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A Computer-Assisted Instruction Program in Mathematics

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A COMPUTER-ASSISTED INSTRUCTION PROGRAM IN MATHEMATICS

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ABSTRACT

In recent years computer-assisted instruction(CAI) has been used in various schools for a wide variety of purposes. This thesis consisted of nine program modules being written in APL(A Programming Language) for the purpose of aiding students in their study of mathematics. The programs covered linear equations, quadratic equations, elementary trigonometry, equations of circles, parabolas, ellipses, hyperbolas and conics in general, and the ratios of limits of polynomials.

The language APL proved very useful in its application for CAI. It had some disadvantages, but these were far outnumbered by its advantages.

A COMPUTER-ASSISTED INSTRUCTION PROGRAM

IN MATHEMATICS

INTRODUCTION

Computer-assisted instruction (CAI) has been used in recent years in elementary schools, in junior and senior high schools, in two year community colleges, in universities and in vocational schools to accomplish a wide variety of purposes. These purposes range from limited assistance, such as repetitive drill and practive work, to revision of the normal curriculum, to total replacement of the teacher[1]. Today's modern computers have the capacity to facilitate individualized instruction, and their flexibility permits a wide-variety of instructional strategies. Many believe that these machines have the potential to enhance the productivity of the individual teacher and improve the quality of the teaching-learning process. CAI is providing opportunities for conceptual learning that are not available in the normal curriculum and relieving the teacher from routine but necessary educational chores.

Supporters of CAI, regarded by some as the most significant educational technology since printing, say that it has the advantage of providing individualized instruction and potentially lower cost than conventional methods, features that would be attractive to many hard-pressed administrators. Two major attempts to demonstrate the value of CAI are in progress under grants from the National Science Foundation. One is the Ticcit (time-shared interactive, computer-controlled information television) system being developed by the Mitre Corporation in conjunction with the University of Texas and Brigham Young University. The other is the Plato (programmed logic for automatic teaching operations) system being developed by the University of Illinois. Neither will be ready for demonstration, however, before the fall of 1973.

Ticcit is a decentralized system built around small computers which are accompanied by a self-contained package of hardware, operating programs, and course materials. Color television is the display medium, and the system is primarily composed of commercially available components. The formalized method of developing CAI "courseware" is achieved through the collaborative efforts of teams of programmers, educational psychologists, and specialists in the subject matter.

In contrast, the Plato system utilizes a large, sophisticated computer (the CDC-6400) in a centralized facility that will serve many schools by way of remote terminals. The terminals consist of a plasma display panel and other hardware specifically designed for CAI. The Plato program uses a more ad hoc approach of allowing teachers to design their own courses with the aid, if necessary, of the Plato staff.

Due to the increasing interest in computer-assisted instruction and because the author would be teaching mathematics at an institution* which stresses individualized instruction, a series of interactive APL (A Programming Language) program modules was developed to aid mathematics students in their learning process. The nine modules developed are:

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1) linear equations
2) quadratic equations
3) trigonometric functions
4) equations of a circle
5) equations of a parabola
6) equations of an ellipse
7) equations of a hyperbola
8) equations of conics
9) limits of ratios of polynomials

Modules were developed in these areas to coincide with an analytical geometry and calculus course that the author would be teaching. However, each module was developed individually so that its use would fit in with the appropriate material in any math course. The level of content, with the exception of the first three modules, coincides with that of Thomas's _Calculus and Analytical Geometry_[2] which the author would be using in the classroom. The programming was done with the idea that other modules could be added easily at a later time.

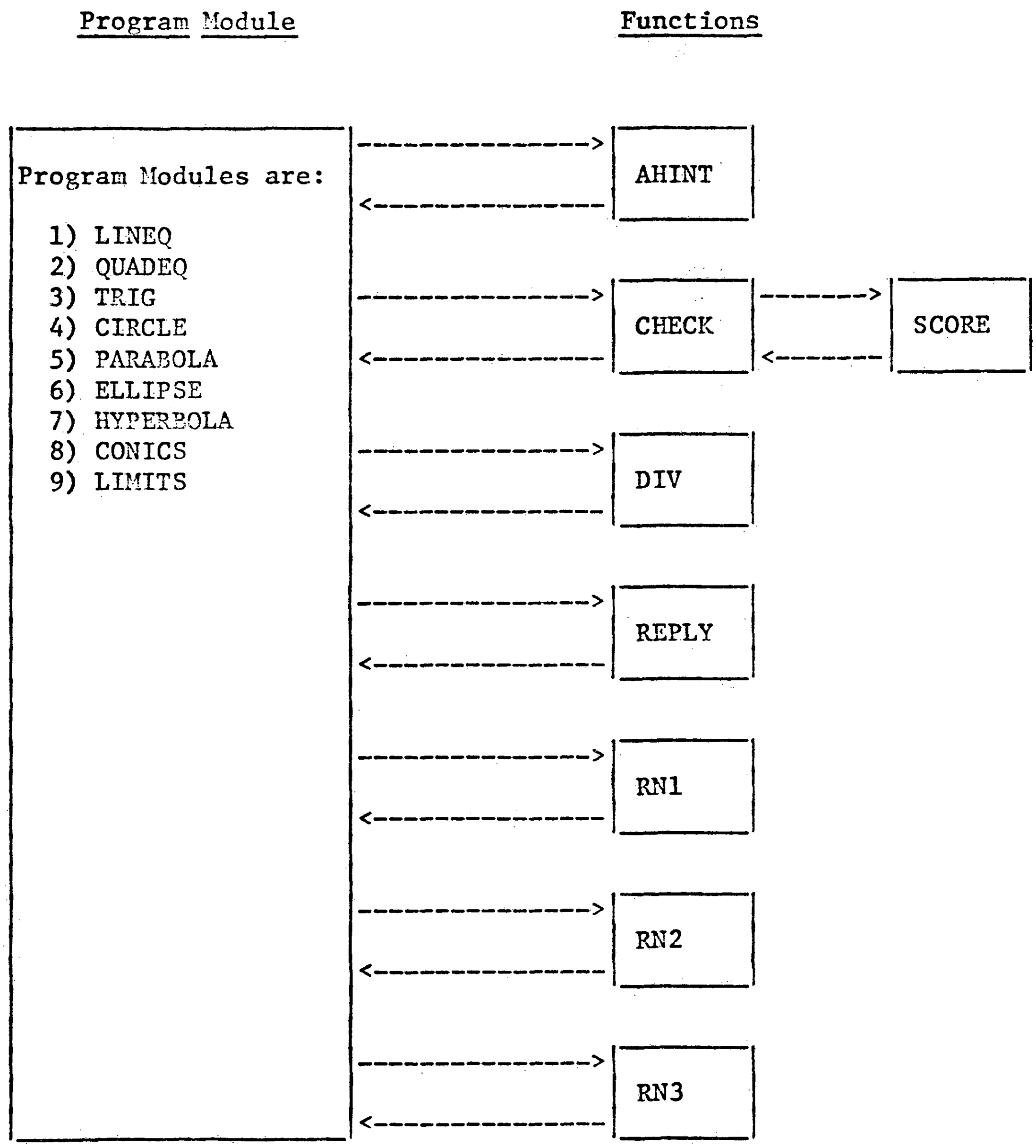
Each module consists of a short description of the material covered by the program, a list of the replies that the student may use, and a series of questions on the material. There are a finite number of problems for each module. After the last question has been answered, the problems are recycled with different parameters. The problems ranged from being difficult to being relatively easy. This was done so that the student would gain confidence by being able to answer some questions. The questions are phrased in such a way that a definite reply has to be given. These questions take the form of fill in the blank, multiple choice, and short answer.

CHAPTER I

DESIGN OBJECTIVES

In planning for the programming of the modules certain ideas were of utmost importance. These were (1) to provide an easy and efficient way for each program to accept all possible replies and (2) to keep repetition of statements to a minimum. The programming structure finally decided upon is shown in the block diagram in Figure 1. In this figure each program module and each function is diagrammed by blocks. Each block is a separate APL function. The arrows between the blocks represent the interaction between the different functions and the modules. The nine different program modules generate only questions and hints, and when the need arises for requesting replies, keeping scores, generating random numbers, and performing other miscellaneous tasks, the appropriate function is called by that module. For example, in generating a question the module LINEQ calls the function RN1 for random coefficients. After the question with these coefficients is printed, LINEQ calls CHECK, which compares the student's reply to the correct answer. A record of this result is kept when CHECK calls SCORE. Control is returned to CHECK and then back to LINEQ.

FIGURE 1


Program Module
Functions
Program Modules are:
1) LINEQ
2) QUADEQ
3) TRIG
4) CIRCLE
5) PARABOLA
6) ELLIPSE
7) HYPERBOLA
8) CONICS
9) LIMITS
AHINT
CHECK
SCORE
DIV
REPLY
RN1
RN2
RN3

The program modules were designed on the premise that the students know no APL other than an introduction, telling how to turn on the machine, how to sign-on, and how to load the program modules. In using these programs the student does need to know, however, that in APL "÷" is used for division, "*" is used for terms raised to a power, and "⎕:" means that the computer is waiting for a reply. The following chapters will include a description of the modules and the techniques used in programming them.

CHAPTER II

DESCRIPTION OF THE PROGRAM MODULES

Table I shows the names of the program modules and how each is loaded into the active workspace of the user.

TABLE I

APL INSTRUCTIONS FOR LOADING EACH MODULE INTO THE WORKSPACE

Name of Module	To load into active workspace type:
1) LINEQ 2) QUADEQ 3) TRIG 4) CIRCLE 5) PARABOLA 6) ELLIPSE 7) HYPERBOLA 8) CONICS 9) LIMITS	)LOAD (public library no.) name of workspace containing module

A description of each module can be found in Appendix A and a description of each function can be found in Appendix B.

After the module has been loaded into the active workspace, the user starts the program by simply typing in the name of the module. The computer responds by typing:

```
ANSWER EITHER YES OR NO.
ARE YOU FAMILIAR WITH ...(then the name of the module)?
```

If the user's reply is NO or anything that does not start with a Y (the program only checks the first letter to see if it is a Y), a short description of the material will be printed. After the description, the computer asks:

ARE YOU FAMILIAR WITH THE REPLIES OF THIS EXERCISE?

If the user's response is anything but a leading Y then the function REPLY is executed. This is the message it prints:

```
 THE COMPUTER WILL ASK YOU QUESTIONS WHOSE ANSWERS
ARE NUMBERS.  YOU ARE TO TYPE IN THE NUMBER.  IF
THE NUMBER IS A FRACTION, HOWEVER, YOU ARE TO USE
THE DIVIDE(÷) SYMBOL AND NOT THE SLASH(/), SO ONE-
HALF IS 1÷2 AND THREE-AND-ONE-THIRD IS 10÷3 .
 IF YOUR REPLY IS INCORRECT THE COMPUTER WILL REPLY
     TRY AGAIN
AND YOU GET ANOTHER CHANCE.  IN FACT, YOU GET 3 TRIES
AT EACH QUESTION.
 IF YOU DO NOT KNOW THE ANSWER TO THE QUESTION YOU
CAN TYPE
     HINT
AND EITHER (1) A COMMENT WILL BE PRINTED OR (2) A
LIST OF FOUR NUMBERS WILL BE PRINTED, WHERE ONE IS
THE CORRECT ANSWER.
 IF YOU DO NOT HAVE THE SLIGHTEST IDEA OF WHAT THE
ABSWER IS TYPE
     HELP
AND THE ANSWER WILL BE GIVEN.
 IF YOU WANT TO KNOW HOW MANY QUESTIONS YOU HAVE BEEN
ASKED TYPE
     QNUMBER
 IF YOU WANT AN XY-AXIS PRINTED FOR SKETCHING FUNCTIONS
THEN TYPE
     GRAPH
 WHEN YOU WANT TO STOP THE EXERCISE TYPE
     STOP
AND A TABULATED RESULT OF YOUR REPLIES WILL BE GIVEN
AND THEN THE EXERCISE WILL TERMINATE.
```

If the user enters NO, this message is not printed, and the computer starts with the first question. The student's normal reply will be a number, but he may use one of the above words. If the user's response is incorrect, the computer will type TRY AGAIN and wait for another reply. If the user's response is correct, however, the computer does one of two things. It either proceeds to the next question or it types VERY GOOD! NOW SEE IF YOU CAN GET THIS ONE. This message is printed at random, but it averages being printed approximately every third correct answer.

When the user wants to terminate the exercise, he types STOP and the following summary is printed out:

```
YOUR RESULTS ARE THE FOLLOWING:
NUMBER OF QUESTIONS                 x
NUMBER ANSWERED ON FIRST TRY        x
NUMBER ANSWERED ON SECOND TRY       x
NUMBER ANSWERED ON THIRD TRY        x
NUMBER NOT ANSWERED                 x
NUMBER OF HINTS YOU RECEIVED        x
NUMBER OF HELPS YOU RECEIVED        x
```

and one of the following:

```
YOUR PERFORMANCE WAS EXCELLENT
YOUR PERFORMANCE WAS VERY GOOD
YOUR PERFORMANCE WAS GOOD
YOUR PERFORMANCE WAS POOR
YOUR PERFORMANCE NEEDS IMPROVEMENT
```

Since the number of hints is not taken into account in measuring the performance of the student, the listing of the number of HINTS can be used as a guide in validating the performance by the student. The number of HELPS gives the number of times that the student asked for an answer.

To determine the performance of the student full credit is given for a correct answer on the first try, half credit for a correct answer on the second try, and one-third credit for a correct answer on the third try. Table II shows the grading scheme.

TABLE II

GRADING SCHEME

Performance	Percentage
1) excellent	for a 90% or better
2) very good	for an 80-89%
3) good	for a 70-79%
4) poor	for a 60-69%
5) needs improvement	for below 60%

To help clarify the preceding descriptions a sample run from the module PARABOLA is given.

```
      PARABOLA
REPLY EITHER YES OR NO.
ARE YOU FAMILIAR WITH THE EQUATIONS OF PARABOLAS?
YES


ARE YOU FAMILIAR WITH THE REPLIES OF THIS EXERCISE?
YES
GIVEN THE EQUATION (X-4)*2 =24(Y-0)
WHAT IS THE X-COORDINATE OF THE VERTEX?
⎕:
      4

WHAT IS THE Y-COORDINATE OF THE VERTEX?
⎕:
      0

WHAT IS THE X-COORDINATE OF THE FOCUS?
⎕:
      HINT
THE ANSWER IS ONE OF THE FOLLOWING:
6   4   0   6
⎕:
      6
TRY AGAIN
⎕:
```

```
      HELP
THE ANSWER IS 4
WHAT IS THE Y-COORDINATE OF THE FOCUS?
⎕:
      6
VERY GOOD! NOW SEE IF YOU CAN GET THIS ONE.
THE DIRECTRIX IS THE LINE Y=_
WHAT NUMBER GOES IN THE BLANK?
⎕:
       6
VERY GOOD! NOW SEE IF YOU CAN GET THIS ONE.
THE GRAPH OF THIS EQUATION OPENS EITHER
      1) UPWARD
      2) DOWNWARD
      3) TO THE RIGHT
      4) TO THE LEFT
IS THE ANSWER 1,2,3 OR 4?
⎕:
      HINT

THE CLUE TO THIS IS THE SIGN OF P AND THE QUADRATIC TERM.
⎕:
      1

GIVEN THE VERTEX V(0,8) AND FOCUS F( 7,8) OF A PARABOLA
AND THE EQUATION (Y-_)*2 =_(X-_)
WHAT NUMBER GOES IN THE FIRST BLANK?
⎕:
      STOP
YOUR RESULTS ARE THE FOLLOWING:
NUMBER OF QUESTIONS                    6
NUMBER ANSWERED ON FIRST TRY           5
NUMBER ANSWERED ON SECOND TRY          0
NUMBER ANSWERED ON THIRD TRY           0
NUMBER NOT ANSWERED                    0
NUMBER OF HINTS YOU RECEIVED           2
NUMBER OF HELPS YOU RECEIVED           1

YOUR PERFORMANCE WAS VERY GOOD
```

CHAPTER III

SOME TECHNIQUES USED IN PROGRAMMING

This chapter is intended for the reader with a knowledge of APL. It is meant to give the reader an insight into some of the techniques used in programming the modules.

For example, the idea of letting the student's word replies be a variable, rather than a character string, was borrowed from IBM'S APLCOURSE*. The word replies that are available to the user and which have been explained previously are HINT, HELP, QNUMBER, GRAPH and STOP. These names are global variables whose values were chosen selectively so that they could never be equal to an answer to an answer to a question in any module. These values are stored in the workspace. The student's reply is compared to the above names, as well as the correct answer, to determine what action is to take place. The function CHECK performs this comparison. Since the function CHECK registers and compares each response, it is called after each question. This is done by the APL statement→0×⍳CHECK=1 where CHECK returns a 1 if the reply was STOP and a 0 for all other cases. If CHECK returns a 1, the program terminates, otherwise CHECK returns a 0 and control of the program continues to the next statement. This one line statement turns out to

* APLCOURSE is an APL program distributed by International Business Machines Corporation(IBM) to teach APL.

be very powerful since it does most of the housekeeping. A call on CHECK determines the student's reply, keeps a record of the reply (CHECK calls SCORE), takes the appropriate action for the respective reply, prints out the summary of the user's responses if appropriate, and determines whether or not the program is to continue to the next question or is to be terminated.

As often as feasible statements were combined to help keep the modules from becoming too lengthy. For example, instead of having three statements to generate three random coefficients (X,Y,Z) for a problem the APL statement X←RN2+0×Y←RN1+0×Z←RN2 combines the statements. Many statements of this type were used in the modules. RN1 and RN2 are two random number generators. RN1 generates a random number from -9 to +9 by the statement RN←¯10+?19. The numbers are restricted to one digit to minimize the computations students must do to work the problem.

Sometimes it was necessary to generate nonzero coefficients. One example comes from QUAD. Equations of the form AX*2 +BX +C = 0 are generated. To insure that the equation is 2nd degree, A cannot be equal to zero. The function RN1 was modified to RN2 and looks like:

```
    ∇RN RN2
[1]  RN←¯10+?19
[2]  →1×⍳RN=0
    ∇
```

The roll operator(?) in the above function is APL's random number generator. The statement ?X returns a random integer from 1 to X inclusive. Each time an APL user signs-on the terminal, he will get the same sequence of random numbers if the same upper limit is specified. To keep from having the coefficients repeated for each problem every time a student reuses a module, a method was needed to alter the seed for the roll operator to a unique starting point every time a program module is loaded and executed. The following two statements which are used in every module have worked well.

```
[1] N←?1+¯1↑(60 60 60 60⊤I20)
[2] N←0×1↑?(Nρ19)
```

In [1] the I-beam 20 function returns a number corresponding to the internal clock of the CPU. It represents the time of day, but to transform it into hours, minutes, seconds and sixtieths of a second requires (60 60 60 60⊤I20). The addition of ¯1↑ before this statement retrieves the number corresponding to sixtieths of a second. Adding 1 to this number insures that the upper limit for the roll operator will always be greater than 0. The random number is then stored in N. In [2] the statement (Nρ19) generates a vector of length N with each element equal to 19. This vector becomes the upper limit for the roll operator and the result is that the roll operator is used N times with the same upper limit of 19. These two statements have, in effect, supplied a new starting point for APL's random number generator. The remaining part of statement [2] results in storing a 0 in N, since N is used elsewhere.

One unforeseen problem of the modules occurs when the answer to a question is a quotient of two integers. For example one question from LINEQ is:

GIVEN $^{-}4X + 6Y + 6 = 0$
WHAT IS THE SLOPE?

The answer is 4÷6 which is .666667 . If the student typed HELP the number .666667 would be printed. Obviously this is not as helpful as 4÷6 . For cases like this two variables are needed. One is called ANS and the other ANSA. ANS is the variable which contains the correct numeric answer and its value is compared to the user's responce. In the example, ANS is .666667 . On the other hand, ANSA is the variable that is printed when the user types HELP. In this example ANSA would be the character string '4÷6'. The problems are generated so that the denominator is never zero. In some cases, however, it may be 1. In generating the questions, if the answer is an integer then ANSA will be the same as ANS. In this case the assignment of the answer to ANS and ANSA (ANS←ANSA←X) is combined with the statement →0×⍳CHECK=1 (which has already been explained) to keep the number of APL statements to a minimum. The APL statement is:

→0×⍳CHECK=1+0×ANS←ANSA←X (where X is the correct answer)

If the answer is a nonintegral quotient, then ANSA uses one of two functions, either DIV or AHINT. The function that is used depends on whether the hint for the question is in the form of a comment or a choice of one of four numbers. If the hint is a comment then the APL statement for ANSA is ANSA←X DIV Y. In the above example from LINEQ, the hint for that question is "THE CLUE TO THIS ONE IS TO PUT THE EQUATION IN THE FORM OF Y=MX+B." In this case ANSA uses the DIV function. The function changes these two integers, X and Y, to the string 'X÷Y' and stores this string in the global variable ANSA. The function DIV involves several techniques. The first uses the encode operator to change the absolute value of the (numeric) numbers X and Y to three single digit numbers--each digit corresponding to the respective 1's, 10's or 100's places of X and Y --by the statements (10 10 10⊤|X). The statements below did this satisfactorily:

```
[1] NA←'0123456789'
[2] NA[1+(10 10 10⊤|X)]
[3] NA[1+(10 10 10⊤|Y)]
```

X and Y are three or fewer digits so only three places are necessary in the encode. Unfortunately a problem results if X or Y is negative. Therefore, the statement (X<0)⍴'-' is catenated to statements [2] and [3]. It catenates a negative sign if one is needed. Thus the DIV function looks like

```
    ∇D←X DIV Y
[1] NA←'0123456789'
[2] D←((X<0)⍴'-'),NA[1+(10 10 10⊤|X)],'÷',((Y<0)⍴'-',
    NA[1+(10 10 10⊤|Y)]
    ∇
```

This works for all cases except for the one in which the two integers are less than three digits in length. Then the leading zeros are unfortunately carried along. For example, '2÷5' would look like'002÷005'. To eliminate this problem each leading 0 is dropped. This is done by the statement T←((1↑T)='0')↓T where T is the string '002' or '005'. This statement is executed twice for each string and before the minus sign is catenated (if necessary), so the final string '2÷5' does indeed look like '2÷5'. The listing of the function DIV can be found in Appendix D.

If the hint for a question involves a choice of one of four numbers and the answer is a nonintegral quotient, then the function AHINT rather than DIV is used. The dyadic function AHINT takes two arguments, X and Y, and like DIV produces the string 'X÷Y' which is stored in ANSA. It also produces one of the following strings which is printed to the user as the hint for the question:

```
     'X÷Y    Y÷X    -X÷Y    -Y÷X'
or
     'Y÷X    -X÷Y   -Y÷X    X÷Y'
```

The second string is a permutation of the first. This permutation was picked so the user could not recognize a pattern in the hint. The techniques in programming AHINT are similar to those used in DIV. The programming is a little easier because the arguments passed to AHINT are always one digit in length.

If the hint for a question does involve a choice of four integers, but the answer is not a quotient, then the other three integers of the hint are picked either at random or in some way related to the problem or both. An example from ELLIPSE shows this:

```
GIVEN AN ELLIPSE WHICH PASSES THROUGH THE ORIGIN, HAS
FOCI AT (-6,-4) AND (6,-4) AND THE EQUATION
(X-_)*2 ÷_ +(Y-_)*2 ÷_ = 1
WHAT NUMBER GOES IN THE FIRST BLANK?
⎕:
     HINT
THE ANSWER IS ONE OF THE FOLLOWING:
0      6      16      4
```

0 is the correct answer, 6 is the value of one of the X-coordinates in the problem, 16 is the value of the Y-coordinate squared, and 4 is a random number.

To keep the answer from being in the same position relative to the other integers every time, a random number is used with the dyadic rotation operator(⌽) to rotate the four numbers. The following APL statement is used in QUADEQ:

```
H←(?4)⌽H←(-X+Y),-|RN2,(-Y),ANS←ANSA←X
```

(X and Y are two parameters used in the problem)

The statement H←(-X+Y),-|RN2,(-Y),ANS←ANSA←X creates a vector of length four which is stored in H. A random number generated by (?4) is used with the dyadic rotation operator(⌽) to rotate the above vector. This final result is stored in H and is printed when a hint is called.

The module LINEQ prints the graph of a straight line as part of two problems. The method of printing the graph originated with Gilman and Rose[3], but was modified to suit these problems. The four statements below are used to generate the graph where the slope of the line and the Y-intercept are variable.

```
[1] X←¯5+⍳9
[2] M←(?2)××RN2
[3] G←(⌽X)∘.=(M×X)+Y←¯4+?7
[4] (18⍴1 0)\' ∘+∘'[G+1+2×0=(⌽X)∘.×X]
```

The first statement is used to set up the X-axis, ranging from -4 to +4 for a total of 9 print positions. The second statement generates a slope which is either ±1 or ±2. Several other slopes were tried but were found to be unsuitable. A slope of 3 or greater creates only 3 or less points to be printed which was not very representative of a straight line. The right part of statement [3], namely Y←¯4+?7, determines the Y-intercept of the line. Statement [3] itself, which involves the use of the outer product, produces a 9×9 matrix of zeros except where the straight line would intersect an element of the matrix. In this case the element becomes a 1. A typical example looks like:

```
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 1 0 0
0 0 0 0 0 1 0 0 0
0 0 0 0 1 0 0 0 0
0 0 0 1 0 0 0 0 0
0 0 1 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0
```

In the right part of statement [4] the matrix G is added to the matrix produced by `1+2×0=(⌽X)∘.×X` giving the matrix:

```
1 1 1 1 3 1 1 1 1
1 1 1 1 3 1 1 1 2
1 1 1 1 3 1 1 2 1
1 1 1 1 3 1 2 1 1
3 3 3 3 3 4 3 3 3
1 1 1 1 4 1 1 1 1
1 1 1 2 3 1 1 1 1
1 1 2 1 3 1 1 1 1
1 2 1 1 3 1 1 1 1
```

It can be seen now that the 3's are centered and form a set of axes. The 2's and 4's lie on the line and the 1's are the other elements of the matrix. Using this matrix as an index on the string ' o+o' causes the 1's to be replaced by blanks, the 2's and 4's by 'o', and the 3's by '+'. The addition of (18ρ1 0)\ operating on this character matrix of blanks, +'s and o's spreads the matrix out so it looks like the following:

```
        +
        +       o
        +     o
        +   o
+ + + + + o + + +
        o
      o +
    o   +
  o     +
```

This graph has a slope of -1 and a Y-intercept of -1.

For the module CONICS a technique is used to order the questions which is different from the other modules. This is due to the fact that the nine questions of CONICS are multiple choice type where each question always requires the same answer regardless of what coefficients are used. For example, question number one reads:

```
GIVEN THE EQUATION 0X*2 +0Y*2 + 1X + ¯4X +6 =0.  THE EQUATION
IS EITHER
1)  A STRAIGHT LINE
2)  A CIRCLE
3)  A PARABOLA
4)  AN ELLIPSE
5)  A HYPERBOLA
6)  NONE OF THESE
IS THE ANSWER 1,2,3,4,5 OR 6?
⎕:
```

The answer to this question is always 1 even though coefficients for the X, Y and constant terms change. Therefore, changing the order of the questions is necessary to help keep the user from recognizing the pattern of the answers. The permuting of the order of the questions involves the dyadic deal operator(?). The deal operator, like the roll operator, has the same seed every time a new workspace is loaded, so the statement I←(|RN2)?9 is used to reinitialize the seed to a new starting point every time CONICS is used. Next, the statement I←9?9 gives a permutation of the integers 1 through 9 --a number corresponding to each question. Thus I becomes an index to nine different labels--one for each question. The following statements show precisely how the questions are selected:

```
[1]  L10:J←0
[2]  I←9?9
[3]  I←I,10
[4]  L0:→(L1,L2,L3,L4,L5,L6,L7,L8,L9,L10)[I[J←J+1]]
```

Statement [1] initializes J to 0 and [2] gives the permutation of the digits 1 through 9. In [3] the number 10 is catenated to this permutation. Since control is returned to L0 after each question has been answered, the right part of statement [4], J←J+1, becomes an index for the next question. This happens by I[J←J+1] returning one of the permuted integers. This integer selects one of the labels L1 through L10. Execution of the corresponding labeled statement follows. After nine questions have been asked J will be incremented by one producing I[10] which corresponds to the label L10. On execution of L10, J becomes reinitialized to 0, a new permutation is produced, and the cycle is repeated.

The module TRIG is unique in that the figure of a triangle is printed for most of the problems. The triangle was made by storing an 8 by 9 matrix of blanks in a variable called TRI and then filling in the correct points with other characters. The figure is shown below:

```
       /|
      /ω|
     /  |
   Z/   |Y
   /    |
  /     |
 / α____|
    X
```

The triangle is stored in the workspace as a global variable and is used by TRIG when necessary.

The various programming techniques discussed previously are unique to the author. However, the remaining programming was approached in a manner typical of any programmer. The programming language APL was selected because it was the only interactive language available. Fortunately, it turned out to be suitable for this application. The language lends itself to compact programming, thus eliminating many programming errors that would occur in other languages. Debugging of the programs was surprisingly easy. This was due to APL's capability of handling independent functions easily. The major disadvantage of APL came from having to use "*" for terms raised to a power. The use of the "*" instead of superscripts made the reading of equations awkward and in some cases difficult.

The advantages of using APL for this application far outnumber any disadvantages. While some of the notation may be awkward in mathematics, the use of APL for CAI could be easily adopted for other areas. For instance, using only the functions CHECK and SCORE a history instructor could easily program a set of lessons on some topic without having to learn much APL. Thus APL is certainly conductive to computer-assisted instruction applications.

APPENDIX A

A DESCRIPTION OF THE MODULES

I. LINEQ

The description of linear equations that is printed out to the user is the following:

```
     LINEAR EQUATIONS
THE GRAPH OF A FIRST DEGREE EQUATION OF THE FORM
     DX+EY+F=0
WHERE D,E AND F ARE REAL NUMBERS AND WHERE EITHER D OR E IS
NOT EQUAL TO ZERO, IS CALLED A STRAIGHT LINE.  SUCH AN EQUATION
IS CALLED A LINEAR EQUATION.
 THE EQUATION IS USUALLY WRITTEN IN THE FORM
     Y=(-D÷E)X+(-F÷E)  OR
     Y=MX+B  WHERE
     M=-D÷E  IS EQUAL TO THE SLOPE AND
     B=-F÷E  IS EQUAL TO THE Y-INTERCEPT.
THIS EQUATION IS SOMETIMES WRITTEN AS
     X=(-E÷D)Y+(-F÷D)  WHERE
     -F÷D  IS EQUAL TO THE X-INTERCEPT.

 THE SLOPE IS DEFINED TO BE THE RATIO
     M=(Y2-Y1)-(X2-X1)
WHERE (X2,Y2) AND (X1,Y1) ARE TWO POINTS.  IF
(X2-X1) EQUALS ZERO THEN THE SLOPE IS SAID TO BE UNDEFINED.
```

The level of the questions for LINEQ is similar to that of Davis's Real Numbers and Elementary Algebra[4] and Munem's, Tschirhart's, and Yizze's Study Guide to Functional Approach to Precalculus[5], which is that of a second year high school algebra course or a first year freshmen elementary math course. There are seven basic problems from which sixteen different questions are generated for each pass through the seven problems. The problems are recycled after each pass with new coefficients giving sixteen new questions. This cycle continues until the user types STOP. A listing of LINEQ is given in Appendix C.

The hints for each question, unless otherwise stated, are a choice of one of four numbers. The problems and questions for LINEQ are listed below:

I) GIVEN 2 POINTS(X2,Y2)=(A,B) AND (X1,Y1)=(C,D) (A,B,C and D are random integers, but A≠C)

1)WHAT IS THE SLOPE?

2)WHAT IS THE Y-INTERCEPT? (This question is skipped the first time the module is used. The intent is to let the user gain confidence by starting with a simple question)

II) GIVEN Y=AX+B (A and B are random integers, A≠0)

3)WHAT IS THE SLOPE?

4)WHAT IS THE Y-INTERCEPT?

III) BELOW IS THE GRAPH OF A LINE

```
        +
        +       o
        +     o
        +   o
+ + + + + o + + +
        o
      o +
    o   +
  o     +
```

WHICH INTERSECTS THE X-AXIS AT A AND HAS A SLOPE=B (A is an integer from -3 to +3 and B is ±1 or ±2).

IF THE EQUATION IS OF THE FORM

Y=_X+_

5)WHAT NUMBER GOES IN THE FIRST BLANK?

6)WHAT NUMBER GOES IN THE SECOND BLANK?

IV) GIVEN THE POINT(X,Y)=(A,B), A SLOPE OF C AND

Y-_=_(X-_)   (A,B and C are random integers, C≠0)

7)WHAT NUMBER GOES IN THE FIRST BLANK?

8)WHAT NUMBER GOES IN THE THIRD BLANK?

9)WHAT NUMBER GOES IN THE SECOND BLANK?

V) GIVEN A Y-INTERCEPT OF A, A SLOPE OF B AND

Y=_X+_   (A and B are random integers, A≠0)

10)WHAT NUMBER GOES IN THE FIRST BLANK?

11)WHAT NUMBER GOES IN THE SECOND BLANK?

VI) GIVEN AX+BY+C=0   (A,B and C are random integers, A≠0 and B≠0)

12)WHAT IS THE SLOPE?

13)WHAT IS THE Y-INTERCEPT?

14)WHAT IS THE X-INTERCEPT?

VII) BELOW IS THE GRAPH OF A LINE

```
  o     +
    o   +
      o +
        o
+ + + + + o + + +
        +   o
        +     o
        +       o
        +
```

WHICH INTERSECTS THE X-AXIS AT A AND THE Y-AXIS AT B   (A is a random number from -3 to +3 including half-intervels and B is a random integer from -3 to +3)

IF Y=_X+_

15)WHAT NUMBER GOES IN THE FIRST BLANK?

16)WHAT NUMBER GOES IN THE SECOND BLANK?

II. QUADEQ

The description of quadratic equations that is printed out to the user is the following:

```
     QUADRATIC EQUATIONS
 THE QUADRATIC EQUATION
     AX*2 + BX + C = 0,    A≠0
WHERE A, B, AND C ARE REAL NUMBERS, AND WHERE X IS
EITHER A REAL OR A COMPLEX NUMBER, HAS TWO SOLUTIONS, NAMELY:
     X=(-B+(B*2 -4AC)*.5)÷2A  AND
     X=(-B-(B*2 -4AC)*.5)÷2A

 THE QUANTITY (B*2 -4AC)*.5 IS KNOWN AS THE DISCRIMINANT AND IT
DETERMINES THE KINDS OF ROOTS OF THE EQUATION.  IF THE DISCRIMINANT
IS:
     1) POSITIVE, THE TWO SOLUTIONS ARE REAL AND UNEQUAL.
     2) ZERO, THE TWO SOLUTIONS ARE REAL AND EQUAL.
     3) NEGATIVE, THE TWO SOLUTIONS ARE UNEQUAL AND COMPLEX.
```

The level of the questions for QUADEQ is the same as that for LINEQ. There are five basic problems from which eight different questions are generated for each pass. The problems are recycled, each time with new coefficients, until the user types STOP. A listing of QUADEQ is given in Appendix C.

The problems and questions for QUADEQ are the following:

I) GIVEN TWO ROOTS X=A,B AND
X*2 +_X +_ =0 (A and B are random integers)

1)WHAT NUMBER GOES IN THE FIRST BLANK?

The HINT for this question is: THE QUADRATIC EQUATION, GIVEN TWO ROOTS, IS FOUND BY MULTIPLYING (X-ROOT1)×(X-ROOT2).

2)WHAT NUMBER GOES IN THE SECOND BLANK?

The HINT is the same as that above.

II) GIVEN X*2 +AX +B = 0 **(A and B are the sum and product of two random integers)**

3)WHAT IS THE SMALLER ROOT OF THE EQUATION?

The HINT **is:** THE SMALLER ROOT IS THE ONE WHICH IS FARTHER TO THE LEFT ON THE REAL NUMBER LINE (-INFINITY....0....+INFINITY).

4)WHAT IS THE LARGER ROOT OF THE EQUATION?

The HINT **is:** THE LARGER ROOT IS THE ONE WHICH IS FARTHER TO THE RIGHT ON THE REAL NUMBER LINE (-INFINITY....0....+INFINITY).

III) THE GRAPH OF AX*2 +BX +C = 0 **(A, B and C are random integers)**

5)OPENS EITHER
1)UPWARD OR
2)DOWNWARD
IS THE ANSWER 1 OR 2?

The HINT **is:** IF THE COEFFICIENT OF THE X-SQUARED TERM IS>0 THE CURVE OPENS UPWARD. IF IT IS<0 IT OPENS DOWNWARD.

IV) GIVEN AX*2 +BX +C = 0 **(A, B and C are random integers, A≠0)**

6)THE SOLUTIONS ARE EITHER
1)UNEQUAL AND COMPLEX
2)REAL AND EQUAL
3)REAL AND UNEQUAL
IS THE ANSWER 1, 2 OR 3?

The HINT **is:** THE VALUE OF THE DISCRIMINATE GIVES THE ANSWER TO THIS QUESTION.

V) GIVEN X*2 +AX +B = 0 **(A and B are the sum and product of two random integers--one positive and one negative)**

7)IF YOU WERE TO GRAPH THIS EQUATION WHERE WOULD IT CROSS THE POSITIVE X-AXIS?

8)WHERE WOULD IT CROSS THE NEGATIVE X-AXIS?

The HINT **for questions 7 and 8 is a choice of one of four numbers.**

III. TRIG

The description of trigonometric functions that is printed out to the user is:

```
          TRIGONOMETRY
     GIVEN A RIGHT TRIANGLE IN ITS STANDARD POSITION IN THE FIRST
    QUADRANT WITH THE ANGLE α AT THE ORIGIN.
                  /|
                 /ω|
         +      /  |
         +  Z/     |Y
         +  /      |
         + /       |
    + + +/ α_______|
         +   X
         +
         +
         +
    PYTHAGORAS'S THEOREM STATES THAT
          Z*2 = X*2 + Y*2
    AND FROM THE DEFINITIONS OF TRIGONOMETRY

     SIN α = SIDE OPPOSITE ÷ HYPOTENUSE = Y÷Z
     COS α = SIDE ADJACENT ÷ HYPOTENUSE = X÷Z
     TAN α = SIDE OPPOSITE ÷ SIDE ADJ.  = Y÷X
     CSC α = HYPOTENUSE    ÷ SIDE OPP.  = Z÷Y = 1÷SIN
     SEC α = HYPOTENUSE    ÷ SIDE ADJ.  = Z÷X = 1÷COS
     COT α = SIDE ADJACENT ÷ SIDE OPP.  = X÷Y = 1÷TAN

     THE SIGNS OF THE TRIG TERMS WOULD CHANGE IF THE TRIANGLE WAS IN
    ANOTHER QUADRANT AND WOULD CHANGE ACCORDING TO THE CORRESPONDING
    VALUES OF X, AND Y IN THAT QUADRANT.  THE TRIG TERMS, THEN, ARE
    CYCLIC AS α IS ROTATED FROM 0 TO 360 DEGREES.
```

The level of TRIG is similar to that of LINEQ. There are seven different problems which ask twenty-eight questions for each pass. Most of the problems involve a right triangle where only two sides are given. These problems have been arranged so that every right triangle is a perfect integer right triangle. This involves the use of the function RN3 which gives six different right triangles. A listing of TRIG can be found in Appendix C.

Unless otherwise stated the HINT for each question is: REFER BACK TO THE CORRESPONDING DEFINITION OF THE TRIG TERM. The problems and questions follow:

I) GIVEN THE RIGHT TRIANGLE

```
         /|
        /ω|
       /  |
     Z/   |Y
     /    |
    /     |
   /α_____|
      X
```

WHERE X=A AND Y=B (A and B are positive integers)

The above triangle is printed for each problem where appropriate, but to prevent redundancy it will be omitted from here on and will be replaced by .....

1)WHAT IS THE VALUE OF Z?

The HINT for this question is: THE CLUE TO THIS ONE IS PYTHAGORAS'S THEOREM.

2)WHAT IS THE VALUE OF TAN α?

3)WHAT IS THE VALUE OF SIN α?

4)WHAT IS THE VALUE OF COS ω?

5)WHAT IS THE VALUE OF COS α?

6)WHAT IS THE VALUE OF SIN ω?

7)WHAT IS THE VALUE OF TAN ω?

8)WHAT IS THE VALUE OF COT α?

9)WHAT IS THE VALUE OF CSC α?

10)WHAT IS THE VALUE OF SEC α?

II) GIVEN THE RIGHT TRIANGLE .....

AND SIN α=A÷B AND Y=A (A and B are two positive integers)

11)WHAT IS THE SIDE OPPOSITE α EQUAL TO?

12)WHAT IS THE HYPOTENUSE EQUAL TO?

The HINT for this one is the same as that for question number 1.

13)WHAT IS THE SIDE ADJACENT α EQUAL TO?

14)WHAT IS THE VALUE OF THE COS α?

15)WHAT IS THE VALUE OF THE TAN α?

III) GIVEN THE RIGHT TRIANGLE .....

AND TAN α=A÷B AND X=B (A and B are positive integers)

16)WHAT IS THE SIDE OPPOSITE α EQUAL TO?

17)WHAT IS THE HYPOTENUSE EQUAL TO?

The HINT is the same as question 1.

18)WHAT IS THE SIDE ADJACENT α EQUAL TO?

19)WHAT IS THE VALUE OF COS α?

20)WHAT IS THE VALUE OF SIN α?

IV) GIVEN THE RIGHT TRIANGLE .....

AND COS α=A÷B AND Z=B (A and B are positive integers)

21)WHAT IS THE SIDE OPPOSITE α EQUAL TO?

22)WHAT IS THE HYPOTENUSE EQUAL TO?

The HINT for this one is the same as that of question 1.

23)WHAT IS THE SIDE ADJACENT α EQUAL TO?

24)WHAT IS THE VALUE OF TAN α?

25)WHAT IS THE VALUE OF SIN α?

V)

26)WHAT IS THE VALUE OF COS(A×PI)? **(A is a positive random integer)**

The HINT **is:** REMEMBER THAT TRIG FUNCTIONS ARE CYCLIC SO YOU NEED TO FIND OUT WHAT QUADRANT THIS WOULD BE IN.

VI)

27)WHAT IS THE VALUE OF SIN(A×PI)? **(A is a positive random integer)**

The HINT **is the same as that above.**

VII GIVEN A RIGHT TRIANGLE .....

WITH THE ANGLE α=B DEGREES **(B is a number equal to 9 times a positive random integer less than or equal to nine)**

28)WHAT IS THE ANGLE ω EQUAL TO?

The HINT **for this question is:** THE CLUE IS THAT THE SUM OF THE INNER ANGLES OF A TRIANGLE=180 DEGREES.

IV. CIRCLE

The description of the equations of a circle that is printed out to the user is:

```
     THE CIRCLE
 THE CIRCLE IS THE LOCUS OF POINTS IN A PLANE AT A GIVEN DISTANCE
(CALLED THE RADIUS) FROM A GIVEN POINT (CALLED THE CENTER).  IF
C(H,K) IS THE CENTER OF THE CIRCLE AND R THE RADIUS THEN THE EQUA-
TION OF THE CIRCLE IS
     (X-H)*2 + (Y-K) = R*2

 AN EQUATION OF THE FORM
     AX*2 + AY*2 + DX + EY + F = 0,    A≠0
CAN OFTEN BE REDUCED TO THE EQUATION OF A CIRCLE BY THE METHOD OF
COMPLETING THE SQUARES.  IN THE ABOVE FORM IT WOULD LOOK LIKE
     (X+D÷2A)*2 + (Y+E÷2A)*2 = (D*2 + E*2 - 4AF) ÷ 4A*2
REMEMBER, HOWEVER, THAT THE RIGHT HAND SIDE MUST BE POSITIVE FOR
THE EQUATION OF A CIRCLE.
```

The level of the questions for CIRCLE is along that of Thomas[2]. There are six basic problems from which sixteen different questions are generated for the first pass; however, the first problem is skipped on each recycle because of its simplicity. Hence there are only thirteen questions for each subsequent pass. New coefficients are generated for each different pass. Unless otherwise stated all hints are just a choice of one of four numbers. A listing of CIRCLE is given in Appendix C.

The problems and questions for CIRCLE are the following:

I) GIVEN A CIRCLE OF RADIUS A WITH ITS CENTER AT THE ORIGIN AND THE EQUATION _X*2 +_Y*2 =_ (A is a random integer not equal to zero)

1)WHAT NUMBER GOES IN THE FIRST BLANK?

2)WHAT NUMBER GOES IN THE SECOND BLANK?

3)WHAT NUMBER GOES IN THE THIRD BLANK?

II) GIVEN (X-A)*2 + (Y-B)*2 = C (A and B are random integers and C is the square of a random integer which is greater than zero)

4)WHAT IS THE X-COORDINATE OF THE CENTER OF THE CIRCLE?

5)WHAT IS THE Y-COORDINATE OF THE CENTER OF THE CIRCLE?

6)WHAT IS THE RADIUS EQUAL TO?

III) GIVEN THE CENTER OF A CIRCLE AS (A,B), A RADIUS OF C AND THE EQUATION (X-_)*2 + (Y-_)*2 = 0 (A, B and C are random integers and C≠0)

7)WHAT NUMBER GOES IN THE FIRST BLANK?

8)WHAT NUMBER GOES IN THE SECOND BLANK?

9)WHAT NUMBER GOES IN THE THIRD BLANK?

IV) GIVEN A CIRCLE WHICH PASSES THROUGH THE ORIGIN, HAS A CENTER AT (A,B) AND THE EQUATION (X-_)*2 + (Y-_)*2 = 0 (A and B are random numbers but B≠0)

10)WHAT NUMBER GOES IN THE FIRST BLANK?

11)WHAT NUMBER GOES IN THE SECOND BLANK?

12)WHAT NUMBER GOES IN THE THIRD BLANK?

V) GIVEN THE EQUATION (X-A)*2 + (Y-B)*2 = C . THE POINT (D,E) IS EITHER
1)INSIDE THE CIRCLE
2)ON THE CIRCLE OR
3)OUTSIDE THE CIRCLE (A, B, D and E are random integers and C is the square of a random integer which is greater than zero)

13)IS THE ANSWER 1, 2 OR 3?

The HINT is: SUBSTITUTE THE VALUES OF X AND Y OF THE POINT INTO THE EQUATION AND MAKE A COMPARISON WITH THE RIGHT HAND SIDE.

VI) GIVEN THE EQUATION X*2 + Y*2 + AX + BY + C = 0 (A, B and C are random integers and B≠0)

14)WHAT IS THE X-COORDINATE OF THE CENTER OF THE CIRCLE?

The HINT **is:** THE CLUE TO THIS ONE IS THE METHOD OF COMPLETING THE SQUARES.

15)WHAT IS THE Y-COORDINATE OF THE CENTER OF THE CIRCLE?

The HINT **is the same as that above.**

16)WHAT IS THE RADIUS-SQUARED TERM EQUAL TO?

The HINT **is:** THE CLUE TO THIS ONE IS THE RIGHT HAND SIDE OF THE EQUATION AFTER THE METHOD OF COMPLETING THE SQUARES HAS BEEN APPLIED.

V. PARABOLA

The description of the equations of parabolas printed out to the user is:

```
     THE PARABOLA
 A PARABOLA IS THE LOCUS OF POINTS IN A PLANE EQUIDISTANT FROM A
POINT (CALLED THE FOCUS) AND A GIVEN LINE (CALLED THE DIRECTRIX).
THE FOCUS IS ON THE AXIS OF SYMMETRY, P UNITS FROM THE VERTEX
WHILE THE DIRECTRIX IS -P UNITS FROM THE VERTEX AND PERPENDICULAR
TO THE AXIS OF SYMMETRY.
IF V(H,K) IS THE VERTEX THEN THE EQUATION OF A PARABOLA IS ONE OF
THE FOLLOWING:
     1) (X-H)*2 =  4P(Y-K)  WHICH OPENS UPWARD
     2) (X-H)*2 = -4P(Y-K)  WHICH OPENS DOWNWARD
     3) (Y-K)*2 =  4P(X-H)  WHICH OPENS TO THE RIGHT
     4) (Y-K)*2 = -4P(X-H)  WHICH OPENS TO THE LEFT

 THE CLUE TO AN EQUATION OF A PARABOLA IS THAT IT IS QUADRATIC IN
ONE OF THE COORDINATES AND LINEAR IN THE OTHER.  WHENEVER THERE IS
THIS TYPE OF EQUATION IT CAN BE REDUCED TO ONE OF THE ABOVE STAN-
DARD FORMS BY COMPLETING THE SQUARE IN THE COORDINATE WHICH APPEARS
QUADRATICALLY.
```

The level of the questions for PARABOLA is the same as that for CIRCLE. There are three basic problems from which fifteen different questions are generated for each pass through the problems. Unless otherwise stated all hints are the choice of one of four numbers. A listing of PARABOLA is given in Appendix C.

The problems and questions for PARABOLA are the following:

I) GIVEN THE EQUATION (X-A)*2 = B(Y-C) (A and C are random numbers and B is four times a random number which is not equal to C)

1)WHAT IS THE X-COORDINATE OF THE VERTEX?

2)WHAT IS THE Y-COORDINATE OF THE VERTEX?

3)WHAT IS THE X-COORDINATE OF THE FOCUS?

4)WHAT IS THE Y-COORDINATE OF THE FOCUS?

5)THE DIRECTRIX IS THE LINE Y=_
WHAT NUMBER GOES IN THE BLANK?

6)THE GRAPH OF THIS EQUATION OPENS EITHER
1)UPWARD
2)DOWNWARD
3)TO THE RIGHT
4)TO THE LEFT
IS THE ANSWER 1, 2, 3, OR 4?

The HINT for this question is: THE CLUE TO THIS IS THE SIGN OF P AND THE QUADRATIC TERM.

II) GIVEN THE VERTEX V(A,B) AND FOCUS (C,B) OF A PARABOLA AND THE EQUATION (Y-_)*2 = _(X-_) (A, B and C are random integers, A≠C)

7)WHAT NUMBER GOES IN THE FIRST BLANK?

8)WHAT NUMBER GOES IN THE SECOND BLANK?

9)WHAT NUMBER GOES IN THE THIRD BLANK?

10)THE DIRECTRIX IS THE LINE X=_
WHAT NUMBER GOES IN THE BLANK?

11)THE GRAPH OF THIS EQUATION OPENS EITHER
1)UPWARD
2)DOWNWARD
3)TO THE RIGHT OR
4)TO THE LEFT
IS THE ANSWER 1, 2, 3 OR 4?

The HINT for this question is the same as that of question 6.

III) GIVEN X*2 + AX + BY + C = 0 (B and C are random integers, C≠0 and A is the product of 2 and a random integer)

12)WHAT IS THE X-COORDINATE OF THE VERTEX?

13)WHAT IS THE Y-COORDINATE OF THE VERTEX?

The HINT is: THIS IS A HARD ONE. TRY PLOTTING THIS ONE AND THEN SEE IF YOU CAN FIGURE IT OUT.

14)WHAT IS THE DISTANCE FROM THE VERTEX TO THE FOCUS?

```
15)THE GRAPH OF THIS EQUATION OPENS EITHER
     1)UPWARD
     2)DOWNWARD
     3)TO THE RIGHT
     4)TO THE LEFT
IS THE ANSWER 1, 2, 3 OR 4?
```

The HINT is the same as that of question 6.

VI. ELLIPSE

The description of equations of ellipses printed out to the user is the following:

```
     THE ELLIPSE
 AN ELLIPSE IS THE LOCUS OF POINTS P(X,Y) THE SUM OF WHOSE DISTANCES
FROM TWO FIXED POINTS (CALLED FOCI) IS CONSTANT.  THE FOCI ARE
ALWAYS ON THE MAJOR AXIS.  IF WE USE THE LETTERS A, B AND C TO
REPRESENT THE LENGTHS OF SEMIMAJOR AXIS, SEMIMINOR AXIS AND HALF-
DISTANCE BETWEEN FOCI, RESPECTIVELY, THEN THE FOLLOWING EQUALITY
HOLDS:
     A*2 = B*2 + C*2
 IF P(H,K) IS THE CENTER, DEFINED AS THE POINT OF INTERSECTION OF
ITS AXES OF SYMMETRY, OF AN ELLIPSE THEN THE EQUATION OF THE ELLIPSE
IS GIVEN BY
     (X-H)*2 ÷ A*2 + (Y-K)*2 ÷ B*2 = 1    OR
     (X-H)*2 ÷ B*2 + (Y-K)*2 ÷ A*2 = 1
DEPENDING ON THE DIRECTION OF THE MAJOR AXIS.

 THE ECCENTRICITY OF AN ELLIPSE IS THE RATIO
     E=C÷A
AND INDICATES THE DEGREE OF DEPARTURE FROM CIRCULARITY.  KEEPING
A FIXED AND VARYING C FROM 0 TO 1, THE RESULTING ELLIPSE WILL VARY
IN SHAPE, BEING CIRCULAR WHEN C=0 AND BECOMING FLATTER AS C IN-
CREASES, UNTIL AT C=A THE ELLIPSE REDUCES TO A LINE SEGMENT JOIN-
ING THE TWO FOCI.
```

The level for ELLIPSE is the same as that of CIRCLE. There are five basic problems which generate eighteen different questions for the first pass. Only one question is skipped for each subsequent pass and that is the third question to problem I. It was skipped on each recycle because of the simplicity of the question. Being asked one time was felt sufficient to get its point across to the student. A listing of ELLIPSE is given in Appendix C.

Unless otherwise stated all hints are just a choice of one of four numbers. The problems and questions for ELLIPSE are the following:

I) GIVEN AN ELLIPSE WITH ITS CENTER AT THE ORIGIN, INTERSECTS THE POSITIVE X-AXIS AT A AND THE POSITIVE Y-AXIS AT B AND THE STANDARD EQUATION X*2 ÷_ + Y*2 ÷_ = 1 (A and B are random integers, both positive and unequal)

1)WHAT NUMBER GOES IN THE FIRST BLANK?

2)WHAT NUMBER GOES IN THE SECOND BLANK?

II) GIVEN AN ELLIPSE WITH ITS CENTER AT C(A,B), A FOCUS AT F(C,B) AND SEMIMAJOR AXIS AT A=D AND THE EQUATION (X-_)*2 ÷_ + (Y-_)*2 ÷_ = 1 (A and B are random integers, but C is the sum of A and another random integer greater than zero. D is the sum of this integer and another random integer which is also greater than zero)

3)WHAT NUMBER GOES IN THE FIRST BLANK?

The HINT given is: THIS ONE IS SO EASY THAT YOU REALLY DON'T NEED A HINT, BUT THE CLUE IS THAT YOU ARE JUST TRANSLATING THE AXIS TO THE CENTER OF THE ELLIPSE.

4)WHAT NUMBER GOES IN THE SECOND BLANK?

5)WHAT NUMBER GOES IN THE THIRD BLANK?

The HINT for this one is the same as question 4.

6)WHAT NUMBER GOES IN THE FOURTH BLANK?

The HINT is: THE CLUE IS THAT A*2 = B*2 + C*2, AND YOU SHOULD KNOW A AND C.

7)WHAT IS THE ECCENTRICITY OF THIS ELLIPSE?

The HINT is: YOU HAVE TO RECALL THAT E=C÷A.

III) GIVEN THE EQUATION AX*2 + BX*2 + CX + DY + E = 0 (A and B are the squares of two unequal random integers greater than zero and C, D and E are combinations of these two random integers and two others. This insures that the above equation can always be broken down to a standard form by the method of completing the square)

8)WHAT IS THE X-COORDINATE OF THE CENTER OF THE ELLIPSE?

The HINT for this one is: THIS EQUATION CAN BE CONVERTED TO THE STANDARD FORM BY THE METHOD OF COMPLETING THE SQUARE.

9)WHAT IS THE Y-COORDINATE OF THE CENTER OF THE ELLIPSE?

The HINT is the same as that above.

10)WHAT IS THE LENGTH OF THE MAJOR AXIS?

The HINT is: THE MAJOR AXIS IS JUST TWICE THE LENGTH OF THE SEMI-MAJOR AXIS A.

11)WHAT IS THE LENGTH OF THE MINOR AXIS?

The HINT is: THE MINOR AXIS IS JUST TWICE THE LENGTH OF THE SEMI-MINOR AXIS B.

IV) GIVEN THE EQUATION (X-A)*2 ÷B + (Y-C)*2 ÷D = 1 (A and C are random integers and B and D are the square of two unequal random integers)

12)WHAT IS THE LENGTH OF THE MAJOR AXIS?

The HINT is the same as that of question 10.

13)WHAT IS THE LENGTH OF THE MINOR AXIS?

The HINT is the same as that of question 11.

14)WHAT IS THE SQUARE OF THE DISTANCE FROM THE CENTER OF THE ELLIPSE TO A FOCUS?

The HINT is: REMEMBER THAT A*2 = B*2 + C*2 AND YOU SHOULD KNOW A AND B.

V) GIVEN AN ELLIPSE WHICH PASSES THROUGH THE ORIGIN, HAS FOCI AT (-A,B) AND (A,B) AND THE EQUATION
(X-_)*2 ÷_ + (Y-_)*2 ÷_ = 1

15)WHAT NUMBER GOES IN THE FIRST BLANK?

16)WHAT NUMBER GOES IN THE SECOND BLANK?

17)WHAT NUMBER GOES IN THE THIRD BLANK?

18)WHAT NUMBER GOES IN THE FOURTH BLANK?

VII. HYPERBOLA

The description of equations of hyperbolas printed out to the user is the following:

```
     THE HYPERBOLA
 A HYPERBOLA IS THE LOCUS OF POINTS P(X,Y) WHOSE DIFFERENCE OF ITS
DISTANCES FROM TWO FIXED POINTS(CALLED FOCI) IS CONSTANT.  THE
FOCI LIE ON ONE OF THE AXES OF SYMMETRY.  IF WE USE THE LETTERS A
AND C TO REPRESENT THE LENGTHS OF HALF-DISTANCE BETWEEN THE VERTICES
AND THE HALF-DISTANCE BETWEEN FOCI, RESPECTIVELY, THEN THE FOLLOW-
ING EQUALITY HOLDS:
     C*2 - A*2 = B*2  OR
     C*2 = A*2 + B*2
 IF P(H,K) IS THE CENTER, DEFINED AS THE POINT OF INTERSECTION OF
ITS AXES OF SYMMETRY, OF A HYPERBOLA THEN THE EQUATION OF THE HY-
PERBOLA IS GIVEN BY
     (X-H)*2 ÷ A*2 - (Y-K)*2 ÷ B*2 = 1  OR
     (Y-K)*2 ÷ A*2 - (X-H)*2 ÷ B*2 = 1
DEPENDING ON WHETHER THE FOCI ARE LOCATED ON THE X-AXIS OR Y-AXIS,
RESPECTIVELY.  LIKEWISE, THE STRAIGHT LINES
     (Y-K) = (B÷A)(X-H) AND (Y-K) = (-B÷A)(X-H)  OR
     (Y-K) = (A÷B)(X-H) AND (Y-K) = (-A÷B)(X-H)
ARE CALLED THE ASYMPTOTES OF THE HYPERBOLA, DEPENDING ON WHICH
RESPECTIVE AXIS THE FOCI ARE LOCATED.
```

The level for HYPERBOLA is the same as that of CIRCLE. There are five basic problems which generate twenty different questions for the first pass through the problems. On subsequent passes the first problem and questions number 2, 4 and 6 of problem number 2 are skipped because of their relative simplicity. A listing of HYPERBOLA is given in Appendix C.

Unless otherwise stated the hint for each question is a choice of one of four numbers. The problems and questions are below:

I) THE GRAPH OF THE EQUATION X*2 -A -Y*2 -B =1 (A and B are random integers not equal to zero)

1)OPENS EITHER

1)UPWARD AND DOWNWARD OR
2)TO THE RIGHT AND TO THE LEFT

IS THE ANSWER 1 OR 2?

The HINT to this question is: IF THE NEGATIVE SIGN IS BEFORE THE X-SQUARED TERM THEN THE CURVE OPENS UPWARD AND DOWNWARD AND IF THE NEGATIVE SIGN IS BEFORE THE Y-SQUARED TERM THEN THE CURVE OPENS TO THE RIGHT AND TO THE LEFT.

II) GIVEN A HYPERBOLA WITH ITS CENTER AT THE ORIGIN, FOCI AT (A,0) AND (-A,0), VERTICES AT (B,0) AND (-B,0) AND THE EQUATION

(X-_)*2 ÷_ -(Y-_)*2 ÷_=_ (B is a positive random integer and A is the sum of B and another positive random integer)

2)WHAT NUMBER GOES IN THE FIRST BLANK?

3)WHAT NUMBER GOES IN THE SECOND BLANK?

The HINT is: REMEMBER THAT A IS THE HALF-DISTANCE BETWEEN THE VERTICES.

4)WHAT NUMBER GOES IN THE THIRD BLANK?

5)WHAT NUMBER GOES IN THE FOURTH BLANK?

The HINT is: REMEMBER THAT C*2 = A*2 + B*2 AND YOU SHOULD KNOW A AND C.

6)WHAT NUMBER GOES IN THE LAST BLANK?

The HINT for this one is: THE EQUATION OF A HYPERBOLA IN ITS STANDARD FORM IS ALWAYS EQUAL TO 1.

7)WHAT IS THE ECCENTRICITY OF THE HYPERBOLA?

The HINT is: THE ECCENTRICITY IS EQUAL TO C-A.

III) GIVEN THE EQUATION A(X-B)*2 -C(Y-D)*2 = F (A and C are the squares of two positive random integers, B and D are two random integers and F is the product of A and C)

8)WHAT IS THE LENGTH BETWEEN THE TWO VERTICES?

The HINT for this question is: REMEMBER THAT A IS THE HALF-DISTANCE BETWEEN THE VERTICES.

9)WHAT IS THE LENGTH BETWEEN THE FOCI SQUARED EQUAL TO?

The HINT is: REMEMBER THAT C*2 = A*2 + B*2 AND YOU SHOULD KNOW A AND B.

10)WHAT IS THE POSITIVE SLOPE OF THE ASYMPTOTES?

The HINT is: THE POSITIVE SLOPE IS B÷A IF THE FOCI ARE ON THE X-AXIS AND A÷B IF THE FOCI ARE ON THE Y-AXIS.

11)WHAT IS THE X-COORDINATE OF THE INTERSECTION OF THE TWO ASYMPTOTES?

The HINT for this question is: THE CLUE IS THAT THE ASYMPTOTES INTERSECT AT THE CENTER OF THE HYPERBOLA.

12)WHAT IS THE Y-COORDINATE OF THE INTERSECTION OF THE TWO ASYMPTOTES?

The HINT for this question is the same as that of the question above.

IV) GIVEN A HYPERBOLA WITH FOCI AT (A,B) AND (A,-B) AND VERTICES AT (A,C) AND (A,-C) AND THE EQUATION
(X-_)*2 ÷_ -(Y-_)*2 ÷_ = 1 (A and C are random integers greater than zero and B is the sum of C and another random integer greater than zero)

13)WHAT NUMBER GOES IN THE FIRST BLANK?

14)WHAT NUMBER GOES IN THE SECOND BLANK?

The HINT is the same as that of question 3.

15)WHAT NUMBER GOES IN THE THIRD BLANK?

16)WHAT NUMBER GOES IN THE FOURTH BLANK?

The HINT for this one is the same as number 5.

17)WHAT IS THE ECCENTRICITY OF THE HYPERBOLA?

The HINT is the same as question number 7.

V) GIVEN THE EQUATION AX*2 -BY*2 + CX + DY + E =0 (A and B are the square of two random integers greater than zero and C,D and E are combinations of these two numbers and two others so that the above equation can always be converted to a standard form by the method of completing the square)

18)WHAT IS THE X-COORDINATE OF THE CENTER OF THE HYPERBOLA?

The HINT is: THIS EQUATION CAN BE CONVERTED TO THE STANDARD FORM BY THE METHOD OF COMPLETING THE SQUARE.

19)WHAT IS THE Y-COORDINATE OF THE CENTER OF THE HYPERBOLA?

The HINT is the same as that above.

20)WHAT IS THE POSITIVE SLOPE OF THE ASYMPTOTES?

The HINT is: THE POSITIVE SLOPE IS B÷A IF THE FOCI ARE ON THE X-AXIS AND A÷B IF THE FOCI ARE ON THE Y-AXIS.

VIII. CONICS

The description of equations of conics printed out to the user is the following:

```
     CONICS
 THE GRAPHS OF THE CIRCLE, PARABOLA, ELLIPSE AND THE HYPERBOLA
ARE CALLED CONICS.  THE PARABOLA, ELLIPSE AND HYPERBOLA HAVE A
COMMON FEATURE CALLED THE 'FOCUS-AND-DIRECTRIX' PROPERTY SINCE
EACH IS DETERMINED BY A GIVEN POINT, CALLED THE FOCUS, A GIVEN
LINE, CALLED THE DIRECTRIX, AND A POSITIVE CONSTANT E, CALLED
THE ECCENTRICITY.  THE CONIC WILL BE A
     1)PARABOLA IF E=1,  OR AN
     2)ELLIPSE IF E<1,≠0 AND A
     3)HYPERBOLA IF E>1
 THE CIRCLE, PARABOLA, ELLIPSE AND HYPERBOLA ARE ALL SPECIAL
CASES OF THE FOLLOWING GENERAL EQUATION OF THE SECOND DEGREE:
     AX*2 + CY*2 + DX + EY + D = 0
 THIS EQUATION, THEN, REPRESENTS
     1)A STRAIGHT LINE IF A=C=0, AND NOT BOTH D AND E VANISH.
     2)A CIRCLE IF A=C≠0.(IN SPECIAL CASES THE LOCUS MAY REDUCE
TO A SINGLE POINT OR NO REAL LOCUS)
     3)A PARABOLA IF THE EQUATION IS QUADRATIC IN ONE VARIABLE
AND LINEAR IN THE OTHER.
     4)AN ELLIPSE IF A AND C ARE BOTH POSITIVE OR BOTH NEGATIVE.
(AGAIN IN SPECIAL CASES THE LOCUS MAY REDUCE TO A SINGLE POINT
OR NO REAL LOCUS)
     5)A HYPERBOLA IF A AND C ARE OF OPPOSITE SIGNS, BOTH DIFFERENT
FROM ZERO.(IN SPECIAL CASES THE LOCUS MAY REDUCE TO A PAIR OF
INTERSECTING STRAIGHT LINES)
```

The level for CONICS is the same as CIRCLE. There are nine different problems which ask nine different questions. The uniqueness of this module is that the questions are not asked in the same order each time the module is used. The trick used in doing this was the APL deal operator(?). In this case it gave in random order the numbers 1 through 9. These numbers were then used to go to the corresponding question.

After each pass the questions would be recycled in different order and with different coefficients. A listing of CONICS is given in Appendix G.

The problems and questions are below (the corresponding correct answer for questions 1 through 8 are underlined):

I) GIVEN THE EQUATION OX*2 + OY*2 + DX + EY + F = 0. (D, E and F are random integers not equal to zero)

```
1)THE EQUATION IS EITHER
     1)A STRAIGHT LINE
     2)A CIRCLE
     3)A PARABOLA
     4)AN ELLIPSE
     5)A HYPERBOLA
     6)NONE OF THESE
IS THE ANSWER 1,2,3,4,5 OR 6?
```

The HINT for this question is: THE CLUE TO THIS ONE IS THE COEFFICIENTS A AND C. The above six choices are printed out to the user for questions 1 through 8.In order to prevent redundancy they will not be printed but will be indicated by .....

II) GIVEN THE EQUATION AX*2 + CY*2 + DX + EY + F = 0 (A and C are equal random integers greater than zero, D and E are random integers greater than or equal to zero and F is a negative random integer not equal to zero)

2)THE EQUATION IS EITHER .....

IS THE ANSWER 1,2,3,4,5 OR 6?

The HINT is the same as that of question number 1.

III) GIVEN THE EQUATION OX*2 + CY*2 + DX + CY + F = 0 (C, D and F are random integers not equal to zero)

3)THE EQUATION IS EITHER .....

IS THE ANSWER 1,2,3,4,5 OR 6?

The HINT is: THE CLUE TO THIS ONE IS THE COEFFICIENTS A AND E.

IV) GIVEN THE EQUATION AX*2 + CY*2 + 0X + 0Y + F = 0 (A and C are unequal random integers greater than zero and F is a negative random integer not equal to zero)

4)THE EQUATION IS EITHER .....

IS THE ANSWER 1,2,3,4,5 OR 6?

The HINT is the same as number 1.

V) GIVEN THE EQUATION AX*2 + CY*2 + DX + EY + F = 0 (A is the square of a random integer not equal to zero, C is the negative of the square of a random integer not equal to zero and D, E and F are combinations of these two and two other random integers so that the equation can always be converted to a standard form by the method of completing the square)

5)THE EQUATION IS EITHER .....

IS THE ANSWER 1,2,3,4,5 OR 6?

The HINT to this question is: THIS EQUATION CAN BE CONVERTED TO A STANDARD FORM BY THE METHOD OF COMPLETING THE SQUARE.

VI) GIVEN THE EQUATION AX*2 + CY*2 + 0X + 0Y +0 = 0 (A is a random integer greater than zero and C is a random integer less than zero)

6)THE EQUATION IS EITHER .....

IS THE ANSWER 1,2,3,4,5 OR 6?

The HINT is: THE CLUE TO THIS ONE IS THE COEFFICIENTS D, E AND F.

VII) GIVEN THE EQUATION AX*2 + CY*2 + DX + EY + F = 0 (A and C are the squares of two unequal random integers greater than zero and D, E and F are combinations of these two integers and two other random integers so that the above equation can always be converted to a standard form by the method of completing the square)

7)THE EQUATION IS EITHER .....

IS THE ANSWER 1,2,3,4,5 OR 6?

The HINT is the same as that of question number 5.

VIII)GIVEN THE EQUATION X*2 + Y*2 + DX + EY + F = 0   (D and E are two positive random integers and F is twice the product of these two integers)

8)THE EQUATION IS EITHER .....

IS THE ANSWER 1,2,3,4,5 OR 6?

The HINT is: THE CLUE TO THIS ONE IS THE COEFFICIENT F.

IX) IF A POINT P(X,Y) IS SUCH THAT ITS DISTANCE PF FROM A FIXED POINT (THE FOCUS) IS PROPORTIONAL TO ITS DISTANCE PD FROMA FIXED LINE (THE DIRECTRIX), THAT IS, SO THAT

PS = A×PF WHERE A IS A CONSTANT, THEN THE LOCUS OF P IS

1)AN ELLIPSE
2)A PARABOLA
3)A HYPERBOLA OR
4)NONE OF THESE

(A is either a number less than 1, but greater than 0, or 1, or an integer greater than 1)

9)IS THE ANSWER 1,2,3 OR 4?

The HINT for this question is: REMEMBER WHAT THE DEFINITION OF ECCENTRICITY IS.

IX. LIMITS

The description of LIMITS that is printed out to the user is below:

LIMITS

THIS SECTION IS CONCERNED WITH THE BEHAVIOR OF THE RATIO OF ALGEBRAIC EXPRESSIONS LIKE (X*2 -2X +4)÷(2X-5) AS X APPROACHES SOME NUMBER OR AS X APPROACHES PLUS OR MINUS INFINITY. IN SOME CASES THE BEHAVIOR OF THE RATIO CAN BE FOUND BY LOOKING AT THE BEHAVIOR OF THE NUMERATOR AND THE DENOMINATOR INDEPENDENTLY. THESE CASES OCCUR WHEN THE DENOMINATOR APPROACHES SOME REAL NONZERO NUMBER. IN THESE CASES THE LIMIT OF THE RATIO OF THE NUMERATOR, A, TO THE DENOMINATOR, B, IS A÷B.

OTHER CASES OCCUR WHEN THE DENOMINATOR APPROACHES ZERO OR WHEN THE DENOMINATOR IS UNBOUNDED(APPROACHES PLUS OR MINUS INFINITY). FOR THESE CASES WE HAVE THE FOLLOWING:

NOTE: LET N REPRESENT THE NUMERATOR, D THE DENOMINATOR, F THE RATIO N D AND A AND B REAL NUMBERS.

1) N→A≠0; D→0; F→PLUS OR MINUS INFINITY DEPENDING ON WHETHER A IS POSITIVE OR NEGATIVE.

2) N→0; D→0; F→0 IF THE DEGREE OF THE NUMERATOR IS GREATER THAN THE DEGREE OF THE DENOMINATOR.
F→ SOME NUMBER IF THE FACTORS APPROACHING ZERO IN THE NUMERATOR AND DENOMINATOR DIVIDE OUT.
F→ INFINITY IF THE DEGREE OF THE DENOMINATOR IS GREATER THAN THE DEGREE OF THE NUMERATOR.

3) N→+ OR -INFINITY; D→+ OR -INFINITY;
F→0 IF THE DEGREE OF THE DENOMINATOR IS GREATER THAN THE DEGREE OF THE NUMERATOR.
F→ SOME NUMBER IF THE NUMERATOR AND DENOMINATOR HAVE THE SAME DEGREE.
F→ INFINITY IF THE DEGREE OF THE NUMERATOR IS GREATER THAN THE DEGREE OF THE DENOMINATOR.

4) N→A; D→+ OR -INFINITY; F→0

THE DESIGNATION OF X→0 FROM THE LEFT OR THROUGH NEGATIVE NUMBERS WILL BE X→0- AND X→0+ WHEN IT APPROACHES 0 FROM THE RIGHT OR THROUGH POSITIVE NUMBERS.

The level of LIMITS is similar to that of CIRCLE, but the reference used was a calculus text by Fobes and Smythe[6] rather than Thomas[2].

There are six different problems which generate eighteen questions for each pass through the problems. On each recycle new coefficients are generated for each problem. A listing of LIMITS is given in Appendix C.

The problems and questions are listed below:

I) GIVEN THE EXPRESSION (X*2 +AX +B)÷(X*2 +CX +D) (A and B are the sum and product of two random integers, J and K, both not equal to zero and C and D, likewise, are the sum and product of two random integers, L and M, both not equal to zero and also not equal to J or K)

1)WHAT IS THE LIMIT AS X→0+ ?

The HINT for this question is: THE CLUE TO THIS ONE IS TO JUST EVALUATE THE EXPRESSION.

2)WHAT IS THE LIMIT AS X→ -J ? (J is the above random integer)

3)WHAT IS THE LIMIT AS X→ -M+ ? (M is the above random integer)

The HINT is: IF THE DENOMINATOR OF AN EXPRESSION GOES TO ZERO AND THE NUMERATOR DOES NOT, THEN THE EXPRESSION GROWS WITHOUT BOUND OR TENDS TO INFINITY.

4)WHAT IS THE LIMIT AS X→1 ?

The HINT is the same as that of question 1.

5)WHAT IS THE LIMIT AS X→ +INFINITY?

The HINT is: THE CLUE TO THIS ONE IS TO DIVIDE NUMERATOR AND DENOMINATOR BY X*2.

II) GIVEN THE EXPRESSION (X*2 +AX +B)÷(X*2 +CX +D) (A and B are the sum and product of two unequal random integers, J and K, both not equal to zero,and C and D are the sum and product of K and another random integer, L, not equal to zero and also not equal to J or K)

6)WHAT IS THE LIMIT AS X→ -K ? (K is the above random integer)

The HINT for this one is: THE CLUE IS TO FACTOR THE NUMERATOR AND DENOMINATOR AND SEE IF A TERM DIVIDES OUT.

7)WHAT IS THE LIMIT AS X→0 ?

The HINT is the same as question 1.

8)WHAT IS THE LIMIT AS X→ -L ? (L is the above random integer)

The HINT for this question is the same as that of question 3.

III) GIVEN THE EXPRESSION (AX*2 +BX +C)÷(DX*2 +EX +F) (A, C, D and F are random integers not equal to zero,and B and E are random integers greater than zero)

9)WHAT IS THE LIMIT AS X→ -INFINITY?

The HINT is the same as question number 5.

10)WHAT IS THE LIMIT AS X→0 ?

The HINT is the same as question 1.

IV) GIVEN THE EXPRESSION (AX-1)÷(X*2 +BX +C) (A is a random integer greater than one, and B and C are the sum of two random integers, J and K)

11)WHAT IS THE LIMIT AS X→ -J+ ?

The HINT for this question is the same as that of question 3.

12)WHAT IS THE LIMIT AS X→+INFINITY?

The HINT is the samw as question number 5.

V) GIVEN THE EXPRESSION (2X*2 +9X +A)÷(X+B) (A is a positive random integer, and B is a random integer not equal to zero)

13)WHAT IS THE LIMIT AS X→ -B+ ?

The HINT for this question is the same as that of question 3.

14)WHAT IS THE LIMIT AS X→ +INFINITY?

The HINT is the same as that of question 3.

15)WHAT IS THE LIMIT AS X→0 ?

The HINT is the same as question 1.

VI) GIVEN THE EXPRESSION 1÷(X-A)*3 (A is a positive random integer)

16)WHAT IS THE LIMIT AS X→ -A+ ?

The HINT is the same as question 1.

17)WHAT IS THE LIMIT AS X→ A+ ?

The HINT for this question is the same as question number 3.

18)WHAT IS THE LIMIT AS X→ A- ?

The HINT is the same as question 3.

APPENDIX B

A DESCRIPTION OF THE FUNCTIONS

The following are the names of the functions and a short description of what they do. A listing of each function is found in Appendix B.

I. AHINT

This function is dyadic. It is used when the hint for a question is a choice of one of four numbers but the numbers themselves are quotients. This function takes its two arguments, A and B, and produces one of the following strings:

'A÷B B÷A -A÷B -B÷A'

or

'B÷A -A÷B -B÷A A÷B'

Which string it produces is determined by a random number generator.

II. CHECK

This is the function called by all modules after each question is asked. It reads the user's reply, compares it against the predetermined replies and generates the appropriate response. It also calls the function SCORE when necessary.

III. DIV

This function is dyadic. It is used when the answer to a question involves a quotient, but the hint for the question is in the form of a comment. The function changes the two numbers, A and B, to the string 'A÷B', and if the user's reply is HELP this string is printed out as the answer.

IV. REPLY

This function prints out the following message:

```
 THE COMPUTER WILL ASK YOU QUESTIONS WHOSE ANSWERS ARE
NUMBERS.  YOU ARE TO TYPE IN THE NUMBER.  IF THE NUMBER IS
A FRACTION, HOWEVER, YOU ARE TO USE THE DIVIDE(÷) SYMBOL
AND NOT THE SLASH(/), SO ONE-HALF IS 1÷2 AND THREE-AND-
ONE-THIRD IS 10÷3 .
 IF YOUR REPLY IS INCORRECT THE COMPUTER WILL REPLY
     TRY AGAIN
AND YOU GET ANOTHER CHANCE.  IN FACT, YOU GET 3 TRIES AT
EACH QUESTION.
 IF YOU DO NOT KNOW THE ANSWER TO THE QUESTION YOU CAN TYPE
     HINT
AND EITHER (1) A COMMENT WILL BE PRINTED OR (2) A LIST OF
FOUR NUMBERS WILL BE PRINTED, WHERE ONE IS THE CORRECT ANSWER.
 IF YOU DO NOT HAVE THE SLIGHTEST IDEA OF WHAT THE ANSWER
IS TYPE
     HELP
AND THE ANSWER WILL BE GIVEN.
 IF YOU WANT TO KNOW HOW MANY QUESTIONS YOU HAVE BEEN
ASKED TYPE
     QNUMBER
 IF YOU WANT AN XY-AXIS PRINTED FOR SKETCHING FUNCTIONS
THEN TYPE
     GRAPH
 WHEN YOU WANT TO STOP THE EXERCISE TYPE
     STOP
AND A TABULATED RESULT OF YOUR REPLIES WILL BE GIVEN AND
THEN THE EXERCISE WILL TERMINATE.
```

V. RN1

This function generates a random integer from −9 to +9.

VI. RN2

This function generates a random integer from -9 to +9 exclusive of zero.

VII. RN3

This function is called only the module TRIG. It generates integer lengths of two sides of a right triangle. Six different combinations are possible thus giving six different triangles.

VIII. SCORE

This function is called by the function CHECK and tallies the different response's given by the user.

APPENDIX C

A LISTING OF THE MODULES

```
      ∇LINEQ[⎕]∇
    ∇ LINEQ
[1]   S←6⍴0
[2]   N←?1+¯1↑(60 60 60 60 ⊤⌶20)
[3]   N←0×1↑?(N⍴19)
[4]   'REPLY EITHER YES OR NO.'
[5]   'ARE YOU FAMILIAR WITH LINEAR EQUATIONS?'
[6]   R←⍞
[7]   →L9×⍳(1↑R)='Y'
[8]   '

            LINEAR EQUATIONS'
[9]   ' THE GRAPH OF A FIRST DEGREE EQUATION OF THE FORM
            DX+EY+F=0  ,'
[10]  'WHERE D,E AND F ARE REAL NUMBERS AND WHERE EITHER D
      OR E IS'
[11]  'NOT EQUAL TO ZERO, IS CALLED A STRAIGHT LINE.  SUCH
      AN'
[12]  'EQUATION IS CALLED A LINEAR EQUATION.'
[13]  ' THE EQUATION IS USUALLY WRITTEN IN THE FORM
            Y=(-D÷E)X+(-F÷E)  OR
            Y=MX+B  WHERE'
[14]  '      M=-D÷E  IS EQUAL TO THE SLOPE AND
            B=-F÷E IS EQUAL TO THE Y-INTERCEPT.'
[15]  ' THIS EQUATION IS SOMETIMES WRITTEN AS
            X=(-E÷D)Y+(-F÷D)  WHERE
            -F÷D IS EQUAL TO THE X-INTERCEPT.'
[16]  '
       THE SLOPE IS DEFINED TO BE THE RATIO
            M=(Y2-Y1)÷(X2-X1)'
[17]  'WHERE (X2,Y2) AND (X1,Y1) ARE TWO POINTS.  IF'
[18]  '(X2-X1) EQUALS ZERO THEN THE SLOPE IS SAID TO BE UND
      EFINED.'
[19] L9:'

      ARE YOU FAMILIAR WITH THE REPLIES OF THIS EXERCISE?'
[20]  R←⍞
[21]  →L0×⍳(1↑R)='Y'
[22]  REPLY
[23] L0:Y←RN2×X←(X*(M≠X←RN2)),M←RN2+1
[24]  'GIVEN 2 POINTS (X2,Y2)=(';1↑X;',';1↑Y;')  AND  (X1,Y
      1)=(';1↓X;',';1↓Y;')'
[25]  'WHAT IS THE SLOPE?'
```

```
[26]  ANS←(-/Y)÷-/X
[27]  H←'THE CLUE TO THIS ONE IS TO KNOW THE DEFINITION OF
      THE SLOPE.'
[28]  →0×⍳CHECK=1
[29]  →L1×⍳N=1
[30]  'WHAT IS THE Y-INTERCEPT?'
[31]  ANSA←ANS←(1↑Y)-ANS×(1↑X)
[32]  H←(?4)⌽H←(X-Y),(-ANS),ANS
[33]  →0×⍳CHECK=1
[34] L1:X←RN2+0×Y←RN1
[35]  'GIVEN Y=';X;'X+';Y
[36]  'WHAT IS THE SLOPE?'
[37]  H←(?4)⌽H←(-X),Y,(-Y),ANS←ANSA←X
[38]  →0×⍳CHECK=1
[39]  'WHAT IS THE Y-INTERCEPT?'
[40]  ANSA←ANS←Y
[41]  →0×⍳CHECK=1
[42]  X←¯5+⍳9
[43]  M←(?2)××RN2
[44]  G←(⌽X)∘.=(M×X)+Y←¯4+?7
[45]  ' BELOW IS THE GRAPH OF A LINE'
[46]  (18⍴ 1 0)\' ∘+∘'[G+1+2×0=(⌽X)∘.×X]
[47]  'WHICH INTERSECTS THE X-AXIS AT ';-Y÷M;' AND HAS A SL
      OPE=';M
[48]  'IF THE EQUATION IS OF THE FORM
             Y=_X+_
      WHAT NUMBER GOES IN THE FIRST BLANK?'
[49]  H←(?4)⌽H←Y,(-Y×M),(-Y),ANS←ANSA←M
[50]  →0×⍳CHECK=1
[51]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[52]  ANS←ANSA←Y
[53]  →0×⍳CHECK=1
[54]  X←RN1+0×Y←RN1+0×M←RN2
[55]  'GIVEN THE POINT(X,Y)=(';X;',';Y;') , A SLOPE OF ';M;
      ' AND'
[56]  'Y-_=_(X-_)'
[57]  'WHAT NUMBER GOES IN THE FIRST BLANK?'
[58]  H←(?4)⌽H←M,X,(X-Y),ANS←ANSA←Y
[59]  →0×⍳CHECK=1
[60]  'WHAT NUMBER GOES IN THE THIRD BLANK?'
[61]  ANS←ANSA←X
[62]  →0×⍳CHECK=1
[63]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[64]  ANS←ANSA←M
[65]  →0×⍳CHECK=1
[66]  M←RN2+0×X←RN1
[67]  'GIVEN A Y-INTERCEPT OF ';X;' , A SLOPE OF ';M;' AND'
[68]  'Y=_X+_'
[69]  'WHAT NUMBER GOES IN THE FIRST BLANK?'
```

```
[70]  H←(?4)⌽H←(-X),X,(M×X),ANS←ANSA←M
[71]  →0×⍳CHECK=1
[72]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[73]  ANS←ANSA←X
[74]  →0×⍳CHECK=1
[75]  X←RN2+0×Y←RN2+0×M←RN1
[76]  'GIVEN ';X;'X+';Y;'Y+';M;'=0'
[77]  'WHAT IS THE SLOPE?'
[78]  ANS←-X÷Y
[79]  H←'THE CLUE TO THIS ONE IS TO PUT THE EQUATION IN THE
       FORM OF Y=MX+B .'
[80]  ANSA←(-X) DIV Y
[81]  →0×⍳CHECK=1
[82]  'WHAT IS THE Y-INTERCEPT?'
[83]  ANS←-M÷Y
[84]  ANSA←(-M) DIV Y
[85]  →0×⍳CHECK=1
[86]  'WHAT IS THE X-INTERCEPT?'
[87]  ANS←-M÷X
[88]  H←'THE CLUE TO THIS ONE IS TO PUT THE EQUATION IN THE
       FORM X=_Y+_'
[89]  ANSA←(-M) DIV X
[90]  →0×⍳CHECK=1
[91]  X←¯5+⍳9
[92]  M←(?2)××RN2
[93]  G←(⌽X)∘.=(M×X)+Y←¯4+?7
[94]  ' BELOW IS THE GRAPH OF A LINE'
[95]  (18⍴ 1 0)\' ∘+∘'[G+1+2×0=(⌽X)∘.×X]
[96]  'WHICH INTERSECTS THE X-AXIS AT ';-Y÷M;' AND THE Y-AX
      IS AT ';Y
[97]  'IF  Y=_X+_
      WHAT NUMBER GOES IN THE FIRST BLANK?'
[98]  H←(?4)⌽H←Y,(-Y×M),(-Y),ANS←ANSA←M
[99]  →0×⍳CHECK=1
[100] 'WHAT NUMBER GOES IN THE SECOND BLANK?'
[101] ANS←ANSA←Y
[102] →0×⍳CHECK=1
[103] →L0
     ∇
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      ∇QUADEQ[⎕]∇
    ∇ QUADEQ
[1]   S←6⍴0
[2]   N←?1+¯1↑(60 60 60 60 ⊤⌶20)
[3]   N←0×1↑?(N⍴19)
[4]   'REPLY EITHER YES OR NO.'
[5]   'ARE YOU FAMILIAR WITH QUADRATIC EQUATIONS?'
[6]   R←⍞
[7]   →L9×⍳(1↑R)='Y'
[8]   '

           QUADRATIC EQUATIONS
       THE QUADRATIC EQUATION
           AX*2+BX+C=0  ,    A≠0'
[9]   'WHERE A,B, AND C ARE REAL NUMBERS, AND WHERE X IS
      EITHER A REAL OR A COMPLEX NUMBER, HAS TWO SOLUTIONS,
      NAMELY:'
[10]  '      X=(-B+(B*2-4AC)*.5)÷2A    AND
           X=(-B-(B*2-4AC)*.5)÷2A'
[11]  '
       THE QUANTITY (B*2-4AC)*.5 IS KNOWN AS THE DISCRIMINA
      NT AND IT
      DETERMINES THE KINDS OF ROOTS OF THE EQUATION.  IF TH
      E DISCRIMINANT IS:'
[12]  '       1) POSITIVE, THE TWO SOLUTIONS ARE REAL AND UNE
      QUAL.
           2) ZERO, THE TWO SOLUTIONS ARE REAL AND EQUAL.'
[13]  '       3) NEGATIVE, THE TWO SOLUTIONS ARE UNEQUAL AND
      COMPLEX.'
[14] L9:'

      ARE YOU FAMILIAR WITH THE REPLIES OF THIS EXERCISE?'
[15]  R←⍞
[16]  →L0×⍳(1↑R)='Y'
[17]  REPLY
[18] L0:X←RN1+0×Y←RN1
[19]  'GIVEN TWO ROOTS, X=';X;',';Y;'  AND
      X*2+_X+_=0
      WHAT NUMBER GOES IN THE FIRST BLANK?'
[20]  H←'THE QUADRATIC EQUATION,GIVEN TWO ROOTS IS FOUND BY
      MULTIPLYING (X-ROOT 1)×(X-ROOT 2).'
[21]  →0×⍳CHECK=1+0×ANS←ANSA←(-X)-Y
[22]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[23]  ANS←ANSA←(-X)×-Y
[24]  →0×⍳CHECK=1
[25]  X←RN1+0×Y←RN1
[26]  M←⌊/X,Y
[27]  G←⌈/X,Y
[28]  'GIVEN  X*2+';(-X)-Y;'X+';(-X)×-Y;'=0
      WHAT IS THE SMALLER ROOT OF THE EQUATION?'
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[29]  H←'THE SMALLER ROOT IS THE ONE WHICH IS FARTHER TO TH
      E LEFT ON THE
      REAL NUMBER LINE(-INFINITY....0....+INFINITY).'
[30]  →0×ιCHECK=1
[31]  'WHAT IS THE LARGER ROOT OF THE EQUATION?'
[32]  H←'THE LARGER ROOT IS THE ONE WHICH IS FARTHER TO THE
       RIGHT ON THE
      REAL NUMBER LINE(-INFINITY....0....+INFINITY).'
[33]  ANS←ANSA←G
[34]  →0×ιCHECK=1
[35]  X←RN2
[36]  'THE GRAPH OF ';X;'X*2+';RN1;'X+';RN1;'=0  OPENS EITH
      ER
           1)UPWARD  OR
           2)DOWNWARD'
[37]  'IS THE ANSWER 1 OR 2?'
[38]  ANS←ANSA←2-X>0
[39]  H←'IF THE COEFFICIENT OF THE X-SQUARED TERM IS >0 THE
       CURVE OPENS UPWARD;
      IF IT IS <0 IT OPENS DOWNWARD.'
[40]  →0×ιCHECK=1
[41]  X←RN2+0×Y←RN1+0×M←RN1
[42]  'GIVEN  ';X;'X*2+';Y;'X+';M;'=0
      THE SOLUTIONS ARE EITHER'
[43]  '      1)UNEQUAL AND COMPLEX
           2)REAL AND EQUAL
           3)REAL AND UNEQUAL
      IS THE ANSWER 1,2 OR 3?'
[44]  ANS←ANSA←2+×((Y*2)-4×X×M)
[45]  H←'THE VALUE OF THE DISCRIMINATE GIVES THE ANSWER TO
      THIS QUESTION.'
[46]  →0×ιCHECK=1
[47]  X←-|RN2+0×Y←|RN2
[48]  'GIVEN  X*2+';-X+Y;'X+';X×Y;'=0
      IF YOU WERE TO GRAPH THIS EQUATION WHERE WOULD IT CRO
      SS THE POSITIVE X-AXIS?'
[49]  H←(?4)⌽H←|X,|RN2,|(X×Y),ANS←ANSA←Y
[50]  →0×ιCHECK=1
[51]  'WHERE WOULD IT CROSS THE NEGATIVE X-AXIS?'
[52]  H←(?4)⌽H←(-X+Y),-|RN2,(-Y),ANS←ANSA←X
[53]  ANS←ANSA←X
[54]  →0×ιCHECK=1
[55]  →L0
    ∇
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```
     ∇TRIG[⎕]∇
    ∇ TRIG;H1;H2
[1]    S←6⍴0
[2]    N←?1+¯1↑(60 60 60 60 ⊤⌶20)
[3]    N←0×1↑?(N⍴19)
[4]    'REPLY EITHER YES OR NO.
       ARE YOU FAMILIAR WITH THE DEFINITIONS OF TRIGONOMETRY
       ?'
[5]    R←⍞
[6]    →L9×⍳(1↑R)='Y'
[7]    '

            TRIGONOMETRY
        GIVEN A RIGHT TRIANGLE IN ITS STANDARD POSITION IN T
       HE FIRST QUADRANT
       WITH THE ANGLE α AT THE ORIGIN.'
[8]    TRI1,TRI;'
       '; 3 5 ⍴'    +'
[9]    'PYTHAGORAS''S THEOREM STATES THAT
            Z*2 = X*2 + Y*2
       AND FROM THE DEFINITIONS OF TRIGONOMETRY'
[10]   '
        SIN α = SIDE OPPOSITE ÷ HYPOTENUSE = Y÷Z
        COS α = SIDE ADJACENT ÷ HYPOTENUSE = X÷Z
        TAN α = SIDE OPPOSITE ÷ SIDE ADJ.  = Y÷X
        CSC α = HYPOTENUSE    ÷ SIDE OPP.  = Z÷Y = 1÷SIN α
        SEC α = HYPOTENUSE    ÷ SIDE ADJ.  = Z÷X = 1÷COS α
        COT α = SIDE ADJACENT ÷ SIDE OPP.  = X÷Y = 1÷TAN α'
[11]   '
        THE SIGNS OF THE TRIG TERMS WOULD CHANGE IF THE TRIA
       NGLE WAS IN ANOTHER
       QUADRANT AND WOULD CHANGE ACCORDING TO THE CORRESPOND
       ING VALUES OF X, AND'
[12]   'Y IN THAT QUADRANT. THE TRIG TERMS, THEN, ARE CYCLIC
        AS α IS ROTATED
       FROM 0 TO 360 DEGREES.'
[13] L9:'
       ARE YOU FAMILIAR WITH THE REPLIES OF THIS EXERCISE?'
[14]   R←⍞
[15]   →L0×⍳(1↑R)='Y'
[16]   REPLY
[17] L0:Z←0×M←RN3
[18]   'GIVEN THE RIGHT TRIANGLE';TRI
[19]   'WHERE X=';X;' AND Y=';Y;'
       WHAT IS THE VALUE OF Z?'
[20]   H←H1←'THE CLUE TO THIS ONE IS PYTHAGORAS''S THEOREM.'
[21]   →0×⍳CHECK=1+0×ANS←ANSA←Z←M
[22]   'WHAT IS THE VALUE OF TAN α ?'
[23]   H←H2←'REFER BACK TO THE CORRESPONDING DEFINITION OF T
       HE TRIG TERM.'
[24]   ANSA←Y DIV X
[25]   →0×⍳CHECK=1+0×ANS←Y÷X
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[26]  'WHAT IS THE VALUE OF SIN α ?'
[27]  ANSA←Y DIV Z
[28]  →0×ιCHECK=1+0×ANS←Y÷Z
[29]  'WHAT IS THE VALUE OF COS ω ?'
[30]  →0×ιCHECK=1
[31]  'WHAT IS THE VALUE OF COS α ?'
[32]  ANSA←X DIV Z
[33]  →0×ιCHECK=1+0×ANS←X÷Z
[34]  'WHAT IS THE VALUE OF SIN ω ?'
[35]  →0×ιCHECK=1
[36]  'WHAT IS THE VALUE OF TAN ω ?'
[37]  ANSA←X DIV Y
[38]  →0×ιCHECK=1+0×ANS←X÷Y
[39]  'WHAT IS THE VALUE OF COT α ?'
[40]  →0×ιCHECK=1
[41]  'WHAT IS THE VALUE OF CSC α ?'
[42]  ANSA←Z DIV Y
[43]  →0×ιCHECK=1+0×ANS←Z÷Y
[44]  'WHAT IS THE VALUE OF SEC α ?'
[45]  ANSA←Z DIV X
[46]  →0×ιCHECK=1+0×ANS←Z÷X
[47]  Z←RN3
[48]  'GIVEN THE RIGHT TRIANGLE';TRI
[49]  'WHERE SIN α =';Y;'÷';Z;'  AND Y=';Y
[50]  I←1
[51] L2:'WHAT IS THE SIDE OPPOSITE α EQUAL TO?'
[52]  →0×ιCHECK=1+0×ANS←ANSA←Y
[53]  'WHAT IS THE HYPOTENUSE EQUAL TO?'
[54]  H←H1
[55]  →0×ιCHECK=1+0×ANS←ANSA←Z
[56]  'WHAT IS THE SIDE ADJACENT α EQUAL TO?'
[57]  H←H2
[58]  →0×ιCHECK=1+0×ANS←ANSA←X
[59]  →L3×ιI=3
[60]  'WHAT IS THE VALUE OF THE COS α ?'
[61]  H←H2
[62]  ANSA←X DIV Z
[63]  →0×ιCHECK=1+0×ANS←X÷Z
[64]  →L4×ιI=2
[65] L3:'WHAT IS THE VALUE OF THE TAN α ?'
[66]  ANSA←Y DIV X
[67]  →0×ιCHECK=1+0×ANS←Y÷X
[68]  →L6×ιI=1
[69] L4:'WHAT IS THE VALUE OF THE SIN α ?'
[70]  ANSA←Y DIV Z
[71]  →0×ιCHECK=1+0×ANS←Y÷Z
[72]  →L5×ιI=2
[73]  →L7×ιI=3
[74] L6:Z←RN3
[75]  'GIVEN THE RIGHT TRIANGLE';TRI
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[76]   'WHERE TAN α =';Y;'÷';X;'  AND X=';X
[77]   I←2
[78]   →L2
[79]  L5:I←3
[80]   Z←RN3
[81]   'GIVEN THE RIGHT TRIANGLE';TRI
[82]   'WHERE COS α =';X;'÷';Z;'  AND Z=';Z
[83]   →L2
[84]  L7:Z←|RN2
[85]   'WHAT IS THE VALUE OF COS(';Z;'×PI)?'
[86]   H←'REMEMBER THAT TRIG FUNCTIONS ARE CYCLIC SO YOU NEE
       D TO FIND OUT
       WHAT QUADRANT THIS WOULD BE IN.'
[87]   →0×⍳CHECK=1+0×ANSA←ANS←(1,¯1)[1+¯1↑(2 2 2 ⊤Z)]
[88]   Z←RN2
[89]   'WHAT IS THE VALUE OF SIN(';Z;'×PI)?'
[90]   →0×⍳CHECK=1+0×ANS←ANSA←0
[91]   Z←9×|RN2
[92]   'GIVEN A RIGHT TRIANGLE';TRI
[93]   'WITH THE ANGLE α =';Z;' DEGREES
       WHAT IS ANGLE ω EQUAL TO?'
[94]   H←'THE CLUE IS THAT THE SUM OF THE INNER ANGLES OF A
       TRIANGLE=180 DEGREES.'
[95]   →0×⍳CHECK=1+0×ANS←ANSA←90-Z
[96]   →L0
     ∇
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```
      ∇CIRCLE[⎕]∇
    ∇ CIRCLE
[1]   S←6ρ0
[2]   N←?1+¯1↑(60 60 60 60 ⊤I20)
[3]   N←0×1↑?(Nρ19)
[4]   'REPLY EITHER YES OR NO.'
[5]   'ARE YOU FAMILIAR WITH THE EQUATIONS OF A CIRCLE?'
[6]   R←⍞
[7]   →L9×⍳(1↑R)='Y'
[8]   '

           THE CIRCLE'
[9]   ' THE CIRCLE IS THE LOCUS OF POINTS IN A PLANE AT A G
      IVEN
      DISTANCE(CALLED THE RADIUS) FROM A GIVEN POINT(CALLED
       THE CENTER).'
[10]  ' IF C(H,K) IS THE CENTER OF THE CIRCLE AND R THE RAD
      IUS THEN
      THE EQUATION OF THE CIRCLE IS
           (X-H)*2 + (Y-K)*2 = R*2'
[11]  '
       AN EQUATION OF THE FORM
           AX*2 + AY*2 + DX + EY + F = 0 ,  A≠0'
[12]  'CAN OFTEN BE REDUCED TO THE EQUATION OF A CIRCLE BY
      THE METHOD
      OF COMPLETING THE SQUARES.'
[13]  ' IN THE ABOVE FORM IT WOULD LOOK LIKE
           (X+D÷2A)*2 + (Y+E÷2A)*2 = (D*2 + E*2 - 4AF) ÷ 4A
      *2'
[14]  ' REMEMBER, HOWEVER, THAT THE RIGHT HAND SIDE MUST BE
      POSITIVE FOR THE EQUATION OF A CIRCLE.'
[15] L9:'

      ARE YOU FAMILIAR WITH THE REPLIES OF THIS EXERCISE?'
[16]  R←⍞
[17]  →L0×⍳(1↑R)='Y'
[18]  REPLY
[19] L0:X←|RN2
[20]  'GIVEN A CIRCLE OF RADIUS ';X;' WITH ITS CENTER AT TH
      E ORIGIN
      AND THE EQUATION _X*2 +Y*2 = _'
[21]  'WHAT NUMBER GOES IN THE FIRST BLANK?'
[22]  H←(?4)⌽H←X,(X*2),(-X),ANS←ANSA←1
[23]  →0×⍳CHECK=1
[24]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[25]  →0×⍳CHECK=1+0×ANS←ANSA←X*2
[26] L1:X←RN1+0×Y←RN1+0×M←|RN2
[27]  'GIVEN (X-';X;')*2+(Y-';Y;')*2=';M*2
[28]  'WHAT IS THE X-COORDINATE OF THE CENTER OF THE CIRCLE
      ?'
[29]  H←(?4)⌽H←Y,M,(M*2),ANS←ANSA←X
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[30]  →0×⍳CHECK=1
[31]  'WHAT IS THE Y-COORDINATE OF THE CENTER OF THE CIRCLE
      ?'
[32]  →0×⍳CHECK=1+0×ANS←ANSA←Y
[33]  'WHAT IS THE RADIUS EQUAL TO?'
[34]  →0×⍳CHECK=1+0×ANS←ANSA←M
[35]  X←RN1+0×Y←RN1+0×M←|RN2
[36]  'GIVEN THE CENTER OF A CIRCLE AS (';X;',';Y;'), A RAD
      IUS OF ';M
[37]  'AND THE EQUATION (X-_)*2 + (Y-_)*2 = _'
[38]  'WHAT NUMBER GOES IN THE FIRST BLANK?'
[39]  H←(?4)⌽H←M,Y,(M*2),ANS←ANSA←X
[40]  →0×⍳CHECK=1
[41]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[42]  →0×⍳CHECK=1+0×ANS←ANSA←Y
[43]  'WHAT NUMBER GOES IN THE THIRD BLANK?'
[44]  →0×⍳CHECK=1+0×ANS←ANSA←M*2
[45]  M←(Y*2)+X×X←RN1+0×Y←RN2
[46]  'GIVEN A CIRCLE WHICH PASSES THROUGH THE ORIGIN, HAS
      A
      CENTER AT (';X;',';Y;') AND THE EQUATION'
[47]  '(X-_)*2 + (Y-_)*2 = _
      WHAT NUMBER GOES IN THE FIRST BLANK?'
[48]  H←(?4)⌽H←Y,M,(X+Y),ANS←ANSA←X
[49]  →0×⍳CHECK=1
[50]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[51]  →0×⍳CHECK=1+0×ANS←ANSA←Y
[52]  'WHAT NUMBER GOES IN THE THIRD BLANK?'
[53]  →0×⍳CHECK=1+0×ANS←ANSA←M
[54]  Z←RN1+0×M←RN1+0×G←|RN2
[55]  X←RN1+0×Y←RN1
[56]  'GIVEN THE EQUATION (X-';Z;')*2 + (Y-';M;')*2 =';G*
      2
[57]  'THE POINT (';X;',';Y;') IS EITHER
           1)INSIDE THE CIRCLE
           2)ON THE CIRCLE   OR
           3)OUTSIDE THE CIRCLE
      IS THE ANSWER 1,2 OR 3?'
[58]  H←' SUBSTITUTE THE VALUES OF X AND Y OF THE POINT INT
      O THE EQUATION
      AND MAKE A COMPARISON WITH THE RIGHT HAND SIDE.'
[59]  ANS←ANSA←2+×(((X-Z)*2)+(Y-M)*2)-G*2
[60]  →0×⍳CHECK=1
[61]  X←RN1+0×Y←RN2+0×M←-|RN1
[62]  'GIVEN THE EQUATION  X*2 + Y*2+';X;'X+';Y;'Y+';M;' =0
      '
[63]  'WHAT IS THE X-COORDINATE OF THE CENTER OF THE CIRCLE
      ?'
[64]  H←'THE CLUE TO THIS ONE IS THE METHOD OF COMPLETING T
      HE SQUARES.'
[65]  →0×⍳CHECK=1+0×ANS←ANSA←-X÷2
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[66]  'WHAT IS THE Y-COORDINATE OF THE CENTER OF THE CIRCLE
      ?'
[67]  →0×⍳CHECK=1+0×ANS←ANSA←-Y÷2
[68]  'WHAT IS THE RADIUS-SQUARED TERM EQUAL TO?'
[69]  ANSA←ANS←(((X*2)+Y*2)-4×M)÷4
[70]  H←'THE CLUE TO THIS ONE IS THE RIGHT HAND SIDE OF THE
       EQUATION AFTER THE
      METHOD OF COMPLETING THE SQUARES HAS BEEN APPLIED.'
[71]  →0×⍳CHECK=1
[72]  →L1
     ∇
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```
      ∇PARABOLA[⎕]∇
    ∇ PARABOLA;RH;HR
[1]   S←6ρ0
[2]   N←?1+¯1↑(60 60 60 60 ⊤I20)
[3]   N←0×1↑?(Nρ19)
[4]   'REPLY EITHER YES OR NO.'
[5]   'ARE YOU FAMILIAR WITH THE EQUATIONS OF PARABOLAS?'
[6]   R←⍞
[7]   →L9×⍳(1↑R)='Y'
[8]   '

           THE PARABOLA
       A PARABOLA IS THE LOCUS OF POINTS IN A PLANE EQUIDIS
      TANT FROM A
      POINT(CALLED THE FOCUS) AND A GIVEN LINE(CALLED THE D
      IRECTRIX).'
[9]   ' THE FOCUS IS ON THE AXIS OF SYMMETRY, P UNITS FROM
      THE VERTEX
      WHILE THE DIRECTRIX IS -P UNITS FROM THE VERTEX AND P
      ERPENDICULAR'
[10]  'TO THE AXIS OF SYMMETRY.
       IF V(H,K) IS THE VERTEX THEN THE EQUATION OF A PARAB
      OLA IS ONE OF
      THE FOLLOWING:'
[11]  '      1) (X-H)*2 =  4P(Y-K)  WHICH OPENS UPWARD
          2) (X-H)*2 = -4P(Y-K)  WHICH OPENS DOWNWARD'
[12]  '      3) (Y-K)*2 =  4P(X-H)  WHICH OPENS TO THE RIGHT
          4) (Y-K)*2 = -4P(X-H)  WHICH OPENS TO THE LEFT'
[13]  '
       THE CLUE TO AN EQUATION OF A PARABOLA IS THAT IT IS
      QUADRATIC IN ONE
      OF THE COORDINATES AND LINEAR IN THE OTHER.  WHENEVER
       THERE IS THIS'
[14]  'TYPE OF EQUATION IT CAN BE REDUCED TO ONE OF THE ABO
      VE STANDARD
      FORMS BY COMPLETING THE SQUARE IN THE COORDINATE WHIC
      H APPEARS QUADRATICALLY.'
[15] L9:'

      ARE YOU FAMILIAR WITH THE REPLIES OF THIS EXERCISE?'
[16]  R←⍞
[17]  →L0×⍳(1↑R)='Y'
[18]  REPLY
[19] L0:X←RN1+0×Y←Y×(M≠Y←RN1)+0×M←RN2
[20]  'GIVEN THE EQUATION (X-';X;')*2 =';4×M;'(Y-';Y;')
      WHAT IS THE X-COORDINATE OF THE VERTEX?'
[21]  H←(?4)⌽H←Y,M,(Y+M),ANS←ANSA←X
[22]  →0×⍳CHECK=1
[23]  'WHAT IS THE Y-COORDINATE OF THE VERTEX?'
[24]  →0×⍳CHECK=1+0×ANS←ANSA←Y
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[25]  'WHAT IS THE X-COORDINATE OF THE FOCUS?'
[26]  →0×⍳CHECK=1+0×ANS←ANSA←X
[27]  'WHAT IS THE Y-COORDINATE OF THE FOCUS?'
[28]  →0×⍳CHECK=1+0×ANS←ANSA←Y+M
[29]  'THE DIRECTRIX IS THE LINE Y=_
      WHAT NUMBER GOES IN THE BLANK?'
[30]  H←(?4)⌽H←(Y+M),(-M),Y,ANS←ANSA←Y-M
[31]  →0×⍳CHECK=1
[32]  ⎕←RH←'THE GRAPH OF THIS EQUATION OPENS EITHER
             1) UPWARD
             2) DOWNWARD
             3) TO THE RIGHT
             4) TO THE LEFT
      IS THE ANSWER 1,2,3 OR 4?'
[33]  H←HR←'THE CLUE TO THIS IS THE SIGN OF P AND THE QUADR
      ATIC TERM.'
[34]  →0×⍳CHECK=1+0×ANS←ANSA←2-M>0
[35]  Y←RN1+0×X←X*(M≠X←RN1)+0×M←RN2+1
[36]  'GIVEN THE VERTEX V(';X;',';Y;') AND FOCUS F(';M;',';
      Y;') OF A PARABOLA
      AND THE EQUATION (Y-_)*2 =_(X-_)
      WHAT NUMBER GOES IN THE FIRST BLANK?'
[37]  H←(?4)⌽H←X,M,(4×M-X),ANS←ANSA←Y
[38]  →0×⍳CHECK=1
[39]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[40]  →0×⍳CHECK=1+0×ANS←ANSA←4×M-X
[41]  'WHAT NUMBER GOES IN THE THIRD BLANK?'
[42]  →0×⍳CHECK=1+0×ANS←ANSA←X
[43]  'THE DIRECTRIX IS THE LINE X=_
      WHAT NUMBER GOES IN THE BLANK?'
[44]  H←(?4)⌽H←M,(X-M),X,ANS←ANSA←X-(M-X)
[45]  →0×⍳CHECK=1
[46]  RH
[47]  H←HR
[48]  →0×⍳CHECK=1+0×ANS←ANSA←4-(M-X)>0
[49]  X←RN1+0×Y←RN2+0×M←RN1
[50]  'GIVEN X*2 + ';2×X;'X + ';Y;'Y + ';M;' =0
      WHAT IS THE X-COORDINATE OF THE VERTEX?'
[51]  H←(?4)⌽H←X,(2×X),(¯2×X),ANS←ANSA←(-X)
[52]  →0×⍳CHECK=1
[53]  'WHAT IS THE Y-COORDINATE OF THE VERTEX?'
[54]  H←'THIS IS A HARD ONE.  TRY PLOTTING THIS ONE AND THE
      N SEE IF YOU CAN FIGURE IT OUT.'
[55]  →0×⍳CHECK=1+0×ANS←((-M)+X*2)÷Y
[56]  'WHAT IS THE DISTANCE FROM THE VERTEX TO THE FOCUS?'
[57]  H←Y AHINT 4
[58]  →0×⍳CHECK=1+0×ANS←|Y÷4
[59]  RH
[60]  H←HR
[61]  →0×⍳CHECK=1+0×ANS←ANSA←2-(-Y)>0
[62]  →L0
    ∇
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      ∇ELLIPSE[⎕]∇
    ∇ ELLIPSE;H1;H2;H3;H4
[1]   S←6⍴0
[2]   N←?1+¯1↑(60 60 60 60 ⊤⍳20)
[3]   N←0×1↑?(N⍴19)
[4]   'REPLY EITHER YES OR NO.
      ARE YOU FAMILIAR WITH THE EQUATIONS OF ELLIPSES?'
[5]   R←⍞
[6]   →L9×⍳(1↑R)='Y'
[7]   '

           THE ELLIPSE
       AN ELLIPSE IS THE LOCUS OF POINTS P(X,Y) THE SUM OF
      WHOSE DISTANCES
      FROM TWO FIXED POINTS(CALLED FOCI) IS CONSTANT.  THE
      FOCI ARE ALWAYS'
[8]   'ON THE MAJOR AXIS.  IF WE USE THE LETTERS A, B AND C
       TO REPRESENT
      THE LENGTHS OF SEMIMAJOR AXIS, SEMIMINOR AXIS AND HAL
      F-DISTANCE BETWEEN'
[9]   'FOCI, RESPECTIVELY, THEN THE FOLLOWING EQUALITY HOLD
      S:
           A*2 = B*2 + C*2
       IF P(H,K) IS THE CENTER, DEFINED AS THE POINT OF INT
      ERSECTION OF ITS'
[10]  'AXES OF SYMMETRY, OF AN ELLIPSE THEN THE EQUATION OF
       THE ELLIPSE
      IS GIVEN BY
           (X-H)*2 ÷ A*2  +  (Y-K)*2 ÷ B*2 = 1  OR
           (X-H)*2 ÷ B*2  +  (Y-K)*2 ÷ A*2 = 1'
[11]  'DEPENDING ON THE DIRECTION OF THE MAJOR AXIS.

       THE ECCENTRICITY OF AN ELLIPSE IS THE RATIO
           E=C÷A
      AND INDICATES THE DEGREE OF DEPARTURE FROM CIRCULARIT
      Y. KEEPING A FIXED'
[12]  'AND VARYING C FROM 0 TO 1 , THE RESULTING ELLIPSE WI
      LL VARY IN SHAPE,
      BEING CIRCULAR WHEN C=0 AND BECOMING FLATTER AS C INC
      RESES, UNTIL AT C=A'
[13]  'THE ELLIPSE REDUCES TO A LINE SEGMENT JOINING THE TW
      O FOCI.'
[14] L9:'

      ARE YOU FAMILIAR WITH THE REPLIES OF THIS EXERCISE?'
[15]  R←⍞
[16]  →L0×⍳(1↑R)='Y'
[17]  REPLY
[18] L0:X←X×(Y≠X←|RN2)+0×Y←1+|RN2
[19]  'GIVEN AN ELLIPSE WITH ITS CENTER AT THE ORIGIN,INTER
      SECTS
      THE POSITIVE X-AXIS AT ';X;' AND THE POSITIVE Y-AXIS
      AT ';Y
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[20]  'AND THE STANDARD EQUATION
            X*2 ÷ _ + Y*2 ÷ _ =1
      WHAT NUMBER GOES IN THE FIRST BLANK?'
[21]  H←(?4)⌽H←(RN2*2),(Y*2),1,ANS←ANSA←X*2
[22]  →0×⍳CHECK=1
[23]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[24]  H←(?4)⌽H←(RN2*2),(RN2*2),1,Y*2
[25]  →0×⍳CHECK=1+0×ANS←ANSA←Y*2
[26] L2:G←|RN2+0×M←|RN2+0×X←RN1+0×Y←RN1
[27]  'GIVEN AN ELLIPSE WITH ITS CENTER AT C(';X;',';Y;') A
       FOCUS
      AT F(';X+M;',';Y;') AND SEMIMAJOR AXIS A=';M+G;' AND
      THE EQUATION'
[28]  '       (X-_)*2 ÷ _ + (Y-_)*2 ÷ _ =1
      WHAT NUMBER GOES IN THE FIRST BLANK?'
[29]  H←H2←'THIS ONE IS SO EASY THAT YOU REALLY DON''T NEED
       A HINT, BUT THE
      CLUE IS THAT YOU ARE JUST TRANSLATING THE AXES TO THE
       CENTER OF THE ELLIPSE.'
[30]  →0×⍳CHECK=1+0×ANS←ANSA←X
[31]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[32]  H←(?4)⌽H←(RN2*2),(X*2),((X+M)*2),ANS←ANSA←(M+G)*
      2
[33]  →0×⍳CHECK=1
[34]  'WHAT NUMBER GOES IN THE THIRD BLANK?'
[35]  H←H2
[36]  →0×⍳CHECK=1+0×ANS←ANSA←Y
[37]  'WHAT NUMBER GOES IN THE FOURTH BLANK?'
[38]  H←H3←'THE CLUE IS THAT A*2=B*2 + C*2, AND YOU SHOULD
      KNOW A AND C.'
[39]  →0×⍳CHECK=1+0×ANS←ANSA←((M+G)*2)-M*2
[40]  'WHAT IS THE ECCENTRICITY OF THIS ELLIPSE?'
[41]  H←'YOU HAVE TO RECALL THAT E=C÷A.        '
[42]  →0×⍳CHECK=1+0×ANS←ANSA←M÷(M+G)
[43]  M←M*(G≠M←|RN2)+0×G←1+|RN2+0×X←RN1+0×Y←RN1
[44]  'GIVEN THE EQUATION ';G*2;'X*2 + ';M*2;'Y*2 + ';G×G×
      2×-X;'X+ ';M×M×2×-Y;'Y+';-(M×M×G×G)+(-G×G×X*
      2)-M×M×Y*2;'=0'
[45]  'WHAT IS THE X-COORDINATE OF THE CENTER OF THE ELLIPS
      E?'
[46]  H←'THIS EQUATION CAN BE CONVERTED TO THE STANDARD FOR
      M BY THE METHOD OF COMPLETING THE SQUARES.'
[47]  →0×⍳CHECK=1+0×ANS←ANSA←X
[48]  'WHAT IS THE Y-COORDINATE OF THE CENTER OF THE ELLIPS
      E?'
[49]  →0×⍳CHECK=1+0×ANS←ANSA←Y
[50]  'WHAT IS THE LENGTH OF THE MAJOR AXIS?'
[51]  H←H1←'THE MAJOR AXIS IS JUST TWICE THE LENGTH OF THE
      SEMI-MAJOR AXIS A.'
[52]  →0×⍳CHECK=1+0×ANS←ANSA←2×⌈/M,G
[53]  'WHAT IS THE LENGTH OF THE MINOR AXIS?'
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[54]  H←H4←'THE MINOR AXIS IS JUST TWICE THE LENGTH OF THE
      SEMI-MINOR AXIS B.'
[55]  →0×⍳CHECK=1+0×ANS←ANSA←2×⌊/M,G
[56]  M←M*(G≠M←|RN2)+0×G←1+|RN2+0×X←RN1+0×Y←RN1
[57]  'GIVEN THE EQUATION (X-';X;')*2 ÷ ';M*2;' + (Y-';Y;')
      *2 ÷ ';G*2;' =1'
[58]  'WHAT IS THE LENGTH OF THE MAJOR AXIS?'
[59]  H←H1
[60]  →0×⍳CHECK=1+0×ANS←ANSA←2×⌈/M,G
[61]  'WHAT IS THE LENGTH OF THE MINOR AXIS?'
[62]  H←H4
[63]  →0×⍳CHECK=1+0×ANS←ANSA←2×⌊/M,G
[64]  'WHAT IS THE SQUARE OF THE DISTANCE FROM THE CENTER O
      F THE ELLIPSE TO A FOCUS?'
[65]  H←'REMEMBER THAT A*2 = B*2 + C*2 AND YOU SHOULD KNOW
      A  AND B.'
[66]  →0×⍳CHECK=1+0×ANS←ANSA←|(M*2)-(G*2)
[67]  X←|RN2+0×Y←RN2
[68]  'GIVEN AN ELLIPSE WHICH PASSES THROUGH THE ORIGIN, HA
      S FOCI
      AT (';-X;',';Y;') AND (';X;',';Y;') AND THE EQUATION
      (X-_)*2 ÷ _ + (Y-_)*2 ÷ _ = 1'
[69]  'WHAT NUMBER GOES IN THE FIRST BLANK?'
[70]  H←(?4)⌽H←X,(Y*2),RN2,ANS←ANSA←0
[71]  →0×⍳CHECK=1
[72]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[73]  H←(?4)⌽H←(Y*2),X,RN2,ANS←ANSA←(X*2)+Y*2
[74]  →0×⍳CHECK=1
[75]  'WHAT NUMBER GOES IN THE THIRD BLANK?'
[76]  H←(?4)⌽H←Y,(X*2),RN2,ANS←ANSA←Y
[77]  →0×⍳CHECK=1
[78]  'WHAT NUMBER GOES IN THE FOURTH BLANK?'
[79]  H←(?4)⌽H←(X*2),Y,RN2,ANS←ANSA←Y*2
[80]  →0×⍳CHECK=1
[81]  →L0
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      ∇HYPERBOLA[⎕]∇
    ∇ HYPERBOLA;H1;H2;H3;H4
[1]   S←6ρ0
[2]   N←?1+¯1↑(60 60 60 60 ⊤I20)
[3]   N←0×1↑?(Nρ19)
[4]   'REPLY EITHER YES OR NO.
      ARE YOU FAMILIAR WITH THE EQUATIONS OF HYPERBOLAS?'
[5]   R←⍞
[6]   →L9×ι(1↑R)='Y'
[7]   '

           THE HYPERBOLA
       A HYPERBOLA IS THE LOCUS OF POINTS P(X,Y) WHOSE DIFF
      ERENCE OF ITS DISTANCES FROM
      TWO FIXED POINTS(CALLED FOCI) IS CONSTANT.  THE FOCI
      LIE ON ONE OF THE AXES OF'
[8]   'SYMMETRY.  IF WE USE THE LETTERS A AND C TO REPRESEN
      T THE LENGTHS OF HALF-DISTANCE
      BETWEEN THE VERTICES AND THE HALF-DISTANCE BETWEEN FO
      CI, RESPECTIVELY, THEN'
[9]   'THE FOLLOWING EQUALITY HOLDS:
           C*2 - A*2 = B*2   OR
           C*2 = A*2 + B*2'
[10]  'IF P(H,K) IS THE CENTER, DEFINED AS THE POINT OF INT
      ERSECTION OF ITS
      AXES OF SYMMETRY, OF A HYPERBOLA THEN THE EQUATION OF
       THE HYPERBOLA IS GIVEN BY
           (X-H)*2 ÷ A*2  -  (Y-K)*2 ÷ B*2 = 1
           (Y-K)*2 ÷ A*2  -  (X-H)*2 ÷ B*2 = 1'
[11]  'DEPENDING ON WHETHER THE FOCI ARE LOCATED ON THE X-A
      XIS OR Y-AXIS, RESPECTIVELY.
      LIKEWISE, THE STRAIGHT LINES'
[12]  '      (Y-K) = (B÷A)(X-H)  AND  (Y-K) = (-B÷A)(X-H)  O
      R
           (Y-K) = (A÷B)(X-H)  AND  (Y-K) = (-A÷B)(X-H)'
[13]  'ARE CALLED THE ASYMPTOTES OF THE HYPERBOLA, DEPENDIN
      G ON WHICH RESPECTIVE
      AXIS THE FOCI ARE LOCATED.'
[14] L9:'

      ARE YOU FAMILIAR WITH THE REPLIES OF THIS EXERCISE?'
[15]  R←⍞
[16]  →L0×ι(1↑R)='Y'
[17]  REPLY
[18] L0:'THE GRAPH OF THE EQUATION  X*2 ÷ ';RN2;' - Y*2 ÷ '
      ;RN2;' = 1
      OPENS EITHER
           1) UPWARD AND DOWNWARD   OR
           2) TO THE RIGHT AND TO THE LEFT.'
[19]  'IS THE ANSWER 1 OR 2?'
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[20]  H←'IF THE NEGATIVE SIGN IS BEFORE THE X-SQUARED TERM
      THEN THE CURVE OPENS UPWARD AND DOWNWARD AND
      IF THE NEGATIVE SIGN IS BEFORE THE Y-SQUARED TERM THE
      N THE CURVE OPENS TO THE RIGHT AND TO THE LEFT.'
[21]  →0×⍳CHECK=1+0×ANS←ANSA←2
[22] L1:X←(|RN2)+Y←|RN2
[23]  'GIVEN A HYPERBOLA WITH ITS CENTER AT THE ORIGIN, FOC
      I AT (';X;',0) AND (';-X;',0),
      VERTICES AT (';Y;',0) AND (';-Y;'0) AND THE EQUATION
      '
[24]  '(X-_)*2 ÷ _  -  (Y-_)*2 ÷ _  = _'
[25]  →L2×⍳N>2
[26]  'WHAT NUMBER GOES IN THE FIRST BLANK?'
[27]  H←(?4)⌽H←RN2,X,(Y*2),ANS←ANSA←0
[28]  →0×⍳CHECK=1
[29] L2:'WHAT NUMBER GOES IN THE SECOND BLANK?'
[30]  H←H1←'REMEMBER THAT A IS THE HALF-DISTANCE BETWEEN TH
      E VERTICES.'
[31]  →0×⍳CHECK=1+0×ANS←ANSA←Y*2
[32]  →L3×⍳N>4
[33]  'WHAT NUMBER GOES IN THE THIRD BLANK?'
[34]  H←(?4)⌽H←RN2,Y,(X*2),ANS←ANSA←0
[35]  →0×⍳CHECK=1
[36] L3:'WHAT NUMBER GOES IN THE FOURTH BLANK?'
[37]  H←H2←'REMEMBER THAT C*2 = A*2 + B*2  AND YOU SHOULD K
      NOW A AND C.'
[38]  →0×⍳CHECK=1+0×ANS←ANSA←(X*2)-Y*2
[39]  →L4×⍳N>6
[40]  'WHAT NUMBER GOES IN THE LAST BLANK?'
[41]  H←'THE EQUATION OF A HYPERBOLA IN ITS STANDARD FORM I
      S ALWAYS EQUAL TO 1.'
[42]  →0×⍳CHECK=1+0×ANS←ANSA←1
[43] L4:'WHAT IS THE ECCENTRICITY OF THE HYPERBOLA?'
[44]  H←H3←'THE ECCENTRICITY IS EQUAL TO C÷A.'
[45]  ANSA←X DIV Y
[46]  →0×⍳CHECK=1+0×ANS←X÷Y
[47]  M←RN1+0×G←RN1+0×X←|RN2+0×Y←|RN2
[48]  'GIVEN THE EQUATION ';Y*2;'(X-';M;')*2-';X*2;'(Y-';G;
      ')*2 = ';X×X×Y×Y
[49]  'WHAT IS THE LENGTH BETWEEN THE TWO VERTICES?'
[50]  H←H1
[51]  →0×⍳CHECK=1+0×ANS←ANSA←X×2
[52]  'WHAT IS THE LENGTH BETWEEN THE FOCI SQUARED EQUAL TO
      ?'
[53]  H←'REMEMBER THAT C*2=A*2 + B*2 AND YOU SHOULD KNOW A
      AND B.'
[54]  →0×⍳CHECK=1+0×ANS←ANSA←(X×X)+Y×Y
[55]  'WHAT IS THE POSITIVE SLOPE OF THE ASYMPTOTES?'
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[56]  H←Y AHINT X
[57]  H←H4←'THE POSITIVE SLOPE IS B÷A IF THE FOCI ARE ON TH
      E X-AXIS AND A÷B IF THE FOCI ARE ON THE Y-AXIS.'
[58]  →0×⍳CHECK=1+0×ANS←Y÷X
[59]  'WHAT IS THE X-COORDINATE OF THE INTERSECTION OF THE
      TWO ASYMPTOTES?'
[60]  H←'THE CLUE IS THAT THE ASYMPTOTES INTERSECT AT THE C
      ENTER OF THE HYPERBOLA.'
[61]  →0×⍳CHECK=1+0×ANS←ANSA←M
[62]  'WHAT IS THE Y-COORDINATE OF THE INTERSECTION OF THE
      TWO ASYMPTOTES?'
[63]  →0×⍳CHECK=1+0×ANS←ANSA←G
[64]  X←|RN2+0×M←(|RN2)+Y←|RN2
[65]  'GIVEN A HYPERBOLA WITH FOCI AT (';X;',';M;') AND (';
      X;',';-M;') AND
      VERTICES AT (';X;',';Y;') AND (';X;',';-Y;') AND THE
      EQUATION
      (X-_)*2 ÷ _   -  (Y-_)*2 = 1'
[66]  'WHAT NUMBER GOES IN THE FIRST BLANK?'
[67]  H←(?4)⌽H←RN2,(Y*2),0,ANS←ANSA←X
[68]  →0×⍳CHECK=1
[69]  'WHAT NUMBER GOES IN THE SECOND BLANK?'
[70]  H←H1
[71]  →0×⍳CHECK=1+0×ANS←ANSA←Y*2
[72]  'WHAT NUMBER GOES IN THE THIRD BLANK?'
[73]  H←(?4)⌽H←RN2,(M*2),Y,ANS←ANSA←0
[74]  →0×⍳CHECK=1
[75]  'WHAT NUMBER GOES IN THE FOURTH BLANK?'
[76]  H←H2
[77]  →0×⍳CHECK=1+0×ANS←ANSA←(M*2)-Y*2
[78]  'WHAT IS THE ECCENTRICITY OF THE HYPERBOLA?'
[79]  H←H3
[80]  →0×⍳CHECK=1+0×ANS←ANSA←M÷Y
[81]  M←|RN2+0×G←|RN2+0×X←RN1+0×Y←RN1
[82]  'GIVEN THE EQUATION ';G*2;'X*2 -';M*2;'Y*2 +';-
      2×G×G×X;'X +';2×Y×M*2;'Y +';(G×G×X×X)+(-M×M×Y×Y)-G×G×
      M×M;' = 0'
[83]  'WHAT IS THE X-COORDINATE OF THE CENTER OF THE HYPERB
      OLA?'
[84]  H←'THIS EQUATION CAN BE CONVERTED TO THE STANDARD FOR
      M BY THE METHOD OF COMPLETING THE SQUARES.'
[85]  →0×⍳CHECK=1+0×ANS←ANSA←X
[86]  'WHAT IS THE Y-COORDINATE OF THE CENTER OF THE HYPERB
      OLA?'
[87]  →0×⍳CHECK=1+0×ANS←ANSA←Y
[88]  'WHAT IS THE POSITIVE SLOPE OF THE ASYMPTOTES?'
[89]  H←H4
[90]  ANSA←G DIV M
[91]  →0×⍳CHECK=1+0×ANS←G÷M
[92]  →L1
    ∇
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      ∇CONICS[⎕]∇
    ∇ CONICS;R1;R2;X2;Y2;X;Y;F;H1;M;G;V;I
[1]   S←6⍴0
[2]   N←?1+¯1↑(60 60 60 60 ⊤I20)
[3]   N←0×1↑?(N⍴19)
[4]   R1←'GIVEN THE EQUATION '
[5]   R2←'THE EQUATION IS EITHER
           1)A STRAIGHT LINE
           2)A CIRCLE
           3)A PARABOLA
           4)AN ELLIPSE
           5)A HYPERBOLA
           6)NONE OF THESE
      IS THE ANSWER 1,2,3,4,5 OR 6?'
[6]   X2←'X*2 +'
[7]   Y2←'Y*2 +'
[8]   X←'X +'
[9]   Y←'Y +'
[10]  F←' = 0 .  '
[11]  H1←'THE CLUE TO THIS ONE IS THE COEFFICIENTS A AND C.
      '
[12]  I←(|RN2)?9
[13]  'REPLY EITHER YES OR NO.
      ARE YOU FAMILIAR WITH THE EQUATIONS OF CONICS?'
[14]  R←⍞
[15]  →L15×⍳(1↑R)='Y'
[16]  '

           CONICS
       THE GRAPHS OF THE CIRCLE,PARABOLA,ELLIPSE AND THE HY
      PERBOLA ARE CALLED CONICS.
      THE PARABOLA,ELLIPSE AND HYPERBOLA HAVE A COMMON FEAT
      URE CALLED THE ''FOCUS-AND-'
[17]  'DIRECTRIX'' PROPERTY SINCE EACH IS DETERMINED BY A G
      IVEN POINT,CALLED THE
      FOCUS, A GIVEN LINE, CALLED THE DIRECTRIX, AND A POSI
      TIVE CONSTANT E, CALLED THE'
[18]  'ECCENTRICITY. THE CONIC WILL BE A
           1) PARABOLA IF E=1, OR AN
           2) ELLIPSE IF E<1,≠0 AND A
           3) HYPERBOLA IF E>1'
[19]  ' THE CIRCLE,PARABOLA,ELLIPSE AND HYPERBOLA ARE ALL S
      PECIAL CASES OF THE
      FOLLOWING GENERAL EQUATION OF THE SECOND DEGREE:'
[20]  '      AX*2 + BXY + CY*2 + DX + EY + F = 0
      SINCE AXES MAY BE ROTATED TO ELIMINATE THE CROSS-PROD
      UCT TERM XY, THERE IS NO'
[21]  'LOSS IN GENERALITY IN ASSUMING THIS HAS BEEN DONE SO
       THE EQUATION LOOKS
      LIKE
           AX*2 + CY*2 + DX + EY + F = 0'
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[22] ' THIS EQUATION,THEN,REPRESENTS
          1)A STRAIGHT LINE IF A=C=0, AND NOT BOTH D AND E
       VANISH.
          2)A CIRCLE IF A=C≠0.(IN SPECIAL CASES THE LOCUS
      MAY REDUCE TO A SINGLE POINT,OR NO REAL LOCUS).'
[23]  '      3)A PARABOLA IF THE EQUATION IS QUADRATIC IN ON
      E VARIABLE AND LINEAR IN THE OTHER
          4)AN ELLIPSE IF A AND C ARE BOTH POSITIVE OR BOT
      H NEGATIVE.(AGAIN IN SPECIAL CASES
      THE LOCUS MAY REDUCE TO A SINGLE POINT OR NO REAL LOC
      US)'
[24]  '      5)A HYPERBOLA IF A AND C ARE OF OPPOSITE SIGNS,
      BOTH DIFFERENT FROM ZERO
      (IN SPECIAL CASES THE LOCUS MAY REDUCE TO A PAIR OF I
      NTERSECTING STRAIGHT LINES).'
[25] L15:'

      ARE YOU FAMILIAR WITH THE REPLIES OF THIS EXERCISE?'
[26]  R←⍞
[27]  →L10×⍳(1↑R)='Y'
[28]  REPLY
[29] L10:J←0
[30]  I←9?9
[31]  I←I,10
[32] L0:→(L1,L2,L3,L4,L5,L6,L7,L8,L9,L10)[I[J←J+1]]
[33] L1:R1;0;X2;0;Y2;RN2;X;RN2;Y;RN1;F;R2
[34]  H←H1
[35]  →L0×CHECK=0×ANS←ANSA←1
[36] L2:R1;M;X2;M←|RN2;Y2;|RN1;X;|RN1;Y;-|RN2;F;R2
[37]  H←H1
[38]  →L0×CHECK=0×ANS←ANSA←2
[39] L3:R1;0;X2;RN2;Y2;RN2;X;0;Y;RN2;F;R2
[40]  H←'THE CLUE TO THIS ONE IS THE COEFFICIENTS A AND E.'
[41]  →L0×CHECK=0×ANS←ANSA←3
[42] L4:G←G*(M≠G←|RN2)+0×M←1+|RN2
[43]  R1;G;X2;M;Y2;0;X;0;Y;-|RN2;F;R2
[44]  H←H1
[45]  →L0×CHECK=0×ANS←ANSA←4
[46] L5:M←|RN2+0×G←|RN2+0×V←RN1+0×Z←RN1
[47]  R1;G*2;X2;-M*2;Y2;¯2×V×G*2;X;2×Z×M*2;Y;(G×G×V×V)+(-M×
      M×Z*2)-G×G×M*2;F;R2
[48]  H←'THIS EQUATION CAN BE CONVERTED TO A STANDARD FORM
      BY THE METHOD OF COMPLETING THE SQUARES.'
[49]  →L0×CHECK=0×ANS←ANSA←5
[50] L6:R1;|RN2;X2;-|RN2;Y2;0;X;0;Y;0;F;R2
[51]  H←'THE CLUE TO THIS ONE IS THE COEFFICIENTS D,E AND F
      .'
[52]  →L0×CHECK=0×ANS←ANSA←6
[53] L7:M←M*(G≠M←|RN2)+0×G←1+|RN2+0×V←RN1+0×Z←RN1
[54]  R1;G*2;X2;M*2;Y2;¯2×V×G*2;X;¯2×Z×M*2;Y;-(M×M×G*
      2)+(-G×G×V*2)-M×M×Z*2;F;R2
[55]  H←'THIS CAN BE CONVERTED TO A STANDARD FORM BY THE ME
      THOD OF COMPLETING THE SQUARES.'
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[56]  →L0×CHECK=0×ANS←ANSA←4
[57] L8:V←|RN2+0×Z←|RN2
[58]  R1;X2;Y2;V;X;Z;Y;2×V×Z;F;R2
[59]  H←'THE CLUE TO THIS ONE IS THE COEFFICIENT F.'
[60]  →L0×CHECK=0×ANS←ANSA←6
[61] L9:M←((0.1×|RN2),1,(1+|RN2))[Z←?3]
[62]  'IF A POINT P(X,Y) IS SUCH THAT ITS DISTANCE PF FROM
      A FIXED POINT(THE FOCUS)
      IS PROPORTIONAL TO ITS DISTANCE PD FROM A FIXED LINE(
      THE DIRECTRIX),THAT IS, SO THAT:'
[63]  '     PF = ';M;'×PD  WHERE ';M;' IS A CONSTANT, THEN
      THE LOCUS OF P IS'
[64]  '      1)AN ELLIPSE
         2)A PARABOLA
         3)A HYPERBOLA  OR
         4)NONE OF THESE
      IS THE ANSWER 1,2,3 OR 4?'
[65]  H←'REMEMBER WHAT THE DEFINITION OF ECCENTRICITY IS.'
[66]  →L0×CHECK=0×ANS←ANSA←Z
     ∇
```

```
      ∇LIMITS[⎕]∇
    ∇ LIMITS;H1;H2;H3;H4
[1]   S←6⍴0
[2]   N←?1+¯1↑(60 60 60 60 ⊤⌶20)
[3]   N←0×1↑?(N⍴19)
[4]   N←0×1↑((|RN2)?9)
[5]   'REPLY EITHER YES OR NO.
      ARE YOU FAMILIAR WITH LIMITS OF RATIOS OF POLYNOMIALS
      ?'
[6]   R←⍞
[7]   →L9×⍳(1↑R)='Y'
[8]   '

             LIMITS
       THIS SECTION IS CONCERNED WITH THE BEHAVIOR OF THE R
      ATIO OF ALGEBRAIC
      EXPRESSIONS LIKE (X*2 -2X +4)÷(2X-5) AS X APPROACHES
      SOME NUMBER OR AS
      X APPROACHES PLUS OR MINUS INFINITY.  IN SOME CASES T
      HE BEHAVIOR OF THE'
[9]   'RATIO CAN BE FOUND BY LOOKING AT THE BEHAVIOR OF THE
       NUMERATOR AND
      THE DENOMINATOR INDEPENDENTLY.  THESE CASES OCCUR WHE
      N THE DENOMINATOR
      APPROACHES SOME REAL NONZERO NUMBER.  IN THESE CASES
     THE LIMIT OF THE RATIO OF'
[10]  'THE NUMBERATOR, A, TO THE DENOMINATOR, B, IS A÷B.
       OTHER CASES OCCUR WHEN THE DENOMINATOR APPROACHES ZE
      RO OR WHEN THE
      DENOMINATOR IS UNBOUNDED(APPROACHES PLUS OR MINUS INF
      INITY).  FOR THESE
      CASES WE HAVE THE FOLLOWING:'
[11]  ' NOTE: LET N REPRESENT THE NUMERATOR, D THE DENOMINA
      TOR, F THE RATIO
      N÷D AND A AND B REAL NUMBERS.'
[12]  '      1) N→A≠0; D→0; F→PLUS OR MINUS INFINITY DEPENDI
      NG ON WHETHER
      A IS POSITIVE OR NEGATIVE.'
[13]  '
          2) N→0; D→0; F→0 IF THE DEGREE OF THE NUMERATOR
      IS GREATER THAN THE
      DEGREE OF THE DENOMINATOR.
                           F→ SOME NUMBER IF THE FACTORS APPRO
      ACHING ZERO IN THE
      NUMERATOR AND DENOMINATOR DIVIDE OUT.
                           F→ INFINITY IF THE DEGREE OF THE DE
      NOMINATOR IS GREATER
      THAN THE DEGREE OF THE NUMERATOR.'
```

```
[14]  '
            3) N→+ OR -INFINITY; D→+ OR -INFINITY;
                              F→0 IF THE DEGREE OF THE DENOMINATO
      R IS GREATER THAN THE
      DEGREE OF THE NUMERATOR.
                              F→ SOME NUMBER IF THE NUMERATOR AND
       DENOMINATOR HAVE THE
      SAME DEGREE.
                              F→ INFINITY IF THE DEGREE OF THE NU
      MERATOR IS GREATER THAN
      THE DEGREE OF THE DENOMINATOR.'
[15]  '
            4) N→A; D→+ OR -INFINITY; F→0'
[16]  '
       THE DESIGNATION OF X→0 FROM THE LEFT OR THROUGH NEGA
      TIVE NUMBERS WILL
      BE X→0- AND X→0+ WHEN IT APPROACHES 0 FROM THE RIGHT
      OR THROUGH POSITIVE NUMBERS.'
[17] L9:'
      ARE YOU FAMILIAR WITH THE REPLIES OF THIS EXERCISE?'
[18]  R←⍞
[19]  →L0×⍳(1↑R)='Y'
[20]  REPLY
[21] L0:'
      NOTE: FOR THIS EXERCISE USE INF TO STAND FOR +INFINIT
      Y AND -INF FOR -INFINITY.

      '
[22] L1:X←(1+(4?9))×(×RN2,×RN2,×RN2,×RN2)
[23]  X←X[1]+0×Y←X[2]+0×Z←X[3]+0×M←X[4]
[24]  'GIVEN THE EXPRESSION (X*2 +';X+Y;'X+';X×Y;') ÷ (X*2
      +';Z+M;'X+';Z×M;')'
[25]  'WHAT IS THE LIMIT AS X→0+ ?'
[26]  H←H1←'THE CLUE TO THIS ONE IS TO JUST EVALUATE THE EX
      PRESSION.'
[27]  ANSA←(X×Y) DIV(Z×M)
[28]  →0×⍳CHECK=1+0×ANS←(X×Y)÷Z×M
[29]  'WHAT IS THE LIMIT AS X→';-X;' ?'
[30]  →0×⍳CHECK=1+0×ANS←ANSA←0
[31]  'WHAT IS THE LIMIT AS X→';-M;'+ ?'
[32]  H←H2←'IF THE DENOMINATOR OF AN EXPRESSION GOES TO ZER
      O AND THE NUMERATOR
      DOES NOT, THEN THE EXPRESSION GROWS WITHOUT BOUND OR
      TENDS TO INFINITY.'
[33]  →0×⍳CHECK=1+0×ANS←ANSA←(×(X-M)×(Y-M)÷(Z-M))×
      1E75
[34]  'WHAT IS THE LIMIT AS X→1 ?'
[35]  H←H1
[36]  ANSA←((1+X)×(1+Y)) DIV((1+Z)×(1+M))
[37]  →0×⍳CHECK=1+0×ANS←((1+X)×(1+Y))÷(1+Z)×(1+M)
[38]  'WHAT IS THE LIMIT AS X→+INFINITY?'
```

```
[39]  H←H3←'THE CLUE TO THIS ONE IS TO DIVIDE NUMERATOR AND
       DENOMINATOR BY X*2.'
[40]  →0×⍳CHECK=1+0×ANS←ANSA←1
[41]  X←(3?9)×(×RN2,×RN2,×RN2)
[42]  X←X[1]+0×Y←X[2]+0×Z←X[3]
[43]  'GIVEN THE EXPRESSION (X*2 +';X+Y;'X+';X×Y;') ÷ (X*2
      +';Z+Y;'X+';Z×Y;')'
[44]  'WHAT IS THE LIMIT AS X→';-Y;' ?'
[45]  H←H4←'THE CLUE IS TO FACTOR THE NUMERATOR AND DENOMIN
      ATOR AND SEE IF A
      TERM DIVIDES OUT.'
[46]  ANSA←(X-Y) DIV(Z-Y)
[47]  →0×⍳CHECK=1+0×ANS←(X-Y)÷(Z-Y)
[48]  'WHAT IS THE LIMIT AS X→0 ?'
[49]  H←H1
[50]  ANSA←(X×Y) DIV(Z×Y)
[51]  →0×⍳CHECK=1+0×ANS←X÷Z
[52]  'WHAT IS THE LIMIT AS X→';-Z;'+ ?'
[53]  H←H2
[54]  →0×⍳CHECK=1+0×ANS←ANSA←(×(X-Z))×
      1E75
[55]  X←RN2+0×Y←RN2+0×Z×RN2+0×M←RN2
[56]  'GIVEN THE EXPRESSION(';X;'X*2 +';|RN2;'X+';Y;') ÷ ('
      ;Z;'X*2 +';|RN2;'X+';M;')'
[57]  'WHAT IS THE LIMIT AS X→-INFINITY?'
[58]  H←H3
[59]  ANSA←X DIV Z
[60]  →0×⍳CHECK=1+0×ANS←X÷Z
[61]  'WHAT IS THE LIMIT AS X→0 ?'
[62]  H←H1
[63]  ANSA←Y DIV M
[64]  →0×⍳CHECK=1+0×ANS←Y÷M
[65]  X←RN2+0×Y←RN2
[66]  'GIVEN THE EXPRESSION (';Z←1+|RN2;'X-1) ÷ (X*2 +';X+Y
      ;'X+';X×Y;')'
[67]  'WHAT IS THE LIMIT AS X→';-X;'+ ?'
[68]  H←H2
[69]  →0×⍳CHECK=1+0×ANS←ANSA←(×((Z×-X)-1)÷(Y-X))×
      1E75
[70]  'WHAT IS THE LIMIT AS X→+INFINITY?'
[71]  H←H3
[72]  →0×⍳CHECK=1+ANS←ANSA←0
[73]  X←RN2+0×Y←|RN2
[74]  'GIVEN THE EXPRESSION (2X*2 +9X+';Y;') ÷ (X+';X;')'
[75]  'WHAT IS THE LIMIT AS X→';-X;'+ ?'
[76]  H←H2
[77]  →0×⍳CHECK=1+0×ANS←ANSA←(×(2×X×X)+(¯9×X)+Y)×
      1E75
[78]  'WHAT IS THE LIMIT AS X→+INFINITY?'
[79]  H←H3
[80]  →0×⍳CHECK=1
```

```
[81]  'WHAT IS THE LIMIT AS X→0 ?'
[82]  H←H1
[83]  ANSA←Y DIV X
[84]  →0×⍳CHECK=1+0×ANS←Y÷X
[85]  'GIVEN THE EXPRESSION 1 ÷ (X-';X←|RN2;')*3'
[86]  'WHAT IS THE LIMIT AS X→';-X;'+ ?'
[87]  H←H1
[88]  ANSA←1 DIV((¯2×X)*3)
[89]  →0×⍳CHECK=1+0×ANS←1÷(¯2×X)*3
[90]  'WHAT IS THE LIMIT AS X→';X;'+ ?'
[91]  H←H2
[92]  →0×⍳CHECK=1+0×ANS←ANSA←1E75
[93]  'WHAT IS THE LIMIT AS X→';X;'- ?'
[94]  →0×⍳CHECK=1+0×ANS←ANSA←-1E75
[95]  →L1
     ∇
```

APPENDIX D

A LISTING OF THE FUNCTIONS

```
      ∇AHINT[⎕]∇
    ∇ H←X AHINT Y;A;B;C;D;BL;NA
[1]   BL←'      '
[2]   NA←'0123456789'
[3]   ANSA←((ANS<0)ρ'-'),NA[1+|X],'÷',NA[1+|Y]
[4]   A←NA[1+|X],'÷',NA[1+|Y]
[5]   B←NA[1+|Y],'÷',NA[1+|X]
[6]   C←'-',NA[1+|X],'÷',NA[1+|Y]
[7]   D←'-',NA[1+|Y],'÷',NA[1+|X]
[8]   →(L1,L2)[?2]
[9]  L1:H←A,BL,B,BL,C,BL,D
[10]  →0
[11] L2:H←B,BL,A,BL,D,BL,C
    ∇
```

```
      ∇CHECK[⎕]∇
    ∇ V←CHECK;D;I
[1]   I←1+V←0
[2]  L0:R←⎕
[3]   →L7×⍳GRAPH=R
[4]   →L6×⍳QNUMBER=R
[5]   →L1×⍳HELP=R
[6]   →L2×⍳HINT=R
[7]   →L3×⍳ANS=R
[8]   →L4×⍳STOP=R
[9]   →L1×⍳4=I←I+1
[10]  'TRY AGAIN'
[11]  →L0
[12] L7:(18ρ 1 0)\' +'[1+0=((⌽D)∘.×D←¯5+⍳9)]
[13]  →L0
[14] L6:'YOU HAVE BEEN ASKED ';N;' QUESTIONS SO FAR.'
[15]  →L0
[16] L3:'VERY GOOD! NOW SEE IF YOU CAN GET THIS ONE.'[⍳
      43×(?19)≤6]
```

```
[17]  →(0,0)[SCORE I]
[18] L2:'THE ANSWER IS ONE OF THE FOLLOWING:'[ι((ρH)<
      30)×35]
[19]  H
[20]  →(L0,0)[SCORE 5]
[21] L1:'THE ANSWER IS ';ANSA
[22]  →(0,0)[SCORE 6]
[23] L4:V←1
[24]  'YOUR RESULTS ARE THE FOLLOWING:'
[25]  'NUMBER OF QUESTIONS                    ';N
[26]  'NUMBER ANSWERED ON FIRST TRY           ';S[1]
[27]  'NUMBER ANSWERED ON SECOND TRY          ';S[2]
[28]  'NUMBER ANSWERED ON THIRD TRY           ';S[3]
[29]  'NUMBER NOT ANSWERED                    ';S[4]
[30]  'NUMBER OF HINTS YOU RECEIVED           ';S[5]
[31]  'NUMBER OF HELPS YOU RECEIVED           ';S[6]
[32]  D←(S[1]+(0.5×S[2])+0.3×S[3])×100÷N
[33]  'YOUR PERFORMANCE WAS EXCELLENT'[ι30×D≥90]
[34]  'YOUR PERFORMANCE WAS VERY GOOD'[ι30×(D≥80)∧D<
      90]
[35]  'YOUR PERFORMANCE WAS GOOD      '[ι30×(D≥70)∧D<
      80]
[36]  'YOUR PERFORMANCE WAS POOR      '[ι30×(D≥60)∧D<
      70]
[37]  'YOUR PERFORMANCE NEEDS IMPROVEMENT'[ι34×D<60]
[38]  →0
    ∇
```

```
      ∇DIV[⎕]∇
    ∇ D←X DIV Y;NA;T
[1]   NA←'0123456789'
[2]   T←((1↑T)='0')↓T←NA[1+(10 10 10 ⊤|X)]
[3]   T←((1↑T)='0')↓T
[4]   D←((X<0)ρ'-'),T
[5]   T←((1↑T)='0')↓T←NA[1+(10 10 10 ⊤|Y)]
[6]   T←((1↑T)='0')↓T
[7]   T←((Y<0)ρ'-'),T
[8]   D←D,'÷',T
    ∇
```

```
      ∇RN1[⎕]∇
    ∇ RN←RN1
[1]   RN←¯10+?19
    ∇
```

```
      ∇RN2[⎕]∇
    ∇ RN←RN2
[1]   RN←¯10+?19
[2]   →1×⍳RN=0
    ∇
```

```
      ∇RN3[⎕]∇
    ∇ RN←RN3;H;D;R
[1]   H← 3 2 ρ 3 4 5 12 9 12
[2]   X←(R←?2)×H[D←?3;1]
[3]   Y←R×H[D;2]
[4]   RN←((X*2)+(Y*2))*0.5
    ∇
```

```
      ∇SCORE[⎕]∇
    ∇ D←SCORE II
[1]   →L0×⍳II=5
[2]   N←N+1
[3]  L0:S[II]←S[II]+1
[4]   D←1
    ∇
```

```
      ∇RN1[⎕]∇
    ∇ RN←RN1
[1]   RN←¯10+?19
    ∇
```

```
      ∇RN2[⎕]∇
    ∇ RN←RN2
[1]   RN←¯10+?19
[2]   →1×⍳RN=0
    ∇
```

```
      ∇RN3[⎕]∇
    ∇ RN←RN3;H;D;R
[1]   H← 3 2 ⍴ 3 4 5 12 9 12
[2]   X←(R←?2)×H[D←?3;1]
[3]   Y←R×H[D;2]
[4]   RN←((X*2)+(Y*2))*0.5
    ∇
```

```
      ∇SCORE[⎕]∇
    ∇ D←SCORE II
[1]   →L0×⍳II=5
[2]   N←N+1
[3]  L0:S[II]←S[II]+1
[4]   D←1
    ∇
```

```
      ∇REPLY[⎕]∇
    ∇ REPLY
[1]     '

       THE COMPUTER WILL ASK YOU QUESTIONS WHOSE ANSWERS AR
      E NUMBERS.'
[2]   'YOU ARE TO TYPE IN THE NUMBER.  IF THE NUMBER IS A F
      RACTION,
      HOWEVER, YOU ARE TO USE THE DIVIDE(÷) SYMBOL AND NOT
      THE SLASH(/),'
[3]   'SO ONE-HALF IS 1÷2 AND THREE-AND-ONE-THIRD IS 10÷3 .
      '
[4]   ' IF YOUR REPLY IS INCORRECT THE COMPUTER WILL REPLY
           TRY AGAIN'
[5]   'AND YOU GET ANOTHER CHANCE. IN FACT, YOU GET 3 TRIES
       AT EACH QUESTION.'
[6]   ' IF YOU DO NOT KNOW THE ANSWER TO THE QUESTION YOU C
      AN TYPE
           HINT'
[7]   'AND EITHER (1) A COMMENT WILL BE PRINTED OR (2) A LI
      ST OF FOUR
      NUMBERS WILL BE PRINTED, WHERE ONE IS THE CORRECT ANS
      WER.'
[8]   ' IF YOU DO NOT HAVE THE SLIGHTEST IDEA OF WHAT THE A
      NSWER IS TYPE
           HELP
      AND THE ANSWER WILL BE GIVEN.'
[9]   ' IF YOU WANT TO KNOW HOW MANY QUESTIONS YOU HAVE BEE
      N ASKED TYPE
           QNUMBER'
[10]  ' IF YOU WANT AN XY-AXIS PRINTED FOR SKETCHING FUNCTI
      ONS THEN TYPE
           GRAPH'
[11]  ' WHEN YOU WANT TO STOP THE EXERCISE TYPE
           STOP'
[12]  'AND A TABULATED RESULT OF YOUR REPLIES WILL BE GIVEN
       AND THEN THE
      EXERCISE WILL TERMINATE.'
[13]  '

      YOUR FIRST QUESTION IS:

      '
    ∇
```

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