress

The total surface tangential stress, \( \tau_o \), is composed of the turbulent Reynolds' stress, \( \tau_t = \rho u'_i u'_j \), the wave induced Reynolds' stress, \( \tau_w = \rho u' v' \), and the viscous stress, \( \tau_v \). Except right near the water surface where \( \lim \tau_v \approx 0 \), the viscous shear can be neglected. Philips (1966) shows that less than 10% of the total momentum transfer is supported by the wave induced Reynolds' stress, since this bends the streamlines over the waves and the wind measurements are usually taken above this height. This yields for a statistically steady and spatially homogeneous sea in the horizontal plane that the momentum exchange is supported mostly by the turbulent Reynolds' stress and that the total stress, \( \tau_o \), is independent of height.

Given the Reynolds' stress, last terms in equations 2.22 and 2.23 the problem now relates to finding an expression for these terms to give closure to the mathematical equation. The momentum flux in the equation for the averaged flow of the air motion yields \( \frac{\tau_o}{\rho} = -u'_i u'_j \) as the important term, and we must find a way to express \( u'_i u'_j \). Properties are generally transferred vertically with rate \( v' \), the vertical velocity of the wind. Upon averaging the wind properties, which is the
most that can be expected for wind input into most models, the length and time scales make \( \bar{U} \approx 0 \) so that the fluctuating velocity, \( \bar{U}' \), does the transfer. There exist several methods for determining this transfer of momentum process. The most tractable method for modeling purposes is the parameterization method. In this method the momentum flux is parameterized by expressions that relate to some readily observed quantity.

The momentum flux can be written as

\[
\frac{\tangle}{\rho} \bar{U}' \bar{\bar{W}} = C_0 \bar{U}' \bar{\bar{W}}
\]

2.28

which can be written as

\[
C_0 = \left( \frac{\nu}{\log ( \frac{h}{\eta})} \right)^2
\]

2.29

when \( \bar{U}_c \) = wind velocity measured at some reference height, \( \bar{U}_c \)

\( C_0 \) = drag coefficient
\( \eta \) = Von Karman's constant
\( \eta_c \) = roughness length

This formulation is particularly suited for modeling purposes and also represents a slightly different meaning. The parameters involved are a function of the wind field, and are not quite relevant in determining the immediate consequence of the wind, the form of spectrum in this wind range. The parameters do determine rates of energy and momentum flux to the waves, but not the limiting configuration (Phillips, 1966). In this
model we are assuming a final configuration, a saturated wave field, and we are just interested in the rate of momentum exchange.

A dimensional argument by Kitaygorodskiy (1973) shows the dimensionless parameters important to find $C_D$ are

$$C_o = C_o \left( \frac{h_z}{h_o}, \frac{z}{L}, \frac{h_z}{\delta_v} \right) \tag{2.30}$$

where $h_z = \text{characteristic height of roughness}$
$\delta_v = \text{thickness of viscous sublayer}$
$z = \text{height above surface}$
$L = \text{Monin-Obukhou length}$

From similarity arguments and the turbulent energy method, it can be shown for modeling that equation 2.30 can be reduced to

$$C_o = C_o \left( \frac{h_z}{h_o} \right) \tag{2.31}$$

This leads to the roughness height being the only important parameter to determine. Since this height is extremely hard to measure because it varies with wind speed, duration, and fetch, and the only roughness regimes established are contested by several authors, the roughness height will be considered a constant. Charnock (1955) suggested on dimensional grounds that

$$h_z = a \frac{u_*^3}{\delta} \tag{2.32}$$

where $a = \text{constant}$
$u_* = \text{friction velocity}$; $u_* = (\tau_{ef})^{1/2}$
such that for a fully rough regime Phillips (1966) and Wu (1968, 1969) establish the value for as .0112. This formulation gives a constant drag coefficient for all wind speeds.

This would not be adequate for the shallow Chincoteague Bay with limited fetch. The drag coefficients can be expected to be lower and, correspondingly, the roughness height will not be quite as large and will vary more. The drag coefficient should take into account the enclosed bay as opposed to the open ocean and Stokes transport which will be large for the time scales of interest (Ianniello, et al, 1975; Longuet-Higgins, 1969). Therefore, a relationship is used that was developed for drag coefficients for drift currents where the effective fetch is small, the wind to depth ratio is larger than oceanic cases, and for which laboratory and field measurements have been correlated. In having a varying drag coefficient, the effects of a low wind will be felt less where the effective transfer of energy among wave numbers is greater for the initial higher frequency waves. The momentum exchange would then be practically representative for modeling purposes, and depend only on wind speed, the most common observed variable. It should be noted that the resulting values should be somewhat less than those
observed on the open ocean, \(1.5 \times 10^{-3} \leq 10^{-3}\) (Pond, 1975).

Substituting equation 2.32 into equation 2.29 and using the value of .0112 for \(a\) yields

\[
\frac{1}{C_0} = \frac{1}{h} \ln \left( \frac{1}{.0112} \right) \text{ F}^2
\]

where \(F = \frac{U_1}{\sqrt{g^2}}\), \(U_1\) = wind speed measured at height \(z\).

This is the equation of Wu (1969) and has been verified through the compiling of laboratory and field data. Some results of equation 2.33 are listed in Table I. The final problem is the actual form of equation 2.29 to use in the model, since \(\overline{C_0}\) is a vector quantity, whereas \(\overline{U}\) is an averaged quantity. Short term averages of the wind data are usually not available or practical for modeling purposes so that larger time averages must be used. The average of the vectoral wind speeds will generally be less than the mean average yielding smaller values of wind stress. For modeling of a tidal cycle, it will be assumed that by taking averages of the vectoral wind speed and considering either the wind as constant or varying over a tidal cycle, that the underestimating will be at most 10%. This yields equation 2.29 in component form as

\[
\begin{align*}
\overline{T_{x,x}} &= C_0 \rho_a \overline{U_x} \overline{|U_x|} \\
\overline{T_{x,y}} &= C_0 \rho_a \overline{U_y} \overline{|U_y|}
\end{align*}
\]
where $U_\theta = \bar{U} \cos(\Theta)$
$U_\phi = \bar{U} \sin(\Theta)$

$\Theta = \text{angle of attack of wind with major axis}$

<table>
<thead>
<tr>
<th>Wind Speed</th>
<th>M/Sec</th>
<th>Knots</th>
<th>Drag Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.3</td>
<td>20</td>
<td></td>
<td>.00105</td>
</tr>
<tr>
<td>8.2</td>
<td>16</td>
<td></td>
<td>.00097</td>
</tr>
<tr>
<td>5.1</td>
<td>10</td>
<td></td>
<td>.00082</td>
</tr>
<tr>
<td>4.1</td>
<td>8</td>
<td></td>
<td>.00076</td>
</tr>
</tbody>
</table>

Table I. Wind Speed and Predicted Drag Coefficients Using Eqn. 2.33

2.3 Mass-Balance and Transport Equation

If we denote the local instantaneous concentration of some dissolved conservative property as $s$ then a mathematical expression can be developed for the conservation of this constituent. By considerations of the conservation of properties we can write

$$\frac{ds}{dt} = 0$$

2.34

where the molecular diffusion terms are neglected since they are many orders of magnitude smaller compared to the turbulent diffusion. Expanding equation 2.34 according to the definition of substantial derivative and adhering to continuity, equation 2.7, the result is

$$\frac{\partial s}{\partial t} - \frac{\partial s}{\partial x} \frac{\partial x}{\partial \gamma} - \frac{\partial s}{\partial \gamma} \frac{\partial \gamma}{\partial \xi}$$

2.35
As the instantaneous field is nearly impossible to calculate, we shall consider the instantaneous motion to be composed of a mean concentration that includes the tidal and non-tidal component of \( s \) and a turbulent fluctuating term, \( s' \), similar to expression 2.4

\[
\frac{\partial s}{\partial t} = -\frac{\partial u s}{\partial x} - \frac{\partial v s}{\partial y} - \frac{\partial w s}{\partial z} - \frac{\partial (u' s')}{\partial x} - \frac{\partial (v' s')}{\partial y} - \frac{\partial (w' s')}{\partial z}
\]

Upon substituting equations 2.4 and 2.36 into equation 2.35 and following similar arguments used in deriving equations 2.20 and 2.21 yields

\[
\frac{\partial s}{\partial t} = -\frac{\partial u s}{\partial x} - \frac{\partial v s}{\partial y} - \frac{\partial w s}{\partial z} - \frac{\partial (u' s')}{\partial x} - \frac{\partial (v' s')}{\partial y}
\]

For a vertically homogeneous estuary it can be shown that the only important terms are the horizontal components. From this equation 3.4 can be written as

\[
\frac{\partial s}{\partial t} = -\frac{\partial u s}{\partial x} - \frac{\partial v s}{\partial y} - \frac{\partial (u' s')}{\partial x} - \frac{\partial (v' s')}{\partial y}
\]

where the bars indicating averages have been omitted for ease since there should be no misunderstanding now that we are considering averaged values. To add closure to equation 2.38 some means must be found to express the turbulent fluctuating terms. Though there is little theoretical backing, it is assumed that these terms can be represented through mixing length theory. Therefore, the non-advective flux terms are assumed to be reducable to the simple Fickian type diffusion terms. Using the Bouisnesq hypothesis that \( u'w' \) can be written as \( \frac{\partial u}{\partial y} \)
we can write a similar relationship for \( u'\) where

\[
\langle u' \rangle = \xi \frac{\partial \xi}{\partial x} \quad \text{2.39}
\]

where

\[
|\tilde{u}'| = \text{const.} \times \ell \times \frac{\partial \tilde{u}}{\partial x}
\]

\[
|\tilde{S}'| = \text{const.} \times \ell \times \frac{\partial \tilde{S}}{\partial x}
\]

such that

\[
\langle u' \rangle = \text{const.} \times |\tilde{u}'| |\tilde{S}'|
\]

\[
= \text{const.} \times \ell^2 \frac{\partial \tilde{u}}{\partial x} \frac{\partial \tilde{S}}{\partial x}
\]

where \( \xi = \text{const.} \times \ell^2 \frac{\partial \tilde{u}}{\partial x} \)

Upon substituting equation 2.39 into equation 2.38, equation 2.40 is obtained

\[
\frac{\partial S}{\partial t} + \frac{\partial S}{\partial x} (uS) + \frac{\partial S}{\partial y} (vS) - \frac{\partial}{\partial x} \left( \xi \frac{\partial S}{\partial x} \right) - \frac{\partial}{\partial y} \left( \xi \frac{\partial S}{\partial y} \right) = 0 \quad \text{2.40}
\]

The above differential equation represents the conservation of a conservative constituent for a vertically homogeneous estuary. Since the velocity inputs to this equation are from the vertically integrated hydrodynamic model, equation 2.40 must also be vertically integrated. Integration over the vertical from the bottom, \( z=-h \), to the free surface, \( z=H \), and denoting \( H=h+\ell \) yields the following equation, where the derivation is the same as for the fluid flow equations

\[
\frac{\partial}{\partial t} \left( HS \right) + \frac{\partial}{\partial x} \left( uHS \right) + \frac{\partial}{\partial y} \left( vHS \right) - \frac{\partial}{\partial x} \left( \xi \frac{\partial S}{\partial x} \right) - \frac{\partial}{\partial y} \left( \xi \frac{\partial S}{\partial y} \right) = 0 \quad \text{2.40}
\]

where

\[
\begin{align*}
\bar{u} &= \frac{1}{H} \int_{-h}^{0} u \, dz \\
\bar{v} &= \frac{1}{H} \int_{-h}^{0} v \, dz \\
\bar{S} &= \frac{1}{H} \int_{-h}^{0} S \, dz \\
D_{xy}, D_{x\gamma}, D_{y\gamma} &= \text{turbulent diffusion-dispersion coefficient}
\end{align*}
\]
2.3.1 Dispersion

Dispersion is caused by turbulent diffusion which passes higher concentration of constituents to lower concentration areas. This, in turn, is dependent upon the hydrodynamic conditions. Fischer (1958) gives a method for predicting dispersion coefficients in natural streams that is based upon the work of Taylor (1950), who looked at turbulent diffusion in pipes. Taylor arrived at a general observational-experimental determined value. Like bottom and surface friction the dispersion coefficients must be determined from field data. Elder (1959) looked at dispersion in turbulent flow like Taylor but did not confine his work to pipes. He arrived at an expression for longitudinal dispersion coefficient and a lateral diffusion coefficient.

Elder found the longitudinal coefficient to be

\[ D_L = 5.93 \frac{H}{\text{u}^*} \]  \hspace{1cm} (2.41)

where \( \text{u}^* \) = shear stress velocity

such that

\[ \text{u}^* = \left( \frac{\tau_x}{\rho} \right)^{1/2} = \bar{u} \delta^{1/2} C^{-1} \]  \hspace{1cm} (2.42)

where \( \tau_x \) = bed shear due to uniform flow  
\( \bar{u} \) = mean velocity  
\( C \) = Chezy coefficient

Combining equations 2.41 and 2.42 yields

\[ D_L = 5.93 \frac{H \bar{u} \delta^{1/2} C^{-1}}{} \]  \hspace{1cm} (2.43)
The lateral dispersion coefficient, $D_-$, was found to be

$$D_- = 2.23 \, u^*$$

Fischer (1958) found larger dispersion coefficients by considering the cross-section of the flow to be divided into stream tubes and calculating the transport due to gradients. By considering it this way, he was not just using average cross-sectional velocity. In this study the equations are solved at discrete points in a grid system and the velocity is considered uniform throughout each grid. Therefore, if the dispersion coefficients are calculated for each grid point with the corresponding velocities, then Elder's analysis can be considered applicable.

Longitudinal dispersion is usually larger than the lateral dispersion which as Leendertse (1970) states, "makes the modeling effort much more difficult, as it makes the dispersion anistrophic". The longitudinal dispersion transport is much smaller than the dispersion by advective transport. This can be shown by the ratio of the two. Due to this, small changes in the longitudinal dispersion coefficient will not effect the solution. Therefore, for computational ease the dispersion will be considered isotrophic. The relationships are those obtained by Leendertse (1970) after numerical and analytical
experiments. The values for the diffusion coefficients are obtained through field data and calibration of the model. The dispersion relationships can be written as

\[ D_x = f(u, C, H) + D_w \] 2.45
\[ D_y = f(v, C, H) + D_w \] 2.46

where \( D_w \) = a diffusion coefficient dependent on wave and wind field.

Equations 2.45 and 2.46 can be rewritten through Elder's analysis and the assumption of isotrophy as

\[ D_x = 5.9 \cdot H \cdot u \cdot \delta^{\frac{1}{2}} \cdot C' \cdot D_w \left( \frac{f^3}{\xi \rho c} \right) \] 2.47
\[ D_y = 5.9 \cdot H \cdot v \cdot \delta^{\frac{1}{2}} \cdot C' \cdot D_w \left( \frac{f^3}{\xi \rho c} \right) \]

where \( D_w \) is estimated using field data.

2.4 The Vertically Integrated Two-Dimensional Dynamic Equations for the Mathematical Model

The vertically integrated mass conservation equation is

\[ \frac{\partial H u}{\partial x} + \frac{\partial H v}{\partial y} = \frac{\partial H}{\partial t} \] 2.48

It will be solved simultaneously with the equations of motion,

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -g \frac{\partial H}{\partial x} + \frac{Ux|Vx|k}{H \rho} - g \frac{u(U^2+V^2)^{\frac{1}{2}}}{Hc^2} \] 2.49

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -g \frac{\partial H}{\partial y} - \frac{Ux|Vx|k}{H \rho} - g \frac{v(U^2+V^2)^{\frac{1}{2}}}{Hc^2} \] 2.50

and mass-transport
25.

\[
\frac{d}{dt} (\eta) + \frac{\partial \eta}{\partial x} + \frac{\partial \ eta}{\partial y} - \eta \frac{\partial}{\partial x} \left( \frac{\partial D}{\partial x} \right) - \eta \frac{\partial}{\partial y} \left( \frac{\partial D}{\partial y} \right) = 2.51
\]

for the dependent variables \(U, V, \eta\) and \(\eta\).
3.0 Introduction

For the fluid flow model Leendertse (1967) uses an alternating scheme for solving the hyperbolic equations. This alternating explicit and implicit scheme is stable as is shown in Appendix A. For the parabolic transport equations, the methods of Leendertse (1970) are those originally used by Peaceman and Rachford (1955) and coded by Hessetal (1975).

To solve the differential equations 2.48, 2.49 and 3.50, finite difference approximations to the differential equations are used. The theory behind this is extensive and will not be gone into in this study except a stability analysis. What the approximations do is to solve the continuous differential equations at discrete points on a grid system. In this study it is done with a space-staggered scheme which reduces the necessary computational time and still renders accurate results. Whether the solutions of the finite difference approximations actually approach the solutions of the differential equations is the crux of the stability analysis. The difference equations are solved at the discrete points.
shown in Figure 3. The water levels, \( h \), and mass densities, \( s \), are computed at integer values of \( n \) and \( m \) while the values of the water depth, \( h \), which were obtained from the Coast and Geodetic Survey Chart 220, and field measurements, are given as input data at half-integer values of \( n \) and \( m \). The \( U \)-velocities are computed at half-integer values of \( n \) and integer values of \( m \). The \( V \)-velocities are computed at half-integer values of \( m \) and integer values of \( n \).

By using this scheme there will be a centrally located spatial derivative for the linear term when the variable is operated on in time. A multioperation method is used where the spatial derivatives and the Coriolis force are alternating forward and backward in time thus making them central or averaged in time over two successive operations (one time step, to be called NST, or two half-steps, each step to be called ISTEP).

The advantage can be seen by looking at the \( x \)-momentum equation and the value of the variable \( U \). For the first time step this value is approximated by a backward difference

\[
\frac{1}{\Delta t} \left( u_{t+\frac{1}{2}} - u_t \right) \approx \frac{1}{\Delta t} \left( u_{t+\frac{1}{2}} - u_{t-\frac{1}{2}} \right) = f_c n (\eta^{t+\frac{1}{2}})
\]

Where in the second time step \( U \) is approximated by a forward difference

\[
\frac{1}{\Delta t} \left( u_{t+1} - u_{t} \right) \approx \frac{1}{\Delta t} \left( u_{t+1} - u_{t+\frac{1}{2}} \right) = f_c n (\eta^{t+\frac{1}{2}})
\]
Figure 3. Location of Variables of Space Staggered Scheme
Another advantage of the schemes used is that the unknown mass densities at time level \( t + \frac{1}{2} \Delta t \) are coupled with only \( x \)-spatial derivatives and unknowns at \( t + \Delta t \) are only coupled with \( y \)-spatial derivatives, this is the advantage of the Peaceman and Rachford (1955) scheme.

3.1 Notation and Approximations for Finite-Difference Equations

To facilitate the writing of the finite-difference expression, we can express a variable, \( F = F(n_{\Delta x}, m_{\Delta y}, n_{\Delta t}) \) on the grid by writing short expressions for averages and differences. These are represented as (for \( x \)-direction only, where \( y \) and \( t \) are the same)

\[
\bar{F}_{x, m} = \frac{1}{2} \left( F[(n+\frac{1}{2})_{\Delta x}, m_{\Delta y}, n_{\Delta t}] + F[(n-\frac{1}{2})_{\Delta x}, m_{\Delta y}, n_{\Delta t}] \right)
\]

\[
\Delta_x = \frac{1}{\Delta x} \left[ F[(n+\frac{1}{2})_{\Delta x}, m_{\Delta y}, n_{\Delta t}] - F[(n-\frac{1}{2})_{\Delta x}, m_{\Delta y}, n_{\Delta t}] \right]
\]

\[
\bar{F}_{x, m} = \frac{1}{4} \left( F[(n+1)_{\Delta x}, (m+\frac{1}{2})_{\Delta y}, n_{\Delta t}] + F[(n-1)_{\Delta x}, (m-\frac{1}{2})_{\Delta y}, n_{\Delta t}] + F[(n+\frac{1}{2})_{\Delta x}, (m+1)_{\Delta y}, n_{\Delta t}] + F[(n-\frac{1}{2})_{\Delta x}, (m-1)_{\Delta y}, n_{\Delta t}] \right)
\]

\[
\Delta_x = \frac{1}{2} \left( F[(n+1)_{\Delta x}, m_{\Delta y}, n_{\Delta t}] - F[(n-1)_{\Delta x}, m_{\Delta y}, n_{\Delta t}] \right)
\]

For a shift in time levels

\[
\Delta_{x,t} F = \frac{1}{\Delta t} \left\{ F[n_{\Delta x}, m_{\Delta y}, (n_{\Delta t} + 1)_{\Delta t}] - F[n_{\Delta x}, m_{\Delta y}, n_{\Delta t}] \right\}
\]

This difference represents the time derivatives if \( n \) is integer.

\[
F_+ = F[n_{\Delta x}, m_{\Delta y}, (n_{\Delta t} + \frac{1}{2})_{\Delta t}]
\]

\[
F_- = F[n_{\Delta x}, m_{\Delta y}, (n_{\Delta t} - \frac{1}{2})_{\Delta t}]
\]

\[
F_{t/2} = \frac{1}{2} \left\{ F[n_{\Delta x}, m_{\Delta y}, (n_{\Delta t} + \frac{1}{2})_{\Delta t}] + F[n_{\Delta x}, m_{\Delta y}, n_{\Delta t}] \right\}
\]

The last average appears in the mass-balance equations for the constituents that use the information from the
time level \( t + \frac{1}{2} \Delta t \) instead of \( t + \Delta t \).

3.2 Finite-Difference Approximations to the Equations

Using the notation of Section 3.1, the difference equations can be written for two time levels. Each operation on the fluid flow takes place over an interval of the two time levels where the time for whole step is listed as \( 2 \Delta T \) in the program. We can write then

**First Half Time Step**

\[
\begin{align*}
\text{x - Momentum} & : u^{n+\frac{1}{2}} = u^n + \frac{1}{2} \Delta t \left[ \nabla \cdot \nabla - \frac{1}{2} \frac{\partial}{\partial t} u^{n+\frac{1}{2}} \right] \\
& - \frac{1}{2} \frac{\partial}{\partial t} \left( \nabla \cdot \nabla \right) u^n - \frac{1}{2} \frac{\partial}{\partial t} \left( \nabla \cdot \nabla \right) u^{n+\frac{1}{2}} \\
& - \frac{1}{2} \Delta t \delta_x \nabla \cdot \nabla - \frac{1}{2} \Delta t \delta_y \left[ (u^n)^2 + (\nabla \cdot \nabla)^2 \right]^{\frac{1}{2}} \\
& - \frac{1}{2} \Delta t k \frac{(U^n)^2}{(h^x + h^y)^n} \\
& \text{at } n+\frac{1}{2}, m
\end{align*}
\]

**Conservation of Mass**

\[
\eta^{n+\frac{1}{2}} = \eta^n - \frac{1}{2} \frac{\partial}{\partial t} \delta_x \left[ (h^x + h^y)^n \nabla \cdot \nabla \right] \\
\text{at } n+\frac{1}{2}, m
\]

**Second Half Time Step**

\[
\begin{align*}
\text{x - Momentum} & : u^{n+1} = u^{n+\frac{1}{2}} + \frac{1}{2} \Delta t \left[ \nabla \cdot \nabla - \frac{1}{2} \frac{\partial}{\partial t} u^{n+1} \right] \\
& - \frac{1}{2} \frac{\partial}{\partial t} \left( \nabla \cdot \nabla \right) u^{n+\frac{1}{2}} - \frac{1}{2} \frac{\partial}{\partial t} \left( \nabla \cdot \nabla \right) u^{n+1} \\
& - \frac{1}{2} \Delta t \delta_x \nabla \cdot \nabla - \frac{1}{2} \Delta t \delta_y \left[ (u^{n+\frac{1}{2}})^2 + (\nabla \cdot \nabla)^2 \right]^{\frac{1}{2}} \\
& - \frac{1}{2} \Delta t k \frac{(U^{n+\frac{1}{2}})^2}{(h^x + h^y)^{n+\frac{1}{2}}} \\
& \text{at } n+\frac{1}{2}, m
\end{align*}
\]

**Conservation of Mass**

\[
\eta^{n+1} = \eta^{n+\frac{1}{2}} - \frac{1}{2} \frac{\partial}{\partial t} \delta_x \left[ (h^x + h^y)^n \nabla \cdot \nabla \right] \\
\text{at } n, m
\]
y - Momentum \( v^{n+1} = v^n - \frac{1}{2} \Delta t \partial_x a^{n-1} - \frac{1}{2} \frac{\Delta t}{\Delta L} \partial_x a^{n-1} \) \( \delta_x v^{n+1} \)
- \( \frac{1}{2} \frac{\Delta t}{\Delta L} v^{n+1} \delta_y v^{n+1} - \frac{1}{2} \Delta t \partial_y \delta_y v^{n+1} - \frac{1}{2} \Delta t \partial_y \delta_y v^{n+1} \)
- \( \frac{1}{2} \frac{\Delta t}{\Delta L} [a^{n-1} g v^{n+1}] \)
- \( \frac{1}{2} \frac{\Delta t}{\Delta L} [\partial_x (a^{n-1})^2] \)
- \( \frac{1}{2} \frac{\Delta t}{\Delta L} \frac{\partial_y (a^{n-1})^2}{(\hat{h}^n - \hat{h}^{n+1})^{\frac{1}{2}}} \)
- \( \frac{1}{2} \frac{\Delta t}{\Delta L} (U_y)^2 \)
\( \alpha + N, M \)

MASS-BALANCE

**First Half Time Step**
\[
\frac{\Delta t}{\Delta L} [S^n h^{n+1}] = 2 \frac{\Delta t}{\Delta L} [S^n (\hat{h}^n h^n)^n] + \frac{1}{2} \frac{\Delta t}{\Delta L} [(h^n h^n)^n u^n S^n] + \frac{1}{2} \frac{\Delta t}{\Delta L} [(h^n h^n)^n v^n S^n]
\]
\[
- \frac{1}{2} \frac{\Delta t}{\Delta L} [(h^n h^n)^n u^n S^n] + \frac{1}{2} \frac{\Delta t}{\Delta L} [(h^n h^n)^n v^n S^n]
\]
\[
- \frac{1}{2} \frac{\Delta t}{\Delta L} [(h^n h^n)^n v^n S^n] - \frac{1}{2} \frac{\Delta t}{\Delta L} [(h^n h^n)^n D^n] - \frac{1}{2} \frac{\Delta t}{\Delta L} [(h^n h^n)^n S^n] + \frac{1}{2} \frac{\Delta t}{\Delta L} [(h^n h^n)^n D^n] - \frac{1}{2} \frac{\Delta t}{\Delta L} [(h^n h^n)^n S^n]
\]
In the first level of the time step, from \( n \Delta t \) to \( n \Delta t + \frac{1}{2} \Delta t \) values of \( h^n, u^n \), and \( v^n \) are obtained from \( \eta^n, U^n \), and \( V^n \) by an implicit operation in \( \eta \) and \( U \) and explicit in \( V \). The values of \( \eta^{n+1}, U^{n+1} \) and \( V^{n+1} \) are computed from \( \eta^{n+1}, U^{n+1} \), and \( V^{n+1} \) where the operation is implicit in \( \eta \) and \( V \) and explicit in \( U \). The procedure for this continues through the desired number of time
steps. In the computational model, the values of the variables at time step $n\Delta t$ are set equal to $u^n$, $v^n$, and another time step is started for the time level $n\Delta t + \frac{1}{2}\Delta t$.

Information from the solution of the fluid flow equations is called off tape for the solution of the mass-balance equations. In the first time level $n$ and $U$ are used to obtain the constituent concentrations $S^n_k$ at time level $n\Delta t - \frac{1}{2}\Delta t$. The results of this are used for the next time level where the computed values from the implicit operation of $\eta_1$ and $V$ are called at time level $n\Delta t + \Delta t$ to obtain the constituent concentrations, $S^{n+1}$ at time level $n\Delta t + \Delta t$.

3.3 Numerical Methods for Solution of Equations

Equations 3.1 and 3.2 yield $U$-velocity and water level information needed for the first time level for the convective-diffusive equations. The $V$-velocity used is from time $n$. Equations 3.4 and 3.5 give the $V$-velocity and water level for the second time level and the $U$-velocity will then be obtained from the $n\Delta t + \frac{1}{2}\Delta t$ time level. Therefore once the values are known, the constituent concentration at time levels $n\Delta t + \frac{1}{2}\Delta t$ and $n\Delta t + \Delta t$ need only the initial and boundary values.
Solving the fluid flow equations, we see that for the implicit operation in the first half time step, equation 3.1, there are three unknown values at time level \( n^{1/2} \), \( u_{n-1/2}^{n+1/2}, u_{n-1/2}^{n+1/2}, \eta_{n}^{n+1/2} \) on line \( m \). Similarly, in equation 3.2 there are three unknowns at time level \( n^{1/2} \), \( u_{n-1/2}^{n+1/2}, \eta_{n+1/2}^{n+1/2}, \eta_{n}^{n+1/2} \) on line \( m \). The solution for these three adjacent values is shown in detail in Leendertse (1967) and interested readers are referred to this reference. The solution process is outlined below and in Appendix B.

The solution is done by elimination and the problem can be stated in matrix form on line \( m \) (after Leendertse, 1971) as

\[
[A] \{F\} = \{B\} \tag{3.9}
\]

The vector \( \{F\} \) contains all the unknown values of \( U \) and \( \eta \) at time level \( n^{1/2} \) and the vector \( \{B\} \) contains the known coefficients previously calculated and the constant coefficients. The matrix \( [A] \) contains all the coefficients of the unknowns \( U \) and \( \eta \) at time level \( n^{1/2} \). Once the values for \( U, V, \) and \( \eta \) are calculated, one applies the initial and boundary values for the transport equations to obtain the solution for mass-transport equations. This is done as in the fluid flow section by use of recursion formulas developed in matrix form.
IV. BOUNDARY CONDITIONS

4.0 Introduction

With some simplifications and planning the boundaries of the problem can be made tractable. The fluid flow boundaries are easily solved but the water quality mass transport boundaries require additional computational methods. This can be seen by looking at two cases. Consider an initial concentration throughout the estuary for some constituent. At the open boundaries a value of the constituent is needed for computations at all times. This poses the question of how much is going out on an ebbing tide and how much is coming in on flood tide. Another question that must be handled is the spatial variation in the vicinity of the sources (outfalls, etc.).

4.1 Fluid Flow Computational Boundaries

There are two possible cases to handle in these computations, landwater boundaries and open boundaries. The land-water boundaries are solved by taking the grid line through the land-water interface at the water level locations and in this way the normal velocity is then given as zero. For open boundaries the water elevations are given as a function of time for the grids at the boundary. Either each grid can be associated with given
input water levels that vary with time or else several grids can be set equal to input data and at the remaining grids the water elevations can be obtained through linear interpolation. The values for the water elevations can be obtained from tide tables, field data, or extrapolation from the interior field. Velocities can also be given as input for the open boundaries but this requires extensive field measurements, therefore, in this study, water elevations were used as open boundary inputs. At the land-water boundary the convective terms give a water velocity that lies outside the computation field. This is handled by taking the convective terms at the boundaries to be equal to zero at the expense of accuracy but to preserve stability in the computational scheme (Leendertse, 1967).

4.2 Mass Transport Boundary Approximations

At the source(s) of mass concentrations certain discontinuities arise. The first is the added water to this grid square. The method for this is to take the discharge in CFS units and the half-time step (time step of one time level) and to calculate the discharge for that time period. The grid size, AL, is then used to find the area of the water in that grid and together the change per time step in height can be found. This depth is then included in computations at the discrete grid point of the
outfall by being added on to the continuity equation water level elevation for that grid point. The next discontinuity encountered is the sharp gradient of mass concentration at the source. Due to the slowly changing variable capability of the finite difference scheme, this results in local instabilities that will propagate as small disturbances. At these points it is assumed that there takes place immediate and complete mixing. In many numerical procedures this is handled by introduction of an artificial viscosity term (Richtmeyer et al, 1968). Leendertse (1970) has shown that this method works but adds additional computational time so this approach is not used. The method used to handle this discontinuity is that of Leendertse (1970) in which upstream averaging is used. In this case the mass density in the convective terms is taken as a spatial derivative rather than the average mass density from the upstream side, i.e. \( S_x S \) rather than \( S^* \). If the convective term is taken as in Equation 3.7 (third and fourth terms) then the spatial derivative operates on the average mass density and this will lead to negative values for the mass concentration since central differencing produces \( S_{n+1} \) \( S_{n-1} \). Slow moving waves resulting from the discontinuity will be propagated downstream. Use of a spatial derivative in
this case preserves stability and the conservation of mass is adhered to. The upstream differencing also adds to the diffusion needed at point sources. Therefore the central difference is used if there are no sources and if there are sources and U is positive then $S$ in Equation 3.7 is taken as $S_{n+1}^{n+1} - S_{n}^{n}$ and if U is negative then the mass concentration $S$ is taken as $S_{n}^{n+1} - S_{n}^{n}$.

At the open boundary the mass concentrations are needed for computations. Due to the extensive field measurements that would be needed to give $S$ as a function of time certain procedures are used to overcome this. Initially, a concentration of some constituent is assumed over the whole bay and at the open boundary. During outflow (ebb) the boundary condition is obtained by extrapolation from the interior field. Therefore, when the next time level is entered there will be values at the boundaries. This procedure can be written as

$$S_{n,m}^{n+1} = S_{n,m}^{n} - u_{n+1,m}^{n} \left( S_{n+1,m}^{n} - S_{n,m}^{n} \right) \frac{\Delta t}{2\Delta x} \tag{4.1}$$

During flooding tide a linear extrapolation from the interior field cannot be used. In this case it is assumed that the concentration changes with time until it reaches some set value over some time period that would be applicable for each constituent. Therefore one can use an exponential, sinusoidal, or linear increase
or decrease to the set value from slack tide. For computational ease a linear relationship is used. This is an arbitrary method but one that can be modified to fit many situations. It also has the advantage that during the verification stage of the model changes can be made readily to achieve the desired results so that predictive runs can be made.
V. MODEL USE AND APPLICATION

5.0 Introduction

The application of the model to an area involves the input and choice of certain parameters. The depth field to be used in the model must be accurate as the model is directed to shallow coastal or estuarine environments. Suitable choice of a time step must be made. This depends upon depth in field, desired degree of accuracy, computational limits, and computer capabilities. The lattice that is used for the model must be thoughtfully selected and is dependent not only on the above mentioned facts, but also on the particular goal of the model. Finally, a suitable choice of Chezy coefficients must be made. The various aspects of the factors for model use are discussed in the following sections.

5.1 Depth Field

As input to the model, the depth for each grid in the computational field is needed. The depth locations are the grid points \( h_{i,j} \) (see Figure 3) and are read from the actual locations for computation which is the center if each grid. All depth information was obtained from the U. S. Coast and Geodetic Survey chart number 1220. If a depth wasn't available at the exact location then the
interpolation from surrounding depths was used. These
depths are all at mean low water and have been adjusted
to the 1929 Geodetic Datum from known locations of
variations from this datum. This was important due to
the shallow depths in Chincoteague Bay. In using mean
Low water as datum, the tidal wave was always of positive
amplitude so that no negative values of water-elevations
entered the computational field. The resulting depth
field was plotted three-dimensionally to give a better
insight into the bathymetry of the bay and to the bay's
circulation (Figure 4). In the actual input depths outside
of the computational field can be entered.

5.2 Time Step

The multioperation method used for the solution of
the difference equations is stable for any time step. The
analytical and numerical work in the stability analysis
has been done for areas of fairly constant topography.
Stability is not guaranteed in regions of rapidly varying
topography, therefore, during modeling it is sometimes
necessary to smooth bottom contours to maintain stability.
Cases still exist where the smoothing of bottom contours
to eliminate steep gradients is not realistic. Therefore,
in the choice of a time step, numerical experiments were
used to determine an economical yet accurate time step.
Figure 4. Bathymetry of Chincoteague Bay
Leendertse (1967) and Sobey (1970) show that though the scheme is stable the accuracy is increased when

\[ \beta = \frac{\Delta t}{L} \sqrt{\frac{g}{h}} \leq 5 \]  \hspace{1cm} 5.1

is less than or equal to five (5). This can be stated more generally from the stability analysis that the imaginary part of \( \beta' \), where \( \beta' \) is the wave number of the calculated numerical wave (the measure of the computed wave deformation), should be less than or equal to five, i.e., the modulus of Leendertse's propagation factor is smaller than unity. Another important aspect to consider with this system is the argument of the propagation factor which is the measure of the calculated phase shift. From Leendertse (1967) the best accuracy is achieved when the value of the nondimensional equation 5.1 is near the value of two (2). A direct result of this is that the tidal wave length of grid size ratio should be on the order of 100 for equation 5.1 near 5 and on the order of 40 for equation 5.1 equal to 2.

The average depth of the bay is six feet, using the value with a grid size of 2,025 feet gives a time step of 728 seconds or less as acceptable. If the maximum depth found in the bay is used, 27 feet, this yields a time step of 343 seconds or less as allowable (or a half
time step of 172 seconds). It could be argued that a larger time step can be used since errors are damped (refer to appendix on stability). Since the inlets are the locations of the forcing functions for the bay and have the largest depths, it is best to use the smaller time step. To evaluate this, numerical experiments were conducted with varying time steps at the grid point (12,2). U-directed velocities and water elevations were stored each time step through a tidal cycle and then calculated for variations, figure 5.

The depth at the grid point (12,2) is 3.6 feet but the surrounding grids have depths of 27, 3.6, 15, and 10.2 feet in counter-clockwise order. These were the maximum gradients to be encountered in the bay and smoothing of the topography would alter the direction of the flow to a large extent. This is due to the fact that the wave upon entering Chincoteague Inlet divides and flows in four directions through separate channels: Toms Cove, Mosquito Creek, Assateague Channel, and Chincoteague Channel. Since these are the main thoroughfares, the actual tidal flows into each channel is important for navigational and flushing purposes.

Numerical experiments showed that one-half time steps of 300 and 270 seconds were too large and
computational instability immediately arose. For the case of $\frac{1}{2} \Delta t = 270$ only one time was completed before instabilities set in, and for the case of 300 seconds, calculations didn't make it through one full time step. The next three $\frac{1}{2}$-time steps investigated were: 210, 150, and 75 seconds. The results are shown in Figure 5.

For the time step of 210 seconds, instabilities formed in the amplitude of the water elevation. This takes place at high water slack when the added mass to the system cannot be advected out of the grid. Physically, the kinetic energy of the system is not being converted to potential energy, the energy method, Richtmeyer and Morton (1967), shows that at particular times an instability can arise especially with the convective-inertia terms that are off-centered (this is particularly true when U is constant and near the boundaries). It is assumed that this is where the instability originates.

The solution for the water elevation is similar for all time steps used except for the instabilities for the 210 half-time step which are damped out and this numerical analysis is compatible with the stability analysis. The larger time steps lead to a lower computed velocity for the "higher components in the tidal wave" (Leendertse, 1967). The difference between the time steps of 150 and 75
Figure 5. Influence of Time Step on Water Elevation and Velocity at Grid Point \((N,M) = (12,2)\)
seconds is approximately 15% but it must be remembered that this is in the area of the steepest gradients. For computational purposes and considering the general smoothness of the bottom topography, a half-time step of 75 seconds will be used to run the model to equilibrium. The results from this will be used as dynamic input for future runs with a half-time step of 150 seconds.

5.3 Computational Lattice Used for Chincoteague Bay

The choice of an appropriate length scale or grid size upon which the difference equations will be solved depends upon several factors. To decide on this grid size one must consider the desired resolution of the velocities and water-levels within the modeled region, bottom and shoreline topography, size of time step, and the computational constraint that there must be at least two grids in each row. A larger lattice would yield shorter computation times at the expense of the accuracy of the computed solution. This, in turn, must be considered with the geometry of the area to be modeled. The relative low tidal velocities in Chincoteague Bay indicates that a large grid size can be used. The grid size is partially determined by the choice of an accurate time step which results in the grid length being bounded,
\[
\frac{L}{\Delta l} \leq 100 \quad \text{if} \quad \frac{\Delta t}{\Delta l} \sqrt{g h_m} \leq 5
\]
\[
\frac{L}{\Delta l} \leq 100 \quad \text{if} \quad \frac{\Delta t}{\Delta l} \sqrt{g h_m} \leq 2
\]

where \( L \) = tidal wavelength
\( \Delta l \) = grid size
\( \Delta t \) = time step, seconds
\( h_m \) = maximum depth modeled

The final constraint that was considered was the two grids per row needed for computation. This was the determining factor in the selection of the grid length. The inlets represent the areas of the tidal forcing which are the main thoroughfares for navigation, and are of complicated bathymetry. Therefore, adequate resolution of these areas is important. To yield this resolution and keep the computational field small, a 23 by 94 field was chosen. This field has 988 computational points (see Figure 6) with a corresponding grid length of 2,025 feet.

This grid length was equal to the width of some parts of the several channels surrounding the inlets but a smaller grid size would have resulted in an increase by 20% of the number of points in the lattice. At the same time a larger grid size could not model the inlet areas accurately, and the circulation patterns within the bay itself would have altered. This grid length is well within the bounded region and the actual lattice is directed 34.1° from North to yield an accurate representation of the Bay's geometry.
Figure 6. Computational Lattice of Chincoteague Bay
OPEN BOUNDARIES; Grids for boundary conditions

Elements not in computational field.
5.4 Chezy Coefficients

The Chezy coefficient is calculated by the use of equation 2.27a. This equation is dependent upon the time varying water elevation and Manning's friction factor n. The calibration of the model involves the adjustment of the friction factor n to achieve proper phase for the tidal wave throughout the Bay. In the frictionally dominant bay, this is perhaps the most important aspect of the model. Results of the variation of the friction factor and methods used will be discussed.

The model study of Harleman and Lee (1969) gave introductory values to use for the Manning's coefficient. The variation of bottom sediment and geometry dictated the use of a formula to obtain a grid row varying friction factor. The method of Hess, et al (1974) was modified for the case of Chincoteague Bay. This equation assumes a linear variation of the n values with grid row M which can be written as

\[ n(M) = n_{avg} (1.3 - 0.6 \frac{M}{\text{Max}}) \]

where \( n_{avg} \) = Ave. Manning's coefficient
\( m \) = Grid Row Number
\( \text{Max} \) = maximum grid number

This equation gave an initial Chezy field that was then modified to fit the tide tables (1975) listed phase shifts for Chincoteague Bay.
Values of the different average Manning's coefficients used during calibration and the resulting phase shifts compared to the tide table data are listed in Table II. The effects of different Manning's coefficients on the propagation velocity on the U-direction are shown in Figure 7. The response of the model is dependent to a large extent on the friction factor. A maximum change in the coefficient of 50% gives a 20% change in the amplitude of the computed velocity. The friction factor should then be chosen carefully with the calibration of the model stressing this point.

The last factor studied was the "updating" of the Chezy values every ten minutes. It was assumed that this would be important due to the shallowness of the Bay and dependence of the diffusion coefficient upon the Chezy value. Experiments showed that the added computational time was not compensated by the slight (approximately 2%) change in the computed parameters. The choice of an accurate Manning's coefficient is more important in achieving accurate results than the updating.
Figure 7. Effects of Manning's Coefficient on Propagation Velocity
U-Velocity: $m = 5$
$n = 14$

Depth: $h = 12$ ft.

Chezy Coefficient: $C = 1.49H^{1/6}$

$N_{ft.}^{1/6}$  $N_{AVG.}$

- $\triangle$ 0.041 0.032
- $\circ$ 0.038 0.030
- $\square$ 0.035 0.028
- $+$ 0.018 0.040
VI. DYNAMICS OF THE MODEL

6.0 Introduction

This section investigates certain aspects of the dynamics of the Bay through the application of the model. The study also yields insights into the capacity and dynamics of the model. The topics investigated in this phase of the model study are: free oscillations in the Bay, with and without friction, the effects of Coriolis force on the circulation in the Bay, and the calibration procedure. Several interesting results were obtained during the calibration stage. The first concerned the dynamics of the Bay and involved the frictional effects on the propagation velocity and amplitude of the tidal wave. The second consisted of the effects of eliminating the explicit scheme in the computational model. The results of this study on the dynamics of the model is represented by comparison of velocity output.

6.1 Free Oscillations

The model was run to determine the natural period of oscillation and the damping effect of friction in Chincoteague Bay. A linear tide was imposed upon the Bay with a 3.4 feet elevation at Ocean City Inlet and zero elevation at Chincoteague Inlet. The model was then run with and without the frictional terms in eq. 2.14. The
former case was run to simulate 20 hours and the latter 6.5 hours. The results of these two tests are shown in Figures 8 and 9.

To determine the natural period of the Bay theoretically the simplified rectangular basin equation, Merian's formula, was used. Since Chincoteague Bay has relatively uniform bathymetry and the shoreline configuration is approximately rectangular this equation should be applicable. The equation as derived from the Laplace equation uses shallow water wave theory. The use of the equation involves some simplifications because water elevation, orbital velocity, period, and wave length expressions obtained from shallow water theory adds some errors as these expressions are approximations. Therefore the two-dimensional Merian formula as derived is, as stated earlier, a simplified expression. Due to longitudinal oscillations being the point of the test the Merian formula is reduced to the one-dimensional case.

\[ T = \frac{L}{\sqrt{gh}} \]  

where \( T \) = period  
\( L \) = bay length  
\( h \) = depth  
\( g \) = gravitational constant

Using a length of 30 nautical miles and considering two cases of average depth, 6.0 feet (the average depth in the central part of the bay) and 4.9 feet (the average depth
Figure 8. Free Oscillation Experiment with Friction Terms
Figure 9. Free Oscillation Experiment Without Friction Terms
FREE OSCILLATION WITHOUT FRICTION
\[ \Delta t = 150 \text{ sec} = \frac{1}{2} \Delta t \]

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>GRID POINTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td>(4, 93)</td>
</tr>
<tr>
<td>△</td>
<td>(9, 46)</td>
</tr>
<tr>
<td>○</td>
<td>(11, 30)</td>
</tr>
<tr>
<td>○</td>
<td>(13, 31)</td>
</tr>
</tbody>
</table>

WATER ELEVATION (FT)

TIME (HOURS)

U-VELOCITY (FT/SEC)

TIME (HOURS)
of all grid points and includes the extensive tidal flats), gives periods of 7.3 and 8.1 hours respectively. The predicted period for the model run with friction is greater than the test run, 21 hours. The test in which the frictional terms were neglected gave a period of 8.6 hours, determined by the average temporal value for each grid point's zero velocity crossing.

Proudman's (1953) work shows that eq. 6.1 generally predicts smaller natural periods than that which is observed and attributes this to the complex geometry of the basins studied. The dynamic response of a basin system is reduced by complex geometry and friction while the resonant frequency of the motions is not as affected. Including the Bay's actual configuration in the calculation of the period by use of the lower average depth showed that this can be important through the model results from the frictionless model run. The model run with friction demonstrated that shallow depths, which cause frictional dominance of the Bay's motion are more important than the configuration and that the resonant frequency can be altered greatly.

Merian's formula, derived from the frictionless Laplace equation, should be applied with caution. If the natural period of a shallow harbor is being investigated,
where the harbor entrance is small compared to the width, an error in the estimation of the harbor's natural period can result. The maximum oscillations at the wall and currents near the centerline of the harbor can act to reinforce the tidal or wave induced oscillation. Merian's formula should not be used for a shallow basin where frictional dominance has been shown to dramatically alter the natural resonant frequency of the basin.

Hess et al's (1974) equation 3.2.1

\[
\frac{\eta(t)}{\eta(\omega)} = e^{-\mu t \sqrt{\omega}}
\]

where \( t \) = time (min)
\( \mu \) = damping factor

yields a damping factor of 4.2 for the frictional case and 1.26 for the model run without friction. The first value represents extreme frictional damping and the second the effects of complex geometry. The period of the deeper Narragansett Bay, average depth 30.0 feet, given by Hess et al (1974) is 4.73 hours with the correspondingly low damping factor of 0.073. The disparity between the damping factors of Chincoteague and Narragansett Bay and the resulting differences in model predicted periods as compared to the theoretical values supports the conclusion that for shallow water bodies friction
Table 2. Computed Tidal Differences with Variations of Manning's Coefficient
### Computed Tidal Differences

<table>
<thead>
<tr>
<th>Location</th>
<th>Chincoteague Channel</th>
<th>Piney Island</th>
<th>Greenbackville</th>
<th>George Island Landing</th>
<th>Assacorkin Island</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed Tidal Differences at Low Water with Respect to Chincoteague Point</td>
<td>36</td>
<td>1.02</td>
<td>2.37</td>
<td>2.51</td>
<td>3.31</td>
</tr>
<tr>
<td>Computed Tidal Differences with Average Manning's Coefficient Used</td>
<td>(0.028^{2})</td>
<td>(0.030^{2})</td>
<td>(0.035^{2})</td>
<td>(0.032^{2})</td>
<td>(0.030^{2})</td>
</tr>
<tr>
<td></td>
<td>(0.035^3)</td>
<td>(0.030^3)</td>
<td>(0.040^3)</td>
<td>(0.035^3)</td>
<td>(0.040^3)</td>
</tr>
<tr>
<td></td>
<td>(0.045^4)</td>
<td>(0.045^4)</td>
<td>(0.045^4)</td>
<td>(0.045^4)</td>
<td>(0.045^4)</td>
</tr>
</tbody>
</table>

1. From Tidal Tables East Coast of North and South America, a United States Department of Commerce Publication
2. Computation of Manning's Coefficient Starting at Grid Row Number \(m = 1\)
3. Additional Changes of Manning's Coefficient Employed
dominates the motions and the dynamic balance is one of friction and the pressure gradient.

6.2 Effects of Coriolis Force

The influence of Coriolis force on the Bay's hydrodynamics was investigated. The model was run for one tidal cycle with and without the Coriolis term. Output consisted of velocity plots of the Bay and current roses at 4 locations. The current rose grid points were chosen such that a large variation in velocity magnitudes could be analyzed for the Coriolis effect.

The results of the current plots showed the largest difference in current direction to be only 10%. The current velocity magnitude changes were small, less than 10%. The relatively small effect of Coriolis force can be attributed to the frictional dominance of the Bay.

Consider the Rossby and Taylor numbers,

\[
\frac{U_o}{L_o} \leq \frac{f D_o}{A_	au},
\]

where

- $U_o$: characteristic velocity
- $L_o$: typical horizontal scale
- $D_o$: typical vertical scale
- $A_	au$: eddy viscosity

Assuming appropriate order-of-magnitude estimates yields $-2$ and $-3$ respective values of $10^2$ and $10^3$. The small Rossby and Taylor numbers imply that the effect of Coriolis force can be large if compared to the non-linear terms but is small
compared to the frictional terms. This was shown through model runs. The small increase in computational time warrants the inclusion of Coriolis force in the model for an increased accuracy of the computed results.

6.3 Calibration

The initial step in model application is the calibration of the model. The tidal input is a pure cosine curve representing the dominant $M_2$ tidal constituent. The average range of the tides obtained from the Tide Tables (1975) were used at Chincoteague and Ocean City Inlet. These are respectively, 3.6 and 3.4 feet, with the Ocean City tide proceeding that at Chincoteague by 35 minutes. The tide ranges also represented the ranges during the period of current meter data for the calibration. The model was run initially to match phase with tidal datum given at six locations in Chincoteague Bay from the Tide Tables (1975). The first stages involved large manipulations of the Manning's coefficient and the results are discussed in section 5.4. During the operation of the model to obtain equilibrium the phase was adjusted by more detailed changes of Manning's coefficient. When the phase was within the accuracy of the Tide Tables the final calibration procedure was carried out.
The final model calibration was done through the use of current meter data collected by the Department of Physical Oceanography for the period August 18-28, 1975. The meters are Braincon 1381 Histogram savonious rotor current meters with a threshold of .08 ft/sec and an accuracy of ±10%. The locations of the nine current meters are shown in Figure 10. This final process consisted of adjustment of local Chezy coefficients to achieve model current velocity phase and amplitude match with observed current meter values. The period from 1700 August 27, 1975 to 0640 August 28, 1975 was selected because this represented a period of calm to low wind velocities, 0-4 knots. Earlier tests with wind as the sole forcing function indicated that these low winds would not alter the dynamics of the system. The results from an early stage of calibration are shown at 3 current meter locations in Figure 11.

An aspect of the model that was investigated during the calibration stage was the effects of by-passing the explicit section of the computational scheme. These results are also shown in Figure 11. Most notable is the resulting phase shift and amplitude change, 12%, that is observed in the test without the explicit operation. The decrease in computational time for the explicit by-pass was
Figure 10. Current Meter Stations Occupied During Aug. 18-26, 1975
Figure 11. Computed Velocity in Calibration Stage vs. Observed and Explicit By-Pass Velocity Profiles
**VELOCITY (ft/sec)**

- **X** OBSERVED
- **o** PREDICTED
- **•** PREDICTED W/O EXPLICIT
  - PREDICTED VALUES NOT ADJUSTED FOR INCREASED AREA OF GRID SCHEME)

![Graphs for Station CB5, V9A, and C21 showing velocity over time (EST)]
18%, from 27 minutes CPU time to 22 minutes CPU time.

The stability of the model was not affected by the by-pass but this can be attributed to the model being in quasi-equilibrium.

The explicit scheme should be included in the computational model when it is to be calibrated, run to equilibrium, and verified. The resulting equilibrium tidal cycle and dynamic starting conditions should then be stored on tape. Solutions of the mass-transport equations for conservative constituents should use this data except in the following cases: 1). if the open boundary conditions are changed, and 2). if the model is run with a wind stress. In the above two cases the dynamics will be altered to an extent that the equilibrium cycle will not adequately model the Bay's motions. The dynamic starting conditions should be used in these situations and the complete model run with the mass-transport equations substituted for the explicit schemes.
VII. VERIFICATION

7.0 Introduction

The purpose of the model is to provide data concerning the dynamics of Chincoteague Bay with predictive capabilities. The previous sections have covered the model use and applications with corresponding insights into the behaviour and dynamics of the Bay and model. The final part is to determine the predictive characteristics of the model by comparing observed field data with predicted parameter distributions. This stage is directed towards the verification of velocity data. The computational model has been designed to yield accurate water elevation histories and of second order accuracy velocity fields. Tide data is not available at this time except for tide table data (Tide Tables, 1975) for which the predicted results are compared in section 7.1. Therefore the verification stage has been done utilizing the velocity field data.

During verification the model response characteristics shall be subjected to a maximum test. This is due to the large spatial extent of Chincoteague Bay, where two-dimensionality is great, and the inclusion of wind stress during the model verification. As a result two factors
are being verified, the tidal model and the complete wind stress expression. Other parameters such as the Bay's residual circulation, particle paths, and wind effects are studied and used during the verification procedure. Discrepancies encountered between the model and the observed data are investigated systematically through the dynamics of the Bay. Adjustments and minor changes are included if the discrepancy lies within the computational model.

7.1 Water Elevations

Water surface elevations in the Bay are controlled by three major factors: friction, tide forcing function, and wind. The location in the Bay determines whether friction or the tidal function is the controlling mechanism. Near the inlets the tide controls the water surface elevations with friction of only secondary importance. In the main body of the Bay friction and wind had more effect on the water surface elevations. This is seen by the 92% decrease in tide height from Chincoteague Inlet to Assacorkin Island (Tide Tables, 1975; model results, Figure 12). This exemplifies the frictional dominance of the Bay but showed that the tidal forcing is important in the Inlet areas. 4.2 and 4.0 feet tide heights were used as boundary inputs for Chincoteague and Ocean City respectively during the verification to match the spring
tides for August 22, 1975 period of field data.

To determine if the system was behaving, qualita-
tively, to wind stress the model was run for varying wind
speeds and directions with the resultant set-up analyzed.
The set-up was downwind with tides ranging from .01 feet
for a 4 knot wind to 2.0 feet for a 20 knot wind. Several
features of this test examined include the wind effects on
the boundary grid points, the areas of return flow, and
changes that could be expected during verification with
wind.

The effects of the wind in the proximity of open
boundaries is seen in Figure 13. This leads to the con-
clusion that tidal heights as open boundary inputs are not
noticably affected by wind. The velocity profiles, Figure
13, indicate that current meter field data should be used
where possible for open boundary inputs when velocity data
is used. The use of river discharge data to obtain velocity
inputs can lead to erroneous results if not adjusted for
wind effects.

The areas of return flow were determined by the
Bay's configuration and mainly lateral movement results.
An imposed wind stress from any direction sets up cross
Bay water elevation differences such that the lateral
pressure gradient is larger than the longitudinal pressure
Figure 12. Computed Water Levels Compared to Tide Table Data
Figure 13. Effects of an Imposed Wind Stress in Locality of Open Boundaries
gradient for the same spatial distance. These surface elevation gradients result in the lateral return flows. Consider the parameter of Csanady (1975)

\[ \frac{fb}{c} \]

where \( b = \text{width} \)
\[ c = gh \]

If this is small then the transverse oscillations become large compared to longitudinal motions, (Csanady, 1975). Representative values cause this parameter to be 10\(^{-2}\) so that the transverse oscillations should dominate. Following the same arguments of Csanady gives a radius of deformation of about 24 miles (or 4 times the width) and period of .6 hours. This leads to the above conclusion that the motions are transverse due to lateral pressure gradients and that the motions are barotrophic modes with a velocity maximum of approximately .2 ft/sec. The model shows that barotrophic lateral motions dominate the wind and residual circulation patterns. The test runs gave the initialization of return flow as .67 hours and a velocity maximum at .16 ft/sec both of which match well with the theoretical results.

The last feature was the change in the tidal height with a wind stress. This is shown in Figure 14 where the
Figure 14. Effect of Wind on Tidal Elevations at Two Grid Points
Figure: Effect of wind on tidal elevations at two grid points that are upwind and downwind.
tidal range differences are those predicted from the tests with wind as the only forcing function. An interesting aspect is the phase shift at grid point (17,24), Greenbackville, where the tidal heights and velocities are large. This leads to the conclusion that phase shifts can be expected with wind stress but the duration of rise and fall would not be as affected. This is especially true in the inlet areas that are not as affected by a wind field.

7.2 Observed and Predicted Velocity Profiles

To achieve maximum correlation with the field data the computations were started at high tide at Chincoteague Inlet boundary and run for 6.21 hours with a vectorally averaged mean wind speed of 10.4 knots from 222°. This was done to achieve the wind developed perurbation of the tidal flow field, corresponding to the time of wind action before the verification period of field data. The model was then run for one tidal cycle with the same wind stress. Output consisted of the vectoral velocity recorded for 7 current meter locations. The predicted velocity magnitude and direction profiles are compared to the field data in Figures 15-21.

The predicted model response characteristics show close agreement with the field data. This is shown through
Figure 15. Computed vs. Observed Velocity at Station V4A, August 22, 1975
Figure 16. Computed vs. Observed Velocity at Station V9A, August 22, 1975
Figure 1. Computed vs. Observed Velocity at Station CB2, August 22, 1975
Figure 18. Computed vs. Observed Velocity at Station C21, August 22, 1975
Figure 19. Computed vs. Observed Velocity at Station C14, August 22, 1975
Figure 20. Computed vs. Observed Velocity at Station CB5, August 22, 1975
FIGURE 20. CURRENT VELOCITY PREDICTIONS VS. OBSERVED
DATE: AUG. 22, 1975
Figure 21. Computed vs. Observed Velocity at Station C11, August 22, 1975
STATION: C11
LOCATION: (N, M) = (11, 45)
DEPTH (C.METER): 4.0 FEET
AVE. WIND: 10.4 KNOTS
DIRECTION: 222.0°
C DRAG: 0.00827
DEPTH (GRID): 60 FEET
OBSERVED VELOCITY: ∆ (ADJUSTED FOR HORIZONTAL WAVE ORBITAL VELOCITY)
PREDICTED VELOCITY: ○

OBSERVED DIRECTION FROM MAGNETIC NORTH:

PREDICTED DIRECTION: ○
the remarkable match between the profiles for all current
meter locations. Most interesting is the directional
predictive results of the model. Except for slight
variations in phase the accuracy of the results are within
10% thus indicating a good representation of the Bay's
circulation. The best results for magnitudes are at the
locations V4A, V9A, and CB2 (Figures 15-17) near Chinco-
teague Inlet, the area least affected by the wind (see
previous section). Outside the inlet area there are
noticeable differences between the observed and predicted
phases at Stations C14 and C21 and magnitudes at Stations
C21, C14, and CB5. The extremely good correlation of
direction at Stations CB5 and C21 combined with the pro-
files from the calibration stage, Figure 11, indicate that
either the wind stress was not being adequately represented
or some other factor was entering into the model results
that was not considered. Therefore, a test was done in
which the drag coefficient in the wind stress formulation
was increased to determine if this would affect the magni-
tude and direction of the velocity. The results from this
test showed a slight increase in magnitude, 5%, with no
change in phase or direction of the predicted velocity
profiles. If one considers the increased water levels and
expected phase shifts shown in Section 7.1 and Figure 14
and then compares the predicted magnitudes of the verification profiles with wind and the calibration profiles without wind a good correlation is seen in the amplitudes. Therefore, the results of the test with the increased drag coefficient, the increase in the water elevation and velocity amplitudes for the verification period, and the good direction correlation between the model results and the observed field data indicates that the wind stress formulation is properly stated. It appears that the discrepancy is outside of the computational model so that another approach was taken.

Because velocities were greatly underpredicted by the model in areas of maximum fetch, an attempt to quantify the wind-wave effect on the savonius rotor meter was made. The modified Sverdrup, Munk, and Bretschneider (SMB) forecasting method of Bretschneider (C.E.R.C., 1973) was used to determine the wave field present during the verification period. The most probable maximum and minimum wave height and period were found for a 10.4 knot wind with a fetch of 17 miles. The average wave height and period were then determined with respective values of .85 feet and 1.6 seconds. This corresponded well to that which has been observed for Chincoteague Bay during that period of time. The average horizontal water particle
velocity was then calculated using linear wave theory. The average value over one period was found to be .61 ft/sec which could explain the discrepancy between field and predicted velocities.

The water particle orbits in shallow water are essentially horizontal to and fro motions. The operation of the savonius rotor meters is such that this movement will be recorded as a progressive forward motion where the retrograde velocity will be added and not subtracted from the recording. Therefore, the average particle velocity was subtracted from the observed velocity and the results are shown at current meter locations C14 and C21 (Figures 18-19). The close agreement of the predicted and observed magnitudes indicates that this may be the factor causing the discrepancy of model results. This shows that a wave field can contaminate current meter data. This is further brought out at Station C11 (Figure 21) where just the adjusted observed velocity values are plotted.

The phase differences can be explained in a similar manner. The test run with a wind stress and no tides gave downward directions for the grids in question. The wave field will dominate the current readings at the change of tide when the velocities are low. This leads to the
rotation of the current meter upwind being delayed with resulting shorter duration of flows against wind, i.e., the wave field dominates the low tidal motions though the net mass transport is small. This is exemplified at Stations CB5 and C21 where the change of direction takes place around the predicted average horizontal orbital velocity.

Subsurface current measurements can be altered to a large extent by a surface wave field. This has been shown in this study and several investigations, (Halpern et al, 1976; Brunard and Lukens (1975), Karweit (1974)). The note of Halpern et al (1976) is a good example of the extent of the contamination of data. Two subsurfaced moored savionus rotor type current meters where analyzed for the influence of the current wave field. The depths of the meters were 3 M and 18 M and these were compared to 43 M current meter data and to each other. The deeper current meter showed much lower velocities, there was a 47% difference in the mean values. This was also true in the comparisons with the 43 M moored current meters. A spectral analysis of the kinetic energy density showed that the 3 M recordings had more energy overall and an accumulation of energy in the higher frequencies. The correlation between the two sets of data for the kinetic
energy density was low for the higher frequencies.

To obtain further insight into this process a Fourier series analysis of current meter data for the calibration and verification periods was done for Stations C14 and V4A. The high frequency waves are filtered out by the method at data collection but an accumulation of energy is still expected because the higher frequency components can transfer energy more efficiently (Lamb, 1945). Higher harmonics can be generated as a tidal wave progresses through shallow water. Therefore, a difference in the energy of the harmonics of the two stations should be noticeable and a further difference at Station C14 was expected for the verification analysis due to the wind-wave affects. The results are seen in Figure 22. The expected broadness of the energy spectrum at Station C14 compared to V4A for the calibration and verification data was found. C14 shows a noticeable increase, 16%, in percent energy of the fundamental for the higher harmonics for the verification period. V4A still displays the approximately same energy banding for this period. Thus, the wind-wave field seems to have contributed energy to the velocity spectrum which is represented by high velocity readings for the upper Bay stations.
Figure 22. Comparison of Magnitude of Harmonics at Two Current Meter Locations, with and without Wind
The large amount of energy in the 17th harmonic, with a period of .73 hours, at both stations is an interesting feature of this analysis. There are two possible explanations of this result. The increase can be attributed to aliasing but a feature of this would be a broader spectrum of energy than the localized phenomenon observed. The current meters effectively filter out the high frequency motions, less than a period of 20 minutes, and the sampling frequency was every 20 minutes so that a definite conclusion cannot be made. The other possible explanation was discussed in Section 7.1. The theoretical fundamental mode of a transverse seiche was shown to be approximately .6 hours in section 7.1. Section 6.1 on free oscillation indicates that this period can be increased because of frictional effects. Considering this and the approximate means used to determine the period the .73 hour period obtained is plausible. The high energy in the 17th harmonic can be caused by either aliasing or the transverse seiche motion and no definite conclusion can be made. This point should be investigated through the use of the tide gauge data at some point in the future.

7.3 Residual Circulation

The understanding of large scale motions in coastal basins involves several concepts. The time dependent
hydrodynamics have been investigated and insights into the Bay's dynamics have been gained. The next question that should be asked is what happens to the Bay's circulation if we average over time scales such that wind, turbulence, and tidal motions are essentially averaged (filtered) out. This is an important concept and can render useful results for diverse studies. The long term flushing of the bay could be studies by noting the net transport. The dynamics of the time-dependent model could be understood better if the residual circulation pattern was known. To obtain the residual circulation the model was run for several tidal cycles and at each grid point the U and V velocities were averaged for the time of computations. The results are shown in the velocity vector plot, Figure 23.

The resultant residual circulation obtained poses an interesting hydrodynamic phenomena. The barotropic motions are essentially transverse geometry controlled motions. There is a net residual circulation out at both inlets and a superelevation in the interior. The inlets reflect the hydraulic head established by the interior superelevation. This is in agreement with Cameron and Pritchard's (1963) statements on vertical homogeneous estuaries and has not been shown for a particular basin in
detail before. The flow pattern indicates that the tidal forcing at Chincoteague Inlet controls the residual circulation. This is dramatically seen at the Northern end of the Bay where there is no net residual circulation. The expected transverse motions are controlled by the configuration of the Bay through the tidal forcing at Chincoteague Inlet. Outside of the inlet areas the velocities show a grid row to grid row change in direction at geometric changes. The flooding tide at Chincoteague Inlet bends to the geometry setting up the 'sine wave' type circulation pattern. The ebbing waters are of lower velocities and from grid row M=35 ebb Northward resulting in an increase in the flood pattern from grid row M=35 on. Before this grid row the lower ebb velocities cannot cancel out the higher flood velocities, thus resulting in the flood dominant circulation pattern exemplified in the velocity vector plots, Figures 24-25, for low and high water at Chincoteague Inlet boundary and the residual circulation plot, Figure 23.

The two current field plots were obtained during the verification run with a wind stress. The wind perturbed tidal flow matches the normal current field without wind very well. The downwind velocities are increased slightly, other perturbations of the tidal flow are discussed in
Figure 23. Velocity Vectors of Residual Circulation in Chincoteague Bay
RESIDUAL CURRENT FIELD IN CHINCOTEAGUE BAY

VELOCITY SCALE
LENGTH

2.0-1.8 FT/SEC
1.7-1.0 FT/SEC
.04-.01 FT/SEC
.009-.001 FT/SEC
Figure 24. Current Field in Chincoteague Bay at Time of Low Water
Figure 25. Current Field in Chincoteague Bay at Time of High Water
Sections 7.1-2. These plots clarify the residual circulation in the Bay. Between M grid rows 36 to 42 there is a convergence of the two tidal waves at high water and divergence at low water. Low water, Figure 24, shows the tidal wave ebbing Northward at M-grid row 35. This will act to reinforce the flooding Chincoteague Inlet wave at high water for a net flux to the interior. This can be attributed to the waters leaving Johnson Bay, (N=16-19, M=35-41), and moving South. These waters are then mixed with the interior water increasing the pressure gradient such that the Northward flow is in turn increased. The net hydraulic head at high water due to the converging waves increases the effective velocities, as shown by the vectors in M grid rows 39-45. The probable initial cause for the above circulation phenomenon is the phase difference of the tides for the two inlets.

The two plots show little flux into Newport Bay and Johnson Bay and is represented by the low residual circulation in these bays. The last interesting point are the two gyres that are in the Bay. Toms Cove has a well defined gyre caused by the incoming water taking the route of least resistance, the deep water next to Fisherman's Point on Assateague Island. The other gyre is seen at the Eastern part of the Bay between M-grid rows 50-56. The
converging waves force part of the Ocean City waters into this section with a return flow resulting in a well defined gyre. This is shown best in Figure 25. The widest portion of the Bay is in this region with a large convergence of width just South of this point. The geometric convergence acts to increase the effective velocities and depth; thus, there is a tendency to keep the same configuration. This results in the arriving Ocean City waters to form the gyre with the above mentioned mechanics. Therefore the velocities are large in the alongshore region of the gyre indicating that the depth regime can be kept intact with little siltation. It cannot be concluded whether the processes stated above act to maintain the configuration controls the dynamic processes. In either case it seems to be a stable dynamic balance with possible evanescent changes resulting from seasonal or wind episodes.

The flushing in the Bay is very poor as shown by the residual circulation, thus the water quality is in a delicate balance. Taking the average residual velocity in the center of the Bay as .2 ft/sec gives 10 days as the time required for a particle to leave the Bay. Since residual velocity decrease towards the North and South this estimate at flushing time is probably low. This time does not
include possible reentrance with a flood tide or transport to areas of no net residual circulation. Pritchard (1963) gives the 50% and 99% renewal times as 9.3 days and 62 days. The 10 day time for particle travel corresponds well with the simple model of Pritchard. These values indicate the extremely poor flushing in the Bay, especially since a contaminant can degrade within this period of time. The poor flushing is especially true in small embayments such as Newport and Johnson Bays which have little or no residual circulation and exchange of waters. In friction dominated basins such as Chincoteague Bay poor flushing can be expected. If a marked particle was tracked through the use of the velocity vector plots the actual flushing time for different regions of the bay could be examined in detail. In accordance with the frictional dominance the length of residence is dependent upon the location in the Bay. This is shown by the percent renewal times. As it takes only 9.3 days for 50% of the water to be renewed as predicted by Pritchard the waters in the center of the Bay take much longer resulting in the 62 days needed for complete renewal.
8.0 Introduction

An important application of the model is the solution of equation 2.35. The method of solution has previously been discussed and the following section covers the results of the experiments with the mass-transport equations. The boundary conditions and the mass-conserving abilities of the model were initially tested. This was achieved by giving the Bay an initial concentration of 27.5 throughout, setting the ocean concentration to 27.5, running the model for a tidal cycle, and letting the open boundaries be 27.5 for all time. The velocity and water level inputs were those from the equilibrium tidal cycle. They were read off tape every one-half time step. The mass throughout the Bay was conserved. This was shown by the printed output which showed no change in the concentration field over a tidal cycle. Experiments with a single and continuous dye dump were conducted. The CPU for one tidal cycle was approximately 6 minutes for one conservative constituent. This is very economical for the returned information on the fate of a dumped conservative constituent at a point source.

97.
8.1 Studies with a Conservative Tracer

Point source simulated dye dump experiments were conducted in the vicinity of N.A.S.A. Wallops sewage outfall that the town of Chincoteague, Virginia uses. To preserve the conservation of mass the 120,000 G.P.D. flow rate of the outfall was reduced to the amount of mass added to a grid square represented by the increase in height by the method set forward in Section 4.2. The resulting increase of mass at grid point (21,13), location of the outfall, was $6.8 \times 10^{-6}$ ft., a negligible amount. Therefore, for the test runs no additional mass was added to the point source grid point. The first experiment consisted of a single dye dump at grid point (21,13). The concentration was 100 ppb corresponding to approximately 40 lbs/day of dye. The second experiment was a continuous dump of 100 ppb every one-half time step for 6.21 hours. This was done to obtain the model's response to large concentration gradients, to test the dispersion capabilities of the model, and to model real time situations of continuous dumping. For both experiments the model predictions, compared at least qualitatively, with dye data collected by the Department of Physical Oceanography in August, 1976 during a dye study at the area. Existing data from this study has not been fully evaluated yet;
Figure 26. Computed Isopleths of Single Dye Discharge
1.33 HOURS

2.67 HOURS

4.0 HOURS

5.33 HOURS

6.67 HOURS

8.0 HOURS
therefore, the model cannot be verified at this point. The diffusion seemed to be adequately represented by the formulation in Section 2.3.1. The maximum and minimum dispersion terms had respective values of $0.78 \times 10^{-1}$ and $0.4 \times 10^{-6}$.

The results from the first test are shown by the isopleths in Mosquito Creek Bay in Figure 26. The concentration fields from 4 ppb to 0.01 ppb represent the net advection while the spreading of the higher concentrations results from dispersion. An important aspect is the relative stable concentration high in the vicinity of the point source. The isopleths move back and forth from the source with the changing tidal direction with little net transport. No dye was lost to the ocean in the ebbing waters. This was borne out in the continuous dump experiment. Once the concentration field reached the proximity of Chincoteague Channel the turn of the tides would transport the dye back into the area of the source while some would be isolated in the area around Chincoteague Island. This is due to the split flow regime and is similar to the process discussed by Pritchard (1960). The steep gradients greatly increased the effective diffusion but the main concentration high stayed in the vicinity of the outfall with almost all the concentration field in Mosquito
Creek Bay, which corresponds to the field observations during the August 1976 dye study. Of the dye lost to Chincoteague Channel none was transported out of the Bay during the 11 hours the model was run but the resulting concentration field was greatly enlarged. This isolation and resulting advection is a major transport phenomena in the Bay but the circulation is such that the concentration is increased throughout the Bay with little lost to the ocean in the ebbing waters. It would be expected that the quantity that would be initially lost to the ocean would return with the flooding waters.

The tidal diffusion is most effective in a vertical homogenous estuary, Pritchard (1960). This is exemplified in Chincoteague Bay where the net advection is small as shown by the tracer studies and the residual circulation. In this type of vertically homogenous shallow water estuary with little fresh water inflow dispersion is the most important process. The dispersion has its greatest influence in areas of high constituent gradients. Dispersion also has a large influence on the net transport when the constituent can be trapped by an irregularity in the shoreline such that it is mixed with waters of a different flow regime. This process has been mentioned above and is illustrated
graphically in the isopleths for the continuous tracer dump, Figure 27. It has also been described by Pritchard but its importance has not been quantified in a model with a corresponding dye study before. The best example is represented at hours 9 and 11. The ebbing waters diverge around Wire Narrows Marsh with a portion of the tracer contaminated water being transported back to Mosquito Creek Bay and the rest being transported in Chincoteague Inlet, with a different flow regime.
Figure 27. Computed Isopleths of Continuous Dye Discharge
IX. SUMMARY AND CONCLUSIONS

The mathematics, methods, response characteristics, and applications of the computational model have been investigated. This section summarizes the observed results of the model use and applications and elucidates the hydrodynamics of the Chincoteague Bay. The computational model used is the state-of-the-art two-dimensional model of coastal waters. The computed results give remarkable resolution with time to the water elevations and velocity fields. The implicit-explicit space staggered scheme is an economical and accurate method for the solution of the two-dimensional vertically integrated hydrodynamic equations of motion. Accurate representation of the computed parameters depends upon an accurate depth field and the proper choice of the time step, computational lattice, and Manning's coefficient.

It was found that the Manning's friction coefficient is perhaps the most important parameter to determine in the model. This is true for all coastal waters and not just shallow bodies such as Chincoteague Bay. This was shown by the model studies of water bodies of greatly varying depths of Leendertse (1967) of the North Sea and Tokyo. Bay and Hess et al's (1974) study of Naragansett Bay.
Previous models yielded accurate transports but the computed velocities can be order-of-magnitude estimates. The close agreement in this study of the computed and observed velocities indicates the accuracy of the model. This can perhaps be attributed to the shallowness of the Bay resulting in essentially barotropic motions where the velocity is approximately uniform over depth and the easily stated boundary conditions. The two-dimensionality and the response to an imposed wind stress shows that the model is quite accurate and versatile. The computational model has much potential for future applications. The ability to handle a wind stress properly in two-dimensions is a major asset.

The conservative constituent water quality model is very economical and conserves mass through the computation. It adequately portrays the dispersion process and can include wind effects on diffusion. The model was shown to handle steep gradients. The open boundary conditions are easily stated and can operate with as many as 15 open boundary points. The boundary conditions can easily be changed to handle more boundary grid points.

The model study of Chincoteague Bay brought out various interesting hydrodynamic characteristics of the Bay and of shallow water bodies. The results corresponded
well with the previous one-dimensional model study of Harleman et al (1969). The area of the superelevation was the same as the area predicted by that study and can be attributed to the non-linear terms. This lends support to the arguments of Cameron and Pritchard (1963) on the dynamics of a vertical homogenous estuary with lateral homogeniety. There is a definite variation of the vertical gradient of the horizontal pressure field with a resulting seaward slope of the pressure surfaces. This has been shown in Section 8.3 to be caused by the asymmetry in the ebb and flood regions on dominance and has been attributed to the linear terms, with an influence from the tide phase difference between the two inlets and inertia. A theoretical investigation on the possible breakdown of the non-linear terms into Reynold stresses would be an interesting aspect of the superelevation. The zone of least tidal influence predicted by their model study is in the same location as the area of no net circulation shown in the residual circulation plot, Figure 23, and represented by extremely low velocities in the computed output. Discrepancies arise in the scale of predicted velocities. Harleman et al's (1969) model does not predict high enough velocities for the Chincoteague Inlet area and too high
velocities for the Ocean City Inlet section. The one-dimensional model cannot give the direction of the flow and thus a good circulation pattern within the Bay.

Wind effects on shallow water bodies are very important in determining the flow field. This is especially true in the interior where the velocities are minimal and the wind is the main method for transport and diffusion. The wind can set up a flow field that can dominate the interior motions with resulting dramatic changes in water elevations, wind surges. These wind surges can greatly reduce the water levels in shallow areas thus making the navigation of these locations dangerous. The shallow coastal bodies can be affected to a large extent by winds as small as 6 knots. This is also very important in the water quality management of these areas. The dominant seasonal winds can greatly decrease the natural flushing and yet can also increase it (i.e. different circulation pattern depending on wind direction). Studies with wind stresses can give insights into proper placement of outfalls. The model results showed the extent of transverse motions in the circulation. It was shown that these motions can perhaps contain a broad spectrum of energy. The tendency to transverse residual motions has great impact on water quality. The essential movements of contaminants
can be confined laterally and thus little flushing would take place. The applied winds will produce transverse return flows rather than an overall longitudinal circulation pattern throughout the Bay. The transverse motions are dependent to a large extent upon the configuration of the Bay. In shallow bays the motions are frictionally dominated thus decreasing the velocities. The resulting motions are then dependent upon the configuration and are lateral, at least in Chincoteague Bay.

The inlet areas are essentially represented by tidal forcing of inertial motions as shown by the Figures 24-25. The interior motions are friction dominated resulting in a dynamic balance of friction and pressure. This results in large sediment accumulation in the Bay because the velocities are not large enough for transport of large grain sizes. Another physical aspect of the frictional balance is the large increase in the resonant period of the water body. Previous sections have shown that this is the dominant mechanism for changing the resonant period rather than the configuration of the water body. It has generally not been stated that friction can control the resonant frequency and this is an important finding from the model results.

The water quality model corresponds quite well, qualitatively, with a dye study by the Department of Physical
Oceanography conducted during August, 1976. The results are not yet available to discern the quantitative ability of the model. An important finding is the extremely poor flushing in the vicinity of Mosquito Creek, the locale of the sewage outfall. Preliminary results indicate that Mosquito Creek is probably a poor location for a sewage treatment plant outfall. Future disposal sources should be placed near the inlets or in the ocean. Velocity vector plots should be obtained every hour in the vicinity of point sources to ascertain the fate of a discharged constituent.

The mechanism of isolation of part of a concentration field by varying flow fields was shown in this test. This aspect is very important in the transport of dissolved constituents, especially if it is a contaminant. The residual circulation, the extent of transverse motions, and the water quality tests all indicate that discharges of contaminants into the Bay can be precarious for the delicate balance of water quality. The high temperature during summer which results in low dissolved oxygen indicates that the Bay cannot handle any increased loadings especially if they are located in a region of poor flushing.
APPENDIX A.

Stability Aspects of the Multi-Operation Difference Scheme

When using numerical schemes for solution of partial-differential equations certain problems arise. The problems are generally put under the label of stability of the difference scheme. The problems, in truth, are much broader than this since a stable scheme can lead to spurious solution modes. There are several questions that one must ask in using a difference scheme: whether the difference equations approach the differential equations; whether the solution of the difference equations on discrete grid points approach, in the limit of $\Delta x, \Delta t \to 0$, the solution of the continuous function differential equations; whether numerical errors introduced through truncation and difference methods amplify or are damped; the amount of deformation in the wave's amplitude and phase of the numerical solution; will the short waves generated by the non-linear terms accumulate; and are mass and momentum conserved. For a detailed analysis of these ideas, the reader is referred to Richtmeyer and Morton (1967) and Leendertse (1967). The major points of Leendertse's analysis will be discussed since they are very important concepts and determine the characteristics of the calculated solutions.
For all of the following discussion the linearized one-dimensional equations stated as
\[ \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0 \]
\[ \frac{\partial u}{\partial t} + g \frac{\partial u}{\partial x} = -f \frac{p}{\partial x} \]  
will be used.

The first concept is the order of approximation of the two systems of equations. This is done by taking the exact solution of the continuous partial-differential equations as a grid function, \([u]_L\), and subtracting from it the solution of the difference equation, \(\bar{u}_L\). This norm, \(\| [u]_L - \bar{u}_L \|\), is the measure of the approximation of the equations and is the same as the order of approximation of the difference equation, if the scheme is stable.

The linearized equations are written out in the chosen implicit form and a Taylor expansion of each equation at time \(t = t + \frac{1}{2} \Delta t\) is written out. Upon neglecting higher order terms, the norm of the approximation is obtained. Leendertse (1967) shows that this system has second order accuracy for all \(\Delta x\) and \(\Delta t\) if \(\Delta x \to 0\) and \(\Delta t \to 0\).

The next aspect that is important is the stability of the difference equations, i.e., numerical errors should not amplify and become unbounded. The method to investigate this is to follow a Fourier expansion of an error wave perturbed on the system. As in the first analysis,
the linearized equations are used with an implicit scheme and the forcing function is assumed known for all grid points as a continuous function.

Consider a line of errors, \( \delta \bar{u}(x) \), and a finite Fourier series of it. Since it is a linear system only one component is considered, \( \bar{A}_n e^{\sigma_n x} \), where \( \bar{A}_n \) is time dependent such that it is written as

\[
\bar{A}_n(t) = \bar{u}^* e^{i \beta_n t}
\]  

(2)

The error wave component becomes

\[
\delta \bar{u}(x, t) = \bar{u}^* e^{i \beta t} e^{i \sigma x}
\]  

(3)

where \( \bar{u}^* = \{ u^*_n \} \)

Equation 3 is substituted into the implicit scheme, letting \( \lambda = e^{i \beta t} \), two homogeneous linear equations in \( \eta^* \) and \( u^* \) are formed. A determinant of this system is solved, since it must vanish identically, and thus, the solution for the eigenvalues are found

\[
\lambda_{1,2} = \frac{1 \pm \frac{i}{\ell} \sqrt{\gamma n \sin(\sigma \ell)}}{1 \pm \frac{t^2}{\ell^2} \gamma n \sin^2(\sigma \ell)}
\]  

(4)

This gives \( |\lambda_{1,2}| < 1 \) and the error wave will decay with time.

This analysis is exactly parallel to investigating the wave solutions of the difference equations since the system is linear and both waves satisfy the same equations.
Consider one component of each solution
\[ \eta(x, t) = \eta_n e^{i(\sigma_n x + \beta_n t)} \]
\[ \psi(x, t) = \psi_n e^{i(\sigma_n x + \beta_n t)} \] (5)

Upon substituting the above into the implicit scheme, the matrix form is arrived at by Leendertse

\[ \begin{bmatrix} A \end{bmatrix} \bar{u}^{n+1} = \begin{bmatrix} G \end{bmatrix} \bar{u}^n \] (6)

where \( \bar{u}^{n+1} = \{ \eta_m^{n+1}, \psi_m^{n+1} \} \)

which can be written as

\[ \bar{u}^{n+1} = \begin{bmatrix} G(t, \xi) \end{bmatrix} \bar{u}^n \] (7)

where \( [G(t, \xi)] \) is the amplification matrix of Lax (Richtmeyer and Morton, 1967, page 67).

For the actual investigation of stability, von Neumann's necessary condition for stability and the sufficient conditions are used (the necessary and sufficient condition is not used since the amplification matrix is not normal which is required to use the necessary and sufficient condition) (Richtmeyer and Morton, 1967). The basic stability analysis of Leendertse then reduces to whether for a finite time step, \( t \), the elements of the amplification matrix are bounded and that

\[ |\lambda_i| \leq 1 + o(\Delta t) \] (8)
such that all but one eigenvalue, equation 4, lie within a unit circle. Therefore, since these conditions hold the system is unconditionally stable.

The final aspect to consider is the rate of convergence of the difference solutions to the exact solutions. A method that produces this is to investigate the ratio of the computed wave with the exact solution after a certain time interval. In this way the amplitude and phase of the components of the computed wave are compared to the physical wave or exact solution.

Leendertse (1967) does this by substituting

\[ \eta = e^{i(\delta x + \beta t)} \]  
\[ \phi = e^{i(\alpha x + \beta t)} \]

(9a)  
(9b)

into the linearized equations which yield

\[ \frac{\beta}{\sigma} = \sqrt{\frac{g}{h}} \]

(10)

Equations 9 are then represented on the grid scheme and substituted into the implicit formulation of the linearized equations. The resulting two equations are solved and the solution is the eigenvalue found earlier, equation 4,

\[ \lambda_{1,2} = e^{i\beta_{1,2} t} = \zeta q_{s} (\mu) \]

(11)
The imaginary parts of $\beta_{1,2}$ are zero and, therefore, $\Re(\beta_1)$ and $\Re(\beta_2)$ are solved for such that $\frac{\beta_{1,2}}{\sigma} < 1$. Therefore, from equation 10 for all values of $t$ the computed wave propagates slower than the real wave such that the computed wave frequency $\beta^1$, is lower. The amplitude of the computed wave does not change as the moduli of the eigenvalues are less than or equal to one.

Finally, the concept of complex propagation factor $\left[ T(\sigma t) \right]$ is used by Leendertse (1957). Since in numerical schemes the amplitude and phase of the computed wave can change, an evaluation of the limits of the changes is profitable and the avoidance of the changes can be accomplished. The propagation factor is the complex ratio of the computed wave to the physical wave after a time in which the physical wave would have traveled over its wavelength. The modulus of the propagation factor is the measure of the decay of the amplitude during this time interval while the argument of $\left[ T(\sigma t) \right]$ is a measure of the computed phase shift. So that

\[
T(\sigma t) = \frac{e^{i(\beta^1 t + \sigma x)}}{e^{i(\beta t + \sigma x)}} = e^{i \gamma (\frac{\beta^1}{\sigma} - 1)}
\]
is the propagation factor. From equation 4 the modulus equals one and the wave decays with time. The argument Leendertse gives as

\[ \text{Arg}\left[\gamma(x,t)\right] = 2\pi \left[ \frac{\sin^{-1}\left(\frac{t^2}{\sqrt{L}} \sin\left(\sigma L\right) \sqrt{L}/(\sigma L)\right)}{t^2 \sqrt{L} \sigma (\sigma L)} - 1 \right] \]  

where a positive value indicates an acceleration of the computed wave. Numerical experiments were conducted by Leendertse (1967) where the phase angle was described for varying values of equation 12 and represented by the dimensionless ratio, L/l. For values of \( \frac{t^2}{\sqrt{L}} \sigma L \leq 5 \) a L/l=40 would give good results with little or no phase shift and for values of \( \frac{t^2}{\sqrt{L}} \sigma L \leq 5 \) then L/l=100 would give adequate resolution.

The essential arguments presented above have been expanded to the non-linear two-dimensional equations by Leendertse (1967) who also conducted numerous tests to check approximations that could not be treated analytically. Some of the tests have been repeated in this paper due to the different modeling areas but the interested reader is referred to Leendertse (1967) for a complete and thorough discussion which is beyond the goal of the thesis.
Solution of Finite Difference Equations and Recursion Formulas

Equations 3.1 and 3.2 can be written as

$$-r_{N-1} u_{N-1}^{n+1} + r_{N+1} u_{N+1}^{n+1} = A_N^n, \text{ all on line } M$$

(1)

$$-r_N v_N^n - r_{N-1} u_{N-1}^{n+1} - r_{N+1} v_{N+1}^{n+1} = B_N^n, \text{ all on line } M$$

(2)

where the r variables contain the coefficients for unknown variables. This can be written in matrix form as

$$
\begin{bmatrix}
    r_{N-1} & 0 & 0 & 0 & 0 \\
    -r_{N+1} & r_{N+2} & 0 & 0 & 0 \\
    0 & -r_{N+1} & r_{N+2} & 0 & 0 \\
    0 & 0 & -r_{N+1} & r_{N+2} & 0 \\
    0 & 0 & 0 & -r_{N+1} & r_{N+2} \\
    \vdots & \vdots & \vdots & \vdots & \vdots \\
    0 & 0 & 0 & 0 & -r_{N+1} \\
    \end{bmatrix}
\begin{bmatrix}
    u_{N-1} \\
    u_{N+1} \\
    u_{N+2} \\
    u_{N+3} \\
    \vdots \\
    u_{N+1} \\
\end{bmatrix}
= 
\begin{bmatrix}
    A_N^n \\
    B_N^n \\
    A_{N+1} \\
    A_{N+2} \\
    \vdots \\
    A_{N+1} \\
\end{bmatrix}
\begin{bmatrix}
    v_N^n \\
    v_{N+1}^{n+1} \\
    v_{N+2}^{n+1} \\
    v_{N+3}^{n+1} \\
    \vdots \\
    v_{N+1}^{n+1} \\
\end{bmatrix}
$$

(3)

The nomenclature is such that $N=J=1$ is the lower bound such that $u_{N-1}^{n+1}$ is a given velocity value, i.e., it is equal to zero at the boundary and $N=I$ is the upper bound such that $u_{N+1}^{n+1}$ is a given velocity. Therefore, assuming $u_{N-1}^{n+1}$ and $u_{N+1}^{n+1}$ are known values, the solu-
tion for the vector \( \{ F \} \) in eq. 3.9 at \( n + \frac{1}{2} \) can be found through a limited series of elimination operations by use of recursion formulas. Physically, this gives one equation with three unknowns for each velocity point \( u_{n+\frac{1}{2}} \) and for each water level point \( h_{n-\frac{1}{2}} \) on line \( k \). For \( N \) water levels this yields \( N-1 \) velocities at time \( n+\frac{1}{2} \) for the unknowns with \( 2N-1 \) equations.

The first equation of the matrix can be written for unknown velocity, \( u_{n+\frac{1}{2}} \), in terms of the unknown water level, \( h_{n+1} \), as

\[
\begin{align*}
u_{n+\frac{1}{2}} &= -R_0 h_{n+1} + S_0 \\
\text{where} \\
R_0 &= \frac{r_{n+1}}{r_{n+\frac{1}{2}}} \\
S_0 &= \frac{B_0 + r_{n+1} h_{n+1}}{r_{n+\frac{1}{2}}} 
\end{align*}
\]

Expanding the second equation and substituting eq. 4 for \( u_{n+\frac{1}{2}} \) yields

\[
-\frac{r_{n+\frac{1}{2}}}{r_{n+1}} \left[ -R_0 h_{n+1} + S_0 \right] + h_{n+\frac{1}{2}} - \frac{r_{n+1} \cdot \frac{1}{2}}{r_{n+\frac{1}{2}}} u_{n+\frac{1}{2}} = \lambda_{n+1}
\]

Expressing the water level as a function of the next velocity gives

\[
h_{n+\frac{1}{2}} = -p_{n+1} u_{n+\frac{1}{2}} + Q_{n+1}
\]
Continuing this process the following recursion formulas can be written for each row $M$

\begin{align}
\eta_{n-1}^N &= -P_N \eta_{n-1}^{N-1} + Q_N \\
\eta_{n-1}^{N-1} &= -R_{N-1} \eta_{n-1}^N + S_{N-1}
\end{align}

The recursion factors, $Q$, $R$, $S$, and $P$ are computed successively from the lower bound, $N=1$, to the upper bound. In the example it has been assumed that the lower boundary is a water boundary with an initial known value of $\eta_{n-1}^N$. Therefore, eqs. 5-5 are used first and $R_1$ and $S_1$ are calculated. $P$-$S$ are next used to find the remaining recursion factors. The velocities and water levels can be found from the recursion formulas, eqs. 9-10, by computing them in descending order. If the lower bound is a land boundary, then eq. 9 is used first at $N+\frac{1}{2}$, i.e., we would have a different leading term in the matrices $\{F\}$ and $[A]$. The process is still the same except the coefficients are changed. The $V$-velocities for the time step can be found explicitly from eq. 9 since all the values will now be known. The operations for the second time level is the same except the implicit operations use $V$
instead of U-velocities. Similar methods and recursion formulas are used for the mass-transport equations.
APPENDIX C

Flow of Program

For future users of the model a diagram is given which shows the flow of the program. The basic program can be divided into several sections: dimensioning, setting of execution parameters, subroutine calls, reading input, set initial configuration of model, and the actual computation sections for each half-time step. This is represented diagramatically in Figure C-1.
Figure C-1. Flow of Program
APPENDIX D

Program User Guide

System Dimensions

The following are the arrays in the common and dimension statements. They are given suitable dimensions for a 32 x 95 lattice. Parameters marked with an asterisk are kept at the same dimensions for all model runs. Parameters without asterisks are dimensioned according to the area being modeled and might be altered for different model runs.

\[
\begin{align*}
NMAX^* &= \text{maximum grid size in N-direction, not to exceed 32} \\
MMAX^* &= \text{maximum grid size in M-direction, not to exceed 32.}
\end{align*}
\]

Dimension \( A^*(\ ) , B^*(\ ) , P^*(\ ) , Q^*(\ ) , R^*(\ ) , S^*(\ ) , F^*(\ ) , \)

\( \text{KONVRT}^*(\ ) , \text{NH}^*(\ ) , \text{NPRINT}^*(\ ) , \text{I PLOT}^*(\ ) \)

The vectors \( A, B, P, Q, R, S \) are used in the implicit operations for the recursion formulas and should be dimensioned \( \text{NMAX} \) or \( \text{MMAX} \) whichever is greater. \( \Gamma \) is the value of the Coriolis parameter and should be dimensioned as the above vectors. \( \text{KONVRT} \) and \( \text{NH} \) are dimensioned \( \text{NMAX} \). The average values of the water elevations and velocities used for printouts are stored temporarily line by line.
in KONVRT.

Common SE( ), SEP( ), V( ), VP( ), U( ), UP( ), C( ), NBD( ), MBD( ), MOBD( ), NOBD( ), H( ), XIA( ), XIB( ), CN( ), CNP( ), IFIELD( ), ZETA( )

SE*( ), SEP*( ) are two-dimensional arrays with general dimensions: SE(NMAX, MMAX). SE represents the water elevations previously calculated and SEP represents the just calculated values for each time step.

V*( ), VP*( ) are two-dimensional arrays with same dimensions as SE( ). V is the velocity in the y-direction along the N-axis and VP represents the newly calculated values.

U*( ), UP*( ) are two-dimensional arrays with same dimensions as V and VP. U is the velocity in the x-direction and UP is the newly calculated values.

CN*( ), CNP*( ) are two-dimensional arrays dimensioned as SE( ). CN represents the concentration field of the dissolved constituent and CNP is the just calculated field.

IFIELD*( ) represents the field of computation points within the N x M lattice and is dimensioned
like $SE(\cdot)$.

$H(\cdot)$ is the two dimensional depth field and is dimensioned as $SE(\cdot)$.

$MBD(\cdot)$, $NB\overline{D}(\cdot)$ store information for the grid boundary used in the computational scheme. They are dimensioned one and a half times the maximum value of $M\overline{M}$ or $M\overline{M}$, whichever is larger.

$XI\overline{A}*(\cdot)$, $XI\overline{B}*(\cdot)$ store water level or velocity information for the open boundaries and are dimensioned double the time steps used in computation. Several more vectors, $XI-(\cdot)$, might be added if these cannot give adequate resolution for the open boundaries or there are more grids points at the open boundaries so that two vectors cannot yield a accurate field.

$MO\overline{B}D(\cdot)$, $NO\overline{B}D(\cdot)$ store values for open boundary information. The dimension of $MO\overline{B}D$ is the number of open boundaries on the grids in the M-direction, plus one. Dimension of $NO\overline{B}D$ operates the same.

### Execution and Computational Parameters

The following are input parameters and characteristics
of the system dimension parameters.

- **AL** = the length of each grid, the distance between each water-level point.
- **AT** = the length of one-half time step in seconds or the time for each implicit-explicit operation. The total computation time is $2 \times \Delta T \cdot \text{MAXST}$.
- **AG** = the acceleration of gravity, feet per second.
- **ANGLAT** = latitude in degrees and decimal fractions of center of modeled area.
- **CMANN** = the average Manning friction coefficient for the computational area.
- **CRHO** = the ratio of the densities of air to water.
- **CDRAG** = the drag coefficient for wind stress. The coefficient is calculated in the main program but is listed as a variable for cases of no wind when the drag coefficient section is bypassed.
- **AQL** = the angle the x-(M)-axis makes with North; used for finding components of wind speed.
- **IPUNCH** = the time step that punched output of SE, U, V can be stored for. Set greater than MAXST if no punched output is desired.
\[\text{MMAX}^* = \text{maximum number of grids in x-direction.}\]

\[\text{MAXST} = \text{the maximum number of time steps to be executed.}\]

\[\text{NI}^* = \text{number of iterations in computation for the nonlinear water level in continuity, usually set equal to one (1).}\]

\[\text{NMAX}^* = \text{maximum number of grids in y-direction.}\]

\[\text{MINDO}^*, \quad \text{NINDO}^* = \text{are equal to the total number of MOBD and NOBD values respectively, plus one.}\]

\[\text{NCARD} = \text{the number of entries for the open boundary tables; if MAXST = 100 then NCARD is set =}\]

\[\text{The XI-( ) tables must be dimensioned at least NCARD since the number of entries in the tables will be NCARD.}\]

\[\text{NSECT} = \text{the value of the dimension of NBD and MBD.}\]

\[\text{PHI} = \text{the actual direction of wind, } 0^\circ-365^\circ (0^\circ=\text{N}).\]

\[\text{SEINV} = \text{this value sets an initial water level for each computation point. It is best to do the first model run at low water slack. Once the model has been run and there is dynamic input to start model set SEINV=999.0 so that the section that sets the initial water level can be bypassed.}\]
QUALT = is set to 0.0 if the operations for the water-quality sections do not want to be entered, otherwise set to any floating point number.

WIOTD = set equal to 1.0 if you want to run the model without tides, otherwise set to any floating point number.

IFIELD* = represents the grids where water levels are to be computed. For each computation point the value of IFIELD=1 otherwise leave blank.

MOBD, NOBD = values give the computational control for the open boundaries. The first number is the M-grid column upon which the open boundary falls, the second numbers gives the lower grid number of the open boundary, and the third numbers give the upper N-grid number of the open boundary, and the last number gives whether the open boundary is on the right-hand (upper) side of M or the left hand (lower) side. The former is set to 1 and the latter is set to 0. NOBD is set up similarly. Example: Consider the following figure:
MBL ( ), NBL ( ) are tables that control the flow of computation. The IFIELD matrix is read in the DIVE subroutine and temporarily stored in M( ). The subroutine sets up the MBD( ), NBD( ) tables and finds the maximum value to which each table goes to (MIND, NID). Example: consider Figure D. a resulting value of a MBD and NBD from the table would be

\[
\text{MBD( )} = [10 | 07 | 01 | 04] \\
\text{NBD( )} = [03 | 02 | 05] \\
[10 | 03 | 07 | 09]
\]
where $a = 10 =$ lower bound tidal height

$20 =$ lower bound velocity

$1 =$ upper bound tidal height

$2 =$ upper bound velocity

So the MBD value indicates that on grid column $M=7$ between $N$ values, $N=1,3$ there are computation points and the 10 indicates that at the end of that column there is an open boundary with prescribed tidal heights.
PROGRAM FOR THE COMPUTATION OF LONG WATER WAVES.

N IS ALONG Y-AXIS IN DIRECTION OF V-VELOCITY
M IS ALONG X-AXIS IN DIRECTION OF U-VELOCITY
SEP(N,M) IS KNOWN WATER LEVEL AT Y=N, X=M
SEP(N,M) TO BE COMPUTED WATER LEVEL
U(N,M) VELOCITY AT Y=N, X=M+1/2
V(N,M) VELOCITY AT Y=N+1/2, X=M
H(N,M) WATER DEPTHS AT X=M+1/2, Y=N+1/2
C(N,M) CHEZY COEF AT X=M, Y=N
AT IS 1/2 TIME STEP OF OPERATION IN SECONDS
AL IS GRID SIZE
AG IS ACCEL. OF GRAVITY
ANGLAT IS LATITUDE OF CENTER OF BAY
MAXST IS NUMBER OF STEPS USED FOR CYCLE
CMANN IS MANNING FRICTION COEF
CDRAG IS DRAG COEFFICIENT FOR WIND STRESS
CHRU IS DENSITY RATIO FOR WIND STRESS TERM
C1-C3 ARE THE CONST. COEFF. FOR THE 2-D EQUATIONS
PHI IS THE ANGLE MAKES WITH TRUE NORTH
WK IS THE WIND VELOCITY IN KNOTS
WM IS THE WIND VELOCITY IN METERS/SEC
W IS THE WIND VELOCITY IN FT/SEC
IPUNCH IS TIME STEP THAT PUNCHED OUTPUT IS DESIRED,
SET GREATER THAN MAXST IF NONE DESIRED
KORIH SUBPROGRAM READS IN OPEN BOUND. VALUES
DIVE SUBPROGRAM READS IN LOCATION OF WATER LEVELS TO BE COMPUTED
DEPTH SUBPROGRAM READS THE WATER DEPTHS
CHEZY SUBPROGRAM CALCULATES CHEZY COEF
FIND SUBPROGRAM PROCESSES DIVE DATA AND SETS NBD, MBD
PRINT SUBPROGRAM PRINTS CALCULATED VALUES AT END OF 2ND HALF STEP
IPLUT PLOTS VELOCITIES AT TIMES DETERMINED BY IPLUT
DIMENSION A(95), B(95), P(95), S(95), R(95), F(95),
1KONVRAT(23), NH(23), TIL(18), NPRINT(96), IPLU(96)
DIMENSION G(30), IJ(30), C(30), ZFLD(30), ZFFD(30), ZGCON(30), ZFOCU(30)
1, ZEDB(30)
COMMON SE(23,95), SEP(23,95), V(23,95), VP(23,95), U(23,95), UP(23,95),
1G(23,95), NBD(160), MB0(160), MJ0(160), NJBD(160), H(23,95),
2XIA(600), XB(600), IFIELD(23,95), LETL(23), CN(23,95), CNP(23,95)

************************************************************

NMAX=23
MMAX=94
ANGLAT = 38.1
AL=2025.
AG=32.2
PHI=0.
WK=0

AQL IS THE ANGLE YOUR LONGITUDINAL AXIS OF THE MODEL
MAKES WITH NORTH, AND IS IN RADIANS (FOR CRINGO, AQL=.59 RADIANS)
AQL=.59
CRHO=.00114
AT=150.0
CMANN=.037
CDRA=.0026
NI=1
MOBD(1)=0111130
MOBD(2)=9404051
MINDO=3
NINDD=1
NSECT=154
OCUNE=31.5
SIDFR=0.0
CONIN=27.5
IPUN=224
IPUNCH=298
NCARD=600
MAXST=298

THIS CARD SET EQUAL TO 999. IF YOU HAVE DYNAMIC INPUT FOR THE
MODEL. WHEN INITIALLY RUNNING THE MODEL THERE WILL BE NO DYNAMIC
STARTING INPUT SO THIS CARD IS SET TO SOME DESIRED WATER LEVEL
AND ALL THE COMPUTATIONS GRIDS WILL HAVE THIS AS AN INITIAL VALUE
FOR STARTING CALCULATIONS. IT IS BEST TO START OUT AT LOW WATER
SLACK AS THE MODEL REACHES EQUILIBRIUM FASTES.

SEINV=999.

THIS NEXT CARD IS SET =0.0 IF YOU DO NOT WANT TO RUN THE WATER
QUALITY SECTION. IT WILL SKIP ALL PARTS THAT INVOLVE WATER QUALITY
IN THE READ SECTION THREE(3) BLANK CARDS SHOULD BE PUT IN FOR
WHERE THE BOUNDARY POINTS OF THE SALINITY DATA ARE NEEDED.
THE CARD IS SET=1.0 IF YOU WANT TO USE WATER QUALITY SUBROUTINE.

QUALT=0.0

NEXT CARD SET =1.0 IF WANT TO LOOK AT DYNAMICS WITHOUT TIDES.
WIDTD=0.0

************

IF WANT TIDES SET TO ANY FLOATING PT. #.
THIS SECTION CALCULATES THE WIND STRESS COEFFICIENT BASED ON
FRGUE #. FR IS THE FRGUE # FOR SCALING AND DEPENDS ON HEIGHT
OF WIND MEASURED (30 FT FOR CHINCOTEAGUE DATA).

IF(WK.EQ.0.0) GO TO 37
CDRA=1
WM=WK*.515
FR=WM/(SQRT(AG*30.0*.3048))
1112
CDR=1/(2.3*ALOG(91.0/(CDRA*FR**2))**2)
IF (ABS(CDRA-CDR).LE.0.0001) GO TO 1111
CDRA=CDR
GO TO 1112
1111 CDRAG=CDR
WRITE (6,1113) CDRAG
1113 FORMAT(1H0,'THE DRAG COEFFICIENT FOR THE WIND STRESS IS',1X,F8.6)

********************************************************************************************************

SET OPEN BOUNDS AS FUNCTIONS OF TABLEVALUES (XIA(K),XIB(K))
OR AS FUNCTIONS OF HALFTIMESTEP NUMBER (K).
AND INITIIZE VARIABLES

STATEMENTS 89 TO 87 ARE CALLED AFTER EACH TIME LEVEL AND KUKIH
DATA(XIA,XIB), IS READ IN AS OPEN BOUNDARY DATA FOR FOR NEXT STEP
WHERE IF NST=10 THEN K=20(HALF-TIME STEP INCREMENTS)

GO TO 87
89 CONTINUE
SEP(11,1)=XIA(K)
SEP(12,1)=XIA(K)
SEP(13,1)=XIA(K)
SEP(5,94)=XIB(K)
SEP(4,94)=XIB(K)
IF (QUALT.EQ.0.0) GO TO 48
**********************************************************************************************

DOWN TO STATEMNT 48 IS THE BOUNDARY CONDITIONS FOR THE WATER-
QUALITY. IT OPERATES ON A LINEAR EXTRAPOLATION DURING EBBY TIDE
AND DURING FLOOD TIDE IT USES THE OCEAN CONCENTRATION (OCONE
WHICH IS SET AT BEGINNING) AS A HIGH AND DECREASES TO SOME SET
VALUE IN 40 TIME STEPS. G(I) REPRESENT THE N GRID POINTS IN THE
OPEN BOUNDARY, D(I) ARE M GRID POINTS AT OPEN BOUNDARIES, AND IJ(I)
ARE M GRID POINTS IMMEDIATELY NEXT TO D(I) GRID POINTS.

DO 45 I=1,30
IF (NST.GT.1) GO TO 47
ZFLD(I)=OCONE-CNPG(I),D(I))
ZFFD(I)=ZFLD(I)/96
ZUCON(I)=CNPG(I),D(I))
ZFUCO(I)=ZUCON(I)+ZFFD(I)
47 IF (U(G(I),D(I)).LT.0.0) GO TO 44
ZE6B(I)=U(G(I),I,J(I)))*AT/AL
IF (ZE6B(I).GE.1.0) GO TO 42
CNPG(I),D(I))=ZE6B(I)*CNPG(I),D(I))+(1-ZE6B(I))CNPG(I),I,J(I))
GO TO 45
42 CNPG(I),D(I))=CNPG(I),I,J(I))
GO TO 45
44 IF (CNPG(I),D(I)).EQ.OCONE) GO TO 46
CNPG(I),D(I))=ZFUCO(I)+ZFFD(I)
GO TO 45
46 CNPG(I),D(I))=OCONE
45 CONTINUE
CONTINUE
IF(ISTEP.EQ.1) GO TO 96
GO TO 301
CONTINUE

INITIALIZE VARIABLES AND CALL SUBROUTINES

FF = 3.1415927 * SIN(ANGLAT*3.1415927/180.)/21600.

THIS SUBROUTINE SETS UP TABLES OF OPEN BOUNDARY DATA THAT WILL
CALLED FOR EACH TIME STEP, THE TABLE VALUES WILL GO UP TO
THE PARAMETER NCARD, I.E., IF NCARD IS = 500 THAN THERE WILL BE
500 TABLE VALUES TO BE READ IN AS OPEN BOUNDARY INPUT.
EX. NCARD=500 THEN NST LESS THAN OR EQUAL TO 250.

CALL KURIH(NCARD,MIOTD)
READ(5,4)(T,J),J=1,16)

IAT=1
NST = 0
C1 = AT*AG/AL
C2 = AT/AL
C3 = AT/4.
C4 = 8.*AT*AG

THIS CARD SETS CONSTANT FOR WIND STRESS TERM WHERE THE 1.687
IS FOR CONVERTING KNOTS TO FEET PER SECOND

C5=2.*CDRAG*CRHU*AT*(1.687)**2
W=WK
PHI=PHI/57.3
WX=W*COS(PHI-AQL)*(-1)
WY=W*SIN(PHI-AQL)
DO 6 N=1,NMAX
DO 8 M=1,MMAX
VP(N,M)=0.0
UP(N,M)=0.0
CN(N,M)=0.0
CNP(N,M)=0.0
V(N,M)=0.
SE(N,M)=0.0
SEP(N,M)=0.0
U(N,M) = 0.
C(N,M) = 0.
H(N,M) = 0.
6 F(N) = FF
CALL DIVE(NMAX,MMAX)
CALL FIND(MIND,NIND,NMAX,NIND,MINDO,NINDO,NSECT)
CALL DEPTH(NMAX,MMAX)

***********************
THIS CARD READS IN TIME STEP VALUES THAT PRINTOUT IS WANTED.

READ(5,25) (NPRINT(N),N=1,96)

THIS CARDS READS IN VALUES OF NST THAT PLOTS ARE WANTED

READ(5,25) (IPLOI(N),N=1,96)

THESE CARDS READ IN VALUES (GRID POINTS) THAT WILL BE USED FOR
THE LINEAR EXTRAPOLATION OF SALINITY BOUNDARY CONDITIONS.
THEY ARE OPEN BOUNDARY GRID POINTS AND THE GRID POINTS
IMMEDIATELY BEFORE THE OPEN BOUNDARY GRID POINTS.

READ(5,43) (G(N),N=1,30)
READ(5,43) (D(N),N=1,30)
READ(5,43) (IJ(N),N=1,30)

43 FORMAT(30I2)
25 FORMAT(16F4,16X)

IF(WIOTO.EQ.1.0) GO TO 1114

READ IN INITIAL VALUES OF U-SE-V, TO USE AS DYNAMIC INPUT

DO 21 M=1,MMAX
21 READ(5,31) (SE(N,M),N=1,12)
DO 51 M=1,MMAX
51 READ(5,31) (SE(N,M),N=13,NMAX)
DO 10 M=1,MMAX
10 READ(5,31) (U(N,M),N=1,12)
DO 11 M=1,MMAX
11 READ(5,31) (U(N,M),N=13,NMAX)
DO 38 M=1,MMAX
38 READ(5,31) (V(N,M),N=1,12)
DO 13 M=1,MMAX
13 READ(5,31) (V(N,M),N=13,NMAX)
1114 CONTINUE

*** This is a continuation of the previous segment. ***

*** This is another continuation. ***

WRITE INITIAL VALUES

WRITE(6,I ) (TITL(J),J=1,1B)
1 FORMAT(1H0,16A4)
IF(WK.EQ.0.) GO TO 18
PHI=PHI*57.3
WRITE (6,5025) WK,WM,PHI
5025 FORMAT(1HO,'FOR THIS RUN THE WIND SPEED IS',2X,'WK= ',F4.1,1X,'KNOTS',2X,'DIR',2X,'WM= ',F4.1,1X,'METERS/SEC',2X,'THE DIRECTION THE WIND IS',2X,'PHI= ',F5.1,' DEGREES')
18 CONTINUE
WRITE(6,12)
12 FORMAT(1H0,'INITIAL DEPTHS IN FEET')
DO 9 M=1,MMAX
9 WRITE(6,6111) M, (H(N,M),N=1,NMAX)
WRITE(6,1) (TITLJ), J = 1,18
6111 FORMAT(1H , 12,2X  , 23(F4.1)),
ISTEP=2
GO TO 500
C**********************************************************************
C
C NEXT 4 CARDS ARE CALLED AFTER EACH TIME THE CALCULATIONS ARE COMPLETED
FOR ISTEP=2. IT SETS ISTEP BACK TO ONE (1) SO THAT IT CAN ENTER THE FIRST
HALF-TIME STEP OPERATIONS, IT SETS YOUR NEW VALUE OF K TO BE USED TO READ
YOUR OPEN BOUNDARY DATA, ADVANCES NST AND CHECKS IF YOU HAVE REACHED
MAXST (WHICH IS THE MAX. TIME STEP THAT YOU ARE RUNNING TO)
C
88 ---ISTEP=1
IAT=IAT+1
NST =NST +1
K=2*NST-1
IF(NST.GT.MAXST) GO TO 1115
C**********************************************************************
C
C SET OPEN BOUND
GO TO 89
C
C**********************************************************************
C
C THIS SECTION DOWN TO STATEMENT 34 IS FOR SETTING AN INITIAL WATER LEVEL
THROUGH THE COMPUTATION FIELD AND AS STATED EARLIER IS ONLY ENTERED WHEN
SEINV IS NOT EQUAL TO 999.
C
IF(QUALT.EQ.0.0.AND.SEINV.EQ.999.) GO TO 34
NUM = 1
IF(NUM.EQ.NIND) GO TO 3
NSRCH =NBD(NUM)/1000000
N =NBD(NUM)/100000 - NSRCH*100
MF =NBD(NUM)/100 -NSRCH*10000 - N*100
L =NBD(NUM) - NSRCH*1000000 - N*10000 -MF*100
NN = N - 1
K = MF
DO 2 M = K,L
IF(QUALT.EQ.0.0) GO TO 74
CNP(N,M)=CONIN
CN(N,M)=CONIN
CONTINUE
IF(SEINV.EQ.999.) GO TO 2
SEP(N,M)=SEINV
SE(N,M)=SEINV
2 CONTINUE
NUM = NUM + 1
GO TO 7
3 CONTINUE
NA=1
5 IF(NA.EQ.MINDO) GO TO 36
M =MOBD(NA)/100000
NBOT =MOBD(NA)/1000 -M*100
NTOP =MOBD(NA)/10 -M*10000 -NBOT*100
DO 32 N=NBOT,NTOP
IF(QUALT.EQ.0.0) GO TO 73
CN(N,M)=CONIN
CNP(N,M)=CONIN
32 CONTINUE
IF(SEINV.EQ.999.) GO TO 32
SEP(N,M)=SEINV
SE(N,M)=SEINV
32 CONTINUE
NA=NA+1
GO TO 5
36 NA=1
33 IF(NA.EQ.MINDO) GO TO 34
N =NOBD(NA)/100000
MLEF =NOBD(NA)/1000 -N*100
MRIG =NOBD(NA)/10 -N*10000 -MLEF*100
DO 35 M=MLEF,MRIG
IF(QUALT.EQ.0.0) GO TO 75
CN(N,M)=CONIN
CNP(N,M)=CONIN
35 CONTINUE
IF(SEINV.EQ.999.) GO TO 35
SE(N,M)=SEINV
SEP(N,M)=SEINV
35 CONTINUE
NA=NA+1
GO TO 33
34 CONTINUE

C ******************************************************************************
C C COMPUTE UP AND SEP ON ROW N (FIRST HALF Timestep)
C
C THIS IS THE 1ST IMPLICIT OPERATION. CALCULATES VALUES OF UP & SE
C ON EACH ROW OF N, SO THAT IT FINDS WHAT GRIDS TO CALCULATE ON BY
C READING OFF THE NBD TABLE.
C
96 NUM =1
100 IF(NUM.EQ.NIND) GO TO 190
C THE NEXT FOUR CARDS TAKE THE VALUE FROM THE NBD TABLE AND BY
TRUNCATION THROUGH ALGEBRA THEY DETERMINE (IN ORDER OF OCCURRENCE)
THE TYPE OF BOUNDARY (OPEN OR CLOSED), THE ROW NUMBER, THE UPPER
BOUND AND THE LOWER BOUND. THIS TYPE SECTION IS USED THROUGHOUT
THE PROGRAM AND IT ALWAYS DOES THE SAME TYPE OPERATION BUT FOR
DIFFERENT CALCULATIONS. BY READING OFF THE NBD OR MBD TABLE
AND USING THAT NUMBER ONE CAN FIND THROUGH ALGEBRA THE TYPE
OF BOUND, THE PARTICULAR ROW OR COLUMN THAT YOU ARE DOING CALCULA
TIONS ON, AND THE UPPER AN LOWER LIMITS IN THAT ROW OR COLUMN FOR
THE COMPUTATION POINTS. EX- IF THERE IS A NUMBER FROM THE NBD
TABLE THAT READS 1037794, THIS MEANS THAT ON N-COLUMN 5 THERE ARE
COMPUTATION POINTS BETWEEN M=77 TO 94 AND THE 10 SHOWS THAT AT
94 THERE IS AN OPEN BOUNDARY.

NSRCH = NBD(NUM)/1000000
N = NBD(NUM)/10000 - NSRCH*100
MF = NBD(NUM)/10 - NSRCH*10000 - N*100
L = NBD(NUM) - NSRCH*1000000 - N*10000 - MF*100
MF = MF-1
NNN = N+1
NN = N -1
IT=1
R(MFF) = 0.0
S(MFF) = 0.0
GAMMA = 0.5

THE NEXT CARD CHECKS IF THE LEADING VALUE FOR THE NBD NUMBER IS
AN 11 OR 10 AND IF IT IS THEN THAT MEANS IT IS AN OPEN BOUNDARY
SO THAT IT COMPUTES THE VELOCITY AT THAT POINT. THIS IS DONE
SINCE THE OPEN BOUNDARIES ARE NOT IN THE COMPUTATION FIELD. THIS
TYPE CHECK AND RESULTING CALCULATIONS ARE DONE FOR EACH SCHEME
FOR EACH TIME STEP. IN THIS WAY THERE ARE COMPUTED VELOCITIES
FOR THE OPEN BOUNDARIES.

IF(NSRCH.GT.11) GO TO 99
MFF = MF-1
IF(TEMP10.EQ.0.) TEMP10= U(NN,MFF)
TEMP11 = U(NN,MFF)
IF(TEMP11.EQ.0.) TEMP11 = U(NNN,MFF)
N=MFF
MMM=MFF
TEMP12 = C5*W*X*ABS(WX)/ (SE(N,M)+SE(N,MMM)+H(N,M)+H(NN,M))
ALPHA = 1.
R(MFF) = C1/(1. + C2*(U(N,MFF)- U(N,MFF))* (1.-ALPHA))
TE1 = U(N,MFF)+C1*SEP(N,MFF)-TEMP12 - U(N,MFF)
TE2 = U(N,MFF)*SQRT(U(N,MFF)**2+ ((V(N,MFF)+V(NN,MFF)**2)/16.)
TE3 = ((SE(N,MFF)+SE(N,MMM)+H(N,MMM)+H(NN,MFF)) * ((C(N,MFF)+C(N,MFF))**
12))**C4
TE5 = (TEMP10-U(N,MFF))
TE6 = GAMMA*C2*(U(N,MFF)-TEMP11)
TE4 = (V(N,MFF)+V(NN,MFF))**25*(AT**F(N)-(1.-GAMMA)**C2*TE5-TE6)
TE7 = (1. + C2*U(N,MFF)-U(N,MFF)* (1.-ALPHA))
S(MFF) = (TE1+TE2/TE3*TE4)/TE7

99 CONTINUE
\[ K = M^2 \]

101  \[ DU 102 \]
\[ M = K, L \]
\[ MM = M-1 \]
\[ MMM = M+1 \]
\[ TEMP9 = SE(N,M) \]
IF (IT \( \cdot \) GT \( \cdot \) 1) TEMP9 = SEP(N,M)
\[ TEMP1 = SE(NNN,M) \]
IF (TEMP1 EQ 0.) TEMP1 = \( 2 \cdot SE(N,M) - SE(NN,M) \)
\[ TEMP2 = SE(NNN,M) \]
IF (TEMP2 EQ 0.) TEMP2 = \( 2 \cdot SE(N,M) - SE(NNN,M) \)
\[ TEMP3 = SE(N,MMM) \]
IF (TEMP3 EQ 0.) TEMP3 = \( 2 \cdot SE(N,M) - SE(N,MM) \)
IF (IT \( \cdot \) EQ \( \cdot \) 1 AND TEMP9 EQ 0.) TEMP9 = SE(N,M)
\[ TEMP4 = SE(N,MM) \]
IF (TEMP4 EQ 0.) TEMP4 = \( 2 \cdot SE(N,M) - SE(N,MM) \)
IF (IT \( \cdot \) EQ \( \cdot \) 1 AND TEMP4 EQ 0.) TEMP4 = SE(N,MM)
\[ TEMP5 = 2 \cdot SE(N,MMM) \]
IF (IT \( \cdot \) EQ \( \cdot \) 1 AND TEMP5 EQ 0.) TEMP5 = SE(N,MMM)
\[ TEMP6 = 2 \cdot SE(N,MMM) \]
IF (IT \( \cdot \) EQ \( \cdot \) 1 AND TEMP6 EQ 0.) TEMP6 = SE(N,MMM)
\[ TEMP7 = 2 \cdot SE(N,MMM) \]
IF (IT \( \cdot \) EQ \( \cdot \) 1 AND TEMP7 EQ 0.) TEMP7 = SE(N,MMM)
\[ TEMP8 = 2 \cdot SE(N,MMM) \]
IF (IT \( \cdot \) EQ \( \cdot \) 1 AND TEMP8 EQ 0.) TEMP8 = SE(N,MMM)
\[ TEMP9 = SEP(N,NM) \]
IF (IT \( \cdot \) EQ \( \cdot \) 1 AND TEMP9 EQ 0.) TEMP9 = SEP(N,NM)
\[ TEMP10 = U(NNN,M) \]
IF (TEMP10 EQ 0.) TEMP10 = U(NNN,M)
\[ TEMP11 = U(NNN,M) \]
IF (TEMP11 EQ 0.) TEMP11 = U(NNN,M)
\[ TEMP6 = AT F(N) \]
\[ \cdot \]
TEMP10 = U(NNN, L)
IF (TEMP10 .EQ. 0.)  TEMP10 = U(NN, L)
TEMP11 = U(NN, L)
IF (TEMP11 .EQ. 0.)  TEMP11 = U(NN, L)
LLL = L + 1
LL = L - 1
MMM = LLL
M = L
TEMP12 = C5 * WX * ABS(WX) / (SE(N, M) + SE(N, MMM) + H(N, M) + H(NN, M))
ALPHA = 0.
TE17 = (C1 * (N, L) + C(N, LLL)) * 2
TE16 = SE(N, L) + SE(N, LLL) + H(N, L) + H(NN, L)
TE15 = 1. - C4 * SQRT(T(U(N, L)) * (V(N, L) + V(NN, L)) * (V(NN, L)) * 2 / 16.) / (TE16 * TE17)
TE14 = - C1 * SEP(N, LLL) + U(N, L) * (TE15)
TE19 = (TEMP10 - U(N, L))
TE18 = .25 * (AT * F(N) - GAMMA * C2 * (U(N, L) - TEMP11) - (1. - GAMMA) * C2 * TE19)
TE20 = V(N, L) + V(NN, L)
TE22 = C1 * (L)
TE21 = (1. + C2 * (AG * P(L) + (U(N, L) - U(N, LLL)) * ALPHA))
UP(N, L) = (TE14 + (TE15 * TE20) + TE22) / TE21

CONTINUE
M = L
DO 106 J = K, L
SEP(N, M) = - P(M) * U(N, M) + Q(M)
UP(N, MM) = - K(M) * SEP(N, M) + S(MM)
106  M = M + 1
IT = IT + 1
IF (IT .LE. NI) GO TO 101
NUM = NUM + 1
GO TO 100
190 CONTINUE
NUM = 1

COMPUTE VP ON COLUMN M (FIRST HALF TIMESTEP)
FIRST EXPLICIT OPERATION THAT USES UP AND SEP VALUES FROM THE
PREVIOUS IMPLICIT OPERATION FOR COMPUTATION ON COLUMNS M, BY
MARCHING UP ROW N=1,NMAX FOR EACH M VALUE.

IF (NUM .EQ. MIND) GO TO 202
M = MBD(NUM) / 1000000
NF = MBD(NUM) / 1000 - M * 100
L = MBD(NUM) - M * 1000000 - NF * 100
LL = L - 1
MM = M + 1
DO 204 N = NF, LL
NN = N - 1
NNN = N + 1
**Beta** = 0.5

\[
\text{BETA} = 0.5
\]

\[
\text{TEMP}4 = C2 \times \left( (1 - \text{BETA}) \times \left(V(\text{NNN}, M) - V(N, M)\right) + \text{BETA} \times \left(V(N, M) - V(\text{NNN}, M)\right)\right)
\]

\[
\text{TEMP}1 = V(N, M) \times 2 \times \left( \left(U(\text{NNN}, M) + U(N, M)\right) + \left(U(\text{NNN}, M) + U(N, M)\right)\right) / 16.
\]

\[
\text{TEMP}2 = \left( \text{SE}(\text{NNN}, M) + \text{SE}(N, M)\right) + H(N, M) + H(N, M) + C(N, M) +
\]

\[
\text{LC}(\text{NNN}, M) \times 2
\]

\[
\text{TEMP}12 = C5 \times \text{WY} \times \text{ABS}(\text{wY}) / \left( \text{SE}(N, M) + \text{SE}(\text{NNN}, M) + H(N, M) + H(N, M)\right)
\]

\[
\text{TEMP}3 = 1 + \text{C4} \times \text{SQRT} (\text{TEMP}1) / \text{TEMP}2 + \text{TEMP}4 + \text{TEMP}12
\]

\[
\text{DELTA} = 0.5
\]

\[
\text{TEMP}10 = V(N, M, M)
\]

\[
\text{IF (TEMP}10 = 0) \text{ TEMP}10 = V(N, M, M)
\]

\[
\text{TEMP}11 = V(N, M, M)
\]

\[
\text{IF (TEMP}11 = 0) \text{ TEMP}11 = V(N, M, M)
\]

\[
\text{TEMP}1 = \left( \left(\text{AT} \times F(N) + (1 - \text{DELTA}) \times C2 \times (\text{TEMP}10 - V(N, M)) + \text{DELTA} \times C2 \times \left(V(N, M) - \text{TEMP}11\right)\right)\right) \times 25
\]

\[
\text{TEMP}3 = 0.0
\]

\[
204 \text{ TE}1 = U(\text{NNN}, M) + U(N, M) + U(N, M) + U(\text{NNN}, M)
\]

\[
\text{TE}2 = C1 \times \left( \text{SE}(\text{NNN}, M) - \text{SE}(N, M)\right)
\]

\[
V(P(N, M) = \text{TEMP}3 \times (V(N, M) - \text{TE}1 \times \text{TE}2) + \text{TEMP}3
\]

\[
\text{IF (MSRCH.EQ.10. OR. MSRCH.EQ.11) GO TO 205}
\]

\[
\text{GO TO 206}
\]

\[
205 \text{ TEMP}10 = V(L, M, M)
\]

\[
\text{IF (TEMP}10 = 0) \text{ TEMP}10 = V(L, M, M)
\]

\[
\text{TEMP}11 = V(L, M, M)
\]

\[
\text{IF (TEMP}11 = 0) \text{ TEMP}11 = V(L, M, M)
\]

\[
L = L + 1
\]

\[
\text{BETA} = 0.5
\]

\[
\text{LL} = L - 1
\]

\[
\text{TEMP}4 = C2 \times \left( (1 - \text{BETA}) \times \left(V(L, M) - V(L, M)\right) + \text{BETA} \times \left(V(L, M) - V(L, M)\right)\right) / 16.
\]

\[
\text{TEMP}1 = V(L, M) \times 2 \times \left( \left(U(L, M) + U(L, M)\right) + \left(U(L, M) + U(L, M)\right)\right) / 16.
\]

\[
\text{TEMP}2 = \left( \text{SE}(L, M) + \text{SE}(\text{LLL}, M) + H(L, M) + H(L, M)\right) \times (C(L, M) + C(L, M)) \times 2
\]

\[
N = L
\]

\[
\text{NNN} = \text{LLL}
\]

\[
\text{TEMP}12 = C5 \times \text{WY} \times \text{ABS}(\text{wY}) / \left( \text{SE}(N, M) + \text{SE}(\text{NNN}, M) + H(N, M) + H(N, M)\right)
\]

\[
\text{TEMP}3 = 1 + \text{C4} \times \text{SQRT} (\text{TEMP}1) / \text{TEMP}2 + \text{TEMP}4 + \text{TEMP}12
\]

\[
\text{DELTA} = 0.5
\]

\[
\text{TEMP}1 = C5 \times \left( \left(\text{AT} \times F(N) + (1 - \text{DELTA}) \times C2 \times (\text{TEMP}10 - V(L, M)) + \text{DELTA} \times C2 \times \left(V(L, M) - \text{TEMP}11\right)\right)\right)
\]

\[
V(L, M) = \text{TEMP}3 \times (V(L, M) - \text{TE}1 \times \text{TE}2) + \text{TEMP}3
\]

\[
L = C1 \times \left( \text{SE}(L, M) - \text{SE}(L, M)\right)
\]

\[
206 \text{ IF (MSRCH.EQ.10. OR. MSRCH.EQ.11) GO TO 207}
\]

\[
\text{GO TO 208}
\]

\[
207 \text{ TEMP}10 = V(N, M, M)
\]

\[
\text{IF (TEMP}10 = 0) \text{ TEMP}10 = V(N, M, M)
\]

\[
\text{TEMP}11 = V(N, M, M)
\]

\[
\text{IF (TEMP}11 = 0) \text{ TEMP}11 = V(N, M, M)
\]

\[
\text{BETA} = 1.0
\]

\[
\text{TEMP}4 = C2 \times (1 - \text{BETA}) \times \left(V(N, M) - V(N, M)\right)
\]

\[
\text{TEMP}1 = V(N, M, M) \times 2 \times \left( \left(U(N, M) + U(N, M)\right) + \left(U(N, M) + U(N, M)\right)\right) / 16.
\]

\[
\text{TEMP}2 = \left( \text{SE}(N, M) + \text{SE}(N, M) + H(N, M) + H(N, M)\right) \times 2
\]

\[
N = N + 1
\]

\[
\text{NNN} = \text{LLL}
\]

\[
\text{TEMP}12 = C5 \times \text{WY} \times \text{ABS}(\text{wY}) / \left( \text{SE}(N, M) + \text{SE}(\text{NNN}, M) + H(N, M) + H(N, M)\right)
\]

\[
\text{TEMP}3 = 1 + \text{C4} \times \text{SQRT} (\text{TEMP}1) / \text{TEMP}2 + \text{TEMP}4 + \text{TEMP}12
\]

\[
\text{DELTA} = 0.5
\]

\[
\text{TEMP}1 = C5 \times \left( \left(\text{AT} \times F(N) + (1 - \text{DELTA}) \times C2 \times (\text{TEMP}10 - V(N, M)) + \text{DELTA} \times C2 \times \left(V(N, M) - \text{TEMP}11\right)\right)\right)
\]

\[
V(N, M) = \text{TEMP}3 \times (V(N, M) - \text{TE}1 \times \text{TE}2) + \text{TEMP}3
\]

\[
L = C1 \times \left( \text{SE}(L, M) - \text{SE}(L, M)\right)
\]

\[
208 \text{ IF (MSRCH.EQ.10. OR. MSRCH.EQ.11) GO TO 209}
\]

\[
\text{GO TO 207}
\]

\[
209 \text{ TEMP}10 = V(N, M, M)
\]

\[
\text{IF (TEMP}10 = 0) \text{ TEMP}10 = V(N, M, M)
\]

\[
\text{TEMP}11 = V(N, M, M)
\]

\[
\text{IF (TEMP}11 = 0) \text{ TEMP}11 = V(N, M, M)
\]

\[
\text{BETA} = 1.0
\]

\[
\text{TEMP}4 = C2 \times (1 - \text{BETA}) \times \left(V(N, M) - V(N, M)\right)
\]

\[
\text{TEMP}1 = V(N, M, M) \times 2 \times \left( \left(U(N, M) + U(N, M)\right) + \left(U(N, M) + U(N, M)\right)\right) / 16.
\]

\[
\text{TEMP}2 = \left( \text{SE}(N, M) + \text{SE}(N, M) + H(N, M) + H(N, M)\right) \times 2
\]
1 \( (C(NF, M) + C(NFF, M)) \times 2 \)

\( N = NFF \)

\( NNN = NF \)

\( TEMP1 = -C5*WY*AAS(WY)/(SE(N,N) + SE(NNN, M) + H(N,M) + H(N,MM)) \)

\( TEMP3 = 1.0 + C4*SQRT(TEMP1)/TEMP2 + TEMP4 + TEMP12 \)

\( TEMP3 = 1.0/TEMP3 \)

\( DELTA = 0.5 \)

\( TEMP1 = 0.25*(AT*F(N)+1.0-DELTA)*C2*(TEMP10-V(NFF,M)) \)

\( 1 + DELTA*C2*(V(NFF,M)-TEMP11) \)

\( TEMP10 = U(NNN,MFF) \)

\( VP(NFF,M) = TEMP3*(V(NFF,M)-TEMP1*(UP(NF,M)+UP(NF,MM))) \)

\( 1 - C1*(SE(NF,M)-SE(NFF,M)) \)

208 CONTINUE

\( NUM = NUM + 1 \)

GO TO 201

202 CONTINUE

IF (QUALT.NE.0.0) GO TO 203

CALL SALT(NST,ISTEP,AL,AG,AT,NMAX,MMAX,NINDO,MINDO,MIND,MIND)

203 CONTINUE

C

**************************************************************************************************

C

PRINT INSTRUCTIONS

C

IF IN FIRST TIME STEP THEN SKIP GOING TO PRINT SUBROUT. AND SET
C
JUST CALCULATED VALUES OF UP, VP, AND SE BACK TO U, V, SE FOR USE
C
AS KNOWN IN FORMATION FOR ISTEP=2 AND GO BACK AND SET BOUNDS AND
C
START CALCULATIONS. IF ISTEP=2 NST=NPRINT THEN CALL PRINT.

C

500 IF (ISTEP-2) 297, 296, 297

296 CONTINUE

IF (NST.EQ.0.0) GO TO 298

GO TO 294

298 CALL CHEZY(NMAX,MMAX,CMNN)

294 CONTINUE

IF (NST.EQ.0.0) AND (IP.EQ.0) GO TO 295

295 CONTINUE

IF (NST.EQ.0.0) AND (JP.EQ.0) GO TO 40

IF (NST.NE.IPLTO(JJ)) GO TO 289

CALL IPLOT(NMAX,MMAX,AT,NST,WK,PHI,WINTD,AL)

40 JP=JP+1

289 CONTINUE

IF (NST.EQ.NPRINT(IP)) GO TO 295

GO TO 297

295 IP = IP+1

CALL PRINT(NST,AT,nK,PHI,NMAX,MMAX,QUALT)

297 NUM = 1

GO 292 N=1,NMAX

GO 292 M=1,MMAX

U(N,M)=UP(N,M)

C(N(N,M)=CNP(N,M)

V(N,M)=VP(N,M)
SE(N,M) = SEP(N,M)
IF(NST.EQ.IPUNCH.OR.NST.EQ.IPUN) GO TO 60
GO TO 61
60 IF(ISTEP.EQ.2) GO TO 62
GO TO 61
62 CONTINUE
DO 63 M=1,MMAX
63 WRITE(7,70) (SE(N,M),N=1,12),NST,M
DO 64 M=1,MMAX
64 WRITE(7,70) (SE(N,M),N=13,NMAX),NST,M
DO 65 M=1,MMAX
65 WRITE(7,70) (U(N,M),N=1,12),NST,M
DO 66 M=1,MMAX
66 WRITE(7,70) (U(N,M),N=13,NMAX),NST,M
IF(QUALT.EQ.0.0) GO TO 61
DO 71 M=1,MMAX
71 WRITE(7,70) (CN(N,M),N=1,12),NST,M
DO 72 M=1,MMAX
72 WRITE(7,70) (CN(N,M),N=13,NMAX),NST,M
70 FORMAT (12F8.2,6X,13,2X,12)
61 CONTINUE
GO TO(299,68),ISTEP
ISTEP=2
K=2*NST
C SET OPEN BOUNDS
GO TO 89
C*****************************************************************************
C COMPUTE VP AND SEP ON COLUMN M (SECOND HALF TIMESTEP)
C*****************************************************************************
301 IF(NUM.EQ.MIND) GO TO 390
MSRCH = MBD(NUM)/1000000
M = MBD(NUM)/10000 -MSRCH*100
NF = MBD(NUM)/100 -MSRCH*10000 -M*100
L = MBD(NUM) -MSRCH*1000000-M*10000 - NF*100
M=M-1
MMM=M+1
LL=L-1
LLL=L+1
NFF=NF-1
R(NFF)=0.0
S(NFF)=0.0
IF(MSRCH.LT.10.0.OR.MSRCH.GT.11) GO TO 319
TEMP10=V(NFF,MM)
IF(TEMP10.EQ.0.0) TEMP10=V(NFF,MM)
TEMP11=V(NFF,MM)
IF(TEMP11.EQ.0.0) TEMP11=V(NFF,MM)
NNN=NF
N=NF
N=NFF
TEMP12=-C5*WY*ABS(WY)/ (SE(N,M)+SE(NNN,M)+H(N,M)+H(N,MM))
DELTA=0.5
BETA=1.*
R(NFF)=Cl/(1.+C2*(V(NF,M)-V(NFF,M))*(1.-BETA))
S(NFF)=(V(NF,M)+C1*SEP(NFF,M)-TEMP12
1-V(NFF,M)*SQRT(V(NFF,M)**2+((U(NF,M)+U(NF,MM))**2)/16.))/
2((SE(NFF,M)+SE(NF,M)+H(NFF,M)+H(NFF,MM))/(C(NFF,M)+C(NF,M))
3*2)*C4-.25*(AT*F(N)+(1.-DELTA)*C2*(TEMP10-V(NFF,M))
4+DELTA*C2*(V(NFF,M)-TEMP11))*(U(NF,M)+U(NF,MM))*/
5(1.+C2*(1.-BETA)*(V(NF,M)-V(NFF,M))
319 CONTINUE
K=NF
IT=1
303 DO 302 N=K,L
NN=N-1
NNN=N+1
TEMP9=SE(N,M)
IF(IT.GT.1) TEMP9=SEP(N,M)
TEMP1 = SE(N,MMM)
IF(TEMP1.EQ.0.) TEMP1 = 2.*SE(N,M) - SE(N,MM)
TEMP2 = SE(N,MM)
IF(TEMP2.EQ.0.) TEMP2 = 2.*SE(N,M) - SE(N,MMM)
TEMP3 = SE(NNN,M)
IF(TEMP3.EQ.0.) TEMP3 = 2.*SE(N,M) - SE(N,NNN,M)
IF(IT.GT.1.AND.TEMP3.EQ.0.) TEMP3=2.*SEP(N,M)-SEP(N,NNN,M)
TEMP4 = SE(N,NNN,M)
IF(TEMP4.EQ.0.) TEMP4 = 2.*SE(N,M) - SE(N,NNN,M)
IF(IT.GT.1.AND.TEMP4.EQ.0.*) TEMP4=2.*SEP(N,M)-SEP(N,NNN,M)
A(N) = SE(N,M) -.5*C2*(H(N,M) +H(NN,M) + SE(N,M) + TEMP1 )*U(N,M)
1M) + .5*C2*(H(N,M) +H(NN,M)+ TEMP2 + SE(N,M))U(N,MM)
P(N) = .5*C2*(H(N,M) +H(NN,M) + TEMP4 + TEMP3 )/(1.+ .5*C2*
1(H(NN,M) + H(N,N,M) + TEMP4 + TEMP3 )#*K(N))
Q(N) = (A(N) + .5*C2*(H(NN,M) + H(NN,MM) + TEMP4 + TEMP9 )*S(NN)
1)/(1.+ .5*C2*(H(NN,M) + H(NN,MM) + TEMP4 + TEMP9 )#*K(N))
IF(N.EQ.L) GO TO 302
DELTA = 0.5
TEMP10=V(N,MM)
IF(TEMP10.EQ.0.) TEMP10 = V(N,MM)
TEMP11=V(N,MM)
IF(TEMP11.EQ.0.) TEMP11 = V(N,MM)
TEMP6 =AT*F(N)+(1.-DELTA)*C2*(TEMP10-V(N,M))
1 +DELTA*C2*(V(N,M)-TEMP11)
TEMP6 = .25*TEMP6
TEMP12=-C5*WY*ABS(WY)/ (SE(N,M)+SE(NNN,M)+H(N,M)+H(N,MM))
TEMP40=0.0
TE1=V(N,M)-TEMP6*(U(N,M)+U(NNN,M)+U(NNN,MM)+U(N,MM))-TEMP12
TE2=SQRT(V(N,M)**2+((U(N,M)+U(NNN,M)+U(N,M)+U(N,N,M)+U(NNN,MM))**2)/16.)
TE3=(SE(N,M)+SE(NN,M)+H(N,M)+H(N,MM))/(C(N,M)+C(NNN,M))**2)*C4
B(N) = TE1 - V(N, M) * (TE2 / TE3) + TEMP40

BETA = 0.5

TEMP1 = 1 + C2 * (AG * P(N) + (1 - BETA) * (V(NNN, M) - V(N, M)) +
1
BETA * (V(N, M) - V(NN, M))

R(N) = C1 / TEMP1

S(N) = (B(N) + C1 * Q(N)) / TEMP1

302 CONTINUE

LLL = L + 1

VP(L, M) = 0.0

IF(MSRCH.EQ.1. OR MSRCH.EQ.11) GO TO 307

GO TO 305

307 CONTINUE

TEMP10 = V(L, MMM)

IF(TEMP10.EQ.0) TEMP10 = V(L, MMM)

TEMP11 = V(L, MMM)

IF(TEMP11.EQ.0) TEMP11 = V(L, MMM)

LLL = L + 1

LL = L - 1

N = L

NNN = LLL

BETA = 0.

TEMP12 = -C5 * WY * ADS(WY) / (SE(N, M) + SE(NNN, M) + H(N, M) + H(N, MMM))

VP(L, M) = -C1 * SEP(LLL, M) + V(L, M) * (1 - C4 * SQRT(V(L, M) ** 2 + ((UIL, M) +
1
UIL, MMM) ** 2)) / ((SE(L, M) + SEP(LLL, M) +
2H(L, M) + H(L, MMM)) * ((C(L, M) + C(LL, MMM) ** 2)) - TEMP12) +
3.25 * (AT * F(N) + (1 - DELTA) * C2 * (TEMP10 - V(L, M)) + DELTA * C2 *
4 * (V(L, M) - TEMP11)) * (U(L, M) + U(L, MMM))

5 * C1 * W(L) / (1 + C2 * (AG * P(L) + BETA * (V(L, M) - V(LL, M))))

305 CONTINUE

N = L

DO 306 J = K, L

NN = N - 1

SEP(N, M) = -P(N) * VP(N, M) * W(N)

VP(N, M) = -R(NN) * SEP(N, M) + S(NN)

306 N = N - 1

IT = IT + 1

IF(IT.LE.NI) GO TO 303

NUM = NUM + 1

GO TO 301

C **************************************************************
C
C COMPUTE UP ON ROW N (SECOND HALF Timestep)
C
390 NUM = 1

340 IF(NUM.EQ.NIND) GO TO 402

NSRCH = NBD(NUM) / 1000000

N = NBD(NUM) / 1000000 - NSRCH * 100

MF = NBD(NUM) / 100 - NSRCH * 10000 - N * 100

L = NBD(NUM) - NSRCH * 1000000 - N * 10000 - MF * 100

NN = N - 1

NNN = N + 1

LL = L - 1
LLL = L + 1
MFF = MF - 1
DO 404 M = MF, LL
MMM = M + 1
MM = M - 1
ALPHA = 0.5

TEMPO = C2 * ((L - ALPHA) * (U(N, MMM) - U(N, M)) + ALPHA * (U(N, M) - U(N, MM)))
TEMP1 = U(N, M) ** 2 + (((V(N, M) + V(N, MMM)) + V(NN, M) + V(NN, MMM)) ** 2) / 16.
TEMP2 = (SEP(N, M) + SEP(N, MMM) + H(N, M) + H(NN, M)) * (C(N, M) + C(N, MMM))
1) ** 2

TEMP12 = -C5 * WX * ABS(nx) / (SE(N, M) + SE(N, MMM) + H(N, M) + H(NN, M))
TEMP3 = 1. * C4 * SQRT (TEMP1) / TEMP2 + TEMP4 + TEMP12

1) GAMMA = 0.5
TEMP10 = U(NNN, M)
IF (TEMP10 .EQ. 0.) TEMP10 = U(NN, M)
TEMP11 = U(NNN, M)
IF (TEMP11 .EQ. 0.) TEMP11 = U(NNN, M)

TEMP1 = AT * F(N) - (1. - GAMMA) * C2 * (TEMP10 - U(N, M))

1) TEMP1 = .25 * TEMP1
TEMP33 = 0.0
404 UP(N, M) = TEMP3 *
1) (U(N, N) + TEMP1 * (VP(N, M) + VP(N, MMM) + VP(NN, M) + VP(NN, MMM)))
2) C1 = (SE(N, MMM) - SE(N, M)) * TEMP33
IF (NSRCH .EQ. 1. OR NSRCH .EQ. 1) GO TO 405
GO TO 406

405 TEMP10 = U(NNN, L)
IF (TEMP10 .EQ. 0.) TEMP10 = U(NN, L)
TEMP11 = U(NNN, L)
IF (TEMP11 .EQ. 0.) TEMP11 = U(NNN, L)

ALPHA = 0.
TEMPO = C2 * ALPHA * (U(N, N) - U(N, L))
TEMP1 = U(N, N) ** 2 + (((V(N, N) + V(N, L)) ** 2) / 16.
TEMP2 = (SEP(N, N) + SEP(N, LLL) + H(N, N) + H(NN, N)) * (C(N, N) + C(N, LLL)) ** 2
M = L
MMM = LLL
TEMPO = -C5 * WX * ABS(nx) / (SE(N, N) + SE(N, LLL) + H(N, N) + H(NN, N))
TEMP3 = 1. * C4 * SQRT (TEMP1) / TEMP2 + TEMP4 + TEMP12

1) GAMMA = 0.5
TEMP1 = .25 * (AT * F(N) - (1. - GAMMA) * C2 * (TEMP10 - U(N, N)) - GAMMA * C2 *
1) (U(N, N) - TEMP11))

UP(N, N) = TEMP3 * (U(N, N) + TEMP1 * (VP(N, N) + VP(NN, N)))
1) C1 = (SE(N, LLL) - SE(N, L))

406 IF (NSRCH .EQ. 10 .OR NSRCH .EQ. 11) GO TO 407
GO TO 408

407 TEMP10 = U(NNN, MFF)
IF (TEMP10 .EQ. 0.) TEMP10 = U(NN, MFF)
TEMP11 = U(NNN, MFF)
IF (TEMP11 .EQ. 0.) TEMP11 = U(NNN, MFF)
ALPHA = 1.
TEMP4 = C2*(1. - ALPHA)*(U(N,MF) - U(N,MFF))
TEMP1 = U(N,MFF)**2 + (V(N,MF) + V(NN,MFF))**2 / 16.
TEMP2 = SEP(N,MFF) + SEP(N,MF) + H(N,MF) + H(NN,MFF) * (C(N, MF) + C(N, MFF))

GAMMA = 0.5
TEMP1 = 0.25*(AT*F(NM) - (1. - GAMMA)*C2*(TEMP10 - U(N,MFF)) - GAMMA*C2*
1(U(N,MFF) - TEMP11))
UP(N,MFF) = TEMP3*(U(N,MFF) + TEMP1*(VP(N,MF) + VP(NN,MF))
1-C1* (SEP(N,MF) - SEP(N,MFF))

CONTINUE
NUM = NUM + 1
GO TO 340

CONTINUE
IF(QUALI.EQ.0.0) GO TO 500
CALL SALT(NST, ISTEP, AL, AG, AT, NMAX, MMAX, NIND, MIND)
GO TO 500

SUBROUTINE KURIH(NCARD, NIUTO)
COMMON SE(23,95), SEP(23,95), V(23,95), VP(23,95), U(23,95), UP(23,95),
1 C(23,95), NBD(160), MBD(160), MBD(3), NSD(2), H(23,95),
2 XI(A(600), X1B(600), X1C(3), X1D(23,95), ETA(23), CN(23,95), CNP(23,95)
IF(WIUTO.EQ.1.0) GO TO 14
WRITE(9,9)
WRITE(6,11)
DO 13 K=1,NCARD
XI(A(K)) = 1.0 - 1.0*COS(.0211*XCOUNT)
X1B(K) = 1.0 - 1.0*COS(.0211*XCOUNT + .0696)
DO 8 K=1,NCARD
8 WRITE(6,12) K, XI(A(K), X1B(K)
7 FORMAT(2F6.3)
9 FORMAT(IH1,12X,35HWRITERLEAS AT STATIONS A THROUGH B)
11 FORMAT(IH0,3X,'K','sx, 'XIA', 'XIB')
12 FORMAT(IH,14,1X,'z(F8.3,2X))
GO TO 16
14 DU 15 K=1,NCARD
XI(A(K)) = 0.0
15 X1B(K) = 0.0
16 CONTINUE
RETURN
SUBROUTINE FINDIMIND, IIND, MMAX, NMAX, MIND, NIND, NSECT)
LOGICAL START
COMMON SE(23,95), SEP(23,95), V(23,95), VP(23,95), U(23,95), UP(23,95),
LC(23,95), NBD(160), MBD(160), NBD(3), NBD(2), H(23,95),
2XIA(600), XIB(600), IFIELD(23,95), ZETA(23), CN(23,95), CNP(23,95)

THIS SUBROUTINE TAKES THE DIVE DATA, WHICH READ IN THE IFIELD DATA
THAT IN TURN TELLS AT WHAT GRID POINTS THE WATER LEVELS, SEP,
ARE TO BE COMPUTED AT, AND WITH DIVE DATA SETS UP TWO TABLES
(NBD, MBD) WHERE MAX VALUES OF EACH TABLE ARE NIND AND MIND. EACH
TABLE SHOULD GO UP TO NMAX OR MMAX SO THAT ALL WATER LEVEL PTS ARE
IN FIELD FOR CALCULATION. EX- IF NUM=70 YOU WOULD READ ACROSS
FROM 70 THE NUMBER 10110205. THE 10 MEANS OPEN BOUNDARY AT END
OF THIS COLUMN. THAT YOU ARE IN N-ROW 11 AND BETWEEN M=02,05 THERE
ARE COMPUTATION POINTS FOR THESE M-VALUES.
NOTE** YOUR DIMENSIONS OF NBD, MBD MUST BE HIGH ENOUGH THROUGH
THE PARAMETER NSECT SO THAT THE N-ROW NUMBERS OF NBD AND THE M-
COLUMN NUMBERS OF MBD GO UP TO THE NMAX AND MMAX VALUES OR ELSE
COMPUTATIONS WILL NOT PROCEED OR OVERFLOW PROBLEMS WILL
DEVELOP. YOUR LAST LISTING IN THESE TWO TABLES SHOULD BE ZERO
IF THEY ARE NOT ZERO SET NSECT HIGHER AND CORRESPONDINGLY SET THE
DIMENSIONS OF NBD AND MBD AT LEAST =NSECT.
DO 1 N = 1, NSECT
NBD(J) = 0
1 MBD(J) = 0
MIND = 1
NIND = 1
DO 2 N = 2, NMAX
START = .TRUE.
DO 3 M = 2, MMAX
IF(.NOT.START) GO TO 4
IF( H(N,M), .EQ. 0.) GO TO 3
NBD(NIND) = M*100 + NBD(NIND)
START = .TRUE.
GO TO 3
IF( H(N,M), .NE. 0.) GO TO 5
NBD(NIND) = M - 1 + NBD(NIND) + 10000*N
GO TO 6
5 IF(M, .NE. MMAX) GO TO 3
NBD(NIND) = M + NBD(NIND) + 10000*N
6 NIND = NIND + 1
START = .TRUE.
CONTINUE
2 CONTINUE
DO 12 N = 2, MMAX
START = .TRUE.
DO 13 N = 2, NMAX
IF(.NOT.START) GO TO 14
END
IF( H(N,M) .EQ. 0.) GO TO 13
MBD(MIND) = N*100 + MBD(MIND)
START = .FALSE.
GO TO 13
14 IF( H(N,M) .NE. 0.) GO TO 15
MBD(MIND) = N - 1 + MBD(MIND) + 10000*M
GO TO 16
15 IF(N .NE. NMAX) GO TO 13
MBD(MIND) = N + MBD(MIND) + 10000*M
16 MIND = MIND + 1
START = .TRUE.
13 CONTINUE
12 CONTINUE
NUM = 1
100 IF(NUM .EQ. NIND) GO TO 300
N = NBD(NUM)/10000
MF = NBD(NUM)/100 - N*100
L = NBD(NUM) - N*10000 - MF*100
MFLEF = MF - 1
LRIG = L + 1
NA = 1
200 IF(NA .EQ. MIND) GO TO 210
M = MBD(NA)/1000000
NBOT = MBD(NA)/1000 - M*100
NTOP = MBD(NA)/10 - M*10000 - NBOT*100
MBERN = MBD(NA) - M*1000000 - NBOT*1000
NTOP = NBD(NA)/10 - M*10000 - MF*100
MFLEF = MF - 1
LRIG = L + 1
NA = NA + 1
GO TO 200
210 NUM = NUM + 1
GO TO 100
300 CONTINUE
NUM = 1
101 IF(NUM .EQ. NIND) GO TO 301
M = MBD(NUM)/10000
NF = NBD(NUM)/100 - M*100
L = NBD(NUM) - M*10000 - NF*100
NBOT = NF - 1
LTOP = L + 1
NA = 1
201 IF(NA .EQ. MIND) GO TO 211
N = NBD(NA)/10000
MLEF = NBD(NA)/1000 - N*100
MRIG = NBD(NA)/10 - N*10000 - MLEF*100
MBERN = NBD(NA) - N*1000000 - MLEF*1000 - MRIG*10
IF((N .GE. NBOT) .AND. (N .LE. NTOP) .AND. (MLEF .EQ. M)) MBD(NUM) = MBD(NUM)
1 + 10000000
IF((N .GE. NBOT) .AND. (N .LE. NTOP) .AND. (LRIG .EQ. M)) MBD(NUM) = MBD(NUM)
1 + 10000000
NA = NA + 1
GO TO 201

211 NUM = NUM + 1
GO TO 101

301 CONTINUE
WRITE(6,20)
DO 22 J = 1, NSECT
WRITE(6,21) J, NBD(J), MOD(J)
22 CONTINUE
20 FORMAT(1H1, 3X, JNMB, 5X, JJNBBD, 7X, JMBBD)
21 FORMAT(1H1, 2X, 14X, 19X, IX, 1X)
RETURN
END

SUBROUTINE DEPTH(NMAX, MMAX)
COMMON SE(23, 95), SEP(23, 95), V(23, 95), VP(23, 95), U(23, 95), UP(23, 95),
1C(23, 95), NBD(160), MOD(160), MOBD(3), NOBD(2), H(23, 95),
2XIA(600), XIB(600), IFIELD(23, 95), ZETA(23), CN(23, 95), CNP(23, 95)
NO = 16
IF(NMAX .LE. 16) NO = NMAX
DO 10 M = 1, MMAX
READ (5, 3) (H(N, M), N = 1, NU)
10 CONTINUE
IF(NMAX .LE. 16) GO TO 12
DO 11 M = 1, MMAX
READ (5, 4) (H(N, M), N = 17, NMAX)
11 CONTINUE
12 CONTINUE
3 FORMAT(1G4.1)
4 FORMAT(1F4.1)
RETURN
END

SUBROUTINE DIVE(NMAX, MMAX)
COMMON SE(23, 95), SEP(23, 95), V(23, 95), VP(23, 95), U(23, 95), UP(23, 95),
1C(23, 95), NBD(160), MOD(160), MOBD(3), NOBD(2), H(23, 95),
2XIA(600), XIB(600), IFIELD(23, 95), ZETA(23), CN(23, 95), CNP(23, 95)
DIMENSION NO(60)
WRITE(6, 5)
DO 1 N = 1, NMAX
NO(N) = N
WRITE(6, 6) (NO(N), N = 1, NMAX)
1 CONTINUE
2 M = 1, MMAX
READ (5, 3) (IFIELD(N, M), N = 1, NMAX)
DO 10 N = 1, NMAX
NOBD(N) = IFIELD(N, M)
IF (NOBD(N) .EQ. 2) NOBD(N) = 0
IF (IFIELD(N, M) .EQ. 1) IFIELD(N, M) = 1
10 CONTINUE
WRITE (6,4) M, (IFIELD(N,M),N=1,NMAX)
DO 2 N=1,NMAX
2 H(N,M) = FLOAT(NBD(N))
RETURN
3 FORMAT(3612)
4 FORMAT(1H12,3X,3612)
5 FORMAT(1H1,10X,21H WATER LEVELS IN FIELD)
6 FORMAT(1H0,2H M,3X,36I2)
END

SUBROUTINE CHEZY(NMAX,MMAX,C0MANN)
COMMON SE(23,95),SEP(23,95),V(23,95),VP(23,95),U(23,95),UP(23,95),
LC(23,95),NBD(160),NBD(160),NBD(160),NBD(160),NBD(160),H(23,95),
2X1A(600),X1B(600),IFIELD(23,95),ZETA(23),CN(23,95),CNP(23,95)
DIMENSION NPRINT(96)
F1=.3
DO 50 I=1,MMAX
M=95-I
IF(M.EQ.40) CMANN=CMANN+.04
IF(M.EQ.15) CMANN=CMANN-.03
F3=CMANN*(1+F1*(1.-(2.*M)/(1. *MMAX))))
DO 40 N=1,NMAX
NN=N-1
MM=M-1
IF(N.EQ.1) GO TO 10
IF(IFIELD(N,M).EQ.0) GO TO 10
20 IF(M.EQ.1) GO TO 30
A=H(N,M)+H(NN,MM)
B=(SE(N,M)*SEP(N,MM))*.5*(SE(NN,MM)+SEP(NN,MM))*.50
GO TO 35
30 A=H(N,M)+H(NN,MM)
B=(SE(N,M)*SEP(N,M))*.50*(SE(NN,M)+SEP(NN,M))*.50
35 A=(A*H(N,M)+H(NN,M))*.25+(B* (SE(N,M)+SEP(N,M))*.50+(SE(NN,M)+SEP(N
1N,M))*.50)*.25
IF(A.LE.0.) GO TO 36
GO TO 38
36 A=(A*H(N,M)+H(NN,M))*.25
38 C(N,M)=1.49*A**((1./6.)/(F3*1.732)
GO TO 37
10 C(N,M)=0.0
37 CONTINUE
40 CONTINUE
50 CONTINUE
C(7,14)=49.
DO 1 M=79,95
C(5,M)=38.
1 C(4,M)=43.0
C(7,15)=36.
C(9,3)=30.
C(7,3) = 78.
C(6,11) = 33.0
C(6,12) = 44.
C(6,13) = 41.
C(6,14) = 41.
C(6,22) = 17.
C(6,23) = 16.
C(18,22) = 16.
C(18,23) = 16.
C(17,22) = 15.
C(17,23) = 16.
C(17,3) = 13.
C(18,25) = 17.
C(7,78) = 35.
RETURN
END

SUBROUTINE PRINT(NST, AT, XK, PHI, NMAX, MMAX, QALT)
COMMON S E (23, 95), SEP(23, 55), V(23, 95), VP(23, 95), U(23, 95), UP(23, 95),
L(23, 95), NOB(160), MBD(160), MOBD(3), NOBD(2), H(23, 95),
2X(600), XIB(600), IFIELD(23, 95), ETA(23), CN(23, 95), CNP(23, 95)
DIMENSION UETA(23), VETA(23), KONVRT(23), IZETA(23), CONVR(23)
TIME = NST
TIME = TIME*2.*AT/3600.
IF(NST .NE. 0.0) GO TO 999
WRITE(6,1) NST, TIME
1 FORMAT(1H1, 'CHEZY VALUES FOR NEXT TIME STEPS', 15, 5X, 'TIME = ', F6.2, '1 HRS')
DO 2 JA = 1, MMAX
2 WRITE(6,3) JA, (C(N, JA), N = 1, NMAX)
3 FORMAT(1H1, 12, IX, 23F4.0)
999 CONTINUE
5020 FORMAT(1H1, 'THAVERAGED SE AND SEP FOR SECOND HALF OF STEP', 15, 5X, 'TIME = ', F6.2, '1 HRS')
WRITE(6, 5020) NST, TIME
DO 6000 M = 1, MMAX
DO 6006 N = 1, NMAX
KONVRT(N) = (S E(N, M) + SEP(N, M)) * 50.
6006 IF(H(N, M), LE, 0.1) KONVRT(N) = 99999
6000 WRITE(6, 6001) M, (KONVRT(N), N = 1, NMAX)
6001 FORMAT(1H1, 12, IX, 23F4.1)
6002 FORMAT(1H1, 12, IX, 23F4.1)
5021 FORMAT(1H1, 'THAVERAGED V AND VP FOR SECOND HALF OF STEP', 15, 5X, 'TIME
1E = ', F6.2, '1 HRS')
WRITE(6, 5021) NST, TIME
DO 6003 M = 1, MMAX
DO 6007 N = 1, NMAX
KONVRT(N) = (V(N, M) + VP(N, M)) * 50.
6007 IF(H(N, M), LE, 0.1) KONVRT(N) = 99999
6003 WRITE(6, 6001) M, (KONVRT(N), N = 1, NMAX)
5022 FORMAT(1H1, 'THAVERAGED U AND UP FOR SECOND HALF OF STEP', 15, 5X, 'TIME
IE = 'F6.2,2 HRS')
WRITE(6,5022) NST,TIME
DO 6004 M=1,MMAX
DO 6008 N=1,NMAX
KONVRT(N)=(U(N,M)+UP(N,M))*50.
6008 IF(H(N,M).LE.0.1) KQNVRT(N)=999999
6004 WRITE(6,6001) M,(KQNVKT(N),N=1,NMAX)
WRITE (6,5023) NST,TIME
5023 FORMAT(1H1,50HAVENAGED VECTORIAL VELOCITY FOR SECOND HALF OF STEP,15,5X,'TIME = ',F6.2,' HRS')
DO 6009 M=1,MMAX
DO 6010 N=1,NMAX
KONVRT(N)=SQRRT(((U(N,M)+UP(N,M))*50.)**2+((V(N,M)+VP(N,M))*50.)**2)
6010 IF(H(N,M).LE.0.1) KQNVRT(N)=999999
6009 WRITE(6,6001) M, (KONVRT(N),N=1,NMAX)
WRITE (6,5024) NST,TIME
5024 FORMAT(1H1,55YOUR VELOCITY TRANSPORT FOR SECOND HALF OF STEP,15,5X,'TIME = ',F6.2,' HRS')
DO 6011 M=1,MMAX
DO 6012 N=1,NMAX
UETA(N)=(U(N,M)+UP(N,M))*50.
VETA(N)=(V(N,M)+VP(N,M))*50.
IF (UETA(N).EQ.0.AND.VETA(N).NE.0.) GO TO 6013
IF (VETA(N).EQ.0.AND.UETA(N).NE.0.) GO TO 6014
IF (UETA(N).EQ.0.AND.VETA(N).NE.0.) GO TO 6015
IF (VETA(N).EQ.0.AND.UETA(N).NE.0.) GO TO 6020
6013 ZETA(N)=(VETA(N)/ABS(VETA(N)))
IF (ZETA(N).GT.0.) GO TO 6023
IF (ZETA(N).LT.0.) GO TO 6024
6023 ZETA(N)=270.0-42.1
GO TO 6012
6024 ZETA(N)=90.0-42.1
GO TO 6012
6014 ZETA(N)=ATAN2(VETA(N),UETA(N))**97.3
GO TO 6019
6015 ZETA(N)=00.0
GO TO 6012
6020 ZETA(N)=UETA(N)/ABS(UETA(N))
IF (ZETA(N).GT.0.) GO TO 6021
IF (ZETA(N).LT.0.) GO TO 6022
6021 ZETA(N)=42.1
GO TO 6012
6022 ZETA(N)=180.0-42.1
GO TO 6012
6019 CONTINUE
IF((ZETA(N).GE.-180.AND.ZETA(N).LT.0.).AND.(VETA(N).LT.0.0)) GO TO 6018
IF(UETA(N).GE.0.0.AND.VETA(N).LT.0.0) GO TO 6016
IF(UETA(N).LT.0.0.AND.VETA(N).GT.0.0) GO TO 6005
6017 ZETA(N)=180-ZETA(N)
IF(ZETA(N).LT.0.0) GO TO 6027
SUBROUTINE SALT(NST,STEP,AL,AG,A,L,NMAX,MMAX,NINO,MINDO,MIND,MIND1)
COMMON SE(23,95),SEP(23,95),V(23,95),VP(23,95),U(23,95),UP(23,95),
  LG(23,95),NBD(160),MBD(160),MBD(31),NUBD(2),H(23,95),
  ZIA(600),XIB(600),IFIELD(23,95),ZETA(23),CN(23,95),CNP(23,95)
DIMENSION A(95),B(95),P(95),Q(95),R(95),S(95)
LOGICAL IEST
JOUT=6
JOU=1.0
C1=AL*AT
C2=AT
C3=AL**2
C7=5.9*SQRT(AG)
C8=AU/AL
C10=2.*C7/(AL**2)
IF(NST.EQ.10*(NST/10)) JB=15
IF(NST.EQ.10*(NST/10)) JA=12
JB=0
JA=0
ER=.0001
MMAXM=MMAX-1
NMAXM=NMAX-1
COMPUTE CNP ALONG ROWS IN SECOND HALF OF TIMESTEP

NUM=1
IF(ISTEP.EQ.1) GO TO 400

203 IF(NUM.EQ.KIND) GO TO 600
MSRCH=MBD(NUM)/1000000
M =MBD(NUM)/100 -MSRCH*10000 -M*100
NF =MBD(NUM)/100 -MSRCH*1000000 -M*10000 -NF*100
IA=MSRCH/10
IA=MSRCH-10*IA
LL=L-1
LP=L+1
NFF=NFF-1
MM=M+1
MM=M-1
N=NFF
GAMMAC=.5*VP(N,M)/(ABS(VP(N,M))**R)+.5
TEMP4=.5*(H(N,N)+H(N,MM)+SE(N,M)+SE(N+1,M))
TEMP3=.5*(H(N,N)+H(N,MM)+SEP(N,M)+SEP(N+1,M))
TEMP2=C3*TEMP4*VP(N,M)
TEMP2=CN*ABS(VP(N,M))*TEMP8**2/(C(N,M)+C(N+1,M)) +DW

DO 220 N=NFF+1
NN=N-1
NNN=N+1
NM=N-1
ALFAC=.5*U(N,N)/(ABS(U(N,N))+EK)+.5
BETAC=-.5*U(N,MM)/(ABS(U(N,MM)))+EK)+.5
DELTAC=-.5*VP(NN,N)/(ABS(VP(NN,N)))+EK)+.5
DELTAC=1.-GAMMAC
GAMMAC=.5*VP(N,N)/(ABS(VP(N,N))+EK)+.5

TEMP1=.25*(H(N,M)+H(NN,M)+H(NN,MM)+H(NN,MM))/AT
TEMP2=.25*(H(N,M)+H(NN,M)+H(NN,MM)+H(NN,MM))/AT
TEMP3=.5*(H(NN,M)+H(NN,MM)+SE(N,M)+SE(NN,M))
TEMP4=.5*(H(N,N)+H(NN,M)+H(NN,MM)+SE(N,M)+SE(NN,M))
TEMP5=.5*(H(N,N)+H(NN,M)+H(NN,MM)+SE(N,N)+SE(N,MM))
TEMP6=.5*(H(N,N)+H(NN,M)+H(NN,MM)+SE(N,N)+SE(N,MM))
TEMP7=TEMP8
TEMP8=.5*(H(N,N)+H(NN,MM)+SEP(N,M)+SEP(N+1,M))

C C TEMP20=C3*TEMP3*VP(NN,M)
C C TEMP21=CN*ABS(VP(NN,M))*TEMP8**2/(C(N,M)+C(NN,M)) +DW
C C TEMP20=TEMP21
C C TEMP21=TEMP23
C C TEMP22=C3*TEMP4*VP(N,M)
C C TEMP23=CN*ABS(VP(N,M))*TEMP8**2/(C(N,M)+C(N+1,M)) +DW
C C TEMP24=C8*TEMP5*U(N,MM)
C C TEMP25=CN*ABS(U(N,MM))*TEMP8**2/(C(N,M)+C(N,MM)) +DW
\[
\begin{align*}
\text{TEMP}_2 &= C_8 \times \text{TEMP}_6 \times U(N, M) \\
\text{TEMP}_7 &= C_{10} \times \text{ABS}(U(N, M)) \times \text{TEMP}_6 \times 2/(C(N, M) + C(N, MMM)) + D \text{W} \\

\text{P}(N) &= -(1 - \text{DELTA}) \times \text{TEMP}_20 + \text{TEMP}_21 \\
\text{Q}(N) &= \text{TEMP}_1 + \text{GAMMAC} \times \text{TEMP}_22 + \text{TEMP}_23 - \text{DELTA} \times \text{TEMP}_20 + \text{TEMP}_21 \\
\text{R}(N) &= (1 - \text{GAMMAC}) \times \text{TEMP}_22 - \text{TEMP}_23 \\
\text{S}(N) &= -C(N, MMM) \times ((1 - \text{DELTA}) \times \text{TEMP}_24 + \text{TEMP}_25) \\
\text{CN}(N, M) &= (\text{TEMP}_2 + \text{ALFAC} \times \text{TEMP}_26 - \text{BETAC} \times \text{TEMP}_24 + \text{TEMP}_27 + \text{TEMP}_25) \\
2 \times \text{CN}(N, MMM) &= (1 - \text{ALFAC}) \times \text{TEMP}_20 - \text{TEMP}_27 \\
\text{TEST} &= \text{.FALSE.} \\
\text{IF}(\text{NUM.EQ.JA}) \quad \text{TEST} &= \text{.TRUE.} \\
\text{IF}(\text{TEST}) \quad \text{WRITE}(\text{JOUT}, 1200) \quad N, M, P(N), Q(N), R(N), S(N) \\
\text{IF}(\text{TEST}) \quad \text{WRITE}(\text{JOUT}, 1200) \quad N, M, \text{TEMP}_1, \text{TEMP}_2 \\
\text{IF}(\text{TEST}) \quad \text{WRITE}(\text{JOUT}, 1200) \quad N, M, \text{TEMP}_20, \text{TEMP}_21, \text{TEMP}_22, \text{TEMP}_23 \\
\text{IF}(\text{TEST}) \quad \text{WRITE}(\text{JOUT}, 1200) \quad N, M, \text{TEMP}_24, \text{TEMP}_25, \text{TEMP}_26, \text{TEMP}_27 \\
\text{IF}(\text{TEST}) \quad \text{WRITE}(\text{JOUT}, 1200) \quad N, M, \text{ALFAC}, \text{BETAC}, \text{GAMMAC}, \text{DELTA} \\
\end{align*}
\]

1200 FORMAT("//,3X,N,=,I2,2X,I2,2X,7(E10.4,2X))

220 CONTINUE

B(NFF) = 0.
A(NFF) = C(NFPMF)
IF(\text{TEST}) \quad \text{WRITE}(\text{JOUT}, 1200) \quad N, M, A(NFF), C(NP, M)

240 N = NF, L
NN = N - 1
F1 = Q(N) - P(N) + B(NN)
A(N) = -(S(N) + P(N) + A(NN)) / F1

245 L = N - 1
N = L
NP = N + 1
CNP(N, M) = A(N) - B(N) + C(NP, M)
IF(\text{TEST}) \quad \text{WRITE}(\text{JOUT}, 1200) \quad N, M, CN(N, M), CNP(N, M)

249 N = N - 1
NUM = NUM + 1
GO TO 208

C

C

\text{COMPUTE CNP ALONG COLUMNS IN FIRST HALF OF TIMESTEP}
C

C

400 IF(\text{NUM.EQ.NIND}) \quad \text{GO TO} \quad 402
\text{NSKCH} = \text{NBD}(\text{NM}) \times 1000000
N = \text{NBD}(\text{NM}) \times 10000 - \text{NSKCH} \times 100
M = \text{NBD}(\text{NM}) \times 100 - \text{NSKCH} \times 1000 - N \times 100
L = \text{NBD}(\text{NM}) \times 100000 - \text{NSKCH} \times 1000000 - N \times 10000 - M \times 100
IA = \text{NSKCH} \times 10
IB = \text{NSKCH} - 10 \times IA
NN = N - 1
NNN = N + 1
LL = L - 1
LLL = L + 1
LP = L + 1
NFF = MF - 1
M = MFF
ALFAC = .5 * (H(N, M) + H(NN, M) + SEP(N, M) + SEP(N, N+1))

TEMP2 = C8 * TEMP4 * UP(N, M)
TEMP3 = C10 * ABS(UP(N, M)) * TEMP8 ** 2 / (C(N, M) + C(N, N+1)) + DW

DO 420 M = MFF, L

MM = M + 1
MM = M - 1
NM = N - 1

ALFAC = .5 * (H(N, M) + H(NN, M) + SEP(N, M) + SEP(N, N+1))
ALFAC = .5 * (H(N, M) + H(NN, M) + SEP(N, M) + SEP(N, N+1))

TEMP4 = .5 * (H(N, M) + H(NN, M) + SEP(N, M) + SEP(N, N+1))

DO 420 M = MFF, L

MM = M + 1
MM = M - 1
NM = N - 1

BETAC = BETAC + .5 * (H(N, M) + H(NN, M) + H(N, N+1) + SEP(N, M))

DO 420 M = MFF, L

MM = M + 1
MM = M - 1
NM = N - 1

DELTA = .5 * V(N, M) / (ABS(V(N, M)) + ER) + .5

DO 420 M = MFF, L

MM = M + 1
MM = M - 1
NM = N - 1

GAMMA = GAMMA + .5 * V(N, M) / (ABS(V(N, M)) + ER) + .5

TEST = .FALSE.

IF (NM = EQ. JB) TEST = .TRUE.
IF (TEST) WRITE (JOUT, 1200) N, M, P(M), Q(M), R(M), S(M)
IF (TEST) WRITE (JOUT, 1200) N, M, TEMP1, TEMP2
IF (TEST) WRITE (JOUT, 1200) N, M, TEMP20, TEMP21, TEMP22, TEMP23
IF (TEST) WRITE (JOUT, 1200) N, M, TEMP24, TEMP25, TEMP26, TEMP27
IF (TEST) WRITE (JOUT, 1200) N, M, ALFAC, BETAC, GAMMA, DELTA
CONTINUE

B(MFF) = 0.
A(MFF) = CNP(N, MFF)
IF (TEST) WRITE (JOUT, 1200) N, M, A(MFF), CNP(N, LP)
DO 440 M = MF, L
MM = M - 1
F1 = Q(M) - P(M) * B(MM)
A(M) = -(S(M) * P(M) * A(MM)) / F1

440 B(M) = R(M) / F1
M = L
DO 445 I = M, L
MP = M + I
CNP(N, M) = A(M) - B(M) * CNP(N, MP)
IF (TEST) WRITE (JOUT, 1200) N, M, CN(N, M), CNP(N, M)

445 M = M - 1
NUM = NUM + 1
GO TO 400
402 CONTINUE
520 CONTINUE
600 CONTINUE
RETURN

SUBROUTINE
SUBROUTINE I PLOT (NMAX, MMAX, AT, NST, K, PHI, W1O0, AL)
COMMUN SE(23, 95), SEP(23, 95), V(23, 95), VP(23, 95), U(23, 95), UP(23, 95),
1C(23, 95), NBD(160), RBD(160), MOBD(1), NOMD(2), H(23, 95),
2X1A(600), X1B(600), 1FIELD(23, 95), ZETA(23), CN(23, 95), CNP(23, 95)
DIMENSION 13UF(4000)
REAL Z
NMAX = 23
MMAX = 94
AL = .025
NST = 74
WK = 10.4
PHI = 0.22
W1O0 = 1.0
AT = 150
DO 20 M = 1, MMAX
20 READ(5, 31) (U(N, M), N = 1, 12)
DO 21 M = 1, MMAX
21 READ(5, 31) (U(N, M), N = 13, MMAX)
DO 22 M = 1, MMAX
22 READ(5, 31) (V(N, M), N = 1, 12)
DO 23 M = 1, MMAX
23 READ(5, 31) (V(N, M), N = 13, MMAX)
31 FORMAT(12F 5.2)
DO 14 M = 1, 94
DO 14 N = 1, 23
UP(N, M) = U(N, M)
14 VP(N, M) = V(N, M)
TIME=NST
TIME=TIME*2*AT/3600
ZST=NST
CALL PLOTS(IBUF,4000)
CALL PLOT(0.0,-5.0,-3)
CALL PLOT(0.0,1.60,-3)
CALL AXIS(0.0,0.0,13HM GRID NUMBER,-13,28.5,0.0,0.0,3.3)
CALL AXIS(0.0,0.0,13HN GRID NUMBER,+13,7.2,90.0,0.0,3.3)
DN=.3
DM=.3
DG 2  M=1,MMAX
DG 2  I=1,NMAX
N=1
IF(MOD(M,2).EQ.0.J) M = (N M A X + 1) / 2
IF(AbS(UP(N,M)).EQ.0.0) GO TO 2
GA=SQR((UP(N,M)**2+VP(N,M)**2))
UZ=0.0
VZ=0.0
IF(AbS(GA)-.05) GO TO 10
CONTINUE
UZ=.5*U(N,M)/GA
VZ=.5*V(N,M)/GA
GO TO 12
CONTINUE
IF(V(N,M).EQ.0.0) GO TO 4
VZ=V(N,M)/GA
GO TO 4
IF(AbS(GA)-.20) GO TO 7
CONTINUE
UZ=U(N,M)/GA
VZ=.5*V(N,M)/GA
GO TO 4
IF(AbS(GA)-.05) GO TO 9
CONTINUE
UZ=U(N,M)/GA
VZ=.5*V(N,M)/GA
GO TO 4
GO TO 10
CONTINUE
UZ=U(N,M)/GA
VZ=V(N,M)/GA
GO TO 4
8 CONTINUE
UZ=U(N,M)/GA
VZ=V(N,M)/GA
4 CONTINUE
BEGINX=M*DM
BEGINY=N*DN
ENDX=BEGINX+UZ
ENDY=BEGINY+VZ
CALL SYMBOL(BEGINX,BEGINY,.07,3,0,,-1)
CALL PLOT(BEGINX,BEGINY,3)
160.
CALL PLOT (ENDX, ENDY, 2)

2 CONTINUE
IF(WGT>D.0) GO TO 5
CALL SYMBOL (20.0, 6.0, 21, 25) H WIND STRESS WITHOUT TIDES, 0.0, 25

5 CONTINUE
CALL SYMBOL (20.0, 5.6, 14, 6) TIME = 0.0, 2
CALL NUMBER (999., 999., 14, TIME, 0.0, 2)
CALL SYMBOL (999., 999., 14, 13) TIME STEP = 0.0, 13
CALL NUMBER (999., 999., 14, ZST, 0.0, 0)
IF(WK.EQ.0.0) GO TO 6
CALL SYMBOL (20.0, 5.3, 14, 13) WIND SPEED = 0.0, 13
CALL NUMBER (999., 999., 14, WK, 0.0, 2)
CALL SYMBOL (999., 999., 14, YKNOTS, 0.0, 6)
CALL SYMBOL (999., 999., 14, 12) DIRECTION = 0.0, 12
CALL NUMBER (999., 999., 14, PHI, 0.0, 2)
CALL SYMBOL (999., 999., 14, 8) DEGREES, 0.0, 8

6 CONTINUE
CALL SYMBOL (25.2, 4.2, 14, VELOCITY SCALE, 0.0, 14)
CALL SYMBOL (24.6, 4.0, 14, LENGTH, 0.0, 6)
CALL PLOT (24.3, 3.97, 3)
CALL PLOT (25.4, 3.97, 2)
CALL SYMBOL (26.0, 4.0, 14, MAGNITUDE, 0.0, 9)
CALL PLOT (26.0, 3.97, 3)
CALL PLOT (26.9, 3.97, 2)
CALL SYMBOL (25.75, 3.82, 14, 70 FT/SEC, 0.0, 14)
CALL SYMBOL (25.00, 3.82, 13, 0.0, -1)
CALL SYMBOL (25.30, 3.82, 13, 0.0, -2)
CALL SYMBOL (25.75, 3.82, 14, 20 FT/SEC, 0.0, 14)
CALL SYMBOL (25.00, 3.82, 13, 0.0, -1)
CALL SYMBOL (25.20, 3.01, 13, 0.0, -2)
CALL SYMBOL (25.75, 3.14, 14, 19.05 FT/SEC, 0.0, 14)
CALL SYMBOL (25.00, 3.40, 13, 0.0, -1)
CALL SYMBOL (25.10, 3.40, 13, 0.0, -2)
CALL SYMBOL (25.75, 3.14, 14, 90 FT/SEC, 0.0, 14)
CALL SYMBOL (25.00, 3.14, 13, 0.0, -1)
CALL PLOT (34.0, 5.0, 999)
RETURN
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VITA

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