1972

Nuclear polarization and charge parameters using the muonic 2s state

Peter Martin

College of William & Mary - Arts & Sciences

Follow this and additional works at: <https://scholarworks.wm.edu/etd>

Recommended Citation

Martin, Peter, "Nuclear polarization and charge parameters using the muonic 2s state" (1972).
Dissertations, Theses, and Masters Projects. Paper 1539623659.
<https://dx.doi.org/doi:10.21220/s2-455r-y906>

This Dissertation is brought to you for free and open access by the Theses, Dissertations, & Master Projects at W&M ScholarWorks. It has been accepted for inclusion in Dissertations, Theses, and Masters Projects by an authorized administrator of W&M ScholarWorks. For more information, please contact scholarworks@wm.edu.

INFORMATION TO USERS

This dissertation was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.
2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.
3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in "sectioning" the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again — beginning below the first row and continuing on until complete.
4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from "photographs" if essential to the understanding of the dissertation. Silver prints of "photographs" may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.

University Microfilms

300 North Zeeb Road
Ann Arbor, Michigan 48106

A Xerox Education Company

72-23,122

MARTIN, Peter, 1942-
NUCLEAR POLARIZATION AND CHARGE PARAMETERS
USING THE MUONIC 2s STATE.

The College of William and Mary in Virginia,
Ph.D., 1972
Physics, nuclear

University Microfilms, A XEROX Company, Ann Arbor, Michigan

NUCLEAR POLARIZATION AND CHARGE PARAMETERS USING THE MUONIC 2s STATE

A Thesis

Presented to

The Faculty of the Department of Physics
The College of William and Mary in Virginia

In Partial Fulfillment

of the Requirements for the Degree of
Doctor of Philosophy

---

by

Peter Martin

September, 1971

APPROVAL SHEET

This thesis is submitted in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

Peter Martin

Peter Martin

Approved, September, 1971

Robert E. Welsh

Robert E. Welsh

Morton Eckhause

Morton Eckhause

John R. Kane

John R. Kane

Richard J. Powers

Richard J. Powers

Raymond W. Southworth

Raymond W. Southworth

Hans C. von Baeyer

Hans C. von Baeyer

PLEASE NOTE:

Some pages may have
indistinct print.

Filmed as received.

University Microfilms, A Xerox Education Company

NUCLEAR POLARIZATION AND CHARGE PARAMETERS USING THE MUONIC 2s LEVEL

TABLE OF CONTENTS

	Page
ABSTRACT	v
PREFACE	2
I. INTRODUCTION	3
A. Historical Developments	4
B. The Muon as a Nuclear Probe	5
C. Charge Distribution	13
D. Nuclear Polarization	18
II. EXPERIMENTAL DETAILS	22
A. Physical Details	22
B. Electronics	24
C. Experimental Procedures	27
D. Energy Evaluation	30
E. Data Analysis	31
F. Isotope Corrections	35
G. Nuclear Parameters	36
H. Relative Intensities	38
III. THEORY	42
A. The Solution to the Dirac Equation	42
B. Group One Corrections	45
1a. Vacuum Polarization	45
1b. Lamb Shift	46
1c. Anomalous Magnetic Moment	47
C. Group Two Corrections	48
2a. Nuclear Polarization	48
2b. Higher Order Vacuum Polarization	52

	Page
D. Group Three Corrections	52
3a. Weak Interactions	53
3b. Electron Shielding	53
3c. Static Hyperfine Effects	53
3d. Dynamic Hyperfine Effects	53
IV. RESULTS	55
A. General Procedure	55
B. The Nucleus ^{120}Sn	56
C. The Nucleus ^{138}Ba	59
D. The Nucleus ^{140}Ce	62
E. The Nucleus ^{142}Ce	64
F. The Nucleus ^{206}Pb	67
G. Summary of Results	72
V. CONCLUSIONS	75
VI. ACKNOWLEDGMENTS	78
VII. REFERENCES AND FOOTNOTES	81
VIII. TABLES	89
IX. FIGURES	94

ABSTRACT

Muonic x-ray transition energies have been determined in ^{120}Sn , ^{138}Ba , ^{140}Ce , ^{142}Ce and ^{206}Pb using a high efficiency Ge(Li) detector. Precise energy measurements, relative intensity measurements, and a comparison of energy splittings have made it possible to identify transitions to and from the 2s state in each of these targets. There is good agreement between theoretical and experimental relative intensities for these transitions, except for ^{206}Pb , where the experimental values are much smaller than expected, and the 3p $1/2$ -2s transition could not be detected at all. These transition energies, together with those from the 2p-1s, 3d-2p and 4d-2p transitions are used to deduce sets of nuclear parameters.

It has been found that for each target except ^{120}Sn , the best agreement between theory and experiment is obtained by applying the theoretical nuclear polarization corrections to all levels with $n > 1$, and allowing the nuclear polarization induced by the 1s muon to vary. The nuclear polarization values determined were $4.5^{+3.0}$ keV for ^{120}Sn , $6.3^{+1.3}$ keV for ^{138}Ba , $5.8^{+1.2}$ keV for ^{140}Ce , $6.3^{+1.4}$ keV for ^{142}Ce and $5.8^{+2.0}$ keV for ^{206}Pb . These results are in good agreement with the most recent theoretical predictions. It is concluded that nuclear polarization is an important effect and should be taken into account in muonic atoms.

In addition, first forbidden electric quadrupole transitions have been detected in all targets except ^{120}Sn , and it is shown that these should be included in the muonic cascade to account for the measured relative intensities.

".....We do not know how big this polarization effect is. It has been lumped into the parameterization of the charge distribution. It is obviously very difficult to determine it and to know you have determined it."

D. H. Wilkinson.
(Summary to Williamsburg Conference on
Intermediate Energy Physics, 1966.)

When a muon is used as a nuclear probe it must interact with the nucleus in order for one to derive any information about the nucleus. One of the results of this electrostatic interaction is called the nuclear polarization. This experiment was designed to measure nuclear charge parameters and the nuclear polarization by detecting and measuring muonic x-ray transitions involving the 2s state, second only to the 1s state in sensitivity to nuclear charge parameters. This goal has been achieved and the results have been analyzed to deduce the nuclear polarization produced by the muon in the 1s state. The results are not entirely unambiguous, but when analyzed in terms of the framework set forth, suggest that there is a good agreement with theoretical predictions.

I. INTRODUCTION

Muonic x-rays were first postulated to exist in the late 1940s and the early 1950s,¹ and then experimentally confirmed in 1953.² These studies enjoyed a period of steady development and investigation until the advent of the solid-state detector in 1964. The superior resolution of the solid-state Ge(Li) detector over the previously used NaI(Tl) crystal stimulated experimental work which was able to probe both the static and dynamic properties of the muonic atom.

Excellent reviews of the early developments are given by Burbridge and de Borde, 1953,³ Rainwater, 1957,⁴ and West, 1958.⁵ More recent reviews which include results using Ge(Li) detectors are by Devons and Duerdoth, 1969,⁶ Wu and Wilets, 1969,⁷ and Scheck and Hüfner, 1971.⁸

Summaries of experimental results can be found in the later reviews or in the reports of the conferences at Williamsburg, 1966,⁹ Ottawa, 1967,¹⁰ and Columbia, 1970.¹¹ After a brief history is given, the value of the muon as a nuclear probe will be demonstrated, together with the reasons for measuring nuclear polarization and the techniques adopted. Succeeding chapters will then cover the theory underlying muonic atoms and nuclear polarization (Chapter III); the experimental methods and data analysis (Chapter II); the results obtained (Chapter IV), and the conclusions and indications of further work suggested by these results (Chapter V).

A. Historical Development

The theoretical basis of muonic atoms was established by Wheeler in 1949,¹ when he presented techniques for evaluating transition energies and transition rates in muonic atoms. This was done in part to explain the experimental evidence of Chang,¹² who had observed γ rays following cosmic rays stopping in metal foils. Wheeler had previously written on the anomalous absorption rates of negative cosmic muons,¹³ which were observed by Conversi et al. in 1947.¹⁴ The idea of muonic atoms had been initially discussed in 1947 by Fermi and Teller,¹⁵ who showed that the stopping and cascade times for a negative muon were far shorter than its mean life. This indicated that muonic atoms should live long enough to be experimentally observed. These theoretical predictions received experimental confirmation in 1953, when Fitch and Rainwater,² were able to measure muonic transition energies in ^{208}Pb to 1% precision using a Na I crystal. This enabled them to detect the fine-structure splitting predicted by Wheeler and to sensibly apply the corrections that Cooper and Henley had evaluated for vacuum and nuclear polarization.¹⁶ As a consequence of this work the radius of the uniform spherical charge distribution used to represent the nucleus was substantially reduced. The usual representation for the uniform radius in fermis is

$$R_0 = r_0 A^{1/3}$$

where A is the nuclear mass number. Prior to the work of Fitch and Rainwater the accepted value was $r_0 = 1.5 \text{ F}$ and was obtained from electronic x-ray data by Schawlow and Townes.¹⁷ The muonic x-ray results predicted $r_0 = 1.2 \text{ F}$ and corroborated the results of Bitter and Feshbach,¹⁸ who had re-evaluated isotope shifts and electron scattering data.

The idea that muonic x-rays would be a useful tool for measuring nuclear charge parameters was originally suggested by Wheeler,^{1,19} and was extended by Hill and Ford.²⁰ Hill and Ford also attempted to fit the experimental data to both uniform and non-uniform charge distributions. They pointed out that the 3d-2p and 2s-2p transitions were highly sensitive to the shape of the nuclear charge distribution and should be used, together with the 2p-1s transitions, to determine nuclear charge parameters. Although they showed that the 2s-2p transitions were far more sensitive than the 3d-2p, it has taken 15 years before the present work,^{21,22} and the work of Anderson et al.²³ have attempted to utilize these charge-sensitive transitions. The main reason for this was the relatively poor energy resolution of the Na I crystals, a barrier which was removed with the development of the solid state Ge(Li) detector.²⁴

The period since then has seen the development of high efficiency Ge(Li) detectors, which allow the detection of these very weak transitions in a reasonable time.

B. The Muon as a Nuclear Probe

The muon is a spin 1/2 particle of intermediate mass (~ 207 electron masses) which can possess a charge of either sign and which can interact with other particles only through electromagnetic, weak and gravitational forces. The latter two can be neglected because of their magnitude, and the negatively charged muon interacts electrostatically with a charged nucleus and can populate Bohr orbits around the nucleus. In the n^{th} level, or Bohr orbit, the energy E_n and the radius r_n are given approximately by

$$E_n = - \frac{mZ^2e^4}{2n^2\hbar^2} \quad 1$$

$$r_n = \frac{n^2\hbar^2}{mZe^2}$$

where e is the electron charge, Ze is the nuclear charge and m is the reduced mass of the muon-nucleus system. If m_μ and m_N are the masses of the muon and the nucleus respectively then

$$m = \frac{m_N m_\mu}{m_N + m_\mu} \quad 2$$

Equation 1 may be used to compare the Bohr radii and energy levels of the muon and the electron. For the same principal quantum number, the muonic energy levels are much greater, whereas muonic radii are much smaller. This leads to a substantial overlap between the muonic and nuclear wave functions. For $Z = 80$ the 1s muonic Bohr radius is ~ 3 F as compared to the experimental nuclear radius of 7 F. The electronic 1s Bohr radius ~ 600 F so that all muonic orbits with $n \leq 14$ will effectively lie within the electron cloud, and the muon will see the bare nuclear charge Ze . Furthermore, because the muon in its lower energy levels spends a considerable fraction of its time within the nucleus, the lower muonic energy levels will be sensitive to the form of the nuclear charge distribution.

The lifetime of a bound negative muon ranges from 2×10^{-6} sec to 10^{-7} sec,²⁵ and since the total time span for atomic capturing of a muon with a momentum of several hundred MeV/c and the subsequent cascade to its ground state, is about 3×10^{-9} sec,¹⁵ muonic atoms should be readily formed. The muon is initially captured into high orbital angular momentum states from which it cascades to the ground state emitting Auger electrons and electromagnetic radiation.²⁶ In the

higher angular momentum states at large n the dominant cascade process is Auger emission, because of the proximity of the electron cloud. From the lower orbits the dominant process is electromagnetic radiation. The relative probabilities of dipole radiation and Auger emission have been discussed by Burbridge and de Borde.³

With the advent of high resolution detectors it has become necessary to account for electric quadrupole transitions,²⁷ the dynamic effects of resonances,^{28,29} and prompt neutron emission induced during the cascade.³⁰ These effects, with the exception of quadrupole transitions, will not be considered in detail.

Once in the ground state the muon can either decay by

$$\mu^- \rightarrow e^- + \nu + \bar{\nu}$$

or can be captured by the nucleus in approximately $4 \times 10^{-7} \left(\frac{82}{Z}\right)^4$ sec.¹

This last mechanism can proceed through the following channels:

1. $\mu^- + (Z, N) \rightarrow (Z-1, N)^* + \nu$ (Non-Radiative Capture)
2. $\mu^- + (Z, N) \rightarrow (Z-1, N)^* + \nu + \gamma$ (Radiative Capture)
3. $\mu^- + (Z, N) \rightarrow (Z-1-\beta, N-\alpha)^* + \alpha n + \beta p + \nu$ (Nucleon emission)

The observations of transitions in the daughter nucleus can provide information on the relative probabilities of the various processes. Experimentally it is found that proton emission following μ^- capture is extremely rare in moderate to high Z nuclei.^{31,32}

Muon wave functions satisfy the Dirac equation:

$$Hu = Eu = (c \vec{\alpha} \cdot \vec{p} + \beta mc^2 - e\phi(r))u \quad 3$$

where $\vec{\alpha}$ and β are Dirac matrices and $\phi(r)$ is the potential experienced by the muon.³³ This equation is solved numerically for a predetermined nuclear charge distribution function, using the method of McKee,³⁴ and yields values for the wave functions u and the energy eigenvalues E .

As a comparison, the energies are also calculated when $\phi(r)$ is evaluated in the field of a point nucleus. For a state with principal quantum number n , and angular momentum ℓ

$$E_{n,\ell} = mc^2 \left[1 + \left(\frac{\alpha Z}{n - |k| + \sqrt{k^2 - \alpha^2 Z^2}} \right)^2 \right]^{-1/2} \quad 4$$

where

$$\alpha = \text{fine structure constant} = e^2/\hbar c$$

$$k = j + 1/2$$

The difference between $E_{n\ell}$ and the numerical solution to equation 3 can be considered a measure of the sensitivity of each muonic level to the finite size of the nucleus. In Table 1 are listed the muonic binding energies ($=mc^2 - E_{n\ell}$) of the lower levels for ^{138}Ba and ^{206}Pb . The muonic levels exhibiting strong finite size effects are the $1s$, $2s$, $3s$, $2p_{1/2}$, $2p_{3/2}$, $3p_{1/2}$, and $3p_{3/2}$ levels. The other levels are experimentally indistinguishable from those for a point nuclear charge. This is demonstrated in Fig. 1 where wave function probability densities for various levels in ^{138}Ba are plotted as a function of radial distance from the center of the nucleus. A typical charge distribution has been superimposed to demonstrate the overlap of nucleus and wave functions. In practice a wave function value occupies a volume $4\pi r^2 dr$, and Fig. 2 shows the linear probability densities ($r^2 \times$ wave functions' probability density), as a function of r , with a charge distribution again superimposed. This demonstrates the finite size effect which is caused by the overlap of the linear probability density and the charge distribution.

Each level experiences a different theoretical dependence on the nuclear charge distribution, and at most, the number of deducible

parameters will be the number of distinct levels that have been measured experimentally. In this context the 2p levels and the 3p levels must each be treated as a single dependence because of the similarity of their linear probability densities (Fig. 2). Until recently the standard method used to derive charge parameters from energy measurements involved measuring the dominant transitions, the 2p-1s doublet and the 3d-2p triplet.³⁵ Theoretical predictions were obtained by solving the Dirac equation for a standard 2-parameter Fermi distribution

$$\rho(r) = \rho_0 [1 + \exp \frac{r-c}{t}]^{-1} [1 + w \frac{r^2}{c^2}] \quad 5$$

where ρ_0 is a normalization constant and c , w and t are variable parameters. To derive acceptable parameters plots of pairs of parameters producing the experimental energy for each transition are made. Because of the experimental error on the energies there is a band of acceptable energies for each transition, and from the intercepts of these bands values of the parameters with probable errors can be measured.³⁶ These plots are called c - t diagrams. This procedure corresponds to measuring dependence of the 1s and 2p muonic states upon the nuclear charge distribution. However, the finite size effect decreases with decreasing A and Z , because of the diminution of the nuclear radius ($\propto A^{1/3}$) and the increase of the muon wave function radius ($\propto Z^{-1}$), so that for lower Z often only a single parameter has been deduced. In order to ask meaningful questions about a third parameter it is essential to have measurements on other transitions, where the experimental uncertainty is only a small percentage of the finite size effect. Because of their large finite size effect the 2s and 3p are the preferred states.

All transitions affected by the finite size of the nucleus exhibit strongest dependence on the parameter c of the charge distribution (Tables II and III). Scheck and Hüfner have determined the dependence of the principal transitions on a nuclear parameter orthogonal to $\langle r^2 \rangle$.⁸ They show $\langle r^2 \rangle$ to be a function of c and t . Excluding 2p-1s transitions they find that the 2s-2p doublet and the 3p-2s doublet are the most valuable transitions for determining parameters other than $\langle r^2 \rangle$.

The effect of the finite size on the individual muonic transitions in ^{138}Ba and ^{206}Pb is shown in Tables II and III respectively. The finite size nuclear transition energies are evaluated by solving the Dirac Equation numerically, using the Fermi charge distribution which gave optimum agreement between theoretical and experimental energy values. No corrections were applied to the energies in the tables. Successive columns then show the difference between these approaches, and the partial derivatives of the energies with respect to the parameters used in the charge distribution. From Table II it is apparent that the 3d-2p transitions and the 2p $3/2$ -2p $1/2$ splitting should be determined to a greater accuracy, if they are to be useful in deducing nuclear parameters. This can be demonstrated when the uncertainty in a charge parameter is expanded in terms of energy derivatives,

$$\begin{aligned} \delta c^2 &= \sum_{\text{energies}} (\delta E_i)^2 \left(\frac{\partial c}{\partial E_i} \right)^2 \\ &= \sum_{\text{energies}} (\delta E_i)^2 \frac{1}{\left(\partial c / \partial E_i \right)^2} \end{aligned} \quad 6$$

Thus transitions having smaller derivatives must have correspondingly smaller experimental errors if they are not to dominate the parameter uncertainty.

The 2s level has not been used in charge determinations until recently³⁷ because of the difficulty in detecting it. Using a cascade program described under "Relative Intensities", it is predicted that only 3.5% of all captured muons will pass through the 3p state, and only 1.5% will pass through the 2s state. Because the selection rules for Auger transitions and electric dipole transitions favor $\Delta l = \pm 1$,³⁸ the outer circular orbits ($l=n-1$) become preferentially populated and the dominant transitions for $n \leq 6$ are between successive circular orbits. The most intense transitions are the 2p-1s doublet (87%), the 3d-2p triplet (80%) and the 4f-3d triplet (71%). Table IV shows the percentage of muons predicted to pass through each state with $n \leq 4$ for ^{140}Ce .

In order to detect the 3p-2s and 2s-2p transition doublets it was necessary to have a large volume detector, with good resolution, as well as a relatively pure and intense muon beam. The choice of targets was governed by the desirability of clean single peaks, with a minimum of isotopic contamination and no hyperfine structure. As a means of positive identification of a transition the following conditions were required:

1. Good agreement between the experimentally measured energy and the theoretical value obtained by using the best available nuclear parameters. The validity of the parameters was checked by comparing the energies of the easily identified principal lines. This was a reasonable requirement since the theoretical predictions showed no other x-ray lines in the vicinity of the 2s-2p and 3p-2s energies for all targets except the ^{206}Pb .

2. The splitting of fine structure components of the two doublets should be in agreement with the fine structure splitting measured

from other transitions. The 2s-2p splitting was readily compared with the 2p-1s splitting. The 3p-2s splitting was compared to the 3p-1s splitting whenever possible, but was not as positive a test since the latter transitions were contaminated by the 3d-1s electric quadrupole transitions and were often statistically limited.

3. Approximate agreement between the theoretical and experimental values of the relative intensities of the fine structure components. For most targets this could also be compared to adjacent intense x-ray peaks as an additional verification.

4. The absence of background from delayed γ rays at the same energy. Since the collected data were divided into two sections, one prompt and one delayed with respect to a muon stop, a direct comparison was easily made. The principal x-ray lines could be used to determine what percentage of an x-ray peak should appear in the delayed spectrum. Each of the four transitions involving the muonic 2s state was unambiguously detected for ^{120}Sn , ^{138}Ba , ^{140}Ce , and ^{142}Ce . For ^{206}Pb three of the transitions were detected but there was some ambiguity with one of them.

Fig. 3 shows typical prompt and delayed spectra obtained when muons are stopped in a target of high or medium Z. It shows the summed spectra for ^{138}Ba and the most prominent features are marked on it. The principal muonic transitions, 2p-1s, 3d-2p, 4f-3d, 5g-4f are clearly visible with the former showing the three sets of peaks, photopeak (PP), single escape peak (SE) and double escape peak (DE), obtained when the energy of an incident photon exceeds the pair-production threshold. The term "photopeak" will be used for the highest energy line as the contribution from pair-production is small. Other prominent lines are

the 511.006 keV positron annihilation γ ray, various γ rays which are clearly visible in the delayed spectrum (where their relative amplitude is considerably larger) and a pulser peak distinguishable by its very narrow width. This width is a measure of the electronic broadening only, since the pulser signal does not pass through the detector.

When a pulse originates in the detector its full width at half maximum amplitude above background (fwhm) varies approximately as $\sqrt{E}$,³⁹ where E is the energy of the incident photon. From this it is generally possible to deduce whether a peak in the spectrum is abnormally broad. This can be indicative of two or more unresolved peaks, or of magnetic dipole hyperfine structure.^{40,41} If such a peak appears in the delayed spectrum it can also be caused by the presence of a nuclear state formed after muon capture.^{31,32,42} This energy dependence of the width was useful for testing our identification of ^{206}Pb lines and Table V shows the fwhm as a function of energy for the various prompt peaks.

C. Charge Distribution

Once the experimental energies have been determined they are compared to a set of theoretical predictions. To obtain these predictions the Dirac equation is solved numerically using a predetermined function for the charge distribution, and these values are then adjusted for the effects of vacuum polarization, Lamb shift, anomalous magnetic moment and nuclear polarization.

From the beginning of muonic x-ray work the question of which functional form to use for the charge distribution has been of considerable interest. The earliest calculations used a uniform spherical charge distribution.^{1,16,19} The idea of using a slightly more physical model,

with no amplitude discontinuity, was initially investigated by Hill and Ford²⁰ using a series of modified Gaussian and modified exponential functions. By adjusting a parameter to produce the experimental value for the 2p-1s transition, they predicted values for the 3d-2p transition and compared these with the experimental results. They concluded that there was no unique distribution and that many forms could give acceptable results. Ford and Wills have since discussed extensively the matching of experimental results with various functional forms of the charge distribution.^{43,44}

They have recently shown that each transition is sensitive to a unique moment $\langle r^\alpha \rangle$ of the charge distribution.⁴⁵ They find the value of α to be essentially model independent, and approximately a linear function of Z , with the zero intercept being an integer value. Their approach did not show a unique moment $\langle r^\alpha \rangle$ for the 2s-2p $_{1/2}$ transitions. Wu and Wilets⁷ explain this discrepancy by expanding the potential generated by the muon, as a power series of distance from the muon over the region of the nucleus, retaining only the most significant term. The behavior of the 2s level suggests that this is not a good approximation for the muonic potential over the nucleus. In an attempt to eliminate the slight functional dependence of the method of Ford and Wills,⁴⁵ Barrett⁴⁶ has used a more general form for the functional expansion than $c + dr^k$. This method can be adjusted to give consistent energies over a wider range of functions than is possible using the form of Ford and Wills.

All the functions so far described have the common disadvantage of having no direct physical derivation. Their use is justified by their convenience and by their giving satisfactory agreement with

experiment. If the shell structure of the nucleus is well known, in addition to its physical properties, it should be possible to synthesize a charge distribution by summing the respective proton wave functions. The first attempts to derive a charge distribution by this method were carried out by Perey and Schiffer,⁴⁷ and Elton and Swift.⁴⁸ They used single-particle wave functions evaluated from a spherical potential, and were able to deduce the saturation effect of the nuclear radius as a function of $A^{1/3}$, (ref. 49). Elton, Webb and Barrett⁵⁰ have recalculated the charge distribution using single particle wave functions in an energy-independent non-local potential. This approach yields excellent agreement with shell separation energies but produces a charge distribution for ^{208}Pb which has a central hump, as opposed to the depression predicted by analysis of electron scattering experiments.⁵¹ A physical approach which may provide the most realistic charge distributions is that due to Vantherin and Brink.⁵² They adopt a Hartree-Fock approach using an effective nucleon-nucleon potential, which includes short range two-body interactions, and zero range three-body interactions. By adjusting the variable parameters to fit some known nuclear properties, good values are obtained for the single particle separation energies, and the charge distribution resembles that obtained experimentally.

For the future it may be possible to relate transition energies to internucleon forces via the Vantherin-Brink theory, or a charge distribution may be specified by a set of moments rather than by the familiar nuclear parameters. The present development of the latter seems inappropriate for these data, since it appears unable to describe adequately transitions involving the muonic 2s state, which is the real strength of these experimental results.

Because of this, the data were analyzed in terms of a generalized Fermi distribution. This simplified the comparisons that were made with electron scattering results, since this is the distribution most commonly used in that form of analysis. A generalized Fermi distribution has four variable parameters corresponding to a radius, a skin thickness, a fall-off parameter and a central depression. It is expressed mathematically as:

$$\rho(r,c,n,w,m) = \rho_0 \frac{1 + w r^2/c^2}{1 + \exp [n^m((r/c)^m - 1)]} \quad 7$$

where ρ_0 is a normalization constant and is evaluated numerically by requiring

$$4\pi \int \rho(r,c,n,w,m) r^2 dr = Ze \quad 8$$

For a normal two-parameter Fermi distribution with $w=0$ and $m=1$, the parameter c represents the distance over which the density falls to half of its maximum value, and n is related to the distance over which the amplitude falls from 90% of its maximum value to 10% (the skin thickness t), by the relation

$$n = 4 \ln 3 c/t \quad 9$$

A non-zero value for the parameter w produces a central hump or depression. The exponent m allows the fall-off rate of the "tail" to be varied independently of the other parameters.

With the inclusion of one nuclear polarization parameter (NP) this gives five independent variable parameters. This is more than can be reasonably fitted simultaneously to the number of measured transitions, and their limited finite size dependence. For each target the following fits were made:

1. Nuclear polarization was assumed zero for all the states and the transitions were fitted to a normal 2-parameter Fermi distribution: (c=variable, n=variable, w=0, m=1, NP=0).

2. The theoretical nuclear polarization correction was applied to each state and the transitions were fitted to a normal 2-parameter Fermi distribution:

(c=variable, n=variable, w=0, m=1, NP=theoretical).

3. The transitions were fitted to a normal Fermi distribution with the 1s nuclear polarization permitted to vary, other levels assuming the theoretical values:

(c=variable, n=variable, w=0, m=1, NP=variable).

4. When corresponding work had been done with electron scattering the experimental value of w was inserted and fits 2 and 3 repeated (c=variable, n=variable, w=experimental, m=1, NP=variable or experimental). In addition fits were attempted in which w, m and the 2s nuclear polarization were also allowed to vary. The results from these fits were insufficiently sensitive to justify applying them to all targets. Also, a considerable number of different fits had been tried with ^{208}Pb , and the resulting parameters were very similar, especially as far as the nuclear polarization was concerned.

These data can be compared with electron scattering results to determine whether introducing nuclear polarization seriously affects the calculated nuclear parameters. For a strict comparison of these two methods the electron scattering data should be at an "equivalent momentum transfer" q_{equ}^2 (ref. 53) where

$$q_{\text{equ}}^2 \approx \sqrt{(3/5)} \frac{6Z}{\langle r^2 \rangle^{1/2} a_{\mu}}$$

and a_μ is the Bohr radius for the muon. At this momentum transfer the elastic electron scattering and the muonic spectrum have the same dependence on second and fourth moments of the charge distribution. A more convincing test is to attempt fits using a different form for the charge distribution, and then to compare the nuclear polarization values from both methods. A distribution used in this work was the three parameter Family II distribution described by Ford and Wills.⁴⁵ This distribution was applied solely to the ^{138}Ba data. Its mathematical form is

$$\rho(r, c, n, w) = \begin{cases} \rho_0(1 + w r^2/c^2)[1 - 1/2 \exp n(\frac{r}{c} - 1)] & r \leq c \\ \rho_0(1 + w) 1/2 \exp n(1 - \frac{r}{c}) & r \geq c \end{cases} \quad 11$$

where the parameters have the same meaning as in the extended Fermi distribution (Equation 7).

D. Nuclear Polarization

Having discussed fitting to one, or possibly two, nuclear polarization parameters, it is appropriate to explain the meaning of nuclear polarization and its relevance to nuclear charge parameters. Nuclear polarization is produced by the muon-nucleus electrostatic interaction. Because of the penetration of the nucleus by the muon, the eigenfunctions of the original system are perturbed and there is an associated change in the energy eigenvalues. In mathematical terms this appears through second order perturbation theory, and can be thought of as the electrostatic interaction exciting the nucleus and the muon into virtual intermediate states.

The earliest discussion of nuclear polarization was by Breit, Arfken and Clendenin in 1950.⁵⁴ They calculated the electronic nuclear polarization for optical isotope shifts. With the advent of muonic atom studies this effect was expected to be much more important because of the larger nuclear overlap by the muon, Cooper and Henley¹⁶ calculated the nuclear polarization for the Fitch and Rainwater results on ^{208}Pb (Ref. 2) and deduced a value of 60 keV for the 1s state. They quoted this as an energy upper limit, and evaluated it using closure over the muon and nuclear states, and an averaged excitation energy. Since 60 keV was the energy resolution of the experiment there was no immediate need to improve the calculations. Successive improved calculations by Lakin and Cohn,⁵⁵ Nuding⁵⁶ and Greiner⁵⁷ reduced the value well below the experimental energy resolution obtainable from a Na I crystal. The work by Joachin⁵⁸ and by Joachin and Demeur⁵⁹ on light nuclei, where the nuclear wave function could be represented by an analytical mathematical function, and by Scheck⁶⁰ demonstrated that important contributions came from intermediate states of the muon and the nucleus in the continuum. The nuclear polarization of the 1s level of ^{208}Pb was then estimated to be in the range 1-6 keV and to increase the binding energy of the state. With the advent of the high resolution Ge(Li) detector in 1964 this effect became extremely important since it could substantially affect the values of the charge parameters deduced. Calculations of nuclear polarization in deformed nuclei using various nuclear models have been performed by Pieper and Greiner,⁶¹ McKinley,⁶² and Chen.⁶³ Nuclear polarization effects in spherical nuclei have been calculated by Cole,⁶⁴ Chen⁶⁵ and Skardhamer.⁶⁶

Because of the similarity of their Feynman diagrams, Cole relates nuclear polarization to inelastic electron scattering and is able to use the experimental scattering cross-section results to deduce nuclear polarization to within a predicted uncertainty of a factor of two. The most comprehensive work on nuclear polarization is by Chen who develops means of dealing with the complete set of intermediate muonic states. His methods will be treated in more detail in the section on theory. A further publication which this work uses is by Skardhamar,⁶⁶ who does a calculation in the Pb region, using a hydrodynamic model to describe the nuclear excitation. His results are in excellent agreement with Chen and include the 3d and the 3p states. All the recent works have made calculations of the nuclear polarization produced when the muon is in a higher state ($n > 1$), and the results are comparable to, or larger than the experimental energy uncertainties, so that they should necessarily be included in the calculation. A recent calculation by Tanabe⁶⁷ uses a different approach which is only applicable to higher states, $n \geq 3$. His results are too small to be of importance for these experimental results and do not agree with those of Skardhamar.

The parameter taken in this work to represent nuclear polarization is that produced by the muon in its 1s state. The principal uncertainty in this quantity is the contribution from monopole modes of excitation for which no experimental measurements exist. The monopole mode also has a considerable contribution to the polarization by the muon in the 2s state, and hence, for ²⁰⁶Pb, this was also allowed to vary.

At the Williamsburg conference in 1966 both Ravenhall and Wilkinson referred to nuclear polarization as one of the major remaining theoretical uncertainties in muonic x-rays. Since then Chen has shown that

corrections may be confidently applied in the evaluation of nuclear parameters. He demonstrated⁶³ that the inclusion of nuclear polarization effects in deformed nuclei produced more realistic and consistent nuclear charge parameters than when it was omitted. It also has a measurable effect on the parameters of spherical nuclei, as will be shown in the results.

A comparison is usually made of the nuclear parameters obtained from muonic atom analysis with those from electron scattering data, but this is not a definitive method of detecting nuclear polarization in muonic atoms, since electron scattering can also induce virtual transitions, and this dispersion effect has yet to be generally applied.^{68,69} Since the radiative corrections have been evaluated by Barrett⁷⁰ with uncertainties <1 keV, and the vacuum polarization by Fricke,⁷¹ the dominant major remaining uncertainty has been that of nuclear polarization. A measurement of nuclear polarization for muonic atoms is therefore of theoretical interest, since apart from the contribution due to the monopole excitation, it should be well understood.

II. EXPERIMENTAL DETAILS

A. Physical Details

The data of this experiment were collected during 4 experimental periods at the NASA Space Radiation Effects Laboratory (SREL) in Newport News, Virginia. Our apparatus was placed at the end of the SREL Meson Channel through which a nominal 200 MeV/c pion beam is transported. A low duty factor external beam was produced by employing a grid of thin carbon filament as the internal target. Events accumulated during the prompt beam spike were rejected by gating off the logic units of the beam telescope with a gate derived from the synchrocyclotron R.F. signal.

The muon beam is produced by pions decaying in flight, and has two predominant muon momenta. These occur when the decay muon is emitted in either the backward or forward direction, with respect to the pion's momentum and are determined by the transport system. The prime experimental concern was the observation of transitions to and from the muonic 2s state. Since these were of low intensity the best possible signal/noise ratio was sought commensurate with high intensity, and this was obtained using the backward muon beam. An initial measurement with the ^{140}Ce target indicated that although the forward beam had twice the intensity it also had greater contamination, and that by selecting the backward muon beam the signal/noise ratio improved from 2:1 to 3.5:1 at 4.2 MeV (^{140}Ce K lines) and from 5:1 to 8.5:1 at 1.3 MeV (^{140}Ce

L lines). From the fact that the range curve (Fig. 4) fell off so rapidly and to such a low value the electron contamination was estimated at less than 5%.

A typical beam telescope was used as is shown in Fig. 5. The scintillation counters used were made from Pilot B plastic scintillant and were mated to Amperex 56 AVP photomultiplier tubes. Counters 1 and 2 were 20 cm x 20 cm x 6 cm, counter 3, which was adjacent to, and the same size as the target, defined muons incident on the target, counter 4 was a large anticoincidence counter and counter 5 served to monitor the efficiency of counter 4. Counter 4 was large enough to detect particles which underwent large angle scattering in the target. Under operating conditions counter 4 was about 98.5% efficient and its efficiency was monitored by the ratio $1235/12345$. A stopping muon was signified by coincident signals from counters 1, 2 and 3 with no signal from 4 ($123\bar{4}$).

Precautions were taken to exclude high Z materials from the region of the detector; the shielding consisted exclusively of borated CH_2 , the counter supporting frame was primarily Al, and the nearest Pb was in the 20 cm x 20 cm collimator located 90 cm upstream from the target. The beam was moderated with 25 cm x 25 cm polyethelene sheets inserted downstream of counter 2. The photon-detector was placed centrally under the target, and was kept beneath the beam level in order to minimize its count rate. The principal detector used was a 7% efficient, coaxially drifted Ge(Li) supplied by Princeton Gamma Tech. Under operating conditions this detector exhibited a resolution of 3.2 keV at 1.3 MeV (^{60}Co) and 3.6 keV at 2.6 MeV (^{56}Co). For some of the later work on

^{120}Sn and ^{206}Pb a larger detector was used, which had an operating resolution of slightly better than 3 keV at 1.3 MeV and an efficiency of 10%.

The detailed chemical composition of the targets, their dimensions and mass are shown in Table VI. All the targets were in powder form except the ^{206}Pb which was in solid sheet form. The powder was made into a target by encapsulating it in a rectangular box made from thin plastic sheet.

B. Electronics

The purpose of the electronic circuitry was to transmit the signal from the Ge(Li) detector to an energy pulse height analyser (PHA), and at the same time to provide timing information of the γ ray relative to the preceding muon stop. The Ge(Li) detector searched for resulting photons for a period of up to 500 nsec after each stopping muon. The timing spectrum (Fig. 6) distinguished between prompt events, such as muonic x rays, where the stopping muon and the resultant photons are coincident, and delayed events, such as nuclear γ rays which can result from muon capture.

Both pieces of information (i.e., energy and timing), were fed into analog to digital converters (ADC) which were gated on after a γ -stop coincidence had been detected. Fig. 7 shows the main logic circuitry used in analyzing the signals. The method for obtaining a stopped muon signal (1234) using the usual combination of discriminators, delays and coincidence units is shown in Fig. 8. The signal from the detector was initially passed through a (room temperature) FET preamplifier (Canberra 1408C), which also received a signal from a reference pulser for stabilization. The pulser signal was fed in through a 1.5 pF

precision capacitor which had a temperature stability of 1 part in $10^6/^{\circ}\text{C}$. These signals were fed directly to the main amplifier system in the counting room via doubly-shielded cable. The main amplifier system consisted of a TC200 amplifier (Tennelec) used to extract timing information, and a second linear amplifier (Tennelec TC203) where the pulse was shaped, and pole-zero and base-line restoration corrections applied, before energy analysis. To extract a timing signal for the pulse, two outputs from the first stage of the TC200 were fed into discriminators and then recombined in a coincidence unit. One discriminator was set to trigger above the noise and was given a wide output while the other triggered in the noise but had a narrow output. Thus an output from the coincidence unit denoted a signal whose magnitude was above the noise, but yet could be given effective leading edge timing. With this method a timing resolution of 100 nsec full width at half maximum (fwhm) over an energy range 0.1 to 8.5 MeV was obtained. This was superior to the timing resolution obtained from a commercial constant-fraction discriminator. Outputs from this coincidence unit were used as one input to the γ -stop coincidence unit, and as the "start" signal of the time to amplitude converter (TAC), which served to provide an analog signal giving the time delay between the stopping muon and the resultant photons. The other input to the γ -stop coincidence unit, and the "stop" signal of the TAC were provided from outputs of a delayed 1234 event. The γ -stop coincidence provided the gating signal for both energy and timing ADCs. The gate generators were logic shaper and delay units (Canberra 1455) and were used to provide a variable width, as well as signal level conversion. Incorporated into the γ -stop coincidence unit was an anti-coincidence signal formed when either of the ADCs was busy, or when

incoming photon signal rates became too high. The latter signal was produced by a commercial pile-up protector (EG&G, model GP 100). The entire logic circuitry could also be gated off by a pulse derived from the synchrocyclotron R.F. signal during the prompt beam "spike".

Since it was possible for a very low-energy pulse to meet all other requirements, but still fail to trigger the base line of the energy PHA, the "sample" input of the PHA was connected to the output of the higher level timing discriminator as the "threshold" control. The effect of this was that any signal meeting the timing requirements was analyzed. This reduced the lower energy threshold from 110 keV to 80 keV.

Data acquisition facilities of the SREL IBM 360-44 were not continuously available for the experiment and thus two data collecting techniques were employed. When they were not available for this experiment, the timing information was analyzed in a 400 channel (TMC) pulse height analyzer. When the 360-44 was used, a Kicksort ADC, set to 1024 channels, was used as the timing digitizer and the data were fed directly to the computer via an IBM 2972/7 interface. The analog energy signal from the TC203 was fed into a Kicksort 8192 channel PHA. The ADC in the Kicksort was able to digitize 0-10 volts into 8192 channels, but since its memory had only 4096 channels it was necessary to accumulate spectra separately into the upper and lower halves of the energy region studied when running without the 360-44. When the computer was available the 13 bits of energy information and the 10 bits of timing information, for each valid event, were transferred to the computer and sorted and stored there.

A block diagram of the data flow is shown in Fig. 9. For each valid event the timing and energy information was transferred through the IBM 2972/7 interface to the computer. The computer sorted these data into two 8192 channel energy arrays, one prompt and one delayed with respect to a muon stop signal, and one 1024 channel timing array. In addition, cumulative prompt and delayed energy analyzers were kept in disc storage. All of these arrays were continuously monitored during each run via printer, point plotter and cathode-ray display. The timing windows for separating the prompt and delayed spectra were determined by inspecting the timing spectrum for the entire energy range and visually judging where the exponential decay due to events following muon capture commenced, (Fig. 6). When running without the 360-44 data acquisition facilities, the timing was adjusted so that the energy PHA would accept signals for 100 nsec after each muon stop. The stored data were initially printed out on a high speed paper printer, but during later runs an incremental magnetic tape unit (CYPHER) was used to store the energy and timing information for each event, and to record the contents of the 4096 channel energy analyzer memory.

C. Experimental Procedures

In order to ensure stability a two-point digital stabilizer (KICKSORT) was used in conjunction with the energy ADC. This held the centers of two chosen peaks fixed, by applying corrections to the gain and slope intercept of the energy ADC conversion. Lines which were used for stabilization were the 136 keV line of ^{57}Co , the 511 keV line from positron annihilation, a reference pulser peak and the 2p-1s muonic x-rays. The advantage of the first three lines is that they can be exactly reproduced in all forms of calibration runs and hence

there is little danger of changing the gain during calibrating between runs. For this reason the 511 keV line and the reference pulser were used whenever possible. The high stability precision reference pulser was constructed from the design of Strauss.⁷² A measure of the stability achieved can be seen in Fig. 10 where the channel variation of the 511 keV line and the 1274 keV line from ^{22}Na are plotted as a function of time. These data are taken from calibration runs during 15 days of the initial experimental run and show a maximum daily fluctuation of less than 100 parts per million.

The system linearity was determined by three independent methods. A precision pulser, with a Kelvin Varley voltage divider, was used to produce a series of equally spaced pulser peaks, which spanned the spectrum. These pulses were fed into the system via a precision capacitor in the preamplifier. To determine the linearity under operating conditions a series of 17 calibration sources was used. Each source was run in conjunction with ^{22}Na which allowed the calibration runs to be compared with each other. A complete listing of the sources used, together with their energies, references and other possible uses is given in Table VII. One of their other uses was the determination of a relative efficiency curve as is shown in Fig. 11. As a check of the efficiency curve a set of calibrated sources was used in a known geometry and absolute efficiency values calculated. There was a good agreement between these methods. A third method of measuring linearity was applicable only at the higher energies, where there is a dearth of well known calibration lines. A photon with energy greater than the threshold for pair production will have two associated peaks, at 511.006 keV and 1022.012 keV below the photopeak, corresponding respectively to the escape of one or both

positron annihilation γ -rays. These 511 keV energy differences were measured from the high energy calibration sources, the muonic K x-rays and from thermal neutron capture in iron, and when used in conjunction with one known energy line provided good energy calibration. The possibility that the measured line separations are not exactly 511.006 keV has been discussed by Strauss.⁷³ Such effects should be negligible for a coaxial detector.

One other parameter essential for the identification of weak or multiple peaks was the peak resolution as a function of energy. Using this, it is possible to distinguish weak peaks in the prompt spectra which may be nearly coincident with muon capture γ -rays. This was especially helpful in the case of ^{206}Pb where the spectrum was rather complex. Table V gives the fwhm of single peaks as a function of energy and is taken from the ^{206}Pb data.

The usual experimental sequence involved collecting data from the target for 7-10 hours, with a short calibration run directly preceding and succeeding each run. The beam was left on during calibration runs but in order to speed data collection the "stop" requirement of the " γ -stop" coincidence unit was removed, and the data were collected "self-gated". The main purpose of these calibration runs was to verify that there had been no substantial gain shift during the target run which might cause peak broadening and a resultant loss of resolution. Once it became apparent that the system was extremely stable, the calibration runs were taken less frequently and the target runs extended in duration.

D. Energy Evaluation

The absolute energies of muonic transitions were determined using two techniques. In order to decrease experimental uncertainties for prominent transitions in the muonic spectra, the K, L and M lines, a "feedthrough" technique was used, which employed the simultaneous accumulation of muonic peaks and precisely known calibration lines. Theoretically it is most important to know the L lines extremely well since their energies are less dependent upon the nuclear parameters but are still far from being point-like. The "feedthrough" technique consisted of determining the approximate energy of the transition, either from previous experimental work or from theory, and then choosing one or two calibration sources whose energies are accurately known and which lie close in energy to the required transition. These sources were then placed near the detector so as to appear in the muonic spectrum by accidental coincidences with muon "stop" signals. Experimentally this technique was convenient since the data accumulated sufficiently rapidly that a fit to the lines from one eight hour run was not statistically limited. In the case of the weak ^{66}Ga source used to evaluate the ^{206}Pb lines this technique was amended, by removing the requirement for a "stop" signal, thus permitting more rapid data accumulation in the calibration peaks. Checks were made to search for possible dependence of the measured energy on the mode of data collection. Provided the deadtime on the PHA was kept within reasonable limits (<25% deadtime), any difference was well within statistical error, independent of whether the beam was on, or whether a photon occurred in coincidence with a muon stop.

In most "feedthrough" runs the chosen calibration lines were within 100 keV of the muonic transitions to be measured. To evaluate the transition energy the system was assumed linear over the small range of interest, the gain calculated from the calibration peaks, and the unknown energy interpolated from the nearest "feedthrough" calibration line. A justification for the validity of this technique was the good consistency found when a line was evaluated from more than one pair of calibration lines, and also the excellent agreement obtained when the energies were evaluated using the preceding and succeeding calibration runs. Some examples of results using feedthrough techniques are given in Fig. 12 and Fig. 13. Fig. 12 shows the L lines of ^{142}Ce evaluated with calibration lines from ^{60}Co and ^{22}Na . Fig. 13 shows the ^{120}Sn muon lines with ThC'' feedthrough.

The remaining muonic transition energies such as the 2s-2p and the 3p-2s lines were determined relative to prominent muonic transitions in the summed spectrum using the gain curve determined from the calibration runs and 511 keV differences. The non-linearity correction was obtained by fitting a variable order polynomial to a variable number of nearest points. In practice a cubic equation was fitted to the closest 6 points but the correction proved to be insensitive to the order of the polynomial or the number of points used.

E. Data Analysis

In fitting the experimental data all major peaks in both prompt and delayed spectra were analyzed using a least squares fitting technique. Since most of the peaks corresponded to γ rays whose natural width was less than our experimental resolution, the mathematical form of the

line shape should represent the response of the apparatus and should ideally be Gaussian.

If $N_i(x)$ represents the theoretical number of counts in channel x , contributed by the i^{th} Gaussian (allowing for the presence of more than one peak in the neighborhood of channel x), then the theoretical form assigned to it was

$$N_i(x) = A_i \exp \left[-1/2(x-X_i)^2/(kw_i)^2 \right] \quad 12$$

and the form of the complete function was

$$N_{\text{theor}}(x) = \sum_{i=1}^j N_i(x) + \text{Background} \quad 13$$

In these formulae

$$k = \text{constant} = 2.35498$$

$$X_i = \text{central channel of } i^{\text{th}} \text{ Gaussian}$$

$$A_i = \text{amplitude in counts of the } i^{\text{th}} \text{ Gaussian}$$

$$w_i = \text{full width at half maximum of the } i^{\text{th}} \text{ Gaussian}$$

In practice the maximum number of Gaussians j was restricted to 5, and in general the number of variables was reduced by equating all the widths w_i to a single variable w . The observed intensity of the transition is given by the area under the Gaussian

$$I_i = kw_i A_i \sqrt{2\pi} \quad 14$$

In order to be able to represent both linear backgrounds and those with a finite curvature, an exponential background shape was assumed:

$$\text{Background} = B \exp [\lambda(x-X_i)]$$

where

$$B = \text{amplitude of background at the center of Gaussian } i$$

$$\lambda = \text{variable exponential constant.}$$

It was found that varying the number of channels over which this function was fitted did not appreciably effect the values of the parameters derived $(A_i, X_i, w_i, B, \lambda)$, provided the range was large enough to allow a good fit to the background function. Exceptions occurred when structure from another Gaussian was obviously being included in the range. As a rule these techniques gave a variance between 0.5 and 1.5, indicating that reasonable theoretical representation had been used for the data.

Prior to one of the runs the detector sustained some neutron damage, and this resulted in exponential tails on the Gaussians, due to incomplete collection of charge. For these data, the variances for a fit to a Gaussian function were very much larger, and in order to represent the data properly a mathematical form of the type suggested by Routti and Prussin⁷⁴ was used. This function contained additional parameters, transition points beyond which the Gaussian function became a decaying exponential, while still preserving functional and differential continuity at the boundaries. For this function

$$N_i(x) = \begin{cases} A_i \exp [p_i(x-X_i)^2] & X_i - V^2 \leq x \leq X_i + U^2 \\ A_i \exp [-p_i V^2(2x - 2X_i + V^2)] & x < X_i - V^2 \\ A_i \exp [-p_i U^2(2X_i - 2x + U^2)] & x > X_i + U^2 \end{cases} \quad 15$$

$$p_i = -1/2(kw_i)^{-2},$$

where V^2 and U^2 are the distances below and above the central channel at which exponential behavior begins. The same functional form was always used to analyze both calibration and data runs.

Transitions involving the 2s state, as well as many of the other higher transitions, were relatively weak and hence essentially unobservable, or at best statistically limited in one ten hour run.

Some method was thus required for summing the data which would not obscure any of the peaks. Simple adding was insufficient since the runs on each target extended for two weeks or more, and a daily gain variation of up to 100 parts per million was determined from the calibration runs. To compensate for this, one run was chosen as the "basis" run, either because there was a calibration source feeding through on accidental coincidences, or because of high consistency between the pre- and post-calibration runs. Within the "basis" run two well defined peaks were chosen and their centers determined. These same two peaks were fitted in each of the runs to be summed. If y_1 and y_2 are the peak centers in the "basis" run, and t_1 and t_2 are the same peak centers in the run to be summed, then the linear transformation

$$C_b = mC_r + k \quad 16$$

where

C_b is a channel in the basis run

C_r is a channel in the run to be summed

$$m = (y_1 - y_2) / (t_1 - t_2)$$

and $k = y_1 - mt_1$

will map the centers t_1 and t_2 onto y_1 and y_2 respectively. From this transformation it was then possible to determine for a particular run what fraction of its channels contributed towards a channel in the "basis" run. After each run was mapped onto the basis run the channels were summed. The data were then in the form of two 8192 channel arrays in which the energies of the dominant peaks were accurately determined from "feedthrough" runs.

F. Isotope Corrections

Once the energies corresponding to peaks in the various spectra had been evaluated they were corrected for isotopic impurities to convert them to energies of the principal isotope. In the cases of ^{120}Sn , ^{138}Ba and ^{140}Ce the targets were of high enough purity ($\approx 98\%$ mono-isotopically pure) that the measured energies could be assigned directly to the transition energies in these targets. In ^{142}Ce and ^{206}Pb the isotopic impurities were sufficiently large that a correction was applied for isotope shifts. The detailed target compositions are shown in Table VI. For the K lines this was relatively easy since transitions from different isotopes were resolved and also since these have been measured accurately by other experimenters.⁷⁵⁻⁷⁷ The $^{140}\text{-}^{142}\text{Ce}$ K isotope shift was measured directly and will be discussed with the results. When the lines had not been detected before or the lines could not be resolved, a different technique was required. To correct for these the Dirac equation was solved numerically using approximate, but reasonable values for the nuclear charge parameters, until a correct K isotope splitting was predicted. The isotope shifts of the other levels were then evaluated from these two solutions.

If $\bar{x}$ is the measured center of the peak and x is the actual center then

$$\begin{aligned}\bar{x} &= (1-\alpha)x + \alpha(x+\delta) \\ &= x + \alpha\delta\end{aligned}\tag{17}$$

where α is the fractional impurity and δ is the isotope shift for the level in question. Since $\alpha < 0.1$ the only states to which a correction could sensibly be applied were the $1s$, $2s$, $2p_{1/2}$ and $2p_{3/2}$ states. The appropriate isotope shifts are given in Table VIII.

G. Nuclear Parameters

As discussed previously not all muonic levels are equally sensitive to the nuclear parameters. Since the parameters are determined by a least squares fitting technique, the inclusion of accurately known transition energies for high ℓ values, which are essentially point-like, or of energies with large errors, will result in an artificially reduced value for the chi-squared test (χ^2). The only transition energies or energy differences used to evaluate the nuclear parameters were those having a substantial finite-size dependence and a small error. In practice this meant transition energies or differences involving the 1s, 2s, 2p_{1/2}, 2p_{3/2}, 3p_{1/2} and 3p_{3/2} states and having errors less than 0.6 keV. The use of energy differences was prompted by the lack of an adequate number of calibration sources above 2 MeV for use in absolute energy determinations. However, an energy difference could still be accurately evaluated because of the independent measurements of the 511.006 keV energy differences. The 4d_{5/2}-2p_{3/2} transition was excluded from the fitting procedure because of its contamination by the 4d_{3/2}-2p_{3/2} transition and by the electric quadrupole transitions 4f_{7/2}-2p_{3/2} and 4f_{5/2}-2p_{3/2}. Table IX shows the combination of various energies which were used to evaluate the nuclear parameters for each target. Tables III and II show the detailed effect of the finite size and also the derivatives of the energies with respect to the parameters of a normal three-parameter Fermi distribution for ²⁰⁶Pb and ¹³⁸Ba respectively. The reduction of finite size effects for lower Z are clear from a comparison of the two tables.

The nuclear parameters and their errors were evaluated by comparing the theoretical and experimental value with a least squares fitting

technique. The theoretical values for the energy levels were obtained by solving the Dirac equation numerically and then applying corrections for first order vacuum polarization, Lamb shift and anomalous magnetic moment. These were calculated from the wave functions derived from the Dirac equation. Further corrections were then applied for higher order vacuum polarization effects, following the methods of Fricke,⁷¹ and in most cases for nuclear polarization following the methods of Chen⁶⁵ or Skardhamar.⁶⁶ The values for these two corrections are shown in Tables X and XI.

For each target, parameters for the two-parameter Fermi charge distribution were obtained: (1) when the nuclear polarization for each level was set at zero; (2) when it was set at the theoretical value; and (3) when the 1s level effect was allowed to vary, the others being set to their theoretical values. It was found that in general, fits did not give convergence when a central depression was included as a third or fourth parameter. However, electron scattering experiments from ^{138}Ba and ^{208}Pb have indicated presence of a central depression.^{51,78} Fits were made including this parameter and also for various values of the parameter m as suggested by Heisenberg *et al.*⁵¹ A similar analysis had been tried on ^{208}Pb , and since the nuclear polarization parameter showed little sensitivity to these additional parameters, only a limited number of fits were tried. Since there appears no theoretical reason to prefer the Fermi distribution (equ. 6), aside from convenience and common usage, the ^{138}Ba analysis was also performed with a Family II distribution⁴³ and with other distributions listed in "Results", to determine whether the Fermi distribution gave markedly better results.

Once a set of parameters had been derived a check was made to determine whether it corresponded to a true minimum. To do this the χ^2 values were recalculated for small increments on either side of the calculated value. In quoting final values consideration was given to the true significance of the figures and they were quoted to the nearest value commensurate with the uncertainties. The errors quoted for the nuclear polarization were increased from the calculated values in order to reflect uncertainties in the nuclear polarization corrections to the other levels, and also the uncertainties in the other theoretical corrections.

H. Relative Intensities

Although not the primary intent of this experiment, measurements were made on the relative intensities of the various muonic transitions. Experimentally the intensity of a given transition was measured by the total number of events detected, which was the area beneath the peak. If $N(E)$ is the number of events detected at energy E then

$$N(E) = \frac{w}{4\pi} N_0(E) \epsilon(E) A(E) \quad 18$$

where w is the average solid angle subtended by the detector to the target

$N_0(E)$ is the total number of γ rays emitted at energy E

$\epsilon(E)$ is the detector efficiency at energy E

and $A(E)$ is an attenuation factor caused because the γ rays are attenuated both in the target and in the intermediate space.

The relative intensity measurements are given by $N_0(E_1)/N_0(E_2)$, and since this is a ratio the solid angle effect can be neglected, it being energy independent. The attenuation factor $A(E)$ was evaluated

from

$$A(E) = \exp [-\lambda_1(E)x_1 - \lambda_2(E)x_2 - \cdots], \quad 19$$

where x_i is the dimension of the intermediate material through which the radiation must pass,

and $\lambda_i(E)$ is the attenuation coefficient of this material. In practice $\lambda_i(E)$ was evaluated from

$$\lambda_i(E) = \left(\frac{\mu_i(E)}{\rho_i} \right) \rho_i \quad 20$$

where ρ_i is the material density and $\mu_i(E)/\rho_i$ values are given in reference.⁷⁹

The calculation of the peak areas $N(E)$ is explained in the section on "Data Analysis". Since isotopic impurities should add a known fraction to each peak, and since only ratios were evaluated, no corrections were made to the relative intensities for isotope contamination.

In order to calculate the detector relative efficiency curve a set of calibration sources having well determined relative intensities and energies, was used, (Table VII). From equation 18 the values of $\epsilon(E)/\epsilon(E_0)$ were evaluated from the calibration runs and hence a relative efficiency curve obtained (Fig. 11). The energy range of this curve is restricted to 0-4.8 MeV because of the lack of higher energy sources. To extend the curves beyond this the relative intensities of the muonic K, L, and M lines for high Z materials are measured and compared with theoretical predictions using a cascade program.⁸⁰

To calculate the efficiency corrections for members of a fine structure multiplet the experimental relative efficiency curve was fitted to a mathematical function,

$$\epsilon(E)/\epsilon(E_0) = (E/E_0)^c \quad 21$$

using a least squares fitting technique, where E_0 is some arbitrary energy which is chosen as a standard, and c is a variable parameter. ($c \approx -1.1$) The theoretical values for the relative intensities were calculated from a cascade program written by J. Hüfner.⁸⁰ He followed the methods of Eisenberg and Kessler^{81,82} who assumed the muons were atomically captured with a known distribution in a high orbital level, and then cascaded through the various muonic levels by Auger electron emission and electric dipole radiation. The transition rates for processes were calculated from the expressions due to de Borde⁸³ and Burbridge and de borde³ which described all bound states with point-like hydrogenic wave functions. Previous work done in this laboratory⁸⁴ suggested that there was no strong dependence of the relative intensities of the lower states upon the initial muonic level chosen for the distribution. Because of this, the initial muonic orbit was chosen at the electronic Bohr radius ($n=14$) and a statistical distribution ($P_\ell = 2\ell+1$) was used.

From the early stages of the analysis it became obvious that electric quadrupole transitions were being observed²⁷ and that the cascade program should be amended to account for these transitions, as well as for the finite-size effect on the muonic wave functions. The theory of electromagnetic transition rates is treated comprehensively in reference 85. Under the non-relativistic long wavelength approximation the electric L-multipole transition probability $P(E_\ell, j \rightarrow j')$ becomes⁶

$$P(E_{\ell}, j \rightarrow j') = 2\alpha \left(\frac{mc^2}{\hbar} \right) J(jj'L) \left(\frac{\Delta E}{mc^2} \right)^{2L+1} M_L^2 \quad 22$$

where

$$J(jj'L) = \frac{(2j'+1)}{\alpha^{2L} \alpha^{2L}} \cdot \frac{L+1}{L(2L+1)[(2L+1)!!]^2} \begin{pmatrix} j & j' & L \\ 1/2 & 1/2 & 0 \end{pmatrix}^2.$$

The last set of parentheses represents a 6-j symbol

$$\text{and } M_L = \langle r^L \rangle = \int_0^\infty r^L (ff' + gg') r^2 dr$$

where ΔE is the transition energy. The muonic Bohr radius and mass are given by a and r respectively, and f , g and f' , g' are the initial and final muon radial wave functions and are obtained from the numerical solution to the Dirac equation.

To account for the effects mentioned above, point nucleus wave functions were used in the cascade calculation only for levels $n > 4$. For $n \leq 4$ the probabilities for electric dipole and electric quadrupole transitions were calculated using equ. 22 and the population of each lower level predicted. For muonic transitions in high Z atoms the probability of Auger emission is extremely small for $n \leq 4$ and was neglected.

The population of each lower level as a percentage of the total number of muons captured into bound atomic states of muonic ^{140}Ce is given in Table IV. The comparison between theory and experiment was used primarily as corroborative identification of the $2s$ state, but was also very useful in the identification and analysis of the electric quadrupole transitions.

III. THEORY

A. The Solution to the Dirac Equation

The muon energy levels are evaluated by solving the eigenvalue equation

$$Hu = Eu \quad 23$$

where H is the Hamiltonian for the whole system, consisting of muon, nucleus, electron cloud and radiation field. It will be shown later that the weak interaction between the muon and the nucleus is negligible, and that for the lower muon levels the electron shielding interaction can also be ignored. To first order the muonic energy levels are given by:

$$[H(\mu) + H(\mu\text{-nucleus})]u = Eu \quad 24$$

where $H(\mu)$ is the Hamiltonian for the free muon

$$H(\mu) = c\vec{\alpha} \cdot \vec{p} + \beta mc^2 \quad 25$$

and $H(\mu\text{-nucleus})$ is the Hamiltonian for the electrostatic muon nucleus interaction. $H(\mu\text{-nucleus})$ is usually given in the form of a central potential $V(r)$. In equation 25 $\vec{\alpha}$ and β are the 4x4 Dirac matrices and u is a 4 - component wave function.³³ Since u is derived from the 2 - component spinors, it is possible to express its components in terms of two radial functions, $f(r)$ and $g(r)$, and an angular dependence expressed in terms of spherical harmonics.

By making the transformation $F(r) = rf(r)$ and $G(r) = rg(r)$ equation 24 can be reduced to two coupled first order differential

equations

$$\begin{aligned}\frac{dF(r)}{dr} &= K \frac{F(r)}{r} - \frac{G(r)}{\hbar c} [E - mc^2 - V(r)] \\ \frac{dG(r)}{dr} &= -K \frac{G(r)}{r} + \frac{F(r)}{\hbar c} [E + mc^2 - V(r)]\end{aligned}\tag{26}$$

where

$$\begin{aligned}K &= -(\ell+1) \quad \text{if } j = \ell + 1/2 \\ &= +\ell \quad \quad \quad = \ell - 1/2\end{aligned}$$

The probability of finding the muon within a radial increment δr at radius r is given by:

$$[F(r)^2 + G(r)^2] \delta r r^{-2} \tag{27}$$

Now m is the reduced mass of the muon-nuclear system and is used to account for the motion really being about the mass center

$$m = \frac{m_\mu m_n}{m_\mu + m_n}$$

Replacing $V(r)$ by the simple electrostatic interaction for two point charges, it becomes

$$V(r) = -\frac{Ze^2}{r} \tag{28}$$

Then equation 26 can be solved exactly to give explicit analytical functions for the energies⁸⁶ and the wave functions.⁸⁷ Because of their slight nuclear overlap this is a good approximation for some of the higher orbital angular momentum states.

For a finite-sized nuclear charge distribution the exact solutions to equation 26 were obtained by using a computer code of McKee,³⁴ which numerically integrates the equations using a Runge-Kutte technique. The potential for the finite size charge distribution is given by:

$$V(r) = -\frac{Ze^2}{r} + \frac{4\pi e}{r} \int_r^\infty p(r')(r'^2 - rr') dr' \quad 29$$

where only the first two terms in the multipole expansion of $V(r)$ have been used. With an initial estimate for the eigenvalue E , the equations were integrated outwards from the origin, and inwards from the infinity. The difference in the ratios $F(r)/G(r)$, obtained by each integration, was evaluated at the muonic Bohr radius. After calculating several values of this difference corresponding to different values of E , an interpolation program was used to evaluate the energy eigenvalue giving a zero difference. Equations 26 were then solved using the energy eigenvalue to obtain the eigenfunctions and the matrix elements necessary to calculate the relative intensity transition rates. Although this evaluates the muon binding energies for the muon-nucleus system, with the nucleus treated as a finite size structure, it is still necessary to account for numerous other effects.

These will be divided into three groups:

1. Effects calculated using the numerical solutions to the Dirac equation.
2. Effects evaluated by other authors.
3. Effects which can be shown to be either too small or absent for the targets used, and which were not applied.

A fourth group which has already been accounted for contains the finite size effect and the use of the reduced muon mass.

Group one contains first order vacuum polarization and other radiative corrections, group two includes nuclear polarization and higher order vacuum polarization, and the third group contains electron shielding, weak interactions, and static and dynamic hyperfine effects.

B. Group One Corrections

1a. Vacuum Polarization

The vacuum polarization effect is the principal quantum electrodynamics effect in muonic atoms. In the presence of an electric field the vacuum becomes polarized through the emission of virtual photons which become virtual electron-positron pairs. These have a "range" of the electronic Compton wavelength $\lambda_e = \hbar/(m_e c) \approx 386$ F, and are important for muonic atoms because of the small Bohr radius for the 1s muon $\alpha_\mu = 250/Z$ F. This correction is represented by the Feynmann graph (a) in Figure 14. To the lowest order in αZ vacuum polarization can be expressed as an additional term $V_{vp}(r)$ in the binding potential $V(r)$, and has the effect of increasing its strength at short distances.^{88,89} In a physical representation, this corresponds to the probe particle, the muon, penetrating the polarized cloud, and observing a larger effective charge than it observed from far off. A multipole expansion⁴⁴ of the vacuum polarization potential gives for the monopole term⁷⁰

$$V_{vp}(r) = \frac{2\alpha e}{3\pi} \int_0^\infty dr' \frac{r'}{r} \left[H\left(\frac{|r'-r|}{\lambda_e}\right) - H\left(\frac{r'+r}{\lambda_e}\right) \right] \rho(r') \quad 30$$

where

$$H(t) = \int_1^\infty dy y^{-2} \exp(-2yt) (1 + 1/2y^{-2}) (1-y^{-2})^{1/2} \quad 31$$

Following the method of Glauber et al.⁹⁰ this may be expanded in a power series in t

$$H(t) = \sum_{k=0}^{\infty} [a_k t^k \exp(-2t) + b_k t^{2k+1} E(2t)] \quad 32$$

where $E(2t)$ is the exponential integral

$$E(2t) = \int_1^\infty \frac{\exp(-2tZ)}{Z} dZ \quad 33$$

The coefficients in equation 32 have been evaluated by Anderson et al.,⁷⁵ and they find that by terminating the expansion at a_4 and b_2 , the error introduced is no larger than 0.01%.

The vacuum polarization energy correction is evaluated from $V_{vp}(r)$ using first order perturbation theory and is given by

$$\Delta E_{vp} = \int_0^\infty V_{vp}(r) [F(r)^2 + G(r)^2] dr \quad 34$$

where $\frac{F(r)}{r}$ and $\frac{G(r)}{r}$ are the eigenfunction from the Dirac equation. This gives an energy correction to the 1s state varying from 67 keV for ^{206}Pb to 35 keV for ^{120}Sn . The other muonic energy levels have much smaller shifts because of the smaller muon-nucleus overlap. It is important to allow for these higher level shifts since the long range of the effect makes them large compared to the experimental errors. Table XII shows the corrections applied for the important levels of each target material.

Because the vacuum polarization effect persists for the higher levels, while finite size effects and the other radiative corrections diminish, it is possible to obtain experimental verification of the theoretical values. Measurements of the principal transitions involving the $n = 4, 5$ and 6 levels have been carried out, and confirm the theoretical predictions on vacuum polarization for these levels.^{91,92} For these measurements it is necessary to correct for the effects of electron shielding and higher order contributions to the vacuum polarization.

1b. Lamb Shift

The Lamb shift is a prime consideration with electronic atoms, but it is also important for the lower levels of muonic atoms.

It is referred to as the self-energy of a particle and physically represents the interaction of a charged particle with the electromagnetic field it produces. To first order in α it can be expressed in the form of a potential energy V_{LAMB} ^{93,94}

$$V_{\text{LAMB}} = \frac{4}{3} \alpha e^2 \lambda_\mu^2 \left[\ln \frac{m_\mu c^2}{2\Delta E} + \frac{11}{24} - \frac{1}{5} \right] \langle \rho(r) \rangle \quad 35$$

where the λ_μ = Compton wavelength for the muon = $\hbar/(m_\mu c)$ $\langle \rho(r) \rangle$ is the space average of the charge density over the nucleus and ΔE is an average excitation energy of the muon calculated by the Bethe sum.^{93,95} This expansion can theoretically only be made under the condition $Z\alpha \ll 1$, which is not valid for ^{206}Pb , but because of nuclear penetration a muon in the lower states sees an effective total charge $Z' < Z$. Barrett et al.⁷⁰ show this expansion to be accurate to within 30%. The average excitation energy of the muon is approximated in their method by equating it to the binding energy of the state under consideration.

The Lamb shift for the muonic 1s state varies from 2.5 keV at $Z=82$ to 1.5 keV at $Z=50$, so that a 30% error could have a substantial effect on the predicted energies. Table XIII shows the Lamb shift for all levels calculated using the method of Barrett. The Lamb shift causes a decrease in the binding energy and principally affects the muonic 1s level.

1c. Anomalous Magnetic Moment

A further energy shift is produced because the magnetic moment of a bound muon is different from that of a free muon.^{96,97} This difference can be expressed in the form of a correction to the potential

energy. The energy shift evaluated by first order perturbation theory gives³⁴

$$\Delta E_{\text{AMM}} = - \frac{\alpha}{2\pi} \kappa_{\mu} \int_0^{\infty} \frac{d\phi(r)}{dr} F(r)G(r) dr \quad 36$$

where $\phi(r)$ is the electromagnetic potential, and $\frac{F(r)}{r}$ and $\frac{G(r)}{r}$ are Dirac eigenfunctions. Table XIV shows the corrections applied to the various states for each target. It is apparent from the small energy shifts that this effect is relatively unimportant, except for the 1s state, and that large percentage errors can be tolerated without seriously effecting the transition energies.

C. Group Two Corrections

This contains nuclear polarization and higher order vacuum polarization, effects calculated without using the solutions to the Dirac Equation.

2a. Nuclear Polarization

Nuclear polarization arises when the muon-nucleus electrostatic interaction excites the system into virtual excited states. In the solution of equation 24 the non-relativistic approximation can be used to calculate corrections since the maximum muon binding energy is approximately 10 MeV and this is very much less than the rest mass of the muon. The total Hamiltonian can then be written

$$H_{\text{tot}} = \frac{p_{\mu}^2}{2m} + V(r) \quad 37$$

In solving equation 26 an approximate form for $V(r)$ was used

$$V(r_{\mu}) = - \int \frac{e \rho(r)}{|\mathbf{r} - \mathbf{r}_{\mu}|} d\mathbf{r} \quad 38$$

The correct form for the potential energy is

$$V(r_\mu) = -e^2 \sum_{p_i} |\bar{r}_{p_i} - \bar{r}_\mu|^{-1} \quad 39$$

where the sum is over all protons in the nucleus and $\bar{r}_{p_i}$ is the coordinate of the i^{th} proton. The difference between these two expressions for the electrostatic potential energy δH , can be treated as a perturbation and the resultant change in energy is called nuclear polarization.

Chen⁶⁵ has shown that the nuclear polarization energy shift may be given by

$$\Delta E = \sum_{\substack{i \neq I_0 \\ \mu \neq \mu_0}} \frac{|\langle I_0 \mu_0 | \delta H | I_\mu \rangle|^2}{E_{I_0} + E_{\mu_0} - E_I - E_\mu} \quad 40$$

E_i is the energy of the state i , $\langle I_0 \mu_0 |$ is the unperturbed wavefunction and the summation is over all possible states except the unperturbed one. When the wavefunction is expressed as a product of separate muon and nuclear wavefunctions ($\langle I_\mu | = \langle I | \cdot \langle \mu |$), the term in equation 40 involving $V(r_\mu)$ becomes zero, since $V(r_\mu)$ is independent of nuclear parameters and the nuclear wavefunctions $\langle I |$ and $\langle I_0 |$ are orthogonal. To further simplify equation 40 $(|\bar{r}_{p_i} - \bar{r}_\mu|)^{-1}$ may be expanded in terms of spherical harmonics⁹⁸ and a new quantity ΔH_ℓ defined by

$$\Delta H_\ell = \sum_{p_i} \frac{4\pi e^2}{(2\ell+1)} \frac{r_<^\ell}{r_>^{\ell+1}} \sum_{m=-\ell}^{m=+\ell} Y_{\ell m}(\Omega_{p_i}) Y_{\ell m}(\Omega_\mu) \quad 41$$

where Ω_{p_i} and Ω_μ are the angular coordinates of the i^{th} proton and the muon respectively, and $r_<$ and $r_>$ are respectively the lesser and greater of $|r_\mu|$ and $|r_{p_i}|$. The nuclear polarization is then given by

$$\Delta E = \sum_{\ell=0}^{\infty} \Delta E_{\ell}$$

$$\Delta E_{\ell} = \sum_{\mu, I \neq I_0} \frac{|M_{I,\mu}^{\ell}|^2}{E_{I_0} + E_{\mu_0} - E_I - E_{\mu}} \quad 42$$

$$M_{I,\mu}^{\ell} = \langle I_0 \mu_0 | \Delta H_{\ell} | I \mu \rangle \quad 43$$

From equation 42 the energy shift for the ground state must be negative. The most recent and comprehensive method of evaluating equation 42 has been developed by Chen⁶⁵ and will be described briefly. The sums over the muonic and nuclear intermediate states are treated from different approaches.

The sum over the muonic intermediate states is treated exactly by using the reference spectrum method. This method calculates the first order correction to the wave function $|\Delta\psi\rangle$

$$|\Delta\psi\rangle = \sum_{\mu} \frac{|I\mu\rangle \langle I\mu | \delta H_{\ell} | I_0 \mu_0 \rangle}{E_{I_0} + E_{\mu_0} - E_I - E_{\mu}} \quad 44$$

by solving the inhomogeneous differential equation

$$(E_{I_0} + E_{\mu_0} - E_I - H_{\mu}) |\Delta\psi\rangle = \delta H_{\ell} |I_0 \mu_0\rangle \quad 45$$

where H_{μ} is the radial Hamiltonian operator for the muon

$$H_{\mu} = -\frac{\hbar^2}{2m_{\mu}} \frac{d^2}{dr_{\mu}^2} + \frac{1}{2} \ell_{\mu}(\ell_{\mu}+1) \frac{\hbar^2}{m_{\mu} r_{\mu}^2} + V(r_{\mu}) \quad 46$$

The energy shift is then calculated from

$$\Delta E_{\ell} = \sum_I \langle I_0 \mu_0 | \delta H_{\ell} | \Delta\psi_I \rangle \quad 47$$

This involves a knowledge of the energies of the nuclear intermediate states E_I , which is not completely available. A comparison with the result obtained using the known discrete muonic levels shows that a

considerable portion of the contribution comes from muonic states in the continuum. These have energies much larger than nuclear excitation energies so that uncertainties in E_I for this states will have very little effect on ΔE_ℓ .

The summation over nuclear states has been calculated both by using closure⁹⁹ with a chosen average nuclear energy $(E_I)_{av}$, and by using calculated theoretical wave functions for the closed shell nuclei. For the nuclear dipole states ($\ell=1$) the wavefunctions evaluated by Gillet et al.¹⁰⁰ and by Kuo et al.^{101,102} were used. Although these wave functions differ considerably, the results are in good agreement, and indicate that principal contributions come from the giant resonance states, which are experimentally and theoretically well understood. Because the result using closure is also in good agreement, the errors associated with the dipole contribution are probably small.

Monopole corrections ($\ell=0$) have been evaluated by Chen from Bloomquist's wavefunctions¹⁰³ and from the closure approximation using various average nuclear energies. Because of a strong energy dependence, and a lack of experimental information on monopole states, these contributions have a large associated uncertainty.

For the higher multipole states the contributions are reliably calculated using the closure approximation. Because of the decreasing radial wavefunction overlap the contributions diminish in magnitude with ℓ . This results in a dominance of contributions from high energy muonic states in the continuum, so that uncertainties in the average nuclear excitation energy have negligible effect and the closure approximation is extremely accurate.

Closure has been used exclusively to calculate the nuclear polarization effect on muonic levels other than the 1s state. The muonic 2s state is the only other one with a large uncertainty produced by the substantial monopole contribution.⁷

A more recent calculation by Skardhamar⁶⁶ uses a hydrodynamic nuclear model, and has included calculations on the n=3 muonic energy levels. There is excellent agreement between these two methods, and because of the extension to n=3 levels, Skardhamar's numbers have been used here. The calculated nuclear polarization shifts for the various levels in each target are given in Table XI.

2b. Higher Order Vacuum Polarization

The vacuum polarization accounted for so far has been evaluated only to the first order in α^2 . With the experimental precision now obtainable it is necessary to account for higher order terms. The corrections applied here are due to the work of Fricke,⁷¹ who has calculated the effects of second and third order contributions graphs. The second order graphs are shown in Figures 5(b), 5(c), 5(d) and 5(e). The effect of production of virtual $\mu^+ - \mu^-$ pairs shown in Figure 5(f) has already been accounted for by Barrett *et al.*⁷⁰ and is represented by the -1/5 in equation 35. Table X shows the values of this effect.

D. Group Three Corrections

These are effects which are either too small to be detected or do not exist for the atoms studied.

3a. Weak Interactions

These are estimated by Hill and Ford²⁰ from the electron-nucleon beta decay interaction, and are probably less than 1eV, and hence completely negligible.

3b. Electron Shielding

For the higher muonic states there will be some overlap between the muon wavefunctions and the electron cloud wave functions. This will have the effect of reducing the effective charge seen by the muon and will hence reduce the binding energies. Barrett *et al.*⁷⁰ have evaluated the effect for the 1s and 5g states of ²⁰⁹Bi and find them to be 4.6eV and 190eV respectively. Since the only level with $n > 3$ which is used to derive nuclear parameters is the 4d level, the electron shielding effect can safely be neglected here.

3c. Static Hyperfine Effects

The term hyperfine effect is applied to an interaction of the muon with some property involving nuclear structure, such as the magnetic dipole moment or the electric quadrupole moment. All the targets used in these experiments are even-even nuclei, and hence have zero magnetic moment, and zero electric quadrupole moment. Thus both the magnetic dipole and the electric quadrupole hyperfine effects are eliminated.

3d. Dynamic Hyperfine Effects

In the normal cascade scheme the muon reaches the 1s state without inducing excitations in the nucleus. If there is a near energy degeneracy between two muonic levels, and an excited nuclear state, it is possible for the nucleus to become excited, and for

a set of compound muon-nucleus states to exist with energy levels different from an unperturbed system. This possibility is explored thoroughly by Scheck¹⁰⁴ and by Scheck and Hüfner,⁸ but did not occur in the targets studied here.

The effects listed under Group 1 were accounted for in every fit to nuclear parameters. When the 1s nuclear polarization effect was included in deriving energies, high order vacuum polarization effects were accounted for in all levels. Theoretical nuclear polarization corrections were also applied to all levels whose nuclear polarization effect was not being treated as a variable.

IV. RESULTS

A. General Procedure

The presentation of the results will be in the form of separate sections for each target used, with an additional section summarizing the results on nuclear polarization. The section on each target will contain details of the specific calibrations used, a complete listing of the x-rays observed, including energies and intensities, and a summary and comparison of the various sets of nuclear parameters derived. The χ^2 per degree of freedom is calculated for each set and is used to compare their relative merits. In the determination of energies, particular care was taken in evaluating those transition energies which were used to derive nuclear parameters. This meant using several alternative methods for determining gains and energy standards. The energies of other peaks were evaluated by only one method and were hence assigned larger errors. A weighted average is quoted for some of the higher transitions in which the spectral line consists of two or more unresolved components. The averaging included all components which contributed more than 10% of the intensity, and weighted them according to their theoretical relative intensities. In the text and figures these will be denoted by the principal quantum numbers of the energy levels involved; that is all unresolved n=6 to n=4 transitions are denoted "6-4", with more complete details given in a table.

In many cases spectral lines were detected whose origins were unknown, but whose energies were accurately measured. When these appear in a figure they are described solely by their energy determination.

With the exception of the nuclear polarization, the errors quoted in the nuclear parameter tables are the diagonal elements of the error matrix. Nuclear polarization was treated differently since, as explained in Chapter 3, there is reason to assign a relatively large uncertainty to the nuclear polarization effect on the 2s level, and this effect was assumed known when deriving sets of nuclear parameters. Once convergence had been obtained with a set of nuclear parameters including a nuclear polarization parameter, the value of the nuclear polarization effect on the 2s level was allowed to vary by 20% in either direction and the nuclear parameters were then recalculated. The average change in the 1s nuclear polarization was then added in quadrature to its uncertainty derived from the error matrix in order to derive a final uncertainty.

B. The Nucleus ^{120}Sn

The ten transition energies and the one energy difference used to derive nuclear charge parameters are given in Table XV, along with theoretical values, previous experimental values, and theoretical and experimental relative intensities. The 3d_{3/2}-2p_{1/2} transition at 1022 keV was used as a standard for intensity measurements since it is an intense single line. A precise comparison can only be made with the work of Ehrlich,¹⁰⁵ since the other numbers were deduced

for either a different isotope,¹⁰⁶ or natural Sn.¹⁰⁷ Full details of the other transitions which were detected and identified are given in Table XVI.

The 2p_{3/2}-2p_{1/2} energy difference given in Table XV was evaluated from the K lines. In addition to its sensitivity to the nuclear parameters this value is important since it determines the exact energy splitting of the 2s-2p doublet. Three independent measurements were made of this quantity. The double escape, single escape and photopeaks of the K lines were used to give a weighted average. The absolute energy values of the K lines were evaluated using ThC'' and ²⁴Na calibration sources feeding through into the spectrum in accidental coincidences with stopping muons. Both these sources have an intense line which falls between the single and double escape peaks of the K lines. The energies of the K lines were then accurately evaluated by using these energy standards and by determining the gain from either a ⁵⁶Co source, or from the 511 keV energy differences. Figure 13 shows the spectrum obtained using the ThC'' as the energy standard.

The energies of the 3d-2p, or L lines, were determined using both ⁴⁶Sc and ⁵⁶Co calibration sources in "feedthrough" runs. Both these sources have γ -ray energies which closely bracket the L lines and in addition the ⁵⁶Co has higher energy lines which were used to evaluate the 4d-2p energies. The four lines involving the muonic 2s state were evaluated by determining the gain with ¹³³Ba, ¹⁹²Ir and ²²Na calibration sources, and then relating them to either the

511 keV positron annihilation peak, or one of the L lines in the summed spectrum. The 2s-2p region of the spectrum is shown in Figure 15 and the 3p-2s region is shown in Figure 16. Peak identifications are shown wherever possible. The 2s-2p lines were identified by their absolute energies, their energy splitting and their relative intensities. The identification of the 3p-2s lines appears fairly conclusive but there were two ambiguities. The first was their anomalous intensities and the second was the presence of a peak at 720.7 keV, whose origin is unknown and which was predominately prompt. All three peaks had contamination from small peaks in the delayed spectrum, but even when these were allowed for, the intensities were higher than predicted theoretically. It is unfortunate that additional identification cannot be obtained on the energy splitting from the 3-1 transitions, but insufficient data were obtained to give reliable fits.

Since no parallel work had been done with electron scattering all the fits to nuclear charge parameters assumed zero central depression, and were restricted to two- and three-parameter Fermi charge distributions. The results of the fits are summarized in Table XVII. It can be seen that any of these distributions adequately describe the experimental data and that a slightly better χ^2 per degree of freedom is obtained by omitting nuclear polarization. The description of the nucleus including nuclear polarization is also satisfactory, and presumably more reasonable, since nuclear polarization seems clearly present at higher Z, and is predicted to increase the binding energy of the 1s state by 4.5 keV in Sn. The associated uncertainty

is 3.0 keV which is dominated by the sensitivity of the parameters to the value assigned to the 2s nuclear polarization. There is a considerable variation in the nuclear parameters obtained with and without nuclear polarization, so that an improvement in the energy determination, especially for the K and L lines, together with results from electron scattering, should determine whether the nuclear polarization effect is in agreement with the theoretical predictions.

C. The Nucleus ^{138}Ba

The transitions measured and identified in ^{138}Ba are shown in Tables XIX and XX. The first table consists of transitions which possess a strong dependence upon the finite nuclear size, and which were used to derive the nuclear parameters. The second table lists the other transitions detected. These were not used to derive nuclear parameters because they either had negligible dependence upon the charge parameters, or the energy uncertainty was too large, or they were members of an unresolved multiplet.

The energies of the K-line double escape peaks were evaluated from a feedthrough run using a ^{24}Na calibration source which has an intense line at 2.75 MeV evaluated to within 120eV (ref. 109).

The ^{24}Na feedthrough run was also used to determine the L-line energies since it has an additional line at 1368 keV. A further feedthrough run was taken using ^{60}Co which has γ -ray energies closely bracketing the ^{138}Ba L lines. There was excellent agreement between the energies evaluated by the Carnegie-Mellon group,⁷⁷ and by this work.

The other transitions in Table XIX were evaluated using a gain curve and relating to either the 511 keV line or the L lines. The region of the 3p-2s transitions is shown in Figure 17. There is very little structure in this region and the 3p-2s lines were easily distinguished. The $8d_{3/2}-3p_{1/2}$ line is energy degenerate with the $3p_{1/2}-2s$ transition, but a theoretical cascade program showed its intensity to be negligible. As further identification of the 3p-2s lines there is excellent agreement between the 3p energy splitting evaluated from this doublet and from the 3p-1s transitions. The identification of the 2s-2p transitions relied more heavily upon the energy splitting and the relative intensities, since there were many peaks in that region. The spectrum is shown in Figure 18 with identification of the peaks wherever possible.

The relative intensity of each peak is quoted with respect to the $3d_{3/2}-2p_{1/2}$ transition. The only exceptions are the 3-1 energy transitions where the uncertainties of the areas of the photopeaks were too large to use reliably, and the numbers quoted are relative intensity within the double escape multiplet. The energies of the 3-1 double escape peaks were measured using the 511 keV difference gain curve and the K-line photopeaks. Their relative intensities do not suggest a strong contribution from electric quadrupole transitions for this low Z, but this is not entirely clear since there were insufficient data to fit the energy-separated $3d_{3/2}-1s$ peak at 5205 keV. The 4-2 transitions do not provide additional information since they are degenerate and the experimental relative intensity has a large uncertainty.

The 10 transitions identified in Table XVIII were fitted to several forms of charge distribution. The most successful fits were obtained using a Fermi distribution, with $m=1$, and a Family II distribution. Following Heisenberg et al.,⁵¹ a sinusoidal variation was added to the Fermi distribution, but using the amplitude they suggest, the energy levels were essentially independent of this form of perturbation. The results obtained for the charge parameters are summarized in Table XX. With the χ^2 per degree of freedom as a measure of the goodness of fit, the best agreement between theory and experiment was obtained by including nuclear polarization effects, and by allowing the $1s$ polarization to vary. Consistently the value obtained for the nuclear polarization increases the binding energy of the $1s$ state by 6 keV, with an associated error of 1.3 keV. This is only in moderate agreement with the prediction of 3.4 ± 0.4 keV of Chen.¹⁰⁹ It is interesting that the same polarization result was obtained whether or not a central depression was included, although a realistic value of the central depression could not be derived from the muonic x-ray energies. The standard central depression used in these calculations for ^{138}Ba was $w=0.375$, as derived by Heisenberg et al.⁷⁸ from electron scattering using an $m=2$ Fermi distribution.

Table XXI gives a comparison of the ^{138}Ba charge parameters obtained from electron scattering, from Thompson's work at Carnegie-Mellon, and from this work. There is good agreement among the results, and it appears that including a central depression in the muonic x-ray analysis improves the agreement between electron scattering and muonic x-ray work, without affecting the conclusion drawn on muonic nuclear polarization.

D. The Nucleus ^{140}Ce

The ten transition energies used to derive nuclear parameters are given in Table XXII. The relative intensities are measured with respect to the $3d_{3/2}-2p_{1/2}$ line, which was set to the theoretical value of 25%. Other transitions observed and identified are shown in Table XXIII.

The K line ($2p-1s$) energies were measured with respect to the ^{24}Na 2.7 MeV line, which was accumulated in accidental coincidence with muon stops, and with respect to ^{56}Co and ^{66}Ga lines which were accumulated with no gating requirement. The energies of the 3-1 double escape peaks were determined using the 2-1 photopeak energies and the gain from the 511 keV differences. These in turn were used to determine the 4-1 energies. The L lines were evaluated using a "feedthrough" run with ^{22}Na and ^{60}Co calibration sources. These sources have well determined γ rays bracketing the lower L lines and within 50 keV of the upper line. The spectrum was similar to that shown in Figure 12, the corresponding spectrum for ^{142}Ce . The other five principal transition energies were evaluated using a gain curve determined by calibration sources, and relating them to either the 511 keV positron annihilation line or to an L line. There was good agreement between the energies obtained in this experiment and those obtained at Carnegie-Mellon.⁷⁷

The relative intensities of the 4-1 transitions are quoted within the multiplet since only the double escape peaks had reasonable statistics. The 3-1 relative intensities were measured with respect to the $3d_{3/2}-2p_{1/2}$ line, but no value is quoted for the $3d_{3/2}-1s$

electric quadrupole transition since the photopeak had insufficient data. Electric quadrupole transitions appear important for this high Z , since it is only by including them that good agreement is achieved between theoretical and experimental relative intensities.

The identification of the 3p-2s doublet was relatively easy, since there was no structure in the delayed spectrum, there was good agreement with initial energy predictions, and the 3p splitting could be compared to that obtained from the 3p-1s doublet.

The region of the 2s-2p transitions in ^{140}Ce is shown in Fig. 19. The 2s-2p identification was complicated since the upper line was almost coincident with a peak at 447 keV, which appeared in both prompt and delayed spectra. This line was identified as a combination of the 8-4 muonic x-ray lines in ^{140}Ce and a μ capture γ ray in ^{137}La (446 $\rightarrow$ 0). A third peak in that vicinity at 438 keV is most likely another delayed γ ray in ^{137}La (446 $\rightarrow$ 10). The detector resolution permitted partial separation of the upper two lines and a successful fit was made to three Gaussians.

The charge parameters were derived for a 3-parameter Fermi distribution. In an earlier analysis²¹ a fit was made with a variable central depression, w . Poor sensitivity to a central depression was indicated by the results obtained, $w=9\pm7$. The fits were made with $w=0$ and the results are shown in Table XXIV. The best fit was obtained by allowing nuclear polarization to vary, although a satisfactory fit was obtained with zero nuclear polarization. The value predicted for nuclear polarization is 5.8 ± 1.2 keV which is somewhat

higher than the theoretical value of 3.6 ± 0.4 keV.¹¹⁰ This fit, however, was much better than one in which the nuclear polarization was held at its theoretical value.

There is good agreement with the parameters derived by Thompson,⁷⁷ who used the theoretical value for nuclear polarization. Our parameter uncertainties are smaller because of the additional transitions used to derive the parameters.

E. The Nucleus ^{142}Ce

Since there was a 7% contamination of ^{140}Ce in the ^{142}Ce target, most peaks in the spectra had two unresolved components, one from ^{140}Ce and one from ^{142}Ce . Energy corrections were applied for this using equation 17 and the isotope shifts in Table VIII. This amounted to 0.2 keV for the 2s level and were negligible (<0.05 keV) for higher levels. No correction was needed for the K lines since the two components were clearly resolved as shown in Fig. 20. The transitions detected and identified are listed in Tables XXV and XXVI. Table XXV contains the energies used in determining nuclear parameters, and, in addition to the usual transitions, it includes the 2p doublet splitting as determined from the K lines. This was included because of the relatively large errors on the K lines. Except for the 3-1 transitions, the relative intensities have all been measured with respect to the $3d_{3/2}-2p_{1/2}$ line intensity. The 3-1 intensities were measured within the multiplet and were derived from the double escape peaks.

Because of the isotope splitting shown in Figure 20 the 2p-1s energies have been determined from the corresponding lines in ^{140}Ce .

The gain was evaluated from the 511 keV energy differences. In making these energy measurements, accurate values were obtained for the 2p splitting, and for 140-142 isotope shift (I.S.) of the two K lines. The latter values are

$$\text{I.S. (2p}_{3/2}\text{-1s)} = 15.36 \pm 0.40 \text{ keV}$$

$$\text{I.S. (2p}_{1/2}\text{-1s)} = 14.90 \pm 0.40 \text{ keV}$$

where the higher energies belong to ^{140}Ce . For these, and the absolute K-line energies, there is moderately good agreement with the work of Thompson.⁷⁷

To evaluate the L lines a feedthrough run was taken using ^{60}Co and ^{22}Na calibration sources. The region of interest is shown in Figure 12. The accuracy of the calibration lines and their proximity to the lines of interest enabled very accurate energy determinations to be made. The other five principal transitions were evaluated by relating them to either the 511 keV line or an L line.

The identification of both 2s-2p and 3p-2s doublets was complicated by the presence of other peaks in the region of interest. The spectrum near the 2s-2p doublet is shown in Figure 21. The lower line, 2s-2p_{3/2}, has a small background peak appearing in the spectrum at the same energy. To account for this, it was assumed that the prompt-to-delayed area ratio was the same for the delayed peak under the 2s-2p_{3/2}, as for the adjacent γ ray at 389 keV. This fraction of the delayed peak at 380 keV was then subtracted from the prompt peak and the intensity error increased proportionately. The upper line, 2s-2p_{1/2}, was overlapped by the muonic 8-4 transitions, as well as being on the lower edge of the very intense 4f_{7/2}-3d_{5/2} line.

In the figure the 4-3 transitions are omitted since they far exceed the scale used. Fits were taken of two Gaussians on an exponential background, as well as three and four Gaussians, the latter including both 4-3 transitions. The peak center remained consistent within the fitted errors, and the mean center was used with a slightly enlarged error.

The 3p-2s region is shown in Figure 22. The 3p-2s peaks were positively identified from approximate energy, energy splitting, and relative intensity. The peak directly above the doublet, corresponds in energy to the lower energy members of the 9-3 transitions. Its intensity is substantially larger than anticipated from a simple cascade program, and it is also surprising that no other peaks appear below and between the 3p-2s lines, which would correspond to 8-3 transitions. Note that the 7-3 transitions appear clearly in the lower part of the spectrum.

The energy values of the 3-1 transitions were determined from the 2p-1s photopeak energies using a 511 keV energy differences. Within the multiplet, relative intensities were calculated from the area ratios, assuming the detector efficiency to be constant over that small range.

The energies were fitted to Fermi distributions and the results are summarized in Table XXVII. The most satisfactory fit was obtained using a variable nuclear polarization which gave a value of 6.3 ± 1.4 keV. This is higher than the theoretical nuclear polarization of 3.6 keV,¹⁰⁹ although a 25% decrease in the polarization effect on the 2s level produces much better agreement between the two polarization

values. Fits were also attempted using a variable central depression. These gave a central depression which was indistinguishable from zero. There were no other accurate nuclear parameter determinations for a comparison, but these follow the anticipated trend of a slightly larger radius than ^{140}Ce .

F. The Nucleus ^{206}Pb

The principal transitions and energy differences which have been accurately determined in ^{206}Pb are shown in Tables XXVIII and XXIX. Table XXVIII shows those which were used to derive nuclear parameters. Table XXIX contains the other principal transitions and energy differences which were measured and identified. Two energy differences were used to help derive the nuclear parameters since the K lines had relatively large errors and since no other transitions which involved the 3p states were positively identified. The 3p-1s transitions were excluded because of their large energy uncertainty. The relative intensities were normalized to the 4f5/2-3d3/2 line at 971 keV and this was assigned the theoretical value of 29%.

Because of the isotopic contamination of the target (see Table VI) it was necessary to apply energy corrections to the deduced energies following Equation 17. The isotope shifts are given in Table VIII, and were either deduced from theoretical predictions or taken from the work of Anderson et al.⁷⁵ Since the work of Anderson et al. on ^{206}Pb was similar to this work a comparison is made with his experimental results whenever possible.

To evaluate the K lines an ungated run was made with a ^{66}Ga source present. The ^{142}Ce target was also placed in the muon beam so that in addition to the ^{66}Ga lines at 4.80, 4.46 and 4.29 MeV, there was a well known muonic x ray line at 4.15 MeV. Unfortunately, the upper two ^{66}Ga lines were poorly determined so that the ^{142}Ce lines were used as the energy standard to evaluate the energies of the K double escape peaks. The 3-1 energies were then deduced from the 2-1 energies using a gain curve determined by the 511 keV energy differences. The determination of detector efficiency at this high an energy is a major problem, but since this was a secondary consideration of the experiment the relative efficiency of the detector was determined up to 3.4 MeV using a ^{56}Co source and then extrapolated linearly on a log-log plot. From experimental and theoretical comparisons at higher energies, it appears that the detector efficiency decreased faster than the extrapolated predictions. Because of this uncertainty in detector efficiency, the relative intensities of the 3-1 energy transitions at 8.4 MeV are only quoted relative to each other.

A ^{56}Co calibration source was run under accidental gating conditions to determine energies of the L and M lines. In addition to bracketing both of these lines it was used to determine the gain and the relative efficiency. Using this gain the 2s-2p lines were evaluated using both the M lines and the double escape of the L lines as energy standards, while the 4-2 lines were related to the L line photopeaks.

The analysis of the transitions involving the 2s states is complicated by the presence of structure in both the prompt and delayed spectra. Figure 23 shows the prompt and delayed spectra in the region of 2s-2p_{3/2} with the Gaussians fitted in both spectra superimposed. The three Gaussians fitted in the prompt spectrum correspond to transitions with energies of 1029, 1038 and 1043 keV respectively. Peaks were fitted in the delayed spectrum with energies of 1038 and 1043 keV. The prompt to delayed ratios of the 1038 and 1043 peaks were consistent with their identification as delayed γ rays. These γ rays were tentatively identified as transitions in the Ge detector (^{70}Ge , 1040 $\rightarrow$ 0) and in ^{205}Pb (1044 $\rightarrow$ 0). The remaining peak at 1029.0 ± 0.45 keV was then identified as being the 2s-2p_{3/2} transition. Although its intensity is less than anticipated, it is consistent with similar results obtained in ^{208}Pb .²²

The 2s-2p_{1/2} region is shown in Figure 24. The initial analysis of the prompt and delayed spectra were in terms of three Gaussians and the resultant areas suggested that there was some prompt contribution to the central peak. However the width of the prompt peaks was anomalously large and the fit was relatively poor, so that the prompt spectrum was reanalyzed in terms of four Gaussians, the central peak being assumed a doublet. This resulted in an improved fit, a width consistent with other adjacent peaks, and better agreement with the peaks fitted in the delayed spectrum. The peaks common to the prompt and delayed spectra have been identified as being at 1208 keV, 1219 keV and 1234 keV, whereas the remaining prompt line is at 1214.8 ± 0.5 keV and is identified as the 2s-2p_{1/2} transition. Its

intensity is in poor agreement with theory but again is consistent with results obtained in ^{208}Pb .²² A further fact supporting both 2s-2p identifications is the 2s-2p doublet splitting, which is in excellent agreement with that determined from the K lines. In the previous three Gaussian fit this energy splitting was inconsistent, but the absolute 2s-2p_{1/2} energy agreed with Anderson *et al.*,⁷⁵ who also obtained a value of 1217.8 keV, but were unable to detect the lower 2s-2p_{3/2} transition.

The region which should contain the 3p-2s peaks is shown in Figure 25. Theoretically they should bracket the 3d_{5/2}-2p_{3/2} double escape peak at 1479 keV, the lower line appearing near channel 1278. No suggestion of a peak could be found in this area. A peak was found corresponding to the 3p_{3/2}-2s transition, but it was wider than expected, and a small peak was found in the delayed spectrum, which could be contributing to this anomalous width. It was possible to fit the prompt peak with two gaussians, but neither corresponded well with the peak in the delayed spectrum. Because of the large uncertainty in the energy of the 3p_{3/2}-2s peak, and because its relative intensity was substantially below the theoretical predictions, or the results obtained in ^{208}Pb , it was omitted from the nuclear parameter analysis.

To compensate partially for the lack of these two transitions, which involved the important 2s and 3p states, an energy difference involving the 3p_{3/2} state was included in the nuclear parameter analysis. This was evaluated from the 3-1 transitions using the 511 keV energy differences. In ^{206}Pb the 3d-1s electric quadrupole transitions are clearly separated from the 3p-1s electric dipole transitions as shown

in Figure 26. The 3d-1s transitions have a comparable intensity to the 3p-1s transition because of the greater population of the initial state and because the probability of E-2 transitions varies as the fifth power of the energy. The energy dependence is clearly evident in a comparison to the 4-2 transitions, where the 4f-2p quadrupole transition intensities are less than half that of the 4d-2p dipole intensities. There is good agreement between theoretical and experimental energy differences, adding credence to the calculations by Skardhamer,⁶⁶ which indicate a substantial nuclear polarization effect in the 3p states. Had this not been included, the 3p_{3/2}-3d_{5/2} energy difference would have been two standard deviations away from the theoretical predictions. All the high-energy relative intensities are lower than predicted but this can probably be attributed to detector efficiency uncertainties. A comparison of quadrupole to dipole relative intensities within a multiplet, which should be approximately independent of detector efficiency, shows them to be in good agreement with theoretical predictions.

The results of nuclear parameter fits are shown in Table XXX. It was found that no useful information could be derived by allowing the central depression to vary, so fits were done with either $w=0$, or with $w=0.14$, the value inferred from electron scattering.⁵¹ The χ^2 per degree of freedom for the fits with nuclear polarization, and with or without a central depression are extremely low. Since a value of 1 is accepted as a good fit it seems that several of the assigned energy errors are overly conservative. However, there is evidence that the inclusion of nuclear polarization produces far

better agreement between theory and experiment than when it is omitted. When treated as a variable parameter it converges to a value of 5.8 ± 2.0 keV. Essentially the same value is obtained when a central depression is included, and both are in excellent agreement with the predictions of Chen⁶⁵ and Skardhamer.⁶⁶ A further fit was obtained by allowing the nuclear polarization effect on the 2s level to be treated as a variable. This again produced an extremely good fit, but the large errors are to be expected since only two transitions involving the 2s state were included. When compared to the nuclear parameters of Anderson et al.⁷⁵ there is poor agreement, which may be attributed to the fact that the fitting conditions are not the same. Anderson et al. did not use a variable nuclear polarization, nor did they apply nuclear polarization corrections to the higher levels.

A fit with $m=2$ again produced nuclear polarization values different from zero, but the large associated errors show that four parameters cannot be accurately determined from this data.

G. Summary of Results

So far each target has been treated as if it were an independent experiment. The purpose of the set of experiments was to investigate the general behavior of nuclear polarization in comparison with theoretical predictions, and this will be considered. The value determined for ^{120}Sn is included in the comparison since it gave a satisfactory fit to the data, and if nuclear polarization is believed to exist for high Z nuclei, then it must surely exist for lower Z , even if its effect is less pronounced. The results obtained with ^{208}Pb (ref.22) will also be considered, since they made up part of this set of experi-

ments and were analyzed in a similar fashion. Theoretical predictions are quoted as a function of Z . Direct comparisons of the theoretical and experimental results, are made in Table XXXI and Fig. 27. The experimental values obtained were independent of the form of the charge distribution used, or whether or not a central depression was included. This last statement should be qualified in that only three forms of distribution were used (Family II, Fermi with and without a central depression) and that two of these were rather similar. The three forms of distribution used for ^{138}Ba are shown in Figure 28. It can be seen from Fig. 27 that the experimental values follow the trend of the theoretical predictions, but tend to be somewhat larger. In ^{208}Pb it was found that the value of the 2s nuclear polarization strongly effected the 1s nuclear polarization and it was hence also allowed to vary. When this was done for ^{206}Pb large errors were obtained for all the parameters. This was probably caused by having insufficient charge sensitive transitions with small errors.

To explain the absence of the 3p1/2-2s transition in ^{206}Pb it is necessary to know what number of counts would be anticipated. By comparison with the ratio of transitions from the 3p states to the 1s state, the 3p1/2-2s transition would have a maximum amplitude of 70 counts on a background of 1700, and should be detectable. These numbers assume a 30% γ -ray contamination of the prompt 3p3/2-2s peak. Because of the difficulty of estimating detector efficiencies at 8.5 MeV the relative intensities of the 3-1 transitions are measured within the multiplet, and no comparison can be made with the relative intensities. Thus it appears that the 3p states are being preferen-

tially depopulated. A possible explanation of this may be the proposal by Scrinavasan and Sundaresin,¹¹⁰ that a muonic np state ($n \geq 3$) may deexcite 60% of the time by the emission of a prompt neutron leaving the muon in the 1s state and hence reducing the number of radiative transitions from the np states.

A point which deserves some discussion is the variation of nuclear polarization among isotopes. Chen has recently done some preliminary calculations on the values of nuclear polarization for the even-even isotopes of Nd. (ref. 111), and these were applied in calculating the isotope shift. He finds no large variation between neighboring even-even isotopes and this is consistent with the results of this work although any small variation would be masked by the experimental uncertainties.

V. CONCLUSIONS

Because of the good agreement between theoretical and experiment values for the lead isotopes, when nuclear polarization is included, it appears that nuclear polarization is a relatively important correction to the nuclear parameters of high Z nuclei. It also appears to give better fits in the region of $Z=58$, although the improvement of fit is not so dramatic.

A measurement of a single isotope or the constraint of a charge parameter fit to theoretical nuclear polarization values may give fits which do not unambiguously give evidence for nuclear polarization. However, in the range of isotopes measured here, the fit to charge parameters consistently seemed to require a nuclear polarization to agree with the data in higher Z elements. Assuming that it is necessary for the higher Z elements, then it should also be applicable to the lower Z elements, and if a consistent set of nuclear parameters are to be deduced then nuclear polarization effects should be included. The results of this work do not give definitive values for nuclear polarization effects in all targets, but they do indicate that the calculations of Chen⁶⁵ and Skardhamar⁶⁶ are more consistent with experimental results, than the predictions of Cole.⁶⁴ In order to reduce the error assigned to the polarization of the $1s$ level, it will be necessary to improve the theoretical uncertainty in the

Lamb shift, at present estimated at 30%. By measuring energies of the higher transitions to better than 100eV it should be possible to evaluate the contribution from high order vacuum polarization, and to compare this with the theoretical predictions, which have been used in this work.

In addition to further calculations of the Lamb shift, more theoretical work on the nuclear polarization of the 3p and 3s states would be most useful, since with the advent of high-efficiency detectors and contemporary data collecting facilities, these levels will undoubtedly be important in future muonic x-ray experiments. This is especially true for the 3s level, since its wavefunction overlap with the nucleus makes it the next most sensitive level beyond the 2s. This may be used to provide independent information on a central depression, to allow a comparison with electron scattering results. The central depression evaluated from electron scattering work seems to agree well with muonic x-ray analysis, and to bring the two sets of nuclear parameters into better agreement without affecting the values deduced for nuclear polarization. This suggests that the most accurate description of a nucleus can be obtained from the combination of electron scattering results and muonic x-ray work.

Future experiments should attempt to decrease the uncertainties on the energies, since it is clear that only this reduction will permit one to obtain more definitive information about the nuclear charge distribution. Transitions of particular interest in future experiments are the 3p-1s transitions, which would provide another

measurement directly dependent upon the $1s$ nuclear polarization. For these transitions it will be necessary to take the electric quadrupole transitions into account since two of the transitions are almost energy degenerate and have comparable intensities.

VI. ACKNOWLEDGMENTS

The author would like to acknowledge the assistance received from the following people:

Dr. Robert E. Welsh, his advisor, for encouragement and advice during the past few years, for experimental aid and guidance and for invaluable advice in the preparation of the manuscript.

Dr. Richard J. Powers, his collaborator, for helping acquire the data, for many discussions on the problems posed by the experiment and their solutions, and a detailed reading of the manuscript.

Mr. George H. Miller, for expert experimental assistance and helpful discussion on the analysis of the data.

Drs. Morton Eckhause and Hans von Baeyer, for helpful discussion on the theoretical aspects of the work and for a careful reading of the manuscript.

Drs. John R. Kane and Raymond W. Southworth, for reading the manuscript.

Dr. David A. Jenkins, for assistance in collecting the data, for helpful discussion on all aspects of the work and for reading the manuscript.

Dr. D. G. Ravenhall, for helpful discussions on the theoretical aspects of nuclear polarization and the experimental data.

Dr. Lawrence Wilets, for helpful discussions on the experimental data and their theoretical implications.

Dr. D. H. Wilkinson, for helpful discussions on the theoretical aspects of nuclear polarization and the experimental data.

Dr. Min-yi Chen, for helpful discussions on the theoretical aspects of nuclear polarization.

Dr. A. R. Kunselman, for assistance in taking the data and helpful discussion on the measurement of relative intensities.

Mr. Stanley G. Hummel and the Physics Department machine shop, for the excellent construction of much of the equipment.

Mr. Donald S. Makowiecki and Mr. Erwin Boiton, for the construction and maintenance of much of the electronics.

The staff of the William and Mary Computer Center, for their patience and help during the data analysis.

Miss Shirley McCallum, for her patient and expert programming assistance.

Dr. Robert T. Siegal and the staff of the Space Radiation Effects Laboratory, for their cheerful and efficient operation of the synchrocyclotron.

Mr. David Holt and members of the Data Acquisition Group at the Space Radiation Effects Laboratory, for providing such efficient data collecting facilities.

Mr. Francis R. Kane, Mr. C. William Lucas, Jr. and Mr. B. Lee Roberts for help in taking data.

Mrs. Gail S. Hunt, for assistance in data collection and for advice on the cascade program.

Mr. George R. Lucas, Jr., Mr. Robert Hancock and Mr. David R. Lee, for help in data analysis.

Mrs. Walter F. Schoellmann, for typing the manuscript and tables.

His wife, Pat, who at appropriate times has calculated, consoled, copied, drawn, earned, encouraged, key-punched, proof-read and typed, all to make his life easier.

VII. REFERENCES AND FOOTNOTES

1. J. A. Wheeler, Rev. Mod. Phys. 21, 133 (1949).
2. V. L. Fitch and J. Rainwater, Phys. Rev. 92, 789 (1953).
3. G. R. Burbridge and A. H. de Borde, Phys. Rev. 89, 189 (1953).
4. J. Rainwater, Ann. Rev. Nucl. Sci. 1, 1 (1957).
5. D. West, Rept. Progr. Phys. 21, 271 (1958).
6. S. Devons and I. Duerdoth, Advances in Nucl. Phys. 2, 295 (1969).
7. C. S. Wu and L. Wilets, Ann. Rev. Nucl. Sci. 19, 529 (1969).
8. F. Scheck and J. Hüfner, to be published.
9. Proceeding of the Williamsburg Conference on Intermediate Energy Physics, Feb. 10-12, 1966. Two volumes printed by the College of William and Mary.
10. Proceedings of the International Conference on Electromagnetic Sizes of Nuclei, Ottawa, May 22-24, 1967.
11. "High Energy Physics and Nuclear Structure" edited by S. Devons, Plenum Press, 1970.
12. W. Y. Chang, Rev. Mod. Phys. 21, 166 (1949).
13. J. A. Wheeler, Phys. Rev. 71, 320 (1947).
14. M. Conversi, E. Pancini and O. Piccioni, Phys. Rev. 71, 209 (1947).
15. E. Fermi and E. Teller, Phys. Rev. 72, 399 (1947).
16. L. N. Cooper and E. M. Henley, Phys. Rev. 92, 801 (1953).
17. A. L. Schawlow and C. H. Townes, Science 115, 284 (1952).

18. F. Bitter and H. Feshbach, Phys. Rev. 92, 837 (1953).
19. J. A. Wheeler, Phys. Rev. 92, 812 (1953).
20. D. L. Hill and K. W. Ford, Phys. Rev. 94, 1617 (1954).
21. D. A. Jenkins, R. J. Powers, P. Martin, G. H. Miller, R. E. Welsh and A. R. Kunselman, Phys. Lett. 32B, 267 (1970).
22. D. A. Jenkins, R. J. Powers, P. Martin, G. H. Miller and R. E. Welsh, to be published in Nuclear Physics.
23. H. L. Anderson, C. K. Hargrove, E. P. Hincks, J. D. McAndrew, R. J. McKee and D. Kessler, Phys. Rev. Lett. 22, 221 (1969).
24. G. T. Ewan and A. J. Tavendale, Can. J. Phys. 42, 2286 (1964).
25. M. Eckhause, R. T. Siegel, R. E. Welsh and T. A. Filippas, Nucl. Phys. 81, 575 (1966).
26. S. S. Gershtein, V. I. Petrukhin, L. I. Ponomarev and Yu. D. Prokoshkin, Usp. Fiz. Nauk 97, 3 (1969) - translation Sov. Phys. Usp. 97, 1 (1969).
27. P. Martin, G. H. Miller, R. E. Welsh, D. A. Jenkins and R. J. Powers, Phys. Rev. Lett. 25, 1406 (1970).
28. H. Backe, R. Engfer, E. Kankeleit, W. U. Schröder, H. K. Walter and K. Wien, p. 166, ref. 11.
29. W. Y. Lee, M. Y. Chen, S. C. Cheng, J. W. Kast, E. Macagno, A. M. Rushton and C. S. Wu, Bull. Am. Phys. Soc. 16, 562 (1971).
30. C. K. Hargrove, E. P. Hincks, G. R. Mason, R. J. McKee, D. Kessler and S. Ricci, Phys. Rev. Lett. 23, 215 (1969).
31. G. Backenstoss, S. Charlambus, H. Daniel, U. Lynen, Ch. v. d. Marlsburg, G. Poelz, H. Povel, K. Springer and L. Tauscher, Nucl. Phys. A162, 541 (1971).

32. G. R. Lucas, Jr., P. Martin, G. H. Miller, R. E. Welsh, D. A. Jenkins and R. J. Powers, to be published.
33. The development of the Dirac Equation will follow that of Bethe and Salpeter in "Quantum Mechanics of One- and Two-Electron Systems".
34. R. J. McKee, Phys. Rev. 180, 1139 (1969).
35. The use of isotope shifts to derive nuclear parameters is deliberately excluded since this experiment was concerned primarily with absolute energies. The isotope shift method essentially derives parameter differences between neighboring isotopes, and is not well suited to derive absolute charge parameters, or absolute values of nuclear polarization.
36. H. L. Acker, G. Backenstoss, C. Daum, J. C. Sens and S. A. de Witt, Nucl. Phys. 87, 1 (1966).
37. The first time the 2s level was used to derive nuclear parameters was by H. L. Anderson et al. (ref. 23). They were only able to detect 2 transitions for ^{206}Pb , and their results are not entirely consistent with the values obtained here. They also applied no nuclear polarization corrections to higher levels.
38. H. Daniel, "Mesic Atoms", Ecole Internationale de la Physique de Particle Elementaire, Herceg-Nov, Yugoslavia, 1969.
39. U. Fano, Phys. Rev. 72, 26 (1947).
40. R. A. Carrigan, Jr., P. D. Gupta, R. B. Sutton, M. N. Suzuki, A. C. Thompson, R. E. Cote', M. V. Prestwick, A. K. Gaigalas and S. Raboy, Bull. Am. Phys. Soc. 13, 65 (1968).

41. R. Baader, H. Backe, R. Engfer, E. Kankeleit, W. U. Schröder, H. Walter and K. Wein, Phys. Lett. 27B, 428 (1968).
42. J. K. MacDonald, Science 167, 1339 (1970).
43. K. W. Ford and J. G. Wills, Los Alamos Scientific Report-LAMS-2387 (1960).
44. K. W. Ford and J. G. Wills, Nucl. Phys. 35, 295 (1962).
45. K. W. Ford and J. G. Wills, LASL Report, LA-DC-10393 (1968).
46. R. C. Barrett, Phys. Lett. 33B, 388 (1970).
47. F. G. Perey and J. P. Schiffer, Phys. Rev. Lett. 17, 324 (1966).
48. L. R. B. Elton and A. Swift, Nucl. Phys. A94, 52 (1967).
49. L. R. B. Elton, "Nuclear Sizes", Oxford University Press, (1961).
50. L. R. B. Elton, S. J. Webb and R. C. Barrett, p. 67 of ref. 11.
51. J. Heisenberg, R. Hofstadter, J. S. McCarthy, I. Sick, B. C. Clark, R. Herman and D. G. Ravenhall, Phys. Rev. Lett. 23, 1402 (1969).
52. D. Vautherin and D. M. Brink, Phys. Lett. 32B, 149 (1970).
53. R. Engfer, Proceedings of International School of Physics, Enrico Fermi Institute, June 1966, Course 38, p. 64.
54. G. Breit, G. B. Arfken and W. W. Clendenin, Phys. Rev. 78, 390 (1950).
55. W. Lakin and W. Kohn, Phys. Rev. 94, 787 (1954).
56. E. Nuding, Z. Naturforsch 12A, 187 (1957).
57. W. Greiner, Z. Physik 164, 347 (1961) and W. Greiner and L. Marshall, Z. Phys. 165, 171 (1962).
58. C. Joachin, Nucl. Phys. 25, 317 (1961).

59. M. Demeur and Ch. Joachin, Nucl. Phys. 17, 329 (1960).
60. F. Scheck, Z. Physik 172, 239 (1963).
61. W. Pieper and W. Greiner, Phys. Lett. 24B, 377 (1967).
62. H. M. McKinley, Phys. Rev. 183, 106 (1969).
63. M. Y. Chen, Phys. Rev. C1, 1176 (1970).
64. R. K. Cole, Jr., Phys. Lett. 25B, 178 (1967) and Phys. Rev. 177, 164 (1968).
65. M. Y. Chen, Phys. Rev. C1, 1167 (1970).
66. H. F. Skardhamar, Nucl. Phys. A151, 154 (1970).
67. K. Tanabe, Phys. Rev. 3A, 1282 (1971).
68. C. Toepffer and W. Greiner, Phys. Rev. 186, 1044 (1969).
69. C. Toepffer and D. Drexel, Phys. Rev. Lett. 24, 1131 (1970).
70. R. C. Barrett, S. J. Brodsky, G. W. Erickson and M. H. Goldhaber, Phys. Rev. 166, 1589 (1968).
71. B. Fricke, Z. Physik 218, 495 (1969) and private communication.
72. M. G. Strauss, L. L. Sifter, F. R. Lenkszus and R. Brenner, I.E.E.E. Trans. Nucl. Sci. Vol. NS-13, 3 (1968).
73. M. G. Strauss, F. R. Lenkszus and J. J. Eichholz, Nucl. Inst. and Meth. 76, 285 (1969).
74. J. T. Routti and S. G. Prussin, Nucl. Inst. and Meth. 72, 125 (1969).
75. H. L. Anderson, C. K. Hargrove, E. P. Hincks, J. D. McAndrew, R. J. McKee, R. D. Barton and D. Kessler, Phys. Rev. 187, 1565 (1969).
76. R. J. Powers, Phys. Rev. 169, 1 (1968).
77. A. C. Thompson, Thesis, Carnegie-Mellon University (1969).

78. J. Heisenberg, R. Hofstadter, J. S. McCarthy, I. Sick, M. R. Yearian, B. C. Clark, R. Herman and D. G. Ravenhall - to be published.
79. Los Alamos Report, LA-3753, Photo Cross Sections from 0.001 to 100 MeV for Elements 1 through 100.
80. We are greatly indebted to J. Hüfner who kindly provided us with a copy of his muon-cascade program.
81. Y. Eisenberg and D. Kessler, Nuovo Cim. 19, 1195 (1961).
82. Y. Eisenberg and D. Kessler, Phys. Rev. 130, 2349 (1963).
83. A. H. de Borde, Proc. Phys. Soc. (London) A67, 57 (1954).
84. W. W. Sapp, Jr., Thesis, The College of William and Mary (1970).
85. J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley and Sons, Inc., New York, 1960) p. 583 ff.
86. W. Gordon, Z. Physik 48, 11 (1928).
87. K. Bechet, Ann. d. Phys. 6, 700 (1930).
88. E. A. Uehling, Phys. Rev. 48, 55 (1935).
89. J. Schwinger, Phys. Rev. 75, 651 (1949).
90. R. Glauber, W. Rarita and P. Schwed, Phys. Rev. 120, 609 (1960).
91. H. L. Anderson, ref. 11, p. 640.
92. G. Backenstoss, S. Charlambus, H. Daniel, Ch. v. d. Malsburg, G. Poelz, H. V. Povel, H. Schmitt and L. Tauscher, Phys. Lett. 31B, 233 (1970).
93. H. A. Bethe, L. M. Brown, J. R. Stehn, Phys. Rev. 77, 370 (1950).
94. R. Karplus, A. Klein and J. Schwinger, Phys. Rev. 86, 288 (1952).
95. G. W. Erickson and D. R. Yennie, Ann. Phys. (N.Y.) 35, 271 and 447 (1965).

96. V. W. Hughes and V. L. Telegdi, Bull. Am. Phys. Soc. 3, 229 (1958).
97. K. W. Ford, V. W. Hughes and J. G. Wills, Phys. Rev. 129, 194 (1963).
98. J. D. Jackson, Classical Electrodynamics (John Wiley and Sons, Inc., New York, 1967).
99. Closure over the nuclear states corresponds to putting

$$\sum_{I, I_0} \langle I_0 | \sum_p f(\vec{r}_p) | I \rangle \langle I | \sum_{p'} f(\vec{r}_{p'}) | I_0 \rangle$$

$$= \sum_i \langle i(\vec{r}) | f(\vec{r}) f(\vec{r}') | i(\vec{r}') \rangle - \sum_{i,j} \langle i(\vec{r}) j(\vec{r}') | f(\vec{r}) f(\vec{r}') | i(\vec{r}') j(\vec{r}) \rangle$$

where $|i\rangle$ and $|j\rangle$ are single particle states below the Fermi level. It was the second term, the exchange term, which was omitted in the original work of Cooper and Henley (ref. 16) and lead to their predicting such a large correction.
100. V. Gillett, A. M. Green and E. A. Sanderson, Nucl. Phys. 88, 321 (1966).
101. T. T. S. Kuo and G. E. Brown, Nucl. Phys. 85, 40 (1966).
102. T. T. S. Kuo and W. D. Markiewicz, Bull. Am. Phys. Soc. 12, 538 (1967).
103. J. Bloomquist, Nucl. Phys. A103, 644 (1967).
104. F. Scheck, Lecture at the SIN-Sommerschule für Mittelenergiephysik, Leysin, Sept. 1969-Ref. TH1092-CERN.
105. R. D. Ehrlich, Phys. Rev. 173, 1088 (1968).
106. H. L. Anderson, C. S. Johnson and E. P. Hincks, Phys. Rev. 130, 2468 (1963).
107. H. L. Acker, G. Backenstoss, C. Daum, J. C. Sens and S. A. de Wit, Nucl. Phys. 87, 1 (1966).

108. J. B. Marion, Nuclear Data A4, 301 (1968).
109. The theoretical values for nuclear polarization were taken from Fig. 12(a), 12(b) and 12(c) of ref. 7.
110. V. Scrinvasan and M. K. Sundaresan, Can. J. Phys. 49, 621 (1971).
111. E. R. Macagno, S. Bernow, S. C. Cheng, S. Devons, I. Duerdoth, D. Hitlin, J. W. Kast, W. Y. Lee, J. Rainwater, C. S. Wu and R. C. Barrett, Phys. Rev. C1, 1202 (1970).
112. C. M. Lederer, J. M. Hollander and I. Perlman, Table of Isotopes (John Wiley and Sons, Inc., New York, 6th Edition, 1967).
113. G. Aubin, J. Barrette, M. Barrette and S. Monaro, Nucl. Inst. methods 76, 93 (1969).
114. A. J. Haverfield, F. M. Bernthal and J. M. Hollander, Nucl. Phys. A94, 337 (1966).
115. H. L. Scott and D. M. Van Patter, Phys. Rev. 184, 1111 (1969).
116. M. E. Phelps, D. G. Sarantites and W. G. Winn, Nucl. Phys. A149, 647 (1970).
117. G. Holm, Arkiv Fysik 37, 1 (1967).

VIII. TABLES

Table Captions

- I. Binding energies in keV of muonic states with $n \leq 3$. The values for a finite nucleus and a point nucleus are obtained by solving the Dirac equation. The finite size effect is the difference between these values.
- II. The effect of finite size on the transition energies in ^{138}Ba and the values of the derivatives with respect to the nuclear parameters. The finite size effect is the difference between the finite nucleus energy and the point nucleus energy, and all energies are measured in keV.
- III. The effect of a finite size nucleus on transition energies and energy difference in ^{206}Pb , and the values of the derivatives with respect to the nuclear parameters. The finite size effect is the difference between the finite nucleus energy and the point nucleus energy. All energies are in keV.
- IV. Relative intensities and level populations for all states with $n < 4$ in ^{140}Ce . Numbers represent the percentage of all captured muons making the given transition, accounting for electric dipole, electric quadrupole and Auger emission. An omitted entry implies no transition allowed or a transition probability < 0.01 .

- V. Variation of the full width at half maximum of peaks in ^{206}Pb as a function of energy. Uncertainties are shown in parentheses.
- VI. Chemical composition and physical properties of the targets.
- VII. Principal lines of the calibration sources used to determine energies, relative efficiencies and linearity.
- VIII. Isotope shifts used for the ^{142}Ce and ^{206}Pb targets. The energies are in keV and the 3d levels are assumed to have zero isotope shift.
- IX. Transition energies and energy differences used to derive nuclear charge parameters for each target.
- X. High order vacuum polarization corrections. They are derived from ref. 71, increase the binding energy of the levels, and are quoted in keV.
- XI. Theoretical nuclear polarization values in keV. With the exception of the 3d level in ^{206}Pb these increase the binding energy of the levels.
- XII. First order vacuum polarization corrections in keV.
- XIII. Lamb shift effects for various targets. Energies are in keV and decrease the binding energy of the states.
- XIV. Energy corrections for the anomalous magnetic moment. The energies are in keV, and a negative sign decreases the binding energy of the state.
- XV. Energies and relative intensities of the transitions used to derive nuclear parameters in ^{120}Sn . A comparison is made with the theoretical predictions and other experimental results. Energies are quoted in keV, and numbers in parentheses denote uncertainties.

- XVI. Additional muonic x-rays measured and identified in ^{120}Sn .
The average energy is obtained by weighing the theoretical energies with the theoretical relative intensities. Energies are quoted in keV, and the relative intensities are measured with respect to the 3d3/2-2p1/2 line (see Table XV.). Errors are given in parentheses.
- XVII. Nuclear charge parameters for ^{120}Sn . No parallel work had been done with electron scattering so the central depression was constrained to be zero. c and t are in Fermis while nuclear polarization is in keV. The associated errors are given in parentheses.
- XVIII. Muonic x-ray transitions used to determine nuclear parameters in ^{138}Ba . Energies for each transition are measured in keV, and the 3d3/2-2p1/2 line is used as the relative intensity standard. Errors are given in parentheses.
- XIX. Other muonic x-rays measured and identified in ^{138}Ba . The average energy is obtained by weighting the theoretical energies with the theoretical relative intensities. Energies are quoted in keV and the relative intensities are measured with respect to the 3d3/2-2p1/2 line (see Table XVIII). Errors are given in parentheses.
- XX. Nuclear charge parameters for ^{138}Ba . The units for c and t are Fermis while nuclear polarization is in keV.
- XXI. Comparison of nuclear parameters obtained for ^{138}Ba by muonic x-rays and electron scattering. The units of c and t are Fermis, while nuclear polarization is in keV. The errors are shown in parentheses.

- XXII. Muonic x-ray transitions used to determine nuclear parameters for ^{140}Ce . Energies for each transition are in keV, and the 3d3/2-2p1/2 transition is used as the relative intensity standard.
- XXIII. Other muonic x-ray transitions observed in ^{140}Ce . All energies are in keV. The relative intensities are measured with respect to the 3d3/2-2p1/2 transition (see Table XXII).
- XXIV. Nuclear charge parameters derived for ^{140}Ce . The units of c and t are Fermis while nuclear polarization is in keV.
- XXV. Energies and relative intensities of the transitions used to derive charge parameters for ^{142}Ce . Errors are shown in parentheses. All energies are in keV.
- XXVI. Other muonic x-ray transitions observed and identified in ^{142}Ce . The average theoretical energy is the average of the theoretical energies weighted by the theoretical relative intensities. The relative intensities are quoted with respect to the 3d3/2-2p1/2 transition (see Table XXV), except for the 3-1 transitions, which are quoted within the multiplet. Energies are in keV.
- XXVII. Nuclear charge parameters derived for ^{142}Ce . Units of c and t are Fermis while nuclear polarization is in keV.
- XXVIII. Energies and relative intensities of the transitions used to derive nuclear charge parameters for ^{206}Pb . Two energy differences between adjacent peaks have also been included. Energies are quoted in keV with errors shown in parentheses.

The $4f5/2-3d3/2$ transition (see Table XXIX) is used as a relative intensity standard.

- XXIX. Other transitions detected in ^{206}Pb . The energies are given in keV and the errors are in parentheses. The 3-1 relative intensities are measured within the multiplet with the $3p3/2-1s$ relative intensity set equal to 1.
- XXX. Nuclear charge parameters derived for ^{206}Pb . Both nuclear polarization values are given in keV, and both c and t are in Fermis.
- XXXI. Comparison of theoretical and experimental nuclear polarization values. With the exception of ^{208}Pb the experimental values were derived from fits to the experimental energies using three variable parameters. For ^{208}Pb the 2s nuclear polarization was also allowed to vary, thus accounting for the larger error.

Table I. Effect of finite size of the nucleus on the binding energies of the $n \leq 3$ muonic states. The energies are in keV, and % is the ratio of the finite size effect divided by binding energy of a finite nucleus.

		^{208}Pb				^{138}Ba			
		Point Nucleus	Finite Nucleus	Finite Size Effect	% of Finite Nucleus Energy	Point Nucleus	Finite Nucleus	Finite Size Effect	% of Finite Nucleus Energy
1s1/2	20991		10606	10385	98	9217	6199	3018	49
2s1/2	5385		3602	1783	50	2330	1875	455	24
2p1/2	5385		4817	568	12	2330	2280	50	2
2p3/2	4837		4632	205	5	2227	2214	13	0.6
3s1/2	2330		1765	565	32	1024	883	141	16
3p1/2	2330		2140	190	9	1024	1006	18	2
3p3/2	2166		2092	74	4	993	988	5	0.5
3d3/2	2166		2173	-7	0.3	993	997	-4	0.4
3d5/2	2122		2130	-8	0.4	984	987	-3	0.3

Table II. The effect of finite size on the transition energies in ^{138}Ba and the values of the derivatives with respect to the nuclear parameters. The finite size effect is the difference between the finite nucleus energy and the point nucleus energy, and all are measured in keV.

	Point Nucleus Energy	Finite a Nucleus Energy	Finite Size Effect	$\frac{\delta E}{\delta c}(\text{keV/fm})$	$\frac{\delta E}{\delta n}(\text{keV})$	$\frac{\delta E}{\delta w}(\text{keV})$
2p3/2-1s1/2	6989.9	3954.4	3035.5	-2210	37	-128
2p1/2-1s1/2	6887.1	3889.8	2997.3	-2160	37	-126
4d3/2-2p1/2	1771.7	1707.7	63.5	-118	2.3	-6
4d5/2-2p3/2	1672.9	1647.0	25.9	-71	1.5	-3.6
3d3/2-2p1/2	1336.6	1272.6	64.0	-117	2.2	-6
3d5/2-2p3/2	1243.2	1217.3	25.9	-71	1.5	-3.6
3d3/2-2p3/2	1233.8	1208.0	25.8	-71	1.5	-3.6
3p3/2-2s1/2	1336.6	880.6	456.0	-350	5.7	-21
3p1/2-2s1/2	1306.1	862.8	443.2	-340	5.5	-20
2s1/2-2p1/2	0	401.1	401.1	+260	-4.0	+16
2s1/2-2p3/2	102.8	336.5	233.7	+306	-4.7	+19
2p3/2-2p1/2	102.8	64.6	38.2	-50	+0.7	-2.4

a calculated with $c = 1.118 \text{ F}$, $n = 11.6$, $w = 0.0$ but no correction other than finite size.

Table III. Effect of a finite size nucleus on transition energies and energy differences in ^{206}Pb . The finite size effect is the difference between point and finite nucleus energies, and all are measured in keV.

	Point Nucleus Energy	Finite a Nucleus Energy	Finite Size Effect	$\frac{\delta E}{\delta c}(\text{keV/fm})$	$\frac{\delta E}{\delta n}(\text{keV})$	$\frac{\delta E}{\delta w}(\text{keV})$
2p3/2-1s1/2	16154.3	5933.4	10120.9	-4450	49	-470
2p1/2-1s1/2	15606.3	5760.9	9845.4	-4235	46	-450
4d3/2-2p1/2	4168.9	3569.2	599.7	-710	9.4	-68
3d3/2-2p1/2	3218.7	2620.7	598.0	-700	9.3	-67
3d5/2-2p3/2	2715.2	2480.3	234.9	-500	6.9	-45
3d3/2-2p3/2	2670.6	2438.2	232.4	-490	6.8	-45
2s1/2-2p1/2	0	1200.8	1200.8	+240	-0.5	35
2s1/2-2p3/2	-548.1	1018.3	1566.4	+450	-3.0	58
2p3/2-2p1/2	548.1	182.5	355.6	-210	2.5	-23
3d3/2-3d5/2	-44.6	38.4	83.0	+160	-2.2	15

a calculated with $c = 1.1065 \text{ F}$, $n = 12.12$, $w = 0.1$

Table IV. Relative intensities and level populations for all states with $n < 4$ in ^{140}Ce . Numbers represent the percentages of all captured muons making the given transition, accounting for electric dipole, electric quadrupole and Auger emission.

[illegible]

Table V. Variation of the full width at half maximum of peaks in ^{206}Pb as a function of energy.

Energy (keV)	Full Width at Half -Maximum (keV)
438	3.8 (0.2)
670	3.8 (0.2)
971	4.5 (0.2)
1405	5.0 (0.35)
1622	5.5 (0.14)
2644	6.8 (0.4)
3430	6.5 (0.4)
4766	8.7 (0.3)
5788	8.7 (0.3)
7450	10.0 (0.4)

Table VI. Chemical composition and physical properties of the targets.

TARGET	^{120}Sn	^{138}Ba	^{140}Ce	^{142}Ce	^{206}Pb
DIMENSIONS	17 x 8 x 0.5 cm	15 x 7.5 x 1.3 cm	15 x 15 x 1.3 cm	7.5 x 7.5 x 1.3 cm	14 x 12 x 1 cm
TOTAL MASS	165 g	185.6 g	694.6 g	108 g	1650 g
CHEMICAL FORM	SnO_2	$\text{Ba}(\text{NO}_3)_2$	CeO_2	CeO_2	RADIOGENIC
ISOTOPIC COMPOSITION	120 98.6%	138 97.2%	140 99.7%	142 92.8%	206 88.0%
	118 0.5%	137 1.8%	142 0.3%	140 7.2%	207 9.0%
	119 0.4%	136 0.9%			208 3.0%

Table VII. A listing of the calibration sources used with the principal lines used in energy determinations, references and usage.

Source	Reference	Energies (keV)	Purpose
^{22}Na	a	511.006 (0.002) 1274.53 (0.10)	Gain calibration, L and M feedthrough
^{46}Sc	b	889.18 (0.10) 1120.41 (0.10)	Gain calibration and L feedthrough
^{57}Co	b	121.97 (0.05) 136.33 (0.04)	Gain calibration and detector efficiency
^{88}Y	a	898.04 (0.04) 1836.13 (0.04)	Gain calibration and detector efficiency
^{192}Ir	a	5 lines between 295 and 612 keV	Gain calibration and detector efficiency
^{182}Ta	b	6 lines between 222 and 1221 keV	Gain calibration
$^{110\text{m}}\text{Ag}$	c	15 lines between 446 and 1562 keV	Gain calibration and detector efficiency
^{54}Mn	a	834.81 (0.03)	Gain calibration and detector efficiency
^{137}Cs	a	661.635 (0.076)	Gain calibration and detector efficiency
$^{177\text{m}}\text{Lu}$	d	13 lines between 105 and 378 keV	Gain calibration and detector efficiency
^{56}Co	e	846.78 (0.06) 1037.90 (0.05) 1238.30 (0.04) 1771.42 (0.07) 2034.92 (0.07) 2598.57 (0.06) +9 other lines between 977 and 3450 keV	L and M feedthrough gain calibration 511 keV energy differences detector efficiency
^{197}Au	a	411.795 (0.009)	Gain calibration

Table VII. (continued).

Source	Reference	Energies (keV)	Purpose
^{24}Na	a	2753.92 (0.12) 1368.526 (0.044)	K and L feedthroughs gain calibration and 511 keV differences
ThC''	a	238.624 (0.009) 583.139 (0.023) 2614.47 (0.10)	K feedthrough, gain calibration and detector efficiency
^{60}Co	a	1173.23 (0.04) 1332.49 (0.04)	L feedthrough, gain calibration and detector efficiency
^{133}Ba	a	4 lines between 276 and 384 keV	Gain calibration and detector efficiency
^{66}Ga	f	4295.61 (0.10) 4806.58 (0.20) +8 other lines between 1039 and 4461 keV	K feedthrough, gain calibration and 511 keV differences

^aRef. 109

^bRef. 113

^cRef. 114

^dRef. 115

^eRef. 116

^fRef. 117

Table VIII. Isotope shifts in Ce and Pb. Energies are in keV and the 3d levels are assumed to have zero isotope shift.

	Ce	Pb ^a	
	140-142	206-207	206-208
1s1/2	15.4	5.0	11.6
2s1/2	2.8	1.3	2.9
2p1/2	0.4	1.5	2.3
2p3/2	0.2	0.6	1.4

^aRef. 75

Table IX. Transition energies and energy differences used to derive nuclear charge parameters.

	^{120}Sn	^{138}Ba	^{140}Ce	^{142}Ce	^{206}Pb
2p3/2-1s1/2	X	X	X	X	X
2p1/2-1s1/2	X	X	X	X	X
4d3/2-2p1/2	X	X	X	X	X
3d3/2-2p1/2	X	X	X	X	X
3d5/2-2p3/2	X	X	X	X	X
3d3/2-2p3/2	X	X	X	X	X
3p3/2-2s1/2	X	X	X	X	
3p1/2-2s1/2	X	X	X	X	
2s1/2-2p1/2	X	X	X	X	X
2s1/2-2p3/2	X	X	X	X	X
2p3/2-2p1/2 ^a				X	X
3p3/2-3d5/2 ^a					X

a Measured as a difference between two peaks.

Table X. High order vacuum polarization corrections in keV.

<u>Level</u>	<u>^{120}Sn</u>	<u>^{138}Ba</u>	<u>^{140}Ce</u>	<u>^{206}Pb</u>
1s1/2	0.65	0.82	0.86	1.75
2s1/2	0.11	0.15	0.17	0.44
2p1/2	0.15	0.21	0.24	0.71
2p3/2	0.15	0.19	0.22	0.64
3p1/2	0.03	0.04	0.06	0.22
3p3/2	0.03	0.05	0.05	0.21
3d3/2	0.03	0.04	0.05	0.19
3d5/2	0.03	0.04	0.05	0.18
4d3/2	0	0	0	0.05
4d5/2	0	0	0	0.05
4f5/2	0	0	0	0.05
4f7/2	0	0	0	0.05

Table XI. Theoretical nuclear polarization effects in keV.

<u>Level</u>	<u>$^{120}\text{Sn}^{\text{a}}$</u>	<u>$^{138}\text{Ba}^{\text{a}}$</u>	<u>$^{142}_{140}\text{Ce}^{\text{a}}$</u>	<u>$^{206}\text{Pb}^{\text{a}}$</u>	<u>$^{206}\text{Pb}^{\text{b}}$</u>
1s1/2	2.7	3.4	3.6	$6.0^{\pm 0.6}$	$6.8^{\pm 2.0}$
2s1/2	0.67	0.7	0.8	$1.2^{\pm 0.2}$	$1.6^{\pm 0.7}$
2p1/2	0.33	0.5	0.6	$1.9^{\pm 0.2}$	$1.9^{\pm 0.3}$
2p3/2	0.33	0.5	0.6	$1.9^{\pm 0.2}$	$1.8^{\pm 0.3}$
3p1/2	-	-	-	-	$0.7^{\pm 0.2}$
3p3/2	-	-	-	-	$0.7^{\pm 0.2}$
3d3/2	-	-	-	-	$-0.03^{\pm 0.02}$
3d5/2	-	-	-	-	0

^a Ref. 65

^b Ref. 66

Table XII. Vacuum polarization effect in keV.

<u>Level</u>	<u>^{120}Sn</u>	<u>^{138}Ba</u>	<u>^{140}Ce</u>	<u>^{142}Ce</u>	<u>^{206}Pb</u>
1s1/2	35.75	41.80	43.96	43.74	67.19
2s1/2	7.74	9.62	10.30	10.26	19.36
2p1/2	10.43	13.78	15.00	14.99	32.31
2p3/2	9.71	12.76	13.86	13.85	29.77
3p1/2	3.27	4.37	4.78	4.77	10.78
3p3/2	3.10	4.13	4.51	4.51	10.25
3d3/2	2.68	3.68	4.05	4.06	10.51
3d5/2	2.61	3.56	3.91	3.91	9.86
4d3/2	1.12	1.55	1.71	1.72	4.58
4d5/2	1.09	1.50	1.66	1.66	4.33
4f5/2	0.90	1.26	1.40	1.40	3.80
4f7/2	0.89	1.24	1.38	1.38	3.70

Table XIII. Lamb shift energy corrections in keV.

<u>Level</u>	<u>^{120}Sn</u>	<u>^{138}Ba</u>	<u>^{140}Ce</u>	<u>^{142}Ce</u>	<u>^{206}Pb</u>
1s1/2	-1.58	-1.78	-1.85	-1.83	-2.31
2s1/2	-0.36	-0.42	-0.44	-0.44	-0.63
2p1/2	-0.10	-0.16	-0.18	-0.18	-0.59
2p3/2	-0.06	-0.11	-0.12	-0.13	-0.45
3p1/2	-0.04	-0.06	-0.08	-0.07	-0.24
3p3/2	-0.03	-0.05	-0.05	-0.05	-0.20
3d3/2	0	0	0	0	-0.02
3d5/2	0	0	0	0	-0.01

Table XIV. Anomalous magnetic moment correction in keV.

<u>Level</u>	<u>^{120}Sn</u>	<u>^{138}Ba</u>	<u>^{140}Ce</u>	<u>^{142}Ce</u>	<u>^{206}Pb</u>
1s1/2	-0.22	-0.27	-0.29	-0.28	-0.45
2s1/2	-0.03	-0.04	-0.04	-0.04	-0.07
2p1/2	+0.07	+0.09	+0.10	+0.10	+0.24
2p3/2	-0.04	-0.06	-0.07	-0.07	-0.20
3p1/2	+0.02	+0.03	+0.03	+0.03	+0.06
3p3/2	-0.01	-0.02	-0.02	-0.02	-0.06
3d3/2	0	+0.01	+0.02	+0.01	+0.06
3d5/2	0	-0.01	-0.01	-0.01	-0.04

Table XV. Principal transitions and energy differences measured in ^{120}Sn .

LEVELS	ENERGY (keV)	OTHER EXPERIMENT	THEORY ^e	RELATIVE INTENSITY	THEORETICAL RELATIVE INTENSITY
2p3/2-2p1/2	45.60(0.15)	45.8(0.2) ^a	45.72		
2p3/2-1s1/2	3454.41(0.33)	3454.5(0.5) ^a 3446.4(6.4) ^b	3454.40	50.8(8.2)	59.70
2p1/2-1s1/2	3408.79(0.37)	3408.7(0.5) ^a	3408.67	28.0(4.5)	31.04
4d3/2-2p1/2	1369.50(0.26)		1369.44	2.7(0.5)	2.54
3d3/2-2p1/2	1022.20(0.20)	1022.6(3.0) ^c	1022.13	25.05 ^d	25.05
3d5/2-2p3/2	982.20(0.20)	982.5(3.0) ^c	982.37	44.7(7.0)	43.91
3d3/2-2p3/2	976.55(0.30)		976.40	5.0(1.12)	4.72
3p3/2-2s1/2	747.15(0.35)		746.16	0.82(0.5)	0.76
3p1/2-2s1/2	733.20(0.35)		733.41	1.1(0.3)	0.49
2s1/2-2p1/2	280.14(0.45)		280.03	1.29(0.4)	0.98
2s1/2-2p3/2	234.50(0.30)		234.30	1.51(0.34)	1.22

^aR. Ehrlich, ref. 105

^bH. L. Anderson et al., ref. 106

^cH. L. Acker et al., ref. 107

^dUsed as a relative intensity standard.

^eCalculated with parameters: c=1.125F, n=11.76, w=0.0.

Table XVI. Other transitions observed in ^{120}Sn .

LEVELS	ENERGY(keV)	AVERAGE THEORETICAL ENERGY	THEORETICAL ^a ENERGY	RELATIVE INTENSITY	THEORETICAL RELATIVE INTENSITY
5d3/2-2p3/2	1529.14(0.8)		1529.95	0.8 (0.4)	0.59
5d5/2-2p3/2	1484.9 (0.7)	1485.36	1485.47 1484.18	1.26(0.3)	1.05 0.10
4f7/2-2p3/2			1327.65		0.37
4d5/2-2p3/2			1326.23		4.32
4d3/2-2p3/2	1325.9 (0.5)	1326.11	1323.71	4.0 (0.7)	0.45
7d5/2-3p3/2			643.91		0.13
7f7/2-3d5/2	642.50(0.7)	642.35	642.25	0.5 (0.2)	0.85
7f5/2-3d3/2			642.02		0.6
6f5/2-3d3/2	595.82(0.7)		596.41	1.3 (0.3)	0.7
6f7/2-3d5/2	590.22(0.6)		590.22	1.32(0.3)	1.0
5f7/2-3d5/2	504.30(0.6)		503.95	4.8 (0.8)	4.78
4f5/2-3d3/2	350.0 (0.3)		349.97	23.1 (3.8)	22.86
4f7/2-3d5/2	345.3 (0.3)		345.24	40.34(7.0)	32.24
6f5/2-4d3/2			248.55		0.72
6f7/2-4d5/2	246.21(0.6)	245.98	246.40	8.7 (1.5)	1.01
6g7/2-4f5/2			246.28		3.44
6g9/2-4f7/2			245.26		4.46

^aCalculated with parameters: c=1.125, n=11.79

Table XVII. Nuclear charge parameters for ^{120}Sn .

TYPE	N.P. Is(keV)	c(F.)	n	t(F.)	w	$\chi^2/\text{Deg. Freedom}$
FERMI	4.5(3.0)	1.1114(0.009)	10.62(0.7)	2.27(0.15)	0 ^a	9.3/7
FERMI	0 ^a	1.125 (0.009)	11.76(0.7)	2.07(0.12)	0 ^a	10.1/8
FERMI	2.7 ^a	1.114 (0.009)	10.85(0.7)	2.23(0.14)	0 ^a	11.43/8

^aHe1d constant during fitting procedure.

Table XVIII. Principal transition energies detected in ^{138}Ba .

LEVELS	ENERGIES(keV)	OTHER EXPERIMENT ^b	THEORY ^c	RELATIVE INTENSITY	THEORETICAL RELATIVE INTENSITY
2p3/2-1s1/2	3988.09(0.27)	3988.26(0.40)	3987.95	65.0 (10.0)	59.76
2p1/2-1s1/2	3922.15(0.30)	3922.25(0.40)	3933.20	31.0 (6.0)	31.23
4d3/2-2p1/2	1720.40(0.30)		1720.51	2.0 (0.4)	2.32
3d3/2-2p1/2	1283.22(0.12)	1283.46(0.28)	1283.30	25.65 ^a	25.65
3d5/2-2p3/2	1227.09(0.15)	1227.28(0.28)	1226.97	47.0 (6.0)	45.06
3d3/2-2p3/2	1218.03(0.23)	1217.44(0.9)	1217.56	5.3 (0.8)	4.70
3p3/2-2s1/2	886.50(0.30)		886.48	1.1 (0.3)	0.83
3p1/2-2s1/2	868.10(0.30)		868.44	0.49(0.13)	0.44
2s1/2-2p1/2	405.41(0.25)		405.54	1.10(0.25)	0.91
2s1/2-2p3/2	339.50(0.30)		339.80	1.27(0.25)	1.15

^aUsed as a standard.

^bA.C. Thompson, ref. 77.

^cCalculated with parameters: $c=1.1182F$, $n=11.62$, $N.P.1s=6.4$

Table XIX. Other transition energies observed in ^{138}Ba .

LEVELS	ENERGY(keV)	AVERAGE THEORETICAL ENERGY	THEORETICAL ENERGY ^b	RELATIVE INTENSITY ^c	THEORETICAL RELATIVE INTENSITY
3d5/2-1s1/2	5215.3 (1.0)	5214.42	5214.90	2.3 (1.0) ^a	1.26
3p3/2-1s1/2			5214.20		2.64
3p1/2-1s1/2	5196.7 (1.0)		5196.18	1.1 ^a	1.15
5d5/2-2p3/2	1859.0 (0.7)		1859.00	2.74(0.5)	1.05
4f7/2-2p3/2			1660.90		0.54
4d5/2-2p3/2	1658.31(0.5)	1658.67	1658.73	4.8 (0.9)	4.03
4d3/2-2p3/2			1654.67		0.40
6f5/2-3d3/2	749.6 (0.7)		750.27	1.13(0.3)	1.04
6d5/2-3d3/2	741.10(0.5)	741.43	741.47	0.56(0.22)	0.17
6f7/2-3d5/2			741.45		1.48
5f5/2-3d3/2	641.3 (0.5)		641.51	2.9 (0.5)	3.36
5f7/2-3d5/2	632.96(0.45)	630.01	633.11	5.0 (0.8)	4.78
5d5/2-3p3/2			632.68		0.43
4f5/2-3d3/2	441.28(0.15)		441.34	23.8 (4.0)	24.88
4f7/2-3d5/2			433.90		35.33
4d5/2-3p3/2	433.72(0.15)	433.76	432.41	37.0 (8.0)	1.43
4f5/2-3d5/2			431.93		1.69

Table XIX. (continued).

LEVELS	ENERGY(keV)	AVERAGE THEORETICAL ENERGY	THEORETICAL ENERGY ^b	RELATIVE INTENSITY ^c	THEORETICAL RELATIVE INTENSITY
8f7/2-4d5/2			417.36		0.20
8g7/2-4f5/2	416.71(0.7)	416.40	417.22	1.03(0.4)	0.43
8g9/2-4f7/2			415.40		0.55
7f5/2-4d3/2			378.65		0.30
7f7/2-4d5/2	373.97(0.5)	374.44	375.05	2.7 (0.5)	0.43
7g7/2-4f5/2			374.87		1.20
7g9/2-4f7/2			373.12		1.55
6f7/2-4d5/2			309.77		1.02
6g7/2-4f5/2	308.83(0.45)	308.78	309.59	8.33(1.2)	3.33
6g9/2-4f7/2			307.97		4.30
5f7/2-4d5/2			201.39		2.86
5g7/2-4f5/2	200.53(0.40)	200.55	201.29	51.0 (7.0)	18.50
5g9/2-4f7/2			199.92		23.95

^aRelative intensities measured within multiplet.

^bCalculated with parameters: c=1.1182F, n=11.62, N.P.1s=6.4.

^cMeasured with respect to the 3d3/2-2p1/2 line.

Table XX. Nuclear charge parameters for ^{138}Ba .

TYPE	N.P.1s(keV)	c(F.)	n	t(F.)	w	$\chi^2/\text{DEG. Freedom}$
FERMI	6.4(1.3)	1.1182(0.003)	11.62(0.35)	2.18(0.07)	0 ^a	9.12/8
FERMI	0 ^a	1.1272(0.003)	12.68(0.30)	2.02(0.05)	0 ^a	11.49/9
FERMI	3.4 ^a	1.1168(0.003)	11.62(0.30)	2.18(0.06)	0 ^a	19.33/9
FERMI	6.2(1.3)	1.0670(0.004)	10.62(0.30)	2.28(0.06)	0.375 ^a	9.19/8
FAMILY II	6.1(0.9)	1.0843(0.003)	8.68(0.15)	2.84(0.05)	0.375 ^a	9.17/8

^aHeld constant during fitting procedure.

Table XXI. Comparison of nuclear parameters for ^{138}Ba .

	c(F.)	n	t(F.)	w	N.P. Is(keV)	m
THIS WORK	1.1182(0.003)	11.62(0.35)	2.18(0.07)	0	6.4(1.3)	1
THIS WORK	1.067 (0.004)	10.62(0.30)	2.28(0.06)	0.375	6.2(1.3)	1
ELECTRON SCATTERING ^a	1.033		2.30	0.375	-	2
THOMPSON ^b	1.122 (0.011)	12.07	2.11(0.15)	0	3.4	1

^aRef. 78

^bRef. 77

Table XXII. Principal transitions detected in ^{140}Ce .

LEVELS	ENERGY (keV)	OTHER EXPERIMENT ^b	THEORY ^c	RELATIVE INTENSITY	THEORETICAL RELATIVE INTENSITY
2p3/2-1s1/2	4171.24(0.11)	4171.21(0.30)	4171.20	67.5 (12.0)	59.76
2p1/2-1s1/2	4097.61(0.13)	4097.68(0.30)	4097.71	33.3 (8.0)	31.23
4d3/2-2p1/2	1846.23(0.30)		1845.98	2.42(0.6)	2.32
3d3/2-2p1/2	1376.41(0.09)	1376.40(0.20)	1376.40	25.65 ^a	25.65
3d5/2-2p3/2	1313.63(0.08)	1313.70(0.20)	1313.74	42.9 (6.0)	45.06
3d3/2-2p3/2	1303.14(0.25)	1303.34(0.50)	1302.91	4.42(0.8)	4.70
3p3/2-2s1/2	935.97(0.45)		935.94	1.27(0.6)	0.83
3p1/2-2s1/2	915.92(0.22)		915.88	0.50(0.15)	0.44
2s1/2-2p1/2	451.22(0.20)		451.28	0.80(0.17)	0.97
2s1/2-2p3/2	377.84(0.14)		377.78	1.30(0.22)	1.15

^aUsed as a standard.

^bA.C. Thompson, ref. 77.

^cCalculated with parameters: $c=1.1142$, $n=11.1$, $N.P.1s=5.8$.

Table XXIII. Other transitions observed in ^{140}Ce .

LEVELS	ENERGY(keV)	AVERAGE THEORETICAL ENERGY	THEORETICAL ENERGY ^b	RELATIVE INTENSITY ^c	THEORETICAL RELATIVE INTENSITY
4d5/2-1s1/2 4p3/2-1s1/2	5948.0 (1.5)	5948.33	5948.26 5948.35	0.66 ^a	0.18 0.50
4p1/2-1s1/2	5940.3 (1.5)		5940.10	0.25 ^a	0.25
3d5/2-1s1/2 3p3/2-1s1/2	5484.54(0.5)	5484.33	5484.36 5484.94	4.8 (1.3)	1.26 2.64
3d3/2-1s1/2	5474.0 (0.8)		5474.11	-	0.79
3p1/2-1s1/2	5464.2 (0.7)		5464.88	1.6 (0.5)	1.15
6d5/2-2p3/2	2107.6 (0.6)		2108.76	0.76(0.28)	0.45
5d5/2-2p3/2	1992.0 (0.4)		1992.00	0.85(0.25)	1.05
4f7/2-2p3/2 4d5/2-2p3/2 4d3/2-2p3/2	1777.22(0.24)	1776.96	1779.52 1777.05 1772.48	3.7 (0.6)	0.54 4.03 0.40
7f7/2-3d5/2 7f5/2-3d5/2 7d5/2-3p3/2	865.34(0.30)	865.69	865.89 865.46 865.45	0.69(0.3)	0.70 0.49 0.10
6f5/2-3d3/2	805.58(0.30)		805.94	0.9 (0.2)	1.04

Table XXIII. (continued 1).

LEVELS	ENERGY(keV)	AVERAGE THEORETICAL ENERGY	THEORETICAL ENERGY ^b	RELATIVE INTENSITY ^c	THEORETICAL RELATIVE INTENSITY
6f7/2-3d5/2	795.49(0.30)	795.71	795.78	3.83(0.6)	1.48
6f5/2-3d5/2			795.11		0.17
5f5/2-3d3/2	688.96(0.20)		689.21	2.4 (0.4)	3.36
5f7/2-3d5/2	679.12(0.20)		679.53	4.1 (0.6)	4.78
4f5/2-3d3/2	474.35(0.15)		474.36	21.5 (3.0)	24.88
4f7/2-3d5/2	465.63(0.15)		465.79	32.2 (4.0)	35.33
8f7/2-4d5/2			447.97		0.20
8g7/2-4f5/2	446.90(0.25)	446.88	447.83	2.26(0.26)	0.43
8g9/2-4f7/2			445.74		0.55
7f5/2-4d3/2	406.1 (0.4)		406.71	0.4 (0.15)	0.30
7f7/2-4d5/2			402.57		0.43
7g7/2-4f5/2	401.42(0.20)	401.43	402.39	2.41(0.35)	1.20
7g9/2-4f7/2			400.37		1.55
6f7/2-4d5/2			332.46		1.02
6g9/2-4f7/2	331.25(0.20)	331.40	330.47	7.1 (0.9)	4.30
6g7/2-4f5/2			332.33		3.33
5f7/2-4d5/2			216.22		2.86
5g7/2-4f5/2	215.20(0.25)	215.28	216.12	41.3 (5.0)	18.50
5g9/2-4f7/2			214.55		23.95

Table XXIII. (continued 2).

LEVELS	ENERGY(keV)	AVERAGE THEORETICAL ENERGY	THEORETICAL ENERGY ^b	RELATIVE INTENSITY ^c	THEORETICAL RELATIVE INTENSITY
8f7/2-5d5/2			233.02		0.12
8g7/2-5d5/2			232.98		0.35
8g9/2-5f7/2	231.13(0.30)	231.94	231.99	2.77(0.37)	0.48
8h9/2-5g7/2			231.92		0.71
8h11/2-5g9/2			231.35		0.88
7g7/2-5f5/2			187.54		1.04
7g9/2-5f7/2	186.2 (0.35)	186.50	186.63	7.6 (1.0)	1.36
7h9/2-5f7/2			186.55		2.86
7h11/2-5g9/2			186.03		3.50

^aRelative intensity measured within the multiplet.

^bCalculated with parameters: $c=1.1142$, $n=11.1$, $N.P.1s=5.8$.

^cMeasured with respect to the 3d3/2-2p1/2 transition.

Table XXIV. Nuclear parameters for ^{140}Ce .

TYPE	N.P.1s(keV)	c(F)	n	t(F)	w	$\chi^2/\text{Deg. Freedom}$
FERMI	5.8(1.2)	1.1142(0.002)	11.1(0.12)	2.29(0.02)	0 ^a	4.47/7
FERMI	0 ^a	1.1253(0.0017)	12.17(0.12)	2.11(0.02)	0 ^a	6.23/8
FERMI	3.6 ^a	1.1135(0.0016)	11.12(0.11)	2.28(0.02)	0 ^a	17.23/8

^a Held constant during the fitting procedure.

Table XXV. Principal transition energies and energy differences measured in ^{142}Ce .

LEVELS	ENERGY(keV)	OTHER EXPERIMENT ^b	THEORY ^c	RELATIVE INTENSITY	THEORETICAL RELATIVE INTENSITY
2p3/2-2p1/2	73.07(0.09)		73.19		
2p3/2-1s1/2	4155.86(0.32)	4156.21(0.50)	4155.96	70.0 (12.0)	59.76
2p1/2-1s1/2	4082.81(0.32)	4083.3 (0.50)	4082.77	37.7 (10.0)	31.23
4d3/2-2p1/2	1846.10(0.50)		1845.43	2.40(0.5)	2.32
3d3/2-2p1/2	1375.80(0.10)		1375.83	25.65 ^a	25.65
3d5/2-2p3/2	1313.47(0.08)		1313.48	48.0 (8.0)	45.06
3d3/2-2p3/2	1303.10(0.25)		1302.64	5.9 (1.2)	4.70
3p3/2-2s1/2	932.55(0.38)		933.17	1.08(0.3)	0.83
3p1/2-2s1/2	913.00(0.30)		913.21	0.50(0.13)	0.44
2s1/2-2p1/2	453.45(0.40)		453.60	1.50(0.40)	0.91
2s1/2-2p3/2	379.90(0.25)		380.40	1.40(0.50)	1.15

^aUsed as a standard.

^bA.C. Thompson, ref. 77.

^cCalculated with parameters: c=1.129, n=12.14, N.P.1s=6.3 keV.

Table XXVI. Other transitions detected in ^{142}Ce .

LEVELS	ENERGY (keV)	AVERAGE THEORETICAL ENERGY	THEORETICAL ENERGY ^b	RELATIVE INTENSITY	THEORETICAL RELATIVE INTENSITY
3d5/2-1s1/2 3p3/2-1s1/2	5469.6 (0.6)	5469.49	5469.44 5469.52	3.0 ^a	1.26 2.64
3d3/2-1s1/2	5459.7 (1.0)		5458.59	0.58 ^a	0.79
3p1/2-1s1/2	5449.2 (0.5)		5449.56	1.15 ^a	1.15
5d3/2-2p1/2	2062.42(0.8)		2062.61	0.69(0.26)	0.59
5d5/2-2p3/2 5d3/2-2p3/2	1993.14(0.8)	1991.56	1991.76 1989.44	1.61(0.47)	1.05 0.10
4f7/2-2p3/2 4d5/2-2p3/2 4d3/2-2p3/2	1777.27(0.5)	1776.70	1779.27 1776.80 1772.23	4.64(0.8)	0.54 4.03 0.40
9f7/2-3d5/2 9d5/2-3p3/2	942.20(0.6)	942.47	942.54 942.15	1.0 (0.2)	0.25 0.05
7f5/2-3d3/2	876.10(0.5)		876.31	1.1 (0.2)	0.49
7f7/2-3d5/2 7f5/2-3d5/2 7d5/2-3p3/2	866.03(0.5)	865.69	865.90 865.47 865.36	1.09(0.2)	0.70 0.49 0.10
6f5/2-3d3/2	804.97(0.7)		804.95	0.85(0.2)	1.04

Table XXVI. (continued 1).

LEVELS	ENERGY (keV)	AVERAGE THEORETICAL ENERGY	THEORETICAL ENERGY ^b	RELATIVE INTENSITY	THEORETICAL RELATIVE INTENSITY
6f7/2-3d5/2 6f5/2-3d5/2	794.84(0.6)	795.70	795.80 795.10	1.42(0.3)	1.48 0.17
5f5/2-3d3/2	688.64(0.5)		689.21	3.39(0.5)	3.36
5f7/2-3d5/2 5d5/2-3p3/2	679.14(0.45)	679.32	679.48 678.13	4.99(0.7)	4.78 0.43
4f5/2-3d3/2	474.24(0.20)		474.31	24.4 (3.5)	24.88
4f7/2-3d5/2	465.46(0.20)		465.79	36.2 (5.0)	35.33
7f7/2-4d5/2 7g7/2-4f5/2 7g9/2-4f7/2	401.07(0.5)	401.38	402.15 402.39 400.38	3.4 (0.5)	0.43 1.20 1.55
6f7/2-4d5/2 6g7/2-4f5/2 6g9/2-4f7/2	331.20(0.45)	331.42	332.47 332.33 330.47	8.5 (1.2)	1.02 3.33 4.30
8f7/2-4d5/2 8g7/2-4f5/2 8g9/2-4f7/2	447.3 (0.5)	446.89	448.00 447.84 445.74	1.0 (0.4)	0.20 0.43 0.55
5f7/2-4d5/2 5g7/2-4f5/2 5g9/2-4f7/2	215.01(0.4)	215.28	216.22 216.13 214.55	51.6 (7.0)	2.86 18.50 23.95

Table XXVI. (continued 2).

^aRelative intensities calculated within the multiplet.

^bCalculated with parameters: $c=1.129$, $n=12.14$, $N.P.1s=6.3$.

Table XXVII. Charge parameters for ^{142}Ce .

TYPE	N.P.1s(keV)	c(F)	n	t(F)	w	$\chi^2/\text{Deg. Freedom}$
FERMI	6.3(0.9)	1.1290(0.0017)	12.14(0.15)	2.13(0.03)	0 ^a	14.56/8
FERMI	0 ^a	1.1400(0.0015)	13.54(0.18)	1.93(0.03)	0 ^a	22.27/9
FERMI	3.6 ^a	1.1256(0.0016)	11.95(0.14)	2.16(0.03)	0 ^a	23.17/9

^a Held constant during fitting procedure.

Table XXVIII. Principal transition energies and energy differences detected in ^{206}Pb .

LEVELS	ENERGY (keV)	OTHER EXPERIMENTS ^a	THEORY ^b	RELATIVE INTENSITY ^c	THEORETICAL RELATIVE INTENSITY
2p3/2-2p1/2	185.5 (0.15)	185.65(0.12)	185.49		
3p3/2-3d5/2	37.47(0.45)		37.67		
2p3/2-1s1/2	5973.6 (0.5)	5974.0 (0.4)	5973.55	45.6 (15.0)	56.28
2p1/2-1s1/2	5788.0 (0.5)	5788.3 (0.5)	5788.05	27.0 (8.0)	30.95
4d3/2-2p1/2	3599.52(0.45)	-	3599.09	0.9 (0.3)	1.80
3d3/2-2p1/2	2644.50(0.40)	2643.8 (0.4)	2644.42	27.3 (6.0)	25.5
3d5/2-2p3/2	2501.60(0.20)	2501.5 (0.4)	2501.73	44.8 (8.0)	45.0
3d3/2-2p3/2	2458.85(0.60)	2458.1 (0.4)	2458.93	4.5 (1.0)	4.40
2s1/2-2p1/2	1214.84(0.50)	1217.8 (0.8)	1214.57	0.6 (0.18)	0.69
2s1/2-2p3/2	1029.03(0.45)	-	1029.07	0.55(0.17)	0.92

^aAnderson et al., ref. 75.

^bCalculated with parameters: $c=1.122F$, $n=12.55$, $N.P.1s=5.8\text{keV}$.

^cMeasured with respect to the 4f5/2-3d3/2 transition.

Table XXIX. Other transition energies and energy differences observed in ^{206}Pb .

LEVELS	ENERGY (keV)	OTHER EXPERIMENTS ^c	THEORETICAL ENERGY ^d	RELATIVE INTENSITY	THEORETICAL RELATIVE INTENSITY
3d5/2-3d3/2	43.01(0.41)		42.83		
3p3/2-3p1/2	47.73(1.10)		47.61		
3p3/2-1s1/2	8512.7 (2.5)		8513.95	1.0 ^a	1.0 ^b
3d5/2-1s1/2	8475.3 (2.4)		8476.28	1.53(0.18) ^a	1.69
3p1/2-1s1/2	8466.5 (2.5)		8466.34	0.39(0.08) ^a	0.45
3d3/2-1s1/2	8432.3 (2.4)		8433.48	1.18(0.14) ^a	1.03
4f5/2-2p1/2	3617.0 (0.8)		3616.36	0.32(0.11)	0.71
4f7/2-2p3/2	3440.00(0.5)		3440.05	1.16(0.3)	1.40
4f5/2-2p3/2 4d5/2-2p3/2	3431.35(0.45)	3429.5(1.4)	3430.85 3431.35	2.1 (0.5)	0.20 3.00
6f5/2-3d3/2	1640.2 (1.0)		1640.13	0.53(0.19)	1.04
6f7/2-3d5/2	1599.6 (0.7)	1598.2(1.0)	1600.06	1.17(0.27)	1.48
3p3/2-2s1/2	1508.8 (1.5)	1507.9(0.8)	1510.32	0.84(0.5)	0.88
5f5/2-3d3/2	1405.2 (0.6)	1404.1(0.4)	1405.40	2.53(0.6)	2.90
5f7/2-3d5/2	1366.75(0.6)	1366.0(0.5)	1366.69	3.95(0.7)	4.10

Table XXIX. (continued).

LEVELS	ENERGY(keV)	OTHER EXPERIMENT ^c	THEORETICAL ENERGY ^d	RELATIVE INTENSITY	THEORETICAL RELATIVE INTENSITY
4f5/2-3d3/2	971.9 (0.3)	971.7 (0.2)	971.97	28.9 ^b	28.9
4f7/2-3d5/2	937.9 (0.3)	937.8 (0.2)	938.32	41.8 (7.0)	39.3
4f5/2-3d5/2	929.6 (0.5)	928.2 (0.5)	929.13	3.1 (0.8)	1.70
4d5/2-3p3/2	892.6 (0.6)		891.90	0.71(0.2)	1.43
6g7/2-4f5/2 6f7/2-4d5/2	671.5 (0.6)	670.9 (0.4)	671.10 670.44	3.3 (0.6)	3.33 1.02
6g9/2-4f7/2	664.0 (0.6)	663.1 (0.4)	663.52	4.2 (0.8)	4.30

^aRelative intensity measured within multiplet.

^bUsed as a standard.

^cExperimental result of Anderson, ref. 75.

^dCalculated with parameters: $c=1.122F$, $n=12.55$, $N.P.1s=5.8$ keV.

Table XXX. A comparison of charge distributions for ^{206}Pb .

TYPE	N.P.1s(keV)	N.P.2s(keV)	c(F.)	n	t(F.)	w	$\chi^2/\text{Deg. Freedom}$
FERMI	5.8(2.0)	1.6 ^a	1.1220(0.0016)	12.55(0.14)	2.32 (0.03)	0 ^a	1.93/7
FERMI	0 ^a	0 ^a	1.1280(0.0011)	13.34(0.11)	2.20 (0.02)	0 ^a	9.6 /8
FERMI	5.6(2.0)	1.6 ^a	1.1003(0.0015)	11.93(0.11)	2.39 (0.02)	0.14 ^a	1.8 /7
FERMI	4.1(3.0)	1.05(0.85)	1.120 (0.004)	12.38(0.3)	2.35 (0.06)	0 ^a	1.50/6
FERMI(m=2)	4.2(3.2)	0.94(0.92)	1.117 (0.005)	2.34(0.04)	-	0 ^a	3.8 /6
FERMI ^b	0 ^a	0 ^a	1.133 (0.004)	13.99(0.56)	2.102(0.079)		

^aHe1d constant during fitting.

^bH. L. Anderson et al., ref. 75.

Table XXXI. Comparison of theoretical and experimental nuclear polarization values.
Energies are in keV.

	^{120}Sn	^{138}Ba	^{140}Ce	^{142}Ce	^{206}Pb	^{208}Pb
COLE	1.6	2.0	2.2			5.3
CHEN	2.7(0.3)	3.4(0.4)	3.6(0.4)		6.0(0.6)	
SKARDHAMAR	-	-	-		6.8(2.0)	
EXPERIMENT	4.5(3.0)	6.3(1.3)	5.8(1.2)	6.3(1.4)	5.8(2.0)	8.9(2.7) ^a

^aThe ^{208}Pb data were analyzed with the 2s nuclear polarization as a variable and this accounts for the larger uncertainty. When analyzed under the same conditions as the ^{206}Pb the uncertainty was 0.8 keV.

IX. FIGURES

Figure Captions

1. Wave function probability densities for the lower muonic energy levels in ^{138}Ba . The units are arbitrary and the charge distribution is superimposed to show its overlap.
2. The linear probability density ($r^2 \times$ wave function probability density) for various levels in ^{138}Ba . The units are arbitrary, and the charge distribution is superimposed.
3. Prompt and delayed spectra for ^{138}Ba with most prominent peaks identified. Both scales are logarithmic.
4. Range curve obtained for ^{140}Ce target.
5. Target geometry, counter and detector positions for the experiment. 1-5 are the scintillation counters used.
6. Timing spectrum of muonic ^{208}Pb events. For other targets the delayed region was as much as 500 nsec and the prompt region as much as 100 nsec.
7. Block diagram of the γ -stop coincidence circuitry. GG=gate generator; DISC=discriminator; COINC=coincidence unit; ADC=analogue to digital converter; PHA=pulse height analyzer; TAC=time to amplitude converter.
8. Block diagram of the beam telescope circuitry. DISC=discriminator; COINC=coincidence unit; • denotes connection in the anti-coincidence mode.

9. Block diagram of the data flow showing alternative connections for the ADC s when used with and without an on-line computer.
10. Fluctuation of two calibration peaks during the experiment. The upper line corresponds to the 1274.55 keV line of ^{22}Na while the lower is the 511 keV line and was used for stabilization. Center uncertainties were less than 0.05 channels.
11. Relative efficiency curve of the Ge(Li) detector in range 0.1 to 5 MeV.
12. Evaluation of ^{142}Ce L lines using ^{22}Na and ^{60}Co calibration source in accidental coincidence with muon stops. Gain is 0.805 keV per channel.
13. Evaluation of ^{120}Sn K lines using a ThC'' calibration source in accidental coincidence with muon stops. Gain is 0.683 keV per channel.
14. Feynman diagrams representing low order contributions to vacuum polarization (a) first order in $e^+ - e^-$, (b), (c), (d), (e) second order in $e^+ - e^-$, (f) first order in $\mu^+ - \mu^-$.
15. Region of 2s-2p transitions in ^{120}Sn . Gain was 0.683 keV per channel and both transitions were detected. Five additional peaks were measured and two were identified as the 6-4 muonic x-ray transition at 246.2 keV and a nuclear γ ray from ^{117}In at 266.4 keV. The latter is probably a transition between the lowest $3/2^-$ and $1/2^-$ states (see ref. 112).
16. Region of 3p-2s transitions in ^{120}Sn . No identification was made of the peak at 720.7 keV.

17. 3p-2s region in ^{138}Ba . In addition to the 3p-2s doublet, delayed γ rays were detected at 848.6 ± 0.4 and 843.0 ± 0.6 keV. These were assigned to transitions in ^{137}Cs ($849 \rightarrow 0$) and ^{27}Al ($842 \rightarrow 0$). The latter is probably a background contamination, while the former, in conjunction with the 395 keV line (see Fig. 18), confirms the level identification in ^{137}Cs by Holm.¹¹⁷
18. 2s-2p region in ^{138}Ba . In addition to the 2s-2p doublet, six other lines were measured. Identifications were assigned to lines at 346.9 ± 0.5 , 373.9 ± 0.5 , 394.6 ± 0.8 and 416.9 ± 0.8 keV. The initial line is probably muonic x-rays from the supporting frame or the detector cap, which are in accidental coincidence with stopping muons.
19. Spectra in the region of the 2s-2p doublet in ^{140}Ce . The 2s-2p $_{1/2}$ line is contaminated by two γ rays at 446.9 ± 0.4 and 438.0 ± 1.0 keV. These have been identified as probable transitions in ^{137}La , $446 \rightarrow 0$ and $446 \rightarrow 10$ respectively (see ref. 112). Gain is 0.805 keV per channel.
20. 2p-1s region for ^{142}Ce target. The peaks from the 7% ^{140}Ce contamination are clearly visible. From these accurate values of the isotope shift were calculated, as well as using the ^{140}Ce peaks as energy standards for the ^{142}Ce peaks.
21. Spectrum in the region of the 2s-2p doublet in ^{142}Ce . Both 2s-2p peaks are contaminated with other lines. The upper falls on the edge of the very intense 4f $_{7/2}$ -3d $_{5/2}$ peak as well as overlapping a small prompt peak, which is probably the muonic 8-4 transition. The lower energy peak has some structure under it in the delayed

- spectrum but by comparing with the adjacent γ ray at 389.8 ± 0.5 keV (origin unknown), it is clear this effect is small.
22. Spectrum in the region of the 3p-2s doublet in ^{142}Ce . Also fitted were the 7-3 transitions and a peak at 942.0 ± 0.7 keV which corresponds in energy to the 9-3 transitions but is much more intense than theoretical predictions.
  23. Spectra in the region of the 2s-2p_{3/2} transition in ^{206}Pb . The Gaussians fitted in the prompt and delayed spectra are superimposed but with false zero levels. The prompt peaks at 1038.0 ± 1.0 and 1043.2 ± 0.8 keV also appear in the delayed spectrum, and the peak at 1029 keV is identified as 2s-2p_{3/2}. The γ rays were tentatively identified as coming from ^{70}Ge (1040 $\rightarrow$ 0) and ^{205}Pb (1044 $\rightarrow$ 0).
  24. Spectra in the region of the 2s-2p_{1/2} transition in ^{206}Pb . The Gaussians fitted in the prompt and delayed spectra are superimposed but with false zero levels. The unidentified peaks which appeared in both prompt and delayed spectra were at 1207.6 ± 0.8 , 1219.1 ± 0.5 , 1227.0 ± 1.5 and 1234.0 ± 0.7 keV. The gain was 1.1 keV per channel.
  25. Region of the 3p-2s doublet in ^{206}Pb . The 3p_{1/2}-2s transition should appear near channel 1278 but no peak could be found. The peak at 1509 keV was wider than predicted, but this could be explained by the presence of a weak line in the delayed spectrum. This peak was tentatively identified as the 3p_{3/2}-2s transition.

26. Prompt spectrum for the 3-1 transitions in ^{206}Pb . The 3d-1s lines are quadrupole transitions. The delayed spectrum has no structure. The gain is 1.1 keV per channel.
27. Variation of nuclear polarization with Z. Wherever available error bars have been included. "+" denotes the theoretical values of Cole, " Δ " those of Skardhamar and "O" those of Chen. The experimental values are denoted "X".
28. Charge distributions derived for ^{138}Ba using a variable nuclear polarization. 1 is a Fermi distribution with $c=1.1182$ F, $n=11.62$ and $w=0$, 2 is a Fermi distribution with $c=1.067$ F, $n=10.62$ and $w=0.375$ and 3 is the Family II distribution with $c=1.0843$ F, $n=8.678$ and $w=0.375$.

FIG 1

FIG. 2

SPECTRA FOR 138 BA DATA BOTH SCALES ARE LOGARITHMIC

FIG. 3

RANGE CURVE FOR BACKWARD
MUON BEAM

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

FIG. 10

FIG. II

3D-2P REGION IN 142 CE 22 NA AND 60 CO IN ACCIDENTAL COINCIDENCE

FIG. 12

120 SN WITH TH C" FEEDTHROUGH

FIG. 13

FIG. 14

2S-2P TRANSITIONS IN 120 SN

GAIN = 0.683 KEV/CHANNEL

FIG. 10

3P-2S TRANSITIONS IN 120 SN

GRIN = 0.683 KEV/CHANNEL

FIG. 16

3P-2S REGION IN 138 BA PROMPT AND DELAYED

GAIN = 0.805 KEV PER CHANNEL

FIG. 17

2S-2P REGION IN 138 BA

GAIN = 0.805 KEV PER CHANNEL

FIG. 18

2S-2P REGION IN 140 CE

GAIN = 0.805 KEV PER CHANNEL

FIG. 19

2P-1S REGION IN 142 CE TARGET HAS 7X 140 CE CONTAMINATION

FIG. 20

2S-2P REGION IN 142 CE

GAIN = 0.005 KEV PER CHANNEL

FIG. 21

3P-2S REGION IN 142 CE

GAIN = 0.805 KEV PER CHANNEL

FIG. 22

2S-2P3/2 REGION IN 206 PB

GAIN = 1.1 KEV PER CHANNEL

FIG. 23

2S-2P1/2 REGION IN 206 PB

GAIN = 1.1 KEV PER CHANNEL

FIG. 24

3P-2S REGION IN 206 PB

GAIN = 1.1 KEV PER CHANNEL

FIG. 25

3-1 TRANSITIONS IN 206 PB

FIG 26

CHARGE DISTRIBUTIONS FOR BAI38

FIG. 28