2012

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Recommended Citation
Kossler, W. J.; Abraham, Nathan; Stronach, C. E.; and Greer, Allan J., Magnetic Fields of Vortices in a Superconducting Thin Film (2012). *12th International Conference on Muon Spin Rotation, Relaxation and Resonance (musr2011)*, 30, 241-244.
10.1016/j.phpro.2012.04.082

Available online at www.sciencedirect.com

SciVerse ScienceDirect

Physics Procedia 30 (2012) 241 – 244

Physics Procedia

12^{th} International Conference on Muon Spin Rotation, Relaxation and Resonance

Magnetic Fields of Vortices in a Superconducting Thin Film

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Abstract

The magnetic fields associated with a single superconducting vortex traversing a thin film are calculated. The formulation of Pearl, which has been used for a geometry in which for z<0 one has a vacuum and for z>0 one has superconducting material, is extended to the case of a thin film. For the thin film, the flux in the mid-plane is less than a flux quantum unless one uses a very large radius. For instance a 100 nm film with a 130 nm penetration depth (λ) has only 80% of a flux quantum within a radius of 1.5 μm = 11.5 $\cdot\lambda$. At 10 μm 96% is enclosed. The magnetic field near the surface in both geometries has a significant radial component. The fields for a vortex array are then obtained by summing the fields from nearby vortices. The measurability of the field distributions is discussed.

Keywords: superconductor, thin-film, vortices, flux-quantum, muSR
PACS: 74.25.Ha, 74.25.Wx, 74.78-w

1. Introduction

With the possibility of using the muon spin rotation technique (μSR) for thin films it has become possible to study superconducting films with thickness comparable to the superconducting penetration depth (λ). Niedermayer et al.[1] have already used μSR to study the internal fields near the surface of a superconductor, but one for which the overall thickness was considerably larger than λ. Kogan[2] has also treated thin films, but was primarily interested in the magnetic behavior near the film edge. Here we present a calculation of the fields produced by vortices in films for which the total superconducting material thickness is comparable to λ.

2. The Calculation

While we could have obtained the internal fields by summing appropriate pancake vortices, see Clem[3], we have instead followed J. Pearl in his treatment of a metal-air interface[4]. While he had a surface separating infinite metal and infinite air regions, we consider a thin film between two infinite air regions. Pearl used Ginzburg-Landau electrodynamics for the superconductor so the vector potential satisfies Eq. 1 in which $\phi_0 = 2.07 \cdot 10^3$ gauss kÅ^2 is the flux quantum.

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doi:10.1016/j.phpro.2012.04.082

$$\nabla \times (\nabla \times \mathbf{A}) + \frac{\mathbf{A}}{\lambda^2} = \frac{\phi_0 \hat{\theta}}{2\pi\lambda^2 r} \tag{1}$$

We are first interested in the field distribution of a single vortex throughout a film of thickness comparable to the penetration depth. The primary differences between our calculation and that of Pearl are that we have two boundaries and that we require symmetry about the thin film's mid-plane. The boundary conditions are that the vector potential and its derivative are continuous across the boundary.

The functional form for the vector potential, $A = f\hat{\theta}$, in Pearl's geometry, for which the material for $z > 0$ is superconducting and for $z < 0$ is vacuum, inside the superconductor is:

$$f_2 = \int_0^\infty \frac{\phi_0}{2\pi\lambda^2} \frac{J_1(\gamma r)}{\gamma^2 + 1/\lambda^2} \cdot \left(1 - \frac{\gamma \exp(-(\gamma^2 + 1/\lambda^2)^{1/2} z)}{\gamma + (\gamma^2 + 1/\lambda^2)^{1/2}}\right) d\gamma \tag{2}$$

Where $J_1(\gamma r)$ is a Bessel function. If we introduce $s = (\gamma^2 + 1/\lambda^2)^{1/2}$, this can be written as:

$$f_2 = \int_0^\infty \frac{\phi_0}{2\pi\lambda^2} \frac{J_1(\gamma r)}{s^2} \left(1 - \frac{\gamma \exp(-sz)}{\gamma + s}\right) d\gamma \tag{3}$$

Outside the superconductor, $z < 0$, the solution is:

$$f_1 = \int_0^\infty \frac{\phi_0}{2\pi\lambda^2} \frac{J_1(\gamma r)}{s^2} e^{\gamma z} \frac{s}{s + \gamma} d\gamma \tag{4}$$

The function f_1 is the solution of:

$$\frac{\partial^2 f_1}{\partial z^2} + \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} r f_1 = 0 \tag{5}$$

The function f_2 is the solution of:

$$\frac{\partial^2 f_2}{\partial z^2} + \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} r f_2 - \frac{1}{\lambda^2} f_2 = -\frac{\phi_0}{2\pi\lambda^2 r} \tag{6}$$

For a thin film of thickness d, the solution inside the superconductor is:

$$f_2 = \int_0^\infty \frac{\phi_0}{2\pi\lambda^2} \frac{J_1(\gamma r)}{s^2} \cdot \left(1 - \frac{\gamma[\exp(-s(z + d/2) + \exp(+s(z - d/2)))]}{s(1 - e^{-sd}) + \gamma(1 + e^{-sd})}\right) d\gamma \tag{7}$$

The solution outside the superconductor and for $z > d/2$ is:

$$f_1 = \int_0^\infty \frac{\phi_0}{2\pi\lambda^2} \frac{J_1(\gamma r)}{s^2} \frac{(1 - e^{-sd}) e^{\gamma(z+d/2)} s}{s(1 - e^{-sd}) + (1 + e^{-sd})\gamma} d\gamma \tag{8}$$

for $z < -d/2$. For $z > d/2$ the exponential is $e^{-\gamma(z-d/2)}$.

3. Results

The general shape of the fields may be seen in the upper portion of Fig. 1. Perhaps the most surprising result is shown in the lower part of Fig. 1. The flux in the film mid-plane contained within a radius of 1.5μm = 11.5 λ for one vortex is shown as a function of film thickness, d, for various penetration depths. All of these curves approach ϕ_0 for large thickness. When d is comparable to λ this flux only approaches ϕ_0 for a very large radius. For d=100 nm and λ=130 nm, 0.8ϕ_0 is enclosed within a radius of 1.5 μm (11.5 λ) and only 0.96ϕ_0 for r = 10μm.

The radial dependence of the magnetic fields near the film's surface is shown in Fig. 2. For a given film thickness and penetration depth we obtained the field components for a single vortex throughout the surrounding film.

Using the results for the field components for a single vortex, we have calculated the field distribution for a triangular array of vortices by summing the fields from individual vortices. We then used these fields to simulate the results for μSR experiments. These are shown in Fig. 3. The upper curves show the cases of the initial polarization transverse to the magnetic field. Though not obvious from the low resolution here, the muons from the mid-plane precess faster. The difference is not great, but would be observable, and would be expected to yield results similar to those of Niedermayer et al.[1]. In the lower graph the initial polarization was assumed along the beam and the detectors were also along the beam axis. A small dc decrease and the occurrence of oscillations is seen for muons stopping near the surface of the film.

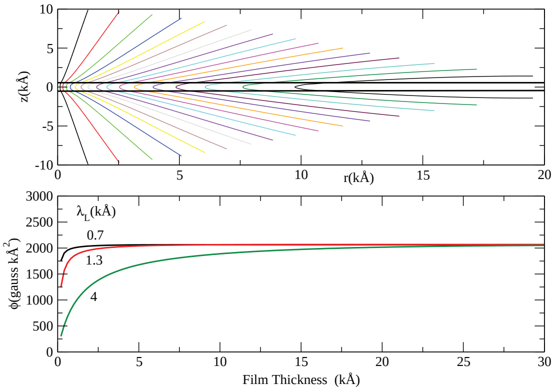

Figure 1: In the upper graph are typical field lines for a thin film of 100 nm and λ=130 nm. One can see the radial component increasing as one is further from the vortex core and nearer the surface. The lower graph shows the flux in the mid-plane contained within a radius of 1.5μm associated with one vortex for three different penetration depths as a function of film thickness.

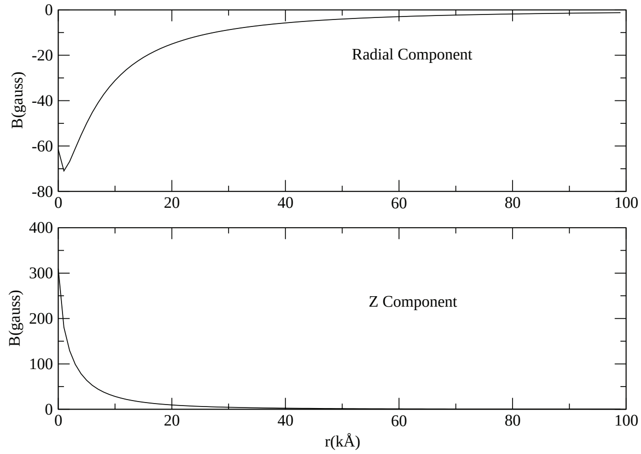

Figure 2: In the upper graph is the radial component of the magnetic field for a single vortex as a function of radius from its core. The component parallel to the vortex core and perpendicular to the plane of the film is shown on the lower graph. These are for near the surface of the superconductor of thickness 100 nm and penetration depth 130 nm.

4. Conclusion

It will be difficult to directly observe the effects of the transverse fields even for the case where the initial polarization is perpendicular to the film. The dc offset or the appearance of oscillations could easily occur for slight misalignments of the sample, detectors, muon beam position, or muon polarization direction.

The reduction of the flux per vortex within a radius of 1.5μm implies a smaller field near the vortex. A consequence of this would be that the second moment of the field distribution, something which is measurable, should be reduced for thin samples. Since this effect depends on the relation of λ to film thickness, the temperature dependence of the second moment of the field distribution will be film thickness dependent. Studying the effects of film thickness on the internal field distributions should be interesting.

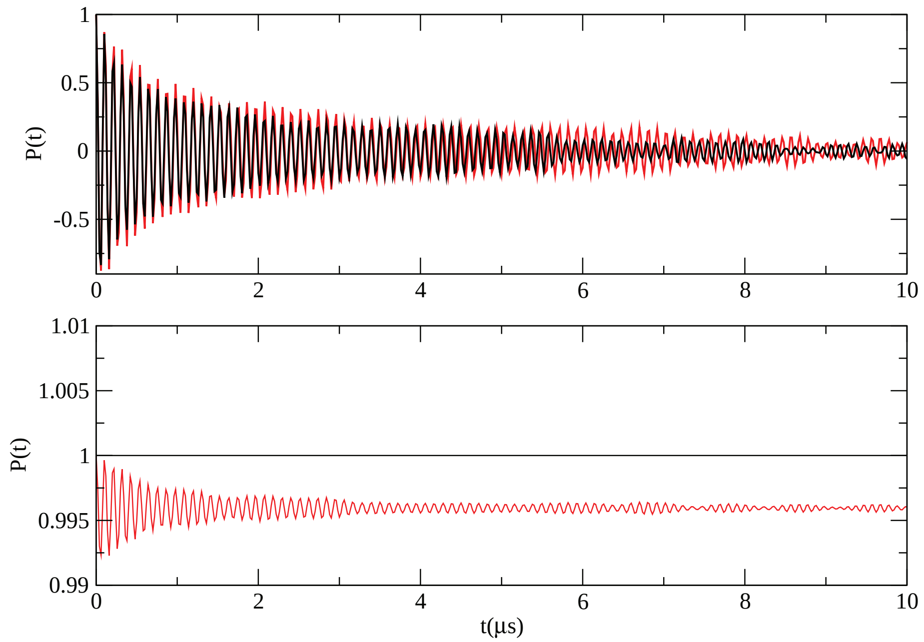

Figure 3: In the upper graph are simulated μSR data for the initial muon polarization transverse to the magnetic field perpendicular to the film. The results for two regions are shown, red: muons stopping near the surface of the film and black: stopping near the center. In the lower graph are simulated μSR data for the initial muon polarization parallel to the magnetic field perpendicular to the film. The polarization is measured along this initial polarization direction. Again red: is for muons stopping near the surface, and black: for muons in the mid-plane

Acknowledgements

C. E. Stronach was supported in part by the Air Force Office of Scientific Research. Conversations with E. Rossi have been very helpful.

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