RDE Model: A Program for Simulating Water Wave Transformation for Harbor Planning

Jerome P.Y. Maa
Virginia Institute of Marine Science

T. W. Hsu
Hydraulics and Ocean Engineering Dept. National Cheng-Kung University

H. H. Hwung
Hydraulics and Ocean Engineering Dept. National Cheng-Kung University

Follow this and additional works at: https://scholarworks.wm.edu/reports

Part of the Marine Biology Commons

Recommended Citation

This Report is brought to you for free and open access by W&M ScholarWorks. It has been accepted for inclusion in Reports by an authorized administrator of W&M ScholarWorks. For more information, please contact scholarworks@wm.edu.
RDE Model:  
A Program for Simulating Water Wave Transformation for Harbor Planning

Special Scientific Report, No. 136

Jerome P.-Y. Maa
School of Marine Science
Virginia Institute of Marine Science
College of William and Mary
Gloucester Point, VA 23062

T.-W. Hsu and H.-H. Hwung
Hydraulics and Ocean Engineering Dept.
National Cheng-Kung University
Tainan, Taiwan, R.O.C.

March 1998
Abstract

The extended mild slope equation, an elliptic type partial differential equation, was solved directly to simulate water wave refraction, diffraction, reflection, shoaling, and harbor resonance for harbor planning purposes. The finite difference method was used to build a banded matrix equation which was then solved directly by using the Gaussian elimination method with partial pivoting and a newly-developed book-keeping procedure. Because this book-keeping procedure changes the large computer memory requirements into a large hard disk size requirement, this model is capable of handling realistic applications efficiently using small computers. Five verification tests were selected to demonstrate the performance of this numerical model. Comparisons between the model results and available experimental or analytical results indicate a satisfactory agreement. Because of the finite difference method and the direct approach in solving the governing equation, this model is simple to maintain, and more importantly, to upgrade for including other processes, e.g., bottom friction, tidal current influence, and spectrum waves, in the future.

The computer codes for the main program, pre- and post-process software are all included in this report. The above codes are also available in a floppy disk, upon requested. The purpose is letting uses to access this method easily, and of course, provide feedback if possible.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.</td>
<td>GOVERNING EQUATIONS</td>
<td>4</td>
</tr>
<tr>
<td>3.</td>
<td>BOUNDARY CONDITIONS</td>
<td>5</td>
</tr>
<tr>
<td>3.1.</td>
<td>Partial Reflection Boundary Condition</td>
<td>7</td>
</tr>
<tr>
<td>3.2.</td>
<td>Given Boundary Condition</td>
<td>9</td>
</tr>
<tr>
<td>4.</td>
<td>NUMERICAL MODEL</td>
<td>12</td>
</tr>
<tr>
<td>5.</td>
<td>MODEL VERIFICATION</td>
<td>14</td>
</tr>
<tr>
<td>5.1.</td>
<td>Infinite Long Breakwater with a gap</td>
<td>14</td>
</tr>
<tr>
<td>5.2.</td>
<td>Finite Length Breakwater</td>
<td>18</td>
</tr>
<tr>
<td>5.3.</td>
<td>Resonance at a Rectangular Harbor</td>
<td>21</td>
</tr>
<tr>
<td>5.4.</td>
<td>Influence of bottom slope and Curvature</td>
<td>24</td>
</tr>
<tr>
<td>5.5.</td>
<td>A Simple but Practical Harbor</td>
<td>27</td>
</tr>
<tr>
<td>6.</td>
<td>DISCUSSION AND CONCLUSIONS</td>
<td>29</td>
</tr>
<tr>
<td>7.</td>
<td>ACKNOWLEDGMENTS</td>
<td>31</td>
</tr>
<tr>
<td>8.</td>
<td>REFERENCES</td>
<td>32</td>
</tr>
<tr>
<td>9.</td>
<td>APPENDIX I, Finite Difference Equations for the Boundary Conditions</td>
<td>1.1</td>
</tr>
<tr>
<td>10.</td>
<td>APPENDIX II, Source Listing of the Main Program and Subroutines RDE.FOR</td>
<td>2.1</td>
</tr>
<tr>
<td>11.</td>
<td>APPENDIX III, Source Listing for the First Post Process Program OUTFORM.FOR</td>
<td>3.1</td>
</tr>
<tr>
<td>12.</td>
<td>APPENDIX IV, Source Listing for the Second Post Process Program for Graphic Presentation, RDEPLOT.M</td>
<td>4.1</td>
</tr>
<tr>
<td>13.</td>
<td>APPENDIX V, Source Listing for a Grid Generation and I/O files Generations</td>
<td>5.1</td>
</tr>
</tbody>
</table>

# TABLE OF FIGURES

I
Fig. 1. Coordinate System and Grid Alignment for the Model .. 6

Fig. 2. Calculation of Wave Phase at Given Boundary Grid Points............................................ 11

Fig. 3. Comparison of Wave Height Contours and Wave Crests for an Infinite-Long Breakwater with a Gap. The top half is a copy from the Shore Protection Manual, the bottom part is model results .......................... 16

Fig. 4. Comparison of Wave Height Contours for a Finite-Length Breakwater with Oblique Incident Waves. (a) Analytical solution from Goda et al. (1971); (b) Model results ... 20

Fig. 5. The Typical Rectangular Harbor. The Displayed Normalized Wave Height Contours is the Response near Harbor Resonance ...................................... 22

Fig. 6. Comparison with Analytical Solution (Solid line) and Model-Calculated Normalized Wave Height at the Harbor End. The dashed line is the model results with the open ocean boundary domain shown in Fig. 5 (12.85 x 18.45 cm). The dashed-dotted line is the model results when the open Ocean boundary domain increased to 19.27 x 30.15 cm ... 22

Fig. 7. Calculated Wave Height Profile along a Wave Flume with Four Sinusoidal Bedforms. The bedforms are given on the bottom ................................................. 26

Fig. 8. Comparison of Calculated and Measured Wave Reflection Coefficient for the Four Sinusoidal Bedforms. The solid line is model results and the circles are experimental data from Davies and Heathershaw (1984) ................. 26

Fig. 9. Comparison of Wave Height Contours in a Simple Harbor (a) Laboratory measurements by Sato et al. (1990); (b) Model Results .................................................. 28
INTRODUCTION

Water wave transformation, i.e., the change in wave height and direction caused by irregular bathymetry and/or structures, plays an important role in harbor planning, navigation safety, as well as shore protection and environmental regulations. There are many numerical models available to simulate wave transformation which can be described by the elliptic mild slope equation (Berkhoff 1972; 1976). These models can be divided into three categories: (1) Using a parabolic approximation to simulate wave refraction, weak diffraction, and shoaling for a relatively large study domain, or (2) Solving a hyperbolic, time dependent, mild slope equation, (3) solving the original elliptic, mild slope equation directly.

The first approach imposes some restrictions, but can be solved relatively fast (e.g., REFDIF-1 by Kirby and Dalrymple 1991). Under this category, numerous studies have been conducted during the past decades (e.g., Kirby 1986a; 1986b; 1988; Panchang et al. 1988; Dalrymple et al. 1989). The second category deals with a transient mild slope equation (Copeland 1985; Madsen and Larsen 1987) and looks for the results at steady state. The computing speed of this approach is approximately the same as to solve the mild slope equation directly (Li 1994b). The advantage is that this approach does not require a huge computer memory. The third category is usually limited to a small study domain,
e.g., harbors, because of the huge memory requirement for large domains.

Improvements on the mild slope equation by adding the effects of steep bottom slopes and bottom curvatures have been established recently (Massel 1993; 1995; Chamberlain and Porter 1995; Porter and Staziker 1995). For harbor planning, this extended mild slope equation can more accurately describe the possible drastic changes of water depth in harbors, and thus, it was selected for this study.

Numerical methods developed in the third category can use finite element method (e.g., Behrendt 1985; Chen and Houston, 1987), or finite difference method. Because of the nature of finite element technique, this kind of model is hard to maintain or upgrade. The available finite difference models all used an iteration method (Li and Anastasiou 1992, Li 1994a; 1994b) because of the low computer memory requirements. A major concern about the iteration method is the convergent rate degrades significantly for a complex boundary geometry. The alternative approach, uses the Gaussian elimination method (Dongarra 1979; Mathews 1987) to directly solve the huge banded matrix equation (hereafter called the direct method), was only possible on main-frame computers with immense core memory (on the order of gigabytes or more). For this reason, using the direct method on small computer for harbor planning purposes has never been
attempted.

Recently, however, a thrifty banded matrix solver (Maa et al. 1997) has been developed. This solver only requires a modest amount of core memory and a large hard disk (which is readily available even for 10 Gigabytes). Thus allowing us to use the direct method to solve the extended mild slope equation with a small computer.

To demonstrate this approach, we first present the governing equation and boundary conditions, followed by the formulation of the finite difference equations (FDE). Applying these FDEs to a study domain resulted in a huge banded matrix equation, which was solved using the thrifty banded matrix solver (Maa et al. 1997). Five cases, in which either analytical solutions or experimental results were available, were selected for model verification.

Only simple harmonic waves were used in this first stage study without considering energy loss caused by bottom friction. The more realistic multidirectional random waves as well as other improvements (e.g., bottom friction, identification of wave breaking, third order approximation for the total passing boundary condition, complex wave reflection coefficient to include phase difference) were left for the next stage.

The computer codes for the main programs, all subroutines, and other pre- and post-processing computer codes are all presented in the Appendices. The most-up-to-date source codes
are available in floppy disk, upon request.

GOVERNING EQUATION

The extended mild slope equation, Eq. 1, (Massel 1995) was selected for this study. Although Eq. 1 can be transformed to the Helmholtz equation and then solved numerically, it was decided to solve the original form to simplify future upgrades.

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{e_0}{b} \left( \frac{\partial h}{\partial x} \frac{\partial \phi}{\partial x} + \frac{\partial h}{\partial y} \frac{\partial \phi}{\partial y} \right) + k^2 (1 + \psi) \phi = 0
\]  

(1)

where

\[
\psi = e_1(kh) \left[ \left( \frac{\partial h}{\partial x} \right)^2 + \left( \frac{\partial h}{\partial y} \right)^2 \right] + \frac{e_2(kh)}{k_0} \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right)
\]  

(2)

\[
e_0 = \frac{kh}{\tanh kh + kh(1-\tanh^2 kh)} \* \left\{ 1 - 3\tanh^2 kh + \frac{2\tanh kh}{\tanh kh + kh(1-\tanh^2 kh)} \right\}
\]  

(3a)

\[
e_1 = \frac{1}{n \tanh kh} \times \frac{1}{24(2kh + \sinh2kh)^2 \cosh^3 kh} \cdot 
\]  

\[
\{ kh[12+16(kh)^2] \cosh kh + 6kh[\cosh3kh + \cosh5kh] + \\
[12+84(kh)^2] \sinh kh + 3[1-4(kh)^2] \sinh3kh - 9\sinh5kh \}
\]  

(3b)
\[ e_2 = \frac{1}{n} \left[ \frac{-4kh \cosh k h + \sinh k h + (kh)^2 \sinh k h + \sinh 3kh}{6(2kh + \sinh 2kh) \cosh^3 k h} - \right. \\
\left. \frac{kh \tanh k h}{2 \cosh^2 k h} \right] \]  

(3c)

\[ n = \frac{1}{2} \left[ 1 + \frac{2kh}{\sinh(2kh)} \right] \]  

(3d)

\( \phi \) is the velocity potential function for a simple harmonic wave flow, \( k_o = 4\pi^2/gT^2 \) is the deep water wave number, \( T \) is the wave period, \( g \) is the gravitational acceleration, \( k = 2\pi/L \) is the local wave number, \( L \) is the local wave length, \( h \) is the water depth, \( \partial h/\partial x \) and \( \partial h/\partial y \) are the bottom slopes in the x and y directions respectively, \( \partial^2 h/\partial x^2 \) and \( \partial^2 h/\partial y^2 \) are the bottom curvatures in the x and y direction, respectively, and \( x \) and \( y \) are the two horizontal coordinates, see Fig. 1.

Using a five-point-approximation, the finite difference equation for Eq. 1 can be written as follows.

\[ (1-d_x)\phi_{i-1,j} + r(r-d_y)\phi_{i,j-1} + \lambda \phi_{i,j} + r(r+d_y)\phi_{i,j+1} + (1+d_x)\phi_{i+1,j} = 0 \]  

(4)

\[ d_x = \frac{e_o}{2h} \frac{\partial h}{\partial x} \]  

(5a)

\[ d_y = \frac{e_o}{2h} \frac{\partial h}{\partial y} \]  

(5b)
where
\[
\lambda = k^2 (1 + \psi) \Delta x^2 - 2 \Delta y^2 - 2
\]  \tag{5c}

\( r = (\Delta x/\Delta y) \), and \( \Delta x \) and \( \Delta y \) are the grid sizes in the x and y directions respectively. Notice that \( \psi, k, \partial h/\partial x, \partial h/\partial y, \partial^2 h/\partial x^2, \) and \( \partial^2 h/\partial y^2 \) can be different at each grid point because of the possible different water depths.

Fig. 1. Coordinate System and Grid Alignment for the Model.
There are only two types of boundary conditions in the simulation of wave transformations: a partial reflection boundary condition and a given boundary condition. These conditions are specified along the border of a study domain, see Fig. 1.

**Partial Reflection Boundary Condition:** The condition described here (Eqs. 6 and 7) is actually a general condition that can be used for (1) total reflection, (2) partial reflection, or (3) total passing through (Behrendt 1985). This is because there is only one difference (in the selection of a constant coefficient, \( \alpha \), in Eqs. 6 and 7) among these three second-order approximations of the boundary conditions.

\[
\frac{\partial \phi}{\partial x} = \pm i \alpha k (\phi + \frac{1}{2 k^2} \frac{\partial^2 \phi}{\partial y^2}), \quad \text{on } \pm x \text{ boundary} \tag{6}
\]

\[
\frac{\partial \phi}{\partial y} = \pm i \alpha k (\phi + \frac{1}{2 k^2} \frac{\partial^2 \phi}{\partial x^2}), \quad \text{on } \pm y \text{ boundary} \tag{7}
\]

here \( i = (-1)^{1/2} \). Equation 6 is applicable to the boundary segments that are perpendicular to the x-axis, where the positive sign is for those segments that have the water grid point on their left side. Equation 7 is applicable to boundary segments that are
perpendicular to the y-axis, where the positive sign is for those segments that have the water grid point on the bottom. When \( \alpha = 0 \), Eqs. 6 and 7 represent total reflection boundary conditions. When \( \alpha = 1 \), these two equations represent total passing through boundary conditions. For \( 0 < \alpha < 1 \), they represent partial reflection boundary conditions. Notice that Eqs. 6 and 7 are only second-order approximations, and thus, reflection waves will be introduced when the approaching waves deviate more than 30 degrees (i.e., \( \beta > 30° \), see Fig. 1) off the normal line of the boundaries, even when \( \alpha = 1 \). For improvements on this issue, Kirby's third-order approximation may be used (Kirby 1989). Unfortunately, even that approximation has its limitations, and further studies are needed.

In order to use the second-order approximation of the boundary conditions at the boundaries, we used an imaginary grid point that is just one grid outside of the study domain. Here the boundary condition for a positive x boundary is used as an example to demonstrate this issue. Considering that the above boundary condition is specified at the boundary grid, I, in the x direction and the water grid point is located on the left side of this point, the finite difference equation for this boundary condition is

\[
\Phi_{I+1,j} - \Phi_{I-1,j} = \beta_x \Phi_{I,j} + \xi_x (\Phi_{I,j+1} - 2\Phi_{I,j} + \Phi_{I,j-1}) \quad (8)
\]
where $\beta_x = 2i\alpha k\Delta x$ and $\xi_x = i\alpha x^2/(k\Delta x)$. Notice that $\phi_{i+1,j}$ is outside the study domain. Because the governing equation at this boundary grid is

$$
(1-d_x)\phi_{i-1,j} + r(r-d_y)\phi_{i,j-1} + \lambda\phi_{i,j} + r(r+d_y)\phi_{i,j+1} + (1+d_x)\phi_{i+1,j} = 0 \tag{9}
$$

these two equations were used to eliminate $\phi_{i+1,j}$ at the imaginary point and the resulting equation for this kind of boundary is as follows.

$$
2\phi_{i-1,j} + [r(r-d_y) + \xi_x(1+d_x)]\phi_{i,j-1} + [\lambda + (\beta_x - 2\xi_x)(1+d_x)]\phi_{i,j} + [r(r+d_y) + \xi_x(1+d_x)]\phi_{i,j+1} = 0 \tag{10}
$$

Similarly, using Eq. 4 with either Eq. 6 or 7, the finite difference equations at the negative x, positive y, and negative y boundary points can be obtained (see Appendix I). For a corner grid point, three equations (i.e., Eqs. 4, 6, and 7) are used to obtain the finite difference equation (in Appendix I) that is applicable at that corner point.

**Given Boundary Condition**: This kind of boundary condition is used at those grid points where input wave information is specified. Because of the possible scatter waves generated from the study domain, the actual velocity potential function is still unknown at these grid points. In other words, there are two velocity potential values at a given boundary grid point, and the outgoing
scatter waves should pass through this boundary and remain unaffected. For this reason, a total passing through boundary condition, eqs. 6 and 7 with \( \alpha = 1 \), is used together with the given wave velocity potential, \( \phi^0 \), as follows (Behrendt 1985).

As an example, we are using the positive x given boundary equation to find the finite difference equation.

Considering that a given boundary is specified at the boundary grid, \( I \), and the water grid point is located on the left of this point, the finite difference equation for this boundary condition, i.e., Eq. 11a with the positive sign, is

\[
\frac{\partial \phi}{\partial x} = \pm ik(\phi + \frac{1}{2k^2} \frac{\partial^2 \phi}{\partial y^2}) + 2ik\phi^g, \quad \text{on} \; \pm x \; \text{boundary} \quad (11a)
\]

\[
\frac{\partial \phi}{\partial y} = \pm ik(\phi + \frac{1}{2k^2} \frac{\partial^2 \phi}{\partial x^2}) + 2ik\phi^g, \quad \text{on} \; \pm y \; \text{boundary} \quad (11b)
\]

\[
\phi_{i+1,j} - \phi_{i-1,j} = \beta^1_x \phi_{i,j} + 4ik\Delta x(\phi^0)_{i,j} + \xi^1_x (\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}) \quad (12)
\]

where \( \beta^1_x = 2ik\Delta x \) and \( \xi^1_x = (ir^2)/(k\Delta x) \). Notice that \( \phi_{i+1,j} \) is outside the study domain. Because the governing equation at this boundary grid is

\[
(1-d_x)\phi_{i-1,j} + r(x-d_y)\phi_{i,j-1} + \lambda\phi_{i,j} + r(x+d_y)\phi_{i,j+1} + (1+d_x)\phi_{i+1,j} = 0 \quad (13)
\]

these two equations are used to eliminate \( \phi_{i+1,j} \) at the imaginary
point and the resulting equation for this kind of boundary is given as follows.

\[
2\phi_{i-1,j} + [r(x-d_y) + \xi^1_x(1+d_x)]\phi_{i,j-1} + [\lambda + (\beta^1_x - 2\xi^1_x)(1+d_x)]\phi_{i,j} + [r(x+d_y) + \xi^1_x(1+d_x)]\phi_{i,j+1} = -4ik\Delta x(1+d_x)(\phi^g)_{i,j}
\]

(14)

Similarly, one can obtain the finite difference equation (see Appendix I) at the negative \(x\), positive \(y\), and negative \(y\) given boundary points.

For a given monochromatic wave with wave height, \(H\), period, \(T\), and direction, \(\theta\) (reference to the boundary), the given wave velocity potential can be calculated as (Behrendt 1985)

\[
\phi^g = Ae^{is} = \frac{igTH}{4\pi} e^{is}
\]

(15)

where \(A\) is the amplitude function and \(S\) is the phase function.

For normally incident waves, the phase function should be the same at all entrance grid points. For convenience and without loss of generality, we may choose \(S = 0\) for this condition. For oblique incident waves, see Fig. 2, the phase function can be calculated as follows.

\[
S(x_L) = \frac{2\pi x_L \sin \theta}{L}, \quad 0 \leq S(x_L) \leq 2\pi
\]

(16)
where \( x_L \) is the local one-dimensional coordinate, \( L \) is the wave length at the boundary location, and \( \theta \) is the angle between the wave direction and the normal vector of the boundary.

**Fig. 2. Calculation of Wave Phase at Given Boundary Grid Points.**

**NUMERICAL MODEL**

Equation 4 and these boundary finite difference equations were applied to all the water grid points in the study domain (Fig. 1), which has MP and NP grids in the x and y directions, respectively. A banded matrix equation can be established as follows

\[
B X = G
\]

(17)

where \( B \) is a banded matrix with a dimension of \( M \times N \), \( N \) is the
length of this banded matrix (same as the number of water grid points), \( M \) is the band width of this matrix, \( \mathbf{x} \) is the unknown column matrix for the wave potential function, and \( \mathbf{g} \) is another column matrix that includes the given boundary conditions. In general, \( M \) varies with the grid alignment as well as the geometry of the study domain. The computer codes were written in such a manner that when the x axis is selected parallel with the longer dimension of the study domain, the band width will be a minimum.

As Behrendt (1985) pointed out in his finite element model, the length of each side of an element should not exceed 1/10 of the wave length in order to solve the elliptic equation. A similar requirement holds for this finite difference model, i.e., \( \Delta x \) and \( \Delta y \) should both less than 1/10 of the wave length. Although \( \Delta x \) and \( \Delta y \) are not required to be the same in this model, the less-than-1/10-wave-length requirement practically limits the choice of \( \Delta x \) and \( \Delta y \). This is because the maximum \( \Delta x \) and \( \Delta y \) is usually desired in practical applications.

The banded matrix equation was solved by using a thrifty banded matrix solver (Maa et al. 1997). The traditional banded matrix equation solver usually requires large computer memory space because it stores the entire banded matrix in memory. For practical applications, \( N \) is usually on the order of \( 10^4 \) - \( 10^5 \) and \( M \) is on the order of \( 10^2 \) - \( 10^3 \). Thus, storing the matrix (using 16 byte complex numbers) alone may require 16 MB to 1.6 GB of
memory.

In the thrifty banded matrix solver, the sparse band matrix is stored in two much-smaller matrices (one is a complex matrix, the other is an integer matrix, each with a size of 5 x N). These two small matrices require much less computer memory, e.g., only 1.2 to 12 MB is required for the previously mentioned banded matrix. Furthermore, the solver constructs the banded matrix equation one block at a time, and then, follows the standard Gaussian elimination method with partial pivoting for forward elimination. It saves the results to the hard disk, and then fetches the information from the two smaller matrices to continue the processes (i.e., constructing a block of the banded matrix equation and performing forward elimination). This procedure continues until the entire banded matrix equation is processed. The back substitution begins by reading the last saved data, also one block at a time, to solve the velocity potential. The back substitution also repeats until all the saved blocks are read and processed. Unlike the virtual memory, this process uses minimal disk I/O, leaving computing time for number crunching. For this reason, the efficiency is high. The computing times for the five cases presented next will demonstrate this point.

MODEL VERIFICATIONS

Five cases, in which either the analytical solution or
experimental results were available, were selected for model verification.

Case 1. Infinite Long Breakwater with One Gap

Blue and Johnson (1949) provided experimental results for this case study, later generalized by Johnson (1953) and then documented in the Shore Protection Manual. In this case, there was an infinitely long straight breakwater with a gap of 2 wave lengths. Waves normally approached this breakwater.

The given wave condition was specified along the gap with the same phase. Along the breakwater, a total reflection boundary condition (Eqs. 6 and 7 with $\alpha = 0$) was specified. On the other three sides, a total passing through boundary condition was assigned ($\alpha = 1$). The grid size, wave parameters, and other parameters used in the model are given in Table 1.

For comparison, the calculated wave heights were normalized by the incident wave height, and then plotted on the left hand side of Fig. 3. The results documented in the Shore Protection Manual were duplicated on the right hand side of Fig. 3. It is obvious that there is an excellent agreement between these two, except at the gap. More discussion of this difference is given in the Discussion section.

The calculated contours of a specified wave phase value (e.g., $S = 3$), which may be used to represent the wave crest lines, were also plotted in Fig. 3 to compare with those documented in the Shore Protection Manual. Excellent agreement
between these two are also obvious.

To insure that the computer codes were correct, we assigned the breakwater and the gap located on the left hand side, right hand side, top side, as well as bottom side of the study domain. The results were identical.

Fig. 3. Comparison of Wave Height Contours and Wave Crests for an Infinite-Long Breakwater with a Gap. The left half is a copy from the Shore Protection Manual, the right half is the model results.
Table 1, Parameters used and Results in the Model Verifications

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>0.05</td>
<td>0.041</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>T (s)</td>
<td>0.8</td>
<td>0.9</td>
<td>varies</td>
<td>varies</td>
<td>0.73</td>
</tr>
<tr>
<td>θ (deg)</td>
<td>0</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>h (m)</td>
<td>0.5</td>
<td>0.4</td>
<td>0.2573</td>
<td>0.1563</td>
<td>0.25-0.102</td>
</tr>
<tr>
<td>Δx (m)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.003212</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>Δy (m)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.003025</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>W x L (m)</td>
<td>10x8</td>
<td>22x9.6</td>
<td>0.4497 x 10x1</td>
<td>2.2x1.2</td>
<td>0.1845</td>
</tr>
<tr>
<td>MP x NP</td>
<td>101x81</td>
<td>221x97</td>
<td>141x62</td>
<td>201x21</td>
<td>111x61</td>
</tr>
<tr>
<td>M</td>
<td>163 (203)</td>
<td>195</td>
<td>125</td>
<td>43</td>
<td>123</td>
</tr>
<tr>
<td>N</td>
<td>8181</td>
<td>21437</td>
<td>4601</td>
<td>4221</td>
<td>6769</td>
</tr>
<tr>
<td>Computing time (s)</td>
<td>42 (62)</td>
<td>180</td>
<td>41</td>
<td>32</td>
<td>24</td>
</tr>
</tbody>
</table>

The computing time is based on a Pentium Pro Personal computer with 200 Mhz speed, 64 MB of memory, and Windows NT operation system. The memory requirement of this model to run the above five cases is about 7 MB. For Case 1, the different computing times represent the results of a different band width caused by selecting x axis parallel to the long (or short) side of the study domain.

W, L are the width and length of the study domain. MP, NP are the grid number in x and y direction, respectively, see Fig. 1.
Case 2, A Finite-length Single Breakwater

Goda et al. (1971) presented their analytical solution for this case study: a two-wave-length long straight breakwater was located in a constant water depth environment. The incident waves approached the breakwater with an angle of 60 degrees; and the analytical solution of normalized wave height distribution is plotted in Fig. 4a. For numerical modeling, the given wave condition was specified along the left border, except at the portion for the breakwater, where a total reflection boundary condition \((\alpha = 0)\) was used. On the other three sides, the total passing through boundary condition \((\alpha = 1)\) was used. All parameters used to simulate this case are also given in Table 1. The model results (contours of the normalized wave height) are plotted in Fig. 4b. Reasonably good agreement can be found at the near field after the breakwater. For the far field, however, the model results are about 20% higher.

It is necessary to point out that the computing domain in the x direction was much larger than that displayed in Fig. 4b. The computing domain in the y direction was also slightly larger than that displayed in Fig. 4b (see Table 1 for the size). The selection of this computing domain was because the two possible boundary conditions on the open boundaries \((\alpha = 0 \text{ for the total reflection boundary condition, or } \alpha = 1 \text{ for the total passing through boundary condition})\) both required a much large simulation.
domain to minimize the influence of reflected waves. Although $\alpha = 1$ was specified along the three open boundaries, reflected waves were introduced because of the second order approximation of the passing through boundary condition and the large incident waves angle (30 degrees off from the normal vectors of the boundaries). Therefore, the unwanted reflected waves propagate into the computing domain and cause error at the far field. It is not clear at this time, however, how much error is contributed from the grid size selection inherent from the finite difference method.

To improve the passing through open boundary condition, a third order approximate (Kirby 1989) is necessary, and that will be the goal for our next stage modifications.
Fig. 4. Comparison of Wave Height Contours for a Finite-Length Breakwater with Oblique Incident Waves. (a) Analytical solution from Goda et al. (1971); (b) Model results.
Case 3. Resonance in a Rectangular Harbor

Many studies of this case have been conducted. In addition to the analytical solution given by Unluata and Mei (1973), there have been many physical model studies (e.g., Ippen and Goda 1963; Lee 1971) as well as numerical studies (e.g., Chen and Mei 1974; Lee 1971; Behrendt 1985). The geometry of this harbor is given in Fig. 5 with harbor length \( E = 0.3212 \) m, harbor width \( B = 0.0605 \) m, and a constant water depth of \( 0.2573 \) m.

In the current model simulation, the entire computation domain is given in Fig. 5. On the left hand side of this domain, the given boundary condition was specified. Along the harbor perimeter, the total reflection boundary condition \( (\alpha = 0) \) was specified. At the top and bottom open boundaries, the total passing through boundary condition was assigned \( (\alpha = 1) \). If this harbor does not exist, the wave height would be doubled at the right side border \( (x = 0.1285 \) m, see Fig. 5) for any given wave condition because of the total reflection boundary condition specified there. This doubled wave height \( (2H_i) \) was used to normalize the calculated wave heights at the harbor end \( (x=0.4497 \) m) shown in Fig. 6. For the harbor geometry given in Fig. 5, Unluata and Mei found that the wave height at the harbor end became very large \( (H/2H_g \approx 8, \) see Fig. 6) when the ratio of harbor length to wave length approached 1/4 \( (2\pi E/L = 1.324) \). This is the first resonance; the second resonance occurred at
Fig. 5. The Typical Rectangular Harbor. The Displayed Normalized Wave Height Contours is the Response near Harbor Resonance.

Fig. 6. Comparison with Analytical Solution (Solid line) and Model-Calculated Normalized Wave Height at the Harbor End. The dashed line is the model results with the open ocean boundary domain shown in Fig. 5 (12.85 x 18.45 cm). The dashed-dotted line is the model results when the open Ocean boundary domain increased to 19.27 x 30.15 cm.
Because an approximation (i.e., the rectangular open boundary) of the exact geometry for the open boundary was used, the model results (dashed line in Fig. 6) cannot be as accurate as the analytical solutions (solid line in Fig. 6). When the size of the open boundary domain (on the left hand side of Fig. 5) increased from 12.85 cm x 18.45 cm to 19.27 cm x 30.15 cm, the performance of this model also changed; see the dash-dotted line in Fig. 6. As pointed out by Chen (personnel communication 1997), a semi-circle open boundary is needed for an accurate simulation of the resonance. The calculated contours of the normalized wave height at a near resonance frequency (see Fig. 5) further demonstrated that the scatter waves propagated outward from the harbor entrance and also showed why a semi-circular open boundary is needed.

Although the performance of this model is not perfect at this time, it does indicate the resonance near the two resonant frequencies (Fig. 6). As the results of the first stage study, we will leave the model as is. We may find a better way to implement the semi-circle open boundary requirement in the next stage study.
Case 4, Influence of bottom slope and Curvature

Davies and Heathershaw (1984) presented the results of a physical model study and analytical solution for evaluating the influence of bottom curvature and steep slope on wave reflection and transmission. In their physical model study, monochromic waves were generated at one end of a one-dimensional wave flume. At the middle of this flume, there was a selected number (1, 2, 4, and 10) of sinusoidal bedform. The average water depth within the sinusoidal bed area was the same as the water depth before and after the sinusoidal bedform. The sinusoidal bedform had an amplitude, $b$, of 0.05 m and length, $L_s$, of 1.0 m. When waves encountered this kind of bed, some of the wave energy reflected back and some passed through (e.g., see Fig. 7). They measured the reflection coefficients, $k_r$ (see Fig. 8 for four sinusoidal bedform), in terms of wave period, water depth, and number of sinusoidal bed forms.

Although this is a one-dimensional case, we can use the two-dimensional model to simulate the wave transformation along this flume. The positive $x$ direction was selected as the wave propagation direction to reduce the band width of the matrix $B$. In the $y$ direction, an arbitrary number of grid points (21 grid points) was selected to represent the channel width. At $x = 0$, the given wave boundary condition was specified. At the flume end, a total passing through boundary condition was specified.
On both the two Y-directional borders, a total reflection boundary condition was used. All other parameters used in the model are given in Table 1.

The model results clearly demonstrated the progress and reflective waves before the bed forms as well as the transmitted waves after the bed forms (Fig. 7). Because of the small $k_r$ (0.083) in this particular case, the normalized transmitted wave height was also close to 1. Wave heights were larger than the incident waves at many places where the bed form were located.

The model also performed very well in identifying the maximum wave reflection; see Fig. 8. This demonstrates a satisfactory performance for this model when encountering irregular bathymetry.
Fig. 7. Calculated Wave Height Profile along a Wave Flume with Four Sinusoidal Bed forms, which are plotted on the bottom.

Fig. 8. Comparison of Calculated and Measured Wave Reflection Coefficient for the Four Sinusoidal Bed forms.
Case 5, A Simple but Practical Harbor

Sato et al. (1990) presented the results (ratio of local and input wave heights) of a physical model study on a small harbor (Fig. 9a). This case was selected because of the small study domain, complicated boundary conditions, and realistic harbor geometry.

The incident wave period was 0.73 s with the wave direction given in Fig. 9a. The given incident wave height was not given but that was immaterial because the measurements were normalized.

Crashed stones were placed on the seaward side of the breakwater in order to dissipate wave energy. For this reason, $\alpha$ was selected as 0.8 to represent a nearly total passing through boundary condition on the seaward face of the breakwater. Total passing through boundary conditions ($\alpha = 1$) were specified for the open sea boundary in both $y$ directional borders. Within the harbor, the vertical wall was capable of reflecting most of the wave energy, and thus, a small $\alpha$ should be selected. It was not documented, however, what the $\alpha$ should be because no experimental data was available. The model results with $\alpha = 0.03$ (Fig. 9b) were quite close to the measurements. When $\alpha = 0$, however, the resulting wave heights at the node points were about 2 times larger than those when $\alpha = 0.03$. Here the node points referred to those locations where the normalized wave heights were large. When using $\alpha = 0.1$, on the other hand, the resulting wave heights were too small to compare with the measurements.
Fig. 9. Comparison of Wave Height Contours in a Simple Harbor (a) Laboratory measurements by Sato et al. (1990); (b) Model Results.
It may be argued that the selection of $\alpha = 0$ with wave energy loss caused by bottom friction may also be able to give results similar to those given in Fig. 9a. This is possible because of the small $\alpha$ used in this case. This argument will be examined in our next stage study when the energy loss caused by bottom friction will be implemented.

DISCUSSION AND CONCLUSIONS

Requiring the grid size to be less than 1/10 of the wave length is the major disadvantage of solving the elliptic equation. Because of the small grid size, the band matrix, $B$, can easily become very large, and thus, require more computing time. For this reason, this method is not recommended when there is no reflection and only weak diffraction. A model using the parabolic approximation should be used instead. The inability to simulate wave reflection and strong wave diffraction, however, prohibits the use of that approach for harbor planning purposes.

The requirement of a small grid size (i.e., less than 1/10 of the wave length) improves the feasibility of using the finite difference method to simulate a complicated harbor geometry. The advantage in using the finite element method, on the other hand, diminishes as the grid size decreases.

One advantage of using the direct method to solve the elliptic equation is that there is no convergence concern even
for a very complicated geometry. In general, convergence is one of the major concerns for irregular geometries using iteration methods.

The most important advantage of using the direct method (i.e., forming a banded matrix equation and solving it using the Gaussian elimination method) to simulate wave transformation processes is depicted next. For a given harbor geometry, different wave conditions only change the column matrix $G$, and does not affect the banded matrix $B$ in eq. 17. This is important because the computing times for the forward elimination and back substitution are proportional to $N^3$ and $N^2$, respectively (Mathews 1987), where $N$ is the length of the banded matrix. Since the given wave conditions do not affect $B$, only a one-time computation of the forward elimination is needed. This can save significant computing time if many wave conditions are involved (usually it is true for harbor planning) for a harbor geometry. To examine the more realistic spectrum waves, this advantage is also important because it can improve the computing efficiency significantly. Improving the computer codes for this purpose, however, is rather straightforward, and will be given in the next stage study.

In conclusion, by using the finite difference method and a simple book-keeping procedure to relax the huge memory requirements, a much simple numerical model for simulating wave
reflection, refraction, diffraction, shoaling, and harbor resonance for harbor planning purposes has been developed. This model can simulate these processes using small computers with reasonable speed. With a more powerful computer, the time required for modeling these processes will be much less.

The effects of bottom friction are not yet included in this model. It can easily be added later because it only slightly affects the construction of the banded matrix equation, while the procedures for obtaining the solution remain the same.

Wave breaking, another important phenomenon, is not examined in this study. This model, however, can give unrealistically large wave heights at the breaking line. This feature can be used in post-processing software to identify the breaking line. After breaking, unfortunately, this model will not be able to continue the simulation because the theory is no longer valid.

ACKNOWLEDGMENTS

Financial support from the U.S. Department of the Interior, Minerals Management Service, Office of International and Marine Minerals, under Cooperative Agreement No 14-35-001-30740 is sincerely acknowledged. Support from the National Science Council of the Republic of China, Contract Number NSC 86-2611-E-006-019 is also sincerely acknowledged. Mr. M.-T. Maa helps to prepare diagrams and proofread the manuscript.
REFERENCES


11, 329-351


Appendix I. Finite Difference Equations for Boundary Conditions.

The extended mild slope equation (Eq. 4) and the second-order approximation of the partial reflection boundary conditions (Eqs. 6 and 7, in finite difference form) are combined together to form the finite difference equation at these boundaries. Notice that the word "right," "left," "top," or "bottom" are used to indicate that the boundary grid point (with an upper case index either I, or J) is on the right, left, top, or bottom side of the water cells. The origin of the Cartesian coordinates is located at the lower-left corner. Equation 10 is repeated here as Eq. I-5 for convenience.

(1) Top side partial reflection boundary.
Considering that this boundary condition is specified at the boundary grid, J, in the y direction and the water grid point is located on the lower side of this point, the finite difference equation for Eq. 7, with a positive sign, is

\[ \phi_{i,j+1} - \phi_{i,j-1} = \beta_y \phi_{i,j} + \xi_y (\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}) \]  

(I-1)

where \( \beta_y = 2i\alpha k \Delta y \) and \( \xi_y = (i\alpha)/(kr^2 \Delta y) \). With Eq. 4 specified for grid point \((i,J)\), the resulting finite difference equation can be found as follows.

\[ [r(r+d_y)\xi_y+(1-d_x)]\phi_{i-1,j} + 2r^2\phi_{i,j-1} + [\lambda+(\beta_y-2\xi_y)r(r+d_y)]\phi_{i,j} \]

\[ + [r(r+d_y)\xi_y+(1+d_x)]\phi_{i+1,j}] = 0 \]  

(I-2)

(2) Bottom side partial reflection boundary.

\[ [r(r-d_y)\xi_y+(1-d_x)]\phi_{i-1,j} + [\lambda+(\beta_y-2\xi_y)r(r-d_y)]\phi_{i,j} \]

\[ + 2r^2\phi_{i,j+1} + [r(r-d_y)\xi_y+(1+d_x)]\phi_{i+1,j}] = 0 \]  

(I-3)

(3) Left hand side partial reflection boundary.

\[ [r(r-d_y)\xi_x(1-d_x)]\phi_{i,j-1} + [\lambda+(\beta_x-2\xi_x)(1-d_x)]\phi_{i,j} \]

\[ + [r(r+d_y)\xi_x(1+d_x)]\phi_{i,j+1} + 2\phi_{i+1,j}] = 0 \]  

(I-4)

1.1
(4) Right hand side partial reflection boundary.

\[ 2\Phi_{I-1,J} + [r(x-d_y) + \xi_x (1+d_x)]\Phi_{I,J-1} + [\lambda + (\beta_x - 2\xi_x) (1+d_x)]\Phi_{I,J} + [r(x+d_y) + \xi_x (1+d_x)]\Phi_{I,J+1} = 0 \] (I-5)

(5) Bottom-Left corner partial reflection boundary.

Considering that this boundary condition is specified at the boundary grid \((I,J)\), and the water grid point is located on the upper right side of this point, using Eqs. 6, 7, and I-3 to eliminate the two velocity potential \((\Phi_{I,J-1} \text{ and } \Phi_{I,J})\) and the resulting finite difference equation is

\[ (\lambda + (\beta_x - 2\xi_x) (1-d_x) + [r(x-d_y) + \xi_x (1-d_x)]\frac{\beta_y - (2+2\xi_x - \beta_x) \xi_y}{1-\xi_x \xi_y} \Phi_{I,J} + \]

\[ \left( [r(x+d_y) + \xi_x (1-d_x)] + [r(x-d_y) + \xi_x (1-d_x)] \frac{1+\xi_x \xi_y}{1-\xi_x \xi_y} \right) \Phi_{I,J+1} + \]

\[ 2\left(1 + \frac{\xi_y [\xi_x (1-d_x) + r(x-d_y)]}{1-\xi_x \xi_y}\right) \Phi_{I+1,J} = 0 \] (I-6)

(6) Top-left corner partial reflection boundary.

\[ (\xi_x (1-d_x) + r(x-d_y) + \frac{1+\xi_x \xi_y}{1-\xi_x \xi_y} [\xi_x (1-d_x) + r(x+d_y)]) \Phi_{I,J-1} \]

\[ (\lambda + (\beta_x - 2\xi_x) (1-d_x) + [r(x+d_y) + \xi_x (1-d_x)]\frac{\beta_y + \xi_y (\beta_x - 2-2\xi_x)}{1-\xi_x \xi_y} \Phi_{I,J} + \]

\[ 2\left(1 + \frac{\xi_y [\xi_x (1-d_x) + r(x+d_y)]}{1-\xi_x \xi_y}\right) \Phi_{I+1,J} = 0 \] (I-7)
(7) Top-right corner partial reflection boundary.

\[ 2 \left( 1 + \frac{\xi_y [r(r+d_y) + \xi_x (1+d_x)]}{1 - \xi_x \xi_y} \right) \phi_{i-1,j} + \]

\[ (r(r-d_y) + \xi_x (1+d_x)) + [r(r+d_y) + \xi_x (1+d_x)] \frac{1 + \xi_x \xi_y}{1 - \xi_x \xi_y} \phi_{i,j-1} + (\lambda + \frac{\beta_y + \xi_y (\beta_x - 2 - 2 \xi_x)}{1 - \xi_x \xi_y}) \phi_{i,j} = 0 \]  \hspace{1cm} (I-8)

(8) Bottom-right corner partial reflection boundary.

\[ 2 \left( 1 + \frac{\xi_y [r(r-d_y) + \xi_x (1+d_x)]}{1 - \xi_x \xi_y} \right) \phi_{i-1,j} + \]

\[ (\lambda + \beta_x - 2 \xi_x) (1+d_x) + [r(r-d_y) + \xi_x (1+d_x)] \frac{\beta_y + \xi_y (\beta_x - 2 - 2 \xi_x)}{1 - \xi_x \xi_y} \phi_{i,j} + \]

\[ (r(r+d_y) + \xi_x (1+d_x)) + [r(r-d_y) + \xi_x (1+d_x)] \frac{1 + \xi_x \xi_y}{1 - \xi_x \xi_y} \phi_{i,j+1} = 0 \]  \hspace{1cm} (I-9)

(9) Top side given boundary

\[ [r(r+d_y) \xi_y^1 + (1-d_x)] \phi_{i-1,j} + 2r^2 \phi_{i,j+1} + [\lambda + (\beta_y^1 - 2 \xi_y^1) r(r+d_y)] \phi_{i,j} + \]

\[ [r(r+d_y) \xi_y^1 + (1+d_x)] \phi_{i+1,j} = -4ik\Delta y r(r+d_y)(\phi^g)_{i,j} \]  \hspace{1cm} (I-10)

(10) Bottom side given boundary.

1.3
\[ [r(r-d_y) \xi_y^1 + (1-d_x)] \Phi_{i-1, j} + [\lambda + (\beta_y^1 - 2\xi_y^1) r(r-d_y)] \Phi_{i, j} + 2r^2 \Phi_{i, j+1} \]

\[ + [r(r-d_y) \xi_y^1 + (1+d_x)] \Phi_{i+1, j} = 4ik\Delta y r(r-d_y)(\Phi^g)_{i, j} \quad (I-11) \]

(11) Left hand side given boundary.

\[ [r(r-d_y) + \xi_x^1 (1-d_x)] \Phi_{i, j-1} + [\lambda + (\beta_x^1 - 2\xi_x^1) (1-d_x)] \Phi_{i, j} + \]

\[ + [r(r+d_y) + \xi_x^1 (1-d_x)] \Phi_{i, j+1} + 2\Phi_{i+1, j} = 4ik\Delta x (1-d_x)(\Phi^g)_{i, j} \quad (I-12) \]

(12) Right hand side given boundary.

This equation is the same as Eq. 14, which is repeated here for convenience.

\[ 2\Phi_{i-1, j} + [r(r-d_y) + \xi_x^1 (1+d_x)] \Phi_{i, j-1} + [\lambda + (\beta_x^1 - 2\xi_x^1) (1+d_x)] \Phi_{i, j} + \]

\[ + [r(r+d_y) + \xi_x^1 (1+d_x)] \Phi_{i, j+1} = -4ik\Delta x (1+d_x)(\Phi^g)_{i, j} \quad (I-13) \]
Appendix II. FORTRAN Source Codes for the main program RDE.FOR

Program RDE

This program solves the extended mild slope equation for simulating water wave refraction, diffraction, reflection, and shoaling.

It uses second order approximation on the governing and boundary conditions. This program uses a direct method to solve the huge Banded matrix equation, which was obtained by doing finite difference on the elliptic equation.

iq,jq : parameters specified for reserving space for 2-D arrays. They should be at least the same as mp and np.
mp,np : number of grid points in x and y direction, respectively. In applications, the x-axis should be parallel to the longer dimension of the water body. The y axis should parallel to the short dimension of the water body. This alignment will generate a smaller banded width.
dx,dy : grid sizes in x and y direction, respectively. In this program, only the equally spaced grid is allowed. But dx do not have to equal to dy.
dlevel: water elev. that added to the bathymetric depth for storm tide.
dmin : min. water depth at a point that this point will be considered as water.
kq : the larger dimension of the banded coefficient matrix. The maximum kq equals to mp*mp if all points in the x-y plane are water points. Practically, it should be much smaller (equal to the length of the total water cells in the study domain because of land cells.
nbc : number of grid points that are assigned as either a passing through, a total reflection, or a partial reflection boundaries.
ngc : number of grid points that are assigned to have a given boundary, e.g., wave height, period, and direction.
id : a 2-D array to store the identification for each grid point. Detail explanation is given in the program.

The following logical numbers are used to read or write files:
1 : CONFILE for Control parameters.
9 : OUTFILE for storage output. This file contains calculation results for all cases. So it may be very big.
11: CHKFILE for storage data for checking purposes. This file erase itself when completed for each case and reused to save disk space.
15: BCDFILE for input ID codes and boundary conditions (given boundary as well as radiation boundary.
16: DEPFILE for input water grid.

2.1
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502, *
iw=1000, jw=1700)
c
character*70 title
ccharacter*20 outfile, confile, chkfile, depfile, bcdfile
ccharacter*11 result
ccharacter*1 id
c
common /files/ depfile, bcdfile
common /waves/ period, waveH, angle0
common /bottm/ dep(iq,jq), id(iq,jq)
ccommon /matrix/ za(kq,5), ia(kq,5), zb(kq)
ccommon /gener/ mp, np, dx,dy, r, yorient, xl, yl, wk0
common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
ccommon /works/ zw(iw,jw), ipvt(kq)
ccommon /given/ nog, ig(ngc), jg(ngc)
ccommon /bound/ nobc, ibc(nbc), jbc(nbc)
ccommon /compx/ zp(ngc), zalp(nbc), phas(ngc)
dimension period_g(85), waveH_g(85), angle0_g(85), dlevel_g(85)
c
zi=(0.0, 1.0)
cgrav=9.8
cpi=3.1415926
c
5 continue
print*,'Key in the Control file name <XXXXX.CON> : '
read(*,'(a20)') confile
open(1, file=confile, form='formatted',status='old', err=10)
go to 20
10 print*,'File not found, or, type not matched '
go to 5
20 read(1,'(a70)') title
print*, title
call time(result)
c
name : a 16 characters string to store the path, name of file.
cLater, three file types (chk, lis, grd) will be appended
to open an input depth grid file and two output files.
cxorient: The angle of x-axis, counted clockwise from The North.
cxl, yl : The global x and y coordinates of the origin of the grid
c
mp,np : Max. grid numbers in x and y direction, respectively.
cdx,dy : grid sizes in x and y direction, respectively.
cncase : Number of cases (wave height, period, angle) in this
ccontrol file
cdlevel: water elevation that added to the bathymetric depth,
c  e.g. storm tide
cdmin : minimum water depth that will be considered as water
c
read(1,'(a)') depfile
c
2.2
the following statement is needed to remove the space on right hand, since MS PowerStation uses all the space declared

```fortran
read(1,'(a)') bcdfile
read(1,'(a)') outfile
read(1,'(a)') chkfile

read(1,*) yorient, xl, yl
read(1,*) mp, np, dx, dy, ncase, dmin
if(mp .gt. iq) then
  print*, 'IQ is smaller than MP, change IQ to ', mp
  stop
end if
if(np .gt. jq) then
  print*, 'JQ is smaller than NP, change JQ to ', np
  stop
end if

r = dx/dy

wave angle: counted clockwise, from North. For example,
if waves come from NE, then the angle will be 45+180=225 degrees.
For waves coming from S, the angle will be 0 degrees.
For waves coming from E, teh angle will be 90 degrees.
The angle given is always reference to North. The angle that

print*, 'The following waveH, period, and angle are included'
do ic=1,ncase
  read(1,*) waveH_g(ic), period_g(ic), angle0_g(ic), dlevel_g(ic)
  write(*,52) ic, waveH_g(ic), period_g(ic), angle0_g(ic),
              dlevel_g(ic)
52 format(' i, waveH(m), T(s), Ang(d),Surge(m)='',i4,2f8.2,2f7.1)
end do

print*, 'Output data goes to ', outfile
open(9, file=outfile, form='formatted', status='unknown')

print*, 'Water depth grid input from ', depfile
call depth_in

print*, 'Boundary condition input from ', bcdfile
call bound_in

do 200 ic=1,ncase

print*, 'Check data goes to ', chkfile
open(11, file=chkfile, form='formatted', status='unknown')

print*, 'case ', ic
period=period_g(ic)
waveH=waveH_g(ic)
```

2.3
angle0 = angle0_g(ic)
dlevel = dlevel_g(ic)

c change water depth, not now, wait until operational

do i=1,mp
    do j=1,np
        dep(i,j) = dep(i,j) + dlevel
    end do
end do

wk0 : wave number at deep water

wk0 = 4.0*pi*pi/(grav*period*period)
write(11, '(a70)') title
write(11, 55) waveH, period, angle0, dlevel
55 format(' waveH (m)=', f8.2, ' T (s)=', f8.2/
* Ang (deg)=', f8.1, ' Surge (m)=', f8.1/
* Given boundary conditions are specified at'/
* i j zph ')

c calculate the given velocity potential based on wave height,
period, direction, and grid locations

zpot = -zi*waveH*grav*period/(4.0*pi)
do k=1,nog
    zp(k) = zpot*exp(zi*phas(k) )
write(11, 60) ig(k), jg(k), zp(k)
60 format(2i7, 2f12.8)
end do

c construct the unknown COLUMN matrix ZX, in the eq. ZA*ZX=ZB
Get each unknown’s location: imap(map), jmap(map)

c map : the number of total unknown, or the length of X. later,
it is reassigned as N

c map=0
do i=1,mp
    do j=1,np
        if(id(i,j) .ne. 'e' ) then
            map = map + 1
            imap(map) = i
            jmap(map) = j
        end if
    end do
end do
if(map .gt. kg) then
    print*, 'Increase KQ to ', map
    stop
end if
if(n .lt. 0.9*kq) then  
  print*  
  print*, '-----------------------------------------------'  
  print*, 'Its better to reduce the length of the Matrix,'  
  print*, 'KQ, & increase the length of matrix ZW: IW,'  
  print*, 'in order to reduce disk I/O'  
  print*,'-----------------------------------------------'  
end if  
print*,'Length of the Banded Matrix is',n  
print*,'The parameter KQ has been declared as',kq

n: length of the banded matrix

n=map

Set up the storage matrices ZA & IA and find out the band width

mu=0
mL=0
do 80 map=1,n  
i=imap(map)  
j=jmap(map)

if(id(i,j) .eq. '0') then  
call domain(i,j,map)  
go to 70  
end if

if(id(i,j) .eq. 'g') then  
call given_bc(i,j,map)  
go to 70  
end if

if(id(i,j) .eq. '1') then  
call bound_t(i,j,map)  
go to 70  
end if
if(id(i,j) .eq. '2') then  
call bound_b(i,j,map)  
go to 70  
end if
if(id(i,j) .eq. '3') then  
call bound_l(i,j,map)  
go to 70  
end if
if(id(i,j) .eq. '4') then  
call bound_r(i,j,map)  
go to 70  
end if
if(id(i,j).eq.'5' .or. id(i,j).eq.'o' .or. id(i,j).eq.'p') then  
call corner_lb(i,j,map)  
go to 70
if(id(i,j).eq.'6' .or. id(i,j).eq.'q' .or. id(i,j).eq.'r') then
  call corner_lt(i,j,map)
go to 70
end if
if(id(i,j).eq.'7' .or. id(i,j).eq.'u' .or. id(i,j).eq.'v') then
  call corner_rb(i,j,map)
go to 70
end if
if(id(i,j).eq.'8' .or. id(i,j).eq.'s' .or. id(i,j).eq.'t') then
  call corner_rt(i,j,map)
go to 70
end if

70 if(id(i,j).ne.'4' .and. id(i,j) .ne. '7' .and. id(i,j).ne.'8'
  * .and. id(i,j) .ne. 'c' .and. id(i,j) .ne. 'd' ) then
  mu_c=ia(map,5) - ia(map,3)
else
  mu_c=0
end if
if(mu_c .gt. mu) mu=mu_c
if(id(i,j) .ne. '3' .and. id(i,j) .ne. '5'.and.id(i,j).ne. '6'
  * .and. id(i,j) .ne. 'a' .and. id(i,j) .ne. 'b' ) then
  mL_c=ia(map,3) - ia(map,1)
else
  mL_c=0
end if
if(mL_c .gt. mL) mL=mL_c
write(11,75) i,j, ia(map,1),ia(map,2),ia(map,3),
  * ia(map,4),ia(map,5), za(map,1), za(map,2),
  * za(map,3), za(map,4),za(map,5), zb(map)
75 format(1x,2i5,3x,5i6,2x,2f7.3, 2x,2f7.3, 2x,2f7.3,
  * 2x,2f7.3, 2x,2f7.3, 2x,2f7.3 )
if(ia(map,1) .lt. 0 .or. ia(map,2) .lt. 0 .or. ia(map,3) .lt. 0
  * .or. ia(map,4) .lt. 0 .or. ia(map,5) .lt. 0) then
  write(*),(78) map,i,j, id(i,j), (ia(map,kk), kk=1,5)
78 format( ' Map, i,j,id=',i10,2i5,a5/ ' ia(map,k)=' ,5i8/
  * ' All 5 IAs should be non-negative, Check !')
stop
end if
continue

close(11)
m=mL+mu+1
Lda=m+ mL
print*,'The band width & length = ', m, n
print*,'Upper band width=','mu
print*, 'Lower band width=','mL
print*, 'Lda for BMSOLVER=','Lda
if(Lda .gt. iw) then
  write(*,90) Lda
2.6
format('Please change parameter IW to >=',i6,' and re-run')
stop
end if

c uses the complex band matrix solver to solve ZA * ZX = ZB
c
call bmsolver(ier_code)
c
output results
c
if( ier_code .eq. 0) then
c
write(9,'(a60)') title
write(9,100) mp,np, dx,dy
100  format(2i5,2f10.6,2f10.3)
write(9,110) yorient,xl,yl
110  format(f10.2,2f10.5)
write(9,120) waveh,period,angle0, dlevel, n
120  format(4f10.3,i10/' i j id dep real imagin')
c
do k=1,n
i=imap(k)
j=jmap(k)
write(9,140) i, j, id(i,j), dep(i,j), zb(k)
140  format(2x,i5,1x,i5,a3,f10.4,2d18.10)
end do
c
end if
200 continue
close(9)
c
print*,result
call time(result)
print*,result
c
stop
c
end
c

subroutine depth_in
c
This sub. reads ASCII depth grids, ID codes
c
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401, nbc=1500, ngc=502)
c
character*70 dtitle,label
character*20 depfile, bcdfile
character*1 id
c
common /files/ depfile, bcdfile
common /bottm/ dep(iq,jq), id(iq,jq)
common /gener/ mp, np, dx,dy, r, yorient, x1, y1, wk0

open(16, file=depfile, form='formatted', status='old')
read(16,25) dtitle
print*, dtitle
read(16,25) label
print*, label
read(16,*) m_dep, n_dep, dx_d,dy_d, x1,y1,czmin,czmax,noz,yorii
write(*,5) czmin, czmax, noz
5 format(' The min., max., and # of raw survey are ',2f8.3,i8)
if(m_dep .eq. mp .and. n_dep .eq. np .and. dx_d .eq. dx .and.
  dy_d .eq. dy .and. yorii .eq. yorient) go to 20
write(*,10) m_dep, n_dep, dx_d, dy_d, yorii,
  mp, np, dx, dy, yorient
10 format(' The following two rows should be the same '/
  5x,2i7, 2f10.3, f10.1/5x, 2i7, 2f10.3, f10.1/
  ' Found when reading depth grid file.'/' Usually this',
  ' means the data file assignment is wrong.')</nstop
20 continue
read(16,25) label
write(*,25) label
25 format(a70)
read(16,25) label
do i=1,mp
read(16,*) ins
if(ins .eq. i) then
  read(16,*) (dep(i,j),j=1,np)
else
  write(*,30) i,ins
30 format(' Sequence wrong for depth input at',2i5)
stop
end if
end do
print*, 'Completed reading water depth matrix, leave depth_in'
close(16)
c return
cend

-----------------------------------------------------

subroutine bound_in

This sub. reads ASCII given and radiation boundary conditions

implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401, nbc=1500, ngc=502)

character*70 dtitle,label
character*20 bcdfile, depfile

2.8
character*1 id

common /files/ depfile, bcdfile
common /bottm/ dep(iq,jq), id(iq,jq)
common /gener/ mp, np, dx, dy, r, yorient, xl, yl, wk0
common /bound/ nobc, ibc(nbc), jbc(nbc)
common /given/ nog, ig(ngc), jg(ngc)
common /compx/ zp(ngc), zalp(nbc), phas(ngc)

open(15, file=bcdfi le, form='formatted', status='old')

read(15,2) dtitle
     2 format(a70)
     print*, dtitle
     print*, label
     read(15,2) label
     read(15,*) m_bcd, n_bcd, dx_b, dy_b, xl, yl, czmin, czmax, noz, yorii
     write(*,5) czmin, czmax, noz
     5 format(' The min., max., and # of raw survey are ',2f8.3,i8)

if(m_bcd .eq. mp .and. n_bcd .eq. np .and. dx_b .eq. dx .and. dy_b .eq. dy .and. yorii .eq. yorient) go to 20
write(*,10) m_bcd, n_bcd, dx_b, dy_b, yorii,
          *   mp, np, dx, dy, yorient
     10 format(' The following two rows should be the same '/
      *      5x,2i7, 2f10.3, f10.1/5x, 2i7, 2f10.3, f10.1/
      *      ' Found when reading depth grid file.'/
          *      ' Usually this',
      *      ' means the data file assignment is wrong.'
     stop
     20 continue

ID code for each grid point

: 0, water interior point
: 1, upper boundary condition
: 2, lower boundary condition
: 3, left boundary condition
: 4, right boundary condition
: 5, left bottom corner B.C.
: 6, left top corner B.C.
: 7, right bottom corner B.C.
: 8, right top corner B.C.
: g, grid point that have a given wave condition
: e, land point
: o, same as 5, also a given B.C. for wave coming from bottom
: p, same as 5, also a given B.C. for wave coming from left
: q, same as 6, also a given B.C. for wave coming from left
: r, same as 6, also a given B.C. for wave coming from top
: s, same as 7, also a given B.C. for wave coming from top
: t, same as 7, also a given B.C. for wave coming from right
: u, same as 8, also a given B.C. for wave coming from right
: v, same as 8, also a given B.C. for wave coming from bottom
The following is an example

\[ j (y) \]

\[
\begin{align*}
&\text{rggggggggggggggggggggggggggggs} \\
&30000000000000000000000000000004 \\
&30000000000000000000000000000004 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cd
&\text{or} \\
&\text{np} \quad \text{eeee61111111111111111111111111} \\
&\text{g111100000000000000000000000000000000} \\
&\text{g000000000000000000000000000000000000} \\
&\text{g000000000000000000000000000000000000} \\
&\text{300000000000000000000000000000000000} \\
&\text{522000000000000000000000000000000000} \\
&\text{eee300000000000000000000000000000000000} \\
&\text{eee300000000000000000000000000000000000} \\
&\text{eee300000000000000000000000000000000000} \\
&\text{1} \quad \text{eeeoggggggggggggggggggggggggggggggggggggg} \\
&\text{1} \quad \text{mp} \quad \text{eeeeee} \quad \text{.111t} \\
&\text{q111100000000000000000000000000000000} \\
&\text{g000000000000000000000000000000000000} \\
&\text{g000000000000000000000000000000000000} \\
&\text{300000000000000000000000000000000000} \\
&\text{522000000000000000000000000000000000} \\
&\text{eee300000000000000000000000000000000000} \\
&\text{eee300000000000000000000000000000000000} \\
&\text{eee300000000000000000000000000000000000} \\
&\text{1} \quad \text{eeeoggggggggggggggggggggggggggggggggggggg} \\
&\text{1} \quad \text{mp} \\
&\text{-------------------------------------------------------------------> i} \\
\end{align*}
\]

```fortran
read(15,2) label
print*,label
nblk=np/75 + 1
do nb=1,nblk
    read(15,2) label
    nb1=(nb-1)*75 + 1
    nb2=nb1+74
    if(nb2 .gt. np) nb2=np
do i=1,mp
    read(15,30) ins1, (id(i,j),j=nb1,nb2)
    format(i5,75a1)
    if(ins1 .ne. i) then
        write(*,35) i,ins1
    format(' Sequence wrong for ID input at',2i5)
    stop
    end if
end do
end do
print*,'Completed reading ID code matrix'
```

nog : number of grid points that a given wave boundary conditions are specified.

ig,jg: one-D arrays to store the grid number of the boundary in I and J directions.

Read the grid number for given boundary points

```fortran
read(15,*) nog
if(nog .gt. ngc) then
    print*,'NOG > NGC, change PARAMETER NGC to ',nog
```

2.10
stop
end if
write(*,40) nog

40   format(i5,' given boundary conditions')
read(15,2) label
print*,label
do i=1,nog
   read(15,50) ins,ig(i),jg(i), id(ig(i),jg(i)), phas(i)
50   format(1x,3i5,4x,a1,f10.5)
if(ins .le. 3) then
   write(*,50) ins,ig(i),jg(i),id(ig(i),jg(i)), phas(i)
end if
end do
print*,'Completed reading Given B.C.s'

nocb : number of grid points that a radiational B.C. is given
ibc,jbc: one-D arrays to store the grid number of the
   boundary in I and J directions
zalp : one-D array to store wave reflection coefficient
   at the boundary. If only one side is subject to
   wave reflection (e.g., ID(i,j)=1, 2, 3, or 4), then
   only the real part will be used.
   If both sides are subject to wave reflection (e.g.,
   ID(i,j)=x, y), then the reflection coefficeint of
   the second side will be stored as the imaginary part.
   At a corner boundary point (e.g., ID=5,6,7,8), the
   reflection coefficient in x direction is store as the
   real part. The reflection coefficient in y direction
   is stored in the imaginary part.

read the radiational boundary points and the reflection coeff.

read(15,* ) nocb
if(nocb .gt. nbc) then
   print*, 'NOBC > NBC, change parameter NBC to ',nocb
   stop
end if
write(*,60) nocb
60   format(i5,' radiational Boundary conditions')
read(15,2) label
print*, label
zi=(0.0, 1.0)
do i=1, nocb
   read(15,*) ins, ibc(i), jbc(i), id_bc, alp_r, alp_i
   zalp(i)=alp_r + zi*alp_i
   if(id_bc.ge.1 .and. id_bc.le.4 .and. alp_i.eq.0.0) go to 70
   if(id_bc.ge.5 .and. id_bc.le.8) go to 70
   write(*,65) i,ibc(i),jbc(i),id_bc, alp_i
65   format(' Boundary condition wrong at i, ibc,jbc= ',3i5/
         *' ID=',i3,' alp_i should = 0, but it is ',f8.3)
   stop
70   continue
if(alp_r > 1.0 .or. alp_r < 0.0 .or. alp_i < 0.0 .or. alp_i > 1.0) then  
   write(*,80) i, ibc(i), jbc(i), alp_r, alp_i  
80 format(' Boundary condition wrong at i, ibc, jbc = ',3i5/  
   ' They should >= 0.0 and <= 1.0, But they =',2f8.3)  
   stop  
end if  
end do  
print*, 'Completed reading radiational B.C.s, leaving BOUND_IN'  
close(15)  
creturn  
end  
c--------------------------------------------------------------------  
csubroutine corner_lb(i,j,map)  
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)  
character*1 id  
parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)  
common /waves/ period, waveH, angle0  
common /bottom/ dep(iq,jq), id(iq,jq)  
common /matrix/ za(kq,5), ia(kq,5), zb(kq)  
common /gener/ mp, np, dx, dy, r, yorient, xl, yl, wk0  
common /unknow/ n, mu, mL, m, Ld, imap(kq), jmap(kq)  
common /given/ nog, ig(ngc), jg(ngc)  
common /bound/ nobc, ibc(nbc), jbc(nbc)  
common /compx/ zp(ngc), zalp(nbc), phas(ngc)  
c It processes the left-bottom corner related 5 elements.  
possible ID layout are as follows:  
| 3 0 0 0 | 3 0 0 0 0 | 3 0 0 0 0 0 0 |  
| 3 0 0 0 | 3 0 0 0 0 | 3 0 0 0 0 0 0 |  
| 3 0 0 0 | 5 2 0 0 0 | 5 2 2 0 0 0 0 |  
| 5 2 2 2 | e e 3 0 0 | e e e 5 0 0 0 |  
| e e 5 2 2 | e e e 5 2 2 |  
|----------------------------------------------------> i  
 
The assignment of 5 points follows the following pattern:  
1: (i-1,j), 2: (i,j-1), 3: (i,j), 4: (i,j+1), 5: (i+1,j)  
give the reflection coefficient, 1.0, from the boundary conditions  
zi=(0.0,1.0)  
ick=1  
if(id(i,j) .eq. '5') then  
do it=1,nobc  
   if(ibc(it) .eq. i .and. jbc(it) .eq. j) then  
      alpha_x= real(zalp(it) )  
      alpha_y=aimag(zalp(it) )  
2.12
ick=0
     go to 10
end if
end do
else
  if(id(i,j) .eq. 'o' ) then
    alpha_x= 0.0
    alpha_y= 1.0
    ick=0
  end if
  if(id(i,j) .eq. 'p' ) then
    alpha_x= 1.0
    alpha_y= 0.0
    ick=0
  end if
end if
10 continue
  if( ick .eq. 1) then
    write(*,20) i,j, id(i,j)
20 format(' Refl. coef. not found in CORNER_LB at',2i5,a4)
  end if
  c
  dk1=wn(dep(i,j), period)
dkh=dk1*dep(i,j)
call e012(dkh, e0, e1, e2)
d_x=e0*( dep(i+1,j)-dep(i,j) )/(2.0*dep(i,j) )
d_y=e0*r*( dep(i,j+1)-dep(i,j) )/(2.0*dep(i,j) )
  c
dhdx2=( (dep(i+1,j)-dep(i,j) )/dx )**2
dhdy2=( (dep(i,j+1)-dep(i,j) )/dy )**2
d2hdx2= 0.0
d2hdy2= 0.0
psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
  c
zbetax=2*zi*alpha_x*dk1*dx
zxix=zi*alpha_x*r*r/(dk1*dx)
zbetay=2*zi*alpha_y*dk1*dy
zxiy=zi*alpha_y/(dk1*r*r*dy)
  c
cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
  c
za(map,1)=0.0
za(map,2)=0.0
za(map,3)=cnmd + (zbetax-2.0*zxix)*((1.0-d_x) +
1     (r*(r-d_y)+zxix*(1.0-d_x) )*(*1.0 - zxix*zxiy))
za(map,4)=r*(r-d_y)+zxix*(1.0-d_x) +
1     (r*(r-d_y)+zxix*(1.0-d_x) )*(1.0+zxiy*zxiy)/(1.0-zxix*zxiy)
za(map,5)=2.0*(1.0 + zxiy*(zxix*1.0-d_x) + r*(r-d_y) )/
1     (1.0 - zxiy*zxiy))
ia(map,1)=0

2.13
ia(map,2)=0
ia(map,3)=map
ia(map,4)=map+1
ia(map,5)=map+idistr(i,j)-1
if(id(i,j) .eq. '5') zb(map)=0.0
if(id(i,j) .eq. '0' .or. id(i,j) .eq. 'p') then
  do it=1,nog
    if(ig(it) .eq. i .and. jg(it) .eq. j) then
      zpotent=zp(it)
      go to 30
    end if
  end do
  write(*,25) i,j
25 format(' Given b.c. location not matched, at LB, i,j=', 2i5)
return
30 if(id(i,j).eq.'p') zb(map) = 4.0*zi*dk1*dx*(1.0+d_x)*zpotent
if(id(i,j).eq.'o') zb(map) = 4.0*zi*dk1*dy*r*(r-d_y)*zpotent
end if
creturn
c--------------------------------------------------------------
c subroutine corner_lt(i,j,map)
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
character*1 id
parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
common /waves/ period, waveH, angle0
common /bottm/ dep(iq,jq), id(iq,jq)
common /matrx/ za(kq,5), ia(kq,5), zb(kq)
common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
common /given/ nog, ig(ngc), jg(ngc)
common /bound/ nobc, ibc(nbc), jbc(nbc)
common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c Give the corner related 5 elements. possible allignment are
as follows:
  j
  | 6 1 1 1 . e e 6 1 1 . . e e e e e 6 1
  | 3 0 0 0 . e e 3 0 0 . . e e e e 6 0
  | 3 0 0 0 . 6 1 0 0 0 . . e e 6 0 0
  | 3 0 0 0 . 3 0 0 0 0 . . e 6 1 0 0 0
  | 3 0 0 0 . 3 0 0 0 0 . 6 0 0 0 0 0
  | . . . . . 3 0 0 0 0 . 3 0 0 0 0 0

----------------------------------------------------------> i

c The assignment of 5 points follows the following pattern:
c 1: (i-1,j), 2: (i,j-1), 3: (i,j), 4: (i,j+1), 5: (i+1,j)
zi=(0.0,1.0)

find the reflection coefficient from the boundary conditions

ick=1
if( id(i,j) .eq. '6') then
  do it=1,nobc
    if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
      alpha_x= real(zalp(it) )
      alpha_y=aimag(zalp(it) )
      ick=0
      go to 10
    end if
  end do
else
  if(id(i,j) .eq. 'q') then
    alpha_x= 1.0
    alpha_y= 0.0
    ick=0
  end if
  if(id(i,j) .eq. 'r') then
    alpha_x= 0.0
    alpha_y= 1.0
    ick=0
  end if
end if
end if
continue
if( ick .eq. 1) then
  write(*,20) i, j, id(i,j)
end if

dk1=wn(dep(i,j), period)
dkh=dk1*dep(i,j)
call e012(dkh, e0, e1, e2)
d_x=e0*( dep(i+1,j)-dep(i,j) )/(2.0*dep(i,j) )
d_y=e0*r*( dep(i,j)-dep(i,j-1) )/(2.0*dep(i,j) )

dhdx2= ( (dep(i+1,j)-dep(i,j) )/dx )**2
dhdy2= ( (dep(i,j)-dep(i,j-1) )/dy )**2
d2hdx2= 0.0
d2hdy2= 0.0
psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)

zbetax=2*zi*alpha_x*dk1*dx
zxix=zi*alpha_x*r*r/(dk1*dx)
zbetay=2*zi*alpha_y*dk1*dy
zxiy=zi*alpha_y/(dk1*r*r*dy)

cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
za(map,1)=0.0
za(map,2)=zxix*(1.0-d_x)+r*(r-d_y) +
  (zxix*(1.0-d_x)+r*(r-d_y))*(1.0+zxix*zxiy)/(1.0-zxix*zxiy)
za(map,3)=cnmd + (zbetax-2.0*zxix)*(1.0-d_x) +
  (r*(r+d_y) + zxix*(1.0-d_x) )*
  (zbetay + zxiy*(zbetax-2.0*zxix-2.0) )/(1.0-zxix*zxiy)
za(map,4)=0.0
za(map,5)=2.0*(1.0 +
  zxiy*(zxix*(1.0-d_x)+r*(r+d_y) )/(1.0-zxix*zxiy) )
c
ia(map,1)=0
ia(map,2)=map-1
ia(map,3)=map
ia(map,4)=0
ia(map,5)=map+idistr(i,j)-1
if( id(i,j) .eq. '6') zb(map)=0.0
if( id(i,j) .eq. 'q' .or. id(i,j) .eq. 'r') then
do it=1,nog
  if(ig(it) .eq. i .and. jg(it) .eq. j) then
    zpotent=zp(it)
go to 30
  end if
end do
write(*,25) i,j
25 format(' Given b.c. location not matched, at LT, i,j=', 2i5)
return
30 if(id(i,j).eq.'q') zb(map) = 4.0*zi*dk1*dx*(1.0+d_x)*zpotent
if(id(i,j).eq.'r') zb(map) = -4.0*zi*dk1*dy*r*(r-d_y)*zpotent
end if
c
return
c--------------------------------------------------------
c subroutine corner_rb(i,j,map)
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
character*1 id
parameter (iq=501, jq=401, kg=80000, nbc=1500, ngc=502)
common /waves/ period, waveH, angle0
common /bottm/ dep(iq,jq), id(iq,jq)
common /matrx/ za(kg,5), ia(kg,5), zb(kg)
common /gener/ mp, np, dx, dy, r, yorient, xl,yl, wk0
common /unknw/ n, mu, mL, m, Lda, imap(kg), jmap(kg)
common /given/ nog, ig(ngc), jg(ngc)
common /bound/ nobc, ibc(nbc), jbc(nbc)
common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c
Give the right-bottom corner related 5 elements:
c
| : 0 0 0 4 : 0 0 0 4 : 0 0 0 0 0 4 |

2.16
The assignment of each 5 points follows the following pattern:
1: (i-1,j), 2: (i,j-1), 3: (i,j), 4: (i,j+1), 5: (i+1,j)

\( zi = (0.0, 1.0) \)
\( ick = 1 \)

find the reflection coefficient from the boundary conditions

If (id(i,j) .eq. '7') then
   do it=1,nobc
      if( ibc(it) .eq. i .and. jbc(it) .eq. j) then
         alpha_x = real(zalp(it) )
         alpha_y = imag(zalp(it) )
         ick = 0
         go to 10
      end if
   end do
else
   if( id(i,j) .eq. 'u') then
      alpha_x = 1.0
      alpha_y = 0.0
      ick = 0
   end if
   if( id(i,j) .eq. 'v') then
      alpha_x = 0.0
      alpha_y = 1.0
      ick = 0
   end if
end if

if( ick .eq. 1) then
   write(*,20) i,j, id(i,j)
20 format(' Refl. coef. not found in CORNER_RB at',2i5,a5)
stop
end if

dk1 = wn(dep(i,j), period)
dkh = dk1*dep(i,j)
call e012(dkh, e0, e1, e2)
d_x = e0*(( dep(i,j) - dep(i-1,j) )/(2.0*dep(i,j) )
d_y = e0*r*(( dep(i,j+1) - dep(i,j) )/(2.0*dep(i,j) )
dhx2 = ( (dep(i,j) - dep(i-1,j) )/dx )**2
dhdx2= 0.0
dhdy2= 0.0

d2hdx2= 0.0
\[
\psi = e_1 (d h x_2 + d h y_2) + e_2 / w k_0 (d^2 h x_2 + d^2 h y_2)
\]

\[
z_{b_{\text{etax}}} = 2 z_i \alpha_x \frac{d k_1}{d x}
\]

\[
z_{x_{i x}} = z_i \alpha_x \frac{r \cdot r}{d k_1 \cdot d x}
\]

\[
z_{b_{\text{etay}}} = 2 z_i \alpha_y \frac{d k_1}{d y}
\]

\[
z_{x_{i y}} = z_i \alpha_y \frac{r \cdot r}{d k_1 \cdot r \cdot r \cdot d y}
\]

\[
\psi = 0.0
\]

\[
n_{\text{cmd}} = d k_1 \cdot d k_1 \cdot (1.0 + \psi) \cdot d x \cdot d x - 2.0 \cdot r \cdot r - 2
\]

\[
\begin{align*}
za(map, 1) &= 2 \cdot (1.0 + z_{x_{i y}} \cdot (r \cdot (r - d_y) + z_{x_{i x}} \cdot (1 + d_x)) / (1.0 - z_{x_{i x}} \cdot z_{x_{i y}})) \\
za(map, 2) &= 0.0 \\
za(map, 3) &= n_{\text{cmd}} + (z_{b_{\text{etax}} - 2.0} \cdot z_{x_{i x}}) \cdot (1.0 + d_x) + \\
1 &= (r \cdot (r - d_y) + z_{x_{i x}} \cdot (1.0 + d_x)) \\
2 &= (z_{b_{\text{etay}}} + z_{x_{i y}} \cdot (z_{b_{\text{etax}} - 2.0} \cdot z_{x_{i x}} - 2.0)) / (1.0 - z_{x_{i x}} \cdot z_{x_{i y}}) \\
za(map, 4) &= r \cdot (r + d_y) + z_{x_{i x}} \cdot (1.0 + d_x) + \\
1 &= (r \cdot (r - d_y) + z_{x_{i x}} \cdot (1.0 + d_x)) \cdot (1.0 + z_{x_{i x}} \cdot z_{x_{i y}}) / (1.0 - z_{x_{i x}} \cdot z_{x_{i y}}) \\
za(map, 5) &= 0.0
\end{align*}
\]

\[
ia(map, 1) = map - \text{idistl}(i, j) + 1 \\
ia(map, 2) = 0 \\
ia(map, 3) = map \\
ia(map, 4) = map + 1 \\
ia(map, 5) = 0
\]

\[
\text{if}( \text{id}(i, j) . \text{eq.} '7') \text{zb}(map) = 0.0 \\
\text{if}( \text{id}(i, j) . \text{eq.} 'u' . \text{or.} \text{id}(i, j) . \text{eq.} 'v') \text{then} \\
doi = 1, n_{\text{og}} \\
\text{if}(i_{\text{g}}(it) . \text{eq.} i . \text{and.} j_{\text{g}}(it) . \text{eq.} j) \text{then} \\
z_{\text{potent}} = z_{p}(it) \\
go \text{ to } 30 \\
end \text{ if} \\
end \text{ do}
\]

\[
\text{write}(*, 25) i, j 
\]

\[
25 \text{ format(' Given b.c. location not matched, at RB, i,j=', 2i5) return} \\
30 \text{ if(id(i, j).eq.'u')} zb(map) = -4.0 \cdot z_i \cdot d k_1 \cdot d x \cdot (1.0 + d_x) \cdot z_{\text{potent}} \text{ if(id(i, j).eq.'v'}) zb(map) = 4.0 \cdot z_i \cdot d k_1 \cdot d y \cdot r \cdot (r - d_y) \cdot z_{\text{potent}} \text{ end if}
\]

\[
\text{return} \text{end}
\]

---

subroutine corner_rt(i, j, map) implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z) character*1 id parameter (iq=501, jq=401, kg=80000, nbc=1500, ngc=502) common /waves/ period, waveH, angle0 common /bottm/ dep(iq,jq), id(iq,jq) common /matrx/ za(kg,5), ia(kg,5), zb(kg) common /gener/ mp, np, dx, dy, r, yorient, xl, y1, wk0 common /unknw/ n, mu, mL, m, Lda, imap(kg), jmap(kg)
Gives the right-top corner related 5 elements. We have 2 possibles as follows:

\begin{verbatim}
|   1 1 1 8   |   1 1 8 e e e e   |   1 1 1 8 e e e e |
|   0 0 0 4   |   0 0 4 e e e e   |   0 0 0 0 8 e e e |
|   0 0 0 4   |   0 0 4 e e e e   |   0 0 0 0 0 8 e e |
|   0 0 0 4   |   0 0 0 1 1 8     |   0 0 0 0 0 4 e e |
|   ....     |   .0 0 0 0 0 0 4   |   .0 0 0 0 0 0 4 e |
\end{verbatim}

The assignment of each 5 points follows the following pattern:
1: (i-1,j), 2: (i,j-1), 3: (i,j), 4: (i,j+1), 5: (i+1,j)

\begin{verbatim}
zi=(0.0, 1.0)
ick=1

find the reflection coefficient from the boundary conditions

if( id(i,j) .eq. '8') then
  do it=1,nobc
    if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
      alpha_x= real(zalp(it) )
      alpha_y=imag(zalp(it) )
      ick=0
      go to 10
    end if
  end do
else
  if( id(i,j) .eq. 't') then
    alpha_x=1.0
    alpha_y=0.0
    ick=0
  end if
  if( id(i,j) .eq. 's') then
    alpha_x=0.0
    alpha_y=1.0
    ick=0
  end if
end if

continue
if( ick .eq. 1) then
  write(*,20) i,j, id(i,j)
20  format(' Refl. coef. not found in CORNER_RT at',2i5,a5)
stop
end if
\end{verbatim}
dk1=wn(dep(i,j), period)
dkh=dk1*dep(i,j)
call e012(dkh, e0, e1, e2)
d_x=e0*( dep(i,j)-dep(i-1,j) )/(2.0*dep(i,j) )
d_y=e0***( dep(i,j)-dep(i,j-1) )/(2.0*dep(i,j) )
dhdx2= ( ( dep(i,j)-dep(i-1,j) )/dx )**2
dhdy2= ( ( dep(i,j)-dep(i,j-1) )/dy )**2
d2hdx2= 0
d2hdy2= 0
psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)

c zbetax=2*zi*alpha_x*dk1*dx
dxix=zi*alpha_x*r*r/(dk1*dx)
zbetay=2*zi*alpha_y*dk1*dy
zxiy=zi*alpha_y/(dk1*r*r*dy)

c cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2

za(map,1)=2*(1.0+zxiy*(r*(r+d_y)+zxix*(1+d_x))/(1.0-zxix*zxiy))
za(map,2)=r*(r-d_y)+zxix*(1.0+d_x) + 1
za(map,3)=cnmd + (zbetax - 2.0*zxix)*(1.0+d_x) + 1
za(map,4)=0.0
za(map,5)=0.0

c ia(map,1)=map-idistl(i,j)+1
ia(map,2)=map-1
ia(map,3)=map
ia(map,4)=0
ia(map,5)=0
if( id(i,j) .eq. '8') zb(map)=0.0
if( id(i,j) .eq. 's' .or. id(i,j) .eq. 't') then
  do it=1,nog
    if(ig(it) .eq. i .and. jg(it) .eq. j) then
      zpotent=zp(it)
goto 30
    end if
  end do
write(*,25) i,j
25 format(' Given b.c. location not matched, at RT, i,j=', 2i5)
return
30 if(id(i,j).eq.'t') zb(map) = -4.0*zi*dk1*dx*(1.0+d_x)*zpotent
if(id(i,j).eq.'s') zb(map) = -4.0*zi*dk1*dy*r*(r+d_y)*zpotent
end if

c return
end

c----------------------------------------------------------------
c
subroutine Given_BC(i,j,map)
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
character*1 id
common /waves/ period, waveH, angle0
common /bottm/ dep(iq,jq), id(iq,jq)
common /matrx/ za(kq,5), ia(kq,5), zb(kq)
common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
common /given/ nog, ig(ngc), jg(ngc)
common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c processes a water cell that is also a given B.C., find
ZA and IA and the coefficient column matrix zb(map).
c j | Top bottom left right
| . g e e .. . 0 0 0 0 0 .. g 0 0 0 .. . 0 0 0 g
| . 0 0 0 0 .. . 0 0 0 0 0 .. g 0 0 0 .. . 0 0 0 g
| . 0 0 0 0 .. g g e e .. g 0 0 0 .. . 0 0 0 g
|-------------------------------------------------------> i
c here, the coefficient for zp(i-1,j), zp(i,j-1), zp(i,j),
zp(i,j+1), and zp(i+1,j) are stored in (map,1), (map,2), (map,3),
(map,4) and (map,5), respectively.
zi=(0.0, 1.0)
dk1=wn(dep(i,j), period)
dkh=dk1*dep(i,j)
call e012(dkh, e0, e1, e2)
do it=1,nog
if(ig(it) .eq. i .and. jg(it) .eq. j) then
  zpotent=zp(it)
go to 10
end if
end do
write(*,8) i,j
8 format('! Given boundary location not matched, at i,j=', 2i5)
stop
c left hand side given B.C.
continue
if(id(i+1,j) .eq. '0') then
  d_x=e0*( dep(i+1,j)-dep(i,j) )/(2.0*dep(i,j) )
d_y=e0**2*( dep(i,j+1)-dep(i,j-1) )/(4.0*dep(i,j) )
dhdx2=(( dep(i+1,j)-dep(i,j) )/dx )**2
dhdy2=(( dep(i,j+1)-dep(i,j-1) )/(2.0*dy) )**2
d2hdx2= 0.0
d2hdy2= 0.0
psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
cnmd = dk1*dk1*(1.0 + psi)*dx*dx - 2.0*r*r - 2.0
zbetax = 2*zi*dk1*dx
zxix = zi*r*r/(dk1*dx)
za(map,1) = 0.0
za(map,2) = r*(r - d_y) + zxix*(1.0 - d_x)
za(map,3) = cnmd + (zbetax - 2.0*zxix)*(1.0 - d_x)
za(map,4) = r*(r + d_y) + zxix*(1.0 - d_x)
za(map,5) = 2.0
ia(map,1) = 0
ia(map,2) = map - 1
ia(map,3) = map
ia(map,4) = map + 1
ia(map,5) = map + idistr(i,j) - 1
zb(map) = 4.0*zi*dk1*dx*(1.0 - d_x)*zpotent
go to 20
end if

c right hand side given B.C.
c
if(id(i-1,j) .eq. '0') then
d_x = e0*(dep(i,j) - dep(i-1,j))/(2.0*dep(i,j))
d_y = e0*r*(dep(i,j) - dep(i-1,j))/(4.0*dep(i,j))
zbetax = 2*zi*dk1*dx
zxix = zi*r*r/(dk1*dx)
dhdx2 = (dep(i,j) - dep(i-1,j))/(dx)**2
dhdy2 = (dep(i,j) - dep(i-1,j))/(2.0*dy)**2
d2hdx2 = 0.0
d2hdyy = 0.0
psi = e1*(dhdx2 + dhdy2) + e2/wk0*(d2hdx2 + d2hdyy)

c
znmd = dk1*dk1*(1.0 + psi)*dx*dx - 2.0*r*r - 2.0
za(map,1) = 2.0
za(map,2) = r*(r - d_y) + zxix*(1.0 + d_x)
za(map,3) = cnmd + (zbetax - 2.0*zxix)*(1.0 + d_x)
za(map,4) = r*(r + d_y) + zxix*(1.0 + d_x)
za(map,5) = 0.0
ia(map,1) = map - idistr(i,j) + 1
ia(map,2) = map - 1
ia(map,3) = map
ia(map,4) = map + 1
ia(map,5) = 0
zb(map) = -4.0*zi*dk1*dx*(1.0 + d_x)*zpotent
go to 20
end if

c bottom side given B.C.
c
if(id(i,j+1) .eq. '0') then
d_x = e0*(dep(i+1,j) - dep(i-1,j))/(4.0*dep(i,j))
d_y = e0*r*(dep(i,j+1) - dep(i,j))/(2.0*dep(i,j))
zbetay = 2*zi*dk1*dy
zxiy = zi/(dk1*r*r*dy)
\begin{verbatim}
dhdl2 = ((dep(i+1,j) - dep(i-1,j)) / (2.0*dx))**2
dhdy2 = ((dep(i,j+1) - dep(i,j)) / dy)**2
d2hdhdx2 = 0.0
d2hdhdy2 = 0.0
psi = e1*(dhdx2 + dhdy2) + e2/wk0*(d2hdhdx2 + d2hdhdy2)

\text{cnmd} = dk1*dk1*(1.0 + psi)*dx*dx - 2.0*r*r - 2.0
\text{za}(map,1) = r*(r - d_y)*zxiy + (1.0 - d_x)
\text{za}(map,2) = 0.0
\text{za}(map,3) = cnmd + (zbetay - 2.0*zxiy)*r*(r - d_y)
\text{za}(map,4) = 2.0*r*r
\text{za}(map,5) = r*(r - d_y)*zxiy + (1.0 + d_x)
\text{ia}(map,1) = map - idistl(i,j) + 1
\text{ia}(map,2) = 0
\text{ia}(map,3) = map
\text{ia}(map,4) = map + 1
\text{ia}(map,5) = map + idistr(i,j) - 1
\text{zb}(map) = 4.0*zi*dk1*dy*r*(r - d_y)*zpotent

\text{go to 20}
\end{verbatim}
print*, 'i,j,map, ia(map,3)=' , i, j, map, ia(map,3)
return
end

----------------------------------------------------------------

subroutine bound_r(i,j,map)
imPLICIT real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
character*1 id
common /waves/ period, waveH, angle0
common /bottm/ dep(iq,jq), id(iq,jq)
common /matrx/ za(kq,5), ia(kq,5), zb(kq)
common /gener/ mp, np, dx, dy, r, yorient, xl, yl, wk0
common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
common /bound/ nobc, ibc(nbc), jbc(nbc)
common /compx/ zp(ngc), zalp(nbc), phas(ngc)

Processes the right side boundary related 4 ZA and IA.
find the reflection coefficient from the B.C.

zi=(0.0,1.0)
do it=1,nobc
if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
  alpha= real(zalp(it) )
go to 10
end if
end do
write(*,8) i, j
8 format(' Reflection coefficient not found in BOUND_R at',2i5)
return

10 dk1=wn(dep(i,j), period)
dkh=dk1*dep(i,j)
call e012(dkh, e0, e1, e2)
d_x=e0***( dep(i,j)-dep(i-1,j) )/(2.0*dep(i,j) )
d_y=e0***( dep(i,j+1)-dep(i,j-1) )/(4.0*dep(i,j) )
dhx2=( (dep(i,j)-dep(i-1,j) )/dx )**2
dhy2=( (dep(i,j+1)-dep(i,j-1) )/(2.0*dy) )**2
\[ d2hdx2 = \frac{\text{dep}(i, j) - 2.0 \times \text{dep}(i-1, j) + \text{dep}(i-2, j)}{(2.0 \times \text{dx} \times \text{dx})} \]
\[ d2hdy2 = \frac{\text{dep}(i, j+1) - 2.0 \times \text{dep}(i, j) + \text{dep}(i, j-1)}{(2.0 \times \text{dy} \times \text{dy})} \]
\[ \psi = e1 \times (dhdx2 + dhdy2) + e2 / \text{wk0} \times (d2hdx2 + d2hdy2) \]
\[ zbetax = 2 \times zi \times \alpha \times dk1 / \text{dx} \]
\[ zxix = zi \times \alpha \times r \times r / (dk1 \times \text{dx}) \]
\[ cnmd = dk1 \times dk1 \times (1.0 + \psi) \times \text{dx} \times \text{dx} - 2.0 \times r \times r - 2.0 \]
\[ za(\text{map}, 1) = 2.0 \]
\[ za(\text{map}, 2) = r \times (r - d_y) + zxix \times (1.0 + d_x) \]
\[ za(\text{map}, 3) = cnmd + (zbetax - 2.0 \times zxix) \times (1.0 + d_x) \]
\[ za(\text{map}, 4) = r \times (r + d_y) + zxix \times (1.0 + d_x) \]
\[ za(\text{map}, 5) = 0.0 \]
\[ ia(\text{map}, 1) = \text{map} - \text{idistl}(i, j) + 1 \]
\[ ia(\text{map}, 2) = \text{map} - 1 \]
\[ ia(\text{map}, 3) = \text{map} \]
\[ ia(\text{map}, 4) = \text{map} + 1 \]
\[ ia(\text{map}, 5) = 0 \]
\[ zb(\text{map}) = 0.0 \]
\[ \text{return} \]
\[ \text{end} \]

---

**Subroutine bound_l(i,j,map)**

```fortran
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
commom /waves/ period, waveH, angle0
common /bottm/ dep(iq,jq), id(iq,jq)
common /matrx/ za(kq,5), ia(kq,5), zb(kq)
common /gener/ mp,np, dx,dy, r, yorient, xl,yl, wk0
common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
common /bound/ nobc, ibc(nbc), jbc(nbc)
common /compx/ zp(ngc), zalp(nbc), phas(ngc)
```

Construct the Trifty storage matrix ZA, & IA when the left hand point (i-1,j) is a Neumann type boundary point.

Assignment of the 5 points follows the following pattern:
1: (i-1,j),  2: (i,j-1),  3: (i,j),  4: (i,j+1),  5: (i+1,j)

find the reflection coefficient from the B.C.

\[ zi=(0.0, 1.0) \]
\[ \text{do it}=1, \text{nobc} \]
\[ \text{if}(\text{ibc(it)} .eq. i .and. jbc(it) .eq. j) \text{ then} \]
\[ \quad \alpha = \text{real}(\text{zalp(it)}) \]
\[ \quad \text{go to 10} \]
\[ \text{end if} \]
\[ \text{end do} \]
\[ \text{write(*,8) i,j} \]
```fortran
8 format(' Reflection coefficient not found in BOUND_L at',2i5)
```

2.25
return
dk1=wn(dep(i,j), period)
dkh=dk1*dep(i,j)
call e012(dkh, e0, e1, e2)
d_x=e0*(dep(i+1,j)-dep(i,j))/({2.0*dep(i,j)})
d_y=e0*r*(dep(i,j+1)-dep(i,j-1))/({4.0*dep(i,j)})
dhdx2={((dep(i+1,j)-dep(i,j))/dx)**2}
dhdy2={((dep(i,j+1)-dep(i,j-1))/(2.0*dy))**2}
d2hdx2={((dep(i+2,j)-2.0*dep(i+1,j)+dep(i,j))/dx)**2}
d2hdy2={((dep(i,j+1)-2.0*dep(i,j)+dep(i,j-1))/dy)**2}
psi=e1*(dhdx2+dhdy2)+e2/wk0*(d2hdx2+d2hdy2)
zbetax=2*zi*alpha*dk1*dx
zxix=zi*alpha*r*r/(dk1*dx)

return
c 10 dk1=wn(dep(i,j), period)
dkh=dk1*dep(i,j)
call e012(dkh, e0, e1, e2)
d_x=e0*(dep(i+1,j)-dep(i,j))/({2.0*dep(i,j)})
d_y=e0*r*(dep(i,j+1)-dep(i,j-1))/({4.0*dep(i,j)})
dhdx2={((dep(i+1,j)-dep(i,j))/dx)**2}
dhdy2={((dep(i,j+1)-dep(i,j-1))/(2.0*dy))**2}
d2hdx2={((dep(i+2,j)-2.0*dep(i+1,j)+dep(i,j))/dx)**2}
d2hdy2={((dep(i,j+1)-2.0*dep(i,j)+dep(i,j-1))/dy)**2}
psi=e1*(dhdx2+dhdy2)+e2/wk0*(d2hdx2+d2hdy2)
zbetax=2*zi*alpha*dk1*dx
zxix=zi*alpha*r*r/(dk1*dx)

c cnmd=dk1*dk1*(1.0+psi)*dx*dx-2.0*r*r-2
za(map,1)=0.0
za(map,2)=r*(r-d_y)+zxix*(1.0-d_x)
za(map,3)=cnmd+(zbetax-2.0*zxix)*(1.0-d_x)
za(map,4)=r*(r+d_y)+zxix*(1.0-d_x)
za(map,5)=2.0
ia(map,1)=0
ia(map,2)=map-1
ia(map,3)=map
ia(map,4)=map+1
ia(map,5)=map+idistr(i,j)-1
zb(map)=(0.0, 0.0)
return
e nd
c subroutine bound_b(i,j,map)
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
character*1 id
common /waves/ period, waveH, angle0
common /bottm/ dep(iq,jq), id(iq,jq)
common /matrx/ za(kq,5), ia(kq,5), zb(kq)
common /gener/ mp, np, dx, dy, r, yorient, xl, yl, wk0
common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
common /bound/ nobc, ibc(nbc), jbc(nbc)
common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c Give the bottom side boundary related 5 ZA and IA.
c find the reflection coefficient from the boundary conditions
c The assignment of each 5 points follows the following pattern:
c 1: (i-1,j), 2: (i,j-1), 3: (i,j), 4: (i,j+1), 5: (i+1,j)
c zi=(0.0,1.0)
do it=1,nobc
if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
   alpha= real(zalp(it) )
go to 10
end if
end do
write(*,8) i,j
8 format(' Reflection coefficient not found in BOUND_B',2i5)
return
c
dk1=wn(dep(i,j), period)
dkh=dk1*dep(i,j)
call e012(dkh, e0, e1, e2)
d_x=e0*(dep(i+1,j) -dep(i-1,j) )/(4.0*dep(i,j) )
d_y=e0*r*(dep(i,j+1) -dep(i,j) )/(2.0*dep(i,j) )
dhdx2= ( (dep(i+1,j) -dep(i-1,j) ) / (2.0*dx) )**2
dhdy2= ( (dep(i, j+1) -dep(i,j) ) / (dy )**2
d2hdx2= ( (dep(i+1,j) -2.0*dep(i,j)+dep(i-1,j) ) / (2.0*dx*dx) )
d2hdy2= ( dep(i,j+2) -2.0*dep(i,j+1)+dep(i,j) )/(2.0*dy*dy )
psi=e1*(d2hdx2+d2hdy2) + e2/wk0*(d2hdx2 + d2hdy2)
zbeta y=2*zi*alpha*dk1*dy
zx iy=zi*alpha/(dk1*r*r*dy)
c
cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
za(map,1)=r*(r-d_y)*zx iy +(1.0-d_x)
za(map,2)=0.0
za(map,3)=cnmd + (zbeta y-2.0*zx iy)*r*(r-d_y)
za(map,4)=2.0*r*r
za(map,5)=r*(r-d_y)*zx iy + (1.0+d_x)
ia(map,1)=map-idistl(i,j)+1
ia(map,2)=0
ia(map,3)=map
ia(map,4)=map+1
ia(map,5)=map+idistr(i,j)-1
zb(map)=0.0
return
end

-------------------------------------------------------------

subroutine bound_t(i,j,map)
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
character*1 id
common /waves/ period, waveH, angle0
common /bottm/ dep(iq,jq), id(iq,jq)
common /matrx/ za(kq,5), ia(kq,5), zb(kq)
common /gener/ mp, np, dx, dy, r, yorient, xl,yl, wk0
common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
common /bound/ nobc, ibc(nbc), jbc(nbc)
common /compx/ zp(ngc), zalp(nbc), phas(ngc)
Give the top side boundary related 5 ZA ans IA.
find the reflection coefficient from the boundary conditions
The assignment of each 5 points follows the following pattern:
1: (i-1,j), 2: (i,j-1), 3: (i,j), 4: (i,j+1), 5: (i+1,j)

zi=(0.0,1.0)
do i=1,nobc
   if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
      alpha = real(zalp(it) )
go to 10
   end if
end do
write(*,8) i,j
8 format(' Reflection coefficient not found in BOUND_T at',2i5)
return

10 dk1=wn(dep(i,j), period)
dkh=dk1*dep(i,j)
call e012(dkh, e0, e1, e2)
d_x=e0*( dep(i+1,j)-dep(i-1,j) )/(4.0*dep(i,j) )
d_y=e0*( dep(i,j)-dep(i,j-1) )/(2.0*dep(i,j) )
dhdlx2= ( (dep(i+1,j)-dep(i-1,j) )/(2.0*dx) )**2
dhdy2= ( (dep(i,j)-dep(i,j-1) )/dy )**2
d2hdx2= ( dep(i+1,j)-2.0*dep(i,j)+dep(i-1,j) )/(2.0*dx*dx )
d2hdy2= ( dep(i,j)-2.0*dep(i,j-1)+dep(i,j-2) )/(2.0*dy*dy )
psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
zbetay=2*zi*alpha*dk1*dy
zxiy=zi*alpha/(dk1*r*r*dy)

cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
za(map,1)=r*(r+d_y)*zxiy + (1.0-d_x)
za(map,2)=2.0*r*r
za(map,3)=cnmd + (zbetay-2.0*zxiy)*r*(r+d_y)
za(map,4)=0.0
za(map,5)=r*(r+d_y)*zxiy + (1.0+d_x)
ia(map,1)=map-idistl(i,j)+1
ia(map,2)=map-1
ia(map,3)=map
ia(map,4)=0
ia(map,5)=map+idistr(i,j)-1
zb(map)=0.0
return
end

subroutine domain(i,j,map)
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401, kq=80000)
character*1 id
common /waves/ period, waveH, angle0
common /bottm/ dep(iq,jq), id(iq,jq)
common /matrx/ za(kq,5), ia(kq,5), zb(kq)
common /gener/ mp, np, dx, dy, r, yorient, xl, yl, wk0

for interior grid points
In this sub., the coefficient matrix zb(map) = 0.
In this system, the coefficient of zp(i-1,j) is stored in (map,1),
the coefficient of zp(i,j-1) is stored in (map,2),
the coefficient of zp(i,j) is stored in (map,3),
and the coefficient of zp(i,j+1) is stored in (map,4),
and the coefficient of zp(i+1,j) is stored in (map,5)

coefficients

dk1=wn(dep(i,j), period)
dkh=dk1*dep(i,j)
call e012(dkh, e0, e1, e2)
d_x=e0*(dep(i+1,j)-dep(i-1,j))/(4.0*dep(i,j))
d_y=e0*r*(dep(i,j+1)-dep(i,j-1))/(4.0*dep(i,j))
dhdx2= ( (dep(i+1,j)-dep(i-1,j))/(2.0*dx) )**2
dhdy2= ( (dep(i,j+1)-dep(i,j-1))/(2.0*dy) )**2
d2hdx2= (dep(i+1,j)-2.0*dep(i,j)+dep(i-1,j))/(dx*dx)
d2hdy2= (dep(i,j+1)-2.0*dep(i,j)+dep(i,j-1))/(dy*dy)
psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)

za(map,1)=1.0-d_x
za(map,2)=r*(r-d_y)
za(map,3)=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
za(map,4)=r*(r+d_y)
za(map,5)=1.0+d_x

relative location

ia(map,1)=map-idistl(i,j)+1
ia(map,2)=map-1
ia(map,3)=map
ia(map,4)=map+1
ia(map,5)=map+idistr(i,j)-1

right hand side

zb(map)= (0.0, 0.0)

return
end

--------------------------------------------------------------------

integer function idistr(i,j)
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401)
character*1 id

c
common /gener/ mp, np, dx, dy, r, yorient, xl, yl, wk0
common /bottm/ dep(iq,jq), id(iq,jq)

c Calculates the distance between points (i,j) & (i+1,j). It shows
many grid points (must be a water point, a radiational boundary
point, or a given boundary point) in between. It counts vertically
from the point (i,j) and up (i,j+1), (i,j+2), ..., (i,np), (i+1,1),
(i+1,2), ..., to (i+1,j). In the matrix equation, this distance
represents the up band width at that particular diagonal element
(i,j).

If this point is a radiational B.C. point, then the distance is
zero.

C
idistr=0
if(id(i,j).eq.'4' .or. id(i,j).eq.'7' .or. id(i,j).eq.'8') then
  return
else
  do 10 jk=1,np
    if( id(i,jk) .eq. 'e' .or. jk .le.j ) go to 10
    idistr=idistr+1
  10  continue

do 20 jk=1,np
  if( id(i+1,jk) .eq. 'e' .or. jk .gt.j ) go to 20
  idistr=idistr+1
20 continue

idistr=idistr+1

end if

return
end

integer function idist1(i,j)
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401)
character*1 id

c
common /gener/ mp, np, dx, dy, r, yorient, xl, yl, wk0
common /bottm/ dep(iq,jq), id(iq,jq)

c Calculate the distance between point (i-1,j) & (i,j). The distance
means how many grid points, which must be a water or a boundary
point, are in between. It counts vertically from (i,j) downward
(i,j-1), (i,j-2) ... (i,1) and restarted from previous i line
(i-1,np), (i-1,np-1) ... to(i-1,j).

In the matrix equation, the distance represents the lower band
If it is a left side radiational boundary point, then distance is zero

```
idistl=0
if(id(i,j).eq.'3' .or. id(i,j).eq.'5' .or. id(i,j).eq.'6') then
    return
else
    do 10 jk=1,np
        if( id(i-1,jk) .eq. 'e' .or. jk .lt.j ) go to 10
    idistl=idistl+1
10 continue
    do 20 jk=1,np
        if( id(i,jk) .eq. 'e' .or. jk .ge. j ) go to 20
    idistl=idistl+1
20 continue
```

```
end if
return
```

```
subroutine BMsolver(ierr_code)
```

```
It solves a complex banded matrix equation ZAF * ZX = ZB
where ZAF is a complex band matrix with dimension (m * n)
ZX and ZB are two complex column matrices with length (n).

Because of the huge size of ZAF, e.g., (300 x 50000), it is
designed to solve this problem using the following two steps.

First, don't use the full size of ZAF, instead, uses two small
matrices: ZA and IA that each only uses 5 x 50000 to save space.

ZA : a complex matrix (5 x n) to store the coefficient matrix in
a matrix equation ZAF * ZX = ZB. Because of using the finite
difference method to solve an elliptic equation, there are
only 5 elements to be saved for the coefficient matrix. The
band width, however, is much much large than 5 with a lot of
"zero." By doing so, we need another matrix
IA : to save the corresponding locations.

Second, uses a working matrix, ZW(IW,JW), and do a systematical
swap between a hard disk and memory. For this reason, be sure
that you do have enough space in your hard disk. For example, a
complex matrix with size of (300 x 50000) requires 120 MB for
storage, if using 4 byte for a real number. If using 8 bytes,
then, 240 MB is needed.
IW, JW: IW should be \( \geq m+ml \), where ml is the lower band width. The size of JW depends on the available computer memory. In general, the larger the JW, the less the disk I/O, and thus, the faster the computing speed. As a rule of thumb, you may select JW = 2\*IW and try to see if your computer has enough memory to run the program.

The procedure follows that given in the subroutine CGBFA & CGBSL from LINPACK. The major difference is just doing it one block at a time, stores the results in hard disk sequentially. After forward elimination, reverse the process by reading the data from hard disk and do back substitute for the solution.

```
implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
parameter (iq=501, jq=401, kq=80000, iw=1000, jw=1700)

character*8 tmpfile
character*3 chrc
character*1 id

common /bottm/ dep(iq,jq), id(iq,jq)
common /matrx/ za(kq,5), ia(kq,5), zb(kq)
common /gener/ mp, np, dx, dy, r, yorient, x1, y1, wk0
common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
common /works/ zw(iw,jw), ipvt(kq)

c abs1(zdum)=abs(real(zdum)) + abs(aimag(zdum))

Gaussian elimination with partial pivoting
NK: # of columns for the working matrix that will be saved.
NW: The number of column needed for working, nw=m-1+nk
NS: An index to show the number of working matrix.

check working matrix dimension
nk=450
nw=m+nk-1

write(*,5) iw, jw, Lda, nw
5 format(' The working matrix, ZW(iw,jw) is ZW(',2i5,')/'
    * ' The best condition should be iw -> Lda= ',i5/
    * ' and jw -> nw = ',i5)
if(nw .gt. jw) then
    write(*,10) Lda, nw
10 format(' IW should be >= ',i5, ' and JW must be >=', i5/
    * ' Please change program and re-run .'/
    * ' If no memory, you can reduce the NK')
ierr_code=1
return
end if
```
clear up the working matrix

```
do i=1,iw
  do j=1,nw
    zw(i,j)=(0.0, 0.0)
  end do
end do
```

construct the first working matrix, ZW, which has a size of (Lda x nw). The thrify storaged matrices are expanded, only to the first (nk x 5) block.

The flag is used to record, only once, when ZA data starts lost. Later it will be used to re-fetch into the working matrix.

```
ns=0
map1=0
iflag=1
ne=nw
if(ne .gt. n) ne=n
```

do map=1,ne
  do i=1,5
    j=ia(map,i)
    if(j .ne. 0) then
      if(j .le. nk+m-1) then
        ids=j-map
        zw(m-ids,j)=za(map,i)
      else
        if(iflag .eq. 1) then
          map1=map
          iflag=0
        end if
      end if
    end if
  end do
end do

Gaussian elimination with partial pivoting

```
j1=min0(n,m)-1
jz=j1
ju=0
nt=0
index=0
```

100 continue

```
if(ju .ne. 0) ju=ju-nk
if(jz .ne. j1) jz=jz-nk
```
c index=0, the first, 2nd, ..., block of matrix.
c =1, the last block of matrix
if(index .eq. 0 .and. ne .ne. n) then
c nc=nk
else
nc=n-ns*nk-1
end if
do k=1,nc
kp1=k+1
kr=ns*nk + k
c find L = pivot index
Lm=min0(mL,n-ns*nk-k)
L=icamax(Lm+1, zw(m,k), 1) + m -1
ipvt(kr)=L+kr-m
c zero pivot implies that this column are all zeros, a singular
c matrix.
if(cabs1(zw(L,k)) .lt. 0.10e-8) goto 120
c interchange if necessary
if(L .ne. m) then
zt=zw(L,k)
zw(L,k)=zw(m,k)
zw(m,k)=zt
print*, '************pivoting'
end if
c compute multipliers
zt=-(1.0d0,0.0d0)/zw(m,k)
call cscal(Lm, zt, zw(m+1,k), 1)
c swap ZB array, if necessary
Lp=ipvt(kr)
zt=zB(Lp)
if(Lp .ne. kr) then
zb(Lp)=zb(kr)
zb(kr)=zt
end if
call caxpy(Lm, zt, zw(m+1,k), 1, zb(kr+1), 1)
c row elimination with column indexing
ju=min0(max0(ju, mu+ipvt(kr)-ns*nk), n-ns*nk)
mm=m
if(ju .ge. kp1) then
    do j=kp1,ju
        L=L-1
        mm=mm-1
        zt=zw(L,j)
        if(L .ne. mm) then
            zw(L,j)=zw(mm,j)
            zw(mm,j)=zt
        end if
        call caxpy(Lm, zt, zw(m+1,k), 1, zw(mm+1,j), 1)
    end do
end if

go to 150
120 continue
    print*,'Zero diagonal element at (L,k)=', L, k
    stop
150 continue

c
Eexcept the last working matrix, write the upper triangular matrix (from j=1,nk) into hard disk. For the last one, i.e., nc<>nk, go to back substitute directly.
c
if(nc .eq. nk) then
    nt=nt+1
    print*,'Writing tem. file #', nt
    tmpfile='t'//chrc(nt)//'.tmp'
    open(12,file=tmpfile,form='unformatted')
    write(12) ((zw(i,j),i=1,m),j=1,nk)
    close(12)
c
    moving the rest working matrix forward to the beginning
c
    do j=nk+1, nk+m-1
        L=j-nk
        do i=1,Lda
            zw(i,L)=zw(i,j)
        end do
    end do

c
    clear the rest working area for reading new working matrix
c
    do j=m, nk+m-1
        do i=1,Lda
            zw(i,j)=(0.0,0.0)
        end do
    end do

c
    read in a full block of ZA and IA, by two steps
c
ns=ns+1
if( (ns+1)*nk+m-1 .lt. n) then
  ne=nk
  index=0

For intermediate blocks, read in ZA and IA by two steps.
First read in the upper triangular matrix that were cut off at the previous time when constructing the working matrix.

if(map1 .ne. 0) then
  do map=map1, m-1+ns*nk
     do i=4,5
        j=ia(map,i)
        if(j .ne. 0) then
           if(j .gt. m-1+ns*nk) then
              ids=j-map
              zw(m-ids,j-ns*nk)=za(map,i)
           end if
        end if
     end do
  end do
end if

now read in next block of ZA and IA

iflag=1
map1=0
do k=1,ne
   map=k + ns*nk + m-1
   do i=1,5
      j=ia(map,i)
      if(j .ne. 0) then
         if(j .le. ns*nk+ne+m-1) then
            ids=j-map
            zw(m-ids,j-ns*nk)=za(map,i)
         else
            if(iflag .eq. 1) then
               map1=map
               iflag=0
            end if
         end if
      end if
   end do
end do

End of read in a block of ZA and IA.

else

This is to read the last block of ZA and IA

ne=n-(m-1+ns*nk)
index=1
if(map1 .ne. 0) then
  do map=map1,m-1+ns*nk
  do i=4,5
    j=ia(map,i)
    if(j .ne. 0) then
      if(j .gt. m-1+ns*nk) then
        ids=j-map
        zw(m-ids,j-ns*nk)=za(map,i)
      end if
    end if
  end do
end do
end if

iflag=1
map1=0
do k=1,ne
  map=m-1+ns*nk+k
  do i=1,5
    j=ia(map,i)
    if(j .ne. 0) then
      if(j .le. m-1+ns*nk+ne) then
        ids=j-map
        zw(m-ids,j-ns*nk)=za(map,i)
      else
        ierr_code=2
        write(*,180)
      end if
    end if
  end do
end do
iflag=1
end if

goto 100

c else

c backward substitution from the last submatrix

c if( nt .eq. 0) nc=n-1
  do kb=1,nc+1
    kr=n+1-kb
    k=nc+2-kb
    zb(kr)=zb(kr)/zw(m,k)
end do
Lm=min0(kr,m)-1
La=m-Lm
Lb=kr-Lm
zt=-zb(kr)
call caxpy(Lm, zt, zw(La,k), 1, zb(Lb), 1)
end do

If one loop can include all elements, i.e., for a small banded matrix, just stops after this.

if( nt .eq. 0 ) go to 400
end if

complete the rest backward substitute
ns=0
200 continue

clear the working matrix for reading new submatrix from disk.
do j=1,nk+m-1
do i=1,m
    zw(i,j)=(0.0,0.0)
end do
end do

print*,'Reading tem. file #', nt
open(14,file=tmpfile,form='unformatted')
read(14) ((zw(i,j),i=1,m),j=1,nk)
close(14,status='delete')
do kb=1,nk
kr=n+(nc+1)-ns*nk-kb
k=nk+1-kb
zb(kr)=zb(kr)/zw(m,k)
Lm=min0(kr,m)-1
La=m-Lm
Lb=kr-Lm
zt=-zb(kr)
call caxpy(Lm, zt, zw(La,k), 1, zb(Lb), 1)
end do
ns=ns+1
nt=nt-1
tmpfile='t'/chr(nt)'/'.tmp'
if(nt .gt. 0) goto 200
400 continue

to restore the original sequence of ZB. Since it starts pivoting at the 2nd. we started at 2nd too for restoring.

print*,'Restore the original sequency'
do 1000 kb=2,n
  k=n+1-kb
  L=ipvt(k)
  zt=zb(L)
  if (L .eq. k) go to 1000
  print*, 'pivoting at L,k,kb=',L,k,kb
  zb(L)=zb(k)
  zb(k)=zt
1000 continue

return
end

character*3 function chrc(nt)

It changes an input integer number NT to character
CHANGQING LI 06/94

character*1 c1
character*2 c2
character*3 c3

i1=nt
i2=i1/10
if(i2 .gt. 0) then
  i3=i2/10
  if(i3 .gt. 0) then
    i4=i3/10
    if(i4 .gt. 0) then
      print*, '-1'
      else
        c3=char(48+i3)//char(48+i2-i3*10)//char(48+i1-i2*10)
        chrc=c3
        write(11,*) c3
      end if
    else
      c2=char(48+i2)//char(48+i1-i2*10)
      chrc=c2
      write(11,*) c2
    end if
  else
    c1=char(48+i1)
    chrc=c1
    write(11,*) c1
  end if

return
end
SUBROUTINE CSCAL(N,ZCA,ZCX,INCX)

PURPOSE
Complex vector scale x = a*x

WRITTEN on Oct. 1, 79, REVISED on Aug. 01, 82

CATEGORY NO. D1A6

KEYWORDS BLAS, COMPLEX, LINEAR ALGEBRA, SCALE, VECTOR

AUTHORS LAWSON, C. L., (JPL), HANSON, R. J., (SNLA)

KINCAID, D. R., (U. OF TEXAS), and KROGH, F. T., (JPL)

DESCRIPTION

BLASSubprogram

Description of Parameters

--Input--
N number of elements in input vector(s)
CA complex scale factor
CX complex vector with N elements
INCX storage spacing between elements of CX

--Output--
CSCAL complex result (unchanged if N .LE. 0)

replace complex CX by complex CA*CX.

For I = 0 to N-1, replace CX(I*INCX) with CA * CX(I*INCX)

REFERENCES: LAWSON C.L., HANSON R.J., KINCAID D.R., KROGH F.T.,
"BASIC LINEAR ALGEBRA SUBPROGRAMS FOR FORTRAN USAGE,"
ALGORITHM NO. 539, TRANSACTIONS ON MATHEMATICAL
SOFTWARE, VOLUME 5, NUMBER 3, SEPTEMBER 1979, 308-323

Routines Called (NONE)

implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
dimension zcx(1)

COMPLEX*8 CA, CX(1)

FIRST EXECUTABLE STATEMENT CSCAL
IF(N .LE. 0) RETURN
NS = N*INCX
DO I = 1,NS,INCX
   ZCX(I) = ZCA*ZCX(I)
END do
RETURN
END

SUBROUTINE CAXPY(N,ZCA,ZCX,INCX,ZCY,INCY)

WRITTEN on Oct. 01, 79, REVISED on April 25, 84

CATEGORY NO. D1A7

KEYWORDS BLAS, COMPLEX, LINEAR ALGEBRA, TRIAD, VECTOR

AUTHOR LAWSON, C. L., (JPL), HANSON, R. J., (SNLA)
KINCAID, D. R., (U. OF TEXAS), KROGH, F. T., (JPL)
PURPOSE Complex computation \( y = a \times x + y \)
DESCRIPTION

B L A S Subprogram
Description of Parameters

--Input--
N number of elements in input vector(s)
ZCA complex scalar multiplier
ZCX complex vector with N elements
INCX storage spacing between elements of CX
ZCY complex vector with N elements
INCY storage spacing between elements of CY

--Output--
ZCY complex result (unchanged if N .LE. 0)

Overwrite complex ZCY with complex ZCA*ZCX + ZCY.
For I = 0 to N-1, replace
ZCY(LY+I*INCY) with ZCA*ZCX(LX+I*INCY) + ZCY(LY+I*INCY), where
LX = 1 if INCX .GE. 0, else LX = (-INCY)*N
and LY is defined in a similar way using INCY.

REFERENCES
LAWSON C.L., HANSON R.J., KINCAID D.R., KROGH F.T.,
*BASIC LINEAR ALGEBRA SUBPROGRAMS FOR FORTRAN USAGE*,
ALGORITHM NO. 539, TRANSACTIONS ON MATHEMATICAL
SOFTWARE, VOLUME 5, NUMBER 3, SEPTEMBER 1979, 308-323
Routines called (NONE)

implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
dimension zcx(1), zcy(1)
COMPLEX CX(1), CY(1), CA
FIRST EXECUTABLE STATEMENT CAXPY
CANORM = ABS(REAL(ZCA)) + ABS(AIMAG(ZCA))
IF(N.LE.0 .OR. CANORM.EQ.0.E0) RETURN
IF(INCX.EQ.INCY.AND.INCX.GT.0) GO TO 20
KX = 1
KY = 1
IF(INCX.LT.0) KX = 1+(1-N)*INCX
IF(INCY.LT.0) KY = 1+(1-N)*INCY
DO 10 I = 1, N
   ZCY(KY) = ZCY(KY) + ZCA*ZCX(KX)
   KX = KX + INCX
   KY = KY + INCY
10 CONTINUE
RETURN
20 CONTINUE
NS = N*INCX
DO 30 I=1, NS, INCX
   ZCY(I) = ZCA*ZCX(I) + ZCY(I)
30 CONTINUE
INTEGER FUNCTION ICAMAX(N,zCX,INCX)

C WRITTEN on Oct. 01, 79, REVISED on Aug. 01, 82
C CATEGORY NO. D1A2
C KEYWORDS BLAS, COMPLEX, LINEAR ALGEBRA, MAXIMUM COMPONENT, VECTOR
C AUTHOR LAWSON, C. L., (JPL), HANSON, R. J., (SNLA)
C KINCAID, D. R., (U. OF TEXAS), KROGH, F. T., (JPL)
C PURPOSE Find the location (or index) of the largest component
C of a complex vector
C DESCRIPTION
B L A S Subprogram
Description of Parameters

--Input--
N number of elements in input vector(s)
zCX complex vector with N elements
INCX storage spacing between elements of CX

--Output--
ICAMAX smallest index (zero if N .LE. 0)

Returns the index of the component of CX having the
largest sum of magnitudes of real and imaginary parts.
ICAMAX = first I, I = 1 to N, to minimize
ABS(REAL(CX(1-INCX+I*INCX))) + ABS(IMAG(CX(1-INCX+I*INCX)))

REFERENCES
LAWSON C.L., HANSON R.J., KINCAID D.R., KROGH F.T.,
*BASIC LINEAR ALGEBRA SUBPROGRAMS FOR FORTRAN USAGE*,
ALGORITHM NO. 539, TRANSACTIONS ON MATHEMATICAL
SOFTWARE, VOLUME 5, NUMBER 3, SEPTEMBER 1979, 308-323

ROUTINES CALLED (NONE)

implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
dimension zcx(1)

COMPLEX CX(1)

***FIRST EXECUTABLE STATEMENT ICAMAX
ICAMAX = 0
IF(N .LE. 0) RETURN
ICAMAX = 1
IF(N .LE. 1) RETURN
NS = N*INCX
II = 1
SUMMAX = ABS(REAL(zCX(1))) + ABS(AIMAG(zCX(1)))
DO 20 I=1,NS,INCX
SUMRI = ABS(REAL(zCX(I))) + ABS(AIMAG(zCX(I)))
IF(SUMMAX .GE. SUMRI) GO TO 10

RETURN
END
SUMMAX = SUMRI
ICAMAX = II
10   II = II + 1
20 CONTINUE
RETURN
END

function wn(depth, period)
   c This function calculate the airy's wave number
   c period : in second.
   c depth : in meter.
   c wavek : in meter^-1
   c
   implicit real*8 (a-h, o-y)
per = 9.8
dol0=depth/(1.56*period*period)
if (dol0 .ge. 0.7) then
   wave=1.56*period*period
   wavekh=6.2832*depth/wave
else
   if (dol0 .ge. 0.01) then
      cgm=6.2832/period
      y=cgm*cgm*depth/per
      sum=1.0 + 0.6666666666*y + 0.3555555555*y*y +
      * 0.1608465608*y*y*y + 0.0632098765*y*y*y*y +
      * 0.0217540484*y*y*y*y*y + 0.00654078*y*y*y*y*y*y
      wavekh=sqrt(y*y + y/sum)
   else
      c long wave
      c=sqrt(per*depth)
      wave=per*period
      wavekh=6.2832*depth/wave
   end if
   end if
   wn=wavekh/depth
   return
end

subroutine e012(x,e0,e1,e2)
   c Massel's subroutine for calculating the three coefficients
   c e0 is for bottom slope term.
   c e1 is for slope square term.
   c e2 is for curvature term.
   c x = kh, is the only input parameter.
implicit real*8 (a-h, o-y)
sh=sinh(x)
ch=cosh(x)
th=tanh(x)
dum=th + x*(1.0 - th*th)
e0=x/dum*(1.0 - 3.0*th*th + 2.0*th/dum)

sh2=sinh(2.0*x)
sh3=sinh(3.0*x)
ch3=cosh(3.0*x)
sh5=sinh(5.0*x)
ch5=cosh(5.0*x)
p=0.5*(1.0+2.0*x/sh2)
tlw=1.0/(24.0*ch**3*(2.0*x+sh2)**2)* (x*(12.0+16.0*x**2)*ch +
  6.0*x*(ch3+ch5) + sh*(12.0+84.0*x**2) +
  3.0*(1.0-4.0*x**2)*sh3 -9.0*sh5)
t2=0.125*(-4.0*x*ch + sh +8.0*x**2*sh + sh3)/ch**3/(2.0*x+sh2) -
  x/2.0*th/ch**2

e1=tlw/p/th
e2=t2/p
return
end
Appendix III. FORTRAN program to generate the input data file for plotting. The input file for this program is the output file generated by RDE.FOR

    program outform
    c
    c This program is the first post-process program for the program
    c RDE.FOR. It converts the one-dimensional output to a two
    c dimensional format for a faster graphic using RDPLT.M
    c
    c In this program, the conversion of x and y coordinates are NOT
    c non-dimensionalized. This is for the convenience of comparison.
    c
    c iq,jq : parameters specified for reserving space for 2-D arrays.
    c They should be at least the same as mp and np.
    c mp,np : number of grid points in x and y direction, respectively.
    c In applications, the x-axis should be parallel to the
    c longer dimension of the water body. The y axis should
    c parallel to the short dimension of the water body. This
    c alignment will generate a smaller banded width.
    c dx,dy : grid sizes in x and y direction, respectively. In this
    c program, only the equally spaced grid is allowed. But
    c dx do not have to equal to dy.
    c tide : water elev. that added to the bathymetric depth for
    c storm tide.
    c dmin : min. water depth at a point that this point will be
    c considered as water.
    c id : a 2-D array to store the identification for each grid
    c point. 0 for water cell; 1 for land cell.
    c
    parameter (iq=330, jq=263)
    c
    implicit real*8 (a-h,o-z)
    character*70 title,label
    character*5 name
    character*12 infile
    character*20 outfile
    character*1 idc
    character*2 chrc
    integer*1 id(iq,jq)
    c
    dimension wh(iq,jq), s(iq,jq), dep(iq,jq), dir(iq,jq)
    c
    pi=3.1415926
    twopi=2.0*pi
    grav=9.8
    c
    Input bathmetry and incident wave condition
    c
    print*, ' Key in the input filename <XXXXX.lis as XXXXX >: '
    read(*,'(a)') name
    c
infile=name // '.lis'
print*, 'Input file name : ', infile
open(1, file=infile, form='formatted', status='old')

icase=0
continue
icase=icase+1
read(1, '(a70)', end=1000) title
print*, title

mp, np : Max. grid numbers in x and y direction, respectively.
 dx, dy : grid sizes in x and y direction, respectively.
 tide : water elev. that added to the bathymetric depth,
 e.g. storm tide
dmin : min. water depth that will be considered as water

read(1, *) mp, np, dx, dy
if(mp .gt. iq) then
  print*, 'IQ is smaller than MP, change IQ to ', mp
  stop
end if
if(np .gt. jq) then
  print*, 'JQ is smaller than NP, change JQ to ', np
  stop
end if

clear the matrices

print*, 'Clean the matrices...
  do i=1, mp
    do j=1, np
      id(i,j)=1
      dep(i,j)=0.0
      wh(i,j)=0.0
      s(i,j)=0.0
      dir(i,j)=0.0
    end do
  end do

yorient: The azimuth of y-axis related to the True North
xl, yl : The x and y coordinates of the origin of the grid

read(1,*) yorient, xl, yl

read(1,*) waveH, period, angle0, tide, map
write(*,30) waveH, period, angle0, tide
30 format(' wave height (m) = ', f8.3/ ' Period (s) = ', f8.2/
  ' Angle (deg) = ', f8.1/ ' Tide (m) = ', f8.2)

read(1,35) label
write(*,35) label
35 format (a50)
  ph_g = grav * period * waveh / (4.0 * pi)

  do n=1,map
      read (1,65) i,j,idc,depth,phr,phi
      65 format (2x,i5,1x,i5,a3,f10.4,2d18.10)
      dep(i,j) = depth
      h = sqrt(phr*phr + phi*phi)
      wh(i,j) = h / ph_g
      s(i,j) = datan2(phi, phr)
      if ( s(i,j) .lt. 0.0) s(i,j) = s(i,j) + twopi
      if(idc .ne. 'e') id(i,j) = 0
      if(i .eq. mp .and. j .eq. np) go to 70
  end do

  mb = 1
  me = mp
  outfile = name // '_' // chrc(icase) // '.out'
  print *, 'outfile name = ', outfile
  open (12, file = outfile, form = 'formatted', status = 'unknown')
  write (12, '(a60)') title
  write (12, 80) mp, np, dx, dy, dmin, yorient, xl, yl
  80 format (2i5, 2f10.5, 2f10.3 / f10.2, 2f10.5)
  write (12, 90) waveh, period, angle0, tide
  90 format (4f10.3)

  write the ID code first

  write (12, 125)
  125 format (' I.D. code for each grid point' / +-------------> i')
  do j=1,np
     jj=np-j+1
     write (12, 130) jj, (id(i,jj), i=mb, me)
  130 format (i5/(40i2))
  end do

  write the wave height

  write (12, 135)
  135 format (' Normalized wave height at each grid point' / * ' +-------------> i')
  do j=1,np
     jj=np-j+1
     write (12, 140) jj, (wh(i,jj), i=mb, me)
  140 format (i5/(10f8.5))
  end do

  write (12, 145)
  145 format (' Wave phase for each grid point' / +-------------> i')
  do j=1,np
     jj=np-j+1
     write (12, 150) jj, (s(i,jj), i=mb, me)
Calculate wave directions. Notice that the phase value only changes from 0 to 2*pi. When it is more than 2*pi, it drop back to a little more than 0. This cause problem for calculate wave direction because it is not a continuous function (it is, but computer can not understand it). So we set up a criterion: when the change of phase is more than 4.0, it means the two grid points just across a drop. We simply change the small phase S to S+2*PI and continue the calculation.

\[
\begin{align*}
\text{do } i=2, \text{mp-1} \\
\quad \text{do } j=2, \text{np-1} \\
\quad \quad \text{dss} = s(i,j+1) - s(i,j-1) \\
\quad \quad \text{if( dss .lt. -4.0) dss=dss+twopi} \\
\quad \quad \text{if( dss .gt. 4.0) dss=dss-twopi} \\
\quad \quad \text{dsdy}=dss/(2*dy) \\
\quad \quad \text{dss} = s(i+1,j) - s(i-1,j) \\
\quad \quad \text{if( dss .lt. -4.0) dss=dss+twopi} \\
\quad \quad \text{if( dss .gt. 4.0) dss=dss-twopi} \\
\quad \quad \text{dsdx}=dss/(2*dx) \\
\quad \quad \text{dir}(i,j)=\text{atan2}(\text{dsdy}, \text{dsdx})
\end{align*}
\]

\[
\begin{align*}
\text{end do} \\
\end{align*}
\]

\[
\begin{align*}
\text{do } i=2, \text{mp-1} \\
\quad \text{do } j=2, \text{np-1} \\
\quad \quad \text{dss} = s(i,np) - s(i,np-1) \\
\quad \quad \text{if( dss .lt. -4.0) dss=dss+twopi} \\
\quad \quad \text{if( dss .gt. 4.0) dss=dss-twopi} \\
\quad \quad \text{dsdy}=dss/dy \\
\quad \quad \text{dss} = s(i+1,np) - s(i-1,np) \\
\quad \quad \text{if( dss .lt. -4.0) dss=dss+twopi} \\
\quad \quad \text{if( dss .gt. 4.0) dss=dss-twopi} \\
\quad \quad \text{dsdx}=dss/(2.0*dx) \\
\quad \quad \text{dir}(i,np)=\text{atan2}(\text{dsdy}, \text{dsdx})
\end{align*}
\]

\[
\begin{align*}
\text{end do} \\
\end{align*}
\]

\[
\begin{align*}
\text{do } i=2, \text{mp-1} \\
\quad \text{dss} = s(i,2) - s(i,1) \\
\quad \text{if( dss .lt. -4.0) dss=dss+twopi} \\
\quad \text{if( dss .gt. 4.0) dss=dss-twopi} \\
\quad \text{dsdy}=dss/dy \\
\quad \text{dss} = s(i+1,1) - s(i-1,1) \\
\quad \text{if( dss .lt. -4.0) dss=dss+twopi} \\
\quad \text{if( dss .gt. 4.0) dss=dss-twopi} \\
\quad \text{dsdx}=dss/(2.0*dx) \\
\quad \text{dir}(i,1)=\text{atan2}(\text{dsdy}, \text{dsdx})
\end{align*}
\]

\[
\begin{align*}
\text{end do} \\
\end{align*}
\]
c
  do j=2, np-1
    dss= s(1,j+1) - s(1,j-1)
    if( dss .lt. -4.0) dss=dss+twopi
    if( dss .gt. 4.0) dss=dss-twopi
    dsdy=dss/(2.0*dy)
    dss= s(2,j) - s(1,j)
    if( dss .lt. -4.0) dss=dss+twopi
    if( dss .gt. 4.0) dss=dss-twopi
    dsdx=dss/dx
    dir(1,j)=atan2(dsdy, dsdx)
  end do

c
  do j=2, np-1
    dss= s(mp,j+1) - s(mp,j-1)
    if( dss .lt. -4.0) dss=dss+twopi
    if( dss .gt. 4.0) dss=dss-twopi
    dsdy=dss/(2.0*dy)
    dss= s(mp,j) - s(mp-1,j)
    if( dss .lt. -4.0) dss=dss+twopi
    if( dss .gt. 4.0) dss=dss-twopi
    dsdx=dss/dx
    dir(mp,j)=atan2(dsdy, dsdx)
  end do

c
  write(12,155)
  155 format(' Wave direction at each grid point'/' +------------> i')
  do j=1,np
    jj=np-j+1
    write(12,160) jj, (dir(i,jj),i=mb,me)
  160 format(i5/(10f8.4))
  end do

c
  write(12,195)
  195 format(' Water depth for each grid point'/' +------------> i')
  do j=1,np
    jj=np-j+1
    write(12,200) jj, (dep(i,jj),i=mb,me)
  200 format(i5/(10f8.3))
  end do

c
  close(12)

c
  go to 25

1000 continue
  stop
end

c
  character*2 function chrc(nt)
  c
  c It changes an input integer number NT to character

3.5
c     character*2 c1
character*2 c2

c     i1=int
     i2=i1/10
     if(i2 .gt. 0) then
       i3=i2/10
       if(i3 .gt. 0) then
         print*,'----------------------------------------'
         print*,'The given integer is > 99, not allowed.'
         print*,'----------------------------------------'
         chrc='1'
       else
         c2=char(48+i2)//char(48+i1-i2*10)
         chrc=c2
       end if
     else
       c2=char(48+i2)//char(48+i1-i2*10)
       chrc=c2
     end if

     return
end
Appendix IV. Matlab program (RDEPLOT.M) to generate graphics as the final output of the program RDE.FOR. The input data files are from the program OUTFORM.FOR.

% Program RDEPLOT.M
% MathLab M file for plotting the outputs of RDE.FOR
% It uses MathLab ver. 4.2C.1 for WINDOWS
%
% Program developed by Jerome P.-Y. Maa at
% Virginia Institute of Marine Science, Feb. 1996
%
clc;
%
% The directory (c:\break\*.out) may change for different application
%
[name,path]=uigetfile('c:\break\*.out','Available Data files');
file=[path,name];
[fid, message]=fopen(file);
if fid == -1
    message
    end
%
% read contour lines information
%
[name,path]=uigetfile('c:\break\*.lvl','Available contour level files');
file_lvl=[path,name];
[fid_lvl, message]=fopen(file_lvl);
if fid_lvl == -1
    message
    end
ftitle=fgetl(fid_lvl);
disp(ftitle);
Nh=fscanf(fid_lvl, '%d', 1);
vh=fscanf(fid_lvl, '%f', Nh);
Ns=fscanf(fid_lvl, '%d', 1);
v_s=fscanf(fid_lvl, '%f', Ns);
ndep=fscanf(fid_lvl, '%d', 1);
v_dep=fscanf(fid_lvl, '%f', ndep);
close(fid_lvl);
%
% read data
%
ftitle=fgetl(fid);
disp(ftitle);
mp=fscanf(fid, '%d', 1);
np=fscanf(fid, '%d', 1);
dx=fscanf(fid, '%f', 1);
dy=fscanf(fid, '%f', 1);
dmin=fscanf(fid, '%f', 1);
dlevel=fscanf(fid, '%f', 1);
message}
disp(['dx= ', num2str(dx), ' dy= ', num2str(dy)]);
xl=fscanf(fid, '%f', 1);
yl=fscanf(fid, '%f', 1);
yorient=fscanf(fid, '%f', 1);
disp(['Yorient= ', num2str(yorient)]);

waveh=fscanf(fid, '%f', 1);
period=fscanf(fid, '%f', 1);
angle0=fscanf(fid, '%f', 1);

disp(fgetl(fid));
disp(fgetl(fid));
fgetl(fid);

% read ID codes

id=[];
for j=1:np
    jj=np-j+1;
    jnk=fscanf(fid, '%d', 1);
    if jnk ~= jj
        disp(['ID Seq. error, j=', int2str(jnk), '<> jj=', int2str(jj)]);
        disp('paused');
        pause;
    end
    a=fscanf(fid, '%d', np);
    id(jj, 1:np)=a';
end

clear a;

% read normalized wave heights

disp(fgetl(fid));
disp(fgetl(fid));
fgetl(fid);
waveh=[];
for j=1:np
    jj=np-j+1;
    nk=fscanf(fid, '%d', 1);
    if nk ~= jj
        disp(['WaveH Seq. error, j=', int2str(nk), '<> jj=', int2str(jj)]);
        disp('paused');
        pause;
    end
    a=fscanf(fid, '%f', mp);
    waveh(jj, 1:mp)=a';
end

% read phases

disp(fgetl(fid));
disp(fgetl(fid));
fgetl(fid);
s=[];
for j=1:np
    jj=np-j+1;
    nk=fscanf(fid, '%d', 1);
    if nk ~= jj
        disp(['Phase Seq. error, j=',int2str(nk),'<>jj=',int2str(jj)]);
        disp('paused');
        pause;
    end
    a=fscanf(fid,'%f', mp);
    s(jj, 1:mp)=a';
end

% read wave direction
% disp(fgetl(fid) );
% disp(fgetl(fid) );
fgetl(fid);
dir=[];
for j=1:np
    jj=np-j+1;
    nk=fscanf(fid, '%d', 1);
    if nk ~= jj
        disp(['Dir Seq. error, j=',int2str(nk),'<>jj=',int2str(jj)]);
        disp('paused');
        pause;
    end
    a=fscanf(fid,'%f', mp);
    dir(jj, 1:mp)=a';
end
% Because x axis is vertical and y axis is horizontal when plotting
% The angle is calculated to reflect this
% dir(jj, 1:mp)=a';
end

% read water depth
% disp(fgetl(fid) );
% disp(fgetl(fid) );
fgetl(fid);
d=[];
for j=1:np
    jj=np-j+1;
    nk=fscanf(fid, '%d', 1);
    if nk ~= jj
        disp(['Depth Seq. error, j=',int2str(nk),'<>jj=',int2str(jj)]);
        disp('paused');
        pause;
    end
    a=fscanf(fid,'%f', mp);
    d(jj, 1:mp)=a';
end
clear a;
fclose(fid);

% find the coordinates of x & y
xmax=(mp-1)*dx;
xmin=0;
ymin=0;
ymax=(np-1)*dy;
axis_ratio=(xmax-xmin)/(ymax-ymin);
clc;

dep_sel=0.5;
disp('After plotted, you may drag a area for zoom in');
disp(' or use left mouse bottom to zoom in 2 times');
disp(' right mouse bottom to zoom out');
disp('The following options are available :');
disp(' 1. select plot area ');
disp(' 2. plot wave height contours');
disp(' 3. plot wave vectors');
disp(' 4. plot water depth contours');
disp(' 5. plot wave phases');
disp(' 6. plot a profile parallel to X axis');
disp(' 7. plot a profile parallel to Y axis');
disp(' -1. exit');
iop=input('please select : ');
if size(iop) == 0 iop=0; end

while (iop+1)

if iop == 1
    disp(['X axis ranged from ',num2str(xmin), ' to ', num2str(xmax), ' km']);
x1= input('Select the min. of X plotting domain : ');
x2= input('Select the max. of X plotting domain : ');
% select Y domain for plotting
    disp(['Y axis ranges from ', num2str(ymin), ' to ', num2str(ymax), ' km']);
y1= input('Select the min. of Y plotting domain : ');
y2= input('Select the max. of Y plotting domain : ');
x=x1:dx:x2;
y=y1:dy:y2;
end

% Plot the normalized wave height contours
if iop == 2
    figure(1);
clf;
zoom off;
axis equal;
set(gca,'linewidth',1);
[cl,hhh]=contour(xc, yc, waveh, v_h);
set(hhh, 'linewidth',2);
clabel(cl, 'manual');
xlabel('x'); ylabel('y');
title('Wave Height Contours', 'fontsize',18);
ans1=input('To: 0.hpgl file; 1.BMP file; 2.gif file : ');
if ans1 == 0 print figure1 -dhpgl; end
if ans1 == 1 print figure1 -dbmp256; end
if ans1 == 2 print figure1 -dgif8; end
zoom on;
end

% plots the vectors
%
if iop == 3
% select X domain for plotting
    ans2=input('Want depth contour plots underlay the phase plot
           <y/n> ? ', 's');
    format short;
    clc;
    nx=input('to skip how many vectors in X domain <3-6>: ');
    dxn=nx*dx;
    is1=xs1/dx+1;
    is2=xs2/dx+1;
    disp(['Selected area from i = ',int2str(is1),' to ',
         int2str(is2)]);
% select Y domain for plotting
    ny=input('to skip how many vectors in y domain <3-6>: ');
    dyn=dy*ny;
    js1=ys1/dy+1;
    js2=ys2/dy+1;
    disp(['Selected area from j = ',int2str(js1),' to ',
         int2str(js2)]);
    xc=xs1:dxn:xs2;
    yc=ys1:dyn:ys2;
% find the sub matrix
figure(2);
clf;
zoom off;
waveh_s=[];
dir_s=[];
dep_s=[];
axis equal;
mv_a=size(xc');
nv_a=size(yc');
mv=mv_a(1);
nv=nv_a(1);
for i=1: mv
    iold=is1+(i-1)*nx;
    for j=1:nv
jold=js1+(j-1)*ny;
waveh_s(j,i)=waveh(jold, iold);
dir_s(j,i)=dir(jold, iold);
dep_s(j,i)=d(jold, iold);
end

u=waveh_s.*cos(dir_s);
v=waveh_s.*sin(dir_s);
quiver(xc, yc, u, v, 2);
set(gca,'aspectratio', [axis_ratio,1]);
set(gca,'xlim', [xs1,xs2], 'ylim', [ys1,ys2]);
xlabel('X (m)'); ylabel('Y (m)');
title('Wave Vectors');
if ans2 == 'y'
    hold on;
    cl=contour(xc, yc, dep_s, v_dep);
end
ans2_2=input('To: 0.hpgl file; 1.BMP file; 2.gif file : ');
if ans2_2 == 0 print figure2 -dhpgl; end
if ans2_2 == 1 print figure2 -dbmp256; end
if ans2_2 == 2 print figure2 -dgif8; end
zoom on;
end

% plot the depth contours

if iop == 4
    figure(3); clf;
    zoom off; grid off;
    xc=xmin:dx:xmax;
    yc=ymin:dy:ymax;
    axis equal;
    [cl,hhh]=contour(xc, yc, d, v_dep);
    set(hhh, 'linewidth',3)
    set(gca,'aspectratio', [axis_ratio,1]);
    set(gca,'xlim', [xmin,xmax], 'ylim', [ymin,ymax]);
    clabel(cl, 'manual');
    xlabel('X (m)'); ylabel('Y (m)');
title('Water Depth Contours');
end

% check if need other type of output
ans3=input('To: 0.return; 1.gif file; 2.BMP file : ');
if ans3 == 1 print figure3 -dgif8; end
if ans3 == 2 print figure3 -dbmp256; end
zoom on;
end

% plot the phase contours

if iop == 5
    ans4=input('Want depth contour plots underlay the phase plot <y/n> ? ', 's');
    figure(4);

4.6
clf;
zoom off;
axis equal;
xc=xmin:dx:xmax;
yc=ymin:dy:ymax;
cl=contour(xc, yc, s, v_s);
set(gca,'aspectratio',[axis_ratio,1]);
set(gca,'xlim', [xmin,xmax], 'ylim', [ymin,ymax]);
clabel(cl, 'manual');
xlabel('X (m)'); ylabel('Y (m)');
title('Wave Phase Contours');
if ans4 == 'y'
    hold on;
    cl=contour(xc, yc, d, v_dep);
end
ans4_2=input('To: 0.return; 1.BMP; 2.gif; 3.HPGL file: ');
if ans4_2 == 1 print figure4 -dbmp256; end
if ans4_2 == 2 print figure4 -dgif8; end
if ans4_2 == 3 print figure4 -dhpgl; end
zoom on;
end

% plot wave height profile parallel to x axis
%
if iop == 6
    ks5=input(['Select a y grid number <0-',int2str(np),'> : ']);
    for i=1:mp
        yas(i)=waveh(ks5,i);
        das(i)=dep_sel - d(ks5,i);
    end
    ans51=input('Experimental data file name : ', 's');
    namei=['\break\berk\', ans51, '.txt'];
    nameo=['\break\', ans51,'.dat'];
    disp(['Data coming from ', namei]);
    [fidi, message]=fopen(namei, 'r');
    if fidi == -1
        message
    end
    ftitle=fgetl(fidi);
    disp(ftitle);
    ndata=fscanf(fidi, '%d', 1);
    ftitle=fgetl(fidi);
    ftitle=fgetl(fidi);
    disp(ftitle);
    xdl=[]; yd=[];
    for j=1:ndata
        a=fscanf(fid,'%f', 2);
        xdl(j)=a(1);
        yd(j)=a(2);
        end
    fclose(fidi);
\[ xd = 25.5 - x_{d1} \]
\[ xw(1) = x_c(3); \quad xw(2) = x_c(mp-3); \quad yw(1) = \text{dep}_sel; \quad yw(2) = \text{dep}_sel; \]

```matlab
figure(5)
cf;
zoom off;
plot(xc, yas, 'r-', 'linewidth', 3);
hold on;
plot(xc, das, 'k-', 'linewidth', 2);
hold on;
plot(xw, yw, 'g-', 'linewidth', 2);
hold on;
plot(xd, yd, 'ro', 'linewidth', 3);
set(gca, 'xlim', [4, 15]);
set(gca, 'ylim', [0, 2.5]);
ans52 = input('To: 0.HPGL file; 1.BMP file; 2.gif file : ');
if ans52 == 0 print figure5 -dhpgl; end
if ans52 == 1 print figure5 -dbmp256; end
if ans52 == 2 print figure5 -dgif8; end
zoom on;
ans52 = input('Save in an ASCII file XPROFILE <y/n> : ', 's');
if ans52 == 'y'
    [fido, message] = fopen(nameo, 'wt');
    if fid == -1
        message
    end
    for i = 1:mp
        fprintf(fido, '%10.4f %10.4f %10.4f
', xc(i), das(i),
            yas(i));
    end
    fclose(fido);
    disp(['data is in file ', nameo]);
end
end

% plot wave height profile parallel to y axis
%
if iop == 7
    ks6 = input(['Select a x grid number <0-', int2str(mp), '> : ']);
    for j = 1:np
        yas2(j) = waveh(j, ks6);
        das2(j) = dep_sel - d(j, ks6);
    end
% save data to a disk file
ans51 = input('Experimental data file name : ', 's');
nameo = ['\break\', ans51, '.dat'];

namei = ['\break\berk\', ans51, '.txt'];
disp(['Data coming from ', namei]);
[fidi, message] = fopen(namei, 'r');
if fidi == -1
    message
```

4.8
end
ftitle=fgetl(fidi);
disp(ftitle);
ndata=fscanf(fidi, '%d', 1);
ftitle=fgetl(fidi);
ftitle=fgetl(fidi);
disp(ftitle);
xd1=[]; yd=[];
for j=1:ndata
    a=fscanf(fid,'%f', 2);
    xd1(j)=a(1);
    yd(j)=a(2);
end
fclose(fidi);
xd=25.5-xd1

% ans61=input('Save in an ASCII file YPROFILE <y/n> : ', 's');
if ans61 == 'y'
    outdata=[yc; das2; yas2];
    [fido, message]=fopen(nameo,'wt');
    if fido == -1
        message
    end
    fprintf(fido,'%10.4f %10.4f %10.4f
',outdata);
fclose(fido);
disp(['Data is in file : ', nameo]);
end;

% figure(6);
clf;
zoom off;
axis equal;
plot(yc, yas2, 'r-', 'linewidth',3);
xw(1)=yc(2); xw(2)=yc(np-2); yw(1)=dep_sel; yw(2)=dep_sel;
hold on;
plot(xw,yw, 'g-', 'linewidth', 2);
xlabel('Y (m)', 'fontsize', 15);
ylabel('Wave Height (cm)', 'fontsize', 15);
hold on;
plot(yc,das2,'k-','linewidth',2);
ans62=input('To: 0.return; 1.BMP file; 2.gif file : ');
if ans62 == 1 print figure6 -dbmp256; end
if ans62 == 2 print figure6 -dgif8; end
zoom on;
end

% plot wave trajectories
%
if iop == 8
    q81=input('Want Depth contours overlap ? <y/n> : ', 's');
    q82=input('Want plot every trajectories? <y/n> : ', 's');

4.9
if q82 == 'n'
    jsk=input(' Skips how many grid points < 2, 3, 4>: ');
else
    jsk=1;
end

ntraj=fix((np-5)/jsk);
figure(8);
clf;
axis equal;
zoom off;
set(gca,'aspectratio', [axis_ratio, 1]);
set(gca, 'xlim', [xmin, xmax], 'ylim', [ymin, ymax]);
xlabel('X (m)'); ylabel('Y (m)');
title('Wave Trajectory Plot');
hold on;

% compute locations of every point in a trajectory
for j=1: ntraj
    xt=[];
yt=[];
k=1;
kp=(j-1)*jsk + 10;
if kp < (np-10)
    x1=xmax;
y1=yc(kp);
    xt(k)=x1;
yt(k)=y1;
    disp([ 'Traj. line #', int2str(j), ', j=', int2str(kp) ] );
    angle=dir(kp, mp);
    for i=1: mp-1
        ii=mp-i;
k=k+1;
x2=x1 - dx;
y2=y1 + dx*sin(angle);
    % find where the intersect point located in next x-grid level
    for L=2: np-1
        if yc(L) <= y2
            Lup=L+1;
            Ldown=L;
        end
    end
    % use linear interpretation to find wave angle and water depth
    dir1=dir(Ldown, ii);
dir2=dir(Lup, ii);
d1=y2 - yc(Ldown);
slope=abs(dir2 - dir1)/dy;
if dir2 > dir1
    angle= dir1 + slope*d1;
else
    angle= dir2 + slope*(dy - d1);
end
    dep1=d(Ldown, ii);
end

4.10
dep2 = d(Lup, ii);
slope = abs(dep2 - dep1)/dy;
if dep2 > dep1
    dep = dep1 + slope*d1;
else
    dep = dep2 + slope*(dy - d1);
end

% if dep >= 0.0
xt(k) = x2;
yt(k) = y2;
x1 = x2;
y1 = y2;
end
% completed one trajectory line
plot(xt, yt, '-');
hold on;
end
% move to the start point of next trajectory line
end
%
hold on;
if q81 == 'y'
    contour(xc, yc, d, v_dep, 'k');
end
% check if need other type of output
ans83 = input('To: 0.return; 1.gif file; 2.HPGL file : ');
if ans83 == 1
    print figure8 -dbmp256;
end
if ans83 == 2
    print figure8 -dhpgl;
end
zoom on;
end
%
clc;
disp('The following options are available : ');
disp(' 1. Select plot area');
disp(' 2. plot wave height contours');
disp(' 3. plot wave vectors');
disp(' 4. plot water depth contours');
disp(' 5. plot wave phases');
disp(' 6. plot a profile parallel to X axis');
disp(' 7. plot a profile parallel to Y axis');
disp(' 8. trajectory plot ');
disp(' -1. exit');
iop = input('please select : ');
if size(iop) == 0 iop = 0; end

4.11
end
%
end
Appendix V. Grid Generation and Control Parameter File Generation Program. The output files are used to run the program RDE.FOR. This is an example to show how to generate data file. For a more complicated cases, a comprehensive program is needed.

```fortran
program bathymetric_matrix_6

  c This program generates the input water depth matrix, I.D. codes
  c for each cell and boundary conditions that are needed for the wave
  c refraction and diffraction program RDE.FOR
  c
  c the classic rectangular harbor, Chen and Mei's configuration

  parameter (iq=400, jq=200, kg=500, Lq=2000)
  character*1 id
  character*13 name
  character*17 grdfile, bcdfile, confile, lisfile, chkfile
  character*30 header
  character*80 title
  complex zalp

  common/const/ mp, np, dx, dy, period, waveh, angle, r, dep_c
  common/array/ depth(iq,jq), id(iq,jq)
  common/bound/ nobc, ibc(Lq), jbc(Lq), zalp(Lq)
  common/given/ nog, ig(kq), jg(kq)

  c mp : mesh number in x direction
  c np : mesh number in y direction
  c period : input wave period
  c waveh : input wave height
  c angle : input wave angle
  c wlength: input wave length
  c
  c dx : spatial grid size in x-direction
  c dy : spatial grid size in y-direction
  c r : dx/dy

  c ID_code for water depth
  c : 0, water interior point
  c : 1, upper boundary condition point
  c : 2, lower boundary condition point
  c : 3, left boundary condition point
  c : 4, right boundary condition point
  c : 5, left bottom corner B.C. point
  c : 6, left top corner B.C. point
  c : 7, right bottom corner B.C. point
  c : 8, right top corner B.C. point
  c : e, land point
```

5.1
c : g, grid point that has a given wave condition
c harbor length h=0.3212m, width=0.0605m,
c
5 print*, 'Select 1. meil21; 2. meil22; 3. meil41; 4. Mei160: '
read(*,*) iption
go to (10, 12, 14, 16) iption
10 name='\break\Mei121'
title=' Classic rectangular harbor, Chen and Mei, MP=121'
  mp=121
  m1=20
  np=62
  n1=21
  n2=41
  go to 20
12 name='\break\mei122'
title=' Classic rectangular harbor, Chen and Mei, Mp=122'
  mp=122
  m1=20
  np=62
  n1=21
  n2=41
  go to 20
14 name='\break\mei141'
title=' Classic rectangular harbor, Chen and Mei, Mp=141'
  mp=141
  m1=40
  np=62
  n1=21
  n2=41
  go to 20
16 name='\break\mei161'
title=' Classic rectangular harbor, Chen and Mei, Mp=161'
  mp=161
  m1=60
  np=101
  n1=41
  n2=61
20 dep_c=0.2573
  period=2.2
  waveh=0.01
  angle=0.0
  dx=0.003212
  dy=0.003025
  call Meil_bc(m1,n1,n2)
c check array size
c
if(mp .gt. iq .or. np .gt. jq) then
  write(*,70) mp, np, iq, jq
70   format(' Given array (mp,np)=',2i5,' is larger than ',
     *   'allocated (iq,jq)=',2i5)
stop  
end if  

grdfile=\'name // \'.grd\'  
open(8,file=grdfile,form=\'formatted\', status=\'unknown\')  
write(8,'(a80)') title  
write(8,100) mp, np, dx, dy  
100 format(\' M N dx dy xL yL zmin\',  
* \' zmax noz yorient\'/ 2i5, 2f10.6,\' 0.000 0.000',  
* \' 0.10 0.10 0 0.0\')  

c  
header=\' Water depth metrix (meter) \'  
write(8,'(a30)') header  
header=\'+-------------------------> j\'  
write(8,'(a30)') header  
do i=1,mp  
write(8,120) i, (depth(i,j), j=1,np)  
120 format(i5/(10f8.4))  
end do  
close(8)  

bcdfile=\'name // \'.bcd\'  
open(8,file=bcdfile,form=\'formatted\', status=\'unknown\')  
write(8,'(a80)') title  
write(8,100) mp, np, dx, dy  
header=' I.D. code for each grid point'  
write(8,'(a30)') header  
header=\'+-------------------------> j\'  
iblock=np/75 + 1  
do ib=1,iblock  
   write(8,'(a30)') header  
   numb1=(ib-1)*75 + 1  
   numb2=numb1+75-1  
   if( numb2 .gt. np) numb2 = np  
do i=1,mp  
   write(8,140) i, (id(i,j), j=numb1,numb2)  
140 format(i4,1x,75a1)  
end do  
end do  

c  
header='Given B.C. at following points'  
phase=0.0  
write(8,'(i5,2x,a30)') nog, header  
write(8,150)  
150 format(' n i j id phase\')  
do i=1,nog  
   write(8,155) i, ig(i),jg(i), id(ig(i),jg(i)), phase  
155 format(1x,3i5, 4x,a1, f10.6)  
end do  

c  
header='Radiation B. C. at points... '  
write(8,'(i5,2x,a30)') nobc, header
write(8,160) format( ' n i j i d r_alp i_alp')
do i=1,nobc
  write(8,170) i, ibc(i), jbc(i), id(ibc(i),jbc(i)), zalp(i)
end do
170 format(1x,3i5,4x,a1,2f8.2)
close(8)
c open an control file for control parameters
c
confile=name // '.con'
lisfile=name // '.lis'
chkfile=name // '.chk'
icase=46

open(8,file=confile,form='formatted')
write(8,'(a80)') title
write(8,(a18/a18/a18/a18/' 0.0 0.0 0.0')
write(8,200) mp, np, dx, dy, icase
200 format(2i5, 2f10.6,i5, ' 0.0')
write(8,210)
210 format(' 0.01 2.20 0.0 0.0'/' 0.010 1.90 0.0 0.0'/' 0.010 1.70 0.0 0.0'/' 0.010 1.50 0.0 0.0'/' 0.010 1.30 0.0 0.0'/' 0.010 1.25 0.0 0.0'/' 0.010 1.20 0.0 0.0'/' 0.010 1.15 0.0 0.0'/' 0.010 1.14 0.0 0.0'/' 0.010 1.13 0.0 0.0'/' 0.010 1.12 0.0 0.0'/' 0.010 1.13 0.0 0.0'/' 0.010 1.11 0.0 0.0'/' 0.010 1.06 0.0 0.0'/' 0.010 1.04 0.0 0.0'/' 0.010 1.00 0.0 0.0'/' 0.010 0.95 0.0 0.0'/' 0.010 0.90 0.0 0.0'/' 0.010 0.80 0.0 0.0'/' 0.010 0.70 0.0 0.0'/' 0.010 0.65 0.0 0.0'/' 0.010 0.62 0.0 0.0'/' 0.010 0.61 0.0 0.0'/' 0.010 0.60 0.0 0.0'/' 0.010 0.59 0.0 0.0'/' 0.010 0.58 0.0 0.0'/' 0.010 0.57 0.0 0.0'/' 0.010 0.56 0.0 0.0'/' 0.010 0.56 0.0 0.0'/' 0.010 0.55 0.0 0.0'/' 0.010 0.55 0.0 0.0'/' 0.010 0.54 0.0 0.0'/' 0.010 0.53 0.0 0.0')
close(8)
c stop
end

------------------------------------------------------------------

subroutine MeiL_BC(m1,n1,n2)
c

generates the water and ID depth matrix

5.4
parameter (iq=400,jq=200, kg=500, Lq=2000)
character*1 id
complex zalp
common/const/ mp, np, dx, dy, period, waveh, angle, r, dep_c
common/array/ depth(iq,jq), id(iq,jq)
common/bound/ nobc, ibc(Lq), jbc(Lq), zalp(Lq)
common/given/ nog, ig(kq), jg(kq)

C first establish the ID code and water depth for the entire grid point

C
C do j=1,np
C   do i=1,mp
C      id(i,j)='0'
C      depth(i,j)=dep_c
C   end do
C end do

C given boundary at the left hand side

C
C nog=0
C do j=2,np-1
C    nog=nog+1
C    ig(nog)=1
C    jg(nog)=j
C    id(ig(nog),jg(nog))='g'
C end do
C nog=nog+1
C ig(nog)=1
C jg(nog)=1
C id(ig(nog),jg(nog))='p'
C nog=nog+1
C ig(nog)=1
C jg(nog)=np
C id(ig(nog),jg(nog))='q'

C Check the Array size
C
C if(nog .gt. kg) then
C   write(*,10) nog, kg, nog
C 10 format(' The # of point for given B.C.=',i6,' > Kq=',i6/
C      *    ' Change the statement to Kq=',i6,' and recompile')
C   stop
C end if

C NOBC : number of boundary points
C
C nobc=0

C top side B.C.
do i=2,m1-1
  nobc=nobc+1
  ibc(nobc)=i
  jbc(nobc)=np
  id(ibc(nobc), jbc(nobc))='1'
  zalp(nobc)=1.0
  end do

do i=m1+1,mp-1
  nobc=nobc+1
  ibc(nobc)=i
  jbc(nobc)=n2
  id(ibc(nobc), jbc(nobc))='1'
  zalp(nobc)=0.0
  end do

c bottom side B.C.
c
  do i=2,m1-1
    nobc=nobc+1
    ibc(nobc)=i
    jbc(nobc)=1
    id(ibc(nobc), jbc(nobc))='2'
    zalp(nobc)=1.0
    end do
  do i=m1+1,mp-1
    nobc=nobc+1
    ibc(nobc)=i
    jbc(nobc)=n1
    id(ibc(nobc), jbc(nobc))='2'
    zalp(nobc)=0.0
    end do

c right side total reflection B.C.
c
  do j=n1+1,n2-1
    nobc=nobc+1
    ibc(nobc)=mp
    jbc(nobc)=j
    id(ibc(nobc), jbc(nobc))='4'
    zalp(nobc)=0.0
    end do
  do j=2,n1-1
    nobc=nobc+1
    ibc(nobc)=ml
    jbc(nobc)=j
    id(ibc(nobc), jbc(nobc))='4'
    zalp(nobc)=0.0
    end do
  do j=n2+1,np-1
    nobc=nobc+1
    ibc(nobc)=ml
    jbc(nobc)=j
    id(ibc(nobc), jbc(nobc))='4'
    zalp(nobc)=0.0
    end do

5.6
jbc(nobc)=j
id(ibc(nobc), jbc(nobc) )='4'
扎lp(nobc)=0.0
end do

c land points
c
do i=m1+1,mp
do j=1,n1-1
   id(i,j)= 'e'
   end do
do j=n2+1,np
   id(i,j)= 'e'
   end do
end do

c right bottom corner b.c.
c
nobc=nobc+1
ibc(nobc)=m1
jbc(nobc)=1
id(ibc(nobc), jbc(nobc) )='7'
扎lp(nobc)=(0.0, 1.0)
nobc=nobc+1
ibc(nobc)=mp
jbc(nobc)=n1
id(ibc(nobc), jbc(nobc) )='7'
扎lp(nobc)=(0.0, 0.0)
c
right top corner b.c.
c
nobc=nobc+1
ibc(nobc)=m1
jbc(nobc)=np
id(ibc(nobc), jbc(nobc) )='8'
扎lp(nobc)=(0.0, 1.0)
nobc=nobc+1
ibc(nobc)=mp
jbc(nobc)=n2
id(ibc(nobc), jbc(nobc) )='8'
扎lp(nobc)=(0.0, 0.0)
c
c check the array assignment
c
if(nobc .gt. Lq) then
   write(*,20) nobc, Lq, nobc
20   format(' The # of point for Radiation B.C.=',i6,' > Lq=',i6/
         *         ' Change the statement to Lq=',i6,' and recompile ')
   stop
end if
return
end