Effects of the small components of the nuclear wave function on threshold pionic disintegration of the deuteron

Etienne Amedee. Delacroix

College of William & Mary - Arts & Sciences

Follow this and additional works at: https://scholarworks.wm.edu/etd

Recommended Citation
https://dx.doi.org/doi:10.21220/s2-cc0x-t908

This Dissertation is brought to you for free and open access by the Theses, Dissertations, & Master Projects at W&M ScholarWorks. It has been accepted for inclusion in Dissertations, Theses, and Masters Projects by an authorized administrator of W&M ScholarWorks. For more information, please contact scholarworks@wm.edu.
INFORMATION TO USERS

This material was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.

2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in "sectioning" the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again — beginning below the first row and continuing on until complete.

4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from "photographs" if essential to the understanding of the dissertation. Silver prints of "photographs" may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.

5. PLEASE NOTE: Some pages may have indistinct print. Filmed as received.

University Microfilms International
300 North Zeeb Road
Ann Arbor, Michigan 48106 USA
St. John's Road, Tyttenham Green
High Wycombe, Bucks, England HP10 9HR
DELCROIX, Etienne Amedee, 1947-
EFFECTS OF THE SMALL COMPONENTS OF THE
NUCLEAR WAVE FUNCTION ON THRESHOLD PIONIC
DISINTEGRATION OF THE DEUTERON.

The College of William and Mary in Virginia,
Ph.D., 1977
Physics, nuclear

University Microfilms International, Ann Arbor, Michigan 48106
EFFECTS OF THE SMALL COMPONENTS OF THE NUCLEAR WAVE FUNCTION
ON THRESHOLD PIONIC DISINTEGRATION OF THE DEUTERON

A Dissertation
Presented to
The Faculty of The Department of Physics
The College of William and Mary in Virginia

In Partial Fulfillment
Of the Requirements for the Degree of
Doctor of Philosophy

by
Etienne Amedee Delacroix
June 1977
APPROVAL SHEET

This dissertation is submitted in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

Etienne Amedee Delacroix

Approved, June 1977

Franz L. Gross
Franz L. Gross

Karl Holinde
University of Bonn, Germany

Carl E. Carlson

Charles F. Perdrisat

Herbert O. Funsten
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>v</td>
</tr>
<tr>
<td>I.  INTRODUCTION AND GENERAL OUTLINE</td>
<td>1</td>
</tr>
<tr>
<td>II. CLASSIFICATION OF RELATIVISTIC CORRECTIONS</td>
<td>25</td>
</tr>
<tr>
<td>III. AMBIGUITY IN THE $\pi N$ COUPLING AND LOW-ENERGY $\pi N$ SCATTERING</td>
<td>49</td>
</tr>
<tr>
<td>IV. THRESHOLD PIONIC DISINTEGRATION OF DEUTERIUM</td>
<td>60</td>
</tr>
<tr>
<td>V.  NUMERICAL RESULTS AND CONCLUSIONS</td>
<td>84</td>
</tr>
<tr>
<td>APPENDIX</td>
<td></td>
</tr>
<tr>
<td>I.  POLE DIAGRAM CONTRIBUTION TO $\pi d \rightarrow pp$</td>
<td>94</td>
</tr>
<tr>
<td>II. MATRIX REPRESENTATION OF NN SPIN HARMONIC FUNCTIONS</td>
<td>102</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>106</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENTS

My deep gratitude goes to the entire William and Mary Physics Department for its continuous, unquestioning support during seven years, thus allowing for a many faceted personal growth.

In particular to Professor Franz Gross for his encouragement and patient guidance throughout this long and difficult apprenticeship.

To Professors Carl Carlson and Edward Remler for numerous enlightening discussions and helpful suggestions.

To Warren Buck and John Hornstein for their essential contributions.

To my friend Rafael for sharing the dark side of things, over and over again.

To Stephanie who performed a miracle.
ABSTRACT

The origin of the various relativistic corrections to the standard N.R. matrix elements for low-energy processes involving the two-nucleon system is examined within the framework of Gross's quasi-potential equation. The leading corrections are seen to arise from the small relativistic components of the NN wavefunction which are generated by the antinucleon degrees of freedom inherently present in a relativistic theory.

The effects of these small components of the nuclear wavefunctions are calculated for threshold pionic disintegration of the deuteron where they are shown to make a measurable contribution. Our model indicates that the inclusion of the small components of the wavefunctions for any mixture of the P,V,-P,S. $\pi N$ couplings requires that the reaction mechanism be treated consistently with a correct description of low energy $\pi N$ scattering.
EFFECTS OF THE SMALL COMPONENTS OF THE NUCLEAR WAVEFUNCTION ON THRESHOLD PIONIC DISINTEGRATION OF THE DEUTERON
I. INTRODUCTION AND GENERAL OUTLINE

This dissertation concerns itself with the application of the relativistic theory of nuclear forces recently developed by Gross\textsuperscript{1} to the description of low energy processes and in particular to threshold pionic disintegration of the deuteron for which we calculate the leading relativistic corrections.\textsuperscript{2} These corrections will be seen to arise from intrinsically relativistic components of the NN wavefunction associated with the antinucleon degrees of freedom inherently present in a relativistic theory.

Because of the difficulties presented by the development of a non-trivial relativistic description of many body systems, nuclear processes have been traditionally analyzed in terms of standard non-relativistic quantum mechanics: cross sections are obtained by calculating between the relevant nuclear wavefunctions, the matrix elements of some transition operators selected to describe the mechanism of interaction of the probe (pion, photon, weak current) with the nuclear system. Figure 1 represents symbolically, in bracket notations, the N.R. matrix elements for some typical mechanisms: Direct absorption of the probe on one nucleon (1a) is generally known as the "impulse approximation". Pions will often scatter on a first nucleon before they can be absorbed by another, a mechanism often called "rescattering" (1b). It is also possible for the probe to interact directly with one of the nuclear mesons, giving rise to what one can properly call a meson
exchange correction (lc) to the impulse approximation

\[
\text{impulse: } \sum_i \langle f | \hat{\psi} \psi | i \rangle
\]  
\[
\text{rescattering: } \sum_{i \neq j} \langle f | \hat{\psi} \psi | i \rangle
\]  
\[
\text{meson exchange: } \sum_{i \neq j} \langle f | \hat{\psi} \psi | i \rangle
\]

\[\text{Fig. 1. N.R. matrix elements of transition operators}\]

These matrix elements involve standard Schrödinger wavefunctions, on mass shell nucleons and integrations over three dimensional space, i.e. they take the general form

\[\int d^3 p \ \psi^*_f \ T \ \psi_i\]

and should be regarded as our best available approximation for the corresponding, and in principle exact, relativistic amplitudes which involve instead covariant interaction vertices, off mass shell relativistic nucleons (i.e. propagating also as antinucleons) and integrations over four dimensional space. These latter quantities should be represented by Feynman diagrams.

While the construction and the evaluation of these diagrams presents considerable difficulties for many nucleon systems, the task remains quite manageable for processes involving only two nucleons.

In such cases Feynman diagrams are still relatively simple (see Fig. 2)
and the relation between covariant NN vertices and standard wavefunctions can be established in a straightforward manner. Figure 2 shows the Feynman diagrams corresponding to the mechanisms of Fig. 1 for reactions involving deuterons.

\begin{align*}
\text{impulse:} & \quad (2a) \\
\text{rescattering:} & \quad (2b) \\
\text{meson exchange:} & \quad (2c)
\end{align*}

Fig. 2. Examples of Feynman diagrams for Deuteron

Disintegration by photons and pions

By allowing a direct comparison of nonrelativistic matrix elements to the reduction of corresponding exact relativistic amplitudes, processes involving the two nucleons system give us the possibility to gain much insight in the nature and the importance of relativistic corrections to the standard description of nuclear processes. In order to illustrate the following discussion we first write down the Feynman amplitude associated with the impulse diagram of Fig. 2a.
In this amplitude, constructed according to the usual Feynman rules, the \( S_F \)'s are Feynman propagators and the \( C \)'s are charge conjugation matrices required to treat this diagram in analogy with a closed fermion loop. \( \Gamma_d \) and \( \Gamma_s \) are covariant NN vertices for the deuteron and for the final scattering state respectively while \( \Gamma_{nn} \) is the pion nucleon interaction vertex which we discuss in Chapter III.

The \( \Gamma_d \) and \( \Gamma_s \) vertices describe phenomenologically the internal dynamics of the two nucleon system. As they must be Lorentz scalars they can be parametrized in terms of all independent scalar products of the four vectors present at the vertex and the four Dirac matrices \( \gamma^\mu \). To each of these scalar parameters is associated a scalar invariant structure function (vertex invariant) whose dynamical content is the relativistic analog of that of the radial wavefunctions which appear in the N.R. matrix elements. While these wavefunctions are normally calculated by solving the Schroedinger equation, the vertex invariants must be calculated by solving a covariant integral equation derived from the well-known Bethe-Salpeter equation, represented diagramatically in Fig. 3.

---

*The Bethe-Salpeter equation can be written in abbreviated notation as

\[
\Gamma' = \int \frac{d^4k}{(2\pi)^4} \, V \Gamma \Gamma
\]
The relation between vertex invariants and radial wavefunctions is an important ingredient of our work and will be stressed as various points of this dissertation.

\[ \begin{array}{c}
\begin{array}{c}
\text{Fig. 3. Diagramatic representation of the Bethe-Salpeter integral equation for the bound state NN vertex.}
\end{array}
\end{array} \]

In Chapter II we analyze all steps of the reduction of Feynman amplitudes for low energy photo and pionic disintegration of the deuteron to their corresponding N.R. matrix elements (see Fig. 4) and show how this reduction generates various types of relativistic corrections whose importance can depend on the nature of the probe and the kinematic regime of the reaction.

where \( G \) is the fully relativistic propagator for the two intermediate nucleons and \( V \) is the sum of all irreducible diagrams contributing to the NN dynamics. By choosing a suitable modification of the propagator \( G = g + (G-g) \) one can rearrange this integral equation into a more manageable three dimensional covariant equation generally known as a quasipotential equation.\(^5\)
We will describe three classes of relativistic corrections to the N.R. matrix elements:

1. **Dynamical Corrections**: The $k_0$ part of the four dimensional loop integral over $dk^4 = dk_0 d^3k$ (such as it appears in the Feynman amplitude 1-2) can be performed by application of the residue theorem to the singularities of the integrand in the $k_0$ complex plane. We can rewrite this four dimensional integral as a sum of three dimensional integrals whose magnitude will decrease with the distance of the associated pole from the origin of the complex plane (see Chap. I for further details). In some well studied cases such as the E.M. disintegration and the E.M. form factor of the deuteron, the dominant contribution comes clearly from a pole which arises as the spectator nucleon $k$ is put on its mass shell (see Fig. 5).

![Diagram](image)

**Fig. 5.** The spectator approximation. The cross indicates that a nucleon is restricted to its mass shell.
Recalling that the N.R. description starts a priori with all nucleons propagating on their mass shell, this spectator contribution can be thought of as the direct relativistic analog of the N.R. matrix element. For this reason the spectator approximation to the impulse diagram of Fig. 3 can be called the relativistic impulse approximation (RIA).

The success of this approximation in E.M. processes has led to the development by Gross of a quasi-potential theory of NN forces in which the modification of the two nucleon propagator appearing in the Bethe-Salpeter equation (see footnote on p. 4) consists in restricting one nucleon to its mass shell.

\[
\Gamma' = \int \frac{d^2 k}{(2\pi)^2} V_g q_0 \Gamma
\]

In the framework of this theory the spectator approximation can be extended to other probes and other mechanisms as well, as we indicate in Fig. 7 for the case of the pionic rescattering diagram.
The contributions from other poles can then be regarded as a particular class of relativistic corrections corresponding to the interactions of shorter range (i.e. "dynamical" corrections) which arise and can be treated naturally by Gross' quasipotential theory. (This is discussed further in Chapter I.)

These corrections however are not the focus of our work and will not be calculated in this dissertation.

We can thus implement the spectator approximation in amplitude (1-2). The propagator of the spectator nucleon appears in this amplitude as

$$ S_F(k) = \frac{(k + m)^T}{M^2 - k^2} = \frac{(k + m)^T}{E_k^2 - k_0^2} \quad (1-3) $$

Application of the residue theorem to the pole $k_0 = E_k$ arising from this singular denominator brings about the prescription

$$ \int \frac{d^4 k}{(2\pi)^4} \frac{i}{E_k^2 - k_0^2} \left\{ \ldots \right\} \rightarrow i \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2E_k} \left\{ \ldots \right\} $$

Fig. 7. Extension of the spectator approximation to pion re-scattering.
while the numerator becomes, for an on shell nucleon

\[
(k' + M)^T = 2M \bar{u}^T(k) u^T(k)
\]  

(1-4)

Carrying these prescriptions in amplitude (1-2) we can rearrange the result in the form

\[
\int d^3k \hbar \left\{ \left( \frac{u^T(k) C \bar{F}_s S_F (\rho' - k)}{\sqrt{(2\pi)^3}} \right) \frac{1}{E_k} \Gamma_W \left( \frac{S_F (0 - k) \Gamma^f_L C \bar{u}^T(k)}{\sqrt{(2\pi)^3}} \right) \right\}
\]

(1-5)

The comparison of this form with the general expression (1-1) for the N.R. matrix elements suggests immediately that we consider

\[
\psi_d(k, \rho - k) = \frac{1}{\sqrt{(2\pi)^3}} S_F (\rho - k) \Gamma^f_L C \bar{u}^T(k)
\]  

\[
\bar{\psi}_s(k, \rho' - k) = \frac{1}{\sqrt{(2\pi)^3}} u^T(k) C \bar{F}_s S_F (\rho' - k)
\]  

(1-6)

To be the direct relativistic covariant analogues of the N.R. wavefunctions \( \psi_d \) and \( \psi_s \) appearing in 1-1.

2. **Recoil Corrections**: While the Feynman amplitudes involve covariant quantities which can in principle be calculated in a frame independent manner (once the scalar invariant structure functions of its vertices are known), non-relativistic wavefunctions are defined in a particular frame of reference. As the extra momentum brought in by the probe has the effect of boosting the final nuclear wavefunction to a moving frame, different from that of the initial nuclear state, recoil
corrections are most naturally seen to arise from the transformation of
that moving wavefunction to its rest frame. The details of the handling
of these corrections rest in a thorough understanding of the relation
between a covariant NN vertex and a frame dependent NN wavefunction in
its standard two component form, as we discuss further in Chapter II.
Recoil corrections, however, can be assumed to be small in low energy
processes such as threshold disintegration of the deuteron and will not
be considered in this work.

3. Corrections arising from the "Small" components or "Pair"

Configurations of the NN wavefunctions: In Feynman diagrams, internal
nucleon lines are described by Feynman propagators $S_p(p)$ which allow for
a fully relativistic mode of propagation of the nucleon, i.e. propagation
of antinucleons as well as nucleons. The following standard decomposi-
tion of the propagator$^7$

$$
S_p(p) = \frac{N}{E} \sum A \left\{ \frac{\gamma (p) \bar{\gamma} (p)}{E_p - P_o} - \frac{\gamma (-p) \bar{\gamma} (-p)}{E_p + P_o} \right\} \quad (1-7)
$$
gives a useful (although frame dependent, i.e. $p^\mu \rightarrow E_p, \bar{p}$) intuitive
picture of this mode of propagation. In this expansion the term in-
volving $u$-spinors can be associated with the propagation of nucleons
and the term involving $v$-spinors with the propagation of antinucleons.

Thus the NN vertices can yield intermediate states containing
virtual antinucleons, states which would not arise in the framework of
a strictly non-relativistic theory. These virtual intermediate states
must correspond to extra, short range configurations in the NN wavefunctions: The role of these intrinsically relativistic components of the wavefunctions in pionic disintegration of the deuteron constitutes the main topic of this work. We now outline in some detail the way in which they arise and comment about their most important properties.

Replacing the propagators in the frame independent relativistic wavefunctions (1-6) by the expansion (1-7) we obtain a frame dependent form in which part of the wavefunction is explicitly related to the antinucleon degrees of freedom, i.e. in the rest frame of the deuteron \(D = (M_d, \vec{0})\) the deuteron wavefunction becomes

\[
\psi(k, M_d) = \sum_{A} \left( \phi_{A}^{+}(k) \psi_{A+1}^{+}(k) + \phi_{A}^{-}(k) \psi_{A+1}^{-}(k) \right) \tag{1-8}
\]

where \(\psi^{+}\) and \(\psi^{-}\) are given by

\[
\psi_{A+1}^{+}(k) = \frac{i}{(2\pi)^3} \frac{M}{E_k} \frac{\bar{u}_{A}(-k) \gamma_{4} C \bar{u}_{A}^{T}(k)}{2E_k - M_d} \tag{1-9}
\]

\[
\psi_{A+1}^{-}(k) = \frac{-i}{(2\pi)^3} \frac{H}{E_k} \frac{\bar{u}_{A}(k) \gamma_{4} C \bar{u}_{A}^{T}(k)}{M_d}
\]

with similar definitions arising for the scattering states. The \(\psi^{+}\) wavefunctions involve nucleons only and must be expected to reduce to a two component form which is the strict analog of the standard N.R. wavefunction. \(\psi^{-}\) involves antinucleons and will reduce to a new type of two component wavefunction which is short ranged and of purely relativistic
origin. In order to carry the reduction of the matrix elements $\bar{\chi} \Gamma \chi$ and $\bar{\psi} \Gamma \psi$ to their two component forms we must next introduce the appropriate NN vertices.

Since the spectator approximation restricts one nucleon to its mass shell, the NN vertices are of the simplest possible kind, known as "restricted" vertices (see Fig. 8)

![Fig. 8. Restricted vertex for the deuteron.](image)

For the deuteron, the most convenient parametrization of the restricted vertex is that introduced by Blankenbecler and Cook (\(\xi\) is the deuteron's polarization 4-vector)

$$\Gamma_d = \Gamma \chi = 6 \frac{\bar{\chi} \chi}{M} \left( H \chi \chi - I \frac{\bar{\chi} \chi}{M} \right)$$

and we have ourselves (ref. 9) introduced similar parametrizations of the restricted vertices for the scattering states corresponding to low J partial waves. For the \(^1S_0\) case which has no polarization we have

$$\Gamma_s = \left\{ F \chi \chi + \frac{(M-N')}{M} H_s \right\} Y^5$$
Using these parametrizations it is a straightforward matter to reduce the matrix elements $\bar{u} \Gamma' \bar{C} \Gamma^T \bar{u}$ and $\bar{v} \Gamma' \bar{C} \Gamma^T \bar{v}$ to their two component forms. The wavefunctions then appear as products of momentum space radial wavefunctions and standard spin-harmonic functions (although cast in their less frequent matrix representation — see appendix II), i.e. cast in the form

$$\sum_L \psi_L^J_{ms} (\hat{k}) \psi_{mA}^J_{mLs} (\hat{k})$$

(1-12)

Non-relativistically the deuteron is known as a $^3S_1 - ^3D_1$ coupled angular momentum state, and indeed the "nucleon" or "positive energy" component $\psi^+$ reduces exactly to the $^3S_1 - ^3D_1$ form

$$\psi^+_{\lambda_n}(k) = \frac{\sqrt{4\pi}}{(2\pi)^{3/2}} \left( u(k) \sigma \cdot \frac{\hat{k}}{\sqrt{2}} + \frac{w(k)}{\sqrt{2}} \left( 3 \hat{r} \cdot \hat{k} \hat{\sigma} \cdot \hat{k} - \sigma \cdot \hat{k} \right) \right) \frac{\lambda_n}{\sqrt{2}}$$

(1-13a)

While the "antinucleon" or "negative energy" component $\psi^-$ reduces to a pair of P states ($^3P_1, ^1P_1$) which have no non-relativistic analogues

$$\psi^-_{\lambda_n}(k) = \frac{-\sqrt{4\pi}}{(2\pi)^{3/2}} \left( V_2 \psi_2(k) i \sigma \cdot \hat{r} \hat{\sigma} \cdot \hat{k} + \psi_2(k) \sqrt{3} \hat{r} \hat{\sigma} \cdot \hat{k} \right) \frac{\lambda_n}{\sqrt{2}}$$

(13b)

Similar results occur for the rescattering states: In the $^1S_0$ case $\psi^+$ reduces to the standard $^1S_0$ form while $\psi^-$ reduces to an associated,
purely relativistic, $^3P_0$ state

$$\Psi^{+}_{4',A}(k) = -\frac{\sqrt{4\pi}}{(2\pi)^{3/2}} y(k) \frac{i\sigma^z}{\sqrt{Z}}$$

$$\Psi^{-}_{\mu_0}(k) = \frac{\sqrt{4\pi}}{(2\pi)^{3/2}} z(k) \sigma^A \frac{i\sigma^z}{\sqrt{Z}}$$

In the course of these reductions, one obtains expressions for the radial wavefunctions in terms of the vertex invariants which in turn can be inverted to give the vertex invariants in terms of the radial wavefunctions.

Thus one sees how vertex invariant functions and momentum space radial wavefunctions are two equivalent sets of functions which carry the same dynamical information. Because these wavefunctions are calculated from the vertex invariants, within the frame of a more general system of equations which contains dynamical information not present in non-relativistic equations, their detailed features, particularly in their short distance, high momentum range, can be quite different from those of the corresponding N.R. wavefunctions. In this light, the standard $u_{N.R.}$ and $w_{N.R.}$ deuteron wavefunctions must be considered as an approximate substitution for the more complete set $(u, w, v_t, v_y)$. One of the results of this work is to illustrate for the case of pionic disintegration of the deuteron how a consistent inclusion of the small components will not only allow these to make contributions of their own but
also will alter the contribution of the standard positive energy wavefunctions. The remaining chapters of this dissertation are concerned with the application of these observations to pionic disintegration of Deuterium.

In Chapter III we discuss the uncertainties which arise from the still standing indetermination over the choice of the relativistic pion nucleon coupling. While both pseudoscalar (P.S.) and pseudovector (P.V.) interactions are known to give more or less the same contribution to the absorption on the standard components of the wavefunctions, (which involve only nucleons, i.e. u-spinors) since, as we show in Chapter III

$$\bar{u} \gamma^s_{\pi N} u \sim \bar{u} \gamma^s_{\pi N} u$$

these couplings are seen to give quite different contributions to the absorption on the small components (which involve antinucleons, i.e. v-spinors), i.e. for an on shell pion we can show that

$$\bar{u} \gamma^s_{\pi N} v \gg \bar{u} \gamma^s_{\pi N} v$$

The \(\gamma^s\) (P.S.) interaction thus giving a much larger result. It is well known that a similar situation arises in low energy \(\pi N\) scattering where the relativistic nucleon pole term gives a very large contribution in \(\gamma^s\) theory and requires the addition of a scalar \(\phi\) meson term in order to fit the experimental data. Such a term is not required in the \(\gamma^s \gamma^s\) (P.V.) theory for which the nucleon pole contribution is suitably
small. We adopt the point of view that a consistent treatment of both couplings can be achieved by implementing the experimental constraints of low energy \( \pi N \) scattering in our model of pionic disintegration.

In Chapter IV, following the suggestive findings of an exact, fully relativistic, preliminary calculation of the nucleon pole diagram contribution to the total cross section for this process over a broad range of energies (this calculation is presented in a separate appendix to this dissertation) we have concentrated our efforts in the threshold region for which we give a detailed calculation of both the pole diagram and the rescattering mechanisms (see Fig. 9), neglecting the final state interactions of the outgoing protons in accordance with the findings of Riska et al.\(^\text{13}\) (See discussion at the end of this introduction)

![Diagram](image)

**Fig. 9.** Nucleon pole and pion rescattering contribution to pionic disintegration of deuterium.
The consequences of the inclusion of the small components of the deuteron wavefunctions are discussed for both pseudoscalar and pseudovector couplings.

All numerical results and the overall conclusion of this work are given in Chapter V.

Before proceeding along our outline we conclude this introduction with a brief comparative review of what is known about threshold disintegration by either photons or pions.

Both reactions have been described quite satisfactorily by N.R. models in which the impulse approximation is supplemented by a rescattering term as shown in fig. 10.

\[
\sum \left< \frac{1}{2} \right| \frac{2}{d} \right> + \left< \frac{1}{2} \right| \frac{2}{d} \right> \\
(a)
\]

\[
\sum \left< \frac{3}{p_1} \right| \frac{1}{d} \right> + \left< \frac{3}{p_1} \right| \frac{1}{d} \right> \\
(b)
\]

Fig. 10. N.R. models of threshold photo (a) and pionic (b) disintegrations of the deuteron.
Threshold photodisintegration presented for a long time a 10% discrepancy between theoretical computations and the highly precise experimental data which was finally resolved when Garf and Huffman calculated the rescattering contribution in terms of the static Chew-Low photoproduction amplitude. An alternative explanation was given concurrently by Riska and Brown who accounted for the bulk of the discrepancy by including a meson exchange effect arising from nucleon-antinucleon pairs. Using $\gamma^5$(pseudoscalar) pion-nucleon coupling, they calculated the N.R. matrix element of the so-called "pair current" represented in fig. 11.

$$\left\langle \left. S_0 \right| \gamma^5 \left| d \right\rangle \right. + \left\langle \left. \gamma^5 \right| \gamma^5 \left| d \right\rangle \right. $$

Fig. 11. N.R. matrix element of the pair current $J_{\text{pair}}$

The relativistic impulse approximation (RIA) for threshold photodisintegration was recently calculated by Dressler and Gross. Their results indicated that, when calculated with $\gamma^5 \pi N$ coupling, the small components of the wavefunctions could also account for the discrepancy. If the $\gamma^b\gamma^5$ (P.V.) $\pi N$ coupling is used, the pair current nearly vanishes while the small component wavefunctions become much
smaller and neither of them makes any observable contribution to the process. In this case however, the minimal E.M. coupling substitution

$$\partial_\mu \rightarrow (\partial_\mu - ieA_\mu)$$

generates a new contact, "seagull" term when implemented in the pseudo-vector $\pi N$ vertex $\bar{\psi} \gamma^5 \gamma^\mu \partial_\mu \phi \psi$. This contact term (fig. 12) gives precisely the main contribution to the photoproduction amplitude used by Gari and Huffman.

$$\begin{align*}
\begin{array}{c}
\text{Fig. 12. Contact "seagull term" arising in the } \\
\text{pseudovector theory.}
\end{array}
\end{align*}$$

Thus all three descriptions can be shown to be equivalent in so far as photodisintegration is concerned.

Consider the time ordered representation of the deuteron's small component wavefunction contribution to the RIA diagram as shown in fig. 13 (a). (The nucleon propagators are replaced by their expansions (1-7) and only the terms relevant to a direct comparison with fig. 11 are kept.)
Fig. 13. Time ordered contribution of the deuteron's small components to the RIA diagram in photodisintegration.

Using the diagramatic representation of Gross' equation given in fig. 6 one can replace the deuteron vertex in Fig. 13a to obtain an amplitude (see Fig. 13b) which is directly comparable to the second term of the N.R. pair current contribution of Fig. 11, if the potential $V$ is approximated by its longest range OPE (one pion exchange) part. A similar analysis of the final state scattering vertex would yield an amplitude analogous to the first term of the N.R. pair contribution. As the non-relativistic calculation used standard wavefunctions, they imply the approximation
which amounts effectively to ignore the **short range** modifications of
the wavefunctions which occur when the small component wavefunctions
are included in a consistent and systematic way. Thus one must conclude
that photodisintegration is not sensitive to details of the short range
part of the wavefunction and does not offer evidence for effects of the
small components *independently* of the choice of the $\pi N$ coupling.

Threshold pionic disintegration presents a striking difference
with the previous process. While the low energy photons yield very
slow nucleons which therefore experience a large $^1S_0$ final state inter-
action, low energy pions must convert their rest mass in kinetic energy
for the outgoing nucleons, giving these a large relative C.M. momentum.

Following the labels of Fig. (10b), conservation of total
four momentum implies

$$\left( \vec{p}_1 + \vec{p}_2 \right)^2 = \left( \vec{q} + \vec{d} \right)^2$$

Expanding these 4-vector scalar products in the C.M. system (i.e. with
$\vec{q} + \vec{d} = 0$ and $\vec{p}_1 = -\vec{p}_2$) and for on-mass shell particles one obtains
easily
$$4E_p^2 = (\mu + M_d)^2 \implies p_{cm}^2 \approx \frac{(\mu+2M)^2}{4} - M^2$$

thus obtaining for the relative C.M. momentum of the final nucleons

$$p_{cm} \approx \sqrt{M_\mu + \frac{\mu^2}{4}} \approx 360 \text{ MeV} / c$$

At such momentum, final state interactions of the outgoing nucleons can be expected to be far less important than in photodisintegration. Recent N.R. calculations of this process have indeed shown that the cross section is largely insensitive to the correlations in the final state wavefunctions. This suggests in particular that the nucleon pole diagram (Fig. 7) would be a good approximation to the impulse amplitude (Fig. 2a). Even though it is well known, as was shown a decade ago by Koltun and Reitan that the rescattering mechanism (see Fig. 10)

![Feynman diagram](image)

Fig. 14. Feynman diagram for the rescattering mechanism in pionic disintegration with negligible final state interactions.
can be expected to give the largest contribution, the impulse contribution is by no means negligible. While the relativistic impulse diagram for photodisintegration yielded overlap integral of wavefunctions over their entire momentum range, thus measuring only their overall relative scales, the pole diagram in threshold pionic disintegration involves no smearing integrals but rather probes directly the details of the deuteron momentum space wavefunctions as $p = 360\text{ MeV/c}$.

Examination of these wavefunctions in the $300-400\text{ MeV/c}$ range (Fig. 15) shows that the $S$-state wavefunctions, which should normally be expected to give by itself the dominant contribution, passes through a zero in the vicinity of that range and in therefore small near $360\text{ MeV/c}$ while the pair configuration wavefunctions are still close to their peaking value.
Fig. 15. *Typical deuteron momentum space wavefunctions for $\chi_5$ coupling.*

These observations suggest that the impulse contributions to the threshold $\Upsilon^+d \rightarrow pp$ reaction is a particularly favorable place to observe unambiguous effects of the pair configurations in the deuteron.

Reporting the discussion of our results and conclusions to Chapter VI we now turn to a more detailed study of the various relativistic corrections referred to in our outline.
II. CLASSIFICATION OF RELATIVISTIC CORRECTIONS

The origin of the various relativistic corrections to the standard non-relativistic matrix elements for low-energy processes can be understood most clearly by extracting these latter quantities from the corresponding Feynman amplitudes and analyzing the extra terms generated in the course of this reduction. In this chapter we focus primarily on the analysis of the reduction of the four-dimensional loop integrals which appear in Feynman amplitudes and show how this analysis leads to a generalization of a spectator-like approximation which restricts one of the nucleons to its mass shell. Subsequent steps of the reduction have been much discussed in Chapter I and are only briefly reviewed here.

1. Dynamical Corrections to the Reaction Mechanisms and Reductions of the 4-Dimensional Integrals

Keeping in mind that all other mechanisms can be analyzed in a similar way, let us consider the impulse diagram of Fig. 16.

Fig. 16. "Impulse" Feynman diagram.
The Feynman amplitude for this diagram can be written as:

\[
M = i \int \frac{d^4 k}{(2\pi)^4} \frac{k \{ \Gamma_{\text{C}}(\Sigma - k, q) \Gamma_{\text{a}}(\Sigma - k) \Gamma_{\text{d}} \} \Lambda^T(\Sigma + k)}{(n^2 - (\Sigma - k)^2 - i\epsilon)(n^2 - (\Sigma - k)^2 - i\epsilon)(n^2 - (\Sigma + k)^2 - i\epsilon)} (-i)^3
\]  

(2-1)

where the short-hand notation \( \Lambda(p) = (p + m) \) was used in the Feynman propagators. This four dimensional loop integral can be reduced to a sum of three dimensional integrals by application of the residue theorem over all singularities of the integrand in the complex energy plane \( k_0 \).

Note, however, that these singularities arise not only from the denominators of the propagators but also from the analytic structure of the NN vertices \( \Gamma_{\text{d}} \) and \( \Gamma_{\text{s}} \) as well. These structures are in principle determined by integral equations of the Bethe-Salpeter type represented diagrammatically in Fig. 17.

![Diagram](image)

Fig. 17. (a) Inhomogeneous Bethe-Salpeter equation for Scattering State.

(b) Homogeneous Bethe-Salpeter equation for Bound State.
Replacing the NN vertices in the diagram of Fig. 16 by these integral equations, one sees that the singular contribution made by the NN vertices to the loop integral in amplitude 2-1 came from the "potentials" \( V \) present in the integral equations, as we illustrate in Fig. 18 for the impulse (a) and meson exchange (b) mechanisms.

![Figures](a) and (b) showing singular contributions from the NN vertices to the loop integrals of typical Feynman amplitudes. Circled arrows indicate loop integration.
(Note that since they contribute to the loop integral only through their potential factors we have dropped the second terms arising from the scattering equation in Fig. 18.)

We can then include the dominant vertex singularities in our analysis by considering the longest range, i.e. O.P.E. (one pion exchange) part of the potential. The typical singularity structure of our integrands can then be related to that of the well known Box diagram of Fig. 19 (a), i.e. we are led to compare the three cases of Fig. 19.
(a) Straight box

\[
\begin{array}{c}
\frac{1}{2} - p \\
\frac{1}{2} - k \\
\frac{1}{2} - p' \\
\frac{1}{2} + p \\
\frac{1}{2} + k \\
\frac{1}{2} + p'
\end{array}
\]

(b) "Impulse"

\[
\begin{array}{c}
\frac{1}{2} - p \\
\frac{1}{2} - p \\
\frac{1}{2} - p + q \\
\frac{1}{2} - p' + q \\
\frac{1}{2} + p \\
\frac{1}{2} + k \\
\frac{1}{2} + p'
\end{array}
\]

(c) "Meson exchange"

\[
\begin{array}{c}
k - p \\
k - p \\
\gamma + p' - k - q \\
\gamma + p' - k - q \\
k - p \\
k - p \\
\frac{1}{2} + k \\
\frac{1}{2} + p' \\
\frac{1}{2} + k \\
\frac{1}{2} + p'
\end{array}
\]

Fig. 19. Box diagram and its extensions appearing in major reaction mechanisms.
Note that the "meson exchange" box involving a pion probe must be associated with the "pion rescattering" mechanism when it proceeds through $\rho$ vector meson exchange as we shall discuss further in the context of our discussion of $\pi N$ scattering in Chapter III.

In the amplitudes corresponding to these diagrams each internal line is associated with a propagator which brings a singular factor having the general form

$$\frac{1}{m^2 - p^2(k) - i\epsilon} = \frac{1}{E_p^2 - p_0^2(k_0) - i\epsilon} = \frac{1}{(E_k - I_{e^2}(k) - i\epsilon)(E_k + I_{e^2} - i\epsilon)} \quad (2-2)$$

in which $m$ and $P(k)$ are the mass and four momentum of the corresponding particle (parametrized as a function of the loop integration variable 4-momentum $k$ as indicated for the various cases of Fig. 19). We are defining energies according to

$$E_p^2(k) = m^2 + |\vec{P}(k)|^2 \quad (2-3)$$

We can then locate the approximate positions of the poles of our integrands in the $k_0$ complex plane by taking suitable static limits under the general assumption that these integrands are dominated by small internal momenta, i.e.

$$\vec{p}, \vec{k} \propto \text{Fermi momentum of the Deuteron} \ll M$$
Thus we take the static limits:

\[ W = M_d \propto 2M \]

\[ E_p \propto E_h \propto E_{q-h} \propto M \]

\[ \omega_{k-p} \propto \mu_\pi \]

The final momentum of the nucleon \( p' \) depends on the nature of the probe. For low energy photons the outgoing nucleons are slow and we can take

\[ \begin{align*}
E_{p'} &\propto M \\
\omega_{p'p} &\propto \mu_\pi
\end{align*} \]

For pions, as we discussed in Chapter I, the final momentum is much higher, i.e. \( p' \propto \sqrt{M \mu} \sim 360 \text{ MeV/c} \) and we must then use

\[ E_{p'} \propto M + \frac{\mu^2}{2M} = (M + \frac{\mu}{2}) \]

\[ \omega_{p'p} \propto \sqrt{\mu'^2 + \mu^2} \propto \sqrt{\mu^2 + M\mu} = 2.77 \mu_\pi \]
As the locations of the meson poles depends on the external nucleon $b$-momenta we will set their relative energies by putting nucleons $(\frac{W}{2} + p)$ and $(\frac{W}{2} + p')$ on their mass shell, an approximation which, for the purpose of this heuristic discussion is consistent with the static limits already assumed. Thus taking $(\frac{W}{2} + p)^2 = (\frac{W}{2} + p')^2 = M^2$ we have

\[ p_0 = E_p - \frac{W}{2} \propto \Delta E' \quad \text{a small positive quantity} \]

\[ p'_0 = E_p - \frac{W}{2} \propto \begin{cases} \Delta E' & \text{for E.M. probe} \\ \frac{\mu}{2} & \text{for pion} \end{cases} \]

The absorption of a pion via the meson exchange mechanism (Fig. 19c), which is an important part of the pion rescattering as we will see later on involves the formation of a $\rho$ meson with mass $m_\rho = 770$. In this case the associated energy reduces to:

\[ \omega_{\rho - p - q} \propto m_\rho + \frac{p'^2}{2m_\rho} \propto m_\rho + \frac{\mu M}{2m_\rho} \approx 6.1 \mu \]

while for the E.M. probe the corresponding energy remains simply $m_\rho$.

Implementing these prescriptions for the various internal lines of the diagrams of Fig. 15 we collect the locations of the corresponding poles in Table (2.4). In this Table $q_0$ must be taken to be $2.22$ MeV = B (i.e. the deuteron binding energy) for an incoming
photon and \( \mu \) for a pion. \( \Delta \mathcal{E} = E_k - \frac{\nu}{2} \) is a positive definite quantity which can increase as the internal momentum \( k^2 \). We must of course emphasize that the static positions are approximate and are solely meant to give the reader a physical insight in the importance of the various contributions. Their exact sizes are dependent on the integrated movements of the associated poles in function of the internal 3-momentum \( |\vec{k}| \).
<table>
<thead>
<tr>
<th>Particle</th>
<th>Momentum</th>
<th>Associated Pair of Poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>nucleon ($\frac{\nu}{2} - k$)</td>
<td>$-E_k + \frac{\nu}{2} + i\xi \sim -\Delta e + i\xi$</td>
<td>$E_k - \frac{\nu}{2} - i\xi \sim 2M - i\xi$</td>
</tr>
<tr>
<td>nucleon ($\frac{\nu}{2} - k$)</td>
<td>$E_k - \frac{\nu}{2} - i\xi \sim +\Delta e - i\xi$</td>
<td>$-E_k - \frac{\nu}{2} + i\xi \sim -2M + i\xi$</td>
</tr>
<tr>
<td>meson (k-p)</td>
<td>$p^0 + \omega_{k-p} - i\xi \sim \Delta e' + \mu - i\xi$</td>
<td>$p^0 - \omega_{k-p} + i\xi \sim \Delta e' - \mu + i\xi$</td>
</tr>
<tr>
<td>nucleon ($\frac{\nu}{2} - k + q$)</td>
<td>$-E_k + \frac{\nu}{2} + q^0 + i\xi \sim q^0 - \Delta e + i\xi$</td>
<td>$E_k - q + \frac{\nu}{2} + q^0 - i\xi \sim 2M + q^0 - i\xi$</td>
</tr>
<tr>
<td>meson (p' - k)</td>
<td>$p^0' - \omega_{p'k} + i\xi \sim \begin{cases} \Delta e' - \mu + i\xi \text{ for } \gamma \ -2.2\mu + i\xi \text{ for } \beta n \end{cases}$</td>
<td>$p^0' + \omega_{p'k} - i\xi \sim \begin{cases} \Delta e' + \mu - i\xi \text{ for } \gamma \ 3.2\mu - i\xi \text{ for } \beta n \end{cases}$</td>
</tr>
<tr>
<td>meson (p' - k - q)</td>
<td>$p^0' - \omega_{p'k} - q^0 + i\xi \sim \begin{cases} \Delta e' - \mu + i\xi \text{ for } \gamma \ -6.6\mu + i\xi \text{ for } \beta n \end{cases}$</td>
<td>$p^0' + \omega_{p'k} - q^0 - i\xi \sim \begin{cases} \Delta e' + \mu - i\xi \text{ for } \gamma \ 5.6\mu - i\xi \text{ for } \beta n \end{cases}$</td>
</tr>
</tbody>
</table>
Closing the $k_0$ integration contours in the lower half planes for all diagrams of Fig. 19 we can then compare the contributions of the various poles present in these half planes (i.e., those associated with $-i\varepsilon$ in Table 2.4) by calculating their residues from the corresponding amplitudes.

(a) **Box diagram of Fig. 19 (a):** The static locations of its poles are displayed on the $k_0$ complex plane diagram of Fig. 20.

$$
\begin{array}{c}
\text{Fig. 20.} \quad k_0 \text{ poles of the box diagram of Fig. 20 (a). Meson poles are indicated by } \times, \text{ nucleon poles are indicated by } \circ (\text{the 2M poles are not indicated in the proper scale).}
\end{array}
$$

The pole structure of Fig. 20 corresponds to a static amplitude of the form

$$
\begin{equation}
\frac{1}{(k_0 - \mu)^2(k_0 + \mu)\left(k_0^2 - \Delta\varepsilon^2\right)(k_0^2 - (2M)^2)} \quad (2-5)
\end{equation}
$$
which gives the three contributions (residues) of Table (2.6).

Table (2.6). Box Diagram Contributions

<table>
<thead>
<tr>
<th>Nearby</th>
<th>Double</th>
<th>Distant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nucleon pole $k_0 = A \varepsilon - i \xi$</td>
<td>$\frac{1}{(\Delta \varepsilon - \mu)^2 (\Delta \varepsilon + \mu)^2 2\Delta \varepsilon (-4M^2)} \approx \frac{1}{2 \Delta \varepsilon \mu^6 (-4M^4)}$</td>
<td>$\frac{1}{(2M - \mu)^2 (2M + \mu)^2 (2M)^2 - (\Delta \varepsilon)^2} \frac{1}{4M} \approx - \frac{1}{4 \mu^5 (-4M^4)}$</td>
</tr>
<tr>
<td>Double Meson pole $k_0 = \mu - i \xi$</td>
<td>$\frac{d}{dk_0} { \frac{1}{(k_0 + \mu)^2 (k_0 - \Delta \varepsilon)^2 (k_0 - (2M)^2)} } \bigg</td>
<td>_{k_0 = \mu}$</td>
</tr>
</tbody>
</table>

The relative sizes of the box diagram contributions from the nearby nucleon pole $\Delta \varepsilon$, the double meson pole $\mu$, and the distant nucleon pole $2M$ are thus respectively

$$1, \quad \frac{3 \Delta \varepsilon}{2 \mu}, \quad \frac{\Delta \varepsilon}{16M} \left( \frac{\mu}{xM} \right)^4$$

Note that the distant nucleon pole at $k_0 = 2M$ clearly makes a negligible
contribution, and, apart from an overall scale factor \((-\frac{1}{4m^2})^{-1}\) we can ignore its role in the remaining diagrams. Note also that the nearby nucleon pole at \(k_0 = \Delta E\) largely dominates over the double meson pole. This can be understood by noticing in Fig. 20 how this pole is enhanced by another nucleon pole located just across the real axis.

Recalling that this \(\Delta E\) pole arises from putting nucleon \((\frac{\omega}{2} + k)\) on its mass shell, this dominance can be seen as a basic heuristic justification for the quasipotential approach (see introduction) to the Bethe-Salpeter equation and particularly for that proposed by Gross. In this context the 4-dimensional \((d^2k)\) box diagram can be seen as a sum of two 3-dimensional pieces \((d^3k)\) as indicated in Fig. 21.

\[\begin{align*}
\text{Fig. 21. Decomposition of the 4-dimensional box diagram in a pair of} \\
\text{3-dimensional pieces. The circle represents the contribution} \\
of \text{all singularities except that of the nearby (spectator-like) nucleon pole.}
\end{align*}\]
represents the contributions from all other poles, must be regarded as a new irreducible diagram corresponding to a short range piece of the two pion continuum contribution to the NN dynamics. Its iteration is also displayed in Fig. 22. Note that a similar treatment can be applied to box diagrams involving heavier mesons as well.

\[ \text{Fig. 22. Iterations of Gross quasipotential equations associated with box diagram contributions.} \]

b) **Impulse diagram:** The static locations of the poles are displayed
on the two diagrams of Fig. 23 (a) and 23 (b) respectively for the E.M. probe and the pionic probe. Note that the pionic case presents a characteristic difference from the E.M. case. The positions of the \((p' - k)\) meson poles and of the \((\frac{V}{2} - k + q)\) are shifted by the pion mass \(\mu = q_0\) as indicated by arrows on Fig. 23 (b).

Fig. 23. Static poles for the impulse diagram of Fig. 20 (b) (approximate scales). The extra nucleon poles introduced by the probe are circled.

The static amplitudes corresponding to these pole structures can be written as, for the E.M. probe: \((q_0 \approx 2.22 \text{ MeV} = B)\)
An amplitude similar to the box case (2-5) except for the two last factors in the denominator which arise from the extra nucleon line \( \frac{w}{c} - k + q \) generated by the incoming probe in Fig. 20 (b). The contributions (residues) from the nearest lower half plane poles are collected in Table (2.9) and likewise, for the pionic case

\[
\frac{1}{(k_0 - \mu)^2 (k_0 - \Delta E) (k_0 - (2M)^2)(k_0 - 2M)(k_0 + \Delta E - B)}
\]

(2-8)

giving the leading contributions collected in Table (2.9).

### Table 2.9. Impulse Diagram Contributions

<table>
<thead>
<tr>
<th>Nucleon pole ( k_0 \cdot \Delta E - i\varepsilon )</th>
<th>E.M. probe</th>
<th>Pion probe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nearest Meson pole ( k_0 - \mu - i\varepsilon )</td>
<td>( \frac{1}{2\Delta E \mu^2 (-4\mu^2) (-2M)(2\Delta E - \mu)} )</td>
<td>( \frac{1}{2\Delta E (2.4\mu^4)(-4\mu^2)(-2M)(2\mu - \mu)} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{\mu^2 (-4\mu^2) (-2M)^2}{15\mu^2 (-4\mu^2) (-2M) \Delta E} )</td>
<td></td>
</tr>
</tbody>
</table>
For the E.M. case the relative sizes of these two contributions are respectively for the nucleon and the meson poles.

\[
\frac{1}{\Delta \epsilon (2\Delta \epsilon - \mu)} \quad \frac{1}{\mu}
\]

while in the pionic case one has instead

\[
\frac{1}{\Delta \epsilon (2\Delta \epsilon - \mu)} \quad \frac{1}{\Delta \epsilon \mu}
\]

Thus recalling that \( \Delta \epsilon = (E_k - \frac{m}{2}) \approx \frac{k^2}{2M} \), one sees that the spectator contributions contains in both cases a factor which can be singular and which will strongly enhance the results of the final three momentum \( k \) integration. Note that this enhancement will be largest for the E.M. case where the singularity occurs for the small value \( \Delta \epsilon = B/2 \sim 1 \text{ MeV} \) making the factor \( (\Delta \epsilon)^{-1} \) further enhance the singularity. In the pionic case the singularity occurs for \( \Delta \epsilon = \frac{k^2}{2} \) and is not enhanced by the factor \( (\Delta \epsilon)^{-1} \) thus bringing a lesser but nevertheless clearly marked spectator dominance for the pion probe.

The contribution from the remaining poles can then be viewed as a particular class of dynamical, relativistic corrections to the reaction mechanisms (by opposition to other corrections which we shall view as corrections to the NN wavefunctions) associated with the new irreducible two pion exchange diagram of Fig. 21 which appear in Gross' quasipotential theory. The size of these corrections is in principle calculable by evaluating the matrix elements of Fig. 25 in which the
transition operators are defined as the contributions from the pion poles to the $k_0$ integrals.

$$\langle \bar{3}P_1 | \quad \quad \quad \quad \quad | d \rangle$$

$$\langle 'S_0 | \quad \quad \quad \quad \quad | d \rangle$$

Fig. 25. **Dynamical relativistic corrections to photo and pionic disintegrations of the deuteron.**

These corrections, which we have called *dynamical corrections* in view of their obvious connection with NN dynamics, should generally be expected to be rather small since they involve a short range mechanism which can make little contributions to overlap integrals involving NN wavefunctions. They will therefore not be calculated in this work.

c) **Meson Exchange Mechanism:** Here again we display the static locations of the poles from the diagrams of Fig. 19 (c) in the complex $k_0$ plane of Fig. 26.
Fig. 26. Static poles for the meson exchange diagrams of Fig. 20 (c).

Comparison with Fig. 20 indicates that for both probes the situation is similar to that of the box diagram, i.e. the nearby nucleon pole at $k_0 = \Delta \varepsilon - i\varepsilon$ and the nearest meson pole at $k_0 = \mu - i\varepsilon$ give contributions whose relative sizes are respectively:

$$1, \quad \frac{\Delta \varepsilon}{\mu}$$

Thus suggesting that a spectator like approximation for the meson exchange mechanism is quite appropriate, although less impressive than for the impulse mechanism. The neglected meson poles generate dynamical corrections analogous to those arising in the impulse mechanism and represented on Fig. 27.
Fig. 27. Dynamical relativistic corrections associated with the meson exchange mechanism.

The analysis presented in this section has thus shown how the spectator approximation can serve as a most appropriate basis for a quasi-potential relativistic description of nuclear processes involving such probes as photons and pions, for which relativistic corrections to the reaction mechanisms, i.e. dynamical corrections, are properly defined and can be expected to be small.

2. Relativistic Corrections to the NN Wavefunctions

a) Recoil corrections

Implementing the spectator approximation in amplitude (2-1) by taking the residue of the pole $k_0 = E_k - \frac{v}{2}$ associated with the spectator nucleon $(\frac{v}{2} - k)$ (see Fig. 16) and using expressions (1-3) and (1-4) from Chapter I we obtain a 3-dimensional integral similar to (1-5) (with a slightly different parametrization of the momenta) which displays the frame independent forms of the wavefunctions defined in p. 9 of Chapter I.
The origin of the recoil correction was discussed in Chapter I. As these corrections cannot be expected to be large in low energy processes (low v/c) they will not be calculated in this work and we content ourselves with a brief outline of the two alternate ways in which they can be treated.

Using the restricted vertices (1-10) and (1-11), the trace appearing in (2-10) can be evaluated in covariant Dirac space, yielding overlap integrals of invariant vertex functions. As these vertex invariants are the same in any reference frame they can be replaced by their expressions in terms of rest-frame wavefunctions (2-11) which are obtainable by the reduction procedure outlined in Chapter I,

giving for the deuteron vertex invariants:

\[
\begin{align*}
    F(k) &= \left\{ u(k) - \frac{W(k)}{\sqrt{2}} + \frac{M}{\sqrt{2}} u_T(k) \right\} \sqrt{4M_d (2E - M_d)} \\
    G(k) &= \left\{ u(k) + \frac{W(k)(2E - M)}{E + M} + \frac{E + M}{M} \sqrt{\frac{3}{2}} u_T(k) \right\} \sqrt{4M_d} \frac{M(2E - M_d)}{E + M} \\
    H(k) &= \sqrt{\frac{3}{2}} u_T(k) \frac{ME}{\mu} \\
    I(k) &= \left\{ u(k) - \frac{W(k)(E + M)}{\sqrt{2}(E - M)} + \frac{(E + M)M_d}{(1 - M_d)^2} \sqrt{3} u_T(k) \right\} \sqrt{4M_d} \frac{M^2 (2E - M_d)}{M_d (E + M)}
\end{align*}
\]
Proceeding in this way one would obtain complicated expressions involving many terms but calculable entirely in terms of rest frame wavefunctions in which recoil corrections are intrinsically mixed with those arising from the small component wavefunctions.

The relativistic calculation of the pole diagrams for pionic disintegration given in Appendix I offers the simplest example of a calculation which includes recoil effects in this manner.

An alternate method, used by Arnold et al. in their calculation of the impulse contribution to the E.M. form factor of the deuteron, allows a complete separation of recoil effects from those of the small components. This method which involves a Lorentz transformation of the frame independent wavefunctions (1-6) into their rest frames by means of spinor boosts and the Wigner-Rotation, is beyond the scope of the present discussion.

b) Contributions of the "small components" or pair configurations NN wavefunctions.

The nature of these intrinsically relativistic pieces of the NN wavefunctions has been described extensively in Chapter I. Proceeding in the manner of p. 11 we expand the Feynman propagators which appear in amplitude (2-10). Taking \( W = M_d \) which amounts to calculating the amplitude in the rest frame of the deuteron and using the spectator prescription \( k_0 = E_k - \frac{M_d}{2} \) we have, at threshold (i.e. for \( q = (p, 0) \)
\begin{align}
S_F (\frac{\mathbf{p}^2 - k^2}{2} - k) &= \frac{N}{E_k} \left\{ \frac{u(-k) \tilde{u}(k)}{i E_k - M_d} - \frac{\nu(k) \tilde{\nu}(k)}{M_d} \right\} \\
S_F (\frac{\mathbf{p}^2 - k^2 + \mu}{2} - k + \mu) &= \frac{N}{E_k} \left\{ \frac{\nu(k) \tilde{u}(k)}{i E_k - (M_d + \mu)} - \frac{\nu(k) \tilde{\nu}(k)}{M_d + \mu} \right\}
\end{align}

(2-12)

where we note that in the second propagator the quantity \((M_d + \mu) = W_{\text{final}}\) has appeared which is the analog of \(M_d\) for the final state wavefunction. Introducing these expressions in amplitude (2-10) and using the definitions (1-9) of Chapter I we can rewrite this amplitude in terms of two component wavefunctions for which we can make a direct comparison with the N.R. matrix elements

\begin{align}
\mathcal{M} &= -i \int d^3p \frac{N}{E_k} \left\{ \psi_s^+ \tilde{u}(-k) \Gamma_{nN} \nu(-k) \psi_d^+ \\
&+ \psi_s^+ \tilde{u}(-k) \Gamma_{nN} \nu(-k) \psi_d^- \\
&+ \psi_s^- \tilde{u}(k) \Gamma_{nN} \nu(-k) \psi_d^+ \\
&+ \psi_s^- \tilde{u}(k) \Gamma_{nN} \nu(-k) \psi_d^- \right\}
\end{align}

(2-13)
The first term of this expression involves only u-spinors and the "positive energy" wavefunctions described in Chapter I and is therefore the direct analog of the N.R. element for the impulse mechanism. The other terms are relativistic corrections arising from the pair configurations of the NN wavefunctions and can be expected to be the only significant relativistic corrections in low energy processes. Similar expressions can be obtained for these contributions to other mechanisms such as the meson exchange and rescattering mechanism and will also be calculated in our analysis of threshold pionic disintegration.

As indicated in Chapter I in the case of threshold photodisintegration these corrections were found to vary considerably with the choice of the $\pi N$ coupling (pseudoscalar $\gamma^5$ or pseudovector $\gamma^\mu \gamma^5$) used in calculating the wavefunctions. For pionic disintegration the $\pi N$ vertex is directly involved in the reaction mechanism through its matrix elements $\bar{u} \Gamma^u_u$, $\bar{u} \Gamma^v_v$ and others which appear in (2-13), and through the model of $N$ scattering required to calculate the rescattering mechanism known to be important in this process.

Thus our study of the relativistic corrections to threshold pionic disintegrations leads us to examine the interdependent relations between the choice of the $\pi N$ coupling, the properties of NN wavefunctions, and the experimental constraints of low energy $\pi N$ scattering to which we address ourselves in this next chapter.
III. AMBIGUITY IN THE $\pi N$ COUPLING AND
LOW ENERGY $\pi N$ SCATTERING

1. Pion Nucleon Vertices

By now, meson and nucleon fields have clearly revealed themselves not to be fundamental fields. Criteria such as renormalizability are thus of no help to make a choice between the pseudovector $\gamma^{\mu} \gamma^\nu$ and pseudoscalar $\rho^5$ $\pi N$ couplings which should both be regarded as phenomenological.

As we have observed in the case of photodisintegration, Lagrangian terms such as the contact seagull term of Fig. 12 can appear or disappear, depending on which $\pi N$ coupling is used thus shifting the emphasis from one to another dynamical mechanism and obscuring the precise role of the pair configurations. We will see that a similar situation arises in pionic disintegration. Consider the pion-nucleon vertex of Fig. 28a:

![Diagram](https://example.com/diagram.png)

**Fig. 28.** a) $\pi NN$ vertex  b) $\pi NN$ vertex
Overall 4-momentum conservation implies \( \vec{q} = (\vec{p}' - \vec{p}) \) for (a) and 
\( \vec{q} = (\vec{p}' + \vec{p}) \) for (b). If both nucleons are on their mass shell, observe first that, (passing the \( \gamma^5 \) through and using \( \gamma u(p) = m u(p) \) and its Dirac conjugate)

\[
\bar{u}(p') \gamma^5 u(p) = \bar{u}(p)'(p' \gamma^5 + \gamma^5 p') u(p) = 2M \bar{u}(p') \gamma^5 u(p)
\]  

(3-1)

This suggests a construction of the P,V, and P.S. vertices using the same coupling constant \( g \) according to

\[
\Gamma^{\text{P.S.}}_{NN} = i g \gamma^5 \quad \Gamma^{\text{P.S.}}_{NN} = i g \gamma^5
\]  

(3-2)

and defines the "on-shell" equivalence of the two couplings. Note that a similar result would occur for a \( (N \rightarrow NN) \) vertex with on shell \( NN \) particles (see Fig. 28b) (using \( \gamma v = -mv \))

\[
\bar{u}(p') \gamma^5 u(p) = \bar{u}(p)'(p' \gamma^5 + \gamma^5 p') u(p) = 2M \bar{u}(p') \gamma^5 u(p)
\]  

(3-3)

This situation is generalized by the well known "equivalence theorem" according to which one can construct a unitary transform (the Dyson
transformation\(^1\) of the P.V. field theoretic Lagrangian which reduces it to the P.S. Lagrangian plus higher order terms in the fields (such as the two pions seagull term of Fig. 29). Note that this theorem does not really imply that the two couplings are strictly equivalent but only their lowest order terms.

![Fig. 29. Two pions seagull term](image)

If one nucleon is off the mass shell, important differences will arise between the matrix elements of the two couplings: Taking \(p\) to be off shell as would be the case for the pole diagram (see Fig. 9) in deuteron pionic disintegration we can reduce (leaving aside the spinor normalization factors which are common to all these matrix elements) these quantities to their two-component forms:

\[
\begin{align*}
\bar{u}(p') \gamma^\sigma u(p) &= \frac{\sigma \cdot p}{E_p + M} - \frac{\sigma \cdot p'}{E_p' + M} \\
\bar{u}(p') \gamma^T \gamma^\sigma u(p) &= \frac{-\sigma \cdot q}{2M} + \frac{q_0}{2M} \left( \frac{\sigma \cdot p}{E_p + M} + \frac{\sigma \cdot p'}{E_p' + M} \right) + \frac{q_0 \sigma \cdot q \cdot p}{2M(E_p + M)(E_p' + M)} \\
\bar{u}(p') \gamma^T u(p) &= 1 - \frac{\sigma \cdot p' \cdot q \cdot p}{(E_p + M)(E_p' + M)} \\
\bar{u}(p') \gamma^T \gamma^\sigma u(p) &= \frac{q_0}{2M} - \frac{\sigma \cdot q \cdot p}{2M(E_p + M)} - \frac{\sigma \cdot p' \cdot q \cdot p}{2M(E_p + M)} + \frac{q_0 \sigma \cdot p' \cdot q \cdot p}{2M(E_p + M)(E_p' + M)}
\end{align*}
\]
At threshold the pion 4-momentum is \( q = (0, 0) \) and the nucleons 3-vectors \( p \) and \( p' \) must be equal and these results become simply

\[
\begin{align*}
\bar{u}(p') \gamma^5 u(p) &= 0 \\
\bar{u}(p') \frac{\not{p} \gamma^5}{2M} u(p) &= \frac{\mu}{2M} \frac{M^2}{m^2} \\
\bar{u}(p') \gamma^5 \not{p} u(p) &\sim 1 \\
\bar{u}(p') \frac{\not{p} \gamma^5}{2M} \not{p} u(p) &\sim \frac{\mu}{2M} 
\end{align*}
\]

(3-5)

We clearly see that, in this kinematic region the vertex involving the small component \( v \) completely dominates the \( \gamma^5 \) theory and that it is still comparable to that involving the nucleon component \( u \) in the \( \gamma^4 \gamma^5 \) theory.

These observations suggest three important remarks:

1) The pair configuration wavefunctions can be expected to be much larger in the \( \gamma^5 \) theory than in the \( \gamma^4 \gamma^5 \) theory.

2) As the \( \pi NN \) vertex appears twice in the nucleon pole contribution to \( \pi N \) scattering, it is very strongly enhanced in the \( \gamma^5 \) theory (by a factor \( (2M/\mu)^2 \approx 200 \)) compared to the \( \gamma^4 \gamma^5 \) theory. This situation reflects the findings of the "equivalence theorem" which requires the addition of a seagull (\( \sigma \)-term) term to bring about complete equivalence. In the next section we present a model of \( \pi N \) scattering that does indeed require the addition of a \( \sigma \)-term to bring the \( \gamma^5 \) theory in line with the data.
3) As these wide differences in the $\pi NN$ vertex will clearly affect the direct absorption mechanism (impulse) in threshold pionic disintegration and since the rescattering mechanism is known to play an important role in this process it is likely that an understanding of these differences can be provided by a consistent inclusion of low energy $\pi N$ scattering in our model of pionic disintegration.

2. Threshold $\pi N$ Scattering

Although much studied over the past three decades low energy $\pi N$ scattering is still far from being understood in terms of a unique well defined theory. It is not our aim to give here an optimal description of the process but rather to review the properties of its main contributing mechanism at threshold in order to provide a consistent treatment of the uncertainty on the $\pi N$ coupling in the reaction $\pi^+ d \rightarrow pp$.

The most obvious contribution to $\pi N$ scattering is that of the nucleon pole diagrams of Fig. 30 which involve directly the $\pi N$ coupling and therefore will present symptoms of the ambiguities described in the previous section.

Another well known mechanism is provided by the exchange of a $\rho$-vector meson as shown on Fig. 31a. This contribution is perhaps the least ambiguous of all as the couplings are well defined and the coupling constants have been measured with reasonable accuracy.\textsuperscript{20}
In order to control the uncertainties arising from the nucleon pole diagram one can then add a $\sigma$-scalar meson exchange (Fig. 31b) such as the one suggested by the chiral invariant $\sigma$-model. Its parameters can be adjusted in order to constrain the model to reproduce the isospin symmetric and antisymmetric combinations $a_+ = 1/3(a_{1/2} + 2a_{3/2})$ and $a_- = 1/3(a_{1/2} - a_{3/2})$ of the experimentally known scattering lengths $a_{1/2}$ and $a_{3/2}$.

As we are solely concerned with threshold, $s$-wave scattering we are not including the $p$-wave $\Delta_{1/2}$ resonance which is known to dominate this process at higher energies.

---

Fig. 30. Direct and crossed nucleon pole contributions to $\pi N$ scattering. $\alpha$; $\beta$ are the isospin indices of the pions.

Following our previous conventions we can write the Feynman amplitude for these diagrams as
\[ m_{\mu\kappa} = \bar{U}(p_\mu) \left\{ \Gamma_{\pi N}^\mu S_F(q_1+p_\mu)\Gamma_{\pi N}' + \Gamma_{\pi N}^\mu S_F(q_2-q_\mu)\Gamma_{\pi N}' \right\} U(p) (-i)^3 \]

(3.6)

at threshold, i.e. for \( p_2 = p_1 = (M, 0) \) and \( q_1 = q_2 = (\mu, 0) \) the nucleon propagators become

\[ S_F(q_1+p_\mu) = \frac{M+(q_1+p_\mu)^2}{M^2-(q_1+p_\mu)^2} = \frac{M + \gamma^0 (M+\mu)}{-2\mu M + \mu^2} \]

\[ S_F(p_2-q_\mu) = \frac{M+(q_2-q_\mu)^2}{M^2-(q_2-q_\mu)^2} = \frac{M + \gamma^0 (M+\mu)}{+2\mu M - \mu^2} \]

(3.7)

Using the mixture \( i \bar{q} (\lambda \gamma^5 + \frac{1-\lambda}{2M} \gamma^\lambda \gamma^5) \) for the \( \Gamma_{\pi N}^\mu \) vertices and with \( d = (\rho_1 - \rho_2) \) giving \( \rho_2 \) at the absorption vertex and \( -\rho_2 \) at the emission vertex this amplitude reduces to (leaving implicit the external spinors \( \bar{u}_2(0) \) and \( u_1(0) \))

\[ g^2 \left\{ (\lambda \gamma^5 - \frac{(1-\lambda)}{2M} \mu \gamma^0 \gamma^5) \left( \frac{M + \gamma^0 (M+\mu)}{-2\mu M + \mu^2} \right) (\lambda \gamma^5 + \frac{(1-\lambda)}{2M} \mu \gamma^0 \gamma^5) \right\} \gamma^\alpha \tau_\alpha \]

(3.8)

\[ g^2 \left\{ (\lambda \gamma^5 + \frac{(1-\lambda)}{2M} \mu \gamma^0 \gamma^5) \left( \frac{M + \gamma^0 (M+\mu)}{+2\mu M - \mu^2} \right) (\lambda \gamma^5 - \frac{(1-\lambda)}{2M} \mu \gamma^0 \gamma^5) \right\} \gamma^\alpha \tau_\alpha \]
passing the $\gamma^5$ through, using $\gamma^{0}u(0) = u(0)$ and defining $\varepsilon = \mu/2M$

the threshold amplitude $M_{\text{poles}}$ reduces further to

$$
\frac{g^2}{2M} \left\{ \frac{(\lambda - \varepsilon(1-\lambda))^2}{1+\varepsilon} T_\beta T_\alpha + \frac{(\lambda + \varepsilon(1-\lambda))^2}{1-\varepsilon} T_\alpha T_\beta \right\}
$$

re-expressing this in terms of isospin symmetric and antisymmetric components, i.e. using the sum and difference of the isospin commutators

$$
\{ T_\beta T_\alpha \}_+ \quad \text{and} \quad \{ T_\beta T_\alpha \}_- \quad \text{we finally obtain the corresponding amplitudes} \quad M^+ \quad \text{and} \quad M^-
$$

$$
M^\pm = \frac{g^2}{2M} \left\{ \frac{(\lambda - \varepsilon(1-\lambda))^2}{1+\varepsilon} \pm \frac{(\lambda - \varepsilon(1-\lambda))^2}{1-\varepsilon} \right\} \left\{ \frac{T_\beta T_\alpha}{2} \right\} \pm \quad (3-10)
$$

which are immediately related to the scattering lengths $a^+$ and $a^-$ for which we take Weinberg's general definition (dimensionless)

$$
\mu a = -\frac{M \mu}{(1 + \mu/M) \pi}
$$

(3-11)

Before comparing these amplitudes to the experimental data we must calculate the $\rho$-meson contribution which we will consider as a stable component of the total amplitude, i.e. from Fig. 31a.
The Λ couples to the pion like a photon but with the $\rho \rightarrow \pi\pi$ decay constant $f$ replacing the electric charge $-e$. Thus we have, dropping the $k^\mu k^\nu /m^2$ term in the vector meson propagator (it would yield a factor $\bar{u}(p_2)\gamma\mu u(p_1) = \bar{u}(p_2)(\not{p_2} - \not{q})u(p_1) \propto 0$ if we neglect at most the neutron-proton mass difference)

$$M_\rho = -\left(i\frac{f}{m_\rho^2}(q_\mu + q_\nu)\epsilon_{\mu\nu\rho\sigma}\right)\left(-\frac{g^{\mu\nu}}{m_\rho^2 - k^2}\right)\left(\frac{g_\rho}{m_\rho} \frac{\gamma_\mu}{x} \gamma_\nu u(p_1)\right)$$

which becomes at the threshold momenta $q_\perp = (\not{p_1}, 0)$, $p_1 = (M, 0)$, and using again $\gamma^0 u(0) = u(0)$, $\bar{u}(0)u(0) = 1$:

$$M_\rho = \frac{\mu f_{\rho\gamma}}{m_\rho^2} i \epsilon_{\mu\nu\rho\sigma} \frac{\gamma^\nu}{x} \frac{-\gamma_\sigma}{x} \left\{ \frac{T_\rho T_\sigma}{z} \right\}$$
As its isospin structure is completely antisymmetric, note that this amplitude contributes only to $a^-$. The $\sigma$-term, on the other hand, is completely symmetric and contributes only to $a^+$. It can be written directly as

$$m_{\sigma} = -\frac{f_\sigma g_\sigma}{m_\sigma^2} \delta_{\rho\alpha} = -\frac{f_\sigma g_\sigma}{m_\sigma^2} \{\frac{\tau_\rho \tau_\alpha}{2}\}^+ \quad (3-14)$$

Collecting these results we obtain general expressions for $b^+ = (1 + \mu/M)^{-1} \mu a^+$ for all mixtures of P.V. - P.S. pion nucleon couplings.

$$b^+ \propto \frac{g_\pi f_\pi \mu}{4\pi m_\sigma^2} - \frac{g_\pi^2}{4\pi} \varepsilon \left(2\lambda^2 + 2\varepsilon^2\right) \quad (3-15)$$

$$b^- \propto \frac{g_\pi f_\pi}{4\pi} \left(\frac{\mu}{m_\rho}\right)^2 - \frac{g_\pi^2}{4\pi} \varepsilon \left(2\lambda^2 \varepsilon - 4\lambda \varepsilon - \frac{\varepsilon^3}{1 - \varepsilon^2}\right)$$

Making use of these results for the two extreme cases $\lambda = 0$ (pure P.V.) and $\lambda = 1$ (pure P.S.) we first compare the pole contributions to the experimental values in Table (3-16) using

$$\frac{g_\pi^2}{4\pi} = 14.5$$

$$\varepsilon = \frac{\mu}{\lambda m} = 0.074$$

$$\varepsilon^2 = 5.5 \times 10^{-2}$$

$$\varepsilon^3 = 4 \times 10^{-3}$$
Table (3-17). Contributions to \( \pi N \) Scattering Lengths

<table>
<thead>
<tr>
<th>Contribution</th>
<th>( b^+ )</th>
<th>( b^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ) = 0 pole</td>
<td>( \sim 0 )</td>
<td>( \sim 0 )</td>
</tr>
<tr>
<td>( \lambda = 1 ) pole</td>
<td>( -\frac{g^2}{4\pi} \tau \epsilon = -2.16 )</td>
<td>( \frac{g^2}{4\pi} \tau \epsilon^2 = 0.16 )</td>
</tr>
</tbody>
</table>

These results indicate that while the pure \( \lambda = 0 \) theory reproduces adequately the \( \pi N \) scattering length without inclusion of \( \sigma \)-exchange (the \( \rho \) by itself giving bulk part of the result) a large amount of \( \sigma \) is required to bring the \( \gamma^5 \) theory in agreement with the data. For models using a mixture of the two couplings the result [3-15] dictates to fix the \( \sigma \)-parameter according to

\[
\frac{g^2 f_\rho}{4\pi m_r^2} = \frac{g^2}{4\pi} \frac{\epsilon^2 \lambda^2}{2m}
\]  

We are now turning to the construction of our model for threshold pionic disintegration of the deuteron and to the question of whether a consistent inclusion of the \( \sigma \)-exchange in the rescattering mechanism can give a satisfactory description of this process for all mixtures of the \( \pi N \) coupling.
IV. THRESHOLD PIONIC DISINTEGRATION OF DEUTERIUM

We now construct a simple model for the threshold (pion 4-momentum $q \rightarrow (\mu, 0)$) reaction which includes the nucleon pole diagram together with pion rescattering (see Fig. 32).

Note that we are neglecting the final state interaction of the two outgoing protons (the reason for this approximation was given on p. 22 of Chapter I) as well as the Coulomb corrections to the initial $\pi d$ state. The primary aim of this model being an examination of the role of small components in this process, we are using the experimental data as an overall constraint rather than seeking precise quantitative agreement.

Fig. 32. Feynman diagrams for the model amplitude. The cross indicates that the particle is put on mass shell.
1. Outline of the Model Amplitude: (use θ = 0)

Our model amplitude for this reaction is constructed from Feynman amplitudes corresponding to the diagrams of Fig. 32. In our conventions each vertex and each propagator is associated with a factor (-1)).

Pole:

\[ M_p = i \left\{ \bar{u}_N(-p) \prod_{i=N} \frac{1}{p_i - m_i} \right\} C \bar{u}(p) \]  \hspace{1cm} (4-1a)

Rescattering:

\[ M_R = i \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{\bar{u}_N(-p) \prod_{i=1} \frac{1}{p_i - m_i} \right\} \prod_{i=1} C \bar{u}(p) \]  \hspace{1cm} (4-1b)

The 4-dimensional integral of \( M_R \) is first reduced to a 3-dimensional integral by application of the spectator-like approximation described in Chapter II, i.e. nucleon \( \frac{W}{2} + k \) is put on its mass shell. The relative energy \( k_0 \) is thus fixed as follows

\[ \left( \frac{W}{2} + k \right)^2 = M^2 \quad \Rightarrow \quad k_0 = (E_k - \frac{W}{2}) \]  \hspace{1cm} (4-2)

and the numerator of propagator \( S_F^{T}(\frac{W}{2} + k) \) reduces to \( 2M(u(k)\bar{u}(k)) \) while its singular denominator brings about the residue theorem prescription.
Note that \( p_0 = E_p - \frac{W}{2} \) since nucleon \( \left( \frac{W}{2} + p \right) \) is a physical on mass shell particle.

Implementing these prescriptions and expanding propagators \( S_p(\frac{W}{2} - p) \) and \( S_p(\frac{W}{2} - k) \) according to expression (1.7) we can rewrite our model amplitudes in terms of the deuteron wavefunctions introduced in Chapter I (they are singled out between parenthesis).

\[
\mathcal{M}_p = (-1) \sum A \left\{ \left( \frac{\mathcal{U}_{d}(-p)}{E_p} \Gamma_{NN} \mathcal{U}_{s}(-p) \right) \left( \frac{M}{E_p} \frac{\mathcal{U}_{d}(-p)}{E_p} \right) \left( \frac{\Gamma_{NN}}{E_p - W} \right) \right\}
\]

\[
\mathcal{M}_R = (+1) \int \frac{d^3k}{(2\pi)^3} \frac{1}{\mu^2 - (\not{p} - \not{k})^2 + (\not{p} - \not{R})^2} \times
\]

\[
\frac{M}{E_k} \sum \left\{ \left( \frac{\mathcal{U}_{d}(-p)}{E_k} \mathcal{H}_{NN} \mathcal{U}_{s}(-k) \right) \left( \frac{M}{E_k} \frac{\mathcal{U}_{d}(-k)}{E_k} \right) \left( \frac{\Gamma_{NN}}{E_k - W} \right) \right\}
\]

\[
+ \left( \frac{\mathcal{U}_{d}(-p)}{E_k} \mathcal{H}_{NN} \mathcal{U}_{s}(k) \right) \left( \frac{M}{E_k} \frac{\mathcal{U}_{d}(k)}{E_k} \right) \left( \frac{\Gamma_{NN}}{W} \right) \left( \frac{\Gamma_{NN}}{E_k - W} \right) \right\}
\]
In each of these amplitudes the second term arises from the pair configuration of the deuteron wavefunction.

Since we are studying the reaction at threshold we restrict ourselves to S-wave incoming pions. The standard selection rules will then restrict the final protons to their $^3P_1$ angular momentum state. We can thus truncate the partial wave expansion of our model amplitudes to that particular state. Using symbolic bracket notations

$$\langle s_1 s_2 p | m | \pi d \rangle = \sum_{J, m, L S} \langle s_1 s_2 p | J M L S \rangle \langle J M L S | m | \pi d \rangle$$

$$\sim \sum_{m} \langle s_1 s_2 p | ^3P_1^{(m)} \rangle \langle ^3P_1^{(m)} | m | \pi d \rangle$$

In this expression the notation $\langle s_1 s_2 p | ^3P_1^{(m)} \rangle$ represents the orthonormal spin harmonic function $y_{J L M S}^{s_1 s_2}(\hat{p})$ for the $^3P_1$ angular momentum state (the construction of the $y_{J L M S}^{s_1 s_2}$ in the matrix representation suitable for use in conjunction with Feynman amplitudes is given in Appendix II).

Thus the relevant object to calculate is the partial wave amplitude $\langle ^3P_1^{(m)} | m | \pi d \rangle$. This is done by projecting our model amplitudes (4.3a and b) on the $^3P_1$ spin harmonic function according to

$$\langle ^3P_1^{(m)} | m | \pi d \rangle = \sum_{s_1 s_2} \int d\Omega_p \, y_{n L = 0}^{I M T} \langle \hat{p} | \tilde{u}_{n L} | \hat{p} \rangle$$

(4.4)
where \( M \) is given in expressions (4-3). The sum over spin will be easily performed by taking the trace of the total matrix once properly reduced to its two component form.

2. Vertices and Wavefunctions

We now define all quantities entering our model amplitude. The pionic currents \( \bar{u} \Gamma^\pi_{\pi N} u \) and \( u^T \Gamma^T_{\pi N} T_{\pi N}^T \) can be taken straightforwardly from the reductions (3-4) of Chapter III, setting the limit \( \vec{q} \to 0 \) and keeping only the lowest order terms in the static limit (\( E \sim M \)).

For the P.V. coupling note that the direct absorption vertex involves the nucleon momentum (see Fig. 32)

Using the momenta of Fig. (32) along with \( \vec{q} = 0 \) in expression (3-2) one has for the direct absorption vertex

\[
\bar{u}(\vec{p}) \Gamma^\pi_{\pi N} u(\vec{p}) = \frac{i \vec{q}}{\lambda M} \mu \left( \frac{\sigma \cdot \vec{p}}{2 M} + \frac{\sigma \cdot \vec{p}}{2 m} \right) \approx \frac{-i \vec{q}}{\lambda M} \frac{\mu}{M} \sigma \cdot \vec{p} \quad (4-5)
\]

while the rescattered pion reaches its absorption vertex with a significant momentum which contributes through the \( \vec{q} \cdot \vec{q} \) term of (3-2) as

\[-\vec{q}^T T_{\pi N} (\vec{p} - \vec{k}) \]. In this case we neglect the nucleon momentum contribution at this vertex which is known from existing calculations to make a small contribution. This approximation will be discussed further in Chapter VI.
along with our numerical results. All results for the required pion absorption vertices are collected in Table (4-7) at the end of this section.

For the sake of clarity we first construct our model for the case of pure $f^\mu f^\nu$ P.V. pion nucleon coupling. The modifications required to extend this model to various mixtures of both P.S. and P.V. couplings do not affect the general structure of the calculations and will therefore be discussed at the end of this chapter.

In this case the rescattering is entirely dominated by the $\rho$ -exchange mechanism whose contribution to the effective interaction $H_{\pi\pi}$ entering expression (4-1b) can be obtained directly from $M_\rho$ in Chapter III (see 3-12). A similar amplitude arises for the $\rho$ -exchange scattering through the pair configuration of the deuteron but with the spinor $u_s(p_1)$ replaced by the antispinor $v_{-s}(p_1)$. Using the parametrization $p_1 = \frac{v}{2} + k$ and $p_2 = \frac{v}{2} - p + q$ of Fig. (32) we reduce the dirac matrix elements $\bar{u} \gamma^\mu u$ and $\bar{u} \gamma^\nu v$ appearing in (3-12) to their two component forms. At threshold and in the static approximations, i.e. setting all $E$'s $\approx M$, keeping only the lowest order terms we have

$$\bar{u}(-p) \gamma^0 u(k) = \left( 1 + \frac{\sigma \cdot F \cdot k}{4M^2} \right) \approx 1$$
$$\bar{u}(-p) \gamma^k u(-k) = \left( -\frac{\sigma^k F \cdot k}{2M} - \frac{\sigma \cdot F \cdot k}{2M} \right) \approx 0 \quad (4-6)$$
$$\bar{u}(-p) \gamma^0 v(+k) = \left( \frac{\sigma^k F}{2M} - \frac{\sigma \cdot F}{2M} \right) \approx 0$$
$$\bar{u}(-p) \gamma^k v(+k) = \left( \sigma^k - \frac{\sigma \cdot F \cdot k}{4M^2} \right) \approx \sigma^k$$
Neglecting the momentum transferred $k^2$ in front of the exchanged mass $m_2^2$ we approximate these amplitudes by the contact interaction $\tilde{u}H_{\pi\mu}^{\sigma} u$ and $\tilde{u}H_{\pi\mu}^{\sigma} v$ given in the following Table where we have collected all the quantities entering our basic model amplitudes (h-3). The wavefunctions are taken from Chapter I (but with the extra normalization factor $\sqrt{m_d}$ used in their calculation by ref. 1) and the $^3P_1$ spin harmonic function is taken from Appendix II. The isospin part of the $M_{3P_1}$ amplitude can be written using the definitions of Fig. 33.

\[ \mathcal{K} \left\{ (-i \frac{T \cdot q^*}{\sqrt{2}})(T \cdot \pi^*_1)(-i \tau_2) \right\} \]

Fig. 33. Isospin Conventions

Pole Diagram

\[ \sum_{\chi} \mathcal{K} \left\{ (-i \frac{T \cdot q^*}{\sqrt{2}}) (i \tau_1 \epsilon_{\theta \phi \chi} \pi^*_1 \pi^*_2 \pi^*_1 \pi^*_{i \chi}) (-i \tau_2) (T \cdot \pi_2) \right\} \]
where $\gamma$ stands for the isospin vector of the final two nucleon state, $\pi_1^+$ and $\pi_2^-$ are the isospin of the incoming pion and rescattered pion respectively. The NN isospin states ($-i \tau_2$) and ($-i \tau_2 \frac{\tau \cdot \pi^*}{\sqrt{2}}$) are matrix representations of standard isospin wavefunctions for the deuteron and the $^3P_1$ state respectively. (See Appendix II)

Summing over the isospin of the rescattered pion (thus allowing for charge exchange rescattering as well) we have

\[
\sum_{k} \pi_2^k \pi_2^k = \delta_{kj} \tag{4.9}
\]

Circulating the $\tau_2$ under the trace and using $\tau_2^2 = 1$ and $\tau_2 \frac{\tau \cdot \pi^*}{\sqrt{2}} = \frac{\pi^*}{\sqrt{2}}$
we perform the traces using rules (4-19) and using also

\[
\epsilon_{\delta, ij} \epsilon_{\delta, e j} = -2 \delta_{\delta, i} \tag{4.10}
\]

we obtain the isospin factors:

**Pole**

\[
-\frac{1}{\sqrt{2}} \kappa (\pi \cdot \pi^* \pi \cdot \pi_1^+) = -\sqrt{2} \frac{\pi^* \cdot \pi_1^+}{\sqrt{2}} = \sqrt{2} \tag{4.11}
\]

**-$\rho$-rescattering**

\[
\frac{i}{\sqrt{2}} \kappa (\pi \cdot \pi^* \epsilon_{\delta, ij} \tau^k \pi_1^+ \delta_{kj} \pi_1^k) = \frac{2i}{\sqrt{2}} (\frac{\pi_1^+}{\sqrt{2}} \epsilon_{\delta, ij} \epsilon_{\delta, e j} \pi_1^i) = 2 \sqrt{2} \pi^* \cdot \pi_1^+ = 2 \sqrt{2} \tag{4.12}
\]
### Table of Reduced Vertices (4-7)

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{u}(-p) \Gamma^{r}_{\bar{r}} u(-p)$</td>
<td>$\frac{g}{2m} \left( \frac{-i\mu}{M} \sigma \cdot \vec{p} \right) \tau \cdot \pi_1$</td>
</tr>
<tr>
<td>$\bar{u}(-p) \Gamma^{r}_{\bar{r}} u(+p)$</td>
<td>$\frac{g}{2M} \left( \frac{i\mu}{M} \right) \tau \cdot \pi_1$</td>
</tr>
<tr>
<td>$\bar{u}^\tau(k) \Gamma^{r}_{\bar{r}} \bar{u}^\tau(-p)$</td>
<td>$\frac{g}{2m} \left( -i \vec{S} \cdot (\vec{p} \cdot \vec{R}) \right) \tau \cdot \pi_2$</td>
</tr>
<tr>
<td>$\bar{u}(-p) H^\rho_{\bar{r}n} u(-k)$</td>
<td>$\frac{g \rho_f}{2m^2} \left( \sigma \cdot (\vec{p} \cdot \vec{R}) \right) \tau_3 \cdot \pi_3 \epsilon_{0,2}$</td>
</tr>
<tr>
<td>$\bar{u}(-p) H^\rho_{\bar{r}n} u(+k)$</td>
<td>$\frac{g \rho_f}{2m^2} \left( \sigma \cdot (\vec{p} \cdot \vec{R}) \right) \tau_3 \cdot \pi_3 \epsilon_{0,2}$</td>
</tr>
</tbody>
</table>

\[
\frac{m}{E_k} \frac{\bar{u}(-k) \Gamma^{r}_{\bar{r}} \bar{u}^\tau(k)}{\sqrt{E_k - W}} = \sqrt{4m^2} (\rho_c \cdot \vec{x} + \rho_c \cdot \vec{y}) (3\rho_c \cdot \vec{x} - \rho_c \cdot \vec{y}) \bar{u}_2 (\tau_2)
\]

\[
-\frac{m}{E_k} \frac{\bar{u}(-k) \Gamma^{r}_{\bar{r}} \bar{u}^\tau(k)}{\sqrt{E_k - W}} = -\sqrt{4m^2} (\rho_c \cdot \vec{x} + \rho_c \cdot \vec{y}) (i \rho_c \cdot \vec{x} + \rho_c \cdot \vec{y}) \bar{u}_2 (\tau_2)
\]

\[
\gamma_1^{\alpha\beta \mu} (\rho) = \sqrt{\frac{3}{16\pi}} (i \rho_c \cdot \vec{x} \cdot \vec{y}) \cdot \bar{u}_2 \left( \frac{\tau_3}{12} \cdot \tau_2 \right)
\]
3. **Reduction of the Partial Wave Amplitude** $\langle 3p_1 | m | \pi^0 d \rangle$

All ingredients of the $\gamma^4 \gamma^5$ model being properly defined and listed for reference in Table (4-7) we proceed to calculate the partial wave amplitude $\langle 3p_1 (m) | m | \pi^0 d \rangle$. Using expressions (4-4) and (4-3) along with the previous tables we first encounter the following expressions:

\[
M_{3p_1}^{\mu \kappa} = \frac{3}{16\pi} \left( \frac{-i g^{12}}{2M} \right) \frac{\mu \sqrt{4\pi M}}{M} \int d\Omega_p \left\{ A + B + C + D \right\}
\]

\[
M_{3p_1}^{\text{recoil}} = \frac{3}{16\pi} \left( \frac{-i g^{12}}{2M} \frac{p_{ \pi^0} q_{\pi^0}}{m_{\pi^0}^2} \right) \frac{\mu \sqrt{4\pi M}}{(8\pi)^3} \frac{1}{\mu^2 - (p^2 - q^2)^2} \left\{ A' + B' - C' - D' \right\}
\]

where $A'B'C'D'$ are weighted traces collected in the following tables:

<table>
<thead>
<tr>
<th>Pole:</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$(\mathcal{L}(\rho) - \frac{W(p)}{r_2}) \left{ \sigma_2 \cdot (\vec{p} \times \vec{x}_3' \times) \cdot \vec{p} \cdot \vec{r}_d \cdot \sigma_2 \right}$</td>
</tr>
<tr>
<td>$B$</td>
<td>$\frac{3}{\sqrt{2}} \frac{\mathcal{L}(\rho)}{W(p)} \left{ \sigma_2 \cdot (\vec{p} \times \vec{x}_3') \cdot \vec{p} \cdot \vec{r}_d \cdot \vec{r}_d \cdot \sigma_2 \right}$</td>
</tr>
<tr>
<td>$C$</td>
<td>$m \sqrt{2} \mathcal{L}(\rho) \left{ \sigma_2 \cdot (\vec{p} \times \vec{x}_3) \cdot (\vec{r}_d \cdot \vec{r}_d) \cdot \sigma_2 \right}$</td>
</tr>
<tr>
<td>$D$</td>
<td>$m \sqrt{3} \mathcal{L}(\rho) \left{ \sigma_2 \cdot (\vec{p} \times \vec{x}_3') \cdot \vec{r}_d \cdot \vec{p} \cdot \sigma_2 \right}$</td>
</tr>
</tbody>
</table>
First we eliminate all $\sigma_2$ matrices by cycling them under the trace and using $\sigma_2^2 = 1$ for the pole traces and $\sigma_2 \sigma_2^T = -\sigma_2^T$ for the rescattering traces. We also rearrange the third scattering matrix by using:

$$
\sigma_2 (k \times \hat{3}_d) \sigma_2 (p^- R) = \sigma_2 (k \times \hat{3}_d) \cdot (p^- R) - (p^- R) \sigma_2 (k \times \hat{3}_d)
$$

(4-17)

thus replacing it by two simpler traces

$$
\sigma_2 (k \times \hat{3}_d) \cdot (p^- R) \ln \left\{ \sigma_2 (p^+ \hat{3}_d) \cdot (p^- R) \right\} - (p^- R)^2 \ln \left\{ \sigma_2 (p^+ \hat{3}_d) \cdot (k \times \hat{3}_d) \right\}
$$

(4-18)

The trace operations are then performed by using the standard trace rules
\[
\begin{align*}
\hbar (\sigma^i \sigma^j \sigma^k) &= 2 i e_{ijk} \quad \text{w.r.t. } A, A', B' \\
\frac{1}{\hbar} (\sigma^i \sigma^j) &= 2 \delta^{ij} \quad \text{w.r.t. } C, C' \\
\hbar \sigma^i &= 0 \quad \text{w.r.t. } B, D, D'
\end{align*}
\]

and we obtain the reduced table 4-12 for which use has been made of the notation

\[a^I b^J c^k e_{ijk} = a \cdot (b \times c)\]

### Table 4-20 reduced traces

<table>
<thead>
<tr>
<th>(a) Pole:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A:</strong></td>
</tr>
<tr>
<td><strong>B:</strong></td>
</tr>
<tr>
<td><strong>C:</strong></td>
</tr>
<tr>
<td><strong>D:</strong></td>
</tr>
</tbody>
</table>
We can then perform the angular part of the integrations appearing in expression (4-14).

While the pole contributions can be treated directly in momentum space, we have found more convenient to calculate the rescattering terms in position space in order to compare our results to those of existing calculations. Turning first to the rescattering term (4-3b) we implement the spectator prescription $k_0 = E_k - \frac{W}{2}$ with the further approximation $E_k - \frac{W}{2} \gg 0$ thus taking for the pion propagator:

$$\frac{1}{\mu^2 - (\not{p} - k_0)^2 + (\not{P} - \not{R})^2} \approx \frac{1}{3/4 \not{P}^2 + (\not{P} - \not{R})^2}$$

(4-21)
and the factor \((q_0 + (p - k_0))\) in \(\frac{e^{i \sqrt{3}}}{\pi \mu} \) becomes simply \(\frac{3}{2} \mu\). (Further comments on this approximation will be made at the end of this chapter.)

The propagator can then be fourier transformed in position space according to

\[
\frac{1}{\frac{3}{4} \mu^2 + (\mathbf{p} - \mathbf{k})^2} = \frac{1}{4 \pi} \int e^{-i (\mathbf{p} - \mathbf{k}) \cdot \mathbf{r}} \frac{e^{i \sqrt{3}/4 \mu \mathbf{r}}}{\mathbf{r}} \, d^3 \mathbf{r} \tag{4-22}
\]

Then, using the general expression

\[
(\mathbf{p} - \mathbf{k}) \int d^3 \mathbf{r} \, e^{i (\mathbf{p} - \mathbf{k}) \cdot \mathbf{r}} \, \mathcal{V}(\mathbf{r}) = i \int d^3 \mathbf{r} \left[ \frac{\nabla}{\mathbf{r}} e^{-i (\mathbf{p} - \mathbf{k}) \cdot \mathbf{r}} \right] \mathcal{V}(\mathbf{r}) \tag{4-23}
\]

where \(\mathcal{V}(\mathbf{r}) = \frac{1}{\mathbf{r}} e^{-\sqrt{3/4} \mu \mathbf{r}}\), and observing the positions of \((\mathbf{p} - \mathbf{k})\) in the rescattering traces \((4-20b)\) one can rewrite them as differential operators acting on the fourier exponential \(e^{-i (\mathbf{p} - \mathbf{k}) \cdot \mathbf{r}}\) which leaves us with the four typical position-space integrals (the last two ones, arising from \(C'\) are associated with \(v_r\)) \((4-24)\) to consider:

\[
(u(k) - \frac{\omega(k)}{r_k}) \int \frac{d^3 \mathbf{t}}{4 \pi} \left\{ (\mathbf{p} \cdot \mathbf{t} - \frac{3}{2} \mathbf{k} \cdot \mathbf{t}) e^{-i (\mathbf{p} - \mathbf{k}) \cdot \mathbf{t}} \right\} \mathcal{V}(\mathbf{r})
\]
Integrating by parts over $d^3r$ and dropping the vanishing ($V(r) \to 0$) surface terms, the differential operators can be made to act on the potential $V(r)$, following the general rules (4-17)

\[ \int d^3n \left( \vec{\alpha} \cdot \nabla e^{-i\vec{q} \cdot \vec{F}} \right) V(n) = - \int d^3n \ \vec{\alpha} \cdot \vec{r} \ e^{-i\vec{q} \cdot \vec{F}} \ \frac{dV(n)}{dn} \]

(4-25)

\[ \int d^3n (\vec{\alpha} \cdot \vec{\beta} \cdot \vec{\gamma} e^{-i\vec{q} \cdot \vec{F}}) V(n) = \int d^3n e^{-i\vec{q} \cdot \vec{F}} \left( \frac{\lambda}{\hbar} V(n) - \frac{\vec{\alpha} \cdot \vec{\beta} \cdot \vec{\gamma} V'(n)}{\hbar} + \frac{\vec{\alpha} \cdot \vec{\beta} \cdot \vec{\gamma} \cdot V''(n)}{\hbar^2} \right) \]

\[ \int d^3n \left( \nabla^2 e^{-i\vec{q} \cdot \vec{F}} \right) V(n) = \int d^3n e^{-i\vec{q} \cdot \vec{F}} \ \nabla^2 V(n) \]
Applying rule (4-25) in expression (4-24) (and leaving aside the $d\mathcal{P}$ integration and all factors external to the integral sign in (4-14)) we obtain for the $S$-state ($u$) rescattering contribution:

$$-3\mu \int \frac{d^3n}{\sqrt{\eta}} e^{ip\cdot n} V'(n) \left( \xi_d \cdot \tilde{p} \cdot \hat{F} - \xi_d \cdot \tilde{p} \cdot \hat{F} \right) \int \frac{d^3k}{(2\pi)^3} \ u(k) e^{i\tilde{K} \cdot \tilde{R}} \tag{4-26a}$$

for the $D$-state ($w$) rescattering contribution:

$$-3\mu \int \frac{d^3n}{\sqrt{\eta}} e^{ip\cdot n} V'(n) \int \frac{d^3k}{(2\pi)^3} e^{i\tilde{K} \cdot \tilde{R} \ u(k)} \frac{w(k)}{\sqrt{\eta}} \ x \left\{ (3\xi_d \cdot \hat{R} \ (\tilde{K} \cdot \xi'_d \hat{F} - \tilde{K} \cdot \xi'_d \hat{F}) - (3 \tilde{p} \cdot \xi'_d \hat{F} - \tilde{p} \cdot \xi'_d \hat{F}) \right\} \tag{4-26b}$$

and for the non vanishing $P$-state ($\nu_t$) rescattering contribution:

$$-\lambda i \int \frac{d^3n}{\sqrt{\eta}} e^{ip\cdot n} \int \frac{d^3k}{(2\pi)^3} e^{i\tilde{K} \cdot \tilde{R} \ \sqrt{\frac{3}{2}} \ \nu_t(k)} \ x \left\{ (\tilde{p} \times \xi'_d) \cdot (\tilde{K} \times \tilde{d}_t) \left[ V''(n) - \frac{2}{\eta} V'(n) \right] - 2(\tilde{K} \times \tilde{d}_t) \cdot \tilde{F} \ \left( \tilde{p} \times \xi'_d \right) \cdot V' \ x \left[ V''(n) - \frac{2}{\eta} V'(n) \right] \right\} \tag{4-26c}$$
Note that the last two $d^3k$ integrals ($w$ and $v_t$) do not depend on $\phi_k$ if we take $r$ as our $z$-direction. One can thus average D and P contributions over $d\phi_k$, using $(z = \cos \theta_k)$

\[
\frac{1}{2\pi} \int d\phi_k \hat{k}^i \hat{k}^j = \hat{\lambda}^i_j z
\] (4.27)

\[
\frac{1}{2\pi} \int d\phi_k \hat{k}^i \hat{k}^j = \left\{ \delta_{ij} \frac{1-z^2}{z} + \hat{\lambda}^i \hat{\lambda}^j D_2(z) \right\}
\] (4.28)

Note that the $\lambda^i \lambda^j P_2(z)$ term makes a vanishing contribution to the $\lambda^i \lambda^j$ term in the D-state amplitude. Note also that the last term of the P-state amplitude vanishes since using (4-19a), the $d\phi_k$ integration yields a $(r \times \hat{\xi}_d)^\dagger \hat{r} = 0$ term, and that $\left( \nabla^2 V(r) - \frac{2}{r} V'(r) \right) = V''(r)$. The D and P states amplitudes are thus reduced further to

(4-29a) and (4-29b)

\[
-3\mu \int \frac{d^3l}{4\pi} e^{-il \cdot \hat{r}} \sqrt{\lambda} \left( \hat{z}_d \cdot \hat{P} \hat{z}_d \cdot \hat{r} - \hat{z}_d \cdot \hat{P} \cdot \hat{r} \right) \int \frac{d^3k}{(2\pi)^3} \frac{w(k)}{\sqrt{2}} (-P_2(z)) e^{i\frac{l \cdot \hat{k}}{2}}
\] (4-29a)

\[
-\frac{\Xi}{2} \int \frac{d^3l}{4\pi} e^{-il \cdot \hat{r}} V'(\lambda) \left( \hat{f} \times \hat{\xi}_d \right)^\dagger \left( \hat{r} \times \hat{\xi}_d \right) \int \frac{d^3k}{(2\pi)^3} \sqrt{2} w(k) P_2(z) e^{i\frac{l \cdot \hat{k}}{2}}
\] (4-29b)
expanding $e^{i \mathbf{k} \cdot \mathbf{r}} = \sum (2\ell + 1) i^\ell \! j_\ell^\dagger (k \mathbf{n}) P_\ell (\mathbf{z})$ in expressions (4-26a) and (4-29), then using the orthogonality property (4-30) of these Legendre polynomials, along with the fourier transforms (4-31)

$$\int \frac{P_\ell (\mathbf{z}) P_m (\mathbf{z}) \, d\mathbf{z}}{\ell + 1} = \frac{\sqrt{2}}{2\ell + 1}$$  \hspace{1cm} (4-30)

$$\frac{\psi_k (\mathbf{r})}{\ell} = \sqrt{\frac{2}{\pi}} \int k^2 dk \int (k \mathbf{n}) \psi_k (k)$$  \hspace{1cm} (4-31)

we bring the momentum space radial wave function into position space and combining (4-26a) and (4-29) the three rescattering contributions reduce to the two integrals (4-23): (apart from external factors and the final $d \cdot \Omega \cdot p$ integration)

$$-\frac{2}{(2\pi)^{3/2}} \int \frac{d^3 \mathbf{n}}{4\pi} e^{-i \mathbf{k} \cdot \mathbf{n}} \left( \mathbf{\hat{z}} \cdot \mathbf{p} \cdot \mathbf{\hat{r}} - \mathbf{\hat{r}} \cdot \mathbf{\hat{r}} \cdot \mathbf{p} \right) \left( \psi_n (\mathbf{n}) + \frac{\psi' (\mathbf{n})}{\psi (\mathbf{n})} \right)$$  \hspace{1cm} (4-32)

$$-\frac{2}{(2\pi)^{3/2}} \int \frac{d^3 \mathbf{n}}{4\pi} e^{-i \mathbf{k} \cdot \mathbf{n}} \left( \mathbf{\hat{z}} \cdot \mathbf{p} \cdot \mathbf{\hat{r}} - \mathbf{\hat{r}} \cdot \mathbf{\hat{r}} \cdot \mathbf{p} \right) \sqrt{\frac{2}{z}} \psi (\mathbf{n})$$

introducing the appropriate spherical harmonics
\[
\begin{align*}
\hat{e}^{-i \mathbf{p} \cdot \mathbf{r}} &= \eta \pi \sum_{\ell m} (-1)^m \left( \begin{array}{c}
\ell \ \ell m \\
\end{array} \right) Y_{\ell m}(\hat{r}) \ Y_{\ell m}^{\star}(\hat{\mathbf{r}}) \ f_{s}(\mathbf{r}) \\
\sqrt{\frac{3}{4\pi}} \ \hat{x} \cdot \mathbf{r} &= \hat{x} \ Y_{10}(\hat{\mathbf{r}}) \\
\sqrt{\frac{3}{4\pi}} \ \hat{p} \cdot \mathbf{r} &= \ Y_{10}(\hat{\mathbf{r}})
\end{align*}
\]

(4-33)

We can integrate over \( d \Omega \) in expressions (4-32) and using the orthogonality of the \( Y_{\ell m} \) and definitions (4-34) we obtain:

\[
\int \frac{d \Omega}{4\pi} \ e^{-i \mathbf{p} \cdot \mathbf{r}} \ (\hat{z}_d \cdot \hat{r} - \hat{z}_d \cdot \hat{x} \ f_{s}(\mathbf{r})) = -i (\hat{\mathbf{z}}_d \cdot \hat{f} \hat{x} \ f_{s}(\mathbf{r}) - \hat{z}_d \cdot \hat{x} \ f_{s}(\mathbf{r})) \ f_{s}(\mathbf{r})
\]

(4-35)

and the final integration over \( d \Omega \) which appear in expression (4-14) can at last be performed on both the pole (using the results of Table (4-20a)) and the rescattering amplitude. Using the prescription (4-36)

\[
\int d \Omega \hat{p} \ f = \eta \frac{\delta_{ij}}{3} \quad \Rightarrow \quad \int d \Omega \hat{z}_d \cdot \hat{p} \ f_{s} = \eta \frac{\delta_{ij}}{3}
\]

(4-36)

yielding a factor \((8\pi/3)i \ \hat{z}_d \cdot \hat{x} \ f_{s}\) for the rescattering (from (4-26))
and \( \frac{16\pi}{3} \xi_0' \) for the pole (from (4-20a)). We have thus reduced the \( \langle 3p_1 | m_l | qd \rangle \) partial wave amplitude to the sum of terms (4-37):

\[
m_{3p_1} = \sqrt{\frac{16\pi}{3}} \left( \frac{\mu \rho \sqrt{4\pi \lambda m}}{M} \right) \left( \frac{ie\hat{g}}{2M} \right) \left\{ P + R \right\} \xi_0' \xi_1'
\]

(4-37)

where we use the definitions

\[
P = \left\{ u(p) - \frac{W(p)}{\sqrt{2}} + \frac{N}{\rho} \sqrt{\frac{2}{\lambda}} v_{\xi}(p) \right\}
\]

(4-38)

\[
R = \frac{\rho \lambda}{4\pi \mu m^2} \frac{N}{\rho} \left\{ \frac{3}{2} I_3 + \frac{1}{\mu} I_4 \right\}
\]

(4-39)

where \( I_3 \) and \( I_4 \) are overlap integrals to be calculated in position space.

\[
I_3 = -\sqrt{\frac{\pi}{2}} \int_0^{\infty} r \, dr \left( u(r) + \frac{W(r)}{\sqrt{2}} \right) V'(r) j_1(\rho)
\]

(4-40)

\[
I_4 = \sqrt{\frac{\pi}{2}} \int_0^{\infty} r \, dr \sqrt{\frac{2}{\lambda}} v_{\xi}(r) V''(r) j_1(\rho)
\]
b. $\gamma^5$ Coupling and $\sigma$-Rescattering

It is now a trivial matter to extend the model described in the previous sections to mixtures of both $\Upsilon N$ couplings. The results of Chapter III dictate the following modifications:

a) Pole contribution

For the $\Upsilon N$ coupling we now take the mixture

$$\Gamma_{\Upsilon N} = ig \left( \lambda \gamma^5 + \frac{1-\lambda}{2\mu} \sigma \gamma^5 \right)$$  \hspace{1cm} (4.41)

and the results $[\bar{u}(p') \gamma^5 u(p)]_{q=0} = 0$ and $[\bar{u}(p') \gamma^5 v(p)]_{q=0} = 1$ of p. 52 indicate that the $S$ and $D$ state wavefunctions $u$ and $w$ do not contribute to the $\lambda \gamma^5$ term while the small components behave just like in the previous section but with the enhancement factor $\frac{2\mu}{\mu}$. Thus we can rewrite the Pole contribution (4.39) as

$$P^I = (1-\lambda) \left\{ u(p) - \frac{w(p)}{\sqrt{2}} + \frac{\gamma}{\mu} \sqrt{\frac{2}{\mu}} \nu(p) \right\} + \lambda \frac{2\mu}{\gamma \mu} \sqrt{\frac{2}{\mu}} \nu(p)$$  \hspace{1cm} (4.42)

b) Rescattering contribution

In order to include pion rescattering consistently with the $\gamma^5 \Upsilon N$ coupling we now add the $\sigma$-exchange contribution to $\bar{u} \Upsilon \gamma^5 \gamma^5 u$
and \( \bar{w} c_{\pi \pi} \) using

\[
\bar{u}(\mathbf{-p}) H_{\pi \eta}^\nu u(-k) = -\frac{\alpha_s f \xi}{m_\rho^2} \eta
\]

\(
\bar{u}(\mathbf{-p}) H_{\pi \eta}^\sigma v(-k) = 0
\) \hspace{1cm} (4-43)

The \( \mathcal{O} \)-term therefore contributes only through the S and D states wavefunctions and yields the same overlap integral as the \( \rho \)-term but with a different weight thus simply adding to the rescattering contribution \( R \) a term

\[
-\frac{\alpha_s f \xi}{4\pi m_\rho^2} \frac{M}{\mu p} I_3
\]

which can be expected to cancel most of the undesirably large \( \lambda v_t(\rho) \) term in (4-42).

Note however, that as the \( \mathcal{O} \)-exchange was meant to control the large yield of both nucleon pole graphs in \( \Pi N \) scattering, one must examine more carefully the role of these particular graphs in the rescattering contribution to pionic deuteron disintegration. This can be done by expanding the rescattering bubble in Fig. 34:
Fig. 34. Role of the nucleon poles contributions to $\pi N$ scattering in pionic deuteron disintegration.

One can see that, while the crossed pole term of Fig. (34c) is already included in the deuteron wavefunctions (dotted circle) and should not be counted again, the direct pole is not and can be regarded as part of the final state interactions as suggested by Fig. (34b).

Although we have assumed, on the basis of the results of N.R. calculations that the $^3P_1$ final state interactions could be neglected, these N.R. models did not include the small components of the wavefunctions and could ignore the problem of controlling the large $\gamma^5$ pair terms in $\pi N$ scattering. Thus the decomposition of Fig. (34) suggests that the $\sigma$-term in the rescattering mechanism should be viewed as controlling the large effects of the $\gamma^5$ pair configurations of both the initial and final state NN wavefunctions.

In order to verify this assumption and to establish the guidelines for a model of deuteron pionic disintegration that could
consistently include the antinucleon degrees of freedom for all mixtures of P.V.-P.S. pion nucleon coupling, we will thus mock up the effect of the final state pair configuration wavefunctions by decomposing the Feynman propagator \( S_p(\vec{w} - k + q) \) in Fig. 3a according to (2-12) and retaining only the antinucleon propagation term

\[
\frac{\nu(k) \bar{\nu}(k)}{\mathcal{M} + \mu}
\]

The diagram of Fig. (3a) reduces to an effective contact rescattering term which yields again the overlap integral \( I_3 \), with a sign opposed to that of the \( P \)-contribution. Thus including all extra terms discussed in this last section of our model amplitude is calculated as

\[
M_{3p} = \sqrt{\frac{\mu}{3}} \left( \frac{\mu \pi}{\mathcal{M}} \right) \left( \frac{\sqrt{2} \mathcal{R}}{2M} \right) \left\{ P' + R' \right\} \bar{\xi} \cdot \bar{\xi}'
\]

with \( P' \) given by (4-42) and \( R' \) written as follows:

\[
R' = \left( \frac{\frac{g_p}{4\pi M^2} \frac{M}{2}}{4\pi M^2} - \frac{g_r}{4\pi M^2} \frac{M}{M^2} - \frac{\lambda(\lambda - 1 M)}{4\pi M^2} \frac{g_{\pi\pi}}{M^2} \right) I_3
\]

\[
+ \left( \frac{\frac{g_p}{4\pi M^2} \frac{M}{M^2}}{4\pi M^2} \frac{M}{M^2} \right) I_4
\]
with \( I_3 \) and \( I_4 \) defined by (4-39) and where the \( \sigma \) parameters are determined by the constraint of \( \pi N \) scattering using expression (3-18) from Chapter III.

In the next and last chapter of this work we collect the numerical results of this analysis for the two fundamental cases \( \lambda = 0 \) (pure P.V. \( \pi N \) coupling) and \( \lambda = 1 \) (pure P.S. \( \pi N \) coupling) and present the general conclusion of this dissertation.
V. NUMERICAL RESULTS AND CONCLUSIONS

1. Definition of the Cross Section

The standard definition for a two body reaction cross section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{p_f}{p_i} \frac{m^2}{16\pi^2 w^2} \frac{1}{(2s_a+1)(2s_b+1)} \sum |m|^{2}$$  \hspace{1cm} (5-1)

where $p_i$ and $p_f$ are the relative C.M. momentum of the initial and final particles, $W$ the total C.M. energy and $s_a, s_b$ the spins of the initial particles. In our case, using amplitude $M_{3p_1}$ \hspace{1cm} (4-43) (with an extra factor 2 arising from the inclusion of the crossed graphs required for a proper antisymmetrization of the final pp state, see Fig. 35) we obtain,
Fig. 35. Antisymmetrization of the final pp state by inclusion of the appropriate crossed graphs simply reduces to an extra factor 2 at threshold.

(Defining the dimensionless variable \( \eta = q/\mu \) and using for \( p_f \) the threshold value \( \sqrt{\mu M} \) given in Chapter I)

\[
\bar{\sigma} = \left\{ \frac{q^2}{3} \left( \frac{\mu}{M} \right)^{3/2} \pi \mu \mathcal{A}_0^2 \right\} \frac{1}{\eta} = \frac{\eta}{\eta} \tag{5-2}
\]

with \( \mathcal{A}_0^2 = (P' + R') \)

where \( P' \) and \( R' \) are the final results (4-42) and (4-45) of Chapter IV.
The standard parametrization of \( \sigma(p^+d \rightarrow pp) \) is most often given in terms of the inverse reaction \( pp \rightarrow n^*d \):

\[
\sigma_{pp \rightarrow nd} = \alpha \eta + \beta \eta^2
\]  

(5-3)

where \( \alpha \) characterizes the production of \( s \)-wave pions and \( \beta \) defines \( p \)-wave pions. Using the principle of detailed balance to relate this cross section to its inverse we have:

\[
\sigma_{nd \rightarrow pp} \sim \frac{2}{3} \frac{\mu_n}{\mu_p} \frac{1}{\eta^2} \sigma_{pp \rightarrow nd} = 4.48 \left( \frac{\alpha}{\eta} + \beta \eta \right)
\]  

(5-4)

according to (5-2) we have thus calculated the coefficient \( \alpha = 4.48 \alpha \) corresponding to \( s \)-wave incoming pions. The second term arises from \( p \)-wave pions and will be dominated by the formation of a \( \Delta_{3/2} \) resonance which is not included in our model. Available experimental data for this process yields\(^2\)

\[
\alpha_{exp} = 240 \pm 20 \mu b
\]

\[
\alpha_{exp} = 1.08 \pm 0.1 mb
\]

\[
A_{exp} = 1.25 \pm 0.05 \text{ GeV}^{-3/2}
\]  

(5-5)
and we can thus compare the result of our calculation to this experimentally determined value of $J^p$.

2. Numerical Results

The deuteron wavefunctions $u(r)$, $w(r)$ and $v_t(r)$ used in evaluating the overlap integrals $I_3$ and $I_{11}$ (see p. 79) appearing in the rescattering contribution $R'$ can be calculated from the general expansion

$$f(r) = \sum_{n=0}^{5} A_m e^{-\left(\alpha_1 + \beta \alpha_2 \right) r} \left( 1 - e^{-\alpha_2 r} \right)$$

Using a particular set of coefficients (given in Table 5-12)

$$\left( A_0, \ldots, A_5, \alpha_1, \alpha_2 \right)$$

for each of the wavefunctions and for a given value of the $\pi N$ coupling mixing parameter $\lambda$. These coefficients were calculated by solving a relativistic quasipotential equation described elsewhere.\(^{11}\) The momentum space wavefunctions $u(p)$, $w(p)$, $v_t(p)$ appearing in the pole contribution $P'$ are then obtained straightforwardly by Fourier transforming (analytically) the position space wavefunctions (5-6).

a) Pure pseudovector $\pi N$ coupling ($\lambda = 0$)

In this case the problem of controlling the large $\gamma^5$ pairs does not arise and we can set the $\gamma$-term to 0 and we obtain the results of Table (5-7)
From these results we can draw our first conclusion. Even in the \( \lambda = 0 \) case, for which the small components have their smallest size, they can still make a non-negligible contribution \( (J_2 + J_4 = .51) \) to our model amplitude for threshold pionic deuteron disintegration both through the impulse \( (J_2) \) and rescattering \( (J_4) \) mechanisms. These contributions appear to be instrumental in cancelling the \( S-D \) state contributions to the pole diagram (thus leaving the \( \rho \) -rescattering as the dominant contribution) and therefore indicate that the small components of the deuteron wavefunctions have measurable effects in this process.

b) Pure pseudoscalar coupling \( (\lambda = 1) \)

In this case the \( \frac{wu(p) - w(0)}{\sqrt{2}} \) makes no contribution to the pole term (see expression (4-42) on p. 80 in which the \( \lambda = 1 \) term vanishes).
which reduces to the $v_t$ term only

$$\left[ \frac{2M}{\mu} \frac{N}{\hbar} \sqrt{\frac{3}{2}} \, V_t(p) \right]_{p = 3\text{c.m.}V} = 21.5 \gg \frac{\alpha}{\epsilon} \quad (5-8)$$

This is a very large value which should be controlled by the cancellation of all the λ dependent terms (including the $\sigma^-$) from $P'$ and $R'$, i.e., we expect that

$$A_{0N\mu} = \frac{2\pi}{\mu} \left\{ \sqrt{\frac{3}{2}} \, V_t(p) - \left( \frac{g_{\sigma} f_{\sigma}}{4\pi \, m_N^2} \frac{\Gamma}{2M} + \frac{g_{\pi}^2}{4\pi M} \right) I_3 \right\} \quad (5-9)$$

will be small ($\ll 1$) if we set the $\sigma^-$-term according to the condition (3-15) of Chapter III. It is clear, however, that we cannot expect more than an approximate cancellation in our model since although the pole contribution is exact, we have made a number of approximations in our treatment of the rescattering diagram. These small corrections can affect sensitively the cancellation in (5-9) because it is multiplied by the large external factor $2M/\mu$.

Implementing the consistency condition (3-15) we take the parameter to be:

$$\frac{g_{\sigma} f_{\sigma}}{m_N^2 \pi} = \frac{g_{\pi}^2}{M} \quad (5-10)$$
and the corresponding result for the $\lambda = 1$ case are given in Table (5-11).

Table (5-11). $\lambda = 1$ Contribution to $\mathcal{A}$

<table>
<thead>
<tr>
<th>$\mathcal{A}_{\lambda=1}$</th>
<th>$\mathcal{A}_{\text{mm}}$</th>
<th>$J_3$</th>
<th>$J_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2M}{\mu} \frac{M}{p} \left{ \left( \frac{r^2}{4} \psi(p) - \left( \frac{g^2}{4\pi} \frac{3}{2} \bar{L}_3 \right) \right) \right}$</td>
<td>$g_f \bar{f} \frac{3}{2} \frac{M}{p} \bar{T}_3$</td>
<td>0.95</td>
<td>1.33</td>
</tr>
<tr>
<td>3.27</td>
<td>$\approx 1.$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This result suggests the following remarks:

It is now clear that inclusion of the $\jmath$-term in the re-scattering mechanism brings about the expected control of the large $\gamma^5$ pair terms. Although the exact result of the cancellation is sensitive to small uncertainties in the rescattering mechanism, the large $v_t(p)$ pole contribution is mostly suppressed.

The fact that the $\rho$-rescattering through the small components of the deuteron ($J_4$) is now making an uncomfortably large contribution brings our attention back to low energy $\pi N$ scattering. Although the $\jmath$-exchange can control the largest discrepancies in
the $\gamma^5$ theory, which occurred for the symmetric $\pi N$ scattering length $b^+$, the $\omega$-meson however cannot affect the antisymmetric scattering length $b^-$ for which a smaller discrepancy remains, (see Table 3,17) which can affect our description of the rescattering mechanism in the $\gamma^5$ model of pionic disintegration.

We believe, however, that a proper inclusion of the static contributions of the $\Delta_{33}$ pole diagrams (see Fig.(36))

![Fig. 36. $\Delta$ Resonance pole contributions to $\pi N$ scattering.](image)

to the $\pi N$ scattering lengths can resolve this discrepancy and that when included in our model of deuteron pionic disintegration, this contribution will bring this model in better agreement with the data.

Our $\lambda = 1$ calculation therefore suggests that it is possible to construct models of pion absorption in nuclei which include the small components of the nucleon wavefunctions (antinucleon degrees of freedom) for all mixtures of the P.S. ($\gamma^5$) and P.V. ($\gamma^5 \gamma^5$) $\pi N$ couplings provided that these models be consistent with the constraints of $\pi N$ scattering.
### Table 5-12. Wavefunctions Coefficients to be Used in Expansion 5-6.

#### \( \lambda = 0 \)

<table>
<thead>
<tr>
<th></th>
<th>( u )</th>
<th>( w )</th>
<th>( v_\tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>45.69</td>
<td>45.69</td>
<td>400.0</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>140.0</td>
<td>70.0</td>
<td>140.0</td>
</tr>
<tr>
<td>( A_6 )</td>
<td>.92598</td>
<td>.050699</td>
<td>-.17927</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>.81055</td>
<td>.94493</td>
<td>1.6937</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>-.34693</td>
<td>-.3728</td>
<td>-5.4284</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>-.63224</td>
<td>14.268</td>
<td>13.969</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>-.40111</td>
<td>-.985</td>
<td>-17.282</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>5.0189</td>
<td>7.1847</td>
<td>7.2570</td>
</tr>
</tbody>
</table>

#### \( \lambda = 1 \)

<table>
<thead>
<tr>
<th></th>
<th>( \kappa_1 )</th>
<th>( \kappa_2 )</th>
<th>( u_\tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa_1 )</td>
<td>45.69</td>
<td>45.69</td>
<td>140.0</td>
</tr>
<tr>
<td>( \kappa_2 )</td>
<td>140.0</td>
<td>70.0</td>
<td>140.0</td>
</tr>
<tr>
<td>( A_6 )</td>
<td>.94927</td>
<td>.050129</td>
<td>.0014786</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>.82048</td>
<td>1.0232</td>
<td>.83822</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>-.14466</td>
<td>-.7450</td>
<td>-.042775</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>-.3.9127</td>
<td>21.974</td>
<td>4.3322</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>-.3.9166</td>
<td>-.37.374</td>
<td>-11.690</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>6.2489</td>
<td>18.666</td>
<td>6.6294</td>
</tr>
</tbody>
</table>
APPENDIX I
POLE DIAGRAM CONTRIBUTION TO PIONIC
DISINTEGRATION OF THE DEUTERON

To gain a global insight in the possible effects of the pair configurations of the deuteron in this process we have performed a completely relativistic calculation of the nucleon pole diagram contribution to the reaction total cross section.

Proper antisymmetrisation of the amplitude under exchange of the final two protons is implemented by calculating the difference of the two diagrams of Fig. A-1. The corresponding Feynman amplitude can be written as (neglecting the phase)
We take the pion absorption vertex to be a mixture of the P.S. and P.V. couplings and the deuteron structure is described by the well known Blankenbecler and Cook vertex, which involves the four vertex invariant functions $F, G, H, I$:

\[ \Gamma_{\text{tot}} = \left\{ \frac{\text{v}^\lambda}{\sqrt{2}} g \left\{ \lambda g^\lambda - (1 - \lambda) \frac{g_{t_u}}{2M} g^u \right\} \right\} \]

\[ \Gamma_{\text{d}} = \left\{ F g^\lambda - G \frac{g_{t_u}}{M} + \frac{(K_{t_u} - M)}{M} (H g^\lambda - I \frac{g_{t_u}}{M}) \right\} \]

In these expressions we are using the labels $t, u$ for bookkeeping purposes. $g$ is the polarization vector of the deuteron. The notation $A \cdot B$ indicates a scalar product of 4-vectors. Note that we are implementing isospin directly by using the charged pion coupling constant $\sqrt{2}g$.

In order to evaluate the cross section from amplitude (A-1) we introduce a covariant expansion of the general amplitude $\bar{u}(p_2)M\bar{u}^T(p_1)$ for the $\pi d \rightarrow pp$ reaction in terms of six reaction invariants $A, B, C, D, E, F'$ (see ref 21)
\[ M = A \gamma^5 \frac{\vec{\xi}}{\xi} \cdot \vec{P} + B \gamma^5 \frac{\vec{\xi}}{\xi} \cdot \vec{Q} + C \gamma^5 \frac{\vec{\xi}}{\xi} \]
\[ + D \gamma^5 \vec{d} \cdot \vec{\xi} + E \gamma^5 \vec{d} \cdot \vec{P} \cdot \vec{\xi} + F \gamma^5 \vec{Q} \cdot \vec{\xi} \]  

(A-1)

in which \( d \) is the deuteron 4-momentum and \( P, Q \) are four vectors defined by
\[ P = \frac{1}{i} (k + \frac{1}{2} \vec{r}) \]
\[ Q = \frac{1}{i} (k - \frac{1}{2} \vec{r}) \]  

(A-5)

The contributions of the Pole amplitude (A-1) to the reaction invariants are then calculated and we obtain the relations of Table (A-9) giving the reaction invariants in terms of the deuteron vertex functions. Throughout this appendix we take the \( z \) direction to be along \( \vec{d} \) in the C.M. with \( \vec{P}_1 \) and \( \vec{P}_2 \) defined as indicated in Fig. (A-2)

![Diagram](image-url)  

Fig. A-2.
Table A-9. Pole Contributions to the Reaction Invariants.

<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
</table>
| \( A_{\mu \nu} = 2 \sqrt{g} \left\{ (G_t + G_u) - (F_t + F_u) + \frac{1}{2M^2} (I(p_t) + I(p_u)) \right\} \) | \[ \]
| \( + \frac{\sqrt{g} (1-\lambda)}{2M^2} \left\{ G(p_t) + G(p_u) - 2(H(p_t) + H(p_u)) \right\} \) | \[ \]
| \( B_{\mu \nu} = 2 \sqrt{g} \left\{ (G_t - G_u) - (F_t - F_u) + \frac{1}{2M^2} (I(p_t) - I(p_u)) \right\} \) | \[ \]
| \( + \frac{\sqrt{g} (1-\lambda)}{2M^2} \left\{ G(p_t) - G(p_u) - 2(H(p_t) - H(p_u)) \right\} \) | \[ \]
| \( C_{\mu \nu} = \frac{\sqrt{g}}{M} \left\{ H(p_t) + H(p_u) \right\} + \frac{\sqrt{g} (1-\lambda)}{2M} \left\{ F(p_t) + F(p_u) - 2(H(p_t) + H(p_u)) \right\} \) | \[ \]
| \( D_{\mu \nu} = \sqrt{g} \left\{ F_t - F_u \right\} + \frac{\sqrt{g} (1-\lambda)}{2M} \left\{ I(p_t) + I(p_u) \right\} \) | \[ \]
| \( E_{\mu \nu} = \frac{\sqrt{g}}{M} \left\{ (G_t + G_u) + \frac{\sqrt{g} (1-\lambda)}{2M^2} \left\{ I(p_t) + I(p_u) \right\} \right\} \) | \[ \]
| \( F_{\mu \nu} = \frac{\sqrt{g}}{M} \left\{ (G_t - G_u) + \frac{\sqrt{g} (1-\lambda)}{2M^3} \left\{ I(p_t) - I(p_u) \right\} \right\} \) | \[ \]

where the notation \( G_{t,u} \) stands for: \[
\frac{G(p_{t,u})}{k_{t,u}^2 - M^2}
\]
The arguments of the deuteron vertex functions appearing in Table (A-9) are defined as (see Fig. A-1)

\[
\begin{align*}
\vec{p}_t &= \frac{\vec{k}_t - \vec{p}_s}{2} = \vec{q} - \vec{p}_s \\
\vec{p}_u &= \frac{\vec{k}_u - \vec{p}_s}{2} = \vec{q} - \vec{p}_s
\end{align*}
\]  \hspace{1cm} (A-6)

They can be calculated in the C.M. frame (for which \(|\vec{q}^2| = |\vec{s}|^2\)) as

\[
\begin{align*}
|\vec{p}_t| &= \sqrt{\frac{q^2}{4} + p^2 - \vec{p}_s \cdot \vec{d}} = \sqrt{\frac{q^2}{4} + p^2 - \vec{p}_s \cdot \vec{d}} \\
|\vec{p}_u| &= \sqrt{\frac{q^2}{4} + p^2 - \vec{p}_s \cdot \vec{d}} = \sqrt{\frac{q^2}{4} + p^2 + \vec{p}_s \cdot \vec{d}} \hspace{1cm} \text{(A-7)}
\end{align*}
\]

Note that the pole factors \((k_t^2 - M^2)^{-1}\) appearing in the next table can be calculated from expressions (A-11) on page 98, giving:

\[
\begin{align*}
(t^2 - M^2) &= (k_t^2 - M^2) = -2 p_0 q_0 + 2 \vec{p} \cdot \vec{q} \cos \theta \\
(u^2 - M^2) &= (k_u^2 - M^2) = \mu^2 - 2 p_0 q_0 - 2 \vec{p} \cdot \vec{q} \cos \theta \hspace{1cm} \text{(A-8)}
\end{align*}
\]

All relevant kinematical quantities will be properly defined in the C.M. frame at the end of this section.
Using expansion (A-4) one can then construct as functions of
the six reaction invariants a set of six independent helicity amplitudes
(ref. Vasavada, Wick) $H_i$ from which the cross section can then be straight-
forwardly computed according to

$$\frac{d\sigma}{d\Omega} = \frac{1}{3} \frac{M^2}{16 \pi^2 W^2} \sum \frac{1}{|H_i|^2} \quad (A-10)$$

Table (A-14) contains the kinematic coefficients required to construct
the helicity amplitudes from the reaction invariants appearing in ex-
pression (A-4).

By using the properties of the Mandelstam variables

$$S = (\vec{q} + \vec{q'})^2 = M_d^2 + \mu^2 + 2(q_0 \cdot d_0)_{\text{lab}}$$
$$t = (\vec{p}_e - \vec{q})^2 = M^2 + \mu^2 - 2(p_0 q_0 - \vec{p}_e \cdot \vec{q})_{\text{CM}} \quad (A-11)$$
$$u = (\vec{p}_e - \vec{q'})^2 = M^2 + \mu^2 - 2(p_0 q_0 - \vec{p}_e \cdot \vec{q'})_{\text{CM}}$$

which are such that

$$S + t + u = \sum |H_i|^2 = 2M^2 + M_d^2 + \mu^2 \quad (A-12)$$
Table A-14. Matrix of Kinematic Coefficients $K_{ij}$ Giving the Helicity Amplitudes in Function of the Reaction Invariants.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F'</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>0</td>
<td>$\frac{p_0 \sin \theta}{\sqrt{2}}$</td>
<td>0</td>
<td>$-\frac{(q^0 - d_1) \sin \theta}{\sqrt{2}}$</td>
<td>0</td>
<td>$-\frac{M_0 p_0 \sin \theta}{\sqrt{2}}$</td>
</tr>
<tr>
<td>$H_2$</td>
<td>0</td>
<td>0</td>
<td>$\frac{p(1 + \cos \theta)}{\sqrt{2}}$</td>
<td>$\frac{Mq(1 + \cos \theta)}{\sqrt{2}}$</td>
<td>0</td>
<td>$\frac{p \sin \theta}{\sqrt{2}}$</td>
</tr>
<tr>
<td>$H_3$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{p(1 - \cos \theta)}{\sqrt{2}}$</td>
<td>$\frac{Mq(1 - \cos \theta)}{\sqrt{2}}$</td>
<td>0</td>
<td>$\frac{b \sin \theta}{\sqrt{2}}$</td>
</tr>
<tr>
<td>$H_4$</td>
<td>0</td>
<td>$-\frac{p_0 \sin \theta}{\sqrt{2}}$</td>
<td>0</td>
<td>$-\frac{(q^0 - d_1) \sin \theta}{\sqrt{2}}$</td>
<td>0</td>
<td>$\frac{M_0 p_0 \sin \theta}{\sqrt{2}}$</td>
</tr>
<tr>
<td>$H_5$</td>
<td>$\frac{b^2 q}{M_d}$</td>
<td>$-\frac{p_0 d_0 \sin \theta}{M_d}$</td>
<td>$-\frac{Mq}{M_d}$</td>
<td>$-\frac{pMq \cos \theta}{M_d}$</td>
<td>$-\frac{M_0 q}{M_d}$</td>
<td>$\frac{M_0^2 p \sin \theta}{M_d}$</td>
</tr>
<tr>
<td>$H_6$</td>
<td>0</td>
<td>0</td>
<td>$\frac{p d_0 \sin \theta}{M_d}$</td>
<td>0</td>
<td>$\frac{b q^0 \sin \theta}{M_d}$</td>
<td>$-\frac{p q_0 d_0 \cos \theta}{M_d}$</td>
</tr>
</tbody>
</table>

$$H_i = \frac{1}{8\pi \rho} \left\{ K_{i1} A + K_{i2} B + K_{i3} C + K_{i4} D + K_{i5} E + K_{i6} F \right\}$$
We can calculate all the kinematical quantities required by Table (A-14) in the C.M. frame solely in terms of rest masses and of the lab frame kinetic energy $E_{\pi}$ of the incoming pion. Taking

\[ q_{0,\text{Lab}} = (\mu + E_{\pi}) \]

one has

\[
\lambda = M^2_d + \mu^2 + 2 M_d (\mu + E_{\pi})
\]

\[
2p_0 = W = \sqrt{\lambda}
\]

\[
p = \sqrt{p_0^2 - M^2}
\]

then using (A-11) and (A-12) one obtains

\[
q_0 = \frac{\lambda + \mu^2 - M^2_d}{4p_0}
\]

\[
|q_1| = \sqrt{q_0^2 - \mu^2}
\]

\[
d_0 = \sqrt{M^2_d + q^2}
\]

Thus all quantities appearing in Table A-14 are properly defined as functions of $E_{\pi}$. 

APPENDIX II

MATRIX REPRESENTATION FOR THE NN SPIN HARMONIC FUNCTIONS

Relativistic calculation of processes involving the NN system are most conveniently performed by considering Feynman diagrams in which the NN wavefunctions appear as vertices along a fermion line, thus requiring a matrix representation of the spin and isospin functions.

The usual direct product representation for the singlet (B-1) and triplet (B-2) NN spin states wavefunctions

\[ \chi_{00} = \frac{\alpha(1)\beta(2) - \beta(1)\alpha(2)}{\sqrt{2}} \]

\[ \chi_{\pm M} = \begin{cases} 
\frac{\alpha(1)\alpha(2)}{\sqrt{2}} & M = 1 \\
\frac{\alpha(1)\beta(2) + \beta(1)\alpha(2)}{\sqrt{2}} & M = 0 \\
\beta(1)\beta(2) & M = -1 
\end{cases} \]  

can be rewritten as matrices by defining the matrix representation of direct products such as \( \alpha(1) \alpha(2) \) according to
\[
\begin{pmatrix}
(a_1) \\
(a_2)
\end{pmatrix}
\otimes
\begin{pmatrix}
(b_1) \\
(b_2)
\end{pmatrix}
= \text{def.}
\begin{pmatrix}
a_1b_1 & a_1b_2 \\
a_2b_1 & a_2b_2
\end{pmatrix}
\] (B-3)

Introducing the polarization vector \( \zeta^{(m)} \) for the triplet states and relating the results to the appropriate \( \sigma \)-matrices one obtains the matrix equivalents of (B-1,2)

\[
\chi^{oo} = \frac{-i\sigma_2}{\sqrt{2}}
\]
\[
\chi^{im} = \frac{i\sigma \zeta^{(m)}\sigma_2}{\sqrt{2}}
\] (B-4)

where \( \zeta^{(i)} = \frac{-i}{\sqrt{2}} (1, i, 0) \), \( \zeta^{(o)} = (0, 0, 1) \), \( \zeta^{(t)} = \frac{i}{\sqrt{2}} (1, -i, 0) \).

These forms can be directly used to construct the isospin part of Feynman amplitude as in p. 66.

For the spin wavefunctions these results must still be coupled to the orbital angular momentum part of the NN wavefunctions \( Y_{LM}(p) \), using standard Clebsch-Gordan coefficients according to

\[
\begin{pmatrix}
Jm,l_s
\end{pmatrix}
_\ell \ell_a \ell_b
= \sum_{m_L,m_s,m_s} C(LSSJ, m_s, m_L) Y_{LM}^{(p)}(\ell) \chi^{Lm_L}_{\ell_a, \ell_b}
\] (B-5)

Calculating this expansion for the \( ^3P_1 \) state which appears in threshold
pionic disintegration of the deuteron we have for $M_J = 1$

\[
Y_{N,0}^{(11,1,0)}(\hat{\vec{p}}) = C(111,10) \gamma_0(\hat{\vec{p}}) \left[ \frac{i}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \sigma \cdot \hat{\vec{x}} \right) \sigma z \right]_{\lambda,\lambda} + C(111,01) \gamma_1(\hat{\vec{p}}) \left[ \frac{i}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \sigma \cdot \hat{\vec{x}} \right) \sigma y \right]_{\lambda,\lambda} \quad (B-6a)
\]

for $M_J = 0$

\[
Y_{N,0}^{(111,1,1)}(\hat{\vec{p}}) = C(111,1,1) \gamma_1(\hat{\vec{p}}) \left[ \frac{i}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \sigma \cdot \hat{\vec{x}} \right) \sigma z \right]_{\lambda,\lambda} + C(111,0,0) \gamma_0(\hat{\vec{p}}) \left[ \frac{i}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \sigma \cdot \hat{\vec{x}} \right) \sigma y \right]_{\lambda,\lambda} + C(111,1,1) \gamma_1(\hat{\vec{p}}) \left[ \frac{i}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \sigma \cdot \hat{\vec{x}} \right) \sigma z \right]_{\lambda,\lambda} \quad (B-6b)
\]

for $M_J = -1$

\[
Y_{N,0}^{(111,1,0)}(\hat{\vec{p}}) = C(111,0,1) \gamma_0(\hat{\vec{p}}) \left[ \frac{i}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \sigma \cdot \hat{\vec{x}} \right) \sigma z \right]_{\lambda,\lambda} + C(111,-1,0) \gamma_0(\hat{\vec{p}}) \left[ \frac{i}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \sigma \cdot \hat{\vec{x}} \right) \sigma z \right]_{\lambda,\lambda} \quad (B-6c)
\]
using the results

\[
\frac{\phi_{\lambda}^{(m)}}{\sqrt{2}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}
\]

\[
\frac{\phi_{\lambda}^{(0)}}{\sqrt{2}} = \frac{\phi_{\lambda}}{\sqrt{2}}
\]

\[
\frac{\phi_{\lambda}^{(-1)}}{\sqrt{2}} = \begin{pmatrix} 1 & 0 \end{pmatrix}
\]

and the cartesian representation of spherical harmonics

\[
\gamma_{l,\pm 1}(\hat{\beta}) = \sqrt{\frac{2}{8\pi}} \left( \hat{\beta}_x \pm \imath \hat{\beta}_y \right)
\]

\[
\gamma_{10}(\hat{\beta}) = \sqrt{\frac{3}{4\pi}} \hat{\beta}_z
\]

we can rewrite all three expressions (B-6) under the single form

\[
\gamma_{\lambda,\mu,\nu}^{(m)}(\hat{\beta}) = \frac{\imath \sqrt{3}}{16\pi} \left[ \left( \imath (\vec{\sigma} \times \hat{\beta}), \frac{\lambda}{\lambda} \right) \sigma_{\nu} \right]_{\lambda,\mu,\nu}
\]

which we have used in our calculation of the \( ^3P_1 \) partial wave amplitude for the reaction \( \pi d \rightarrow pp \).
REFERENCES

18. See for example Schweber, Bethe and de Hoffman, "Mesons and Fields"