Prompt gamma rays from pion absorption on C, N, Na, S, and Ca

Carey Elliott Stronach

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PROMPT γ RAYS FROM π- ABSORPTION

ON C, N, Na, S, AND Ca

A Dissertation

Presented to

The Faculty of the Department of Physics
The College of William and Mary in Virginia

In Partial Fulfillment

Of the Requirements for the Degree of

Doctor of Philosophy

by

Carey E. Stronach

1975
APPROVAL SHEET

This dissertation is submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

_Carey E. Stromach_

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Approved, December 1975

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ABSTRACT

The spectra of nuclear $\gamma$ rays emitted in coincidence with $\pi^-$-absorption events in carbon, nitrogen, sodium, sulfur, and calcium, were taken with a Ge(Li) detector. Yields of excited states of daughter nuclei were determined from photopeak areas, recoil momenta were computed for states which displayed Doppler broadening, and the mean number of $\gamma$ rays produced per $\pi^-$ stop was measured with a spectrum stripping analysis.

Gross features of the spectra of daughter nuclei from $^{32}$S and $^{40}$Ca are explained by a model which assumes a combination of absorption on pairs plus small residual excitation, and uniform distribution of the pion's rest energy in the nucleus, followed by statistical evaporation. Faissevulation formula. Three to five $\gamma$ rays were emitted per pion stopped in $^{32}$S and $^{40}$Ca.

Excitation of $J^P, T = 0^+$, 1 states of (A-2) daughter nuclei from carbon and sulfur was not observed. Some preference for excitation of high-spin states was seen. One-nucleon removal was strongly suppressed except in the case of the sodium target.

An analysis of Doppler broadening indicated that nuclei of $^{12}$C in the $4439$-keV state produced from $\pi^-$ absorption on $^{14}$N have mean recoil momenta of $(169 \pm 6)$ MeV/c. This agrees with a simple calculation which assumes absorption on a deuteron. Ratios of np- to pp-removal yields were found to be about 1 for $^{32}$S and $^{40}$Ca, and about 3 for $^{23}$Na. Theory predicts 3, which can be reconciled with the $^{32}$S and $^{40}$Ca results by correcting for enhancement of $2^+$ states of even-even nuclei.

Three-nucleon removal is prominent for both $^{32}$S and $^{40}$Ca, and there are anomalously large yields for states corresponding to transfer of the odd neutron to the $f_{7/2}$ shell.

Appreciable yields are also seen for removal of the equivalent of one or more alpha particles from $^{32}$S and $^{40}$Ca. $^{28}$Si produced from $^{32}$S, and $^{36}$Ar and $^{26}$Si from $^{40}$Ca display large recoil momenta. These two, together with the observation of the production of high-spin states, are suggestive of absorption on larger aggregates of nucleons, perhaps alpha clusters.

Comparisons are also made with various other experimental results and theories.
PROMPT $\gamma$ RAYS FROM $\pi^-$ ABSORPTION

ON C, N, Na, S, AND Ca
I. INTRODUCTION

Pions have been used as probes of nuclear structure since shortly after their discovery in 1947.\(^1\) Because of the properties of pions (see Table I), their interactions with nuclei emphasize some nuclear features not amenable to study through use of the more usual nuclear probes, such as nucleons, alpha particles, and electrons. Reactions which are of particular interest in this regard include double charge exchange,\(^2\) and \(\pi^-\) absorption, which is the topic of this study.

Pion absorption is a relatively unique process in that it involves a large excitation (140 MeV) of the absorbing nucleus, or portion thereof, with no corresponding momentum transfer. The concomitant dynamical restrictions give it potential value in the study of short-range correlations within nuclei.\(^3\) The usefulness of this reaction in nuclear structure studies has manifested itself only gradually, however, for two reasons.

First, the intensity and spatial and momentum resolution of pion beams have been poor, which has severely limited the accuracy of experiments based upon measurement of out-going particles. Analysis of the spectra of nuclear \(\gamma\) rays emitted in prompt coincidence with \(\pi^-\) absorption, which is the technique used in this study, partially overcomes these difficulties because the energy resolution of the final states of the nuclei is several keV, compared with several MeV for charged-particle spectroscopy of pions. In the future higher intensities
and hence better resolution will be available at the LAMPF, TRIUMPF, and SIN accelerators.

Secondly, in pion absorption there are competing processes, and final-state interactions tend to distort and mask what begins, at least in theory, as a few-body problem. In order to account for these factors many more details of the reactions, involving the behavior of both pions and nucleons, must be known. Therefore, as far as uncovering nuclear structure information is concerned, $\pi^-$ absorption becomes inextricably entwined with a broad range of reactions, where the value of each in terms of the understanding of nuclear structure it provides is strongly coupled to the understanding provided by each of the others.

The remainder of this chapter will address itself to the questions regarding pion-nucleus interactions and nuclear structure which will hopefully be illuminated by these experiments, to the experimental and theoretical background which provides the basis on which results of these experiments may evolve into new knowledge, and to the specific purposes and anticipated results of each of the experiments which collectively constitute this study.

A. Pion-Nucleus Interactions

The interaction of the low- and medium-energy pions with nuclei is dominated by the $\Delta(1232)$ resonance, with $J = \frac{3}{2}$ and $T = \frac{3}{2}$. For pion-nucleon interactions at this resonance the familiar prediction

$$\frac{\sigma(\pi^+ p \rightarrow \pi^+ n)}{\sigma(\pi^- p \rightarrow \pi^- n)} / \sigma(\pi^- p \rightarrow \pi^0 n) = \frac{q}{1/2}$$
is found to be valid, which supports the hypothesis that the $\Delta(1232)$ is a pure $J = \frac{3}{2}$, $T = \frac{3}{2}$ state.

At very low energy the pion-nucleon scattering cross section is very small. The interaction of low-energy negative pions with nuclei is dominated by capture into atomic orbitals followed by absorption by the nuclei.

Chew and Low$^5$ obtained a pion-absorption interaction Hamiltonian of the form

$$H_{\text{in}} = -\frac{1}{\mu} \sum_{i=1}^{A} \vec{\sigma}_i \cdot \vec{\tau}_i \left[ \vec{\nabla} \phi_\alpha(x_i) - \frac{i}{\hbar m} \left\{ \Pi_\alpha(x_i) \vec{\nabla}_i \right\} \right], \quad (1-2)$$

where $f$ is the coupling constant, $\mu$ is the pion mass, $m$ the nucleon mass, the $i$ index refers to nucleons, $\alpha$ refers to the pion, and $\Pi_\alpha(x_i)$ is the canonical conjugate of the pion wave function $\phi_\alpha(x_i)$. By using the familiar second-order perturbation-theory expression for the reaction probability,

$$P = 2\pi \left| \sum_n \frac{\langle f | H | n \rangle \langle n | H | i \rangle}{E_n - E_i} \right|^2 \rho, \quad (1-3)$$

where $\rho$ is the density of final states. Taking the appropriate solution of the Klein-Gordon equation as the initial state of the orbiting pion, and assuming a two-body residual nuclear force of the form

$$V_{i\neq k} \left( \vec{x}_i - \vec{x}_k \right) = A \delta \left( \vec{x}_i - \vec{x}_k \right), \quad (1-4)$$

Huguenin$^6,7$ obtained
\[ P = \frac{2.25 \, m^2}{2 \pi} \sum_{\alpha \beta} \left| \left< \psi_{F, \alpha \beta} \left| H(\tilde{x}_1) F(\tilde{x}_i - \tilde{x}_i') \right| \delta(\tilde{x}_1 - \tilde{x}_2') \right| \psi_I \right|^2 \]  

(1-5)

as the absorption probability, where \( \psi_I \) is the initial nuclear state, \( \alpha \) refers to the two emitted nucleons, \( \beta \) to the residual nucleus, and \( F \) is the function

\[ F(\tilde{x}_i - \tilde{x}_i') = \frac{e^{-\sqrt{m}|\tilde{x}_i - \tilde{x}_i'|}}{|\tilde{x}_i - \tilde{x}_i'|} \]  

(1-6)

where the primed and unprimed coordinates are the nucleon coordinates in the initial and final states, respectively. Further calculation reduced Equation (1-5) to

\[ P = f^2 \int d^3 x \rho_p \left[ 3.1 \left| \nabla \psi \right|^2 \left( \rho_p + \rho_N \right) + 0.46 \left| \psi(\tilde{x}) \right|^2 \left( \rho_p + 3 \rho_N \right) \right] \]  

(1-7)

where \( \rho_p \) and \( \rho_N \) are the proton and neutron densities, respectively. Numerical evaluation then gave the absorption rates from S and P orbitals

\[ P_S \approx 7.77 \times 10^{15} Z^4 \, \text{sec}^{-1} \]  

(1-8)

\[ P_P \approx 4.41 \times 10^{10} Z^6 \, \text{sec}^{-1} \]  

(1-9)

Huguenin claims that absorption on one nucleon is extremely unlikely because of the energy-momentum mismatch. Spector, however, by writing
the interaction as a sum of single-particle operators and using distorted waves for the outgoing nucleons, obtained a rate for one-particle emission smaller than that for two-particle emission by a factor of only 4.3. See Chapter IV, Section B.

Eisenberg and Letourneaux have calculated angular correlations and energy distributions for \( \eta \eta \) and \( \eta \rho \) pairs emitted following \( \pi^-\) absorption on correlated pairs in \( ^{12}\text{C} \) and \( ^{16}\text{O} \). They pointed out that such calculations encounter difficulties in four areas: (1) differences inside the nucleus between pion atomic wave functions and those predicted from non-relativistic hydrogen-like wave functions, (2) correlations between initial-state nucleons, (3) reliable descriptions of the pion-nucleon interaction, and (4) the effects of final-state interactions between outgoing nucleons and the residual nucleus. Their calculations predicted that back-to-back (180°) emission of two nucleons is suppressed by final-state interactions, and that \( \rho \rho \) pairs should be removed about twice as frequently as \( \eta \rho \) pairs, which is in disagreement with experiment, including this work (see Chapter IV, Section B).

It has been suggested that \( \pi^-\) absorption occurs not only on nucleon pairs, but on larger clusters as well, particularly alpha clusters. Kolybasov and Tsepow claim that the comparatively crude data on angular correlations, spectra, and absolute capture rates can be explained if allowance is made for both the two-nucleon mechanism and an alpha-particle mechanism. The fundamental problem of absorption on a cluster, whether it be a quasi-deuteron, an alpha particle, or any other correlated group of nucleons, is to determine whether the absorption is on a preformed cluster or if a dynamical correlation of the nucleons occurs during the direct capture process. Although the evidence
is not conclusive, some experimental results, including some results from this work, suggest that a limited amount of preformation might occur.\(^1\) Unfortunately, final-state interactions, statistical evaporation, and enhancement of \(2^+\) states of even-even nuclei complicate the interpretation of the \(\gamma\)-ray spectra.

Some probes other than pions are also useful, at least potentially, in the study of short-range correlations in nuclei. The most obvious of these are kaons, which undergo absorption in a manner similar to pions, and medium-energy \(\gamma\) rays which, because of their energy-momentum "mismatch", cannot absorb on single nucleons. The difficulty with kaons is the low quality of kaon beams, which are currently as poor in comparison to pion beams as pion beams are in comparison to those of protons and alpha particles. The medium-energy photonuclear experiments are very difficult to perform and are generally not as advanced as the pion experiments. Several experiments have analyzed the quasi-deuteron effect \((\gamma, np)\),\(^1\) but here final-state interactions also make interpretation difficult.

It has been suggested by Bernabeu et al.,\(^1\) that muon capture in the limit of very small neutrino energy is very much like pion capture and leads to the emission of correlated nucleon pairs. They claim that if such experiments should turn out to be practical they may provide an effective means of studying the off-shell behavior of pions in hadronic matter.

Experiments with more conventional probes, such as \((p, pd)\), and the pickup reactions \((d, \alpha)\), \((p, t)\), and \((p, ^3\text{He})\)\(^1\) have some value in determining momentum distributions. Pickup and stripping reactions, as well as pion absorption, can be utilized, in principle, to
determine two-particle coefficients of fractional parentage. However, pickup (and stripping) reactions differ from pion absorption in an important way. Pions sample the nucleon-pair relative coordinate differently than do pickup reactions (in pickup the transferred particles are in a bound state). Thus the requirements on the relative wave functions of the nucleons are quite different for pickup and stripping reactions as compared with pion absorption.

B. Nuclear Structure

The shell and collective models have been most successful in accurately describing a wide variety of nuclear phenomena. These models are amply described in the literature\textsuperscript{19-21} and require no discussion here. There are residual interactions between nucleons which are not included in the shell and collective models. One example which is well understood is the energy gap resulting from pairing of like nucleons in even-even nuclei.\textsuperscript{22} Short-range correlations, which involve correspondingly large momenta, are relatively poorly understood and are difficult to explore experimentally. The observation of the quasi-deuteron effect in photonuclear reactions does indicate the existence of such correlations, but quantitative analysis is made difficult by final-state interactions.

It was pointed out by Ericson\textsuperscript{23} that the momentum distribution of an ejected pair is related to the Fourier transform of the wave function of the pair's center-of-mass coordinate. Thus the nature of such correlations is amenable to analysis by measurement of the Doppler broadening of \( \gamma \) rays from the appropriate residual nuclei resulting from \( \pi^- \) absorption.
As already mentioned in Section A, there also is evidence for larger correlated aggregates of nucleons, perhaps alpha clusters. There are models of nuclear structure using alpha clusters as geometric building blocks, but their range of application is limited, and the cluster model is not designed to replace the more detailed predictions of the shell model.

Although a pure cluster model is unlikely to give an accurate description of all nuclear properties, there remains the possibility that cluster formation may be relatively more important near the nuclear surface.

$\pi^-$ absorption should be sensitive to the presence of alpha or other heavy clusters near the nuclear surface. This expectation is based upon the requirement that pions absorb near the nuclear surface on aggregates of at least two nucleons. Because four separated nucleons in a region of relatively low density of nuclear matter have more energy than if they are clustered into a quasi-alpha particle, clusters are expected to form near the nuclear surface. A four-nucleon cluster contains six pairs. Therefore, if clusters exist, pions should preferentially absorb on them.

Indications of absorption on heavy clusters may include large yields for the removal of one or more equivalent alpha particles, large momentum transfers in such processes, excitation of high-spin states, and four-particle-four-hole effects indicative of direct reactions on four-nucleon groups. Evidence of such phenomena in these experiments is discussed in Chapter IV, Section B.
C. Purposes of the Work

The general purpose of this study was to utilize the observed prompt $\gamma$-ray spectra from daughter nuclei produced by $\pi^-$ absorption to learn as much as possible about the reaction and nuclear structure. This technique has both strengths and weaknesses. Among the strengths are (1) all bound excited states of daughter nuclei produced with adequate yields will be observed, no matter how complex the particle-emission processes generating those states may be, (2) the experiments are usually not biased in favor of a particular reaction channel, (3) recoil momenta of daughter nuclei can frequently be measured, (4) energy resolution is excellent, (5) pion beams of relatively poor spatial and momentum resolution may be utilized, (6) no vacuum system is needed, and (7) states resulting from neutron emission can be seen.

Weaknesses of this technique include (1) the inability to observe ground-state transitions, (2) the inability to observe daughter nuclei which do not have bound excited states, such as many of those relatively far from the maximum-stability curve, (3) the inability to measure recoil momenta of nuclei in states with long life-times ($\tau \gg 1$ picosecond), (4) occasional ambiguities in $\gamma$-ray identifications, (5) the low efficiencies of Ge(Li) detectors, and (6) the difficulty of obtaining total yields for odd-even and odd-odd daughter nuclei. On balance, the $\gamma$-ray technique is an effective means of studying nuclear structure, especially when it is viewed as a complement to experiments which measure the spectra of outgoing charged particles and neutrons.

The purpose of the $\pi^- + {}^{12}\text{C}$ experiment was to measure the relative excitations of $^{10}\text{B}$ and $^{10}\text{Be}$, and to look for one-particle
emission by observation of any $^{11}_B$ lines which might have been present. Excitation or, more correctly, non-excitation of the $0^+, 1$ state of $^{10}_B$ (see Chapter IV, Section B1) was also of interest.

Measurement of the recoil momentum spectrum of $^{12}_C 2^+$ daughter nuclei was the purpose of the $\pi^- + ^{14}_N$ experiment. From this the momentum distribution of the absorbing np pair was determined assuming a one-step quasi-free process.

There were several reasons for performing the $\pi^- + ^{32}_S$ and $\pi^- + ^{40}_Ca$ experiments. These included measurement of np/pp removal ratios, comparison with predictions of a statistical evaporation code, and a search for evidence of alpha clustering.

The $\pi^- + ^{23}_Na$ experiment was performed primarily to determine what systematic differences may exist between spectra of daughter nuclei from odd-even targets as compared with the even-even targets $^{32}_S$ and $^{40}_Ca$. The np/pp removal ratio was of particular interest because here there was no $2^+$ enhancement (see Chapter IV, Sections Al, B4c, and B6b) to bias the result. Also, would equivalent alpha emission be observed strongly, or would equivalent triton removal be dominant instead? Another reaction channel of interest here was single-proton removal, a $d_{3/2}$ proton being the unpaired particle. In this case would it be strongly suppressed as according to Huguenin, or nearly comparable to two-nucleon removal as predicted by Spector?

These questions, and a number of others which arose in the course of the analysis, are discussed in light of the experimental results in Chapter IV.
II. EXPERIMENTAL APPARATUS

A. The SREL Synchrocyclotron

These experiments were performed at the Space Radiation Effects Laboratory $^{31}$ synchrocyclotron in Newport News, Virginia, using the negative pion beams supplied to the meson and NMA-1 experimental areas.

The SREL machine is a frequency-modulated cyclotron with the accelerating frequency modulated according to a frequency program given by a variable capacitance. This is done by a large tuning fork, the prongs of which form part of a variable capacitor, and which vibrates at $5\frac{1}{4}$ Hz, each vibration corresponding to the acceleration of a group of protons from zero to maximum energy. $^{32}$ The acceleration frequency thus varies from 29.8 MHz to 17.0 MHz over each cycle, and the internal beam protons are accelerated to a kinetic energy of 590 MeV.

Negative and positive pions are produced by reactions of the form

$$ p + ^{12}C \rightarrow ^{11}C + 2p + \pi^- \quad (1) $$

in an internal "harp" target consisting of 9-micron-thick filaments of carbon located approximately 2.2 meters from the center of the cyclotron. $^{33}$ The cross section for $\pi^-$ production in carbon at a laboratory
energy of 600 MeV is approximately $5 \mu b/MeV \text{ steradian}^{34}$ and typical values are shown in Figure 1.

Of the five experiments described in this dissertation, one, $\pi^- + ^{14}N$, was performed in the meson channel and the others were done in the NMA-1 area. The layouts of these beam channels are shown in Figure 2. Both of these channels emit 180-MeV/c pions which are focused by a pair of quadrupole magnets and then momentum selected by a bending magnet. The magnet settings are given in Table 2 and pion flux as a function of bending-magnet setting is given in Figure 3. A beam study conducted prior to these runs$^{35}$ indicated that it was $(85.3 \pm 7.0)\%$ pions, the remainder being muons and electrons. This is reasonably consistent with the one indirect measurement of beam composition done in these experiments, which found a $(6.1 \pm 1.2)\%$ yield of the 787-keV 2p-1s muonic X rays from $^{40}$Ca in the $\pi^- + ^{40}$Ca experiment. Since this muonic X-ray line has a relative intensity of $(83.6 \pm 1.7)\%$, the observed yield would correspond to $(7.3 \pm 1.4)\%$ muons in the beam.

B. Detection of Stopped Pions

The 180-MeV/c negative pions from either the meson channel or the NMA-1 channel entered the experimental apparatus as shown in Figures 4 and 5. The first element was an energy degrader made of borated paraffin and sheets of aluminum. Optimum degrader thickness was determined by taking a range curve and thus maximizing the stopping rate, as shown in Figure 6.

Following the degrader the pions passed through two scintillation detectors S1 and S2, each consisting of a $1/8"$-thick piece of Pilot B scintillator connected through an adiabatic light pipe to a
photomultiplier tube. AND pulses from these two counters indicated that pions had reached the target, which was immediately behind S2. Scintillator S3, immediately behind the target, was in anticoincidence with the S1-S2 AND. Thus a $12\bar{3}$ signal indicated a pion stop.

The targets for the five experiments were: (1) a 10 x 10 x 1" block of graphite, (2) three jars of Li NH$_2$, which served as a nitrogen target, (3) six cylinders of metallic sodium 8" long by 2" diameter in thin plastic bags hanging parallel to each other, (4) a $1\frac{1}{8}$" thickness of powdered sulfur in a thin plastic bag, and (5) a 2" thickness of calcium turnings in a thin plastic bag.

The targets were hung with the target plane at a $45^\circ$ angle with respect to both the pion beam and the normal line from the beam to the Ge(Li) detector. This increased the target thickness seen by the beam and decreased that seen by the Ge(Li). Corrections for target thickness are discussed in Chapter III.

C. The Gamma-Ray Spectroscopy System

$\gamma$ rays emitted in coincidence with stopped $\pi^-$ were detected with a 40 cm$^3$ Princeton Gamma-Tech Ge(Li) detector. It was cooled by a 30-liter liquid-nitrogen dewar and was placed to one side of the target so that the face of the Ge(Li) cryostat was approximately 14" from the center of the target, which was coincident with the center of the pion beam line. The 2000-volt negative bias required by the detector was supplied by a Fluke 405B power supply connected through an RC network (RC = 60 sec). The AC input to the power supply was regulated by a Sorensen R1010 AC regulator. See Figure 7.
A 6" x 6" scintillator was placed immediately in front of the Ge(Li) and was used as a charged particle anticoincidence. However, there were no perceptible differences between spectra taken with and without this detector in anticoincidence.

The gated $\gamma$-ray signals were digitized by a multichannel analyzer (a Kickeort 711 for the C, N, and S targets, and a Northern Scientific NS 720 for the Na and Ca targets). Spectra associated with two different timing arrangements (see Section D) were routed into separate 2048-channel analyzer regions. Overall system gain was stabilized by means of an Ortec 419 precision pulse generator and a digital stabilizer (a Kickeort 353 N or a Northern Scientific NS 454 for the respective analyzers).

The $\gamma$-ray energy analysis system was calibrated for energy and efficiency with radioactive sources, primarily $^{60}$Co and $^{228}$Th. Logic pulses from the constant fraction discriminator gated the analyzer so that it would count all $\gamma$ rays observed by the Ge(Li). This eliminated the possibility of a gain shift arising from turning the analyzer gate off. The energy calibrations were double checked with the positions of $\gamma$-ray peaks which were expected to be strongly excited in the respective experiments. The 511-keV line was, of course, common to all runs. All of the experiments were consistent within 2 keV with the energy calibrations obtained from sources, which are given in Table 3.

The efficiency calibration was performed with two separate $^{60}$Co sources, each of which had a strength of 5 $\mu$ curies ($\pm10\%$) on January 20, 1971, $^{37}$ for determining absolute efficiencies at 1173 and 1332 keV. A $^{228}$Th source was used to determine relative efficiencies over a much wider range, 238 to 2614 keV.
All three sources were provided by the SREL equipment pool. The efficiency curve so obtained from source runs taken on June 6, 1974 is given in Figure 8. This is in good agreement with the efficiency function of the same detector as determined by B. J. Lieb, who performed a prior experiment.\textsuperscript{38}

Several factors contributed to possible errors in the absolute detector efficiency. These include (1) the 10% uncertainty in the strength of the $^{60}\text{Co}$ sources, (2) the $\gamma$ rays from $\pi^-$ interactions came from all parts of a 6" x 6" section of the target and not just from its geometrical center, as was the case in the source runs (this could increase the efficiency by as much as 5%), (3) slight variations in the Ge(Li) position with respect to the target resulting from its being moved when the liquid-nitrogen dewar was refilled (a 1 cm difference could introduce an error of 7%), and (4) an analyzer dead time which varied from 0 to 3% depending on source intensity. Although addition of these possible errors in quadrature gave a net error of 13%, to be conservative, an error of $\pm 20\%$ was assigned to the absolute efficiency calibration. Relative efficiencies, however, were much more precise, being limited by the statistical errors in the individual $\gamma$-ray peak areas and the accuracy of the computer fitting procedure.

D. Logic and Timing Circuits

A block diagram of the electronics employed in these experiments is given in Figure 9. The central feature was the $\gamma$-ray spectroscopy system described in Section C. The purpose of the auxiliary logic and timing circuitry was to discriminate $\gamma$ rays in prompt coincidence
with pion absorption events (1234) from background radiation and decays of long-lived (τ > 5 nsec) states of daughter nuclei. This was done by sending γ-ray pulses from the amplifier to a constant fraction discriminator (CFD), which produced NIM logic pulses when the γ signals reached a constant fraction of their maximum heights, thus reducing considerably the time slewing which would have occurred with an ordinary discriminator.

A time-to-amplitude converter (TAC) accepted pion stop pulses (1234) as START signals, and the CFD output as STOP signals. The resulting timing spectra, such as that for the sulfur experiment shown in Figure 10, were accumulated by a Nuclear Data multichannel analyzer. These spectra contained large peaks about 40 nanoseconds wide corresponding to prompt events, with flat regions corresponding to accidentals on both sides.

The prompt-event peak and a broad flat region preceding it were selected by two single-channel analyzers (SCA's) as the "on-time" and "off-time" sections, respectively. The SCA's produced logic pulses which routed the prompt and accidental γ-ray pulses into their respective 2048-channel halves of the energy analyzer.

In order to maintain digital stabilization the pulser discriminator output was fed into the off-time routing through a logical OR connection. A similar OR for the CFD output was activated for calibration runs with sources.

The analyzer was gated by the output of an OR logic unit with inputs from both the on-time SCA and the off-time/pulser logic unit.

The accumulated counts from S1, from the 12 logical AND, and from the 123 and 1234 AMTI's were recorded on scalers and these were
read at the end of each run. The number of $\pi^-$ stops in each experiment is given in Table 4. The $^{123}\text{U}$ output was also connected to a rate-meter, which was used in maximizing the beam rate through fine adjustments in magnet currents.

E. Data Collection

1. $\pi^- + ^{12}\text{C}$

This experiment was first performed in December, 1971 with a Nuclear Data analyzer which was borrowed from the Laboratory of Nuclear Science of the Massachusetts Institute of Technology. The data were written onto magnetic tape and transferred to cards by the IBM 360/44 computer at SREL, and were analyzed as described in Chapter III. When this analysis was performed, however, it indicated a large, broad peak at 1023 keV, which corresponds to the first $T = 1$ level of $^{10}\text{B}$. The excitation of such a state was in disagreement with a conjecture based upon the results of an analogous experiment on $^{16}\text{O}$, in which the first $T = 1$ state of $^{14}\text{N}$ was only very weakly excited. Because of suspicions as to the nature of this peak, the carbon experiment was repeated in August, 1972. A two-week run provided sufficient statistics to resolve the apparent large broad peak into two peaks, the 1023-keV $^{10}\text{B}$ line, which was now reduced in area to only marginally more than that necessary to account for feeding from higher levels, and the 1013-keV line of $^{27}\text{Al}$, apparently a contaminant resulting from excitation of the aluminum casing of the Ge(Li) detector and other aluminum structures in the experimental area. This $^{27}\text{Al}$ peak appears in the spectra of all five reactions studied in this work. The details of the data analysis are given in Chapter III.
2. \( \pi^- + ^{14}_N \)

This experiment was first performed in April, 1971, with the equipment described earlier in this chapter, except that a Kicksort ADC and the SREL IBM 360/44 computer were used as an analyzer. The results indicated that the 4439-keV line of \(^{12}\)C was strongly excited, in fact it was the only line observed excepting contaminants. However, the timing was bad and the on-time and off-time spectra were indistinguishable. Therefore the experiment was repeated in January, 1973, with the Kicksort 711 analyzer. The results were consistent with the earlier run and are given in Chapter III.

An attempt was made prior to this 1973 run to fabricate a suitable liquid nitrogen target. Such a target would require both thin walls (so that the pions would stop in the nitrogen and not in the walls) and sufficient insulation to prevent rapid boiling. Several styrofoam containers were fabricated, but the seals came apart when they were filled with nitrogen.

After having these difficulties it was decided to use the solid powder \( \text{Li NH}_2 \) as a nitrogen target. This has the advantages of sufficient density at room temperature (1.178 gm/cm\(^3\)), and no bound excited states of any possible daughter nuclei resulting from \( \pi^- \) absorption on lithium and (of course) hydrogen. Thus all \( \gamma \) rays resulting from \( \pi^- \) absorption on the target could be associated with nitrogen.

\( \text{Li NH}_2 \) does, however, have the disadvantage that the absolute yields of states of daughter nuclei from absorption on \(^{14}_N\) could not be determined accurately. This is because an unknown fraction of the pions absorb on the lithium and hydrogen, and the yields from \(^{14}_N\) interactions
cannot be normalized to the stopped pion count. One could use the Fermi-Teller "Z-law", which predicts that 47.3% of the stopped \( \pi^- \) will absorb on \(^{14}\text{N}\) nuclei. However, the apparent existence of meso-atomic processes and large mesic molecules invalidates use of the Z-law in systems in which these phenomena occur. Their occurrence in \(^6\text{Li NH}_2\) would render the 47.3% figure invalid. But the main purpose of the \( \pi^- + ^{14}\text{N} \) experiment was to measure the recoil-momentum distribution of the \(^{12}\text{C}\) daughter nuclei, which was unaffected by the uncertainty in absolute yields resulting from use of the \(^{6}\text{Li NH}_2\) target.

3. \( \pi^- + ^{23}\text{Na} \)

This experiment was initially performed in August, 1973, but the results were of limited value because of poor energy resolution (approximately 10 keV at 1332 keV), poor timing, and poor statistics. Therefore it was run again in June, 1974, without these difficulties.

The chemical activity of sodium is, of course, well known, and it presented some minor problems. When the target cylinders arrived in August, 1973, they were immediately placed in thin plastic bags, a few drops of kerosene were added, and the bags were then tightly sealed. This kept them in good condition through the 1973 run, but they were badly oxidized by June, 1974. The cylinders were thus removed from the bags, the oxide was scraped off, and they were then immediately sealed in new plastic bags.

4. \( \pi^- + ^{32}\text{S} \)

The sulfur experiment was performed in October, 1972. No difficulties were experienced except that, because this was done on a
parasite basis, the scaler bank was removed by the prime user and there was no record of the number of pion stops. However, the measured stopping rate (50 K/sec) was multiplied by the running time (20 hours) to obtain an estimate of the total pion stops (see Table 3).

The width of the prompt peak in the timing spectrum was approximately 25 nanoseconds. This was the best timing resolution obtained in this set of experiments, the typical value in the other runs being 40 nanoseconds.

5. $\pi^- + ^{40}$Ca

The first attempt at this experiment was made in April, 1971, immediately following the first attempted $^1$H N run. The calcium data were likewise afflicted with the bad timing which rendered the nitrogen results invalid. A second attempt was made in December, 1971, immediately after the first $^{12}$C experiment, but the statistics were so poor that only the very strongest low-energy peaks could be identified. A third, successful run was performed in May, 1974.

The target consisted of calcium turnings which were poured into thin plastic bags and tightly sealed. No significant oxidation was observed.
III. DATA ANALYSIS

The determination of which states of which daughter nuclei are excited in the $\pi^-$ absorption processes is outlined in Section A and a detailed peak-by-peak study of each spectrum is then given in Section B. These results are used in Section C to compute the mean numbers of protons, neutrons, and nucleons emitted in these processes. Doppler-broadened peaks are considered in Section D, and Section E is a spectrum-stripping analysis in which the total number of $\gamma$ rays produced is determined from the Compton cross section.

A. Procedures Followed in Spectral Analyses

Peaks were noted in both on- and off-time spectra which had been plotted by the William and Mary IBM 360/50 computer. Peak energies were computed from the energy calibration relations given in Chapter II, Section C. The spectra were then double-checked by examination of all regions where peaks, both from $\pi^-$ absorption on the targets and from contaminants, were expected. Tables of nuclear energy levels of known $\gamma$ rays by energy and nucleus were used extensively in these searches. Observed levels were checked for consistency with known branching ratios and if these were in gross disagreement with the data, the assignments were rejected.
After all the peaks in a given spectrum were compiled and, if possible, identified, they were curve-fitted in order to accurately determine the height, center, width, background, and area of each. These were computed with the aid of the POLYFIT program in use at the College of William and Mary, which utilizes a Gauss-Seidel least-squares fit.

The POLYFIT function is

\[
f(x) = \sum_{j=1}^{N} A_j \exp\left[-\frac{2.773(x-C_j)^2}{F_j^2}\right] + B \exp[S(x-C_1)],
\]

where \(N\), which is the number of gaussian functions being fitted to the given data region, can be no greater than 6. In this combination of gaussian peaks with an exponential background

- \(A_j\) is the amplitude of the \(j^{th}\) gaussian peak,
- \(C_j\) is the center channel of the \(j^{th}\) gaussian,
- \(F_j\) is the full-width-at-half-maximum of the \(j^{th}\) gaussian,
- \(C_1\) is the center channel of the first gaussian,
- \(B\) is the amplitude of the background at channel \(C_1\), and
- \(S\) is a measure of the slope of the exponential background.

The parameters \(A_j, C_j, F_j, B,\) and \(S\) were varied so as to minimize the \(\chi^2\), given by

\[
\chi^2 = \sum_{i=1}^{K} \left[ \frac{Z_i - f(x_i)}{\sqrt{Z_i}} \right]^2,
\]

where \(Z_i\) is the number of counts in the \(i^{th}\) channel, and \(K\) is the number of channels in the fitting region. Trial values of \(A_j, C_j, F_j, B,\) and \(S\) were supplied to the computer with the data and program, and
successive $\chi^2$ minimization iterations were performed until each parameter changed by no more than 0.001% in the next following computation. In a few cases this status could not be realized in 100 iterations and the process was stopped. Such peaks were refitted using different trial values until a convergent fit was obtained, or until it became obvious that this was impossible. If no convergent fit was obtained the non-converging fits were examined and the best taken. If none were deemed acceptable the parameters were estimated by eye. In the rare instances where this was necessary errors were assigned very cautiously so that the true values of the parameters would certainly fall within the error limits.

The quantity

$$V = \frac{\chi^2}{N - P},$$

(3-3)

where $N$ is the number of channels in the fit and $P$ is the number of parameters used, is called the variance of the fit. It measures the quality of the fit with respect to an ideal fit, which would have a variance of 1.000. Fits with variances greater than 2.0 were done over with different initial guesses and/or variables being held constant (see below), but in some cases desirably small variances were impossible to obtain. In such situations the best fit curves were used only if they faithfully reproduced the data. If not, hand fitting was employed. Particular attention was paid to the background fit because an overshoot or undershoot of this curve could radically alter the computed yield.
Areas of given peaks were computed from the relation

\[ \text{AREA}_j = 1.064 A_j F_j \]  \hspace{1cm} (3-4)

These areas were used in computing the raw yields of the respective states.

The raw yield of a given \( \gamma \) ray (by raw it is meant that no corrections for branching and feeding have been done) is given by

\[ \text{Raw Yield (\%)} = \frac{\text{No. } \gamma \text{ rays produced}}{\text{No. } \gamma \text{ rays stopped}} \times 100. \]  \hspace{1cm} (3-5)

In order to extract the number of \( \gamma \) rays produced from the area of a given peak, it must be corrected for Ge(Li)-detector efficiency and \( \gamma \)-ray absorption in the target. Detector efficiency was accounted for by dividing the area by the value of the efficiency \( \varepsilon \) of the Ge(Li) system at the energy of the \( \gamma \) level given in Figure 8 of Chapter II.

Target thickness corrections were made by calculating the correction factor

\[ \eta = \frac{I_0}{I} = e^{\mu l^{1/2}} \]  \hspace{1cm} (3-6)

for each of the targets as a function of \( \gamma \)-ray energy, where

\[ \mu = \frac{0.693}{t^{1/2}} \]  \hspace{1cm} (3-7)
being the characteristic half-thickness for γ-ray absorption by the target material. These values were taken from a graph in the Nuclear Data Tables, which is shown in Figure 11. $l_{1/2}$ is the actual effective half-thickness of the target expressed in gm/cm$^2$. This figure is given for each of the targets in Table 5. $\eta$ was calculated as a function of $\gamma$-ray energy in steps of 200 keV, these values were plotted, and the best smooth curves were drawn through the points. The values of $\eta$ for each of the $\gamma$-ray peaks were taken from these graphs. See Figure 12.

The actual number of $\gamma$ rays produced corresponding to a given transition is thus given by

$$N_{\gamma} = \text{Area} \times \left( \frac{\eta}{E} \right).$$ \hspace{1cm} (3-8)

The number of $\pi^-$ stopped, $N_{\pi}$, was obtained from the total 123 count by correcting for muon contamination of the beam (see Chapter II, Section A). Since muons were found to constitute 7.3% of the beam,

$$N_{\pi} = \frac{123}{927}. \hspace{1cm} (3-9)$$

The percentage raw yields, $\frac{N_{\gamma}}{N_{\pi}} \times 100$, were then calculated.

After the raw yields for transitions were determined, the corrected yields for individual states were obtained from consideration of the branching ratios for transitions to lower levels, and feeding from higher levels. The tables of Endt and van der Leun$^{42}$ were used extensively in these analyses. As noted previously, at least approximate agreement with the published branching ratios was required for a
transition identification to be accepted. Occasionally one transition from a given level could not be seen because it was either (1) too low in energy, (2) covered up by a strong line, or (3) below observation threshold because of small branching ratio. In these cases the identifications were accepted if there were no ambiguities. The possibility that many additional higher levels were excited but were below the threshold of observation may account in whole or in part for the relatively large yields of the $2^+$ levels of even-even nuclei, because higher levels tend very strongly to cascade through these states (see Chapter IV, Section B).

B. Determination of Yields of States of Daughter Nuclei

In this section the analyses of the individual lines of the experimental spectra are given, except for nitrogen, which is in Section D.

1. Carbon

The on-time and off-time spectra from the December, 1971, $\pi^- + ^{12}$C experiment are shown in Figures 13 and 14, respectively. Four peaks which could be associated with $\pi^-$ absorption were observed, and their energies, identifications, areas, corrected areas, and raw yields are given in Table 6. The fit of the 717-keV peak is shown in Figure 13.

A peak was observed at 2154 keV, which corresponds to the transition from the 2154 keV state of $^{10}$B to its ground state, but the statistics were too poor to obtain a fit. Its contribution was inferred from the other transitions of the 2154 keV state which were seen, and from the branching ratios of Young and Hornyak$^{47}$ to be 1.14%.
The raw yields were corrected for branching and feeding and the transition scheme in Figure 16 was inferred. From this, subtractions for feeding were made. With errors added in quadrature, this gave the direct yields listed in Table 7. Note that the excitation of the 1740-keV \( (0^+, 1) \) level is only marginally above its statistical error. Adding the corrected yields and their errors (in quadrature) gives a net yield for \( ^{10}\text{B} \) from \( \pi^- \) absorption on \( ^{12}\text{C} \) of \( (13.83 \pm 0.40)\% \).

2. Sodium

The on-time and off-time spectra from the June, 1974 \( \pi^- + ^{23}\text{Na} \) experiment are shown in Figures 17 and 18, respectively. The peaks which could be associated with prompt \( \gamma \) rays from \( \pi^- \) absorption on \( ^{23}\text{Na} \) are listed, along with their values of \( N_y \), \( N_y \times \left( \frac{\gamma_E}{E} \right) \), and raw yields, in Table 8. As an example, the fit for the 1614- and 1633-keV peaks from \( ^{19}\text{F} \) and \( ^{20}\text{Ne} \), respectively, is given in Figure 20, and the resultant corrected yields are in Table 9. These are summarized by daughter nuclei in Table 10, and by mass number \( A \) of daughter nuclei in Table 11. The latter is presented in histogram form in Figure 21. The total yield, summed over all observed states of daughter nuclei, was \( (31.3 \pm 2.1)\% \).

3. Sulfur

The on-time and off-time spectra from the October, 1972, \( \pi^- + ^{32}\text{S} \) experiment are shown in Figures 22 and 23, respectively. The peaks which could be associated with prompt \( \gamma \) rays from \( \pi^- \) absorption on \( ^{32}\text{S} \) are listed, along with their values of \( N_y \), \( N_y \times \left( \frac{\gamma_E}{E} \right) \), and raw yields, in Table 12. The fit for the 1779-keV and 1809-keV peaks from \( ^{28}\text{Si} \) and \( ^{26}\text{Mg} \), respectively, is given in Figure 24.
The raw yields were corrected for branching and feeding. The level schemes for $^{30}$P, $^{29}$Si, and $^{26}$Mg, shown in Figure 25, were used in these computations, which produced the corrected yields in Table 13. These are summarized by daughter nuclei in Table 14, and by mass number $A$ of daughter nuclei in Table 15. The information in the latter is also presented in histogram form in Figure 26. The total yield, summed over all observed states of daughter nuclei, was $(37.5 \pm 1.2)\%$.

4. Calcium

The on-time and off-time spectra from the May, 1974, $\pi^- + ^{40}$Ca experiment are shown in Figures 27 and 28, respectively. The peaks which could be identified with prompt $\gamma$ rays from $\pi^-$ absorption on $^{40}$Ca are listed, along with the corresponding values of $N_\gamma$, $N_{\gamma-x}(\%E)$, and raw yields, in Table 16. One peak, with a 3.14% yield at 117.4 keV, could not be identified. Three apparent peaks, too small to give convergent fits, were also observed. These were the 2523-keV line from the first excited state of $^{39}$K, the fifth excited state of $^{37}$Cl (3710 keV), and the sixth excited state of $^{35}$Cl (3162 keV). The fit for the 1729-keV ($^{37}$Cl) and 1779-keV ($^{28}$Si) peaks is given in Figure 29, and that for the 1970 keV ($^{36}$Ar) peak is in Figure 30.

The corrections for branching and feeding in $^{39}$Ar, $^{38}$Ar, and $^{36}$Ar required explicit addition and subtraction of raw yields, and the level schemes of these nuclei are given in Figure 31. These corrections produced the yields given by level, daughter nucleus, and mass number in Tables 17, 18, and 19, respectively. The yield by mass-number histogram is Figure 32. The total yield, summed over all observed states of daughter nuclei, was $(29.9 \pm 1.4)\%$. 
There is an ambiguity in the assignment of the 1267-keV peak to the first excited state of $^{39}$Ar. The first excited state of $^{31}$P (1266 keV), the second excited state of $^{30}$Si ($3498 - 2235$ keV = 1263 keV), and the first excited state of $^{29}$Si (1273 keV) are all sufficiently close to 1267 keV that any one, or some combination thereof, could produce the observed peak. Therefore this peak may not arise from production of $^{39}$Ar.

C. Computation of Mean Numbers of Nucleons Emitted in $\pi^-$ Absorption

The average numbers of protons, neutrons, and nucleons emitted per observed $\pi^-$ absorption reaction are given by

$$
\underline{\# \, P} = \frac{\sum_i Y_i \Delta Z_i}{\sum_i Y_i}, \quad (3-10)
$$

$$
\underline{\# \, N} = \frac{\sum_i Y_i \Delta N_i}{\sum_i Y_i}, \quad \text{and} \quad (3-11)
$$

$$
\underline{\# \, N} = \frac{\sum_i Y_i \Delta A_i}{\sum_i Y_i}, \quad (3-12)
$$

where the $Y_i$ are the yields for the particular states, and $\Delta Z_i$, $\Delta N_i$, and $\Delta A_i$ are the numbers of protons, neutrons, and nucleons, respectively, by which the daughter state is removed from the target nucleus. These were computed from the observed yields for sodium, sulfur, and calcium, and the results are listed in Table 20.
D. Analysis of Doppler-Broadened Lines

Doppler broadening of γ-ray lines is observed when the emitting nuclei are moving rapidly, and hence the line shapes of such transitions contain implicitly the recoil momentum distribution of the nuclei,\(^{39}\) which is equal to the sum-momentum of the particles ejected by the nuclei in the reaction leading to the excited state. The γ-ray energy shift is

\[
\Delta E = \frac{E_y K \cos \theta}{M c},
\]

(3-13)

where \(E_y\) is the energy of the unshifted peak, \(K\) the momentum of the emitting nucleus, \(M\) its mass, and \(\theta\) the angle between the nuclear velocity and the detector direction. If it is assumed that the momentum distribution of the recoiling nuclei is spatially isotropic, and that the γ rays are emitted isotropically with respect to the momenta (this is strictly true only for decay of \(J = 0, \frac{1}{2}\) states, but is expected to be approximately valid in general\(^ {48}\)), then, for each value of \(K\), a line that would have been a delta function if \(K = 0\) becomes a rectangle between the limits \(\pm \frac{E_y K}{M c}\). The actual line is a superposition of such rectangles weighted by the distribution of momenta. The intensity of a peak at a distance \(\Delta \xi'\) from its center is made by contributions from all momenta greater than

\[
k' = \frac{M c \Delta \xi'}{E_y}.
\]

(3-14)
All the contributions from $K < K'$ are found closer to the center than $AE'$. So the intensity at a distance $AE'$ from the peak's center is

$$N(\Delta E') \propto \int_{K'}^{\infty} \frac{1}{K} \frac{d^n}{dK} dK,$$

the $1/K$ factor giving the relation between $K$ and the height of the corresponding rectangle (base x height = constant area).

Suppose the momentum distribution is given by

$$\frac{d^n}{dK} \propto K^2 e^{-K^2/Q^2},$$

whose corresponding function

$$\frac{d^3n}{dK^3} = \frac{1}{K^2} \frac{d^n}{dK},$$

is the momentum distribution for an $N = L = 0$ harmonic-oscillator wave function. Then

$$N(\Delta E') \propto e^{-K'^2/Q^2},$$

and

$$Q = \frac{M_c (FWHM)}{2 \sqrt{\ln 2} E_y}$$

is the momentum corresponding to the maximum value of $|d^n/dK|$, FWHM being the full-width at half-maximum of the peak. A related quantity, the mean momentum, is
If the lifetime of the state is sufficiently short that slowing down of the excited nucleus is negligible, this expression will give the mean recoil momentum provided the other previously stated assumptions are valid.

Some broadened lines of physical interest are, unfortunately, from transitions of states whose lifetimes $\tau$ are not negligible compared with the mean slowing-down time $\alpha$ of the nucleus in the target material. Although the actual broadening of these peaks is less than what it would be if there were no slowing down, the mean recoil momentum can be unfolded from these peaks if

$$\alpha \geq \tau .$$  

(3-21)

There are two somewhat different approaches to this problem which, as will be seen, give results which differ by no more than 20% in the cases studied.

Blaugrund derives an expression for the ratio of the observed mean width of a peak $\Delta E_y$ to the mean width $\Delta E_{y0}$ of the peak if produced by $y$ rays from nuclei moving with their full initial velocities in the direction of the detector (note that $\Delta E_y$ and $\Delta E_{y0}$ are defined differently from the $\Delta E$ and $\Delta E'$ discussed previously):

$$\bar{P} = \sqrt{\ln 2} \, Q = \frac{Mc(FWHM)}{2 \, E_y} .$$  

(3-20)
\[
\frac{\Delta E_y}{\Delta E_{y0}} = \frac{1}{1 + \tau/\alpha} - \frac{4}{kE_0} \left( 1 + \frac{A_2}{A_1} \frac{\partial n}{\partial n} \right) \frac{\tau/\alpha}{1 - (\tau/\alpha)^2},
\]

which is valid in the range

\[
\tau/\alpha \lesssim 0.3 \text{ to } 0.5. \tag{3-23}
\]

The mean slowing-down time is given by Lindhard, Scharff, and Schiøtt as

\[
\alpha (\text{p sec}) = \frac{A_1}{Z_1} \frac{A_2}{Z_2} \frac{Z}{\gamma} \frac{0.0198}{\rho} , \tag{3-24}
\]

where

\[
Z^{2/3} = Z_1^{2/3} + Z_2^{2/3}, \tag{3-25}
\]

and

\[
\frac{Z}{\gamma} = Z_1^{1/6}. \tag{3-26}
\]

\(A_1\) and \(A_2\) are the atomic mass number of the projectile and target atoms, respectively, \(Z_1\) and \(Z_2\) are the corresponding charge numbers, and \(\rho\) (gm/cm\(^3\)) is the density of the target material. Blaugrund gives

\[
K = \frac{0.0793 Z_1^{1/2} Z_2^{1/2} (A_1 + A_2)^{3/2}}{Z_1^{1/6} A_1^{3/2} A_2^{1/2}}. \tag{3-27}
\]
and

\[ E_0 = 10.2 \frac{g_m}{A_1} \frac{E}{m c^2}, \]  \hspace{1cm} (3-28)

where

\[ g_m = \frac{1.63 \times 10^3 A_1 A_2}{Z_1 Z_2 Z^3 \sqrt{(A_1 + A_2)}}. \]  \hspace{1cm} (3-29)

\( E \) is the initial kinetic energy of the recoiling nucleus, and \( MC^2 \) is the rest energy of the electron. Non-zero values of \( G \) account for changes in direction of the projectile with respect to the detector, assuming initial velocity directly toward it.\textsuperscript{51} The pion-absorption process, however, has no preferred spatial direction and hence \( G = 0 \) for these interactions. One thus has

\[ \frac{\Delta E_Y}{\Delta E_{Y0}} = \frac{1}{1 + \gamma/\alpha} - \frac{4}{k E_0} \frac{\gamma/\alpha}{1 - (\gamma/\alpha)^2}. \]  \hspace{1cm} (3-30)

Solution of this equation requires an iteration procedure because the \( E_0 \) is dependent upon the initial momentum, which is unknown:

\[ E = \frac{\rho^2}{2m}. \]  \hspace{1cm} (3-31)

The iteration can be done because the second term is much smaller than the first. \( \frac{1}{1 + \gamma/\alpha} \) can be computed without knowing \( E_0, \Delta E_{Y0} \) calculated, a mean momentum determined, and hence \( E_0 \) computed. This in turn is substituted into Eq. 30, \( \Delta E_Y/\Delta E_{Y0} \) recomputed, a new \( E_0 \)
obtained, and the process is repeated until successive mean momenta agree within 1%. This typically required 3 or 4 successive iterations.

A slightly different method, developed by Lewis, alters Eq. 19:

\[
Q = \frac{M_c (FWHM)}{E_y A},
\]

so

\[
\bar{p} = \sqrt{\ln 2} \frac{M_c (FWHM)}{E_y A},
\]

where \( A \) is an attenuation function computed by Lewis, which is dependent upon the ratios \( (\alpha/\gamma) \) and \( (K_n/Q) \), and which is given for several values of \( (K_n/Q) \) in Figure 33. \( K_n \) is a characteristic momentum related to the slowing down of projectiles through collisions with target atoms, and is given by

\[
K_n (MeV/c) = 0.537 A, \frac{Z^{5/2}}{\gamma^{1/2}} \left[ \frac{A_1 Z_1 Z_2}{A_2 (A_1 + A_2)} \right]^{1/4}.
\]

By iterating from the graph in Figure 33 the value of \( A \) can be estimated to within about 10%, and the mean recoil momentum then calculated from Eq. 33.

Before any of these calculations could be carried out, the measured width of the broadened line had to be corrected for system resolution. This was done by measuring the width of a nearby unbroadened peak \( \Delta E_n \) (from either a radioactive source or a state known to
have a long mean lifetime, \( \tau > 10\alpha \) and subtracting this in quadrature from the measured broadened-line width \( \Delta E_b \):

\[
\Delta E_\gamma = \sqrt{(\Delta E_d)^2 - (\Delta E_n)^2} \quad (3-35)
\]

In these experiments six lines were clearly Doppler broadened. Four, the peaks from the first \( 2^+ \) levels of \(^{36}\text{Ar} \) (1970 keV) and \(^{28}\text{Si} \) (1779 keV) and the first excited state of \(^{37}\text{Cl} \) (1727 keV) in the \(^{40}\text{Ca} \) experiment, and that from the \( 2^+ \) level of \(^{26}\text{Si} \) in the \(^{32}\text{S} \) experiment, required correction for slowing down. The other two, from the \( 2^+ \) level of \(^{12}\text{C} \) (4439 keV) in the \(^{14}\text{N} \) experiment, and the \( 3/2^- \) level of \(^{39}\text{Ar} \) (1267 keV) in the \(^{40}\text{Ca} \) experiment, have sufficiently short mean lifetimes that no such corrections were necessary. In addition, the 1719-keV transition from seventh to third excited states of \(^{21}\text{F} \) in the \(^{23}\text{Na} \) data appeared to be broadened, but the presence of contaminating lines on both sides made quantitative analysis impossible. If better statistics are obtained from a future \( \pi^- + ^{23}\text{Na} \) experiment this peak could probably be resolved from the contaminants with sufficient accuracy for a mean recoil momentum calculation.

As discussed in Chapter II, Section E, the only peak in the \( \pi^- + ^{14}\text{N} \) spectrum which can be associated with \( \pi^- \) absorption on \(^{14}\text{N} \) is the 4439-keV level of \(^{12}\text{C} \). The on-time and off-time spectra from this experiment are given in Figures 34 and 35. The fitting of the double-escape peak of the first excited state of \(^{12}\text{C} \) is given in Figure 36. The FWHM is \((134.7 \pm 4.6)\) keV. The double-escape peak was used because of the better statistics and the presence of several dropped channels in the photopeak (the addition of 512 counts seemed to correct this).
The 2614-keV peak from a $^{228}\text{Th}$ source had a FWHM of 9.7 keV when measured by the analysis system immediately before this run. The broadening of an otherwise sharp (delta function) peak at 4439 keV could thus be conservatively estimated at about $(11 \pm 2)$ keV. The natural width of the $^{12}\text{C} \, 2^+$ double-escape peak is then

$$\sqrt{(134.7)^2 - (11)^2} \rightarrow (134.2 \pm 4.6) \text{ keV}.$$ 

This gives a mean recoil momentum

$$\overline{p} = (169 \pm 6) \text{ MeV/c} \quad ; \quad \frac{1}{\chi} = (0.857 \pm 0.029) \text{ fm}^{-1}.$$ 

The 1779-keV line from $^{28}\text{Si}$ in the $\pi^- + ^{32}\text{S}$ data was broadened, but surmounted by a narrow spike apparently resulting from feeding from long-lived higher levels and/or transitions of $^{28}\text{Si}$ daughter nuclei which had already completely stopped in the target. See Figure 24. This narrow spike (the two center channels of the 1779-keV peak) was subtracted and the remaining broadened peak was found to have a FWHM of $(23.6 \pm 3.0)$ keV. An unbroadened peak at 1369 keV (the first excited state of $^{24}\text{Mg}$) had a FWHM of $(7.2 \pm .2)$ keV and the 2614-keV line from a $^{228}\text{Th}$ source had a FWHM of $(11.0 \pm 2.0)$ keV. By taking ratios a resolution FWHM of $(8.6 \pm 6.0)$ keV was obtained for 1779 keV. Subtracting in quadrature gave $\Delta E_y = (21.9 \pm 3.1)$ keV. The Blaugrund $^{14}9$ technique for slowing-down corrections was then applied. For $^{28}\text{Si}$ in $^{32}\text{S}$, $\alpha = 1.73$ picoseconds, and the mean lifetime of the state is 0.68 picoseconds. Four successive iterations of Eq. 30 converged to $\Delta E_y^0 = 32.7$ keV. Blaugrund claims 20% accuracy for his technique. Computing the mean recoil momentum with Eq. 20, and adding errors in quadrature gave $\overline{p} = (240 \pm 28)$ MeV/c.
The Lewis \(^{48}\) technique gives fair agreement with that of Blaugrund. The characteristic momentum \(K_n\) for \(^{28}\)Si in \(^{32}\)S is 77 MeV/c, and \(K_n/Q \approx 0.4\). The attenuation factor (Figure 33) \(A \approx 0.92\), and Eq. 33 gives \(\bar{p} = (290 \pm 30)\) MeV/c, which exceeds the Blaugrund calculation value by 17%.

There were four peaks in the \(\pi^- + ^4\)Ca spectrum which clearly displayed Doppler broadening. The first excited state of \(^{39}\)Ar (1267 keV) had a FWHM of \((20.5 \pm 0.9)\) keV. Subtracting system resolution in quadrature gave a corrected FWHM of \((20.1 \pm 0.9)\) keV. The lifetime of this state is not known, but it was assumed to be sufficiently short that slowing-down corrections would not be necessary. Applying Eq. 20, \(\bar{p} = (291 \pm 13)\) MeV/c; \(\frac{1}{\lambda} = (1.47 \pm 0.07)\) fm\(^{-1}\). Approximately 25% of this line's strength arises from feeding by the long-lived second excited state (1517 keV), provided that it is correctly identified as arising from \(^{39}\)Ar (see Chapter IV, Section B3). No sharp central spike was present, however, and subtraction of the feeding contribution was not practical. The true value of the mean recoil momentum for this state may thus be slightly higher than the 291 MeV/c calculated. On the other hand, as mentioned in Chapter IV, Section B3, this peak may not even arise from \(^{39}\)Ar, as ambiguities exist with states from three other possible daughter nuclei.

The first excited state of \(^{37}\)Cl (1727 keV) had a FWHM of \((17.4 \pm 1.2)\) keV which, when corrected for system resolution, became \((15.9 \pm 1.4)\) keV. This level has a mean lifetime of 0.185 picosecond, so \(\tau/\alpha \approx 0.1\), and, from Lewis' graph \(^{48}\) (Figure 33), the attenuation factor \(A = 1.38\). Then, applying Eq. 33, \(\bar{p} = (191 \pm 13)\) MeV/c; \(\frac{1}{\lambda} = \)
The Blaugrund formula gives $\bar{p} = (175 \pm 13) \text{MeV}/c$; $\frac{1}{\lambda} = (0.89 \pm 0.07) \text{fm}^{-1}$, which is smaller than the Lewis method value by 8.4%.

The next peak to be considered was the first excited state of $^{28}$Si (1779 keV) in the $\pi^- + ^{40}$Ca spectrum (see Figure 29). The measured FWHM was $(26.6 \pm 2.0) \text{keV}$, which, with correction for system resolution became $(25.7 \pm 2.8) \text{keV}$. $\alpha = 1.58$ picoseconds, and the Blaugrund iteration procedure gave

$$\Delta E_{\gamma_0} = (39.0 \pm 3.9) \text{keV},$$

so that

$$\bar{p} = (286 \pm 30) \text{MeV}/c ; \quad \frac{1}{\lambda} = (1.45 \pm 0.15) \text{fm}^{-1}.$$

Lewis' technique, with $K_n = 78.8 \text{MeV}/c$, $K_n/Q \approx 0.23$, and $\alpha/\tau = 2.32$, gave an attenuation factor $A = 0.87$, and

$$\bar{p} = (360 \pm 50) \text{MeV}/c ; \quad \frac{1}{\lambda} = (1.83 \pm 0.25) \text{fm}^{-1},$$

which is 21% larger than the value obtained with the Blaugrund relations.

22% of the peak from the first excited state of $^{36}$Ar (1970 keV) resulted from feeding from the long-lived ($\tau > 3$ picoseconds) second excited state. This narrow spike was subtracted from the data and it was refitted, with a FWHM = $(40.4 \pm 2.9) \text{keV}$. Correction for system resolution reduced this to $(39.8 \pm 3.2) \text{keV}$. The characteristic slowing-down time $\alpha = 1.70$ picoseconds, and the mean lifetime $\tau = 0.415$ picoseconds. The Blaugrund calculation converged to
After three iterations.

Application of the Lewis technique, with $K_n = 115\,\text{MeV/c}$ and $K_n/Q \approx .22$, predicted an attenuation factor $A = 1.16$ and hence

$$\bar{p} = (485 \pm 40)\,\text{MeV/c} ; \quad \frac{1}{\lambda} = (2.46 \pm .20)\,\text{fm}^{-1}$$

which are 9.3% greater than the Blaugrund method results.

All of the mean recoil momenta calculated from Doppler-broadening measurements are summarized in Table 21.

E. Spectrum Stripping Analysis

A useful bit of information which may be used in formulating an understanding of ($\pi$, $\gamma X$) reactions is the average number of nuclear $\gamma$ rays emitted per pion interaction. This number, when considered in conjunction with yields or cross sections for production of particular states and, hopefully, the spectra of charged particles and neutrons, should help determine the excitation of nuclei in pion reactions, and indicate whether many reaction products are created which do not produce $\gamma$ rays directly observable as photopeaks. The statistical limitations on ($\pi$, $\gamma X$) spectra resulting from pion beams of low intensity and poor spatial and momentum resolution (the latter being especially disadvantageous in $\pi^-$ stopping experiments because the ability of a relatively thin target to stop incident pions is quite
sensitive to the energy spread of the pions) preclude resolution of photopeaks from weakly-excited states, which may well be numerous.\textsuperscript{53}

In order to measure the total number of nuclear $\gamma$ rays produced in a given experiment, the gross shape of the $\gamma$ spectrum, with all photopeaks neglected, was assumed to be the sum of a large number of approximately rectangular Compton spectra from Compton scattering of $\gamma$ rays in the Ge(Li) detector. Because the total Compton area for a $\gamma$ ray is much larger than that of the corresponding photopeak, and because analysis of the gross features of the spectra does not require taking small differences of large numbers over a small number of channels, this approach should provide a more accurate determination of the total number of $\gamma$ rays produced than does a sum of photopeak yields.

The contribution to the area $A$ of a spectrum from Compton-scattered $\gamma$ rays of incident energy $E$ is

$$dA = \sigma(E) dN,$$  \hspace{1cm} (3-36)

where $\sigma(E)$ is the Compton cross section at energy $E$ and $dN$ is the number of incident $\gamma$ rays of energy $E$. For simplicity the Compton spectra were assumed to be rectangles extending from the low-energy end of the spectrum out to the photopeak energy $E$. From visual surveys of Compton spectra from the Ge(Li) employed in these experiments, it was estimated that this approximation would introduce errors no greater than 10%. With this assumption, the height of a particular Compton spectrum from incident $\gamma$ rays of energy $E$ is
\[ \frac{d\mathcal{H}}{dE} = \frac{dA}{E} = \frac{\sigma(E) dN}{E}. \]  

(3-37)

The height of the gross spectrum at energy \( E \) would then be

\[ \mathcal{H}(E) = \int \frac{\sigma(E)}{E} dN \]

(3-38)

\[ = \int \frac{\sigma(E)}{E} \frac{dN}{dE} dE, \]

(3-39)

where \( \frac{dN}{dE} \) is the number of incident \( \gamma \) rays with energy between \( E \) and \( (E + dE) \). Differentiating this relationship gives

\[ \frac{d\mathcal{H}}{dE} = \frac{\sigma(E)}{E} \frac{dN}{dE}, \]

(3-40)

so

\[ \frac{dN}{dE} = \frac{E}{\sigma(E)} \frac{d\mathcal{H}}{dE}, \]

(3-41)

and the total number of incident \( \gamma \) rays is then

\[ N = \int \frac{dN}{dE} dE = \int \frac{E}{\sigma(E)} \frac{d\mathcal{H}}{dE} dE. \]

(3-42)
In order to use these relations to obtain the $\gamma$-ray spectra $dN/dE$ and the total number of $\gamma$ rays $N$, the Compton cross section had to be computed as a function of energy, and the absolute efficiency of the system had to be found. The former was done by a FORTRAN program which computed the Compton cross section from the Klein-Nishina formula\textsuperscript{54}

$$\frac{\phi}{\phi_0} = \frac{3}{4} \left\{ \frac{1 + \gamma}{\gamma^3} \left[ \frac{2 \gamma (1 + \gamma)}{1 + 2 \gamma} - \ln (1 + 2 \gamma) \right] + \ln (1 + 2 \gamma) - \frac{1 + 3 \gamma}{(1 + 2 \gamma)^2} \right\}, \quad (3-43)$$

where $\phi_0 = \frac{8 \pi}{3} \kappa_0^2 = 6.65 \times 10^{-25}$ cm$^2$, the Thomson cross section, and $\gamma$ is the ratio of the $\gamma$-ray energy to the rest energy of the electron. $\frac{\phi}{\phi_0}$ was calculated in 10-keV steps from 10 keV to 10 MeV by the Virginia State College IBM 360/30 computer.

The absolute-efficiency determination was made by analyzing a spectrum from a $^{228}$Th source taken with the experimental analysis system. The areas of the 583-keV and 2614-keV photopeaks and their corresponding Compton spectra were measured. The ratio of Compton area to photopeak area was 5.76 at 583 keV and 16.93 at 2614 keV. The ratio of these values is 2.93. The ratio of the quotients of relative Compton cross sections to photopeak efficiencies was also computed:

$$\frac{\phi (2614)}{\phi (583)} \times \frac{\text{Eff} (583)}{\text{Eff} (2614)} = 2.48,$$

where the efficiencies were taken from the calibrations given in Chapter II, Section C. The 15% difference between the two results, 2.93 and 2.48, which ideally should be equal, gives a rough estimate of the
accuracy of the calibration for these calculations. The product of the Compton spectrum–photopeak area ratios and the absolute photopeak efficiency from Figure 8 was $1.87 \times 10^{-4}$ at 583 keV and $1.01 \times 10^{-4}$ at 2614 keV. Dividing through by the values of $\frac{\phi}{\phi_0}$ from the Klein-Nishina formula, .4081 and .1883, respectively, gave corresponding factors of $4.59 \times 10^{-4}$ and $5.35 \times 10^{-4}$. These differ by the aforementioned 15%, so their average, $4.97 \times 10^{-4}$, was taken as the best estimate of the conversion ratio to divide into the $N$ obtained from Eq. 42c (where the $\sigma'(E)$ is the relative value $\frac{\phi}{\phi_0}$) to get the absolute number of $\gamma$ rays.

Another factor for which corrections were made was the number of prompt $\gamma$ rays not associated with $(\pi^-, \gamma X)$ processes. Non-prompt $\gamma$ rays were easily accounted for by subtracting off the contributions from the off-time spectra, but the correction for prompt background was more difficult. The approach taken was to subtract the prompt contribution of a $\gamma$ spectrum from a $\pi^-$ absorption experiment on a target from which no nuclear $\gamma$ rays would be produced. This would ideally be either hydrogen, helium, or lithium, because there are no bound excited states of any possible daughter nuclei from $\pi^-$ absorption on these elements. However, none of these targets was available, and the $\gamma$ spectrum from the $\pi^- + ^{12}$C run was used instead. $^{12}$C, being a light nucleus, has very few possible bound excited states of daughter nuclei from $\pi^-$ absorption, and three of these can be accounted for (see Section B). Figure 37 gives the dN/dE spectrum from the $\pi^- + ^{12}$C data, as calculated from Eq. 41 using steps of 100 channels. Integrating over this spectrum gives a total of $5.29 \times 10^{10}$ $\gamma$ rays, or $12.2 \gamma$ rays.
per $\pi^-$ stop. Limiting the summations to the energy ranges in the $^{32}\text{S}$ and $^{40}\text{Ca}$ spectra gives 5.1 and 8.0 prompt background $\gamma$ rays, respectively. But the contributions to the Compton spectrum from the four nuclear $\gamma$ rays seen in the $\pi^- + ^{12}\text{C}$ run must not be counted. These had a net yield of 17.5%, so the prompt background figures for $^{32}\text{S}$ and $^{40}\text{Ca}$ are reduced to 4.9 and 7.8 $\gamma$ rays per $\pi^-$ stop, respectively.

The $dN/dE$ spectra calculated from Eq. 41 for the $\pi^- + ^{32}\text{S}$ and $\pi^- + ^{40}\text{Ca}$ data are given in Figures 38 and 39, with $\gamma$-ray totals of $5.6 \times 10^{10}$ and $1.45 \times 10^{11}$, respectively. These gave 8.9 $\gamma$ rays per $\pi^-$ stop for $^{32}\text{S}$ and 12.3 $\gamma$ rays per $\pi^-$ stop for $^{40}\text{Ca}$. Subtraction of prompt background gave 4.0 nuclear $\gamma$ rays per $\pi^-$ stop in the $^{32}\text{S}$ case, and 4.4 in the $^{40}\text{Ca}$ case.

Because of the succession of approximations it is difficult to assign limits of accuracy to these figures with precision, but an estimate can be made by adding in quadrature the 15% error in determining Compton efficiency, the 20% estimated error in the absolute efficiency calibration, and another 15% error to account for inaccuracies in taking differences from the data to compute the $dN/dE$ spectra. A net error of $\pm$ 30% is obtained. Thus this Compton analysis indicates that between 3.0 and 5.4 nuclear $\gamma$ rays are emitted, on average, following each $\pi^-$ absorption interaction on $^{32}\text{S}$ or $^{40}\text{Ca}$.
IV. DISCUSSION OF RESULTS

In this chapter the results of these experiments will be compared with other experimental data and with theoretical models in order to extract information on the nature of pion-nucleus interactions and nuclear structure. Experimental comparisons include other $\pi^-$ absorption studies, pion-nucleus interactions near the $\Delta$ (1232) resonance, kaon absorption, proton and alpha particle reactions at intermediate energies, and photonuclear reactions in the same energy range.

The data analyzed in Chapter III, as well as other experimental results (especially the interaction of $\pi^+^{56}$ and $\pi^-^{57}$ with calcium near the $\Delta$ (1232) resonance), contain a wealth of detailed information, such as the presence of certain spin states and the absence of others, ratios of two-nucleon removal yields and cross sections, and strong excitation of even-even nuclei. These details will be considered in Section B, after consideration of the gross features of the spectra of daughter nuclei in terms of equilibrium distribution of the nuclear excitation energy and ensuing statistical evaporation in Section A.

A. Gross Features of Spectra

1. Statistical Evaporation

One of the most striking characteristics of the spectra of states of daughter nuclei from $\pi^-$ absorption on the medium-mass targets studied (Na, S, and Ca, particularly the latter two) is the wide range
in the mass numbers of the daughters. The absorption of a $\pi^-$ by a
given nucleus is observed to result in a spectrum of daughter nuclei
differing from the target by $\Delta A$ values as small as two and as great as
twenty. For example, $\pi^-$ absorption on $^{40}$Ca produces daughter nuclei
from $^{38}$Ar to $^{20}$Ne with comparable yields (see Table 18). Qualitatively
similar results have been obtained for the interaction of both $\pi^+$ and
$\pi^-$ with calcium near the $\Delta (1232)$ resonance. These are shown in histo-
gram form in Figures 40 and 41, respectively.

In order to gain understanding of the general nature of pion
absorption simple models were employed. Among these were (a) excitation
of the target nucleus with $Z' = Z - 1, N' = N + 1$, to $140$ MeV; (b) a
quasi-impulse approximation with two nucleons removed leaving an excita-
tion of the resulting daughter nucleus of about $30$ MeV; (c) a combination
of a and b in varying proportions; and (d) a combination of c plus an
admixture of single-nucleon removal with about $70$-MeV residual nuclear excitation. All of these were constructed from results obtained from a
statistical evaporation calculation which will now be described.

The nuclear evaporation code ALICE, devised by M. Blann and F.
Plasil, is based upon the standard Weisskopf-Ewing form of the stat-
istical theory of nuclear reactions, with the distribution of levels
in residual nuclei assumed to be described by a function

$$
\rho(E', J) = (2J + 1) \rho(E') ,
$$

in which $E'$ is the excitation energy of the residual nucleus and $J$ is
its angular momentum. The probability of particle emission is given by
\[ P_i(E^*, \varepsilon) \, d\varepsilon = \gamma_i \, \sigma_{\text{inv}} \, \varepsilon \, \frac{\rho(E')}{\rho(E^*)} \, d\varepsilon, \quad (4-2) \]

where \( P_i(E^*, \varepsilon) \, d\varepsilon \) represents the probability per unit time that a nucleus excited to energy \( E^* \) will emit particle \( i \) with an energy between \( \varepsilon \) and \( (\varepsilon + d\varepsilon) \).

\[ \gamma_i = \frac{g_i m_i}{\pi \hbar^2 \, \rho_i^3} \quad (4-3) \]

where \( g_i \) is the number of spin states of particle \( i \), and \( m_i \) is its reduced mass. \( \rho(E')/\rho(E^*) \) is the ratio of the energy level densities of the residual nucleus at energy \( E' \) and the initial nucleus at \( E^* \). The inverse cross section \( \sigma_{\text{inv}} \) is the cross section for capture of particle \( i \) with kinetic energy \( \varepsilon \) by the residual nucleus at excitation \( E' \) to form the initial nucleus with excitation energy \( E^* \).

The distribution of level densities in nuclei with excitation energy \( E \) is assumed to be

\[ \rho(E) = C (E - \delta)^{-2} \exp \left[ 2\sqrt{a(E - \delta)} \right], \quad (4-4) \]

where \( \delta \) is a displacement in the ground-state energy designed to account for odd-even effects.

Before Eq. 2 and 4 could be used for excitation-function calculations, values of \( \sigma_{\text{inv}}, a, \) and \( \delta \) had to be chosen from experimental data. Blann and Merkel\(^66\) took \( a = 7 \text{ MeV}^{-1} \), \( \delta = 0 \text{ MeV} \) for odd-odd nuclides, \( \delta = 1.4 \text{ MeV} \) for odd-even nuclei, and \( \delta = 2.8 \text{ MeV} \) for even-even
nuclei.\(^{68}\) The inverse cross sections were determined by assuming them equal to ground-state capture cross sections\(^{69}\)

\[
\sigma_{\text{inv}}^{\text{t}} (0, E) = \sigma_{\text{inv}}^{\text{t}} (E, E),
\]

and the \(\sigma_{\text{inv}}^{\text{t}} (0, E)\) were taken to be the optical-model total nonelastic cross sections, except for neutron nonelastic cross sections. Neutron optical-model parameters from elastic scattering correspond to fairly transparent nuclei. Since the nuclei considered in these calculations are highly excited they should be more opaque than ground-state nuclei. Thus the imaginary potential was arbitrarily deepened by about 15 MeV for low-energy neutrons.\(^{66-68}\)

A correction is necessary to account for excitation energy which is tied up as rotational kinetic energy and thus not available for generating intrinsic states.\(^{70,71}\) This can be done by decreasing the excitation energy by the average classical rotational energy

\[
\overline{E_{\text{rot}}} = \frac{\sum_{l=0}^{\infty} (2l+1) \frac{T_l(E) L(l+1)}{\ell^2/2\hbar^2} \partial R_{\text{rigid}}}{\sum_{l=0}^{\infty} (2l+1) T_l(E)},
\]

where the transmission coefficients \(T_l (E)\) were calculated from the optical-model potential and the rigid-body moment of inertia was calculated assuming \(R = 1.2 \ A^{1/3}\) fermis.

The input to the ALICE program included the charge, mass, and spin of the target nuclei and the incident particles, the excitation energy deposited in the nuclei, the binding energies for all emitted particles, and the optical-model parameters. The calculation proceeded in the following steps:
(a) The first particle to be emitted was taken to be a neutron. The probabilities for its emission were calculated for all possible kinetic energies in 0.5-MeV steps, with the requirement that the residual nucleus have at least 1.0-MeV excitation.

(b) A second neutron was selected for emission and, for the maximum excitation remaining after emission of the first neutron, the probabilities were calculated for emission of the second neutron at all possible kinetic energies. These were multiplied by the probability for producing the maximum excitation after the first neutron emission.

(c) Process b was repeated until all possible energies of the second particle had been calculated for all possible energies of the first particle, and the result stored.

(d) The second particle was changed to p, d, t, $^3$He, and $^4$He, and steps b and c repeated.

(e) The first particle was changed to p, d, t, $^3$He, and $^4$He, and steps b, c, and d repeated.

The output included the cross sections for production of daughter nuclei of given Z and A values as functions of the angular momentum of the compound nuclei, plus the total cross section for each (Z, A) combination summed over the angular momentum states.

The all-too-obvious first step in this analysis was to run the program with an energy deposition of 140 MeV. This was done for both $^{32}$S and $^{40}$Ca, and these results are shown in Figures 42 and 43, respectively. These obviously do not agree with the data in Figures 26 and 32. The spectra produced by the ALICE code did not include the relatively large yields observed experimentally for $\Delta A = 1, 2, 3,$ and $4.$
On the other hand, it produced larger yields than observed for large $\Delta A$ values, especially in the $^{32}$S case. Another difficulty, related to finer details of nuclear structure, was the discrepancy in yields for even-even versus odd-even and odd-odd nuclei between the data and the code results. The data show relatively large yields for even-even nuclei, especially for large $\Delta A$ values, while the code results show no special preference. The importance of this feature, which is discussed later in this section and in Section B, is minimized for the present in the ALICE analysis, and average yields over regions in which $\Delta A$ varied by four were used as criteria instead of the rapidly fluctuating yields from nucleus to nucleus.

Because $\pi^-$ absorption by nuclei is believed to occur predominantly on nucleon pairs the next process to be considered was removal of two nucleons accompanied by excitation of the residual nucleus. For comparison with the $^{32}$S data the ALICE code was run for $^{30}$P excited by 10, 15, 20, 25, 30, 35, 45, 55, and 65 MeV (Figure 44), and for $^{30}$Si excited by 10, 15, 20, 25, 30, 35, 45, 55, 65, and 75 MeV (Figure 45). The $^{40}$Ca data were compared with $^{38}$K excited by 15, 20, 25, and 30 MeV (Figure 46), and with $^{38}$Ar excited by 15, 20, 25, 30, 35, 40, 45, and 55 MeV (Figure 47). None of these taken singly or in combination give qualitative agreement with the experimental data. This suggests that absorption of a $\pi^-$ by a pair of nucleons followed by quasi-impulse-approximation emission of the same two nucleons is by itself not an adequate description of the process. A combination of two-nucleon absorption and statistical evaporation plus, possibly, absorption on alpha clusters, $^{72,73}$ nuclear Auger effects, $^{74}$ and intranuclear cascades $^{75}$ may
be required to accurately predict the yields of daughter nuclei. For some of these processes no satisfactory quantitative theories exist, and, in any event, attempts to sum contributions from all of them would involve a set of adjustable parameters approaching in number the data points (yields of different daughter nuclei).

A simple model, deliberately limited to statistical evaporation (uniform absorption of 140 MeV) and quasi-impulse-approximation processes (removal of two nucleons with minimal residual excitation), was constructed in order to determine if such a combination could give qualitative agreement with the data. Inspection of Figures 44 through 47 indicated that excitation energies of 30 MeV gave the best representation of the small-$\Delta A$ ends of the experimental spectra. It was noted that the probability for the reaction would not merely follow a delta function at 30 MeV, but would include a range of energies, so the net yields for the two-nucleon absorption process were summed from gaussian distributions 20-MeV wide centered on 30 MeV. It was also assumed that absorption occurs twice as often on np pairs as on pp pairs.\textsuperscript{76} The yields thus obtained were added in varying proportions to those produced assuming 140-MeV absorption. Those generated for $\pi^-$ absorption on $^{32}\text{S}$ from a 75\% weighting for two-nucleon absorption plus a 25\% weighting for 140-MeV statistical evaporation are shown in Figure 48, while Figure 49 gives the results for a 50-50 mixture. Analogous yields for $\pi^-$ absorption on $^{40}\text{Ca}$ are given in Figures 50 and 51. The 50-50 mixtures give much better fits to the data, and, of these, the fit is considerably better for $^{40}\text{Ca}$ than for $^{32}\text{S}$. This is not surprising because the ALICE code tends to give better results for heavier nuclei in which shell
effects are not as prominent. In summary, qualitative agreement with data is given by a model of \( \pi^- \) absorption including roughly equal components of statistical evaporation following absorption of the pion's rest mass by the nucleus as a whole, and approximately 30-MeV residual excitation of \((A-2)\) nuclei following removal of two nucleons by \( \pi^- \) absorption on the pair.

A similar model, suggested by R. G. Winter, considered three possible processes in pion absorption, the two already utilized plus ejection of one nucleon combined with reabsorption of the other, whose kinetic energy (roughly 70 MeV) is then shared by all the remaining nucleons. It was estimated through a crude geometrical analysis of this situation that, given such a model, reabsorption of one nucleon would occur in about 50% of the events, and that direct ejection of both nucleons and reabsorption of both would be approximately equally likely. Applying this to \(^{32}\)S, the spectra of daughter nuclei generated by the ALICE code for 140 MeV into \(^{32}\)S, 75 MeV into \(^{30}\)Si, and 30 MeV into \(^{30}\)P were combined in a 25-50-25 ratio to form the yield histogram of Figure 52. This spectrum, like all the ALICE histograms, contains an obvious anomaly at \( \Delta A = 0 \) because Blann and Plasil constructed the program to correspond to collisions of energetic particles (a 5-MeV kinetic energy was assumed for the \( \pi^- \)) rather than absorption at rest. The agreement with experimental data is about as good as that for the two-energy 50-50 mixture. The two-energy mixture gives better agreement for \( \Delta A = 1 \) and 3, but the three-energy mixture gives better agreement in the \( \Delta A = 10 \) region.
The same analysis was done for $^{40}$Ca by combining 140 MeV into $^{40}$Ca, 55 MeV into $^{38}$Ar, and 30 MeV into $^{36}$Kr in the same 25-50-25 ratio to form Figure 53. Here the agreement with experiment is generally not quite as good as for the two-energy model. It is much worse for $A = 37$, but slightly better for $A = 36$ and 32. The region around $\Delta A = 12$ is somewhat better described by the two-energy model. In both of these cases, $^{32}$S and $^{40}$Ca, an arbitrary reshuffling of the ratios would likely give better agreement for the three-energy mixture, but the arbitrary nature of such an adjustment would reduce its significance. In summary, qualitative agreement with experiment is seen for this model also, but on the whole not quite as good as for the two-energy model provided the ratio is varied to obtain the best fit.

As already mentioned, there is an obvious enhancement in the yields of the even-even daughter nuclei. This results from cascading of almost all higher levels through the first $2^+$ state. Detailed consideration is given to this in Section B, where it is determined through analysis of both theory and experiment, that the observed yields of odd-even and odd-odd daughter nuclei are approximately one-third what they would be if they were enhanced like the even-even nuclei. In order to account for this systematic difference, the yields of the most strongly-excited state of the odd-even and odd-odd daughter nuclei were multiplied by three and the resulting yield spectra were tabulated. These results are shown for $\pi^- + ^{32}$S in Figure 54 and for $\pi^- + ^{40}$Ca in Figure 55. This correction greatly improves the agreement with the results from the ALICE code where a mixture of 140-MeV absorption and quasi-impulse-approximation processes were assumed, especially in the case of $^{32}$S,
where the statistics were better. There is an obvious anomaly for the A = 37 yield from \(^{40}\text{Ca}\), however. This is suggestive of a direct interaction and is discussed in Section B, parts 2 and 5.

A further test of the validity of the statistical evaporation analysis is to compare experimental yields with computed yields as simultaneous functions of \(N\) and \(Z\) of the daughter nuclei. A strong divergence of the experimental and theoretical curves which follow the maximum yield for given values of \(A\) would indicate that statistical evaporation does not provide an adequate description of pion absorption.

The observed yields for \(\pi^- + ^{32}\text{S}\) as functions of \(N\) and \(Z\) are listed in Table 22, and the same yields corrected for enhancement are listed in Table 23. A similar list of the ALICE predictions (using the 50-50 mixture of \(^{140}\text{MeV}\) absorption and quasi-impulse-approximation processes) is given in Table 24. Since the ALICE results are to an arbitrary scale, the yields were normalized to the experimental yield of the \((Z - 2, N - 2)\) nucleus, and those less than 0.01\% were omitted from this table. Analogous lists for \(\pi^- + ^{40}\text{Ca}\) are given in Tables 25, 26, and 27. It is obvious that the general trend of the theory matches the experiment in both cases. Experiment and theory agree that maximum yields lie slightly to the neutron-rich side of the \(N = Z\) line for both \(^{32}\text{S}\) and \(^{40}\text{Ca}\), and, in this representation, the ALICE code appears to give an adequate description of \(\pi^-\) absorption.

2. Other Computer Simulations

Another approach to understanding the processes occurring in medium-energy nuclear reactions is the intranuclear cascade calculation. In this analysis the incoming particle is assumed to interact with one
nucleon at a time. This is reasonable at medium-to-high energies because in this region the projectile's wavelength is considerably smaller than both its mean free path and the mean internucleon distance. The nucleus is assumed to be a sphere of constant density with no consideration of the possible existence of cluster, or of shell effects. With these assumptions intranuclear cascade calculations are, like the ALICE code, more appropriate for medium-to-heavy nuclei than for light nuclei.

Intranuclear cascade calculations proceed by Monte Carlo methods. The position of impact of the incident particle is chosen at random, the depth of its first collision is selected, this selection being weighted by the mean free path, and the struck nucleon is chosen using the Z/N ratio. Its momentum is chosen from the Fermi gas momentum distribution, and the scattering angle is selected through weighting by the nucleon- (or pion-) nucleon differential cross section. If what results is permitted by the Pauli exclusion principle the process is allowed and the resulting particles are followed through all ensuing collisions until all the particles have left the nucleus. This entire procedure is repeated many times over until sufficient statistics have been obtained for meaningful comparison with experiment. 1000 cascades are typically required. A comparison of intranuclear cascade calculations with experiment for 380-MeV protons on $^{75}$As gives good results. A code written by H. W. Bertini was adapted by B. J. Lieb for application to interactions of pions with nuclei at the $\Delta (1232)$ resonance. Lieb obtained reasonably good agreement with experiment for $\pi^- + ^{27}$Al.

Harp et al. have developed an intranuclear cascade code for use with pion projectiles near the $\Delta (1232)$ resonance which explicitly includes isobar capture,
\[ \Delta + N_1 \rightarrow N_2 + N_3 \quad (4-7) \]

and isobar-nucleon exchange scattering,

\[ \Delta_1 + N_1 \rightarrow \Delta_2 + N_2 \quad (4-8) \]

They calculated mass yields as functions of \( \Delta A \) for \( 65\text{-MeV } \pi^\pm + ^{65}\text{Cu} \), obtaining agreement with radiochemical measurements\(^83\) within a factor of two for the cross sections for production of all observed daughter nuclei from \(^{62}\text{Zn} \) to \(^{45}\text{Ti} \). Zeev Fraenkel has calculated cross sections for production of daughter nuclei from \( 220\text{-MeV } \pi^- + ^{40}\text{Ca}, ^{32}\text{S}, ^{28}\text{Si}, ^{27}\text{Al}, \) and \(^{24}\text{Mg} \).\(^84\) His results, along with the published results\(^57\) of the experiments done at SREL, are shown in Table 28. However, there was an error in the efficiency calibrations of some of the experiments,\(^85\) and further inaccuracies in curve fitting of the photopeaks in the \(^{40}\text{Ca} \) data have been discovered.\(^86\) Thus this comparison should only be made after a careful reanalysis of these data. J. P. Schiffer has pointed out\(^87\) that data taken at the Los Alamos Meson Physics Facility do not agree with the Fraenkel calculations. However, the gross structure of the unnormalized spectra of daughter nuclei for both \( \pi^+ \) and \( \pi^- \) on \(^{40}\text{Ca} \) near the \( \Delta(1232) \) resonance taken at SREL,\(^56,57\) is similar to that for \( \pi^- \) absorption and suggests that calculations of the form of the intranuclear cascade codes may be useful in developing further understanding of the \( \pi^- \) absorption process.
3. Spallation Interpretation

Ullrich, Engelhardt, and Lewis have analyzed their spectra of prompt \( \gamma \) rays from \( \pi^- \) absorption on \( ^{16}_6O, ^{19}_9F, ^{31}_1P, \) and \( ^{40}_2Ca \) in terms of an empirical spallation formula developed by Rudstam:

\[
\sigma(z,A) \propto \exp \left[ p(A - r(z - sA)^2) \right], \tag{4-9}
\]

where \( p, r, \) and \( s \) are empirical parameters developed by Rudstam.

Ullrich et al. plotted the sum of the yields from each of the targets which corresponded to particular \( \Delta A \) values (with two even-even and two odd-even targets this apparently reduced the effect of apparent preferences for even-even daughter nuclei) as a function of \( \Delta A \) on semi-logarithmic graphs. \( p = .14 \), which corresponds to a projectile energy of 500 MeV, gave the best fit to the data. \( p = .43 \), which corresponds to 140 MeV, did not agree with their data. Ullrich et al. claim that the comparison indicates strong similarity between reactions resulting from \( \pi^- \) absorption at rest and spallation reactions, but that their data correspond to spallation data with 500-MeV projectile energy, which implies that the \( \pi^- \) absorption process, because of its unique nature, is much more effective at removing nucleons than are other reactions in the same energy range.

The yields of even-even daughter nuclei from the \( \pi^- \) absorption experiments on \( ^{32}_1S \) and \( ^{40}_2Ca \), as analyzed in Chapter III, are plotted semi-logarithmically as functions of \( \Delta A \) in Figure 56. Only the even-even yields were used because the \( 2^+ \) enhancement of these nuclei gives assurance that the total yields of these are observed. Lines corresponding to \( p = .43 \) (140 MeV) and \( p = .14 \) (500 MeV) are drawn in each. The
$^{32}\text{S}$ data give better agreement with the $p = .43$ line, while the $^{40}\text{Ca}$ data give better agreement with $p = .14$, although neither agreement is very good. This suggests that $\pi^-$ absorption on $^{32}\text{S}$ proceeds by direct interactions more frequently than is the case with $^{40}\text{Ca}$, where statistical evaporation is apparently more dominant. Discussion of this analysis in terms of nuclear structure details is given in Section B.

Garrett and Turkevich have fitted their analysis of 65-MeV $\pi^\pm + ^{65}\text{Cu}$ done by radiochemistry with Rudstam's formula. They found that their data fitted the formula almost as well as data taken with proton and alpha-particle projectiles. 65-MeV pions were found to correspond in average energy deposition to protons of about 300 MeV. Significantly, Garrett and Turkevich had to ignore their results for small $\Delta A$ to get acceptable agreement with the spallation formula. This disagreement, and that for small $\Delta A$ in the results for $\pi^-$ absorption in this work, suggest that direct reactions predominate in the small-$\Delta A$ cases.

Lewis has suggested further that, in light of their agreement with Rudstam's spallation formula and the fact that the daughter nuclei observed from $\pi^-$ absorption fall near the maximum stability curve, $\pi^-$ absorption can be explained simply as spallation. This argument is weakened, however, since experiments which observe de-excitation $\gamma$ rays are inherently biased toward nuclei near the maximum stability curve because those nuclei possess many more bound excited states. Further, although some of the gross features of the spectra are explained by the spallation interpretation, processes much like those observed in direct nuclear reactions are clearly evident in the detailed structure discussed in Section B.
4. Interpretation of Spectrum-Stripping Analyses

For the cases of $\pi^-$ absorption on $^{32}$S and $^{40}$Ca a simplified spectrum stripping was performed on the overlapping Compton continua which constitute the gross forms of the $\gamma$-ray spectra (Chapter III, Section E). Although this analysis is admittedly crude, it did imply that roughly three to five nuclear $\gamma$ rays are emitted per $\pi^-$ absorption on these targets. Since the sums of observed raw yields for these experiments are 41% ($^{32}$S) and 32% ($^{40}$Ca), about a factor of ten smaller, either many (presumably small or very broad) photopeaks were not observed, or the Compton analysis calibration was grossly incorrect. The latter is quite unlikely, however, because the Compton calculation is very tightly coupled to the photopeak calibration and, even if the latter were incorrect, the ratio between Compton and photopeak yields would remain the same. Thus the observed photopeaks apparently represent a relatively small fraction of the total number of nuclear $\gamma$ rays. Some support for this interpretation can be drawn from the work of Hornyak et al., who observed nuclear $\gamma$ rays following the interaction of $^{140}$MeV alpha particles with $^{27}$Al and a number of isobars of Sm. From the $^{27}$Al experiment they identified 93 distinct $\gamma$ lines, corresponding to 2/3 of the geometric cross section (780 mb). Because of the intensity of the alpha beam and its spatial and momentum resolution they were able to accumulate spectra with photopeak areas 100 times greater than those obtained for analogous transitions in pion reactions with light- and medium-mass nuclei. Thus many weak transitions which are below the threshold of observation in the pion experiments could be seen in the alpha-particle reactions. The implication is that many $\gamma$ transitions are
present in the $\pi^-$ absorption reactions which are below the threshold of observation. These could possibly, among other things, fill in the yield spectra for odd-even and odd-odd daughter nuclei which, unlike even-even nuclei, do not cascade predominantly through the first excited state. Furthermore, since at least three $\gamma$ rays are produced per $\pi^-$ stop, the nuclei must be highly excited in order to, on average, cascade through two or more excited states in the de-excitation process.

In even-even nuclei cascades through the $2^+$ states predominate and give a reliable measure of the total excitation of those particular nuclei. Since even-even nuclei make up 25% of all possible combinations of nucleons, multiplication of these yields by four might give an estimate of the true production of bound excited states of daughter nuclei. For $\pi^- + ^{32}\text{S}$ this gives .85 such events per $\pi^-$ stop, for $\pi^- + ^{40}\text{Ca}$, .77 events per $\pi^-$ stop. Since the total yield of daughter nuclei must sum to 100%, since absorption reactions going directly to ground states of daughter nuclei should constitute only a small fraction of events, and since radiative pion capture gives yields of no more than a few percent, the sums of even-even $2^+$ yields multiplied by four do give values consistent with expected total yields. The remaining factor of four is consistent with previous experiments because excited nuclei have been found, on average, to emit from two to fourteen de-excitation rays, depending on the mode of excitation and the target nucleus. For example, Muelhauser observed from two to six $\gamma$ rays emitted in $(n,\gamma)$ reactions on nuclei ranging from Na to Hf. Grover and Gilat found averages of seven to fourteen $\gamma$ rays emitted following heavy-ion
interactions on several heavy nuclei near 100 MeV. Heavy-ion reactions, of course, tend to produce excited nuclei in high-spin states, thereby increasing the number of successive de-excitations necessary to reach the ground state. In light of this the observation of three to five \( \gamma \) rays produced per \( \pi^- \) absorption is most reasonable and consistent with the interpretation that odd-even and odd-odd daughter nuclei are produced with yields roughly equal to those for even-even nuclei.

If this description is correct it would tend to strengthen the argument that the production of daughter nuclei far removed in mass from the parent results from statistical evaporation and not from nuclear structure effects, such as alpha clustering. This is not the only interpretation, however, and a somewhat contradictory approach to the Compton analysis result, which does involve detailed features of nuclear morphology, will be considered in Section B.

The basic information which can be gleaned from this examination of the gross features of the \( \Delta A \) spectra is that successful models of the absorption processes require a wide range of excitation energies. This in turn suggests that a number of competing reactions may be required to fully describe pion absorption by complex nuclei. It may be possible to unravel some of these by studying fine details of the experimental spectra, which are considered in Section B.

B. Detailed Features of Spectra

1. The \( 0^+ \), \( 1^- \) Selection Rule

One of the most striking features of the \( \pi^- \) absorption spectra is the absence of excitation of \( J^P, T = 0^+, 1^- \) states in all cases.
attempted so far. Specifically, in the $\pi^- + ^{12}$C experiment the $0^+, 1$ state at 1740 keV in $^{10}$B has a net corrected yield of only $(14 \pm 12)\%$, compared with $(9.46 \pm .31)\%$ and $(4.23 \pm .23)\%$ for the other observed states in $^{10}$B. This small excitation is well within the error bounds for feeding from higher states. In the $\pi^- + ^{32}$S experiment the $0^+, 1$ state at 677 keV in $^{30}$P is not seen, although three higher levels of $^{30}$P are observed. Unfortunately no such determination could be made in the $\pi^- + ^{40}$Ca experiment because the $0^+, 1$ state of $^{38}$K is at 131 keV, below the lower threshold for $\gamma$-ray observation in that run.

An extremely small excitation of the $0^+, 1$ 2311-keV level of $^{14}$N was observed by Kossler et al. for $\pi^- + ^{16}$O. They explained the phenomenon as follows. The pion-nucleus interaction Hamiltonian is taken to be:

$$H \propto \sum_{i=1}^{A} \tau_{-i} \vec{\sigma}_{i} \cdot (\vec{V}_{\pi} - \vec{V}_{i}).$$

Pion absorption on light- and medium-mass nuclei occurs predominantly from $\ell = 1$ orbitals and thus the gradient resulting from the $\vec{V}_{\pi}$ term is finite at the origin, while the $\vec{V}_{i}$ gradient terms are multiplied by the pion wave function, which is zero at the origin and small throughout the nucleus. Thus the interaction Hamiltonian may be rewritten as

$$H \propto \sum_{i=1}^{A} \tau_{-i} \vec{\sigma}_{i} \cdot \vec{V}_{i} G_{\pi} \vec{f}_{i},$$

where $G_{\pi}$ is the pion wave function.
with it now operating only on nucleon coordinates. For a given m value of the pion wave function the matrix element to be evaluated reduces to

$$\langle n_n, \text{core} | \tau_- \sigma_z | p_n, \text{core} \rangle \propto \int d \tau \psi_{\text{core}}(0^+, 1) \psi_{pn}(L_f; S_f, T=1) \sigma_z \gamma_- \otimes \psi_{\text{core}}(0^+, 1) \psi_{pn}(L_i = 0; 0^+, 1).$$

(4-12)

The wave function $\psi_{pn}$ for the interacting pair must have spins anti-parallel in order to be in a $0^+$ state. $\tau^-$ converts the proton into a neutron. The $\sigma_z$ operator requires the two final-state neutrons to have parallel spins: $S_f = 1$. There is no angular part to the interaction so $L_f = L_i = 0$. However, an $S$ state of two neutrons with parallel spins is forbidden by the Pauli principle. Thus $0^+, 1$ states should not be generated by direct-reaction pion absorption, which agrees with the results of the $\pi^-$ absorption experiments on $^{16}$O, $^{12}$C, and $^{32}$S.

2. High-Spin States

The spectra of states of daughter nuclei also display some preference for excitation of high-spin states in certain cases. This seems to be most prominent in the $^{23}$Na results. Here the $9/2^+$ level at 2866 keV of $^{21}$Ne is fairly strongly excited, $(3.20 \pm .36)_E$, while the $5/2^+$ and $7/2^+$ levels below it are not seen. A search was made to determine if this might be the result of a cascade from an even higher $J$ state, but no photopeaks corresponding to such states could be found. The $11/2^+$ yrast level at 4432 keV was not excited to the threshold of observation. Other prominent excitations of relatively high spin states include the $5/2^-$ level of $^{19}$F, which was excited almost five times more strongly
than the $3/2^+$ and $3/2^-$ levels. Here, however, the $5/2^-$ level is lower than the other two and feeding from unobserved higher levels could have contributed to the $5/2^-$ yield. A reverse preference is seen for $^{20}$Ne, where the $2^+$ level at 1633 keV is much more strongly excited than the $4^+$ level at 4245 keV. Of course the preference of any unobserved higher states for cascading through the $2^+$ level could easily account for this observation.

No decisive preference for high-$J$ states was observed in the $\gamma^- + 32\text{S}$ results. To the contrary, the observed levels of $^{29}\text{Si}$ display a preference for the $3/2^+$ state over the $5/2^+$ and $7/2^-$ states, although the $7/2^-$ state is excited about 60% more strongly than the $5/2^+$ state. The $3/2^+$ state is the first excited state and it could contain some contribution from higher unobserved levels. Also, it is close in energy to the first excited state of $^{31}\text{P}$ (1266 keV) and the second excited state of $^{30}\text{Si}$ (4398 keV - 2235 keV = 1263 keV), and, although the peak was found to be precisely at 1273 keV and one-particle emission is expected to be strongly suppressed, there could be contributions from these other two levels. The relatively strong excitation of the $7/2^-$ state of $^{29}\text{Si}$ is interesting because (1) it is analogous to the production of the $7/2^-$ state of $^{37}\text{Ar}$ from $^{40}\text{Ca}$, both of these involving removal of two protons and one neutron, and (2) because it apparently involves transfer of a neutron from the $sd$ shell to the $f_{7/2}^-$ shell. Calculations of enhancement factors due to feeding were made for $^{29}\text{Si}$, assuming an initial equal population of all states of the nucleus and using known branching ratios$^{98}$ to calculate the relative probability for each possible transition. This gave a 12.1% probability to the $7/2^-$
state, 40.7% to the $5/2^+$ state, and 32.0% to the $3/2^+$ state. The observed yields are not in proportion with these calculations, thereby suggesting that direct nuclear reactions are involved.

As already mentioned, the $7/2^-$ level of $^{37}$Ar is observed 7.2 times more strongly than the $1/2^+$ level following $\pi^-$ absorption on $^{40}$Ca. Here the enhancement calculation was done with two different assumptions. It was done once assuming equal population of all levels. This gave a ratio of

$$
\left( \frac{7/2^- \rightarrow 3/2^+}{1/2^- \rightarrow 3/2^+} \right) = \frac{1}{16}.
$$

The calculation was repeated assuming population of states proportional to $(2J + 1)$. This gave a ratio of

$$
\left( \frac{7/2^- \rightarrow 3/2^+}{1/2^- \rightarrow 3/2^+} \right) = 3.71,
$$

which, although better than the calculation without the $(2J + 1)$ factor, is still a factor of two smaller than the observed ratio. It is also most interesting to note that an almost identical ratio, 7.3, was observed for the interaction of 190-MeV $\pi^+$ with $^{40}$Ca.$^{56}$ Excitation of this level involves transfer of one neutron to the $f_{7/2}^-$ shell in conjunction with removal of two protons and one neutron. The mechanism responsible for this effect is not obvious, but one possibility is capture of the pion on a surface alpha cluster with one of the three resulting neutrons being recaptured by the nucleus. The eight configurations available in the $f_{7/2}^-$ shell would make production of the $7/2^-$ state relatively likely. Further discussion of possible clustering phenomena will be given later in this section.
3. Single-Nucleon Removal

Single-nucleon removal is, as expected, strongly inhibited. This is because conservation of energy and momentum would require the absorbing proton to have a momentum of at least 500 MeV/c, which is far above the average Fermi momentum of bound nucleons. Therefore sizeable yields of $\Delta A = 1$ nuclei should have to result from such effects as recapture of one of the nucleons composing the absorbing group. In these experiments no states corresponding to single-nucleon emission were observed from $^{12}\text{C}$ or $^{1\text{h}}\text{N}$. In the case of $^{32}\text{S}$ the 752-keV $1/2^+$ state of $^{31}\text{Si}$ is seen with a very small yield, $(0.22 \pm 0.10)\%$. Production of this state would involve knockout of one proton along with absorption of the pion's charge by another proton. An alternate and much simpler explanation is that the $^{31}\text{Si}$ is produced by $\pi^-$ absorption on $^{33}\text{S}$ and/or $^{34}\text{S}$, which together constitute 5% of natural sulfur. In fact, multiplying the observed yield by 20 gives 4.4%, which is a most reasonable yield for two- or three-nucleon removal.

Significant yields of $\Delta A = 1$ nuclei appear to be present in the $^{23}\text{Na}$ and $^{40}\text{Ca}$ spectra. In the case of $^{23}\text{Na}$ the first $2^+$ and $4^+$ levels of $^{22}\text{Ne}$ are seen with yields of $(1.02 \pm 0.64)\%$ and $(1.45 \pm 0.60)\%$, respectively. Sodium is monoisotopic, so these must result from one-nucleon emission. It is interesting to note that it is the even-even neighbor, $^{22}\text{Ne}$, rather than the odd-odd $^{22}\text{F}$ and/or $^{22}\text{Na}$, which is observed, and that it involves proton removal. Thus no mechanism of charge exchange between nucleons is involved, and the absorption is on the odd particle. Also, the higher-spin state is slightly more excited, but not beyond the range of the error bars. It may be that single-nucleon
emission, although suppressed, is observed in this case because of $2^+$ enhancement. If so, the yields of the $2^+$ (and $4^+$) states represent the entire production of $^{22}\text{Ne}$. This would be impossible for an even-mass target.

The apparent single-nucleon emission from $^{40}\text{Ca}$ is most difficult to explain. Peaks corresponding to the $3/2^-$ 1267-keV and $3/2^+$ 1517-keV states of $^{39}\text{Ar}$ are seen with respectable yields, $(2.74 \pm 0.38)$% and $(1.38 \pm 0.26)$%, respectively. States of $^{39}\text{K}$ are not seen, however. If $\Delta A = 1$ processes do occur with $^{40}\text{Ca}$, $^{39}\text{K}$ is the expected daughter nucleus because it corresponds to simple proton knockout. Production of $^{39}\text{Ar}$ requires knockout of one proton plus absorption of the pion's charge by a second proton which remains in the nucleus, the same kind of process necessary to produce $^{31}\text{Si}$ from $^{32}\text{S}$. In the case of calcium, however, the relatively large yields preclude assignment of $^{39}\text{Ar}$ production to $\pi^-$ absorption on $^{42}\text{Ca}$, $^{43}\text{Ca}$, $^{44}\text{Ca}$, $^{46}\text{Ca}$, and $^{48}\text{Ca}$, which collectively constitute only 3% of natural calcium. The most plausible explanation of the apparent presence of the $3/2^-$ level of $^{39}\text{Ar}$ is ambiguity with photopeaks of about the same energy from other daughter nuclei. A large peak $(45 \pm 3$ mb) is, in fact, seen at 1267 keV for 190-MeV $\pi^+$ on $^{40}\text{Ca}$, which, because of the charge conservation requirement, cannot produce $^{39}\text{Ar}$.56 This peak was ascribed to some combination of the first $3/2^+$ state of $^{31}\text{P}$ (1266 keV), the second $2^+$ state of $^{30}\text{Si}$ (3498 keV - 2235 keV = 1263 keV), and/or the first $3/2^+$ state of $^{29}\text{Si}$ (1273 keV). The spread in energies of these levels could account for some or all of the apparent Doppler broadening of the 1267-keV peak.
If this peak is due to these other possible daughter nuclei there remains the question of the $3/2^+$ level of $^{39}\text{Ar}$ at 1517 keV. No ambiguity in this assignment could be found. However, the stated yield was extrapolated from the raw yield for one branch which has a 45% branching ratio. The other branch, which goes to the 1267-keV level, could not be seen because its $\gamma$ ray is below the lower-energy threshold of this experiment. The observed raw yield of $(0.62 \pm 0.16)\%$ could conceivably be due to some obscure transition which was not identified. Thus the $\Delta A = 1$ yield from $^{40}\text{Ca}$ is uncertain and difficult to account for with complete consistency.

It has been claimed by Spector that because of nuclear distortion of pion waves single-nucleon emission is not substantially suppressed. He calculated a ratio of one-nucleon emission to two-nucleon emission from $\pi^{-}$ absorption on $^{16}\text{O}$ of 0.23. From the experimental results of Kossler et al. a ratio of 0.06 can be calculated. The five experiments described in this work give ratios of 0.0 ($^{12}\text{C}$), 0.0 ($^{14}\text{N}$), 0.30 ($^{23}\text{Na}$), 0.00 - 0.02 ($^{32}\text{S}$), and 0.00 - 1.10 ($^{40}\text{Ca}$). The average over the five targets thus lies somewhere between 0.06 and 0.28, with a value near the lower end of the range being more likely. If the 1267-keV line in the calcium data is not from $^{39}\text{Ar}$, these results clearly do not agree with Spector's prediction. If the $^{39}\text{Ar}$ is present, then the anomalously large ratio for single-nucleon emission from $^{40}\text{Ca}$ would have to be attributed to some unknown factor apparently unique to $\pi^{-}$ absorption on $^{40}\text{Ca}$. Ullrich et al. do not report any excitation of $^{39}\text{Ar}$ in their $\pi^{-}$ absorption experiment on $^{40}\text{Ca}$. 
4. Two-Nucleon Removal

The suggestion that the study of reactions in which pion absorption results in the emission of two nucleons should yield information about nuclear structure is not new. In fact, many experiments, some of which will be discussed later in this section, have been performed in which two-nucleon emission has been studied. The analysis of these reactions through study of the spectra of de-excitation γ rays, especially determination of the momentum spectra of nucleon pairs from measurements of Doppler broadening, however, is a relatively recent development.

a. π⁻ + ¹²C

In the cases of two of the targets studied, ¹²C and ¹⁴N, the only daughter states observed correspond to two-nucleon emission, and in both cases np pairs are removed. This is not to imply, however, that emission of two nucleons is the dominant reaction mode. In the case of ¹²C the ¹⁰B yields add up to only (13.83 ± .04)%.

This leaves about 86% of the π⁻ stops to be accounted for. Significantly, perhaps, ⁸Be, which corresponds to equivalent alpha-particle removal, is not bound and its production cannot be observed through this technique. Also, because very light nuclei have relatively few bound excited states, ground states collectively constitute a considerably greater fraction of the available reaction channels for the ¹²C and ¹⁴N targets than in the cases of medium-mass nuclei such as ³²S and ⁴⁰Ca.

The three states of ¹⁰B observed from π⁻ absorption on ¹²C all have lifetimes which are much too long for their photopeaks to
display any Doppler broadening. This is unfortunate because determination of pair-momentum spectra from this reaction will require explicit angular correlation measurements of the outgoing neutrons in conjunction with γ-ray detection.

b. Recoil Momentum of $^{12}\text{C}^*$ from $\pi^- + ^{14}\text{N}$

The production of the Doppler-broadened 4439-keV level of $^{12}\text{C}$ from $\pi^-$ absorption on $^{14}\text{N}$ is one of the most interesting results from these experiments because p-shell structure is sufficiently understood to permit fairly believable calculation of the pair-momentum distribution, which can be compared with experiment.

The reaction rate for production of $^{12}\text{C}$ with recoil momentum between $K$ and $(K + dK)$ is

$$
\frac{dN}{dK} \propto \left| \left< \alpha | H | i \right> \right|^2 q K^2 \, d\Omega_q \, d\Omega_K \, ,
$$

(4-13)

where the interaction of Eq. (4-11) will be used for the Hamiltonian, and $q K^2 \, d\Omega_q \, d\Omega_K$ is the phase space factor. The $^{14}\text{N} \, 1^+$ wave function is written as a product of the $^{12}\text{C} \, 2^+$ wave function and the appropriate sum of two-particle terms:

$$
\left| i \right> = \left| ^{14}\text{N} \right> = \sum_{\alpha J_c \beta J_p} \{ \psi_{12\text{C}}(\alpha J_c) \, \psi_{\text{p}}(\beta J_p) \} \, \phi_{12\text{C} \, \alpha J_c \beta J_p} \, ,
$$

(4-14)

where $J_c$ is the angular momentum of the $^{12}\text{C}$, $\alpha$ refers to the other quantum numbers necessary to specify its wave function, $J_p$ is the angular momentum of the nucleon pair, $\beta$ refers to the other quantum numbers of the pair wave function, and the $\phi_{12\text{C} \, \alpha J_c \beta J_p}$ are the expansion coefficients. The final state is given by
\[ |f\rangle = \psi_{l_2 c}(1, 2^+) e^{i \hat{q} \cdot \hat{R}} e^{i \hat{R} \cdot \hat{R}_{i-12}} \chi(s_1 s_2) \]  

(4.15)

where \((1, 2^+)\) refers to the first \(2^+\) state, where \(q\) is the relative momentum of the outgoing nucleons, and \(\chi(s_1 s_2)\) is their spin function with the two spins coupled to zero. Putting these together, averaging over initial states, and summing over final states gives

\[
\frac{dN}{dK} \propto q K^2 \sum_{m_i m_f} \left| \sum_{\alpha J_c \beta J_p} C(\alpha J_c \beta J_p) \right| \psi_{l_2 c}^\ast(1, 2^+)_{m_f} \otimes e^{-i(\hat{q} \cdot \hat{R} + \hat{R} \cdot \hat{R}_{i-12})} \chi(s_1 s_2) \otimes \sigma_{m_m} \tau \otimes \left\{ \psi_{l_2 c}(\alpha J_c) \psi_{\beta J_p}(\beta J_p) \right\}^{1+} d\hat{R}_{i-12} d^3 \tau d^3 R \bigg|_m \frac{d \Omega_q}{d \Omega_K}.
\]  

(4.16)

The integral over the spatial coordinates generally involves a Jacobian, but for the purposes of this calculation it simply becomes a proportionality factor. Doing the integral over the core and then the appropriate sum gives:

\[
\sum_{\alpha J_c} C(\alpha J_c \beta J_p) \psi_{l_2 c}^\ast(1, 2) \psi_{l_2 c}(\alpha J_c) d\hat{R}_{i-12} = \sum_{\alpha J_c} C(\alpha J_c \beta J_p) \delta_{l_1, \alpha} \delta_{l_2, J_c} = C(\alpha J_c \beta J_p).
\]  

(4.17)

Writing out the coupling coefficients for the two-particle wave function now gives:
The next step is to write out the two-particle wave function in terms of the total orbital angular momentum \( \Lambda \) and its projection \( M_\Lambda \), along with its spin function. This gives terms of the form:

\[
\psi_p(\beta \mathbf{J}_p) \Rightarrow \psi_p(\eta \Lambda \mathbf{J}_p) = C_{\Lambda}^{\Lambda \mathbf{J}_p} \psi(\eta \Lambda M_\Lambda) \chi_{m_s}^{\mathbf{J}_p}
\]

(4-19)

Because \( \beta \) includes \( \Lambda \) among its quantum numbers, and \( \Lambda \) is now written explicitly, the other quantum numbers of the pair will now be collectively labelled \( \eta \). These turn out to be the principal and orbital-angular-momentum quantum numbers for the relative \( (nL) \) and center-of-mass \( (NL) \) parts of the pair's wave function. Also, to be consistent with the \( \Lambda \eta \) notation, the expansion coefficients will henceforth be written as \( C(\mathbf{J}_p \Lambda \eta) \).

Evaluating the spin bracket gives

\[
\left\langle \chi_0^{\mathbf{J}_p \mathbf{K}} \sigma m_\pi \mid \chi_{m_s}^{\mathbf{J}_p} \right\rangle = \delta_{m_s, m_\pi}
\]

(4-20)

A typical term is now
\[
\int e^{-i(\hat{q} \cdot \hat{r} + \hat{k} \cdot \hat{R})} \psi(\Lambda M_\Lambda) C(J_P \Lambda \eta) C_{m_f}^{2} J_P \frac{1}{m_i-m_f} m_i \otimes C_{m_f}^{\Lambda} m_i-m_f d^3r d^3R.
\]

Doing the integral, which yields the Fourier transform of the initial-state wave function, and reintroducing the summation yields

\[
\frac{dN}{dK} \propto g K^2 \sum_{m_f m_i m_f} \sum_{\Lambda M_\Lambda} \psi(q, K)_{M_\Lambda}^{\Lambda} C(J_P \Lambda \eta) \otimes C_{m_f}^{2} J_P \frac{1}{m_i-m_f} m_i C_{m_f}^{\Lambda} m_i-m_f m_f d \Omega q d \Omega K,
\]

where the substitution \( M_\Lambda = m_i - m_f - m_f \) has been made in the last Clebsch-Gordan coefficient.

Doing the integrals over \( d \Omega q \) and \( d \Omega K \) will eliminate cross terms in \( \Lambda, \ell, \) and \( L \). The sum over \( m_f \) with fixed \( \Lambda \) fixes \( J_P \), and the sums of the squares of Clebsch-Gordan coefficients give constants which factor out. The only sum inside the bracket involves terms with different values of \( n \) and \( N \), but with the same values for \( \ell \) and \( L \) (because of the orthogonality properties of the angular integrals). The reaction rate thus becomes

\[
\frac{dN}{dK} \propto g K^2 \sum_{J_P \Lambda} \left| \sum_{\eta} C(J_P \Lambda \eta) R_{J_P \Lambda \eta}(q, K) \right|^2,
\]

where \( R_{J_P \Lambda \eta}(q, K) \) is the appropriate radial part of the shell-model wave function, and the \( \eta \) sum is over the \( (n \ell, NL) \) values, but with the restriction that the terms of the sum have different values of \( n \) and \( N \), but the same values of \( \ell \) and \( L \).
The coefficients and radial wave functions must still be written explicitly. In a $j$-$j$ coupling scheme Cohen and Kurath obtained the two-particle wave function:

\[
\left\{ a_1 \left| \frac{3}{2} \frac{3}{2} \frac{3}{2} \right> + a_2 \left| \frac{3}{2} \frac{1}{2} \frac{1}{2} \right> + a_3 \left| \frac{3}{2} \frac{3}{2} \frac{1}{2} \right> \\
+ a_4 \left| \frac{3}{2} \frac{1}{2} \frac{1}{2} \right> + a_5 \left| \frac{1}{2} \frac{1}{2} \frac{1}{2} \right> \right\}
\]

where the notation is $|j_1 j_2 j\rangle$, and $a_1 = -0.0016$, $a_2 = -0.1033$, $a_3 = -0.0471$, $a_4 = 0.1242$, and $a_5 = -0.0201$. Before this function can be broken up into relative and center-of-mass coordinates it must be transformed from $j$-$j$ coupling to $L$-$S$ coupling via the relation:

\[
\begin{aligned}
\left| l_1 s_1 \frac{1}{2} l_2 s_2 \frac{1}{2} j_1 \frac{3}{2} \frac{3}{2} \frac{3}{2} j_2 \frac{1}{2} \frac{1}{2} \frac{1}{2} J M \right> &= \sum_{l S} \sqrt{(2j_1+1)(2j_2+1)(2L+1)(2S+1)} \otimes \\
\left\{ l_1, l_2, L, s_1, s_2, S \right\} \left| l_1, l_2, L, s_1, s_2, S, J, M \right>
\end{aligned}
\]  

After evaluating sixteen 9-$j$ symbols through use of relations in Edmonds and Rotenberg et al., the terms reduce to

\[
\left\{ -0.0016 |112, 13\rangle + 0.0084 |111, 12\rangle - 0.0596 |112, 02\rangle \\
- 0.0730 |112, 12\rangle - 0.2276 |110, 11\rangle \\
- 0.0032 |111, 01\rangle - 0.0423 |112, 11\rangle + 0.0945 |111, 11\rangle \right\}
\]

where the ket notation is $|l_1 l_2 L, S J\rangle$.

The next step was to transform the wave functions into products of relative and center-of-mass functions represented by the kets $|n l, N L, ASJ\rangle$, where $n$ and $l$ are the principal and orbital-angular-momentum quantum numbers for the relative part of the two-particle wave function, and $N$ and $L$ are the analogous quantum numbers for the center-of-mass part.
The Moshinsky transformation was performed and the following normalized function resulted:

\[
\left\{ -0.6022 \left| 00,10,011 \right> + 0.6022 \left| 10,00,011 \right> + 0.3537 \left| 01,01,111 \right> \\
-0.1931 \left| 00,02,212 \right> + 0.1931 \left| 02,00,212 \right> - 0.1576 \left| 00,02,202 \right> \\
+ 0.1576 \left| 02,00,202 \right> - 0.1119 \left| 00,02,211 \right> \\
+ 0.1119 \left| 02,00,211 \right> + 0.0314 \left| 01,01,112 \right> - 0.0120 \left| 01,01,101 \right> \\
- 0.0041 \left| 00,02,213 \right> + 0.0041 \left| 02,00,213 \right> \right\}
\]

\( (4-27) \)

The pion-absorption interaction involves that portion of the nuclear wave function which has short-range correlations associated with it. The relative S-wave gives the required strong overlap. Because of this requirement and the further fact that \( l = 0, S = 1 \) terms dominate the results of the Moshinsky transformation, all other terms were dropped, leaving

\[
\left\{ -0.6022 \left| 00,10,011 \right> + 0.6022 \left| 10,00,011 \right> - 0.1931 \left| 00,02,212 \right> \\
- 0.1119 \left| 00,02,211 \right> - 0.0041 \left| 00,02,213 \right> \right\}
\]

\( (4-28) \)

Apply Eq. \((4-23)\) to obtain the rate:

\[
\frac{dN}{dk} \propto \left\{ \left| -0.6022 \left| 00,10,011 \right> + 0.6022 \left| 10,00,011 \right> \right|^2 \\
+ \left| -0.1931 \left| 00,02,212 \right> \right|^2 \\
+ \left| 0.1119 \left| 00,02,211 \right> \right|^2 + \left| 0.0041 \left| 00,02,213 \right> \right|^2 \right\} q^2 k^2
\]

\( (4-29) \)
The last term is clearly negligible and may be dropped. Using the appropriate radial functions and normalizing once more gives

\[
\frac{dN}{dK} \propto q^2 K^2 \left\{ 0.79 \left[ R_{oo}(\sqrt{2} q) R_{10}(\frac{K}{\sqrt{2} q}) - R_{10}(\sqrt{2} q) R_{oo}(\frac{K}{\sqrt{2} q}) \right]^2 + 0.18 \right\} .
\]

Because the relative momentum of the two outgoing nucleons is so great, about 370 MeV/c, the first and last terms are negligible, and the relation reduces to that obtained by Kossler et al.\textsuperscript{39} for \( \pi^- + {}^{16}_0 \text{N} \), provided the function is scaled to the rms radius of \( ^{14}_0 \text{N} \), which is 2.48 fermis.\textsuperscript{106}

The necessary radial harmonic oscillator wave function is\textsuperscript{107}

\[
R_{oo} \propto e^{-\alpha^2 r^2/2},
\]

where

\[
\alpha = \sqrt{\frac{m \omega}{\hbar}},
\]

which can be computed from\textsuperscript{107}

\[
\langle r^2 \rangle_{nl} = \alpha^{-2} \left[ 2(n-1) + l + 3/2 \right].
\]

With \( \langle r^2 \rangle^{1/2} = 2.48 \) fermis,

\[
\alpha = 0.6376 \text{ fm}^{-1} = 125.6 \text{ MeV/c},
\]

which provides the scaling factor for the recoil momentum. Thus the momentum distribution may be written as

\[
\frac{dN}{dK} \propto K^2 e^{-K^2/2}
\]
in these units. The related distribution

\[ \frac{d^3N}{dK^3} = \frac{1}{K^2} \frac{dN}{dK} \propto e^{-K^2/2} \]  \hspace{1cm} (4-36)\]

These relations predict a mean recoil momentum of 148 MeV/c, compared with the experimental result of (169 ± 6) MeV/c, a difference of 12%, which is very good agreement in light of the various approximations made in these calculations. The calculated forms of dN/dK and d³N/dK³, along with the experimental results, are plotted in Figs. 57 and 58, respectively.

The preceding calculation was done without introducing any correlation function. The same form would be obtained by assuming a cluster model for the pair of absorbing nucleons, thereby taking an n = 0, L = 0 wave function for its center-of-mass motion. This result is similar to that of Kossler et al., who measured a mean recoil momentum of 145 MeV/c for π⁻ + 16O. Their analogous calculation using the same requirement on the pair (np cluster with n = 0, L = 0) predicted a mean recoil momentum of 126 MeV/c, which differed from their measurement by 13%.

c. np to pp Removal Ratios

Apart from recoil-momentum spectra, the other quantity of physical interest which can be extracted from yields for two-nucleon emission is the ratio of np knockout to pp knockout, nn knockout being forbidden by charge conservation. A simple model which assumes π⁻ absorption on relative S-wave nucleons with equal probability for a triplet pair or a singlet pair gives an np/pp ratio of 3.0. Nordberg et al. have measured this ratio for a wide range of targets by detecting outgoing protons and neutrons and obtained an average of (3.9 ± 1.0).
In the π⁻ absorption experiments described in this work only np removal was observed for ¹²C and ¹⁴N, giving a ratio of infinity. Nordberg et al. measured ratios of (2.3 ± 0.8) and (3.7 ± 1.1) for these two targets, respectively. It is possible that γ rays corresponding to pp removal were emitted, but were below the threshold for detection in these experiments. Ground-state transitions are also possible.

The ³²S and ⁴⁰Ca experiments gave np/pp removal ratios much smaller than predicted or observed by Nordberg et al., ³²S giving (0.61 ± 0.12) and ⁴⁰Ca giving (0.88 ± 0.18). However, both of these are even-even nuclei, and pp removal produces even-even daughter nuclei which display the familiar enhancement of detection of 2⁺ states. There is no such enhancement for states of the odd-odd nuclei resulting from np removal. Therefore an inherent bias by a factor of about three (see below) toward 2⁺ states of even-even nuclei can explain this apparent discrepancy. This explanation is strengthened by the results of the ²³Na experiment, for which there is no enhancement because both modes of two-nucleon removal result in odd-even daughters. For this case the ratio is (2.58 ± 1.32), which is in good agreement with both the experimental and theoretical results of Nordberg et al. Although the ²³Na results alone are not conclusive, if values near 3 are observed for other odd-even targets and values near 1 for even-even targets, this would constitute a good measure of the effect of 2⁺ enhancement in even-even daughter nuclei.

An analysis was made of enhancement of low-lying transitions in which equal population of all bound excited states was assumed. It found that, on average, about 75% of all initial excitations resulted in transitions through the 2⁺ state of even-even nuclei, but that only about 25%
resulted in transitions through that low-lying level (usually the first
or second excited state) which received the most feeding from higher
levels for the odd-even and odd-odd cases. The ratio of the enhancements
for the two cases is about 3, which agrees with the ratio of the np/pp
removal ratios observed for odd-even and even-even target nuclei.

d. Other Two-Nucleon-Removal Studies

Kopaleishvili\textsuperscript{96} has analyzed the interactions (\(\pi^-\), nn) and
(\(\pi^-\), np) with stopped pions, and the high-energy photoeffect (\(\gamma\), np) for
\(^{12}\text{C}\) and \(^{16}\text{O}\) targets. The energy spectra of out-going nucleons were found
to agree with predictions for S absorption, but not P absorption. He also
obtained an np/pp removal ratio of 2 to 3, which agrees with Nordberg et al.
and the \(^{23}\text{Na}\) results from this study. Arthur et al.\textsuperscript{108} studied the (\(\pi^+\), 2p)
reaction on \(^2\text{H}\), \(^6\text{Li}\), \(^{14}\text{N}\), and \(^{16}\text{O}\) with 70-MeV pions. Their results gave
good quantitative agreement with calculations made using the plane-wave
impulse approximation with the assumption that absorption takes place on a
quasi-deuteron cluster with the rest of the nucleus acting as a spectator.

Another class of experiments with potential for measuring short-
range two-nucleon correlations is those involving medium-energy electro-
magnetic probes. If a photon of energy between, say, 50 and 400 MeV is
absorbed on a nucleus, the energy-momentum imbalance is such that the ab-
sorption cannot take place on a single nucleon,\textsuperscript{109} and photo-ejection of
a single nucleon is therefore suppressed. But two nucleons colliding in
the nucleus may have large individual momenta even though the sum may be
very small. An electric dipole interaction on a neutron-proton pair can
bring about their emission near 180\textdegree\ opposite to each other, so that the
energy-momentum mismatch is overcome. This process, known as the
quasi-deuteron effect, is similar to $\pi^-$ absorption and can, in theory, be used to measure short-range correlations in nuclei. Gari and Hebach, however, have shown that gauge invariance contributions (interaction of photons with internal nucleon lines, with exchanged mesons, etc.) to correlations make up the most important part of processes below pion threshold. These considerations explain the dominance of $(\gamma, np)$ reactions over the $(\gamma, pp)$ and $(\gamma, nn)$ modes because of the symmetry between protons and neutrons of the gauge terms. This analysis also gives good agreement with experimental angular distributions for $(\gamma, p)$ and $(\gamma, n)$ reactions. A number of $(\gamma, np)$ experiments have been performed which suggest the presence of deuteron clusters in nuclei. A similar experiment by Heimlich et al. utilized quasi-elastic electron scattering to measure cross sections for the reactions $^6\text{Li}(e, e'p)$ and $^6\text{Li}(e, e'd)$ near 2.5 GeV. Their $(e, e'p)$ results gave good agreement with a model assuming short-range nucleon-nucleon correlations, and the $^6\text{Li}(e, e'd)$ data agree well with predictions of an ad cluster model of $^6\text{Li}$.

5. Three-Nucleon Removal

Three-nucleon removal was observed from $\pi^-$ absorption on $^{23}\text{Na}$, $^{32}\text{S}$, and $^{40}\text{Ca}$. Reference has already been made to these with regard to excitation of high-$J$ states. The large observed yield of $^{20}\text{Ne}$ from $^{23}\text{Na}$, $(7.56 \pm 0.54)\%$, is surely due in part to enhancement of the $2^+$ state of an even-even nucleus, but could be interpreted as evidence for the presence of a triton cluster in the $^{23}\text{Na}$ nucleus. The relatively large yields for $7/2^-$ states in $^{29}\text{Si}$ and $^{37}\text{Ar}$ from the $^{32}\text{S}$ and $^{40}\text{Ca}$ targets, respectively, especially the latter, and the large cross section (36.2 mb) for production
of the $7/2^-$ state of $^{37}$Ar from 190-MeV $\pi^+$ on $^{40}$Ca are noteworthy because they imply that 3-nucleon removal from these nuclei, especially $^{40}$Ca, usually includes transfer of a fourth nucleon, presumably the odd neutron, from the sd shell to the $f_{7/2}^-$ shell. An intriguing hypothesis is that this results from $\pi^-$ absorption on an alpha cluster with one of the cluster nucleons being retained by the daughter nucleus.

Castleberry et al. observed yields of $13 \pm 2\%$ and $2.5 \pm 0.5\%$ for deuteron and triton emission, respectively, by $^{40}$Ca following $\pi^-$ absorption. Although they did not veto events in which other particles were emitted in coincidence with deuterons or tritons, the large relative abundance of energetic deuterons and tritons they observed suggests that some direct process(es), rather than pure statistical evaporation, may be involved. The anomalously large yield for the $7/2^-$ state of $^{37}$Ar (2.1 times larger relative to the $^{36}$Ar yield than predicted by the ALICE code even without accounting for the $2^+$ enhancement of $^{36}$Ar) observed in this study is further evidence of a direct process leading to 3-nucleon removal from $\pi^-$ absorption on $^{40}$Ca.

If a heavy cluster is removed in a direct reaction a relatively large recoil momentum would be expected. Unfortunately the mean lifetime of the $7/2^-$ state of $^{37}$Ar is $(6.3 \pm 0.2)$ nanoseconds, and the $\gamma$-ray line is consequently not Doppler broadened. Decisive analysis of this mechanism may thus require identification and measurements of energy spectra and angular correlations of outgoing protons, neutrons, deuterons, and tritons in coincidence with the $\gamma$ rays.
6. Removal of Four or More Nucleons

a. Four-Nucleon Removal

Four-nucleon removal was found to be quite prominent from the medium-mass targets $^{32}$S and $^{40}$Ca, was present, but with modest yield, from $^{23}$Na, and was not observed from $^{12}$C and $^{14}$N (there are no bound states of $^8$Be, so it cannot be observed from $^{12}$C with this technique). As listed in Table 13, the yield of the first excited state of $^{28}$Si from $\pi^-$ absorption on $^{32}$S is $(6.79 \pm 0.44)\%$, about 75% of the summed yield for all states observed in two-nucleon removal, which is not especially large, but the $^{28}$Si peak is Doppler broadened with a mean recoil momentum near 265 MeV/c. A detailed calculation of the momentum distribution due to four-particle removal from $^{32}$S is not practical because of the complexity of the wave functions and the lack of four-nucleon coefficients of fractional parentage. However, in a harmonic-oscillator potential the momentum distribution has the same functional form as the radial part of the wave function $^{118}$ scaled by the root-mean-square radius $R_{\text{rms}}$ of the nucleus. Thus scaling the results from $^{14}$N and $^{16}$O by holding $KR_{\text{rms}}$ constant, where $K$ is the mean recoil momentum, should provide a reasonable estimate of the mean recoil momentum:

$$\left(\frac{14}{32}\right)^{1/3} (14.8) = 11.2 \, \text{MeV/c}.$$  \hspace{1cm} (4-37)

This is less than half the measured value. A similar result is found for the first excited state of $^{36}$Ar from $^{40}$Ca, which was observed with a yield of $(4.58 \pm 0.62)\%$, and a mean recoil momentum of about 460 MeV/c. Here
scaling $K R_{\text{rms}}$ is in even more obvious disagreement than in the $^{32}\text{S} - ^{28}\text{Si}$ case:

$$\left(\frac{14}{40}\right)^{1/3} (148) = 104 \text{ MeV/c}$$  \hspace{1cm} (4-38)

is less than one-fourth the observed value. These large recoil momenta suggest that reactions involving removal of energetic heavy aggregates may be involved, such as absorption on clusters, or cascade processes, including final-state pickup, rather than boil-off of successive particles in random directions.

Two states of $^{36}\text{Ar}$, the $2^+$ at 1970 keV and the $3^-$ at 4178 keV, were observed in the $^{40}\text{Ca}$ experiment. The ratio of the raw yields is

$$\frac{\text{Raw Yield } (2^+ \rightarrow 0^+)}{\text{Raw Yield } (3^- \rightarrow 2^+)} = 4.63.$$  \hspace{1cm} (4-39)

To compare this with a statistical population of states, enhancement factors were computed using the known branching ratios for the excited states of $^{36}\text{Ar}$ with three different models: (1) level populations proportional to $(2J + 1)$, (2) equal level populations, and (3) no spin dependence, but with an energy weighting which assumed $\pi^-$ absorption on two nucleons, 30-MeV excitation of the daughter nucleus, and boil-off of two neutrons, each taking off 7 MeV of binding energy and 2 MeV of kinetic energy. This left a state weighting factor

$$WF_S \propto e^{-E_s/12 \text{MeV}}.$$  \hspace{1cm} (4-40)

The three models gave $(2^+ + 0^+)/ (3^- + 2^+)$ ratios of 1.93, 2.44, and 2.45,
respectively, all of which are smaller than the observed value of 4.63. Since the $3^-$ level and the levels which feed it involve population of the negative-parity $f_{7/2}$ shell, and since the $2^+$ level and those which feed it directly, rather than the $3^-$ level, involve population of the positive-parity sd shell, the results suggest that sd-shell states of $^{36}$Ar are preferentially produced by $\pi^-$ absorption on $^{40}$Ca. This is in contrast to the production of $^{37}$Ar, where transfer to the $f_{7/2}^-$ shell is dominant. A plausible explanation of these two observations may be made by assuming that production of both comes about primarily by $\pi^-$ absorption on an alpha cluster. In the $^{37}$Ar case one of the neutrons from the cluster is re-captured in the higher $f_{7/2}^-$ shell, while in the $^{36}$Ar case all four cluster nucleons are removed with relatively small excitation of the residual $^{36}$Ar nucleus.

b. $\Delta A > 4$

Removal of more than four nucleons in $\pi^-$ absorption has been described fairly successfully in terms of statistical evaporation in Section A. The most striking characteristic of this region of the $^{32}$S and $^{40}$Ca spectra is the dominance of the even-even nuclei, especially those with $Z = N$. The observation of these daughter nuclei may arise from the enhancement of detection of the $2^+$ states. If there were no threshold of observation for individual lines it is possible that yields of states of odd-even and odd-odd nuclei in this region would sum to comparable magnitudes. As mentioned in Section A, this interpretation is buttressed by the results of Hornyak's ($\alpha$, $\gamma\chi$) experiments\(^{91}\) and the analysis of the Compton continua in Chapter III of this work. However, the relatively
large yields of $N = Z$ nuclei for large $\Delta A$ can be interpreted as resulting from alpha clustering in the target nuclei, but at the price of introducing an inconsistency with the sums of the measured yields.

It was pointed out by Nishimura and Arima in an analysis of nuclear kaon absorption that if a meson absorbs on an alpha cluster (or any heavy multi-nucleon aggregate), the excited states of the residual nuclei are likely to have very high spins. Because of these high spins the nuclei will prefer alpha emission to neutron emission. If this interpretation is correct the results of the analysis of the Compton continua would have to be viewed as resulting from many $\gamma$ rays per even-even daughter nucleus and relatively few from odd-even and odd-odd nuclei. Alpha-cluster absorption has some similarity to heavy-ion reactions in its ability to excite high-spin states of the residual nuclei, which emit many $\gamma$ rays per de-excitation. Thus this process could account for the observed number of $\gamma$ rays per stopped pion. However, it would mean that the yields of daughter nuclei, both observed and unobserved, would sum to substantially less than 100%. Therefore this hypothesis cannot be accepted, but a more modest preference for alpha-cluster absorption, say, less than 20%, would be within the limits of error of the experiments.

The 1779-keV $2^+$ state of $^{28}$Si produced in the $^{40}$Ca experiment displays considerable Doppler broadening, with a mean recoil momentum between 286 and 360 MeV/c, depending upon which slowing-down correction is used. In the interaction of 190-MeV $\pi^+$ with $^{40}$Ca this state is observed with a very large cross section (87.5 mb) and a mean recoil momentum of about 500 MeV/c. These results suggest that absorption upon and emission of heavy aggregates are involved, perhaps alpha clusters, more exotically,
$^{28}\text{Si}$ and $^{12}\text{C}$ subgroupings in $^{40}\text{Ca}$. It is also noteworthy that in studies of the interaction of $220\text{-MeV }\pi^-\text{ and }190\text{-MeV }\pi^+\text{ with }^{51}\text{V}$, a nucleus in the same mass range as $^{40}\text{Ca}$, but which is odd-even with $N - Z = 5$, no significant Doppler broadening of any transition lines of daughter nuclei was observed. Another interesting result from $190\text{-MeV }\pi^+\text{ on }^{40}\text{Ca}$ is the observation that, when coincidence of the incoming $\pi^+$ and the de-excitation $\gamma$ rays with outgoing pions $30^\circ$ from the beam line was required, the shapes of some of the Doppler-broadened peaks were radically different from those in the spectrum accumulated without this additional requirement. Some peaks which had a gaussian shape without outgoing $\pi^+$ coincidence were rectangular in the spectrum taken with the outgoing pion coincidence. Unfortunately statistics were relatively poor when this latter requirement was applied, and no quantitative results are yet available.

c. Comparison with Other Experiments

Preference for removal of the equivalent of one or more alpha particles has been observed in other pion-nucleus experiments. Large cross sections were measured for single or multiple $2p + 2n$ removal from $^{28}\text{Si}$, $^{32}\text{S}$, and $^{40}\text{Ca}$ bombarded by $220\text{-MeV }\pi^-$ at the Space Radiation Effects Laboratory.$^{57}$ As mentioned in Section A, there was an error in the efficiency calibration, and some peaks should be reanalyzed, but it remains clear that relative to other daughter nuclei, those corresponding to the removal of from one to five alpha-particle equivalents constitute a considerable, if not dominant, fraction of the observed reaction cross section. The same can be said of the results from the bombardment of $^{40}\text{Ca}$ with $190\text{-MeV }\pi^+$. Lieb and Funsten$^{121}$ found, using the de-excitation
γ-ray technique, that the 4439-keV $2^+$ state of $^{12}$C was the most strongly excited (16.1 mb) of all daughter states resulting from the interaction of 230-MeV π$^-$ with $^{16}$O. Kossler et al., $^{39}$ in a similar experiment with stopped π$^-$ found this state populated with a 0.84% yield, which is larger than any other yields except for $^{14}$N. Ullrich et al. $^{74}$ found a yield for this state of 4.0% by the same method. They also measured the Doppler broadening, assuming the momentum distribution to be

$$\rho_L(K) \propto K^{2L} e^\frac{-K^2}{2Q^2_L}$$

(4-h1)

With $L = 2$, they obtained $Q^2 = (77 \pm 5)$ and $(72 \pm 9)$ MeV/c for the double-escape and photopeaks, respectively, which agree very well with the calculation of Balashov et al., $^{122}$ who predicted a value of 65 MeV/c for $Q^2$. Depending upon the nuclear potential function used $Q^2$ should be from 1.0 to 1.3 times the value of $Q^2$. $^{123}$

Ullrich et al. $^{74}$ have also measured yields for π$^-$ absorption on $^{24}$Mg, $^{31}$P, and $^{40}$Ca, as mentioned in Section A. Their measured yields for all residual nuclei from $^{40}$Ca are at least a factor of three smaller than the values determined in this work. They reported that for both π$^-$ absorption and for 60-MeV π$^-$, cluster removal is comparable to other reaction channels, but that it seems to be dominant near the Λ(1232) resonance.

H. E. Jackson et al. $^{124}$ measured the γ-ray spectra resulting from the bombardment of $^{60}$Ni and $^{28}$Si with 500-MeV/c π$^-$. They found strong spectral lines corresponding to one- and two-alpha removal and, in
the case of $^{60}\text{Ni}$, a weaker line corresponding to three-alpha removal. A more recent set of experiments has looked at $\gamma$ rays from 220-MeV $\pi^+$ and $\pi^-$, and 100-MeV $\pi^+$ on $^{58}\text{Ni}$ and $^{60}\text{Ni}$, where it was found that nuclei with the same neutron excess as the target (nuclei differing from it by an integral number of alpha particles) appear with significantly larger cross sections than other even-even nuclides. Specifically, for 220-MeV $\pi^+$ and $\pi^-$ incident on $^{58}\text{Ni}$, the summed cross sections for production of $N - Z = 2$ nuclei were found to be 131 and 135 mb, respectively, while the analogous summed cross sections for production of $N - Z = 4$ nuclei were only 58 and 94 mb. In contrast, for the same projectiles incident on $^{60}\text{Ni}$, the $N - Z = 2$ nuclei were seen with cross sections of 67 and 49 mb, but the $N - Z = 4$ nuclei were seen with cross sections of 99 and 152 mb.

Ashery et al. have used the de-excitation $\gamma$-ray method to measure cross sections for daughter nuclei produced by bombardment of $^{27}\text{Al}$ and $^{28}\text{Si}$ with 70-MeV $\pi^-$ and 25, 70, and 100-MeV $\pi^+$. They found large cross sections for equivalent alpha removal for both nuclei and both pion charges.

Some $\pi^-$-interaction studies which measured the outgoing charged particles also provide evidence in favor of alpha clustering. Particularly noteworthy is that of Castleberry et al., who measured the energy spectra and yields for $^1\text{H}$, $^2\text{H}$, and $^3\text{H}$ emission from a number of light- and medium-mass nuclei following $\pi^-$ capture. They found that these spectra and yields are quite similar to those obtained from $\pi^-$ absorption on $^4\text{He}$, which suggests that absorption on alpha clusters may play a significant role in $\pi^-$ absorption on these nuclei. Two experiments have also been done
to measure the spectra of alpha particles emitted by nuclei following $\pi^-$ interactions. The motivation for these was to test the hypothesis that absorption on one alpha cluster produces a high-$J$ daughter which, because of its high spin, de-excites by emission of low-energy alpha particles instead of neutrons.\textsuperscript{28} Comiso et al.\textsuperscript{127} found that alphas emitted in prompt coincidence with $\pi^-$ stops in $^{12}\text{C}$ were predominantly of low energy (less than 10 MeV), and that $(1.00 \pm 0.07)$ alphas were produced per $\pi^-$ stop. Doron et al.\textsuperscript{128} measured the spectrum of alpha particles emitted following the bombardment of $^{27}\text{Al}$ with $70$-MeV $\pi^-$ and found a cross section of $(6 \pm 2)$ mb/sr for the alpha energy range $5.5 - 30$ MeV. They obtained preliminary agreement with the predictions of a cascade evaporation model.

Experimental studies using bombarding particles other than pions have also given results which are suggestive of alpha clustering in nuclei. Of particular interest is an experiment by Barnes et al.\textsuperscript{129} who stopped negative kaons in Ni, Cu, Si, and Al, and observed the de-excitation $\gamma$ rays. Instead of seeing a predominance of neutron-poor isotopes of the $(Z-1)$, $(Z-2)$, and $(Z-3)$ elements, as predicted by their evaporation code (this conflicts with the predictions of the ALICE code used in this work), they found for Ni, removal of one, two, or three equivalent alpha particles, and for Cu, removal of a triton plus zero, one, or two alphas. These results are similar to those found by Jackson et al.\textsuperscript{103} for fast pions incident on nickel isotopes. For the lighter targets Barnes et al. found single-proton removal and removal of one or two equivalent alphas to be the prominent features.
Evidence for alpha clustering in $^{197}$Au nuclei has been obtained by Adler et al., who studied the $^{197}$Au($\gamma, \alpha$)$^{193}$Ir reaction with 500-MeV bremsstrahlung. Their data agreed well with a calculation which assumed the presence of four alpha clusters on the $^{197}$Au, each with a mean kinetic energy of 16 MeV.

A number of heavy-ion experiments give results which are suggestive of alpha clustering. See, for example, reference 131. One of particular interest, because it involves even-even sd-shell nuclei, is a study of the $^{32}$S($^{16}$O, $^{12}$C)$^{36}$Ar reaction by Charlton and Robson. Their differential cross section agrees well with a calculation assuming direct transfer of an alpha particle in an excited state.

7. Alpha-Clustering Theory

Two theoretical analyses of pion-nucleus scattering which are based on nuclear cluster models have given results which agree with experiment to within 10 to 20%. Hufner et al. computed pion-nucleus scattering lengths for all the stable nuclei with $A \leq 20$ assuming alpha clustering. For example, for $^{12}$C they assumed

$$a_s(\pi^{12}C) = 3 \cdot a_s(\pi\alpha) + 3 \cdot 2 \cdot a_s^2(\pi\alpha)/d + 3 \cdot 2 \cdot a_s^3(\pi\alpha)/d^2 + \ldots$$

(4.12)

where $d$ is the distance between cluster centers. Agreement with experiment was better than to within 20%, with best agreement for $N = Z$ even-even nuclei. Yam has computed the forward amplitude for pion-nucleus scattering in the $\Delta(1232)$ region using Glauber formalism and a cluster model of the nucleus. Agreement to within 10% was found for light nuclei up to
but poor agreement was obtained for $^{27}\text{Al}$ and $^{32}\text{S}$. The agreement was very poor for a nucleon model all across the mass spectrum. This is not surprising, however, because use of the cluster model sidesteps a very difficult calculation involving infinitely many scatterings on strongly correlated nucleons by using the scattering data on the clusters, which approximate the characteristics of strongly-correlated nucleons.

Theoretical models of cluster phenomena range from pure cluster theories which postulate that all even-even $N=Z$ nuclei consist of non-varying geometrical configurations of alpha clusters, to hybrid schemes which treat alpha clustering as relatively small perturbations superimposed upon a basic shell-model picture. Typical of the former, are the models of Hauge et al.,$^{24}$ who calculated that, for example, $^{20}\text{Ne}$ should be a $D_{3d}$ distorted tetrahedron, $^{24}\text{Mg}$ a bitetrahedron, $^{28}\text{Si}$ a $D_{3d}$ oblate structure, and $^{40}\text{Ca}$ an octahedron of alphas surrounding a tetrahedral $^{16}\text{O}$ core. Similarly, Inopin et al.$^{135}$ predict clustering of $^{12}\text{C}$, $^{20}\text{Ne}$, $^{24}\text{Mg}$, $^{28}\text{Si}$, $^{32}\text{S}$, and possibly $^{40}\text{Ca}$, but claim a shell structure for $^{16}\text{O}$. They also predict intensified clustering in excited rotational states.

A lengthy study of Chevarier et al.$^{12}$ of proton-, deuteron-, $^{3}\text{He}$-, and $\alpha$-induced reactions on $^{58}\text{Co}$, $^{60}\text{Ni}$, $^{67}\text{Ga}$, $^{119}\text{Sb}$, and $^{210}\text{Po}$ found that a pre-equilibrium reaction model which assumed preformation of alpha clusters gave good agreement with measured cross sections. They defined a quantity $P_\alpha$, the preformation factor for four nucleons to be correlated as an alpha particle, and found it to range between 0.1 and 0.8 for the nuclei studied, with an average value of about 0.2. They also pointed out that the largest part of the calculated emission arises from the diffuse
edge of the nucleus. This agrees very well with calculations made by Clark and Wang,27 who concluded that 20% clustering is possible when the nuclear matter density is less than 90% of the density of the nuclear center.

In summary, there is a considerable body of evidence which indicates that some measure of alpha clustering exists in nuclei, especially even-even N = Z isotopes. Some of the results of the π⁻-absorption experiments described in this work tend to strengthen this conjecture, in particular (1) the anomalously large yields for ΔA = 3 from $^{32}$S and $^{40}$Ca and the particular ($l^T/2^-$) states excited, (2) large recoil momenta of some daughter nuclei which correspond to removal of one or more alpha particles, and (3) the tendency in some cases toward excitation of high-spin states. However, even though clustering may be the simplest explanation of these phenomena, they can be explained by other processes, such as intranuclear cascades and final-state interactions. Further, the bulk of the data, including the large observed yields for even-even daughter nuclei, can be explained quite adequately by statistical evaporation plus enhancement of the $2^+$ states of even-even nuclei. More evidence is needed before the argument in favor of alpha clustering can be considered compelling.136

Pion absorption, because of the energy-momentum mismatch, should be a valuable technique in the continuing search for clustering phenomena. Careful comparisons between π⁻-absorption data and fast-pion reactions should be valuable because of the difference in momentum transfer and in reaction times, which are on the order of $10^{-19}$ sec. for 1s and $10^{-16}$ sec. for 2p absorption,137 and about $3 \times 10^{-23}$ sec. for a pion traversing a
medium-mass nucleus near the Δ(1232) resonance. In a sense the fast pion takes a snapshot of the nucleus, while the pion absorbing from atomic orbit senses its complete wave function.

C. Conclusions

The data from the experiments described in this work contain an enormous amount of information. Unfortunately, however, interpretation of these data is made difficult because (1) the strong-interaction dynamics of the initial absorption mechanism are not well understood, and (2) the relatively-messy final-state interactions of the daughter nuclei with the emitted nucleons must be unfolded from the data in order to obtain information on the absorption process. In spite of these difficulties, which are generally shared by other techniques designed to probe short-range nuclear correlations, there are a number of conclusions, some definite and some tentative, which can be drawn from this study:

1. Gross features of the spectra can be explained by a combination of quasi-impulse-approximation reactions and statistical evaporation.

2. The wide range of energy deposition needed to accurately describe the gross features of the spectra suggests that π⁻ absorption involves a variety of reaction processes.

3. The yields of even-even daughter nuclei from $^{32}\text{S}$ and $^{40}\text{Ca}$ give fair agreement with the predictions of Rudstam's spallation formula. π⁻ absorption appears from this analysis to be more effective in exciting $^{40}\text{Ca}$ nuclei than $^{32}\text{S}$.

4. Two-nucleon removal does not excite $0^+$, 1 states of daughter nuclei. This is in agreement with a calculation assuming direct absorption on a relative S-state np pair.
5. Although not conclusive, there seems to be some preference for excitation of high-spin states of daughter nuclei. This may be a result of pion-absorption on heavy multinucleon aggregates in the target nuclei, alpha clusters being one possibility.

6. Single-nucleon emission is suppressed relative to 2-, 3-, and 4-nucleon emission, and may be totally absent in the cases of even-even targets, although ambiguities in γ-ray identification make the results somewhat inconclusive.

7. Two-nucleon emission is the only process observed from π⁻ absorption on \(^{12}\)C and \(^{14}\)N (this experiment was not sensitive to most larger values of ΔA because of the lack of bound excited states in this mass region), and is a prominent, but not dominant, part of the spectra from \(^{23}\)Na, \(^{32}\)S, and \(^{40}\)Ca. Thus the early hypothesis that π⁻ absorption should result almost solely in two-nucleon emission is not valid, and the processes are more complex.

8. Two-nucleon emission from \(^{14}\)N produces \(^{12}\)C in its first 2⁺ state with a recoil momentum of \((169 \pm 6)\) MeV/c, which agrees to within 12% with the value predicted by a calculation which assumes π⁻ absorption on a relative S-wave triplet-state np pair.

9. The ratio of np to pp removal was observed to be about 1 for the even-even targets \(^{32}\)S and \(^{40}\)Ca, and about 3 for the odd-even target \(^{23}\)Na. The latter agrees with a simple model for the process. Assuming this to be valid, the ratio of the ratios for even-even and odd-even targets agrees with a statistical calculation of the enhancement of 2⁺ states.
of even-even nuclei, which predicts these states to be enhanced by a factor of about 3 over low-energy states of odd-even nuclei.

10. Three-nucleon emission is comparable in yield to two-nucleon emission for \( \pi^- \) absorption on \(^{23}\text{Na},^{32}\text{S}, \) and \(^{40}\text{Ca}, \) and is anomalously large in comparison with predictions of the statistical evaporation code. Its presence is striking for \(^{23}\text{Na}, \) where the daughter is \(^{20}\text{Ne}, \) which has been hypothesized to have an alpha-cluster structure.\(^{24,135}\)

11. Three-nucleon removal from \(^{40}\text{Ca} \) produces \(^{37}\text{Ar} \) daughter nuclei predominantly with one neutron transferred to the \( f_{7/2} \) shell. Four-nucleon removal produces \(^{36}\text{Ar} \) with nucleons remaining in the sd shell more frequently than predicted by statistics. These results strengthen the hypothesis that some absorption on alpha clusters occurs with \(^{40}\text{Ca}.\)

12. Four-nucleon emission is comparable to 2- and 3-nucleon emission for the \(^{32}\text{S} \) and \(^{40}\text{Ca} \) targets, but is relatively suppressed in the case of \(^{23}\text{Na}. \) Multiple equivalent-alpha emission is also present in the \(^{32}\text{S} \) and \(^{40}\text{Ca} \) results.

13. From three to five \( \gamma \) rays are emitted per \( \pi^- \) stop on \(^{32}\text{S} \) and \(^{40}\text{Ca}. \) This may result from excitation of high-energy states of a wide variety of daughter nuclei, only a small fraction of which are observed as photopeaks.

14. The spectra of daughter nuclei from \( \pi^- \) absorption are qualitatively similar to spectra from bombardment with fast pions in both charge states near the \( \Delta(1232) \) resonance. The latter appear to be somewhat more effective in producing multiple-equivalent-alpha removal, however.
It is customary, in concluding a study such as this, to suggest where future studies in the field may be directed in order to clarify questions left unresolved by the current work. Some of the questions motivating this study have been clearly answered, such as whether $0^+$, $1^-$ states are excited in $\pi^-$ absorption, the mean recoil momentum of $^{12}_C^{2+}$ from absorption on $^{14}_N$, and the range of excitation energies resulting from absorption on $^{32}_S$ and $^{40}_Ca$. Throughout this analysis hints in favor of some degree of $\pi^-$ absorption on alpha clusters have appeared, but, however strained, other interpretations of these results can be made, and the evidence for alpha clustering cannot be considered compelling. Taken in the light of other evidence for cluster structures in nuclei, it does lend some weight to the clustering hypothesis. This question of clustering has become one of the prime questions of concern to physicists studying nuclear structure with medium-energy probes, and $\pi^-$ absorption, along with pion-nucleus interactions at higher energies, with all its inherent problems, is at least as useful a tool in these studies as are other medium-energy probes.

Analysis of de-excitation $\gamma$-ray spectra taken in prompt coincidence with incident pions, either fast or stopped, has brought precise energy resolution and accurate recoil-momentum measurements to the study of pion-nucleus reactions. It has its weaknesses, however, in that some reactions cannot be observed because of the lack of bound excited states of the expected residual nuclei, and because in many cases the reaction channel cannot be uniquely determined. For example, the production of $^{32}_S$ from $^{40}_Ca$ might result from the ejection of as few as two particles or as
many as eight. In order to sort these different processes it will be necessary to detect the outgoing particles. \((\pi, 2p), (\pi, np),\) and \((\pi, 2n)\) experiments have been performed,\(^{76,108}\) and, of course, \((\pi, \gamma X)\), as in this study and elsewhere. In the studies which observe only outgoing nucleons (and pions) it is generally impossible to determine the state of the residual nucleus, whereas in the \(\gamma\)-ray studies the nature of the outgoing particles is not uniquely determined. These difficulties will be overcome, naturally, if coincidence observations of \(\gamma\) rays and particles are made. Unique identification of states of residual nuclei taken in coincidence with identification, energy spectra, and angular distributions and correlations of outgoing particles, should give better evidence as to the importance of preformed alpha clusters in nuclei.\(^{138}\)

Experiments in which coincidences of de-excitation \(\gamma\) rays and outgoing particles are taken will require pion beams of greater intensity, and better momentum and spatial resolution, than those available up to the present. Higher-efficiency \(\gamma\)-ray detectors are also desirable. Some of the new generation of medium-energy accelerators, such as the SIN cyclotron in Villigen, Switzerland, and the TRIUMPF cyclotron near Vancouver, B.C., should meet these requirements. The LAMPF accelerator at Los Alamos, N.M. is an extremely valuable tool for pion physics research, but it is unlikely to be of value for this particular type of experiment because of its low duty factor.

When coincidence experiments of the nature just described become a reality, hopefully in the near future, studies of pion-nucleus reactions should reach their full potential as superior probes of the detailed structure of atomic nuclei.
Table 1
Properties of Pions

A. Charge
\[ q(\pi^\pm) = \pm e = \pm 1.6 \times 10^{-19} \text{ coul.} \]
\[ q(\pi^0) = 0 \]

B. Mass
\[ m(\pi^\pm) = 139.6 \text{ MeV/c}^2 \]
\[ m(\pi^0) = 135.0 \text{ MeV/c}^2 \]

C. Spin
\[ J(\pi^\pm) = 0 \]

D. Parity
Pions have odd intrinsic parity in the convention where the intrinsic parity of nucleons is defined to be even.

E. Isospin
\[ T(\pi^\pm) = 1 \]
\[ T_3(\pi^\pm) = 1, \quad T_3(\pi^0) = 0, \quad T_3(\pi^-) = -1 \]

F. G - Parity
The G-parity operator is defined to be the product of the charge conjugation and charge symmetry operators: \[ G = C e^{i\pi T_2}, \]
where \[ e^{i\pi T_2} \] represents a 180° rotation in isospin space around the \[ T_2 \] axis, that is, it changes \[ T_3 \] to \[ -T_3 \]. Pions have negative G-parity.
Table 1 - (continued)

G. Mean Lifetimes

\[ \tau (\pi^\pm) = 2.60 \times 10^{-8} \text{ sec.} \]
\[ \tau (\pi^0) = 0.84 \times 10^{-16} \text{ sec.} \]

H. Decay Modes

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \quad 100\% \]
\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad 100\% \]
\[ \pi^0 \rightarrow 2\gamma \quad 98.8\% \]
\[ \pi^0 \rightarrow \gamma + e^+ + e^- \quad 1.2\% \]

These properties were taken from references 29 and 30.
Table 2

Magnet Current Settings

<table>
<thead>
<tr>
<th>Magnet</th>
<th>Current (amps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quad In</td>
<td>90</td>
</tr>
<tr>
<td>Quad Out</td>
<td>90</td>
</tr>
<tr>
<td>Bender</td>
<td>110</td>
</tr>
</tbody>
</table>

Table 3

Energy Calibrations

\[ E = a(\text{channel no.}) + b \]

<table>
<thead>
<tr>
<th>Target</th>
<th>Date</th>
<th>( a(\text{keV/ch.}) )</th>
<th>( b(\text{keV}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{12}\text{C})</td>
<td>Dec. 1971</td>
<td>1.387</td>
<td>+226.</td>
</tr>
<tr>
<td>(^{23}\text{Na})</td>
<td>June 1974</td>
<td>3.2676</td>
<td>+9.74</td>
</tr>
<tr>
<td>(^{32}\text{S})</td>
<td>Oct. 1972</td>
<td>2.949</td>
<td>-89.0</td>
</tr>
<tr>
<td>(^{40}\text{Ca})</td>
<td>May 1974</td>
<td>3.753</td>
<td>+141.1</td>
</tr>
</tbody>
</table>
Table 4

Number of $\pi^-$ Stops (123)

<table>
<thead>
<tr>
<th>Target</th>
<th>Date</th>
<th>$\pi^-$ Stops (123)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\text{C}$</td>
<td>Dec. 1971</td>
<td>$4.34 \times 10^9$</td>
</tr>
<tr>
<td>$^{14}\text{N}$</td>
<td>Jan. 1973</td>
<td>$8.27 \times 10^8$</td>
</tr>
<tr>
<td>$^{23}\text{Na}$</td>
<td>June 1973</td>
<td>$2.82 \times 10^9$</td>
</tr>
<tr>
<td>$^{32}\text{S}$</td>
<td>Oct. 1972</td>
<td>$6.34 \times 10^9$</td>
</tr>
<tr>
<td>$^{40}\text{Ca}$</td>
<td>May 1974</td>
<td>$1.185 \times 10^{10}$</td>
</tr>
</tbody>
</table>

Table 5

Effective Half-Thicknesses of Targets

<table>
<thead>
<tr>
<th>Target Material</th>
<th>$\ell_{1/2} (\text{gm/cm}^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>2.78</td>
</tr>
<tr>
<td>Sodium</td>
<td>7.8</td>
</tr>
<tr>
<td>Sulfur</td>
<td>4.21</td>
</tr>
<tr>
<td>Calcium</td>
<td>6.2</td>
</tr>
</tbody>
</table>
Table 6
Peaks in the $\pi^- + ^{12}\text{C}$ Spectrum

<table>
<thead>
<tr>
<th>E(keV)</th>
<th>Identification</th>
<th>$(N_Y \pm \Delta N_Y)$</th>
<th>$(N_Y \pm \Delta N_Y) \times \eta/E$</th>
<th>Raw Yields (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>414</td>
<td>$^{10}\text{B} 2154 \text{ keV} \rightarrow 1740 \text{ keV}$</td>
<td>2,790 ± 53</td>
<td>6.82 x 10^7 ± 1.30 x 10^6</td>
<td>1.57 ± .03</td>
</tr>
<tr>
<td>717</td>
<td>$^{10}\text{B} 717 \text{ keV} \rightarrow 0$</td>
<td>12,380 ± 168</td>
<td>5.51 x 10^8 ± 7.46 x 10^6</td>
<td>12.69 ± .17</td>
</tr>
<tr>
<td>1023</td>
<td>$^{10}\text{B} 1740 \text{ keV} \rightarrow 717 \text{ keV}$</td>
<td>1,128 ± 77</td>
<td>7.42 x 10^7 ± 5.06 x 10^6</td>
<td>1.71 ± .12</td>
</tr>
<tr>
<td>1436</td>
<td>$^{10}\text{B} 2154 \text{ keV} \rightarrow 717 \text{ keV}$</td>
<td>761 ± 116</td>
<td>6.62 x 10^7 ± 1.01 x 10^7</td>
<td>1.52 ± .23</td>
</tr>
</tbody>
</table>

Total $\pi^-$ Stots = 4.34 x 10^9
Table 7

Corrected Yields for the $^1_{0B}$ Levels Observed
in the $\pi^- + ^{12}_C$ Experiment.

$\pi^- \text{ Absorption on } ^{12}_C$

<table>
<thead>
<tr>
<th>Daughter Nucleus</th>
<th>$J^P, T$</th>
<th>Energy (keV)</th>
<th>Yield (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{10}_B$</td>
<td>$1^+, 0$</td>
<td>717</td>
<td>9.46 ± .31</td>
</tr>
<tr>
<td>$^{10}_B$</td>
<td>$0^+, 1$</td>
<td>1740</td>
<td>.14 ± .12</td>
</tr>
<tr>
<td>$^{10}_B$</td>
<td>$1^+, 0$</td>
<td>2154</td>
<td>4.23 ± .23</td>
</tr>
</tbody>
</table>

Total Yield = (13.83 ± 40)%
### Table 8

Peaks in the $\pi^- + ^{23}\text{Na}$ Spectrum

<table>
<thead>
<tr>
<th>E(keV)</th>
<th>Identification</th>
<th>$(N\gamma \pm \Delta N\gamma)$</th>
<th>$(N\gamma \pm \Delta N\gamma) \times \eta/\varepsilon$</th>
<th>Raw Yields (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>350</td>
<td>$^{21}\text{Ne}$ 1 → 0</td>
<td>204 ± 28</td>
<td>$7.62 \times 10^6 \pm 1.04 \times 10^6$</td>
<td>.27 ± .04</td>
</tr>
<tr>
<td>495</td>
<td>$^{17}\text{F}$ 1 → 0</td>
<td>45 ± 18</td>
<td>$2.26 \times 10^6 \pm 9.14 \times 10^5$</td>
<td>.08 ± .03</td>
</tr>
<tr>
<td>657</td>
<td>$^{20}\text{F}$ 1 → 0</td>
<td>315 ± 72</td>
<td>$2.04 \times 10^7 \pm 4.68 \times 10^6$</td>
<td>.72 ± .17</td>
</tr>
<tr>
<td>869</td>
<td>$^{17}\text{O}$ 1 → 0</td>
<td>229 ± 45</td>
<td>$1.90 \times 10^7 \pm 3.75 \times 10^6$</td>
<td>.67 ± .13</td>
</tr>
<tr>
<td>1117</td>
<td>$^{21}\text{Ne}$ 5 → 2</td>
<td>526 ± 60</td>
<td>$5.50 \times 10^7 \pm 6.27 \times 10^6$</td>
<td>1.95 ± .22</td>
</tr>
<tr>
<td>1238</td>
<td>$^{19}\text{F}$ 3 → 1</td>
<td>351 ± 40</td>
<td>$4.01 \times 10^7 \pm 4.60 \times 10^6$</td>
<td>1.42 ± .16</td>
</tr>
<tr>
<td>1274</td>
<td>$^{22}\text{Ne}$ 1 → 0</td>
<td>591 ± 53</td>
<td>$6.95 \times 10^7 \pm 6.21 \times 10^6$</td>
<td>2.47 ± .22</td>
</tr>
<tr>
<td>1398</td>
<td>$^{21}\text{Ne}$ 2 → 1</td>
<td>330 ± 58</td>
<td>$4.28 \times 10^7 \pm 7.52 \times 10^6$</td>
<td>1.52 ± .27</td>
</tr>
<tr>
<td>1459</td>
<td>$^{19}\text{F}$ 4 → 0</td>
<td>54 ± 42</td>
<td>$7.17 \times 10^6 \pm 5.58 \times 10^6$</td>
<td>.25 ± .19</td>
</tr>
<tr>
<td>1554</td>
<td>$^{19}\text{F}$ 5 → 0</td>
<td>63 ± 49</td>
<td>$9.22 \times 10^6 \pm 7.18 \times 10^6$</td>
<td>.33 ± .26</td>
</tr>
<tr>
<td>1614</td>
<td>$^{19}\text{Ne}$ 5 → 0</td>
<td>294 ± 60</td>
<td>$4.30 \times 10^7 \pm 8.76 \times 10^6$</td>
<td>1.52 ± .31</td>
</tr>
<tr>
<td>1633</td>
<td>$^{20}\text{Ne}$ 1 → 0</td>
<td>$1456 \pm 104$</td>
<td>$2.13 \times 10^8 \pm 1.52 \times 10^7$</td>
<td>7.56 ± .54</td>
</tr>
<tr>
<td>1719</td>
<td>$^{21}\text{F}$ 7 → 3</td>
<td>362 ± 71</td>
<td>$5.47 \times 10^7 \pm 1.07 \times 10^7$</td>
<td>1.95 ± .38</td>
</tr>
<tr>
<td>1730</td>
<td>$^{21}\text{F}$ 3 → 0</td>
<td>98 ± 37</td>
<td>$1.53 \times 10^7 \pm 5.70 \times 10^6$</td>
<td>.54 ± .20</td>
</tr>
<tr>
<td>Energy</td>
<td>Level</td>
<td>$^2$X</td>
<td>Transition</td>
<td>$E$</td>
</tr>
<tr>
<td>--------</td>
<td>-------</td>
<td>-----</td>
<td>------------</td>
<td>-----</td>
</tr>
<tr>
<td>1748</td>
<td>2</td>
<td>$^{21}$Ne</td>
<td>$2 \rightarrow 0$</td>
<td>72 ± 15</td>
</tr>
<tr>
<td>1983</td>
<td>1</td>
<td>$^{18}$O</td>
<td>$1 \rightarrow 0$</td>
<td>272 ± 39</td>
</tr>
<tr>
<td>2072</td>
<td>6</td>
<td>$^{21}$F</td>
<td>$6 \rightarrow 0$</td>
<td>194 ± 98</td>
</tr>
<tr>
<td>2085</td>
<td>2</td>
<td>$^{22}$Ne</td>
<td>$2 \rightarrow 1$</td>
<td>226 ± 94</td>
</tr>
<tr>
<td>2312</td>
<td>1</td>
<td>$^{14}$N</td>
<td>$1 \rightarrow 0$</td>
<td>140 ± 78</td>
</tr>
<tr>
<td>2612</td>
<td>2</td>
<td>$^{20}$Ne</td>
<td>$2 \rightarrow 1$</td>
<td>115 ± 49</td>
</tr>
<tr>
<td>4439</td>
<td>1</td>
<td>$^{12}$C</td>
<td>$1 \rightarrow 0$</td>
<td>316 ± 90</td>
</tr>
<tr>
<td>4555</td>
<td>4</td>
<td>$^{17}$O</td>
<td>$4 \rightarrow 0$</td>
<td>73 ± 32</td>
</tr>
</tbody>
</table>
Table 9
Corrected Yields from the $\pi^- + ^{23}$Na Experiment by Level.

$\pi^-$ Absorption on $^{23}$Na

<table>
<thead>
<tr>
<th>Daughter Nucleus</th>
<th>Z</th>
<th>A</th>
<th>$J^P, T$</th>
<th>Energy (keV)</th>
<th>Yield (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>10</td>
<td>22</td>
<td>$2^+$</td>
<td>1275</td>
<td>1.02 ± 0.64</td>
</tr>
<tr>
<td>Ne</td>
<td>10</td>
<td>22</td>
<td>$4^+$</td>
<td>3356</td>
<td>1.45 ± 0.60</td>
</tr>
<tr>
<td>Ne</td>
<td>10</td>
<td>21</td>
<td>$9/2^+$</td>
<td>2866</td>
<td>3.20 ± 0.36</td>
</tr>
<tr>
<td>Ne</td>
<td>9</td>
<td>21</td>
<td>$2^+$</td>
<td>1633</td>
<td>6.63 ± 0.67</td>
</tr>
<tr>
<td>Ne</td>
<td>9</td>
<td>20</td>
<td>$4^+$</td>
<td>4245</td>
<td>0.93 ± 0.40</td>
</tr>
<tr>
<td>Ne</td>
<td>9</td>
<td>19</td>
<td>$3^+$</td>
<td>656</td>
<td>0.72 ± 0.17</td>
</tr>
<tr>
<td>Ne</td>
<td>10</td>
<td>19</td>
<td>$3/2^-$</td>
<td>1614</td>
<td>1.52 ± 0.31</td>
</tr>
<tr>
<td>F</td>
<td>9</td>
<td>19</td>
<td>$3/2^-$</td>
<td>1554</td>
<td>0.32 ± 0.26</td>
</tr>
<tr>
<td>F</td>
<td>9</td>
<td>19</td>
<td>$3/2^+$</td>
<td>1459</td>
<td>0.25 ± 0.19</td>
</tr>
<tr>
<td>F</td>
<td>9</td>
<td>19</td>
<td>$5/2^-$</td>
<td>1346</td>
<td>1.42 ± 0.16</td>
</tr>
<tr>
<td>O</td>
<td>8</td>
<td>18</td>
<td>$2^+$</td>
<td>1982</td>
<td>1.67 ± 0.24</td>
</tr>
<tr>
<td>F</td>
<td>9</td>
<td>17</td>
<td>$1/2^+$</td>
<td>495</td>
<td>0.08 ± 0.03</td>
</tr>
<tr>
<td>O</td>
<td>8</td>
<td>17</td>
<td>$3/2^-$</td>
<td>4554</td>
<td>1.02 ± 0.45</td>
</tr>
<tr>
<td>O</td>
<td>8</td>
<td>17</td>
<td>$1/2^+$</td>
<td>871</td>
<td>0.67 ± 0.13</td>
</tr>
<tr>
<td>N</td>
<td>7</td>
<td>14</td>
<td>$0^+, 1$</td>
<td>2313</td>
<td>1.00 ± 0.56</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>12</td>
<td>$2^+$</td>
<td>4439</td>
<td>4.29 ± 1.22</td>
</tr>
</tbody>
</table>

Total Yield = (31.3 ± 2.1)\%
Table 10. Yields from the $\pi^- + ^{23}\text{Na}$ Experiment by Daughter Nuclei

<table>
<thead>
<tr>
<th>Daughter Nucleus</th>
<th>% Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{22}\text{Ne}$</td>
<td>2.47 ± 0.88</td>
</tr>
<tr>
<td>$^{21}\text{Ne}$</td>
<td>3.20 ± 0.36</td>
</tr>
<tr>
<td>$^{21}\text{F}$</td>
<td>5.14 ± 0.98</td>
</tr>
<tr>
<td>$^{20}\text{Ne}$</td>
<td>7.56 ± 0.54</td>
</tr>
<tr>
<td>$^{20}\text{F}$</td>
<td>7.2 ± 0.17</td>
</tr>
<tr>
<td>$^{19}\text{Ne}$</td>
<td>1.52 ± 0.31</td>
</tr>
<tr>
<td>$^{19}\text{F}$</td>
<td>1.99 ± 0.36</td>
</tr>
<tr>
<td>$^{18}\text{O}$</td>
<td>1.67 ± 0.24</td>
</tr>
<tr>
<td>$^{17}\text{F}$</td>
<td>0.08 ± 0.03</td>
</tr>
<tr>
<td>$^{17}\text{O}$</td>
<td>1.69 ± 0.47</td>
</tr>
<tr>
<td>$^{14}\text{N}$</td>
<td>1.00 ± 0.56</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>4.29 ± 1.22</td>
</tr>
</tbody>
</table>
Table 11. Yields from the $\pi^- + {}^{23}\text{Na}$ Experiment by Mass Number $A$

<table>
<thead>
<tr>
<th>$A$</th>
<th>% Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>2.47 ± .88</td>
</tr>
<tr>
<td>21</td>
<td>8.34 ± 1.04</td>
</tr>
<tr>
<td>20</td>
<td>8.28 ± .57</td>
</tr>
<tr>
<td>19</td>
<td>3.51 ± .48</td>
</tr>
<tr>
<td>18</td>
<td>1.67 ± .24</td>
</tr>
<tr>
<td>17</td>
<td>1.77 ± .47</td>
</tr>
<tr>
<td>14</td>
<td>1.00 ± .56</td>
</tr>
<tr>
<td>12</td>
<td>4.29 ± 1.22</td>
</tr>
</tbody>
</table>
Table 12
Peaks in the $\pi^- + ^{32}$S Spectrum

<table>
<thead>
<tr>
<th>$E$(keV)</th>
<th>Identification</th>
<th>$(N_Y \pm \Delta N_Y)$</th>
<th>$(N_Y \pm \Delta N_Y) \times \eta/\varepsilon$</th>
<th>Raw Yields (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>440</td>
<td>$^{23}$Na 1 $\rightarrow$ 0</td>
<td>2650 $\pm$ 197</td>
<td>8.21 $\times$ 10$^7$ $\pm$ 6.11 $\times$ 10$^6$</td>
<td>.130 $\pm$ .10</td>
</tr>
<tr>
<td>586</td>
<td>$^{25}$Mg 1 $\rightarrow$ 0</td>
<td>1300 $\pm$ 126</td>
<td>5.64 $\times$ 10$^7$ $\pm$ 5.48 $\times$ 10$^6$</td>
<td>.089 $\pm$ .09</td>
</tr>
<tr>
<td>678</td>
<td>$^{30}$P 1 $\rightarrow$ 0</td>
<td>600 $\pm$ 122</td>
<td>3.02 $\times$ 10$^7$ $\pm$ 6.13 $\times$ 10$^6$</td>
<td>.48 $\pm$ .10</td>
</tr>
<tr>
<td>709</td>
<td>$^{30}$P 2 $\rightarrow$ 0</td>
<td>1000 $\pm$ 229</td>
<td>5.23 $\times$ 10$^7$ $\pm$ 1.20 $\times$ 10$^7$</td>
<td>.83 $\pm$ .19</td>
</tr>
<tr>
<td>745</td>
<td>$^{30}$P 3 $\rightarrow$ 2</td>
<td>650 $\pm$ 131</td>
<td>3.60 $\times$ 10$^7$ $\pm$ 7.27 $\times$ 10$^6$</td>
<td>.57 $\pm$ .12</td>
</tr>
<tr>
<td>752</td>
<td>$^{31}$Si 1 $\rightarrow$ 0</td>
<td>250 $\pm$ 113</td>
<td>1.39 $\times$ 10$^7$ $\pm$ 6.29 $\times$ 10$^6$</td>
<td>.22 $\pm$ .10</td>
</tr>
<tr>
<td>777</td>
<td>$^{30}$P 3 $\rightarrow$ 1</td>
<td>150 $\pm$ 92</td>
<td>8.63 $\times$ 10$^6$ $\pm$ 5.30 $\times$ 10$^6$</td>
<td>.14 $\pm$ .09</td>
</tr>
<tr>
<td>1013</td>
<td>$^{27}$Al 2 $\rightarrow$ 0</td>
<td>780 $\pm$ 160</td>
<td>5.90 $\times$ 10$^7$ $\pm$ 1.21 $\times$ 10$^7$</td>
<td>.93 $\pm$ .19</td>
</tr>
<tr>
<td>1129</td>
<td>$^{26}$Mg 2 $\rightarrow$ 1</td>
<td>600 $\pm$ 96</td>
<td>5.05 $\times$ 10$^7$ $\pm$ 8.07 $\times$ 10$^6$</td>
<td>.80 $\pm$ .13</td>
</tr>
<tr>
<td>1238</td>
<td>$^{19}$F 3 $\rightarrow$ 1</td>
<td>410 $\pm$ 92</td>
<td>3.80 $\times$ 10$^7$ $\pm$ 8.52 $\times$ 10$^6$</td>
<td>.60 $\pm$ .13</td>
</tr>
<tr>
<td>1273</td>
<td>$^{29}$Si 1 $\rightarrow$ 0</td>
<td>3600 $\pm$ 146</td>
<td>3.40 $\times$ 10$^8$ $\pm$ 1.38 $\times$ 10$^7$</td>
<td>5.37 $\pm$ .22</td>
</tr>
<tr>
<td>1369</td>
<td>$^{24}$Mg 1 $\rightarrow$ 0</td>
<td>1826 $\pm$ 125</td>
<td>1.84 $\times$ 10$^8$ $\pm$ 1.26 $\times$ 10$^7$</td>
<td>2.90 $\pm$ .20</td>
</tr>
<tr>
<td>1483</td>
<td>$^{30}$P 8 $\rightarrow$ 3</td>
<td>403 $\pm$ 94</td>
<td>4.50 $\times$ 10$^7$ $\pm$ 1.05 $\times$ 10$^7$</td>
<td>.71 $\pm$ .17</td>
</tr>
<tr>
<td>1516</td>
<td>$^{28}$P 5 $\rightarrow$ 0</td>
<td>350 $\pm$ 82</td>
<td>3.93 $\times$ 10$^7$ $\pm$ 9.18 $\times$ 10$^6$</td>
<td>.62 $\pm$ .14</td>
</tr>
</tbody>
</table>
Table 12 - (continued)

<table>
<thead>
<tr>
<th>Energy</th>
<th>Isotope</th>
<th>Transition</th>
<th>$A$ (MeV)</th>
<th>$B$ (MeV)</th>
<th>$C$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1595</td>
<td>$^{29}\text{Si}$</td>
<td>$5 \rightarrow 2$</td>
<td>740 ± 110</td>
<td>$8.82 \times 10^7 \pm 1.31 \times 10^7$</td>
<td>1.39 ± .21</td>
</tr>
<tr>
<td>1611</td>
<td>$^{25}\text{Mg}$</td>
<td>$3 \rightarrow 0$</td>
<td>200 ± 98</td>
<td>$2.40 \times 10^7 \pm 1.18 \times 10^7$</td>
<td>.38 ± .19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>or</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$^{25}\text{Al}$</td>
<td>$3 \rightarrow 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1633</td>
<td>$^{20}\text{Ne}$</td>
<td>$1 \rightarrow 0$</td>
<td>830 ± 116</td>
<td>$9.96 \times 10^7 \pm 1.39 \times 10^7$</td>
<td>1.57 ± .22</td>
</tr>
<tr>
<td>1779</td>
<td>$^{28}\text{Si}$</td>
<td>$1 \rightarrow 0$</td>
<td>3293 ± 212</td>
<td>$4.30 \times 10^8 \pm 2.77 \times 10^7$</td>
<td>6.79 ± .44</td>
</tr>
<tr>
<td>1809</td>
<td>$^{26}\text{Mg}$</td>
<td>$1 \rightarrow 0$</td>
<td>2131 ± 174</td>
<td>$2.82 \times 10^8 \pm 2.30 \times 10^7$</td>
<td>4.45 ± .36</td>
</tr>
<tr>
<td>2028</td>
<td>$^{29}\text{Si}$</td>
<td>$2 \rightarrow 0$</td>
<td>951 ± 88</td>
<td>$1.41 \times 10^8 \pm 1.31 \times 10^7$</td>
<td>2.22 ± .21</td>
</tr>
<tr>
<td>2235</td>
<td>$^{30}\text{Si}$</td>
<td>$1 \rightarrow 0$</td>
<td>2160 ± 203</td>
<td>$3.52 \times 10^8 \pm 3.31 \times 10^7$</td>
<td>5.55 ± .52</td>
</tr>
<tr>
<td>2260</td>
<td>$^{30}\text{P}$</td>
<td>$8 \rightarrow 1$</td>
<td>351 ± 82</td>
<td>$5.80 \times 10^7 \pm 1.36 \times 10^7$</td>
<td>.92 ± .22</td>
</tr>
<tr>
<td>2726</td>
<td>$^{30}\text{P}$</td>
<td>$6 \rightarrow 0$</td>
<td>298 ± 95</td>
<td>$6.02 \times 10^7 \pm 1.91 \times 10^7$</td>
<td>.95 ± .30</td>
</tr>
</tbody>
</table>
Table 13. Corrected Yields from the $\pi^- + ^{32}\text{S}$ Experiment by Level

$\pi^-$ Absorption on $^{32}\text{S}$

<table>
<thead>
<tr>
<th>Daughter Nucleus</th>
<th>$J^P, T$</th>
<th>Energy (keV)</th>
<th>Yield (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{31}\text{Si}$</td>
<td>$1/2^+$</td>
<td>752</td>
<td>.22 ± .10</td>
</tr>
<tr>
<td>$^{30}\text{P}$</td>
<td>$2^+, 1$</td>
<td>2938</td>
<td>2.20 ± .38</td>
</tr>
<tr>
<td>$^{30}\text{P}$</td>
<td>$2^+$</td>
<td>2733</td>
<td>.95 ± .30</td>
</tr>
<tr>
<td>$^{30}\text{P}$</td>
<td>$2^+$</td>
<td>1454</td>
<td>.00 ± .23</td>
</tr>
<tr>
<td>$^{30}\text{P}$</td>
<td>$1^+$</td>
<td>709</td>
<td>.26 ± .23</td>
</tr>
<tr>
<td>$^{30}\text{Si}$</td>
<td>$2^+$</td>
<td>2235</td>
<td>5.55 ± .52</td>
</tr>
<tr>
<td>$^{29}\text{Si}$</td>
<td>$7/2^-$</td>
<td>3624</td>
<td>1.56 ± .24</td>
</tr>
<tr>
<td>$^{29}\text{Si}$</td>
<td>$5/2^+$</td>
<td>2028</td>
<td>.96 ± .30</td>
</tr>
<tr>
<td>$^{29}\text{Si}$</td>
<td>$3/2^+$</td>
<td>1273</td>
<td>5.24 ± .22</td>
</tr>
<tr>
<td>$^{28}\text{P}$</td>
<td></td>
<td>1516</td>
<td>.62 ± .14</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>$2^+$</td>
<td>1779</td>
<td>6.79 ± .44</td>
</tr>
<tr>
<td>$^{27}\text{Al}$</td>
<td>$3/2^+$</td>
<td>1013</td>
<td>.96 ± .20</td>
</tr>
<tr>
<td>$^{26}\text{Mg}$</td>
<td>$2^+$</td>
<td>2938</td>
<td>.89 ± .14</td>
</tr>
<tr>
<td>$^{26}\text{Mg}$</td>
<td>$2^+$</td>
<td>1809</td>
<td>3.65 ± .38</td>
</tr>
<tr>
<td>$^{25}\text{Mg or } ^{25}\text{Al}$</td>
<td>$7/2^+$</td>
<td>1611</td>
<td>.38 ± .19</td>
</tr>
<tr>
<td>$^{25}\text{Mg}$</td>
<td>$1/2^+$</td>
<td>585</td>
<td>.89 ± .09</td>
</tr>
<tr>
<td>$^{24}\text{Mg}$</td>
<td>$2^+$</td>
<td>1369</td>
<td>2.90 ± .20</td>
</tr>
<tr>
<td>$^{23}\text{Na}$</td>
<td>$5/2^+$</td>
<td>440</td>
<td>1.30 ± .10</td>
</tr>
<tr>
<td>$^{20}\text{Ne}$</td>
<td>$2^+$</td>
<td>1633</td>
<td>1.57 ± .22</td>
</tr>
<tr>
<td>$^{19}\text{F}$</td>
<td>$5/2^-$</td>
<td>1346</td>
<td>.60 ± .13</td>
</tr>
</tbody>
</table>

Total Yield = (37.5 ± 1.2)%
Table 14. Yields from the $\pi^- + ^{32}\text{S}$ Experiment by Daughter Nuclei

<table>
<thead>
<tr>
<th>Daughter Nucleus</th>
<th>% Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{31}\text{Si}$</td>
<td>.22 ± .10</td>
</tr>
<tr>
<td>$^{30}\text{P}$</td>
<td>3.41 ± .58</td>
</tr>
<tr>
<td>$^{30}\text{Si}$</td>
<td>5.55 ± .52</td>
</tr>
<tr>
<td>$^{29}\text{Si}$</td>
<td>7.76 ± .44</td>
</tr>
<tr>
<td>$^{28}\text{P}$</td>
<td>.62 ± .14</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>6.79 ± .44</td>
</tr>
<tr>
<td>$^{27}\text{Al}$</td>
<td>.96 ± .20</td>
</tr>
<tr>
<td>$^{26}\text{Mg}$</td>
<td>4.54 ± .40</td>
</tr>
<tr>
<td>$^{25}\text{Mg}^*$</td>
<td>1.27 ± .21</td>
</tr>
<tr>
<td>$^{24}\text{Mg}$</td>
<td>2.90 ± .20</td>
</tr>
<tr>
<td>$^{23}\text{Na}$</td>
<td>1.30 ± .10</td>
</tr>
<tr>
<td>$^{20}\text{Ne}$</td>
<td>1.57 ± .22</td>
</tr>
<tr>
<td>$^{19}\text{F}$</td>
<td>.60 ± .13</td>
</tr>
</tbody>
</table>

*This assumes the 1611-keV peak is from the third excited state of $^{25}\text{Mg}$.
Table 15. Yields from the $\pi^- + ^{32}\text{S}$ Experiment by Mass Number $A$

<table>
<thead>
<tr>
<th>$A$</th>
<th>$%$ Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>$0.22 \pm 0.10$</td>
</tr>
<tr>
<td>30</td>
<td>$8.96 \pm 0.78$</td>
</tr>
<tr>
<td>29</td>
<td>$7.76 \pm 0.44$</td>
</tr>
<tr>
<td>28</td>
<td>$7.41 \pm 0.46$</td>
</tr>
<tr>
<td>27</td>
<td>$0.96 \pm 0.20$</td>
</tr>
<tr>
<td>26</td>
<td>$4.54 \pm 0.40$</td>
</tr>
<tr>
<td>25</td>
<td>$1.27 \pm 0.21$</td>
</tr>
<tr>
<td>24</td>
<td>$2.90 \pm 0.20$</td>
</tr>
<tr>
<td>23</td>
<td>$1.30 \pm 0.10$</td>
</tr>
<tr>
<td>20</td>
<td>$1.57 \pm 0.22$</td>
</tr>
<tr>
<td>19</td>
<td>$0.60 \pm 0.13$</td>
</tr>
<tr>
<td>E(keV)</td>
<td>Identification</td>
</tr>
<tr>
<td>--------</td>
<td>----------------</td>
</tr>
<tr>
<td>333</td>
<td>(^{38}\text{K}) (2 \rightarrow 1)</td>
</tr>
<tr>
<td>671</td>
<td>(^{38}\text{Cl}) (1 \rightarrow 0)</td>
</tr>
<tr>
<td>1267</td>
<td>(^{39}\text{Al}) (1 \rightarrow 0, \text{or})</td>
</tr>
<tr>
<td>31_p</td>
<td>(^{31}\text{P}) (1 \rightarrow 0, \text{or})</td>
</tr>
<tr>
<td>30_Si</td>
<td>(^{30}\text{Si}) (2 \rightarrow 1, \text{or})</td>
</tr>
<tr>
<td>29_Si</td>
<td>(^{29}\text{Si}) (1 \rightarrow 0).</td>
</tr>
<tr>
<td>1369</td>
<td>(^{24}\text{Mg}) (1 \rightarrow 0)</td>
</tr>
<tr>
<td>1410</td>
<td>(^{37}\text{Ar}) (1 \rightarrow 0)</td>
</tr>
<tr>
<td>1517</td>
<td>(^{39}\text{Ar}) (2 \rightarrow 0)</td>
</tr>
<tr>
<td>1571</td>
<td>(^{38}\text{K}) (3 \rightarrow 1)</td>
</tr>
<tr>
<td>1611</td>
<td>(^{37}\text{Ar}) (2 \rightarrow 0)</td>
</tr>
<tr>
<td>1633</td>
<td>(^{20}\text{Ne}) (1 \rightarrow 0)</td>
</tr>
<tr>
<td>1644</td>
<td>(^{38}\text{Ar}) (3 \rightarrow 1)</td>
</tr>
<tr>
<td>1727</td>
<td>(^{37}\text{Cl}) (1 \rightarrow 0)</td>
</tr>
</tbody>
</table>
Table 16 - (continued)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1779</td>
<td>$^{28}_\text{Si}$</td>
<td>1 $\rightarrow$ 0</td>
<td>1809 ± 278</td>
<td>$2.61 \times 10^8 \pm 4.02 \times 10^7$</td>
</tr>
<tr>
<td>1809</td>
<td>$^{26}_\text{Mg}$</td>
<td>1 $\rightarrow$ 0</td>
<td>527 ± 123</td>
<td>$7.75 \times 10^7 \pm 1.81 \times 10^7$</td>
</tr>
<tr>
<td>1970</td>
<td>$^{36}_\text{Ar}$</td>
<td>1 $\rightarrow$ 0</td>
<td>4291 ± 410</td>
<td>$6.92 \times 10^8 \pm 6.61 \times 10^7$</td>
</tr>
<tr>
<td>2028</td>
<td>$^{29}_\text{Si}$</td>
<td>2 $\rightarrow$ 0</td>
<td>544 ± 152</td>
<td>$8.84 \times 10^7 \pm 2.46 \times 10^7$</td>
</tr>
<tr>
<td>2127</td>
<td>$^{34}_\text{S}$</td>
<td>1 $\rightarrow$ 0</td>
<td>985 ± 119</td>
<td>$1.69 \times 10^8 \pm 2.04 \times 10^7$</td>
</tr>
<tr>
<td>2168</td>
<td>$^{38}_\text{Ar}$</td>
<td>1 $\rightarrow$ 0</td>
<td>1210 ± 123</td>
<td>$2.12 \times 10^8 \pm 2.15 \times 10^7$</td>
</tr>
<tr>
<td>2208</td>
<td>$^{36}_\text{Ar}$</td>
<td>2 $\rightarrow$ 1</td>
<td>835 ± 171</td>
<td>$1.49 \times 10^8 \pm 3.05 \times 10^7$</td>
</tr>
<tr>
<td>2230</td>
<td>$^{32}_\text{S}$</td>
<td>1 $\rightarrow$ 0</td>
<td>1370 ± 160</td>
<td>$2.48 \times 10^8 \pm 2.89 \times 10^7$</td>
</tr>
</tbody>
</table>
Table 17. Corrected Yields from the $\pi^- + {}^{40}\text{Ca}$ Experiment by Level

$\pi^-$ Absorption on $^{40}\text{Ca}$

<table>
<thead>
<tr>
<th>Daughter Nucleus</th>
<th>$J^P$, $T$</th>
<th>Energy (keV)</th>
<th>Yield (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{39}\text{Ar}$ (or $^{31}\text{P}$, $^{30}\text{Si}$, or $^{29}\text{Si}$)</td>
<td>$3/2^-$</td>
<td>1267</td>
<td>$2.74 \pm 0.38$ ?</td>
</tr>
<tr>
<td>$^{39}\text{Ar}$ (?)</td>
<td>$3/2^+$</td>
<td>1517</td>
<td>$1.38 \pm 0.26$ ?</td>
</tr>
<tr>
<td>$^{38}\text{K}$</td>
<td>$1^+, 0$</td>
<td>461</td>
<td>$0.92 \pm 0.09$</td>
</tr>
<tr>
<td>$^{38}\text{K}$</td>
<td>$1^+, 0$</td>
<td>1700</td>
<td>$0.66 \pm 0.15$</td>
</tr>
<tr>
<td>$^{38}\text{Ar}$</td>
<td>$2^+$</td>
<td>2168</td>
<td>$1.33 \pm 0.25$</td>
</tr>
<tr>
<td>$^{38}\text{Ar}$</td>
<td>$3^-$</td>
<td>3810</td>
<td>$0.46 \pm 0.17$</td>
</tr>
<tr>
<td>$^{38}\text{Cl}$</td>
<td>$5^-$</td>
<td>671</td>
<td>$0.38 \pm 0.14$</td>
</tr>
<tr>
<td>$^{37}\text{Ar}$</td>
<td>$1/2^+$</td>
<td>1410</td>
<td>$0.26 \pm 0.17$</td>
</tr>
<tr>
<td>$^{37}\text{Ar}$</td>
<td>$7/2^-$</td>
<td>1611</td>
<td>$1.86 \pm 0.16$</td>
</tr>
<tr>
<td>$^{37}\text{Cl}$</td>
<td>$1/2$</td>
<td>1727</td>
<td>$1.80 \pm 0.24$</td>
</tr>
<tr>
<td>$^{36}\text{Ar}$</td>
<td>$2^+$</td>
<td>1970</td>
<td>$4.58 \pm 0.62$</td>
</tr>
<tr>
<td>$^{36}\text{Ar}$</td>
<td>$3^-$</td>
<td>4178</td>
<td>$1.34 \pm 0.28$</td>
</tr>
<tr>
<td>$^{34}\text{S}$</td>
<td>$2^+$</td>
<td>2127</td>
<td>$1.42 \pm 0.17$</td>
</tr>
<tr>
<td>$^{32}\text{S}$</td>
<td>$2^+$</td>
<td>2230</td>
<td>$2.10 \pm 0.24$</td>
</tr>
<tr>
<td>$^{29}\text{Si}$</td>
<td>$5/2^+$</td>
<td>2028</td>
<td>$0.79 \pm 0.22$</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>$2^+$</td>
<td>1779</td>
<td>$2.21 \pm 0.34$</td>
</tr>
<tr>
<td>$^{26}\text{Mg}$</td>
<td>$2^+$</td>
<td>1809</td>
<td>$0.65 \pm 0.15$</td>
</tr>
<tr>
<td>$^{24}\text{Mg}$</td>
<td>$2^+$</td>
<td>1369</td>
<td>$2.65 \pm 0.23$</td>
</tr>
<tr>
<td>$^{20}\text{Ne}$</td>
<td>$2^+$</td>
<td>1633</td>
<td>$2.39 \pm 0.78$</td>
</tr>
</tbody>
</table>

Total Yield = $(29.9 \pm 1.4)\%$
Table 18. Yields from the $\pi^- + {}^{40}\text{Ca}$ Experiment by Daughter Nuclei

<table>
<thead>
<tr>
<th>Daughter Nucleus</th>
<th>% Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{39}\text{Ar(?)}$</td>
<td>$4.12 \pm .46(?)$</td>
</tr>
<tr>
<td>$^{38}\text{K}$</td>
<td>$1.58 \pm .17$</td>
</tr>
<tr>
<td>$^{38}\text{Ar}$</td>
<td>$1.79 \pm .30$</td>
</tr>
<tr>
<td>$^{38}\text{Cl}$</td>
<td>$.38 \pm .14$</td>
</tr>
<tr>
<td>$^{37}\text{Ar}$</td>
<td>$2.12 \pm .23$</td>
</tr>
<tr>
<td>$^{37}\text{Cl}$</td>
<td>$1.80 \pm .24$</td>
</tr>
<tr>
<td>$^{36}\text{Ar}$</td>
<td>$5.92 \pm .68$</td>
</tr>
<tr>
<td>$^{34}\text{S}$</td>
<td>$1.42 \pm .17$</td>
</tr>
<tr>
<td>$^{32}\text{S}$</td>
<td>$2.10 \pm .24$</td>
</tr>
<tr>
<td>$^{29}\text{Si}$</td>
<td>$.79 \pm .22$</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>$2.21 \pm .34$</td>
</tr>
<tr>
<td>$^{26}\text{Mg}$</td>
<td>$.65 \pm .15$</td>
</tr>
<tr>
<td>$^{24}\text{Mg}$</td>
<td>$2.65 \pm .23$</td>
</tr>
<tr>
<td>$^{20}\text{Ne}$</td>
<td>$2.39 \pm .78$</td>
</tr>
</tbody>
</table>
Table 19. Yields from the $\pi^- + ^{40}\text{Ca}$ Experiment by Mass Number A

<table>
<thead>
<tr>
<th>A</th>
<th>% Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>39(?)</td>
<td>$4.12 \pm .46(%)$</td>
</tr>
<tr>
<td>38</td>
<td>$3.75 \pm .37$</td>
</tr>
<tr>
<td>37</td>
<td>$3.92 \pm .33$</td>
</tr>
<tr>
<td>36</td>
<td>$5.92 \pm .68$</td>
</tr>
<tr>
<td>34</td>
<td>$1.42 \pm .17$</td>
</tr>
<tr>
<td>32</td>
<td>$2.10 \pm .24$</td>
</tr>
<tr>
<td>29</td>
<td>$.79 \pm .22$</td>
</tr>
<tr>
<td>28</td>
<td>$2.21 \pm .34$</td>
</tr>
<tr>
<td>26</td>
<td>$.65 \pm .15$</td>
</tr>
<tr>
<td>24</td>
<td>$2.65 \pm .23$</td>
</tr>
<tr>
<td>20</td>
<td>$2.39 \pm .78$</td>
</tr>
</tbody>
</table>
Table 20. Mean Numbers of Protons, Neutrons, and Nucleons Emitted per Observed $\pi^-$ Absorption.

<table>
<thead>
<tr>
<th>Target</th>
<th>$\overline{#p}$</th>
<th>$\overline{#n}$</th>
<th>$\overline{#N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na</td>
<td>$2.11 \pm .27$</td>
<td>$2.14 \pm .30$</td>
<td>$4.25 \pm .56$</td>
</tr>
<tr>
<td>S</td>
<td>$2.73 \pm .12$</td>
<td>$1.89 \pm .09$</td>
<td>$4.63 \pm .21$</td>
</tr>
<tr>
<td>Ca</td>
<td>$3.96 \pm .35$</td>
<td>$2.96 \pm .32$</td>
<td>$6.92 \pm .66$</td>
</tr>
</tbody>
</table>
Table 21. Mean Recoil Momenta Calculated from Analyses of Doppler Broadened Lines

<table>
<thead>
<tr>
<th>Target</th>
<th>Daughter</th>
<th>State</th>
<th>Mean Recoil Momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{14}_N$</td>
<td>$^{12}_C$</td>
<td>$2^+$</td>
<td>4439 keV</td>
</tr>
<tr>
<td>$^{40}_{Ca}$</td>
<td>$^{39}_Ar(?)$</td>
<td>$3/2^-$</td>
<td>1267 keV</td>
</tr>
</tbody>
</table>

**Short-Lived States**

<table>
<thead>
<tr>
<th>Target</th>
<th>Daughter</th>
<th>State</th>
<th>Mean Recoil Momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{32}_S$</td>
<td>$^{28}_Si$</td>
<td>$2^+$</td>
<td>1779 keV</td>
</tr>
<tr>
<td>$^{40}_{Ca}$</td>
<td>$^{37}_Cl$</td>
<td>$(1/2)$</td>
<td>1727 keV</td>
</tr>
<tr>
<td>$^{40}_{Ca}$</td>
<td>$^{28}_Si$</td>
<td>$2^+$</td>
<td>1779 keV</td>
</tr>
<tr>
<td>$^{40}_{Ca}$</td>
<td>$^{36}_Ar$</td>
<td>$2^+$</td>
<td>1970 keV</td>
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**States Corrected for Slowing Down with Blaugrand Procedure**

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**States Corrected for Slowing Down with Lewis Procedure**
Table 22. Variation of Observed % Yields for $\pi^- + ^{32}S$ with N and Z

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Table 23. Variation of Observed % Yields for $\pi^- + ^{32}\text{S}$ with N and Z

Including Enhancement Correction

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Table 24. ALICE Predictions for $\pi^- + ^{32}\text{S}$ on an N versus Z Grid, 50% 140 MeV on $^{32}\text{S}$, 50% 30 MeV on $^{30}\text{P}$.

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Table 25. Variation of Observed % Yields for $\pi^- + {}^{40}\text{Ca}$ with $N$ and $Z$

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N = Z
Table 26. Variation of Observed % Yields for $\pi^- + ^{40}\text{Ca}$ with N and Z

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Table 27. ALICE Predictions for $\pi^- + {}^{40}\text{Ca}$ on an N versus Z Grid, 50%

140 MeV on $^{40}\text{Ca}$, 50% on 30 MeV on $^{38}\text{K}$

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Table 28. Comparison of 220-MeV $\pi^{-}$ $^{40}$Ca Data with Fraenkel Calculation

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<td>- 610</td>
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EXPERIMENTAL APPARATUS

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VIII. VITA

Carey Elliott Stronach was born on August 8, 1940, in Boston, Massachusetts. He moved to Petersburg, Virginia with his parents in 1942, attended the Petersburg public schools, and graduated from Petersburg High School in 1957. He then entered the University of Richmond and in 1961 received the Bachelor of Science degree in physics and mathematics. He pursued graduate study in physics at the University of Virginia and received the Master of Science degree in 1963. His master's thesis was entitled Photoprotons from Aluminum. He continued his studies at Virginia, but withdrew in 1965 at the time of his mother's terminal illness.

He was appointed instructor of physics at Virginia State College in 1965 and was promoted to assistant professor in 1966. He began doctoral studies at the College of William and Mary in 1970, while continuing to teach at Virginia State. He was a National Science Foundation Science Faculty Fellow during the 1971-72 academic year, and resumed teaching duties at Virginia State in September, 1972.

He is a member of the Phi Beta Kappa, Sigma Xi, Sigma Pi Sigma, and Pi Mu Epsilon honorary fraternities, and belongs to the American Physical Society, the American Association of Physics Teachers, the American Association for the Advancement of Science, the National Institute of Science, and the Physics Club of Richmond. He has been active in academic and community affairs, having served as president of the Virginia
State College Chapter of the American Association of University Professors, and as president of the Petersburg Area Chapter of the Virginia Council on Human Relations. He is currently corresponding secretary of the Petersburg City Democratic Committee, and is a member of the Commission on Community Relations Affairs of the City of Petersburg.

He is married to the former Joan Alice Venner, a native of Halifax, Nova Scotia. They have two sons, John, 8, and Howard, 3.