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Ballistics

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BALLISTICS.

Ballistics is the science that treats of the motion of projectiles. It is divided into interior and exterior ballistics.

Interior ballistics treats of the motion of the projectile while still in the bore of the gun. It includes a study of the mode of combustion of the powder, the pressure developed and the velocity of the projectile along the bore of the gun. Sarrau, an eminent French engineer, first put the study of interior ballistics on a scientific basis. He developed formulas based on thermo dynamic principles connecting the velocity, pressure and travel of the projectile in a gun. Much useful information tending to improvement in both powder and guns has been obtained by a study of interior ballistics.

By means of formulas to be developed we may, having given the dimensions of the gun, weight of charge and projectile and granulation of the powder, calculate the curves of velocity and pressure in the bore of the gun as functions of travel of the projectile or as functions of the time. We may thus determine the muzzle velocity to be expected from a given charge, or we may determine the charge that will produce a required muzzle velocity.

By further application of the formulas we may design the granulation of the powder for a given gun so as to obtain the highest possible muzzle velocity with a given weight of projectile while keeping the pressure along the bore within the limits imposed by the strength of the walls of the gun and the weight of charge low enough for convenient loading.

When a new gun is to be designed, the formulas enable us to calculate the size of powder chamber and length of gun that will be necessary in order that the required muzzle velocity may be obtained within the limit of pressure for which the gun is to be designed.

After determining the powder pressure curve for the gun we may, by the principles of gun construction design the walls of the gun to withstand the expected pressure at each point.

In 1743 Benjamin Robins described, before the Royal Society of England, experiments that he had made to determine the velocities of musket balls when fired with given charges of powder. To measure the velocities he invented the ballistic pendulum, which consisted of a large block of wood suspended so as to move freely. The bullet was fired into the block of wood, and the velocity impressed upon the pendulum was measured. By equating the expressions for the quantities of motion in the bullet before striking the pendulum, and in the pendulum after receiving the bullet, the velocity of the bullet was obtained. Robins also invented the gun pendulum which consisted of a gun mounted to swing as a pendulum. As a result of these experiments Robins announced the following, The temperature of explosion is at least equal to that of red-hot iron; the maximum pressure exerted by the powder gases is equal to about 1000 atmospheres; the weight of the permanent gases is about three-tenths that of the powder, and their volume at atmospheric pressure and temperature about 240 times that occupied by the charge.

Dr. Charles Hutton, Professor in the Royal Military Academy, Woolwich, continued Robins's experiments, 1773 to 1791, improving and enlarging the ballistic pendulum so that it could receive the impact of one pound balls. He verified Robins's deductions as to the nature of the gases, but put the temperature of explosion at double that of Robins, and the maximum pressure at 2000 atmospheres. Hutton produced a formula for the velocity of a spherical projectile at any point of the bore, upon the assumption that the combustion of the charge is instantaneous and that the expansion of the gas follows Mariette's law- no account being taken of the loss of heat due to work performed - a principle which, at that time, was unknown.

In 1760 the Chevalier D'Arcy made the first attempt to determine dynamically the law of pressure in the bore by successively shortening the length of the barrel and measuring the velocity of the bullet for each length. The pressures were determined from the calculated accelerations.

In 1792 Count Rumford, born in the United States, tried to make direct measurement of the pressure exerted by fired gunpowder by measuring the maximum weights lifted by different charges fired in a small but very strong wrought-iron mortar. He determined a relation existing between the pressure of the powder gases and their density. The maximum pressure that would be exerted by the gases from a charge that completely filled the chamber was as calculated by Rumford, about 100 tons to the square inch. Noble and Abel, in their later experiments, arrived at 43 tons per square inch as the maximum pressure under these conditions. Their value is now accepted as being about right. The great difference in the two determinations is probably due to the fact that Rumford deduced his value for the maximum pressure from experiments with small charges that did not fill the chamber, so that the energy of the gases was greatly increased by the high velocity they attained before acting on the projectile.

From 1857 to 1860 General Rodman of the Ordnance Department, United States Army, conducted the experiments that resulted in the change of form of powder grains and their variation in size according to the caliber of the gun. He devised the pressure gauge for directly measuring the maximum pressures of powder gases. General Rodman was also the author of the principle of interior cooling of cast-iron cannon, by the application of which principle the metal surrounding the bore of a gun was put under a permanent compressive strain which greatly increased the resistance of the gun to the interior pressures.

In 1875 Sarrau first proposed his mathematical formulas connecting the travel and velocity of the projectile and the pressure developed in the bore of the gun. These formulas were based on the use of black powder. When smokeless powder began to be used it was found that the results of calculations with these formulas were not correct.

About 1904 Gossot and Liouville published new formulas of interior ballistics based on those of Sarrau, but adapted to smokeless powder.

In investigating the subject of interior ballistics we will consider, first, the quantity of energy released per pound of powder as the charge is burned; second, the lineal rate of burning of the powder; third, the effect of different forms of granulation on the relation between the lineal rate of burning of the powder and the rate of burning of the charge; and, fourth, the various ways in which the energy resulting from the burning of the charge is utilized.

Our service smokeless powder is manufactured from nitrocellulose of an average percent of nitrogen of about 12.60, by colloiding it with a solvent of ether-alcohol. After granulation the bulk of this solvent is dried out, but 2% to 6% may remain in the powder, the amount depending upon the size of the grain.

The alcohol used in the solvent contains 5% water, which usually remains in the powder after drying. Additional water may be absorbed from the air during the process of making and handling. Recent lots of powder also contain .4% to .8% of a stabilizer, and .30 caliber rifle powders contain, in addition, 1% of graphite.

The energy per pound of pure nitrocellulose of 12.60% nitrogen is 1442000 ft.lbs. per pound. If an inert material is mixed with the nitrocellulose the energy per pound of the resulting material will be less than that of pure nitrocellulose. Of the materials entering powder water is considered as having the same effect as the same percentage of inert matter. Alcohol has a greater effect than inert matter. Practical tests indicate that the effect of 1% alcohol in reducing the energy per pound of nitrocellulose is equal to the effect of 2.5% inert matter. The effect of 1% stabilizer is equal to the effect of 4% inert matter. The effect of 1% graphite in reducing the energy per pound of nitrocellulose is equal to that of 2.5% inert matter.

The percentage of nitrogen in the nitrocellulose used is carefully determined for each lot of powder, as is also the percentage of moisture and volatiles remaining in the finished powder. Moisture and volatiles consist principally of water and alcohol in equal percentages.

The specific rate of combustion of smokeless powder is the rate of combustion in inches per second, measured perpendicular to the burning surface, when the powder is burning in its own gas at a pressure of one pound per square inch.

Theoretical considerations lead to the belief that the specific rate of combustion is directly proportional to the energy per pound of powder , and that it is inversely proportional to the sum of the following quantities:

(a) The energy required to heat a pound of powder from the temperature it may have at any time to 356°F., the ignition temperature.

(b). The energy required to evaporate the water and alcohol from the powder.

It is assumed that this evaporation from successive layers takes place before they burn.

Modern smokeless powder is granulated in regular geometrical forms convenient for manufacture by pressing through a die or rolling between rollers. The principal forms in use are, the solid rod of considerable length; short or long cylinder with axial perforation; short or long flat strips, and cylinder with seven axial perforations. In all these granulations the least burning thickness is called the web of the grain.

It is known that smokeless powder burns faster at high pressures than at low pressures. Under constant pressure, as in the air, a grain of modern smokeless powder burns in parallel layers and with uniform velocity in directions perpendicular to all the ignited surfaces. Under variable pressure as in a gun, powder burns with a variable velocity, but under that condition it still burns in parallel layers in the same manner as at constant pressure. As the grains burn over their entire surface they will usually be entirely consumed when a thickness equal to one-half their least dimension has been burned from each of the bounding surfaces.

The initial volumes of any of the grains usually used are easily figured from their dimensions. The dimensions of grains are measured in inches and their volumes are given in cubic inches.

The initial burning surface of a grain is the area of the total outside surface of the grain including the surfaces of the ends and the perforations , if any.

The volume of one pound of solid powder may be obtained by dividing the volume of one pound of water, 27.68 cu.in., by the density of the powder or representing by V_p the volume of one pound and by δ the density of the powder we have $V_p = \frac{27.68}{\delta}$

The number of grains of powder is the volume of one pound of solid powder divided by the initial volume of one grain. Let n_p repersent the number of grains per pound and V_g the volume of one grain and we have $n_p = \frac{V_p}{V_g} = \frac{27.68}{\delta V_g}$

The initial burning surface per pound is the initial burning surface of one grain multiplied by the number of grains per pound. Let S_p represent the initial burning surface per pound and S_g the initial burning surface per grain and we have $S_p = n_p S_g = \frac{27.68 S_g}{\delta V_g}$

For any granulation the volume of a grain after a thickness r has been burned may be determined by adding or subtracting 2r, the total thickness burned on opposite sides, to or from the original grain. Subtract the new volume from the original volume and we obtain the volume burned.

The best form of granulation, from a ballistic point of view, is that which, with a given weight of charge, will give the projectile the highest muzzle velocity, within the limit of maximum pressure for which the gun was designed. To obtain the highest muzzle velocity it is necessary that all the powder be burned as soon as possible after the projectile starts moving, that is, that the web thickness of the grains be as small as possible, so as to allow the full weight of the gases to act on the projectile for the longest possible distance.

Distribution of powder in the gun.

Let p = the weight of the projectile in pounds;
p_1 = the weight of the gun and recoiling parts in pounds;
v = the velocity of the projectile in feet per second;
v_1 = the velocity of the gun and recoiling parts;
u = the travel of the projectile in feet, with reference to the gun, when the velocity of the projectile is v;
c_2 = the weight of powder burnt in pounds when the velocity of the projectile is v, and its travel with reference to gun is u;
A = area of cross-section of bore in square inches.
c = the total weight of charge in pounds;

We have represented by n' the energy given off on the combustion of one pound of powder. The energy given off when c pounds of powder are burned is $n' c_2$.

This energy is distributed as follows:

(a) Energy of translation of the projectile.
(b) Energy of rotation of the projectile.
(c) Energy of translation of the gun (recoil).
(d) Energy of translation of the unburnt charge and gases.

The total weight of the unburnt charge and gases is always that of the original charge and it is assumed as uniformly distributed between the face of the breech block and the base of the projectile.

(e) Energy consumed in overcoming the passive resistance of the projectile. This resistance arises from the friction of the projectile against the walls of the bore and of the rotating band against the driving edges of the lands. In the first stages it also arises from the cutting of grooves in the rotating band by the lands. In rifling with increasing twist a resistance is caused by the change in form of the grooves as the projectile moves through the bore.
(f) Energy lost as heat to the gun.
(g) Energy that remains in the gas as sensible or latent heat.

The energy $n' c_2$ must at each instant be equal to the sum of the above energies.

The energy of translation of the projectile is $\frac{pv^2}{g2}$.

Energy of rotation of the projectile is as follows:

Let ϕ = the mean angle of twist of the rifling. The velocity of rotation of the projectile in radians per second is then $\frac{2v \tan\phi}{d_1}$,

where d_1 is the diameter of the bore of the gun in feet. The energy of rotation of the projectile is therefore

$$\frac{p}{g} k_2^2 \frac{(2v \tan\phi)^2}{2d_1^2} = \frac{p}{g}\left(\frac{2k_2}{d_1}\right)^2 \tan^2\phi \frac{v^2}{2}$$

where k is the radius of gyration of the projectile in feet.

The value of $\left(\frac{2k_2}{d_1}\right)^2$ may be assumed as follows, though it varies somewhat for projectiles of different dimensions:

For solid shot .50
For small cavity shot .53
For shell .58

The energy of translation of the gun is equal to $\frac{p_1}{g}\frac{v^2}{2}$

To get a relation between v and v assume that the gun is recoiling freely, that is , it is not retarded by the carriage. This is nearly so for most carriages while the projectile is in the bore. Also assume that the unburnt charge and gases are uniformly distributed throughout the bore and that their mean velocity in the direction of the projectile is $\frac{v}{2}$. Equating momenta, neglecting the momentum due to the rotation of the projectile, we have, $\frac{pv}{g} = \frac{p}{g} v + \frac{\frac{1}{2}c}{g} v$ or $v_1 = \frac{2p+c}{2p_1} v = e v$.

The energy of translation of the gun therefore becomes $= \frac{p_1}{g} e^2 \frac{v^2}{2}$.

Energy of Translation of the Unburnt Charge and Gases.- The total weight of these is C, and they are assumed uniformly distributed throughout the bore behind the projectile. Let C_x be the weight of charge and gases between the section of 0 velocity, considered at the breech of the gun, and a section forward of this whose velocity is v_x. The weight of the latter elementary section is dC_x and its energy is $\frac{dC_x}{g}\frac{v_x^2}{2}$.

Since the charge and gases are assumed as uniformly distributed throughout the bore, we have the relation, $\frac{C_x}{C} = \frac{v_x}{v}$, hence $C_x = \frac{Cv_x}{v}$ and $dC_x = \frac{C}{v} dv_x$ and the energy of the elementary section considered reduces to $\frac{C}{gv}\frac{v}{2} dv$. That is d(energy of motion of weight of charge and gases, C) $= \frac{C}{gv}\frac{v_x^2}{2} dv_x$

Integrating between limits $v_x = 0$ and $v_x = v$, we have, Energy of motion of C $= \frac{C}{3g}\frac{v^2}{2}$.

A number of experiments made at Watertown Arsenal of forcing projectiles through the bases of guns of various calibers, and measuring the resistance encountered indicate:

(a) For most guns, the highest resistance occurs at the start, when grooves are being cut in the rotating band by the rifling. The resistance then drops greatly, but generally again increases, rising to a maximum at a travel of two to four calibers and then decreasing rather uniformly to the muzzle.

(b) The speeds at which the projectiles were forced through bores were all very low, but an increase of speed seemed to reduce the resistance.

(c) Greater width and diameter of band increased the resistance.

(d) The starting resistance and mean passive resistance expressed in pounds per square inch on base of projectile both decrease as the caliber increases.

When a gun is fired the driving edges of the lands of the rifling exert a pressure against the edges of the lands left in the rotating band. This pressure produces additional friction which is not produced in slow motion ofthe projectile through the bore.

The general of the passive resistance curves obtained from these experiments leads to the belief that the values of the passive resistance in pounds per square inch on base of projectile, maybe taken for a given charge as at each instant equal to the product of the accelerating pressure in pounds per square inch on the base of projectile by a constant factor. The value of this constant factor will be different for different charges. Let P_p be the pressure per square inch required to overcome the passive resistance.

The energy lost in foot pounds by passive resistance over a travel u is therefore $\int P_p A \, du$.

Thus far we have considered all of the energies entering the problem in which heat has been converted into mechanical work.

The sum of these energies may be considered as the effect of the action of the mean total pressure P_1 per square inch, acting on the cross-section of the bore for a distance u, that is we have the following equation:

$$\int PA du = \frac{p}{g}\frac{v^2}{2} \quad \frac{p}{g}\left(\frac{2K}{d}\right)^2 \tan^2\varphi \frac{v^2}{2} + \frac{p}{g}e^2\frac{u^2}{2} + \frac{c}{3g} \quad \frac{v^2}{2} + \int P_p A \, du \ .$$

Placing $p' = p\left[1 + \left(\frac{2K}{d}\right)^2 \tan^2\varphi\right] + p_1 e^2 + \frac{c}{3}$ we have $\int PA \, du = \frac{p'}{g}\frac{v'}{2} + \int P_p A \, du$

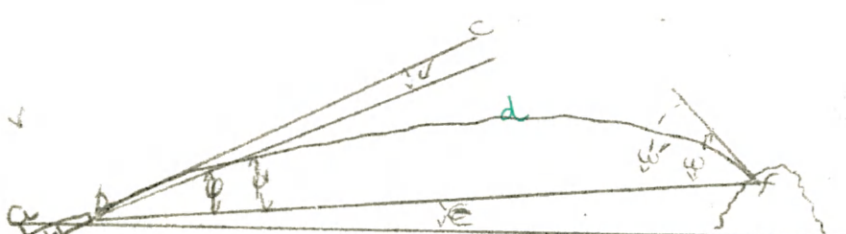
p' may be called the reduced weight of the projectile.

In the design of a new gun the caliber, weight of projectile and muzzle velocity are determined by the use to which the gun is to be put.

Having obtained, or assumed, the weight of projectile and muzzle velocity the problem of determining the size of powder chamber and length of gun required becomes purely one of exterior ballistics.

Exterior ballistics treats of the motion of a projectile after it has left the piece.

In the discussions the dimensions of the gun are considered negligible in comparison with the trajectory.



The Trajectory, bdf, is the curve described by the center of gravity of the projectile in its movement.

The Range, bf, is the distance from the muzzle of the gun to the target.

The Line of Sight, abf, is the straight line passing through the sights and the point aimed at.

The Line of Departure, bc, is the prolongation of the axis of the bore at the instant the projectile leaves the gun.

The Plane of Fire, or Plane of Departure, is the vertical plane through the line of departure.

The Angle of Site, ϵ, is the angle made by the line of sight with the horizontal.

The Angle of Departure, φ, is the angle made by the line of departure with the line of sight.

The Quadrant Angle of Departure, $\varphi+\epsilon$, is the angle made by the line of departure with the horizontal.

The Angle of Elevation, φ', is the angle between the line of sight and the axis of the piece when the gun is aimed.

The Jump is the angle j through which the axis of the piece moves while the projectile is passing through the bore. The movement of the axis is due to the elasticity of the parts of the carriage, to the clearances in the trunnion beds and other bearings of the carriage, and in some cases to the action of the elevating device as the gun recoils. The jump must be determined by experiment for the individual piece in its particular mounting. It usually increases the angle of elevation so that the angle of departure is greater than that angle.

The Angle of Fall, ω, is the angle made by the tangent to the trajectory with the line of sight at the point of fall.

The Point of Fall, or, Point of Impact, is the point at which the projectile strikes.

The Striking Angle, ω', is the angle made by the tangent to the trajectory with the horizontal at the point of fall.

Initial Velocity is the velocity of the projectile at the muzzle.

Remaining Velocity is the velocity of the projectile at any point of the trajectory.

Drift, kf, is the departure of the projectile from the plane of fire, due to the resistance of the air and the rotation of the projectile.

As far as concerns the equations to be used in the work of exterior ballistics, all trajectories may be placed in one of the following two classes:

Direct Fire, which pertains to small angles of departure, usually not above 15°.

Curved Fire, which pertains to all angles of departure greater than about 15°. This class embraces the so-called High-angle Fire.

Curved Fire is generally limited in practice to velocities lower than those used in direct fire.

For convenience of discussion curved fire is considered as firings with elevations 15° and 40° and high-angle fire as firings with elevations above 40°.

The projectile as it comes from the muzzle of the gun has impressed upon it a motion of translation and a motion of rotation about its longer axis. The guns of our service are rifled with a right-handed twist, and the rotation of the projectile is, there-fore, from left to right when regarded from the rear. After leaving the gun the projectile is a free body acted upon by two extraneous forces, gravity and the resistance of the air.

When the projectile first issues from the gun, its longer axis is tangent to the trajectory. The resistance of the air acts along the tangent, and is at first directly opposed to the motion of translation of the projectile.

The longer axis of the projectile tends to remain parallel to itself during the passage of the projectile through the air, but the tangent to the trajectory changes its inclination, owing to the action of gravity. The resistance of the air, acting always in the direction of the tangent, becomes inclined to the longer axis of the projectile, and in modern projectiles its resultant intersects the longer axis at a point in front of the center of gravity.

The resultant effect of the resistance of the air on the rotating projectile is a precessional movement of the point of the projectile to the right of the plane of fire. After the initial displacement of the point to the right the direction of the resultant resistance changes slightly to the left with respect to the axis of the projectile and produces a corresponding change in the direction of the precession, which diverts the point of the projectile slightly downward.

If the flight of the projectile were continued long enough the point would describe a curve around the tangent to the trajectory; but actually the flight of the projectile, except in mortar firings at angles of elevation exceeding about 66°, is never long enough to permit more than a part of this motion to occur.

When the point of the projectile leaves the plane of fire the side of the projectile is presented obliquely to the action of the resistance of the air, and a pressure is produced by which the projectile is forced bodily to the right out of the plane of fire. It is to this movement that the greater part of the deviation of the projectile is due.

The departure of the projectile from the plane of fire, due to the causes above considered, is called the drift. It may be analytically that the drift of the projectile increases more rapidly than the range. The trajectory is, therefore, a curve of double curvature, convex to the plane of fire.

The trajectory ordinarily considered is the projection of the actual curve upon the vertical plane of fire. This projection so nearly agrees with the actual trajectory that the results obtained are practically correct; and the advantage of considering it, instead of the actual curve, is that we need consider only that component of the resistance of the air which is directly opposed to the motion of translation.

To determine the accuracy of a gun at any range and under any conditions a number of shots are fired under the given conditions. The point of fall of each shot is plotted and the mean is found. This is called the center of impact.