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A MATHEMATICAL APPROACH TO DEPURATION

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ABSTRACT

Two equations can be written to describe the change in bacterial concentrations in the shellfish and the water in depuration units. Analytical solutions of these equations for special conditions and numerical solutions for general conditions indicate that there will always be an initial, rapid decrease in bacteria in the oysters during depuration. With suitable flow rates and loading rates, the die off will be exponential and several orders of magnitude reduction can be achieved within 72 hours. Both very low flow rates and very low loading rates increase the residence time of water in the tank, and therefore depuration will occur slowly after about 24 hours. In addition, the bacterial levels at 72 hours may be quite high for the case of very low flow rates. Further improvements and verification of the model are desired, but use of the model can aid in the design and operation of depuration plants now.

INTRODUCTION

Although researchers over the years have investigated and described the various biological functions and environmental factors which are important to the process of depuration, to the author's knowledge, very little work has been done to incorporate these findings into a unified theory of depuratioı.. As a first step in that process, the present study attempts to describe depuration from a phenomenological point of view. The author is aware that this approach neglects many of the physiological aspects of the process and that the model system under consideration is a highly simplified version of the real world situation. Furthermore, no attempt was made to duplicate the results of depuration experiments. Rather the purpose of this study is to determine whether or not the simple model can simulate depuration in general. It is the author's opinion that the mathematical analysis can lead to a better understanding of the interactions which occur and provide a means of evaluating the hydraulic design of flow systems for depuration units. It is hoped that future work can expand the model to incorporate more parameters and to make the model more realistic. But the first need is to demonstrate that the model is a sound one.

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EQUATIONS OF DEPURATION

Two equations are needed to describe the total depuration process: one for the shellfish and one for the water in the tank. The first equation tells how the number of bacteria in a shellfish varies with time, and the second equation accounts for changes in the concentration of bacteria in the water. These equations are:

$$dE/dt = -k\,E + p\,f\,c \qquad (1)$$
$$dc/dt = (+k\,E - p\,f\,c - q\,c)\,N/V \qquad (2)$$

The first equation says that the time rate of change of E, the number of bacteria per shellfish, is equal to a fixed percentage, k, of the bacteria present (negative because they are excreted) and a fixed percentage, f, of the bacteria in the water, c, which is pumped through the shellfish at the volumetric rate, p. The second equation says that the time rate of change of the concentration of bacteria in the water is equal to those excreted by the shellfish minus those filtered out by the shellfish and those lost from the system. Note that it is assumed that the incoming water has been completely disinfected so there is no term for incoming bacteria. Also since q is the flow of water through the tank *per shellfish,* this term, as well as the other two, must be multiplied by N, the number of shellfish in the tank, and divided by V, the volume of water in the tank to put everything in terms of concentration. All terms in the equations, brief descriptions, units and typical values are listed in Table 1.

A better understanding of these equations can be gained if specific cases are considered. For this study, the depuration of fecal coliforms by the eastern oysters during summer conditions will be used. Assumptions made to relate numbers and volumes of oysters, and other factors are given in Table 2. With a few exceptions, these values have been taken from the Public Health Service publication entitled "Depuration Plant Design" (Furfari, 1966). It should be noted that the equations are written for "ideal" oysters whose behavior matches the average values of a set of real oysters. For the purposes of this study, this idealization does not limit the usefulness of the equations.

TABLE 1. *Symbols Used in Depuration Equations*

Term	Description	Unit
E	number of coliforms per shellfish	$\frac{\text{MPN}}{\text{oyster}}$
c	number of coliforms per volume of water (concentration)	$\frac{\text{MPN}}{\text{100 ml}}$
t	time	hour
k	decay rate	1/hour
f	filtering factor	1
q	flow of water through tank per shell fish (specific flow rate)	$\frac{\text{liters}}{\text{hour-oyster}}$
N	number of shellfish in tank	oysters
V	volume of water in tank	liters
t_{res}	residence time of water	hour

TABLE 2: *Environmental and Behavioral Constants*

1 bushel	=	225 oysters
1 oyster	weighs	25 grams
Temperature	=	20-25°C
Pumping rate	=	10 liter/hour
Specific flow rate	=	$\frac{\text{1 gallon}}{\text{minute-bushel}} = \frac{\text{1 liter}}{\text{hour-oyster}}$
Decay rate	=	0.17/hour
Filtering factor	=	0.005

During the initial stages of depuration, the bacterial concentration in the water will be zero or very small. This situation could also arise if the flow around the oyster completely removed all fecal matter. For these cases, equation (1) reduces to

$$dE/dt = -k E \qquad (3)$$

for which the solution is $E = E_o e^{-kt}$. In other

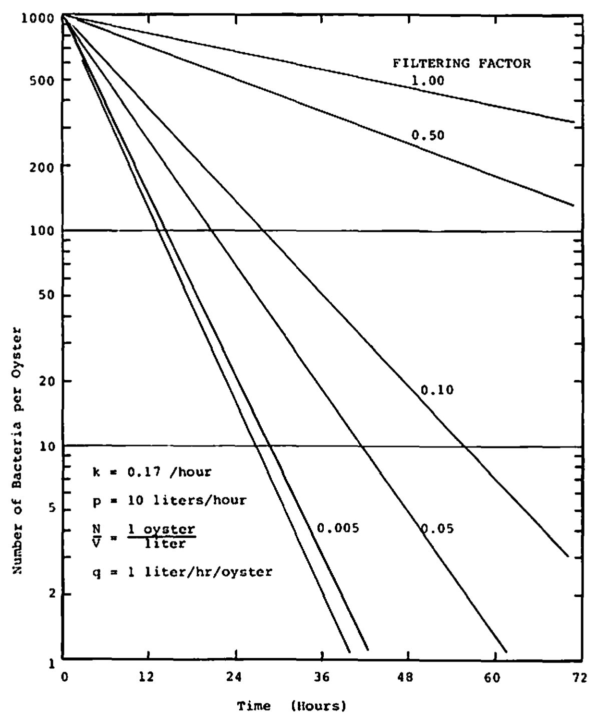

FIG. 1. *Oyster Depuration for Various Filtering Factors.*

words, when there is no feed back of bacteria to the oyster, the decline in bacterial levels within the oysters will be exponential. Thus one could determine the value of k by conducting experiments in which the ambient concentrations are kept very, very low. For this study, k has been assumed equal to 0.17 per hour, or in other words, every hour 17% of all bacteria in the oyster are voided to the water. This means that concentrations will be reduced to one tenth of the original concentration in 13.5 hours, and to 1% of the original value in 27 hours. (Fig. 1).

When natural waters have a relatively constant bacterial level, an equilibrium is reached such that the number of bacteria excreted by an oyster equals the number ingested from the water it pumps. That is, $dE/dt = 0$ and

$$k E_e = pfc_e \qquad (4)$$

or

$$\frac{E_e}{c_e} = \frac{pf}{k} \qquad (5)$$

Equation 5 says that the equilibrium concentration factor (E/c) is equal to the product of the pumping rate, p, and the filtering factor, f, divided by the decay rate, k. This relationship appears to be sound physiologically, because studies have shown that when the water temperature increases from 10C to 20C, the pumping rate and the concentration ratio both increase. (Furfari, 1966). If we assume that this relationship does hold, then equation (5) presents a means of determining the filtering factor, f. If there is an ambient bacterial concentration of 330 MPN/100 ml (or 3300 MPN/liter) then the bacterial concentration in the oyster for summer conditions and Virginia growing areas will be around 4000 MPN/100 grams. (Reference to Hope & Wiley, 1961 in Furfari). If an average oyster weighs 25 grams, then E will be 1000 MPN/oyster. Thus the concentration ratio as defined above is about 0.3 and the filtering factor is 0.005. In other words, the oyster ingests 0.5% of the coliforms in the water which it pumps. Pumping rate has been assumed to be 10 liters per hour.

If all factors other than E and C are assumed to be constant with respect to time, then several analytical methods of solution are available. For this study, the coupled equations (1) and (2) were transformed to finite difference form and programmed on a Hewlett Packard 9800 desk top calculator. The time interval for integration was 0.1 hour. Die off curves for several values of the filtering factor are shown in Figure 1. It is interesting to note that for the assumed flow rate and biomass to volume ratio, the decay of bacteria in the oyster is always exponential. Also depuration occurs, albeit slowly, even if 100% of the bacteria are filtered by the oyster from the water it pumps. For the rest of this study, it will be assumed that $k = 0.17$ per hour, $p = 10$ liters per hour and $f = 0.005$. From a mathematical point of view, the curves shown in Figure 1 appear to be reasonable, but certainly there is need for verification of these biological coefficients. Equation (2) describes the changes in concentration levels in the water in the depuration tank. It includes the two factors which are amenable to control by the designer and operator of a depuration plant: N/V, the biomass to volume ratio or the loading rate, and q, the specific flow rate. The effect of these factors can be illustrated by an analysis similar to that done for Equation (1).

During the initial phase of depuration, E is very large and c is small. Thus the term, kE, is the dominant one and c will increase rapidly. Typically the maximum value for c is attained in the first or second hour of depuration, and it decreases slowly there after. However, E declines rapidly and the kE term quickly becomes so small that it can be neglected during the later stages of depuration. Since the specific flow rate, q, is 1 liter per hour and pf is only 0.05 liters/hour, Equation (2) can be approximated by

$$dc/dt = \frac{Nqc}{V} = (1/t_{res})\, c \qquad (6)$$

where t_{res} = the mean residence time for a parcel of water flowing through the tank, defined as the quotient of the water volume and the total flow rate, which in turn is equal to the specific flow rate times the number of oysters.

$$t_{res} = V/Q = V/Nq \qquad (7)$$

The solution to equation (6) will contain the term $e^{-t/t_{res}}$. In other words, the rate of change of the concentration in the tank will be a function of the residence time of the water.

It should also be noted that if there is no growth of bacteria and if there is some finite flow of water through the tank, there will be a continual loss of bacteria from the system. Thus the only equilibrium which can occur is when $c = 0$. The rate of decay for low flow rates and large residence times will be very slow, so that depuration will not occur over a practicable period of time. One might criticize the model for not including a term for the growth of bacteria since fecal pellets and other detritus can provide a suitable medium for growth to occur. However, any growth of bacteria in the tank will slow down and interfere with the depuration process. Thus, the condition of no growth is the one which is desired and operating procedures should be designed to remove the biodeposits at frequent intervals.

OPERATIONAL CONTROLS

The two factors which are directly under the control of a depuration plant operator are the flow of water through the tank and the loading rate. Several model runs were made to examine the behavior of the system to variation in these two factors. In Figure 2, the die off curves for a range of specific flow rates are shown. If the specific flow rate is very small, as illustrated by the curve for 0.01 liters/hour/oyster, there is an initial period of rapid decay, followed by an intermediate transtition period and a final slow decay. In the initial stages the die off will be exponential with the decay rate equal to k. In the final stages the die off will be exponential as well, but the decay rate will be smaller and equal to $1/t_{res}$. Since q is small, the residence time is large and the inverse of the residence time will be small too. It is important to note that in addition to the slow die off, the reduction during a 72 hour period is very small. Less than an order of magnitude reduction occurs during this period.

For a specific flow rate of 1 liter/hr/oyster, the reduction in decay rate is much less pronounced but still present. The bacterial level is reduced to less than 1% of the initial count within 60 hours. As the flow rate is increased further, there are additional increases in the decay rate. But above 10 liters/hr/oyster only marginal increases in depuration rate are achieved for large increases in flow rate.

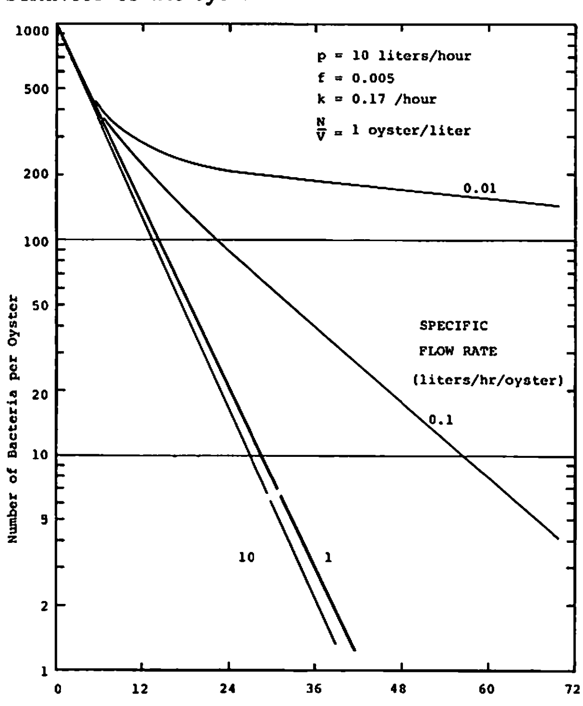

FIG. 2. *Oyster Depuration for Various Specific Flow Rates.*

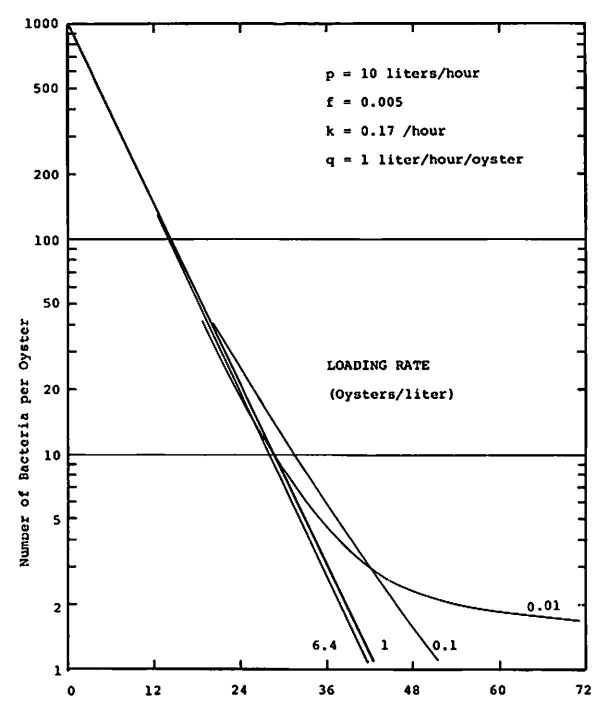

FIG. 3. *Oyster Depuration for Various Loading Rates.*

The loading rate shows variations somewhat similar to those for the specific flow rate. The maximum value which can be achieved is a bushel of oysters in a bushel of water or 6.4 oysters/liter of water. Only a modest variation in depuration is seen if the loading rate is reduced to 0.1 oysters/liter. If the loading rate is reduced by an additional order of magnitude to 0.01 oysters/liter, the final stage of depuration occurs at a slow rate. For this situation, the bacterial levels are reduced to less than 1% of the original value within 48 hours before the reduction in decay rate takes effect.

The two cases of low specific flow rate and low loading rate are related in that each has a long residence time. In each case an initial period of rapid die off will be followed by a period of very slow depuration. With extremely low flow rates, bacterial levels in the water can be high, whereas for low loading rates, a large volume of water is available to dilute the bacteria given off by the oysters. For example, when $q = 0.01$ liter/hr/oyster and $N/V = 1$ oyster/liter, the water concentration at 24 hours is 65 MPN/100 ml (Fig. 2). But when $q = 1$ liter/hr/oyster and $N/V = 0.01$ oyster/liter the bacterial level in the water is only 0.8 MPN/100 ml at 24 hours (Fig. 3). For both cases, the bacteria in the water will be removed from the tank slowly, but the higher levels present with low flow rates will have the effect of maintaining higher levels in the oysters.

DISCUSSION

The sample runs show several features of the depuration process which have a bearing on the operation of a depuration plant. First, there is an initial rapid decay in bacterial levels no matter what flow rate and loading rate are used. Therefore, one possible means of achieving suitable reductions in coliform counts is to hold the oysters with no through flow. At periodic intervals the water should be flushed from the tank and it would be replaced by clean water. Bacterial reductions for several holding times are given in Table 3. All frequencies for removing and renewing the water appear to work, so that biological factors should be used to choose the best frequency. For example, the oysters may not begin to pump until a half hour after being immersed, so that a longer interval would be better. Also, some

TABLE 3. *Reduction in Bacterial Levels With No Flow Through Tank*

Time	Hour	2 Hrs.	3 Hrs.	4 Hrs.	6 Hrs.
0	1000	1000	1000	1000	1000
12	142	145	152	162	186
24	20	21	23	26	35
36	3	3	3	4	7

means will be needed to maintain dissolved oxygen concentrations. Submerged air diffusers appear to be a very efficient and cost-effective method.

The model indicates that the oysters can never be packed too densely in the tank. Implicit in the model is the assumption that the water in the tank is mixed rapidly so that the concentration in the water does not vary significantly throughout the tank. Additionally, the removal of fecal matter is very important. Thus the need for a good circulation in the tank, rather than considerations of depuration rate, may dictate a limit to the loading rate. But from a mathematical point of view, increased loading rates will not hinder the depuration.

The specific flow rate is much more difficult to characterize. If a high flow rate is used, a great deal of energy will be expended unnecessarily. On the other hand, if a very low flow rate is used, not only will the depuration rate be slow but also the reduction in coliform counts will be insufficient. The best course of action is to run tests in the tank to document the rate of depuration for various flow rates. The model could then be used to determine a specific flow rate that insured the necessary reduction in bacterial levels without wasting energy.

In summary, the mathematical approach to depuration can be very fruitful. There is need for input from biologists as to the acceptability of the various assumptions and simplification, the correct values for the biological coefficients and the best means of improving the model. However, even at this stage, the combination of actual depuration runs and mathematical analysis can aid in the design and operation of depuration plants.

LITERATURE CITED

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