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Nonlocal Lorentz-violating Modifications of QED

Qian Niu
William & Mary

Nonlocal Lorentz-violating Modifications of Quantum Electrodynamics

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by

Qian (Cooper) Niu

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(Honors or non-Honors)

Advisor: Prof. Chris Carone

Prof. David Armstrong

Prof. Joshua Erlich

Prof. Paul Davies

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Abstract

We consider nonlocal Lorentz-violating theories, with infinite-derivative quadratic terms. The nonlocal modifications in the form of exponential damping in the propagator lead to a better convergence of amplitudes than in the local theories. Moreover, the nonlocal Lorentz-violating theories are ghost-free and unitary when formulated in Minkowski space. We compute the loop effects assuming one-parameter and two-parameter nonlocal functions. By comparing the lower bound of the nonlocality scale with the Planck scale, we rule out these theories. We then review a more general argument, developed in Ref. [12], that a microscopic theory with Lorentz violation around the Planck scale has no suppression at low energy. Additionally, we consider the possibility of suppressing Lorentz violation in these theories when there is a hard cutoff.

Acknowledgement

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Contents

1. Introduction

A. Background

The criterion of simplicity and beauty is an inexhaustible source of inspiration for physicists to construct new theories. Although beauty and simplicity are not generically scientific principles, these aesthetic judgments can often end up being confirmed by empirical facts. Historically, simplicity within heliocentrism inspired Copernicus to abandon the geocentric model of Ptolemy, and this simple heliocentrical model is widely validated by later scientific investigations.

In modern physics, aesthetics also motivates physicists to go beyond the Standard Model (SM) of particle physics. One of the most mentioned concepts is the assumption of naturalness, *i.e.*, all dimensionless parameters of a physical theory should take values of order 1 unless a more detailed theory exists. A more precise account often involves the idea of effective field theories (EFT), which are approximations that include the appropriate degrees of freedom to describe physical phenomena below a specified energy scale, while ignoring underlying substructure. Because the cutoff is the largest dimensional scale is the theory, all parameters of a natural EFT are required to be of order unity in units of the cutoff. This implies that a natural EFT would prefer ratios between its free parameter and its cutoff with values like 1.23 rather than 123000 or 0.000123. Because the SM cannot be valid up to arbitrarily high energies, it can be incorporated into the framework of EFT. Therefore, the free parameters of the SM are thus expected to be of order unity in units of the cutoff, under the assumption of naturalness.

However, if we examine the SM from the perspective of naturalness, various fine-tuning problems and hierarchy problems emerges. The most famous one is about the mass of the Higgs boson. Since the masses of the fundamental particles are not predicted by the SM, they are empirically measured and plugged back into the theory. In 2012, the Higgs mass was measured at the Large Hadron Collider (LHC) at CERN to be $M_{\text{Higgs}} \approx 125$ GeV [3]. The alarm of naturalness is immediately triggered due to the tremendous gap between the Higgs mass and the Planck mass $M_{\text{Planck}} \sim 10^{19}$ GeV. The Higgs mass is modified by quantum corrections that are sensitive to heavy particles, and the upper bound is defined by Planck scale. Thus, the Higgs mass is either much higher than the experimental result or the result of a fine-tuning cancellation. Both situations lead to a naturalness problem. More specifically, the experimentally observed mass, in a textbook treatment [7], is given by the following renormalization condition

$$m^2 = m_0^2 + \Sigma(m^2),$$

where m_0 is the bare mass parameter in the Lagrangian and $\Sigma(m^2)$ is the loop quantum correction of the mass. For a theory with a cutoff Λ, the lowest-order loop quantum correction $\Sigma(m^2)$ can be approximated by $\Lambda^2/(4\pi)^2$. In the case of the Higgs mass, we have $m = 125$ GeV and Λ of the Planck scale $\Lambda \sim M_{\text{Planck}} \sim 10^{19}$ GeV. This implies an incredible fine-tuning cancellation between the quadratic radiative corrections and the bare mass, i.e., $m_0^2 \approx (1 + 10^{-34})\Lambda^2/(4\pi)^2$. This fine-tuning is considered unnatural and an explanation of why the Higgs mass can be naturally maintained to be hierarchically smaller than the Planck scale or any other large cutoff scale Λ is required [15].

The hierarchy problem motivates many proposed extensions of the minimal Standard Model. One popular approach to solve the hierarchy problem is to remove the quadratic divergence in the Higgs mass. In quantum field theory, field interactions are represented by divergent loop Feynman diagrams. To reduce the divergence, one can arrange for a cancellation of the part of loop amplitudes that blows up most quickly, by adding new particles. One of the most popular solutions, offered by supersymmetry (SUSY), follows this line of thinking. In SUSY, quantum corrections cancel between partners and superpartners due to a minus sign associated with fermionic loops. Hence, the quadratic divergence in loop diagrams is reduced under such a mechanism [15]. As an alternative to the SUSY solution, Ref. [13] considers nonlocal higher-derivative models in the form of exponential damping of the propagators and demonstrates the potential applications in addressing the hierarchy problem. The nonlocal higher-derivative theories are appealing because they are expected to be ghost-free and has finite scattering amplitudes. However, these theories have complications related to unitarity in Minkowski space.

Relaxing the requirement of Lorentz invariance has been proposed in Ref. [1] as a possible way to avoid these complications about unitarity. Lorentz Violation (LV) has been widely studied in the last two and a half decades [11]. A comprehensive EFT framework of all possible Lorentz-violating terms, the Standard Model Extension (SME) [9, 10], has been developed by Colladay and Kostelecky. It incorporates within the Standard Model all the possible renormalizable Lorentz-violating terms, while retaining $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge symmetry and the standard fields. A variety of experiments, ranging from collider physics to astrophysics, have placed strong bounds on the possible LV coefficients in the SME. Therefore, the SME can provide a convenient framework for us to categorize our Lorentz-violating effects and compare with the experimental constraints.

B. Framework

We consider Lorentz-violating modifications of the nonlocal higher-derivative theory which avoid problems with unitarity and present a phenomenological study. Our interests in nonlocal Lorentz-violating theories with infinite-derivative quadratic terms are motivated by several reasons. For example, consider a nonlocal, Lorentz-invariant theory of N real scalar fields of mass m with $O(N)$ symmetry [1],

$$\mathcal{L} = -\frac{1}{2}\phi^2 \hat{F}(\Box)^{-1}(\Box + m^2)\phi^a - \frac{1}{8}\lambda_0(\phi^a\phi^a)^2, \tag{1.1}$$

where $a = 1...N$, λ_0 is the dimensionless quartic coupling, $\Box \equiv \partial_\mu\partial^\mu$ and

$$\hat{F}(\Box) = \exp(-\eta\Box^n), \tag{1.2}$$

where η determines the nonlocality scale (in local theories, $\eta = 0$ and $\hat{F} = 1$); n is a positive, even integer such that $\hat{F}$ provides a convergence factor regardless of whether the theory is formulated in Minkowski or Euclidean space. With the additional exponential factor, the momentum-space propagator

$$\tilde{D}_F(p) = \frac{i\hat{F}(-p^2)}{p^2 - m^2 + i\epsilon} \tag{1.3}$$

decreases significantly with momentum and thus leads to more convergent amplitudes than in the local theory with $\hat{F} = 1$.

Nonlocal theories can also avoid the complications related to unitarity in higher-derivative local theories. Although local theories, like the Lee-Wick Standard Model [4], can also have more convergent amplitudes, such theories have to confront the unappealing ghosts, *i.e.*, particles with wrong-sign kinetic and mass terms. Hence, special treatment would be required to compute S-matrix elements to maintain the unitarity of the theory [5], but these extra assumptions to retain unitarity can be avoided if we choose $\hat{F}$ to be an entire function, like Eq. (1.2). Apart from the pole at $p^2 = m^2$, our choice of $\hat{F}$ will cause no additional pole in Eq. (1.3).

However, complications related to unitarity still arise in ghost-free nonlocal theories when formulated in Minkowski space. Ref. [1] studied two-into-two scattering to all orders in the quartic coupling in the large-N limit. The theory defined by Eq. (1.1) and Eq. (1.2) with $n = 2$ is shown to be non-unitary when it is defined in Minkowski space. To avoid both the ghosts and the unitarity problem of Lorentz-invariant nonlocal theories, we consider the Lorentz-violating theories. An alternative form of $\hat{F}$ is chosen, in which the Laplacian operator replaces the d'Alembertian operator [2],

$$\hat{F}(\nabla) = \exp(\eta \nabla^2). \tag{1.4}$$

Since a specific frame is preferred in which the Lagrangian is invariant only under spatial rotations, this new theory breaks Lorentz-invariance. The absence of time derivatives in the Laplacian operator avoids the unitarity problem that appears when the Lorentz-invariant theory is formulated in Minkowski space since Wick rotation is still allowed when evaluating loop diagrams [1].

If the nonlocality represented by Eq. (1.4) is relevant in nature, one would expect that modification of gauge-invariant quantities in the standard model would lead to significant lower bounds on the nonlocality scale $\eta^{-1/2}$ due to the stringent experimental constraints on the violation of Lorentz invariance. In the present work, we apply nonlocal Lorentz-violating modifications to quantum electrodynamics. We insert the nonlocal function of ordinary derivatives in the middle of the kinetic energy terms for the photon, which gives us

$$\mathcal{L}_{KE} = -\frac{1}{4} F_{\mu\nu} \hat{F}(\nabla^2)^{-1} F^{\mu\nu}, \tag{1.5}$$

where the field strength is defined by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and is invariant under a local gauge transformation.

This paper is organized as follows. In Sec. 2, we calculate the effects of a nonlocal Lorentz correction with one free parameter on the electron self-energy diagram. The Lorentz-violating amplitude we find is independent of the nonlocal parameter η. This motivates us to consider a nonlocal function with two free parameters as a possible path to obtain a coefficient-dependent operator that allows us to parametrically suppress Lorentz-violating effects. In Sec. 3, we perform the similar computation with a two-parameter nonlocal Lorentz-violating function. In Sec. 4, we review a more general argument that a microscopic theory with Lorentz violation appearing in the regulator of loop diagrams will unsuppress Lorentz-violating effects at low energy [12]. To avoid this conclusion, we consider cutting off our theory with a momentum scale Λ and derive the bound on the nonlocality scale in Sec. 5. In the final section, we summarize our conclusions.

2. One-Parameter Lorentz-Violating Function

A. Photon Propagator

In this section, we derive the Feynman rule for photon propagators by repeating the textbook treatment in [6] and show how the form of $\hat{F}$ in Eq. (1.4) modifies it with a nonlocality parameter. Starting from the nonlocal Lorentz-violating Lagrangian defined in Eq. (1.5), we can rewrite the equation as:

$$\mathcal{L}_{KE} = \frac{1}{2} A_\mu [(g^{\mu\nu}\Box - \partial^\mu \partial^\nu) e^{-\eta \nabla^2}] A^\nu \tag{2.1}$$

The photon propagator $D^\rho_{F\nu}$ can be derived from the inverse of this operator:

$$[(g^{\mu\nu}\Box - \partial^\mu \partial^\nu) e^{-\eta \nabla^2}] D^\rho_{F\nu}(x-y) = i g^{\mu\rho} \delta^{(4)}(x-y), \tag{2.2}$$

from which we see that $D^\rho_{F\nu}$ has no solution since the operator $[(g^{\mu\nu}\Box - \partial^\mu \partial^\nu) e^{-\eta \nabla^2}]$ has no inverse. This problem can be fixed by introducing a gauge-fixing term $-\frac{1}{2\xi}(\partial^\mu A_\mu) e^{-\eta \nabla^2} (\partial^\nu A_\nu)$ into the Lagrangian, where the parameter ξ is the gauge parameter which determines which gauge we are working in. Thus, the Lagrangian is then written as

$$\begin{aligned} \mathcal{L}_{KE} &= -\frac{1}{4} F_{\mu\nu} e^{-\eta \nabla^2} F^{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu) e^{-\eta \nabla^2} (\partial^\nu A_\nu) && (2.3) \\ &= \frac{1}{2} A_\mu [(g^{\mu\nu}\Box - \partial^\mu \partial^\nu) e^{-\eta \nabla^2}] A^\nu - \frac{1}{2\xi}(\partial^\mu A_\mu) e^{-\eta \nabla^2} (\partial^\nu A_\nu) && (2.4) \\ &= \frac{1}{2} A_\mu e^{-\eta \nabla^2} [(g^{\mu\nu}\Box - (1 - \frac{1}{\xi}) \partial^\mu \partial^\nu] A_\nu. && (2.5) \end{aligned}$$

All our discussions are conducted in position space so far. In momentum space, the operator would be $e^{\eta\vec{k}^2}[(g^{\mu\nu}k^2 - (1-\frac{1}{\xi})k^\mu k^\nu]$, and the modified photon propagator satisfies:

$$e^{\eta\vec{k}^2}[(-g^{\mu\nu}k^2 + (1-\frac{1}{\xi})k^\mu k^\nu]\tilde{D}^{\rho}_{F\nu} = ig^{\mu\rho}. \tag{2.6}$$

The solutions of $\tilde{D}^{\rho}_{F\nu}$ have to be of the form $\mathcal{A}g^{\rho}_{\nu} + \mathcal{B}k^\rho k_\nu$. We then plug this back into Eq. (2.6) to solve for the undecided coefficients $\mathcal{A}$ and $\mathcal{B}$. Their expressions are then given by

$$\mathcal{A} = -\frac{ie^{-\eta\vec{k}^2}}{k^2} \quad \text{and} \quad \mathcal{B} = \frac{ie^{-\eta\vec{k}^2}(1-\xi)}{k^4}. \tag{2.7}$$

This gives us the nonlocal modified photon propagator:

$$\tilde{D}^{\mu\nu}_{F} = \frac{-i}{k^2}e^{-\eta\vec{k}^2}[g^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2}]. \tag{2.8}$$

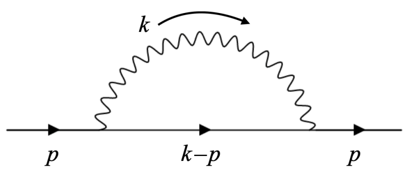
In the following discussion, we will work in the Feynman gauge,

$$\tilde{D}^{\mu\nu}_{F} = \frac{-ig^{\mu\nu}}{k^2}e^{-\eta\vec{k}^2}, \tag{2.9}$$

where the gauge parameter ξ is chosen to be 1. This choice simplifies our calculations. Notice that the nonlocal photon propagator differs from the local photon propagator only by a factor of $e^{-\eta\vec{k}^2}$. This is nothing but the modification function defined in Eq. (1.4) in momentum space. In Sec. 3, we proceed by similar argument.

B. Electron Self-Energy

Tests of Lorentz invariance include probing the relation between energy and 3-momentum of an isolated particle, which is given by the position of the pole of the particle's full propagator. We will calculate the effect of loop corrections on this relation. At one-loop order, we apply the modified Feynman rule to the electron self-energy diagram following the textbook treatment [6],



$$= \frac{i(\not{p}+m)}{p^2-m^2}[-i\Sigma_2(p)]\frac{i(\not{p}+m)}{p^2-m^2}, \tag{2.10}$$

where the self-energy amplitude is

$$\begin{aligned} -i\Sigma_2(p) &= \int \frac{d^4k}{(2\pi)^4}(-ie\gamma^\mu)\frac{i(\not{p}-\not{k}+m)}{(p-k)^2-m^2+i\epsilon}(-ie\gamma^\nu)\frac{-ig^{\mu\nu}}{k^2+i\epsilon}e^{-\eta\vec{k}^2} \\ &= -e^2\int \frac{d^4k}{(2\pi)^4}\frac{(-2\not{p}-\not{k})+4m}{[(p-k)^2-m^2+i\epsilon](k^2+i\epsilon)}e^{-\eta\vec{k}^2}. \end{aligned} \tag{2.11}$$

In the second line, we can contract the indexes $\gamma^\mu(\not{p}-\not{k}+m)\gamma_\mu = -2(\not{p}-\not{k})+4m$. Notice we cannot drop the term linear in k in the numerator as the textbook's odd function integral argument, because the nonlocal function breaks the symmetry of oddness. Then, we want to combine the denominator factors into a single higher degree polynomial such that we can treat it as a spherically symmetric integral. This can be achieve by using Feynman parameterization,

$$\frac{1}{AB} = \int_0^1 \frac{1}{[xA+(1-x)B)]^2}, \tag{2.12}$$

where x is a Feynman parameter.

The denominator in Eq. (2.11) can be written as:

$$\frac{1}{[(p-k)^2-m^2]k^2} = \int_0^1 dx \frac{1}{[k^2+(p^2-2pk+k^2-m^2-k^2)x]^2}$$
$$= \int_0^1 dx \frac{1}{[k^2+p^2x-2xp\cdot k-m^2x]^2}. \tag{2.13}$$

Now, we define a shifted momentum $k \to k + xp$ and complete the square in the denominator. This turns Eq. (2.11) into

$$i\Sigma_2(p) = -e^2 \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \frac{-2\not{p}+2\not{k}+2x\not{p}+4m}{(k^2-\Delta+i\epsilon)^2} e^{-\eta(\vec{k}+x\vec{p})^2}. \tag{2.14}$$

where we define $\Delta \equiv m^2x - p^2x + x^2p^2$ for simplicity. This integral has two poles at $k_0 = \sqrt{\vec{k}^2+\Delta} - i\epsilon$ and $k_0 = -\sqrt{\vec{k}^2+\Delta} + i\epsilon$. Since the poles are in the top-left and bottom-right quadrant of the k_0 complex plane, the integrals over the real axis and the imaginary axis are equal. We use the Wick rotation to transform from Minkowski space to Euclidean space via restricting the time component of momentum to the imaginary axis. After the Wick rotation, we can neglect the $i\epsilon$ and set $\epsilon = 0$. Substituting $k^0 \to ik_E^0$, we have

$$i\Sigma_2(p) = -ie^2 \int \frac{d^4k_E}{(2\pi)^4} \int_0^1 dx \frac{-2\not{p}+2\not{k}+2x\not{p}+4m}{(k_E^2+\Delta)^2} e^{-\eta(\vec{k}+x\vec{p})^2}. \tag{2.15}$$

We will be interested in extracting bounds from experimental processes involving the physical electron, which satisfies the on-shell condition $p^2 = m^2$ at lowest order in the coupling e^2. For this reason, we may substitute $p^2 = m^2$ in Eq. (2.15) consistently at one-loop order in perturbation theory. Hence, $\Delta = x^2m^2$ is always positive and thus the factor of $(k_E^2+\Delta)$ is also positive. This allows us to exponentiate the denominator using a Schwinger parameter u, the general form of which is given by

$$\frac{1}{A^n} = \frac{1}{(n-1)!}\int_0^\infty du\ u^{n-1}e^{-uA}. \tag{2.16}$$

In our case, we have

$$\frac{1}{(k_E^2+\Delta)^2} = \int_0^\infty du\ ue^{-u(k_E^2+\Delta)}. \tag{2.17}$$

The entire function can be transformed into an exponential form,

$$i\Sigma_2(p) = -e^2\int\frac{d^4k}{(2\pi)^4}\int_0^1 dx\int_0^\infty du(-2\not{p}+2\not{k}+2x\not{p}+4m)\ ue^{-u(k_E^2+\Delta)-\eta(\vec{k}+x\vec{p})^2}. \tag{2.18}$$

We can further shift the momentum $\vec{k}\to\vec{k}+\frac{\eta x\vec{p}}{u+\eta}$ so that the quantity that is exponentiated in the integrand is spherically symmetric,

$$\begin{aligned} i\Sigma_2(p) = -ie^2\int_0^1 dx\int_0^\infty du\ u\ e^{-\frac{\eta u x^2|\vec{p}|^2}{u+\eta}-u\Delta} \\ \times\int\frac{d^4k_E}{(2\pi)^4}(-2\not{p}+2x\not{p}+4m+2\not{k}+\frac{2\eta x\vec{\gamma}\cdot\vec{p}}{u+\eta})\ e^{-uk_E^2-\eta\vec{k}^2}. \end{aligned} \tag{2.19}$$

This form makes it easy to see how to throw away terms that are odd in k. The momentum integrals can be then evaluated by treating them as Gaussian integrals. Eventually, we get

$$-i\Sigma_2(p) = -\frac{ie^2}{(4\pi)^2}\int_0^1 dx\int_0^\infty du\frac{u^{1/2}}{(\eta+u)^{3/2}}e^{-\frac{\eta u x^2|\vec{p}|^2}{u+\eta}-u\Delta}\left[\frac{2\eta x}{u+\eta}\vec{\gamma}\cdot\vec{p}-2\not{p}+2x\not{p}+4m\right]. \tag{2.20}$$

We focus on the first Lorentz-violating term in square brackets in Eq. (2.20), which will be sufficient for us to rule out this model. That is

$$-i\Sigma_{LV} = -\frac{ie^2}{(4\pi)^2}\int_0^1 dx\ 2\eta x\ \vec{\gamma}\cdot\vec{p}\,\mathcal{I}, \tag{2.21}$$

where we denote the Schwinger parameter integral as

$$\mathcal{I} = \int_0^\infty du \frac{u^{1/2}}{(\eta+u)^{5/2}}\exp\left[-\frac{\eta u x^2|\vec{p}|^2}{u-\eta} - u\Delta\right]. \tag{2.22}$$

We can extract the η dependence by defining $\tilde{u} = u/\eta$,

$$\mathcal{I} = \frac{1}{\eta}\int_0^\infty d\tilde{u} \frac{\tilde{u}^{1/2}}{(1+\tilde{u})^{5/2}}\exp\left[-\eta(\frac{\tilde{u}x^2|\vec{p}|^2}{\tilde{u}+1} + \tilde{u}\Delta)\right]. \tag{2.23}$$

Via numerical integration, we check that the integral has a smooth limit as $\eta \to 0$. We can set $\eta \to 0$ at the limit and evaluate the integral analytically, which gives us

$$\Sigma_{LV} = \frac{e^2}{(4\pi)^2}\frac{2}{3} \sim 10^{-4}. \tag{2.24}$$

Notice that Σ_{LV} is independent of the nonlocal parameter η, which would lead our theory to danger. In the next subsection, we will explain how the electron self-energy function gives us effective operators that violate Lorentz invariance and can be used to rule out this theory.

C. Experimental Bound on Nonlocality

We employ an effective field theory framework, provided by the SME of Colloday and Kostelecky, to study the bound on nonlocality and matching the corresponding Lorentz-violating effects to the nonlocality scale in our toy model. A general form for the quadratic sector of a renormalizable Lorentz and CPT-violating Lagrangian describing a single massive spin-$\frac{1}{2}$ Dirac fermion is [17]

$$\mathcal{L} = \frac{1}{2} i\bar{\psi}\Gamma^\nu \overleftrightarrow{D}_\nu \psi - \bar{\psi}M\psi, \tag{2.25}$$

where

$$\Gamma^\nu = \gamma^\nu + c^{\mu\nu}\gamma_\mu + d^{\mu\nu}\gamma_5\gamma_\mu + e^\nu + if^\nu\gamma_5 + \frac{1}{2}g^{\lambda\mu\nu}\sigma_{\lambda\mu}, \tag{2.26}$$

$$M = m + a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}. \tag{2.27}$$

The gamma matrices $1, \gamma_5, \gamma^\mu\ \gamma_5\gamma^\mu, \sigma^{\mu\nu}$ are defined by the same conventions as in Lorentz invariant theories. The covariant derivative is defined as $D^\nu = \partial^\nu + ieA^\nu$, and $A\overleftrightarrow{D^\nu}B \equiv A(D^\nu B) - (D^\nu A)B$. In the context of the standard-model and QED extensions, the parameters $a_\mu, b_\mu, c_{\mu\nu}, \ldots, H_{\mu\nu}$ are determined by expectation values of Lorentz tensors arising from spontaneous Lorentz breaking in a more fundamental theory.

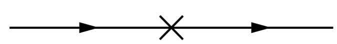
Since experiments in QED often involve relativistic electron-positron collisions, we can simplify the Lagrangian in the fermion sector. Given a relativistic limit, effects from the non-derivative couplings associated with the coefficients $a_\mu, b_\mu, H_{\mu\nu}$ are trivial when comparing to other Lorentz-violating coefficients [17]. In this case, it is reasonable to neglect the effects of the coefficients $a_\mu, b_\mu, H_{\mu\nu}$. To further simplify the Lagrangian, we neglect possible effects from the CPT violating coefficients $e_\mu, f_\mu, g_{\lambda\mu\nu}$. In the exact QED limit of the standard-model extension, these coefficients can be set to zero because they break the $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge symmetry. Furthermore, in most of the real world situation, the colliding electrons and positrons are unpolarized in experiments. Since the γ_5 coupling, in the $d^{\mu\nu}$ field term, induces a correction of opposite sign for χ_L and χ_R, effects from $d_{\mu\nu}$ are zero in the unpolarized beams.

Therefore, the only remaining coefficient is $c_{\mu\nu}$, and the corresponding field operator produces the dominant Lorentz-violating effects. The effective Lagrangian for the fermions can be simplified to be

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \left(g_{\mu\nu} + c_{\mu\nu}\right) \bar{\psi} \gamma^{\mu} \left(\overleftrightarrow{\partial}^{\nu} + 2iqA^{\nu} \right) \psi - m\bar{\psi}\psi, \tag{2.28}$$

$$\delta\mathcal{L}_{\text{eff}} = \bar{\psi}\big(ic_{\mu\nu}\gamma^{\mu}\partial^{\nu} - m\big)\psi - ec_{\mu\nu}\bar{\psi}\gamma^{\mu}\psi A^{\nu}. \tag{2.29}$$

From the effective Lagrangian, we can obtain the Feynman rule for $c_{\mu\nu}$, treating it as a perturbative interaction:



$$= -iec_{\mu\nu}\gamma^{\mu}p^{\nu}. \tag{2.30}$$

This can be compared to the loop amplitude in Eq. (2.24) to extract $c_{\mu\nu}$. Unlike the specific theory we have defined using a nonlocal Lorentz-violating function, the SME categorizes all possible forms of Lorentz and CPT violation in the language of effective field theory. As a complete effective field theory, the SME provides complete predictive power. Hence, it is convenient to find the corresponding LV effect for each specific theory or experiment. We are interested in the experimental bound on $c_{\mu\nu}$ and determine whether the nonlocality scale η is acceptable in a physical theory.

The information about the terms in the restriction of the minimal SME to quantum electrodynamics (QED) in Riemann spacetime is given by the following table [10].

$c_{\mu\nu}$	Bound	System	Ref.
c_{TT}	$(-4 \text{ to } 2) \times 10^{-15}$	Collider Physics	Altschul (2010)b*
c_{TX}	$(-30 \text{ to } 1) \times 10^{-14}$	Collider Physics	Altschul (2010)b*
c_{TY}	$(-80 \text{ to } 6) \times 10^{-15}$	Collider Physics	Altschul (2010)b*
c_{TZ}	$(-11 \text{ to } 1.3) \times 10^{-13}$	Collider Physics	Altschul (2010)b*

Table 1: Disentangled bounds on the boost invariance violation components of c. The Z-axis points along the Earth's rotation axis; the X-axis points toward the vernal equinox point on the celestial sphere; and the Y-axis is set by the right hand rule. Time in these coordinates is T.

We are interested in the experimental bound on c_{TT}. The best bound is given by the instantaneous synchrotron spectrum [16],

$$|c_{TT}| < 4 \times 10^{-15}. \tag{2.31}$$

Thus the Lorentz-violating contribution to the fermion propagator is

$$\Sigma_{\text{LV}} \sim 10^{-15}. \tag{2.32}$$

The experimental bound on the Lorentz-violating operator is much smaller than that of our theory of order $\sim 10^{-4}$. Due to the lack of η dependence, it is impossible to suppress the loop correction. Therefore, we can conclude that the nonlocal theory defined by Eq. (1.4) and Eq. (1.5) can be ruled out by the experimental bound on Lorentz violation.

3. Two-Parameter Lorentz-violating Function

A. Photon Propagator

As we have shown in Sec. 2, the one-parameter nonlocal function leads to a parameter-independent LV operator and thus an unbounded LV effect. Given our main goal to damp the loop effect at high momentum via nonlocality parameters, it is natural to consider multiple-parameter nonlocal functions that generate parameter-dependent operators. In particular, we are interested in the following form of nonlocal function

$$\hat{F}(\nabla) = \frac{1 + c_0}{1 + c_0 e^{-\eta \nabla^2}}, \tag{3.1}$$

where we introduce a new nonlocal parameter c_0 (a local theory at $c_0 = 0$ or $\eta = 0$). Notice that the nonlocal effects of this function also vanish when the photon momentum goes to zero. Similar to our discussion in Sec. 2, the nonlocal photon propagator can be obtained from transforming $\hat{F}(\nabla)$ into momentum space. This gives us

$$\tilde{D}_F^{\mu\nu} = \frac{-ig^{\mu\nu}}{k^2} \frac{1 + c_0}{1 + c_0 e^{\eta \vec{k}^2}}. \tag{3.2}$$

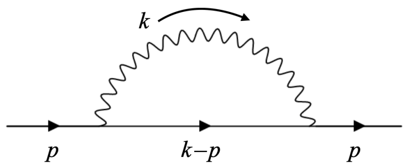
Here we also choose Feynman gauge to simplify the calculation.

B. Electron Self-Energy

Given result in Sec. 2, we can simplify our calculation by recognizing the geometric relation between the nonlocal coefficient terms in Eq. (3.2) and Eq. (1.4). That is

$$\frac{1 + c_0}{1 + c_0 e^{\eta \vec{k}^2}} = \frac{1 + c_0}{c_0} \sum_{j=0}^{\infty} \left(- \frac{1}{c_0} \right)^j e^{-\eta_j \vec{k}^2}, \tag{3.3}$$

where $\eta_j \equiv \eta(j+1)$. This form reduces the problem to a sum of integrals, and each integral has the same form as Eq. (2.11), which we have already evaluated. The electron self-energy diagram is



$$= \frac{i(\not{p}+m)}{p^2-m^2}[-i\Sigma_2(p,\eta,c_0)]\frac{i(\not{p}+m)}{p^2-m^2}, \tag{3.4}$$

where the self-energy amplitude is

$$-i\Sigma_2(p,\eta,c_0) = -e^2\int \frac{d^4k}{(2\pi)^4}\frac{(-2(\not{p}-\not{k})+4m)}{((p-k)^2-m^2)k^2}\left(\frac{1+c_0}{c_0}\sum_{j=0}^{\infty}\big(-\frac{1}{c_0}\big)^j e^{-\eta_j\vec{k}^2}\right) \tag{3.5}$$

$$= \frac{1+c_0}{c_0}\sum_{j=0}^{\infty}\big(-\frac{1}{c_0}\big)^j\left[-e^2\int\frac{d^4k}{(2\pi)^4}\left(\frac{-2(\not{p}-\not{k})+4m}{[(p-k)^2-m^2]k^2}\right)e^{-\eta_j\vec{k}^2}\right] \tag{3.6}$$

$$= \frac{1+c_0}{c_0}\sum_{j=0}^{\infty}\big(-\frac{1}{c_0}\big)^j[-i\Sigma_2'(p,\eta_j)]. \tag{3.7}$$

The $-i\Sigma'(p,\eta_j)$ has the exactly same form as Eq. (2.11), which gives us

$$-i\Sigma'(p,\eta_j) = -\frac{ie^2}{(4\pi)^2}\int_0^1 dx\int_0^{\infty} du\frac{u^{1/2}}{(\eta_j+u)^{3/2}}e^{-\frac{\eta_j u x^2|\vec{p}|^2}{u+\eta_j}-u\Delta}\left[\frac{2\eta_j x}{u+\eta_j}\vec{\gamma}\cdot\vec{p}-2\not{p}+2x\not{p}+4m\right]. \tag{3.8}$$

Furthermore, the evaluation of Lorentz-violating contribution in the case where $\eta\to 0$ can follow the similar logic in Sec. 2 and gives

$$-i\Sigma'_{LV}(p,\eta_j) = -\frac{ie^2}{(4\pi)^2}\frac{2}{3}. \tag{3.9}$$

Plugging this back to $-i\Sigma(p, \eta, c_0)$ give us

$$-i\Sigma_{LV} = \frac{1+c_0}{c_0} \sum_{j=0}^{\infty} \big(- \frac{1}{c_0} \big)^j \Big[- \frac{ie^2}{(4\pi)^2} \frac{2}{3} \Big]. \tag{3.10}$$

The geometric series can be easily summed, so we get

$$\Sigma_{LV} = \frac{e^2}{(4\pi)^2} \big(\frac{2}{3}\big) \; \sim 10^{-4}. \tag{3.11}$$

Once again, the nonlocal function leads to a coefficient-independent Lorentz-violating effect such that the loop correction has no suppression. Since the magnitude of the Lorentz violation is exactly the same as Eq. (2.24), the two-parameter nonlocal model is also ruled out by the experimental bound in Eq. (2.32). Since limitations on our choices of nonlocal functions are few and the most hopeful choices have already been ruled out by the same reason, it is thus reasonable for us to doubt whether the nonlocal function in any form would have an unsuppressed amplitude.

4. General Argument

A. Lorentz Invariance Fine-tuning

Without specifying the form of Lorentz-violating damping, Ref. [12] argues that the Lorentz violation has no suppression by powers of E/E_p at low energies if the theory violates Lorentz invariance at the Planck Scale. The Lorentz violation can be cancelled by finding Lorentz-violating counterterms in the Lagrangian. However, given the observed Lorentz invariance at low energy, the counterterms must be fine-tuned, which is unacceptable in a fundamental theory. Such a conclusion indicates that all attempts of finding nonlocal Lorentz-violating functions in any forms will fail to reduce the divergence in loop corrections. In Sec. 2 and Sec. 3, our choices of one-parameter and two-parameter nonlocal functions offer two special cases to support this conclusion.

In this section, we review the general argument given by Ref. [12]. This argument is demonstrated via a simple Yukawa theory of a scalar field and a Dirac field, which is used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields. In this case, it is reasonable to determine whether a theory has a fine-tuning problem from the suppression condition of Lorentz violation effects in the Yukawa theory. The Lagrangian density of interest is given by

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_0^2\phi^2 + \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi + g_0\phi\bar{\psi}\psi. \tag{4.1}$$

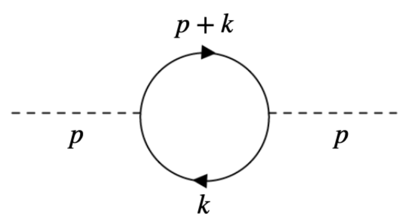
The fermion propagator can be modified as follow

$$\frac{i}{\gamma^\mu k_\mu - m + i\epsilon} \longrightarrow \frac{i\hat{F}(|\vec{k}|^2/\Lambda^2)}{\gamma^\mu k_\mu - m + i\epsilon}, \tag{4.2}$$

where Λ is a cutoff and $\hat{F}(|\vec{k}|^2/\Lambda^2)$ satisfies $\hat{F}(0) = 1$ and $\hat{F}(\infty) = 0$. In the following discussion, we will denote the propagator as $\mathcal{G}(k) = \hat{F}(|\vec{k}|^2/\Lambda^2)\mathcal{S}_{\rm F}(k)$ where $\mathcal{S}_{\rm F}$ is the ordinary fermion propagator. Moreover, note that we are using $|\vec{k}|^2/\Lambda^2$ as argument of the LV cutoff function $\hat{F}$ instead of $|\vec{k}|/\Lambda$ used in Ref. [12]. This choice is motivated by simplicity and clearly it does not affect the general behaviour of the final result.

B. Scalar Self-Energy

In this subsection, we compute the scalar self-energy function following Ref. [18]. The Feynman diagram of scalar self-energy is given by



The one loop correction is

$$\Pi_2(p) = -\frac{i\lambda^2}{16\pi^4}\int d^4k\ \hat{F}(|\vec{k}|^2/\Lambda^2)\hat{F}(|\vec{k}+\vec{p}|^2/\Lambda^2) \times \frac{\mathrm{tr}[(\not{k}+m)(\not{k}+\not{p}+m)]}{(k^2-m^2+i\epsilon)\,[(k+p)^2-m^2+i\epsilon]}. \tag{4.3}$$

The dominant Lorentz violation at low energy can be characterized by

$$\xi = \left[\frac{\partial^2\Pi_2(p)}{\partial(p^0)^2} + \frac{\partial^2\Pi_2(p)}{\partial(p^1)^2}\right]_{p=0}. \tag{4.4}$$

Since we preserve the rotational symmetry in the Lorentz transformation, it is reasonable to consider only p^0 and p^1 derivatives without losing generality. We can use chain rule to get $\xi = [(p^0)^2 + (p^1)^2]\Pi_2''(p)$. For Lorentz invariant theories, we expect a cancellation at $p = 0$, which leads to $\xi = 0$. However, if the Lorentz invariance is slightly violated, ξ will have a nonzero value. For an arbitrary derivative of $\Pi_2(p)$, we have

$$\left[\frac{\partial}{\partial p^a}\frac{\partial}{\partial p^b}\Pi(p)\right]_{p=0} = -\,i\lambda^2\int\frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\left[\left(\frac{\partial}{\partial p^a}\frac{\partial}{\partial p^b}G(k-p)\right)G(k)\right]\Bigg|_{p=0} \tag{4.5}$$

$$= i\lambda^2\int\frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\left[\frac{\partial G(k)}{\partial k^a}\frac{\partial G(k)}{\partial k^b}\right], \tag{4.6}$$

where in the last line we have used that $\partial/\partial p^a = -\partial/\partial k^a$, performed an integration by parts and discarded a boundary term. Then, it is convenient to Wick rotate $k^0 \to ik_E^0$ to work in Euclidean space and to compute first the integral over k_E^0, since the unknown cutoff function $\hat{F}$ does not depend on it. After some algebra, it becomes

$$\left(\frac{\partial^2\Pi(p)}{\partial p^0\partial p^0}\right)_{p=0} = \lambda^2\int\frac{d^3k}{(2\pi)^3}\hat{F}^2\left(-\frac{1}{E_k^3}+\frac{m^2}{E_k^5}\right) \tag{4.7}$$

$$\left(\frac{\partial^2\Pi(p)}{\partial p^i\partial p^i}\right)_{p=0} = -\lambda^2\int\frac{d^3k}{(2\pi)^3}\left[\frac{(k^i)^2}{\Lambda^2}\left(-\frac{8(\hat{F}')^2}{\Lambda^2E_k^3}k^2-4\frac{\hat{F}\hat{F}'}{E_k^5}\left(-k^2+2m^2\right)\right)+\frac{5m^2\hat{F}^2\left(k^i\right)^2}{E_k^7}-\frac{\hat{F}^2}{E_k^3}\right], \tag{4.8}$$

where we abbreviate $k = |\vec{k}|$ and $E_k = \sqrt{k^2+m^2}$. We then use the spherical symmetry of integrals to evaluate ξ, which is

$$\xi = -\frac{\lambda^2}{2\pi^2}\int_0^\infty dkk^2\left[\hat{F}^2\frac{m^2}{E_k^7}\left(\frac{5}{3}k^2-E_k^2\right)-\frac{8k^4}{3\Lambda^4E_k^3}(\hat{F}')^2+\frac{4k^2\left(k^2-2m^2\right)}{3\Lambda^2E_k^5}\hat{F}\hat{F}'\right]. \tag{4.9}$$

Note that $\left(\frac{\partial^2\Pi(p)}{\partial p^i\partial p^i}\right)$ is actually independent of i. This expression can be simplified further by defining the dimensionless variable $y = k^2/\Lambda^2$. As a result,

$$\xi = -\frac{\lambda^2}{2\pi^2}\left[-\frac{4}{3}\int_0^\infty dyy^{5/2}(\hat{F}')^2\frac{1}{(y+m^2/\Lambda^2)^{3/2}}+\frac{2}{3}\int_0^\infty dyy^{3/2}\hat{F}\hat{F}'\frac{y-m^2/\Lambda^2}{(y+m^2/\Lambda^2)^{5/2}}\right]. \tag{4.10}$$

The terms containing derivatives are evaluated by assuming that $\hat{F}'$, $\hat{F}''$ are sharply localized in the region $y = k^2/\Lambda^2 \approx 1$. This implies that we neglect the mass contributions $m^2 \ll k^2 \approx \Lambda^2$ in the corresponding integrals. In this limit, we get

$$\xi = \frac{\lambda^2}{6\pi^2}\left[1 + 4\int_0^\infty dy y \hat{F}'(y)^2\right]. \tag{4.11}$$

Switching our convention back to the origin Lorentz-violating modification function *,i.e.,* $\hat{F}(k/\Lambda)$ and $x = k/\Lambda$ gives

$$\xi = \frac{\lambda^2}{6\pi^2}\left[1 + 2\int_0^\infty dx x \hat{F}'(x)^2\right]. \tag{4.12}$$

The Lorentz violation is not suppressed by the ultraviolet scale Λ, which agrees with our conclusions in Sec. 2 and Sec. 3. Since this result is applicable to self-energy diagram for all fields, we consider the top-quark Higgs coupling as an example to determine the magnitude of ξ. In the standard model, the top-quark Higgs coupling λ_t has the magnitude of order one, so we have $\xi \geq 10^{-2}$ immediately from Eq. (4.12). On the other hand, the term ξ can also be interpreted as a modification of the velocity of light since the dispersion relation can be corrected by $E^2 - p^2 - m^2 - \Pi(p) = 0$. In a perturbative expansion, the speed of light is modified by a factor of $1 + \xi/4 + \mathcal{O}(\xi^2)$, which is tightly bounded by [19]

$$\delta c \leq 10^{-20}. \tag{4.13}$$

This is clearly in conflict with the magnitude of Eq. (4.12). We can thus conclude that the radiative corrections with Lorentz-violating functions would induce Lorentz violations that are much larger than the already established bounds. To produce an acceptable size of the radiative correction, it is inevitable to introduce some fine-tuning cancellation, which leads to a naturalness problem.

5. Nonlocal Lorentz violation with A Hard Cutoff

A. Momentum Cutoff

We have shown that Lorentz-violating terms with a coefficient proportional to the coupling of the interactions are not suppressed by inverse powers of the Lorentz-violating scale. In the literature, possible ways to avoid this outcomes have been discussed. Ref. [20] argues that non-perturbative treatments might lead to Lorentz violations that do not necessarily imply observational consequences at low energy, while Ref. [21] argues again this proposal by pointing out that they are based on a non-generic special property in some model of Lorentz violation.

In this section, we introduce a momentum scale Λ to cut off the toy model defined by Eq. (1.4) and check whether it could be a loophole for evading unsuppressed Lorentz-violating effects. Given the detailed calculation in Sec. 2, we start from Eq. (2.21),

$$-i\Sigma_{LV} = -\frac{ie^2}{(4\pi)^2}\int_0^1 dx\, 2\eta x\, \vec{\gamma}\cdot\vec{p}\,\mathcal{I}, \tag{5.1}$$

where we denote the Schwinger parameter integral as

$$\mathcal{I} = \int_0^\infty du \frac{u^{1/2}}{(\eta+u)^{5/2}} \exp\left[-\frac{\eta u x^2|\vec{p}|^2}{u+\eta} - u\Delta\right]. \tag{5.2}$$

The η dependence can be extracted by $\tilde{u} = u/\eta$. We can implement the high momentum scale by setting the lower limit to be $\frac{1}{\eta\Lambda^2}$ in the $d\tilde{u}$ integral,

$$\mathcal{I} = \frac{1}{\eta}\int_{\frac{1}{\eta\Lambda^2}}^\infty d\tilde{u} \frac{\tilde{u}^{1/2}}{(1+\tilde{u})^{5/2}} \exp\left[-\eta\Big(\frac{\tilde{u}x^2|\vec{p}|^2}{\tilde{u}+1} + \tilde{u}\Delta\Big)\right]. \tag{5.3}$$

We can evaluate this integral by setting $\eta \to 0$. As a result,

$$\Sigma_{LV} = \frac{e^2}{(4\pi)^2}\left(\frac{2}{3} - \frac{2}{3(1+\eta\Lambda^2)^{3/2}}\right). \tag{5.4}$$

From the result of the experimental bound on Σ_{LV} in Sec. 2, the dimensionless coefficient $\eta\Lambda^2$ is expected to be small. We can expand Eq. (5.4) around small $\eta\Lambda^2$,

$$\Sigma_{LV} \approx \frac{e^2}{(4\pi)^2}(\eta\Lambda^2). \tag{5.5}$$

From the experimental constrain on the SME coefficient, we can find that the bound on nonlocality $\eta\Lambda^2$ is of order $\sim 10^{-12}$. Thus, the nonlocality scale $\eta^{-1/2}$ must be 6 order magnitude larger than the momentum cutoff Λ. Although the cutoff might suppress the Lorentz-violating effect into an experimentally viable magnitude, the 12 order difference between the nonlocality scale square and the cutoff square violates the naturalness criterion that physical scales should be no greater than the cutoff. Therefore, introducing a momentum cutoff is not an ideal solution either.

6. Conclusion

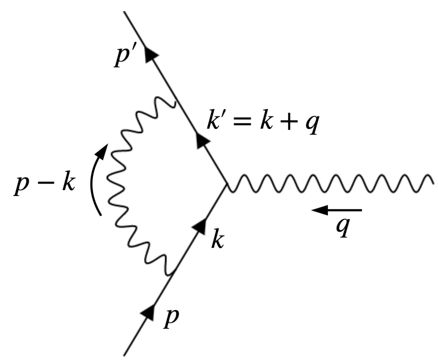
In this thesis, we have considered infinite-derivative theories in which the quadratic terms are nonlocal and Lorentz-violating. Our initial motivation is to avoid the complications related to unitarity when formulated in Minkowski space. We present a phenomenological study of these theories by computing the loop effects, which receive modifications from our choices of nonlocal functions.

Two toy models are considered, namely one-parameter and two-parameter nonlocal functions, and their electron self-energy functions are calculated. By comparing the lower bound of the nonlocality scale with the experimental bounds in the SME, we conclude that both nonlocal functions fail to reduce the level of divergence in the loop effects because the Lorentz violation operators in our nonlocal theories are unsuppressed. The unsuppressed Lorentz violation at low energy is discussed in the literature. We revisit an argument given by Collins, Perez, Sudarsky, Urrutia, and Vucetich through a simple toy model of a scalar field coupled to a fermion field via a Yukawa interaction.

In the last section, we propose a way to avoid the unsuppressed low-energy Lorentz violation. We show that a momentum cutoff might reconcile a Lorentz invariance violating high momentum scales and a suppression of Lorentz invariance violations at low momenta. However, this is only true when the nonlocality parameter defines an energy scale that is much larger than the cut off scale for the theory, violating one of the standard criteria for naturalness.

A. Calculation of Vertex Function

It is also possible to use the vertex function to compute the Lorentz violation effects in our theory defined by Eq. (1.4). In this note, we are going to evaluate the one-loop effect on the electron vertex function described by the following Feynman diagram,



Applying the Feynman rules to order α, we can find

$$\begin{aligned}&\bar{u}(p')\delta\Gamma(p',p)u(p)\\&=\int\frac{d^4k}{(2\pi)^4}\frac{-ig_{\nu\rho}[e^{-\eta\vec{k}^2}]}{k^2}\bar{u}(p')(-ie\gamma^\nu)\frac{i(\not{k}'+m)}{k'^2-m^2}\gamma^\mu\frac{i(\not{k}+m)}{k^2-m^2}(-ie\gamma^\rho)u(p)\end{aligned} \tag{A.1}$$

It can be evaluated by exponentiating the denominator, with Schwinger parameter, and proceed as the textbook treatment. The result is

$$\begin{aligned}\bar{u}(p')\delta\Gamma(p',p)u(p) &= -\frac{2e^2}{(4\pi)^2}\int_0^\infty du\int_0^1 dxdydz\,\delta(x+y+z-1)\\ \times\,\bar{u}(p')&\left(\frac{1}{(u+\eta)^{3/2}}\,e^{-\frac{u\eta(y\vec{q}-z\vec{p})^2}{u+\eta}-u\Delta}\left[u^{3/2}\mathcal{C}+\gamma^\mu\frac{1}{4}u^{1/2}+\gamma^\mu\frac{u^{3/2}[3u+3\eta+2\eta^2(y\vec{q}-z\vec{p})^2]}{4(u+\eta)^2}\right]\right)u(p),\end{aligned} \tag{A.2}$$

where $\mathcal{C}=\gamma^\mu[(1-x)(1-y)q^2+(1-4z-z^2)m^2]+\frac{i\sigma^{\mu\nu}q_\nu}{2m}[2m^2z(1-z)]$.

We focus on the last term in the bracket. That is

$$\delta\Gamma_{LV}\left(p',p\right) = \int_0^\infty du\gamma^\mu \frac{u^{3/2}}{4(u+\eta)^{7/2}}\left[3\eta + 2\eta^2(y\vec{q}-z\vec{p})^2\right]\exp\left[-\frac{u\eta(y\vec{q}-z\vec{p})^2}{u+\eta} - u\Delta\right] \tag{A.3}$$

The evaluation of this integral is similar to the electron self-energy diagram. The η dependence can extract by redefining $\tilde{u} = u/\eta$, and we can set $\eta \to 0$. After some algebra, the amplitude becomes

$$\delta\Gamma_{LV} = -\frac{2e^2}{(4\pi)^2}(\frac{3}{10}) \quad \sim 10^{-4}. \tag{A.4}$$

This also leads to an unsuppressed Lorentz-violating effect. Since the model has already been ruled out from our previous consideration, we will not undertake a detailed decomposition and phenomenological study of Eq. (A.4) in terms of SME operators.

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