Teacher instructional practices designed to meet the individual learning needs of mathematically gifted/talented students in middle school Algebra I

Virginia Caine Tonneson
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TEACHER INSTRUCTIONAL PRACTICES DESIGNED TO MEET
THE INDIVIDUAL LEARNING NEEDS OF MATHEMATICALLY
GIFTED/TALENTED STUDENTS IN MIDDLE SCHOOL ALGEBRA I

A Dissertation
Presented to
The Faculty of the School of Education
The College of William and Mary in Virginia

In Partial Fulfillment
Of the Requirements for the Degree
Doctor of Philosophy

by
Virginia Caine Tonneson
March 2011
TEACHER INSTRUCTIONAL PRACTICES DESIGNED TO MEET
THE INDIVIDUAL LEARNING NEEDS OF MATHEMATICALLY
GIFTED/TALENTED STUDENTS IN MIDDLE SCHOOL ALGEBRA I

by

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Approved March 2011 by

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DEDICATION

To my husband, Larry, and my children, Matt and Jen,

for their patience, love, and steadfast support throughout this endeavor,

and to my parents, Phil and Doris Caine,

for their love and encouragement and their lifetime of dedication to education.
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ABSTRACT

This descriptive study sought to determine the ways in which seven middle school Algebra I teachers from a suburban school district modified their instructional practices for the gifted/talented students in their classes. The researcher observed each teacher for approximately four hours and evaluated their effectiveness in various teaching behaviors using a modified Classroom Observation Scale-Revised (VanTassel-Baska et al., 2005a). The researcher also interviewed the teachers to ascertain how they modified the pace and challenge of the course, their differentiation strategies, and other ways they supported their gifted/talented students. By conducting a case study of each teacher and then by using a cross-case analysis, the researcher discovered common themes in how these teachers addressed the needs of their gifted/talented students.

The researcher found that the teachers did very little aimed specifically at the gifted/talented segment of the class. Nonetheless, most of the gifted students generally were engaged. This level of engagement may be attributed to the fact that the course already had challenge and rigor built in, the pace was fairly quick, and the teachers provided a supportive environment where the students felt free to take risks. Despite their concerns about the increasing number of students taking Algebra I in middle school, and rather than lowering the level of the course to accommodate struggling students, the teachers kept the rigor of the course high and provided the gifted/talented students with adequate attention. Because of this, the needs of the gifted/talented students appeared to be met to some extent.

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THE INDIVIDUAL LEARNING NEEDS OF MATHEMATICALLY

GIFTED/TALENTED STUDENTS IN MIDDLE SCHOOL ALGEBRA I
Chapter 1: The Problem

In their hallmark 1980 publication, *Agenda for Action*, the National Council of Teachers of Mathematics (NCTM) made the observation, “The student most neglected, in terms of realizing full potential, is the gifted student of mathematics. Outstanding mathematical ability is a precious societal resource needed to maintain leadership in a technological world” (NCTM, 1980, p. 18). Little has changed over the past three decades; in fact, some would argue that the child most left behind under the No Child Left Behind (NCLB) Act (2001) is the gifted student. Plucker, Burroughs, and Song (2010) pointed out that the relatively small percentage of American students scoring in the top levels on achievement tests suggests that “children with advanced academic potential are being under-served” (p. 1). Dr. Camilla Benbow, Vice Chair of the National Mathematics Advisory Panel and National Science Board member, expressed a similar sentiment during her keynote address at the 2009 National Curriculum Network Conference stating, “In NCLB, we don’t hold schools accountable for the achievement of the students at the top.” In fact, National Assessment of Educational Progress (NAEP) scores reveal that gains by low-achieving students are outpacing those of high-achieving students by a factor of two or three to one (Loveless, 2008a). Furthermore, gifted/talented students are often the last students to garner teachers’ attention. On a national survey of 900 teachers in grades 3 through 12 conducted by the Fordham
Institute, 81% said they give one-on-one attention to academically struggling students, but only 5% give it to advanced students (Farkas & Duffet, 2008).

The lack of focus on the needs of gifted/talented children is compounded in mathematics by another issue – the increasing number of non-gifted students taking Algebra I in middle school. Over the past decade, U.S. schools have started pushing students toward taking algebra coursework prior to entrance into high school (U.S. Department of Education, 1997, 2008b). Part of this is driven by the achievement scores of American students on international assessments. For example, on the 1995 Trends in International Mathematics and Science Study (TIMSS), U.S. twelfth grade students outscored only 2 of the 21 TIMSS countries on the mathematics general knowledge assessment. Researchers discovered that the TIMSS mathematics content equated to a seventh grade level for most TIMSS nations, but a ninth grade level for the United States. Part of the reason for this disconnect was because the U.S. does not cover algebraic topics until much later than other countries (National Center for Education Statistics, 1998). Additionally, educators have begun to realize the key role algebra plays in a student’s secondary education, providing access to higher-level math and science courses. These elements have combined to create a steady increase in enrollment in middle school Algebra I. In fact, some states have set actual enrollment goals. California has mandated that all eighth graders take Algebra I (O’Connell, 2008) while Virginia has a more modest goal of 45% (Virginia Board of Education, 2007). The result is that middle school Algebra I classes that had previously been relatively homogeneous – reserved for those students who were of high ability – now increasingly contain students with average or below mathematical aptitude.
Teachers have felt the impact of these increasingly heterogeneous classes. Rather than being able to move through the material at the pace and level of complexity needed for advanced students, they may now find themselves trying to reach both high-ability students and those for whom algebra is a real struggle within the same class. In addition, because teachers have to be concerned with all students passing the end-of-course standards test, they may find themselves focused more on the students at the lower end of the spectrum. To engage all the students in the class requires the teacher to have a thorough understanding of the unique abilities of each student.

In the case of mathematically gifted/talented students, these abilities may not be fully appreciated because differences between gifted and average students go beyond their abilities to simply discover answers more quickly and accurately. In fact, cognitive neuroscience research suggests that the brains of mathematically gifted children may actually be somewhat different from those of average ability children both quantitatively and qualitatively (O’Boyle, 2008). In other words, not only do students gifted in mathematics develop mathematical abilities more rapidly than average students, but their brains may actually process mathematical problems differently. They also have more efficient memories (Perleth et al., 2000) and are better able to manipulate numerical and spatial information in their working memories (USDOE, 2008b). Similarly, gifted students have exceptional insightfulness (Davis & Rimm, 2004) which enables them to omit seemingly essential steps in mathematical problem solving (Krutetskii, 1976). Additionally, metacognitive knowledge and control appear to develop earlier in gifted children than in their average-ability peers (Schraw & Graham, 1997). Furthermore, mathematically gifted students are able to use strategies more flexibly and consistently.
and to apply strategies to novel situations with much greater ease than their age mates (Carr et al., 1996). They tend to focus on the conceptual underpinnings of problems rather than their surface features, and are able to abstract similarities in the structures of problems which allows them to generalize mathematical principles much more quickly than their peers (Grouws, Howald, & Colangelo, 1996; Krutetskii, 1976; Sriraman, 2003). Finally, their conception of mathematics is unlike that of average students. Although mathematically advanced students recognize the role of algorithms and facts, they place more emphasis on the underlying principles and concepts. Essentially, they tend to view mathematical knowledge as “a coherent system of important ideas and the relationships among them” (Grouws, Howald, & Colangelo, 1996, p. 22), while less advanced students view mathematics more as formulas, facts, and procedures.

This is not to say, however, that all mathematically gifted/talented students are alike. In fact, the differences in students who score in the 97th percentile and above on grade-appropriate tests are as great as the differences within the general student population as a whole (Mills, Ablard, & Gustin, 1994). Some mathematically gifted students may use their visual-spatial abilities to solve problems, others may use their logic, while still others may use a combination of the two (Krutetskii, 1976). In addition, some may have advanced computational skills, while others may not (Kalbfleisch, 2008b). Furthermore, gifted students may also differ in their modes of learning, motivation, and interests (National Association for Gifted Children, 2008).

All of these factors, therefore, make it essential that teachers understand how to differentiate to meet the unique needs of the gifted/talented students in their classes. Although there are many differentiation techniques including enrichment, acceleration,
flexible grouping, and various methods of adjusting activities and questions, finding the right approach requires the teacher to have a thorough understanding of each student’s needs, as well as the time and willingness to make the necessary modifications. Unfortunately, however, teachers of gifted students who are strong in many areas of good teaching oftentimes fall short in their differentiation practices for these students (VanTassel-Baska, 2004).

In fact, many teachers feel unprepared to address the needs of mathematically gifted/talented children overall. Two separate national surveys pointed out the limited amount of gifted training teachers receive. Farkas and Duffet (2008) found that 65% of teachers had little or no training during their teacher preparation programs in meeting the needs of gifted students, while Archambault et al. (1993) found that 61% of teachers had never had staff development in gifted education. This is important because the factor that affects teachers’ attitudes toward gifted students the most is whether they have studied gifted education (Plunkett, 2000).

The role of the teacher cannot be overstated as the teacher is the most important school-related factor affecting student learning (Wright, Horn, & Sanders, 1997). Several studies have shown that teacher quality is important in student learning gains in mathematics (e.g. Gordon, Kane, & Staiger, 2006; Nye, Konstantopoulos, & Hedges, 2004; Rivkin, Hanusheck, & Kain, 2005; Sanders & Rivers, 1996). Teacher quality is especially important in the case of a gifted/talented child as Sanders and Horn (1998) found that the highest achieving students only made adequate gains when taught by the top 20% of teachers, those considered to be the most highly effective. They concluded that:
the majority of the brightest students fail to achieve to their potential year after year and, in the long run, attain a level of achievement far below that of their more fortunate peers who have benefited from the most effective teachers. (Sanders & Horn, 1998, p. 254)

Similarly, Wright, Horn, and Sanders (1997) found that the highest achieving students actually made the lowest academic achievement gains, possibly due to an inability to progress at their own pace, a lack of challenge in curricular materials, and the concentration of the teacher’s instruction aimed at the average or below average students, rather than the students at the top. These factors point out the need for a teacher who not only understands gifted/talented students, but one who also understands and is willing to differentiate for them so that they can reach their potential.

**Statement of Purpose**

The purpose of this study was to determine the ways in which middle school Algebra I teachers modified their Algebra I course for their gifted/talented students. Because of the cognitive differences between gifted and average students, mathematically promising students need more challenging material to stimulate their thinking and they need to move through basic material at a quicker pace than other students. Gifted/talented students also need to have a supportive environment where the teacher provides them the attention, assistance, and modeling they need to achieve higher levels of learning.

**Research Questions**

This study used observations and interviews to answer the following research questions:
1. To what extent do middle school Algebra I teachers modify the pace of instruction for their gifted/talented students?

2. In what ways do middle school Algebra I teachers increase the level of challenge for their gifted/talented students?

3. What differentiation strategies do middle school Algebra I teachers use to meet the needs of their gifted/talented students?

4. In what ways do middle school Algebra I teachers provide a supportive environment for their gifted/talented students?

**Significance of the Study**

This study was significant for several reasons. There is a lack of research related to gifted/talented students in middle school Algebra I classroom settings, so this study adds to the body of knowledge related to the topic. This is especially important because a heterogeneous classroom setting is the environment in which a gifted/talented student will mostly likely experience Algebra I. In addition, this study helped teachers to become more aware of the extent to which they were differentiating to meet the needs of the gifted/talented students in their own classrooms. Research consistently reveals that “accelerated and demanding instruction is needed for these students to reach their full potential in mathematics” (USDOE, 2008b, p. 4-109), and the study helped teachers to assess whether they were providing such instructional modifications. It also made them aware of the attention they provided to their gifted/talented students in relation to the time they spent with lower-achieving students. Like other students, gifted/talented children have their own unique needs that deserve to be addressed and this study made teachers more aware of whether they were dividing their attention toward their students equitably.
Similarly, it assisted administrators in becoming cognizant of how teachers were meeting the needs of their gifted/talented students as well as some of the challenges they faced in teaching students with differing abilities. It provided data that may be used as a needs assessment for educators to plan professional development sessions pertaining to differentiation and gifted education. Learning more about gifted education and how to differentiate to meet the needs of gifted/talented students is important because teachers who are trained in gifted education are more likely to consider individual variance in their instruction, understand how to provide challenging opportunities, allow students to express themselves in a variety of ways, and to foster high-level thinking (NAGC, 2008). This may benefit teachers not only in their dealings with gifted/talented students, but also in helping all students in the class to achieve their potential.

**Justification**

Much has been written about educating gifted/talented students and the importance of differentiating for them. Roger’s (2007) synthesis of the research since 1861 revealed 168 research studies and 358 literature articles on instructional differentiation. Several authors specifically address differentiation in K-12 mathematics classrooms (e.g. Diezmann & Watters, 2000, 2002b; Johnson, 2000; Kim, 2006; Reed, 2004; Rotigel, & Fello, 2004; Stepanek, 1999; Tieso, 2002; Waxman, Robinson, & Mukhopadhyay, 1996; Wilkins, Wilkins, & Oliver, 2006). However, despite this quantity of research, there have been very few studies that address mathematically promising students in Algebra I. In fact, a search of the literature revealed only two studies dealing specifically with gifted/talented students in Algebra I within a normal classroom setting. Matthews and Farmers (2008) studied the factors that affected the Algebra I achievement of gifted
students in grades 7 through 9 while Sriraman (2003) explored the relationship between mathematical giftedness and the ability to generalize and problem solve in an accelerated ninth grade algebra class.

A few other studies touch on the topic peripherally. Pajares (1996) explored self-efficacy beliefs and problem solving among gifted students in middle school algebra classes; Cunningham (1983) examined self-instruction training of gifted and non-gifted students in grade 9 and 10 algebra classes; and Rolle (2008) conducted a qualitative study of a high-achieving seventh grade pre-algebra class to determine the teacher’s habits of practice. In addition, Stanley, Benbow, Lubinski, and their associates have written about algebra as part of the Study of Mathematically Precocious Youth (SMPY) which was begun at Johns Hopkins University in 1971 (Stanley, 1991); however, research conducted on SMPY participants as well as those in other talent search programs do not look at gifted/talented students in the context of a typical middle school classroom. This lack of research pertaining to the environment in which most mathematically gifted/talented students find themselves indicated the need for further investigation in this area.

**Operational Definition of Key Terms**

The following key terms were used in this study:

- *Challenging tasks* (or level of challenge): Challenging tasks refer to mathematical problems or activities that are more complex or abstract than tasks given to average students. Teachers may make tasks more challenging by increasing the obstacles to problem solving (such as removing some of the information given in a problem or using more “difficult” numbers), requiring students to think at higher levels, or by requiring students to examine and
understand a concept in greater depth in order to solve the problem. Providing students with enrichment activities, tiered assignments, and open-ended questions and assignments are three differentiation techniques that help to increase the level of challenge. The measurement of the level of challenge relied on the teacher’s subjective judgment and the researcher’s observations.

- **Gifted/talented students**: Students who are capable of high performance in a domain by virtue of their exceptional capabilities. Because researchers and educators use different criteria for categorizing students as “gifted,” in this study, the terms gifted, high-ability, and promising were used to characterize these students. Similarly, because many educators and researchers do not specify whether the students they refer to as “gifted” are globally gifted or gifted in a certain domain, use of the term gifted was assumed to include those students who have high abilities in mathematics. Whenever possible, this study referred to gifted/talented students using the same terminology as the work under consideration.

- **Mathematically gifted/talented students**: Students who are capable of high mathematical performance by virtue of their exceptional abilities. There is no universally accepted set of criteria by which a student is identified as mathematically gifted, so the terms mathematically gifted, mathematically precocious, and mathematically promising were used to characterize these students. The term gifted was frequently used to describe these students when discussing them in a mathematical context.
• **Middle school**: The middle school in this study was a public school which contained students in grades 6, 7, and 8.

• **Middle school algebra teachers**: Middle school algebra teachers are those teachers that teach a recognized Algebra I course containing the content required by the state. Teachers of Pre-algebra, Advanced Algebra, and Algebra II were not included in this study.

• **Pace**: Pace refers to the speed at which the material in a course is covered. The amount of instructional time a teacher spends explaining a concept and the amount of practice time a student requires to achieve mastery of the concept impact the pace. Acceleration and curriculum compacting are two methods of differentiation that affect pace.

• **Supportive environment**: A supportive environment for gifted/talented students includes the teacher providing them equitable attention and appropriate scaffolding, pressing them for explanations for their problem-solving techniques and the meaning of their ideas, and encouraging them to perform up to their abilities. Flexible grouping and providing students with alternatives and choices are two differentiation techniques that help to create a supportive environment. The measurement of whether the environment was supportive relied on the teacher’s subjective judgment and the researcher’s observations.

**Delimitations of the Study**

This study had several delimitations:

• It only looked at Algebra I teachers at the middle school level because the vast majority of gifted/talented students take the course prior to entrance into high
school. The findings may or may not be similar to what one would find with high school Algebra I teachers.

- The study was limited to teachers in the southeastern Virginia area. Because each state has different Algebra I requirements and different criteria by which they identify gifted students, the subject matter and mix of students within the classroom may not be similar to what one would find in other areas of the state and nation.

Limitations of the Study

This study also had several limitations:

- The findings were based on a limited number of observations and those observations may not have been typical of the normal classroom environment. Teachers may have had a variety of reasons for participating in this study, and their behavior may not have revealed their true interest or disinterest in the research. Furthermore, the teachers were aware that the researcher was specifically interested in the gifted/talented students in the classroom and this may have led to some artificiality in the way the teachers dealt with the students. It should also be noted that teachers' interest and proficiency in differentiation may have been impacted by their education in giftedness and teaching experience.

- Pre- and post-observation discussions with the teachers were limited in time according to the teachers' schedules.
• Interviews relied on teachers being willing to openly share details about their instruction and classroom. Teacher responses may have been influenced by their concern with portraying their classroom or their instruction in a certain light.

• Identification of giftedness varies across school districts and so students identified as gifted in one district might not necessarily be identified as gifted in another district. Additionally, because many school districts simply identify students as being gifted overall rather than in a specific domain, the gifted students in this study may or may not have been mathematically gifted.

• The overall sample of teachers is this study was atypical. All teachers were “highly qualified,” and all had Bachelor’s degrees in Mathematics or related areas (Economics and Accounting). Four had Master’s degrees in Education or mathematics-related fields (Mathematics and Economics), while two of the others were enrolled in Master’s of Education graduate programs. While level of educational attainment does not translate into teacher effectiveness, the results of this study may have been somewhat different had the researcher used a sample of teachers who were not so well-versed in their content area.

• The “average” students referred to in this study were, in fact, one or two years above grade level since ninth grade was considered the normal year for a student to take Algebra I in this school district. The results of this study may have been somewhat different had the researcher conducted it in a district with an “algebra for all” policy in middle school where an “average” student was truly average.

Because of these issues and the limited nature of the study, it may not be generalizable to populations that differ significantly from the sample.
Chapter 2: Relevant Literature

This chapter contains a review of the literature relevant to exploring the question of how and to what degree middle school Algebra I teachers modify their course to meet the needs of their mathematically gifted/talented students. This literature review will show how the research questions from the study are related to the extant knowledge, identify the gaps in the literature, and help to place this study into its broader scholarly context.

To frame the context of this investigation, the literature review will begin with a focus on algebra in the curriculum. It will review the research on the importance of algebra, access to algebra, the timing of algebra, and the impact of taking algebra prior to high school. Next, in order to determine whether teachers are meeting the needs of mathematically gifted/talented algebra students, it is important to identify what those needs really are. As such, the second major section will examine the concept of mathematical giftedness, differences among mathematically gifted/talented students, and the differences between mathematically gifted/talented students and their non-gifted peers. It will look specifically at differences in their brains, memory, insightful thinking, metacognition, strategy knowledge and use, problem solving, and their conceptions of mathematics. Finally, the literature review will examine the instructional implications these differences have for gifted/talented students and the various types of differentiation strategies that have been shown to be effective at meeting these needs.
Algebra in the Curriculum

Algebra has been a cornerstone of mathematics for centuries. During the Middle Ages, Middle Eastern and Asian mathematicians pioneered its use as an efficient way to solve equations, and by the 17th century, it was recognized as essential in promoting advancements in all branches of science and math (USDOE, 2008a). Although algebra has been a part of the curriculum in the United States since the academies of the early 1800s (Ornstein & Hunkins, 2008), the timing of when it is most appropriate to take algebra has never been fully resolved. For example, in 1895 the Committee of Fifteen recommended algebra be taught in seventh and eighth grades (Ornstein & Hunkins, 2008), but until fairly recently, most students waited to take Algebra I until high school. Over the past decade, however, schools in the United States have started pushing students toward taking algebra coursework prior to entrance into high school.

Importance of algebra. Part of the push to take algebra is related to the key role algebra plays in a student’s secondary education. For example, Usiskin (1995) talked about algebra as the language of mathematics. Smith (1996) characterized algebra as a “gatekeeper” course for advanced mathematics and science and pointed out that for a student to grasp the complexities advanced courses require, an understanding of algebraic concepts was essential. Gamoran and Hannigan (2000) determined that all students benefit from taking algebra, even those with very low prior mathematics achievement. Added to this mix is the idea that access to algebra might be a means to close the White-minority achievement gap. Because algebra facilitates entrance into higher-level math and science, Moses (1995) termed it “the new civil right” (p. 53) and Steen (1999) talked about it as being “an invaluable engine of equity” (p. 6).
The National Mathematics Advisory Panel likewise acknowledged the importance of algebra, focusing a large portion of their *Reports of the Task Groups and Subcommittees* (USDOE, 2008b) on the topic. The Panel was established by President Bush in 2006 to look into the state of mathematics in the United States (Executive Order, 2006). He charged the panel with using the best scientific evidence available to find ways to improve the mathematics performance of U.S. students. The introduction to the National Math Panel’s final report pointed out that “while the presidential charge contains many explicit elements, there is a clear emphasis on the preparation of students for entry into, and success in, algebra” (USDOE, 2008b, p. 1-1). The report emphasized the importance of algebraic thinking throughout a child’s early education, pointing out that a major goal of K-8 mathematics should be the development of certain skills that form the “Critical Foundations of Algebra” (USDOE, 2008c, p. xvii).

**Early access to algebra.** By the late 1990s, the issue of when students should take algebra had begun to receive national attention. In 1997, Secretary of Education Richard Riley wrote, “The key to understanding mathematics is taking algebra or courses covering algebraic concepts by the end of the 8th grade. Achievement at that stage gives students an important advantage in taking rigorous high school mathematics and science courses” (USDOE, 1997). The following year, President Clinton issued a call to improve mathematics in this nation. He stated:

Students must challenge themselves and take the most advanced math and science courses they can. . . . Around the world, middle [school] students are learning algebra and geometry. Here at home, just a quarter of all students take algebra before high school. (USDOE, 1998, para. 26)
Over the past decade, schools have responded to this call for earlier access to algebra. In 1996, approximately 25% of U.S. eighth grade students were enrolled in Algebra I or a subsequent course (USDOE, 1997), but by 2005, that number had increased to 39% (USDOE, 2008c). Despite this increased access, some educators question whether early algebra is appropriate for all students.

**What research says about the timing of algebra.** One of the topics the National Math Panel examined was the timing of algebra coursework. Although panel members reviewed over 16,000 studies, only the six mentioned below met their criteria for high-quality research and addressed the long-term benefits for taking Algebra I prior to ninth grade.

Lee, Burkam, Chow-Hoy, Smerdon, and Geverdt (1998) used data from the High School Effectiveness Supplement of the National Educational Longitudinal Study and found that students who took courses lower than Algebra I scored lower overall on mathematics achievement in grade 12 than did students who took Algebra I or higher level courses. Examining the issue from the opposite view, Jones, Davenport, Bryson, Bekhuis, and Zwick (1986) used High School and Beyond (HS & B) data from over 9,000 students to show that overall student mathematical achievement by their senior year was strongly related to the number of mathematics classes a student took at the Algebra I level and beyond. This is important because Smith (1996), also using HS & B data, found that access to algebra prior to ninth grade increased the amount of math the students and their teachers expected them to take in high school, and so it socialized them into actually taking more mathematics courses. These additional math courses resulted in higher math achievement and attainment in high school.
In fact, Ma (2005) looked at just that. He used the data from the Longitudinal Study of American Youth, a six-year panel study of over 3,000 students, to measure student mathematical growth from grade 7 to grade 12. He found that students who gained early entrance into a formal algebra course at the seventh or eighth grade level had improved mathematical achievement in the four mathematical areas he examined (basic skills, quantitative literacy, algebra, and geometry) compared to those students who were not accelerated. Even more significant is the fact that the rates of growth among the students who were accelerated despite the fact that they were low achievers to begin with outpaced the rates of growth of both low achievers and high achievers who were not accelerated. This held true regardless of the student’s individual characteristics or those of his family or school. Ma (2000), using the same data set, looked specifically at achievement on a year-to-year basis. After controlling for prior mathematical achievement, socioeconomic status, gender, and age, he found that students who took Pre-algebra or higher-level courses in grade 7 had higher achievement in grade 8 than those students not enrolled in the advanced courses. Similarly, students who were enrolled in Algebra I or higher math courses in grade 8 scored higher on grade 9 achievement tests than those students not enrolled in the advanced courses. Finally, Wilkins and Ma (2002) found that students who took Algebra I prior to ninth grade had significantly higher rates of growth in their mathematical content knowledge than did their peers. After reviewing the research, the National Math Panel’s recommendation for the timing of algebra was that schools should “prepare more students than at present to enroll in such a course by Grade 8” (USDOE, 2008b, p. 23) but that students need prerequisite skills prior to taking the course. Cathy Seeley, the former president of the National Council of Teachers of Mathematics,
cautioned, “The move to push algebra down has to be approached carefully and thoughtfully. The solution is complex” (cited in Fratt, 2006).

**The impact of early algebra.** Whether the move to place more students into algebra in middle school is working is open to debate. Looking strictly at numbers, it appears that this initiative has been successful. As mentioned earlier, almost 40% of our nation’s middle school students are currently enrolled in an algebra course compared to only 25% a decade ago (USDOE, 2008c). In fact, more eighth graders across the nation take algebra than any other math course (Loveless, 2008b). The movement has also increased advanced math course-taking. Spielhagen (2006a, 2006b) confirmed Smith’s (1996) findings that students who take algebra early stay in the math pipeline longer. Furthermore, the Brown Center on Educational Policy at the Brookings Institution (Loveless, 2008b) conducted a study using restricted-use 2005 National Assessment of Educational Progress (NAEP) data files which allowed them to match student course-taking with NAEP scores. They found that in 2000, 26.7% of the eighth graders taking the NAEP were in Algebra I, Geometry, or Algebra II, but by 2005, that number had increased to 36.6%. They surmised that “the campaign for algebra by eighth grade clearly succeeded in boosting the number of American youngsters enrolled in tougher mathematics courses” (Loveless, 2008b, p. 5).

If, on the other hand, one takes a more critical look at factors behind the algebra initiative, it is apparent that it has not been as successful as it appears. Since President Clinton’s call to action, poor U.S. international performance in mathematics has continued. U.S. eighth grade students ranked 15th out of 46 industrialized countries on the 2003 TIMSS, and on the 2006 Program for International Student Assessment (PISA),
U.S. 15-year olds ranked 25th out of 30 developed nations in math literacy and problem solving (Baldi, Jin, Skemer, Green, & Herget, 2007). These data sets do not allow one to determine the course-taking profiles of the students, so it is not possible to compare their achievement based on when they took Algebra I. What is clear from the data, however, is that the impact goes beyond mathematics. The American Institutes for Research (2008) cited poor international mathematics scores as one of the reasons U.S. science performance in the upper grades has declined in recent years.

On the home front, although math scores on the 2007 NAEP actually increased slightly, they still showed that only 32% of eighth graders and 23% of twelfth graders were at or above the proficient level (USDOE, 2007). In fact, using the restricted-use data files previously mentioned, the Brown Center (Loveless, 2008b) discovered that 7.8% of the eighth grade students enrolled in Algebra I or above actually scored in the lowest 10% on the 2005 NAEP. Because 11 points on the test is considered to be equivalent to a year's learning, this equates to 120,000 eighth graders enrolled in an advanced math class who know about the same amount of math as a typical second grader. The magnitude of this finding is especially significant when seen from the view of a teacher who must somehow modify his or her instruction to address the needs of a class full of students who may have a range of mathematical abilities anywhere from that found in early elementary school to that found in high school. Loveless pointed out that in an advanced math class (Algebra I, Geometry, or Algebra II) of 26 students, a teacher can expect to have two students who perform at a grade level several years below what is expected. These misplaced students take time and attention away from the students who are truly prepared for such higher-level math.
In fact, a national survey of 743 Algebra I public school teachers conducted by the National Opinion Research Center found that over half of the teachers believed that mixed-ability classes were a serious (23%) or moderate (28%) problem (USDOE, 2008b). Similarly, the survey discovered that 62% of teachers found the single most challenging aspect of teaching Algebra I successfully was “working with unmotivated students” (USDOE, 2008b, p. 9-26). Many of these unmotivated students are those who struggle with algebra, so not only do teachers have to deal with academic issues, but they must also contend with students who have poor attitudes. This takes even more attention away from those students who are ready – and eager – to learn algebra.

Summary. Research has shown that 1) all students benefit from taking algebra at some point in their mathematics career; 2) students who take algebra prior to ninth grade take more math courses; and 3) overall mathematical achievement is related to the number of math classes a student takes from the Algebra I level and beyond. This suggests that we should be offering algebra in eighth grade to far more students than are currently enrolled. In fact, the National Mathematics Advisory Panel (USDOE, 2008b) determined that:

Research evidence, as well as the experience of other countries, supports the value of preparing a higher percentage of students than the U.S. does at present to complete an Algebra I course or its equivalent by Grade 7 or 8. . . . (p. 3-47)

On the other hand, the literature also reveals that many of the students currently being placed in algebra classes are not prepared for that level of mathematics. In fact, the National Math Panel caveated their recommendation concerning algebra in eighth grade by specifying that “students must be prepared with the mathematical prerequisites for this
course” (USDOE, 2008c, p. 23). The impact these unprepared students have on schools, teachers, and particularly higher-ability students is an area that needs more research.

There are other gaps in the literature on algebra as well. According to the National Math Panel (US DOE, 2008b), there is no research that identifies a sequence of math topics across grades that assures algebra success, nor are there studies pertaining to the effectiveness of a single-subject versus an integrated approach to algebra. There is also a need for additional studies on the long-term impact of increased numbers of students taking Algebra I prior to high school. International achievement tests suggest that this move has done little to improve our rankings in mathematics.

To understand how the move toward increasing the enrollment of students in middle school Algebra I may impact teachers and their efforts to meet the needs of mathematically gifted/talented students, it is important to first understand the ways in which these students differ from typical students. The next section will specifically address mathematically gifted/talented students.

Findings related to the issue of increasing the number of students taking algebra in middle school can be found at Table 1. A more detailed description of the studies is at Appendix A.
### Table 1

*Algebra in the Curriculum Research Matrix*

<table>
<thead>
<tr>
<th>References</th>
<th>Importance of Algebra</th>
<th>Early Access</th>
<th>Research on Timing</th>
<th>Impact of Access</th>
</tr>
</thead>
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<td>X</td>
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<td></td>
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<tr>
<td>Gamoran &amp; Hannigan (2000)</td>
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<td></td>
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<td>Jones et al. (1986)</td>
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<td>X</td>
<td>X</td>
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<td>Loveless (2008a)</td>
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<td>Ma (2005)</td>
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<td>Steen (1999)</td>
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<td>Usiskin (1995)</td>
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<td>Wilkins &amp; Ma (2002)</td>
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**Mathematically Gifted/Talented Students**

Teachers are often quick to identify their mathematically gifted/talented students as those who are able to rapidly and accurately solve math problems. They are often independent, self-directed workers for whom new mathematical concepts come easily. While this simple characterization may be accurate for some mathematically precocious students, “mathematical promise cannot be equated either with student achievement or
with performance on computational algorithms” (House, 1999, p. 4). Mathematical giftedness is a much more complex construct.

**What is mathematical giftedness?** Researchers over the years have come up with various conceptions of the term *mathematical giftedness*. One of the first to look specifically at mathematical giftedness was Vadin Krutetskii, a Russian psychologist who conducted longitudinal studies to explore the structure and nature of children’s mathematical abilities. He concluded that students who were gifted in mathematics viewed the world through a mathematical lens, paying attention to the quantitative and spatial relationships around them. He defined mathematical giftedness as “a unique aggregate of mathematical abilities that opens up the possibility of successful performance in mathematical activity” (Krutetskii, 1976, p. 77). More recently, the National Council of Teachers of Mathematics defined mathematically promising students as “those who have the potential to become leaders and problem solvers of the future” (House, 1999, p. 3). Others prefer to use a list of abilities as indicators of mathematical precocity. Wieczerkowski, Cropley, and Prado (2000) suggested that divergent thinking was essential, but not sufficient, for true giftedness and that “mastery of basic facts, speed, accuracy, rapid recall of material from memory and similar factors are also part of mathematical giftedness” (p. 418). Similarly, Waxman, Robinson, and Mukhopadhyay (1996) explained that gifted students had a “rapid and intuitive understanding” (p. 3) of mathematics, while House (1999) said they had a quick mastery of new learning, analytical and original thinking, and the ability to concentrate and work independently. Not only are definitions and characterizations of mathematical giftedness widespread, but researchers have classified the students in their studies as “gifted” using
various criteria. The National Mathematics Advisory Panel (USDOE, 2008b) considered students at or above the 90th percentile on standardized achievement tests as gifted, while Garofalo (1993) considered students with an Iowa Test of Basic Skills mathematics score in the 99th percentile as gifted. The students in Steiner’s (2006) study were identified as gifted using the state’s criteria, while the “gifted” students in Threlfall and Hargreaves’ (2008) study were simply identified by their teacher as being in the top 10% of math ability. Sternberg (2004) recognized this inconsistency, stating that “a bad habit of much of the gifted field is to do research on giftedness or worse, identify children as gifted or not gifted without having a clear conception of what it means to be gifted” (p. xxiii).

This overall lack of agreement on the terminology and criteria that should be used to characterize students with high abilities is of concern for several reasons. Most schools identify their students as “gifted” rather than “mathematically gifted” and they may base their determination of giftedness based on overall indicators of their cognitive ability. This could mean the student may or may not have exceptionally high abilities in mathematics. In fact, Benbow and Minor (1990) pointed out that “reliance on global indicators of intellectual functioning may exclude too many nonverbally gifted students, who appear to be less balanced than verbally gifted students in their cognitive development” (p. 21). In addition, the lack of agreement on what “gifted” or “mathematically gifted” really mean makes it difficult for researchers to compare findings of studies. Appendix B shows the criteria by which the researchers cited in this section identified their sample as gifted. The following section describes some of the differences that may be found among those students who have been labeled as “gifted.”
Differences among gifted students. Many researchers use the term *gifted* to include all precocious children including those who are globally gifted or “notationally gifted” (Winner, 2000b, p. 164) as well as those who are gifted in a certain domain. Benbow and Minor (1990) pointed out, however, that all gifted students are not alike, stating that “verbal and mathematical precocity are distinct forms of intellectual giftedness” (p. 25). They found that students who were verbally precocious scored higher on general intelligence and verbal tests, while mathematically gifted students scored higher on tests of memory, spatial ability, speed, mechanical comprehension, and nonverbal reasoning. Likewise, Brody and Stanley (2005) explained that gifted students vary in their cognitive profiles and can be strong in one domain, but not another. They pointed out that many educators equate giftedness with a high general ability, but argue that “the measurement of specific aptitude [e.g., math or verbal] has been found to be much more useful educationally than general IQ for identifying precocity” (p. 28). This view, while somewhat similar to Gardner’s (2005) conception of multiple intelligences, stops short of giving equal weight to all the domains he labels as “intelligences.”

Similarly, Winner (2000b) proposed that uneven abilities between mathematical and verbal abilities “may be the rule, not the exception” (p. 164). Benbow and Minor (1990) found that while exceptionally high verbal ability increased the likelihood of having high mathematical ability, the opposite was not true. Exceptionally high mathematical ability did not indicate high verbal ability, which suggests that mathematically gifted students may have more uneven cognitive profiles than verbally precocious children. Furthermore, in a study of over 1,000 gifted students, Achter, Lubinski, and Benbow (1996) found that 42% of the students who scored in the top .5% on the SAT had math
and verbal scores more than one standard deviation apart, while 72% of the students in
the top .01% had at least a one standard deviation difference between the two measures.
In a later study, Lubinski, Webb, Morelock, and Benbow (2001) found that for the top
3% of adolescent students in general ability, the correlation between the math portion of
the SAT (SAT-M) and verbal portion (SAT-V) was approximately $r = .55$.

Similarly, mathematically precocious children do not constitute a homogeneous
group. Their preferred modes of learning, motivation, and interests may differ (NAGC,
2008). Likewise, their abilities and prior knowledge may vary widely (Armstrong, 1992;
Davis & Rimm, 2004). In fact, the differences in students who score in the top three
percentiles on grade-appropriate tests are as great as the differences within the general
student population as a whole (Mills, Ablard, & Gustin, 1994). Moreover, an IQ range of
more than 63 points (137 to beyond 200) can be found within the top 1% of students
(Lubinski & Benbow, 2006). Some mathematically talented students are much stronger
in concept development than they are at computation and so they often demonstrate an
uneven pattern of mathematical development and understanding (Rotigel & Fello, 2004).
Krutetskii (1976) specifically pointed out that computation abilities are “not obligatory in
the structure of mathematical giftedness” (p. 351). In fact, many gifted students are not
quicker than average students in basic math facts (Kalbfleisch, 2008b).

Several researchers have attempted to label the different ways in which individuals
may be mathematically gifted. Krutetskii (1976) proposed that there were geometric
types of students who thought in visual-pictorial terms and had high spatial abilities;
analytical types who tended to think in verbal-logical terms; and harmonic types who
used both spatial and logical approaches to problem solving. Sowell, Zeigler, Bergwall,
and Cartwright (1990) distinguished between mathematically gifted students as those who were able to solve problems normally given to older students and those who solved problems using qualitatively different thinking processes. Diezmann and Watters (2002a) categorized mathematically precocious students as analytically gifted and spatially gifted. Finally, Sak (2009) proposed seven different forms of mathematical giftedness consisting of analysts, creators, knowledge experts, creative analysts, expert analysts, creative experts, and masters. Regardless of how one identifies the various strengths mathematically gifted students possess, it is important to remember that each individual is unique, and therefore, may vary substantially in their interest and approach to mathematical tasks (Diezmann, 2005).

Despite these individual differences among mathematically gifted students, as a group they have several differences that distinguish them from their non-gifted peers. Gifted individuals not only differ from average students in quantitative aspects such as the speed with which they develop their abilities, but they also differ qualitatively in the way in which they process information (Gorodetsky & Klavir, 2003; Winner, 2000a). These differences are apparent in gifted students’ brain structure and activity; their memory, insight, and metacognition; the way they strategize and solve problems; and even in the way they conceptualize mathematics.
Table 2

**Similarities and Differences Among Mathematically Gifted Students Based on the Extant Research**

<table>
<thead>
<tr>
<th>Similarities</th>
<th>Differences</th>
</tr>
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<tbody>
<tr>
<td>Physical brain development</td>
<td>Modes of learning</td>
</tr>
<tr>
<td>Areas of the brain used for processing</td>
<td>Interests</td>
</tr>
<tr>
<td>Quick cognitive processing</td>
<td>Motivation</td>
</tr>
<tr>
<td>Excellent working memory for numerical information</td>
<td>Prior knowledge</td>
</tr>
<tr>
<td>Exceptional insightfulness</td>
<td>Strength of computational abilities</td>
</tr>
<tr>
<td>Early development of metacognitive knowledge and control</td>
<td>Overall abilities (IQ spread may be quite large)</td>
</tr>
<tr>
<td>Use of higher-level strategies; use strategies more consistently and flexibly</td>
<td>May or may not be verbally gifted</td>
</tr>
<tr>
<td>Ability to make generalizations</td>
<td>Problem-solving approaches (spatial, logical, or both)</td>
</tr>
<tr>
<td>Focus on conceptual underpinnings of problems</td>
<td></td>
</tr>
<tr>
<td>View math in a manner similar to professional mathematicians</td>
<td></td>
</tr>
</tbody>
</table>

**Cognitive neuroscience.** One way to examine the differences between mathematically gifted and non-gifted students is to look outside the field of gifted education. Cognitive neuroscience offers a unique approach to studying giftedness, although this approach has limitations because it takes the subject out of the context in which they normally demonstrate their unique abilities (Kalbfleisch, 2008a). Although the cognitive development of gifted students is not fully understood (Steiner, 2006), research into “the brain bases of mathematical thinking . . . is important for understanding how math giftedness comes to be” (M. O’Boyle, personal communication, March 22, 2010). Cognitive neuroscience research suggests that the brains of mathematically gifted children are, in fact, quantitatively and qualitatively different from those of average ability children (O’Boyle, 2008).
**Physical differences in the brain.** Shaw et al. (2006) studied over 300 children and adolescents to explore the relationship between their brain development and their IQ. Using magnetic resonance imaging, they found that children with superior intelligence—which they defined as having an IQ in the range of 121 to 149—had a markedly different brain development than children with IQs below 120, particularly related to the plasticity and rate of change in the thickness of the cerebral cortex. The cortex in gifted children’s brains reached its peak thickness later than it did in non-gifted children, suggesting that “high-level circuitry” (U.S. Department of Health and Human Services, 2006, para. 1) in the brain had a longer time to develop. This is important because abstract thought, information processing, and other executive functions take place within the cerebral cortex. This part of the brain also thins faster during a gifted child’s late teens which may indicate that unused neural connections are withering as the brain streamlines its operations (USHHS, 2006, para. 1). These findings suggest that nature plays an important role in giftedness.

**Processing differences in the brain.** Not only are there differences in the physical development of the brain between average and mathematically gifted children, but there are also differences in how and where the brain processes information. There have been several studies using electroencephalograms, functional magnetic resonance imaging, and behavioral tasks to assess the timing and location of processes within the brain (e.g., O’Boyle, Benbow, & Alexander, 1995; O’Boyle et al., 2005; Singh & O’Boyle, 2004). These studies have found that mathematically gifted children have enhanced right hemisphere development. In fact, their right hemispheres have been shown to be equally effective at what would normally be considered a left hemisphere task, such as processing
verbal inputs (O’Boyle, 2008). Their brains also rely more on the right hemisphere’s visual-spatial capabilities, suggesting the importance of visual imagery for high-level mathematical reasoning (O’Boyle & Benbow, 1990). Mathematically gifted students’ brains also seem to be more efficient at passing information between the two hemispheres than those of average children (O’Boyle, 2008). In fact, it is this “enhanced brain connectivity” (Kalbfleisch, 2008b, p. 155) that seems to contribute most strongly to giftedness in math.

These brain differences also suggest that mathematically gifted/talented students may not use the same cognitive strategies as average children. For example, when students were asked to mentally rotate a three-dimensional shape, the amount of brain activation for mathematically gifted children was several times greater than for average children and the activated areas were distributed in very different locations—specifically in areas that are known to mediate working memory, spatial attention, and executive functions—suggesting enhanced processing resources (O’Boyle et al., 2005). These findings may help explain why several educational studies have shown that gifted students tend to process information more rapidly than average students (e.g., Geary & Brown, 1991; Krutetskii, 1976; Swanson, 2006). In fact, Perleth, Schatz, and Monks (2000) stated that “the superiority of gifted children may be attributed to higher cognitive efficiency, i.e. to a higher basic speed of information processing and a higher level of automation” (p. 304). Some researchers suggest that the cumulative effect of faster processing yields a vastly increased knowledge base, greater cognitive proficiency, and more sophisticated intellectual skills in gifted children (Steiner & Carr, 2003). Similar to
findings related to the physical development of the brain, these findings also suggest the importance of nature in the development of gifted students' cognitive abilities.

**Cognitive functions.** Researchers and educators alike have noted several differences in the cognitive functioning of average and mathematically gifted/talented students. These differences are apparent in the two groups' memory, insightful thinking, metacognition, strategy knowledge and use, and problem solving approaches.

* Differences in memory. O'Boyle et al.'s (2005) findings lend support to the recurrent theme in gifted education literature that gifted students have high-capacity memories and are faster at retrieving information from memory than their average peers (e.g., Davis & Rimm, 2004; Schneider, 2000; Silverman, 1993). Perleth et al. (2000) pointed out that “the efficiency of the memory system is considered to be the main cause . . . [of the] differences in the achievement of gifted, average, and retarded children” (p. 304). These efficiency differences are apparent within the short-term, long-term, and working memories. In addition, Geary and Brown (1991) found a “nearly adult-like long-term memory organization of basic facts” (p. 404) in the mathematically gifted third and fourth grade students they studied, while Dark and Benbow (1990) found that mathematically gifted students were able to represent and manipulate material in their working memories better than other students.

Other studies have shown that working memory capacity appears to be domain specific. For example, Dark and Benbow (1990) gave 80 seventh grade students and college undergraduates two recall tasks, one involving a series of numbers and the other involving characters located in various spaces. They also gave 64 seventh and eighth grade students two recall tasks involving lists of letters, digits, words, and locations, as
well as numbers paired with letters (Dark & Benbow, 1991). Their studies found that mathematically gifted students had a larger working memory span for numbers than verbally gifted students or average students. They also had an enhanced ability to manipulate numerical and spatial information in their working memory, outperforming both verbally gifted and college students in these areas. Similarly, Robinson, Abbott, Berninger, and Busse (1996) found working memory abilities to be domain specific in their study of over 300 mathematically precocious kindergarten and first grade children. The National Mathematics Advisory Panel (USDOE, 2008b) made a similar observation. They pointed out that mathematically gifted students appear to have an enhanced ability to retrieve numerical and spatial – but not verbal information – from long-term memory and an enhanced ability to manipulate it in their working memory. These memory advantages are important as they play a role in a student’s insightful thinking, metacognition, and strategy use.

**Differences in insightful thinking.** Another distinctive characteristic of gifted students is their exceptional insightfulness (Davis & Rimm, 2004). Krutetskii (1976) explained that mathematical insight allows mathematically gifted students to omit many of the seemingly essential links in a logical train of thought. When solving a problem, they have an ability to think in “curtailed structures” (Krutetskii, 1976, p. 273), and “the problem solves itself” (p. 274). The result is a significantly shortened processing time.

There are three basic cognitive sub-processes—*selective encoding, selective comparison*, and *selective combination*—that form the basis of insightful thinking (Davidson & Sternberg, 1984). Selective encoding involves sorting out and encoding the information that is relevant to solving a problem; selective comparison involves
comparing new information to material that was previously learned and retrieving from memory only the material that is relevant; and selective combination involves assembling the relevant pieces of information together in the working memory to come up with a solution (Lohman, 2000; Steiner & Carr, 2003). Davidson and Sternberg (1984) conducted a study using mathematical insight problems and found that gifted elementary students performed better than average students in all three areas. Of particular interest was the fact that gifted students performed selective comparison spontaneously. The researchers did not distinguish between students who were mathematically and verbally gifted, and therefore, it is unknown whether spontaneous selective comparison is related to an overall high cognitive ability or to a specific domain.

More recently, Gorodetsky and Klavir (2003) found that gifted and average middle school students actually used different cognitive sub-processes than their non-gifted peers to solve insight problems. The gifted students used selective encoding and selective combination, whereas the average students focused more on selective comparison and retrieval. Gorodetsky and Klavir proposed that retrieval is an additional sub-process involved in problem solving. It involves the activation of the concepts that allow individuals to interpret problems in their own terms. They found that gifted students were able to retrieve and reorganize new problems into familiar terms automatically. These findings suggest that gifted/talented students use a faster, more efficient cognitive process and they may use different reasoning to arrive at a solution.

**Differences in metacognition.** Another area in which mathematically gifted/talented students differ from their non-gifted peers is in their metacognitive skills. Metacognition consists of two components, oftentimes distinguished as metacognitive knowledge and
**metacognitive control.** Metacognitive knowledge involves declarative knowledge (knowledge about one’s own cognitive abilities), procedural knowledge (knowledge about cognitive strategies), and conditional knowledge (knowledge about why and when to use procedures) (Schraw & Graham, 1997). Metacognitive control includes the processes that control one’s thinking or learning and includes components such as error detection and correction, inhibitory control, planning, and resource allocation (Fernandez-Duque, Baird, & Posner, 2000). Both metacognitive knowledge and metacognitive control appear to develop earlier in gifted children than in their average peers (Schraw & Graham, 1997). This is beneficial because having more metacognitive knowledge leads to better metacognitive control, and better metacognitive control leads to the acquisition of new metacognitive knowledge (Schraw & Dennison, 1994).

These metacognitive abilities contribute to the high performance of gifted individuals (Steiner & Carr, 2003). For example, gifted students are more likely to use their metacognitive skills to find solutions to complex problems for which they lack a suitable solution schema. On the other hand, average students tend to be more impulsive when searching for solutions and seem to lack the self-corrective processes that gifted children have (Gorodetsky & Klavir, 2003). These differences in metacognition favoring gifted students have been found with preschool children, elementary students, and adolescents (Steiner & Carr, 2003). In fact, some gifted elementary students use metacognitive strategies commonly found in adult experts (Shore, 2000). Furthermore, gifted students also tend to observe their own metacognitive behaviors more accurately than average students (Robinson, 2000).
Despite these findings, the relationship between giftedness and specific aspects of metacognition is not entirely clear. Alexander, Carr, and Schwanenflugel (1995) pointed out the importance of identifying the specific aspect of metacognition one is researching as there appear to be different giftedness effects related to the different components of metacognition. Their review of the literature on metacognition indicated that although gifted students seem to have a better declarative knowledge than their non-gifted peers, there did not appear to be significant differences in their cognitive monitoring ability, an aspect of procedural metacognitive knowledge that helps individuals recognize their limitations and abilities of their cognitive processes when they perform a task. They suggested that “average intelligence may be all that is required for cognitive monitoring to occur” (Alexander et al., 1995, p. 17). In fact, Carr, Alexander, and Schwanenflugel (1996) suggested a ceiling effect for certain aspects of metacognition, stating that IQs higher than about 115 do not make an independent significant contribution to differences in metacognitive functioning or achievement. Instead, high achievement is believed to involve the development of good metacognitive knowledge aligned with good strategy use within a domain of expertise. (p. 215)

They pointed out, however, the need for further research on their ceiling hypothesis. These researchers have found, however, that there are other aspects of metacognition that show a definite relationship with giftedness. Declarative metacognitive knowledge and far transfer show a monotonic relationship with giftedness, while the spontaneous use of simple and complex strategies shows a more accelerated relationship with the effects of giftedness increasing with age (Alexander et al., 1995).
**Differences in strategy knowledge and use.** Some researchers believe that a student’s actual use of a strategy “is the ultimate criterion for determining whether children possess usable metacognitive knowledge” (Carr et al., 1996, p. 213), but the relative contribution of metacognition, intelligence, and knowledge base to a student’s strategy selection is unclear (Alexander et al., 1995). In general, however, research has shown that mathematically gifted students have better strategy knowledge, and are better able to transfer this knowledge to novel situations than are non-gifted students (Carr et al., 1996). They seem to understand the types of tasks on which a strategy is effective, the amount of effort a particular strategy took when using it on a previous task, and the likelihood of a particular strategy being successful on other problems. They also seem to use strategies more flexibly (Davis & Rimm, 2004). In addition, gifted students become increasingly better at their spontaneous use of strategies within a domain as they get older compared to their non-gifted peers (Alexander et al., 1995). This is not to say that gifted students use strategies that are unique only to them; however, there are clear differences in the speed and fluency with which different strategies are invoked and used (Shore, 2000).

**Variability.** Siegler (1996) likened children’s acquisition of new strategies to the metaphor of overlapping waves with “a gradual ebbing and flowing of the frequencies of alternative ways of thinking with new approaches being added and old ones being eliminated” (p. 86). In other words, students rely on a variety of strategies over time to solve various problems, with the less effective strategies gradually disappearing from their repertoire. The elimination of ineffective strategies may help explain the fact that gifted students show a higher level of consistency—and therefore lower variability—in
their strategy use and are able to “adapt readily to cognitive tasks, consistently using a strategic approach that yields optimal performance” (Coyle et al., 1998, p. 284). Because gifted students are able to inhibit task-irrelevant information, they have the cognitive resources available to execute the optimal approach, whereas less able students may not be able to do so (Coyle et al., 1998). For example, Swanson’s (2006) research on the cognitive processes that underlie mathematical precociousness showed that average children had to use some of their limited working memory capacity to resist interference from material that was not central to various tasks, whereas mathematically gifted students were better able to inhibit non-relevant information from entering the working memory, thus leaving room to process other information.

*Use of higher-level strategies.* This does not mean that gifted students do not show some variability in their strategy selection, but when faced with an unfamiliar task, they quickly develop and rely on higher-level strategies while average students tend to rely on lower-level strategies (Steiner, 2006). For example, gifted elementary math students may use strategies similar to those used by older children (Geary & Brown, 1991). In fact, these students may use strategies frequently found in adult experts, such as working with a plan and organizing their knowledge in a hierarchical manner (Shore, 2000). This suggests that teachers need to be aware that their mathematically gifted/talented students may approach tasks using atypical or unexpected strategies.

*Differences in problem solving.* Differences in strategy use also impact how mathematically gifted/talented and average students approach and solve problems. Sriraman (2003) found that gifted high school students employed a consistent problem-solving approach of using simpler cases to model the solution method for more complex
problems. These gifted students were consistently able to understand problem situations, assess the given information, and identify and understand the assumptions related to the problems. In contrast, average students approached problems inconsistently, had a difficult time comprehending the problem situations, used the numbers given in the problem without regard for their importance, and did not understand the assumptions or made assumptions up.

Problem-solving focus. These differences between average and mathematically gifted/talented students may reflect the dissimilar ways in which the two groups view mathematical problems. Average students tend to associate each problem-solving technique with a particular problem and to focus on the surface features of the problem rather than its conceptual underpinnings, whereas mathematically talented students consider a variety of problem-solving techniques and look for the conceptual connections among them and the problem at hand (Grouws, Howald, & Colangelo, 1996). In fact, Grouws et al. (1996) found that almost all of the mathematically talented high school students in their study believed that finding solutions to one type of mathematics problem could help with finding solutions to other types of problems, while less than half of the average students believed that to be the case. Similarly, Garofalo (1993) found that mathematically gifted seventh graders had a meaning-oriented approach to problem solving, while the average students had a number-oriented approach. This suggests that not only do gifted students look beyond the numbers for the deeper connections tying mathematical concepts together, but their insightful thinking and ability to inhibit irrelevant information may make their solution approaches different from typical students.
Ability to generalize. There is also a difference in the way mathematically
gifted/talented students are able to generalize and reflect on problems. Krutetskii (1976)
found that gifted children were able to come up with broad generalizations “on the spot”
(p. 249), whereas average students were only able to form isolated, concrete associations
related to the given problem. Although average students were able to gradually turn
these associations into generalizations, it required additional time and practice to do so.
Likewise, Sriraman (2003) found that gifted students were able to abstract the similarities
in the structure of problems, verbalize the common principles using analogies, and come
up with plausible examples and non-examples that fit the generalization. In contrast,
average students focused on superficial similarities in the wording of the problems. Their
comparisons between problems were inconsistent and they had difficulty articulating
generalizations. This implies that a teacher must provide additional opportunities for
typical students to see connections between problems—time that might be put to better
use for gifted students by providing them with enrichment or acceleration opportunities.

Differences in conception of mathematics. Not only do they solve problems in a
different ways, but mathematically gifted/talented students view mathematics as a
discipline differently than typical students. Grouws et al. (1996) found that
mathematically talented students believed the real utility of mathematics was in the
underlying concepts, principals, and generalizations, whereas the average students
viewed mathematics as simply implementing procedures. Furthermore, talented students
saw mathematics as “a sense-making process which establishes mathematical knowledge
through personal reflection and justification” (p. 25). In fact, almost twice as many
talented as average students felt that by independently trying to solve problems, one
could learn mathematics. Non-gifted students, on the other hand, relied more on outside sources such as the teacher or textbook rather than on internal reflection to justify their answers. In short, gifted/talented students seem to view mathematics as “a way of thinking” (Sriraman, 2003, p. 163) rather than simply operations on numbers.

The gifted/talented student’s view of mathematics is similar in many ways to that of a professional mathematician. Sriraman (2004b) studied the creative process among mathematicians and found that they spent a considerable amount of time preparing to solve the problem, used an informal trial-and-error approach guided by their intuition, looked for examples and non-examples to gain insight into the problem, and needed time for incubation and the resulting illumination. He then conducted a study with mathematically gifted ninth grade students and found that they approached unfamiliar math problems using techniques very similar to those of professional mathematicians (Sriraman, 2004a). Likewise, they demonstrated some of the personal characteristics found in professional mathematicians including persistence (Diezmann & Watters, 2002b; Waxman et al., 1996), flexibility (Shore & Kanevsky, 1993), and the ability to reverse a logical train of thought (Krutetskii, 1976). These habits of mind are important because they “provide the dispositions necessary to do the skillful thinking required within and beyond the classroom walls” (Costa, 2003, p. 327).

**Summary.** Overall, educational research and cognitive neuroscience have shown that mathematically gifted/talented students are cognitively different in several ways from their non-gifted peers. They have differences in the physical structure of their brains, use different parts of their brain to process material, and are able to process material quicker. They can manipulate mathematical material better in their working memories, and they
have more insightful thinking and generally better metacognitive skills than average students. They quickly develop and rely on higher-level strategies, approach problem solving by looking at the structures of problems, and have unique abilities to generalize. They view mathematics through a lens of conceptual connections rather than as numbers and procedures, and they approach complex problems in ways similar to professional mathematicians, sometimes requiring additional time to plan their solution approach.

Because of these cognitive differences, mathematically gifted/talented students need more challenging material to stimulate their thinking and need to move through basic material at a quicker pace than other students. Research consistently reveals that “accelerated and demanding instruction is needed for these students to reach their full potential in mathematics” (USDOE, 2008b, p. 4-109). Gifted/talented students also need to have a supportive environment where the teacher provides them the attention, assistance, and modeling they need to achieve higher levels of learning. The next section will discuss various differentiation strategies that teachers can use to help facilitate these needs.

Despite the things we do know about mathematically gifted/talented students, there are many shortfalls in the literature. There has been little communication between the gifted and cognitive development fields resulting in few cognitive development studies conducted with gifted students (Steiner & Carr, 2003). Similarly, the National Mathematics Advisory Panel (USDOE, 2008b) noted that there are “only a handful” (p. 4-109) of studies that examine the cognitive processes underlying mathematically gifted students’ accelerated learning. Several of these studies are over a decade old, and although they are still valid, advancements in cognitive neuroscience suggest that updated
research is warranted. In addition, much of the research uses an arbitrary IQ score or test score to identify gifted students, treating them as a homogeneous group, when we know that there are differences among and between students gifted in a certain domain. Additionally, while many studies compare gifted/talented students to their average age mates, there are relatively few that are longitudinal in nature. Similarly, there are relatively few that compare gifted students to older students with a similar mental level (Steiner, 2006). Finally, there is a lack of research on how gifted students develop strategies; the relative contributions of intelligence, metacognition, and knowledge base to strategy use; and what causes gifted children to use particular strategies (Steiner & Carr, 2003; Steiner, 2006).

The research matrix at Table 3 identifies the literature that contributes to the knowledge base on mathematical giftedness. A detailed description of the studies is at Appendix B.
Table 3

*Mathematical Giftedness Research Matrix*

<table>
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<tr>
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Mathematically Gifted/Talented Students’ Needs and Differentiation Strategies to Address Them

Mathematically gifted/talented students’ needs. Because we know that mathematically gifted/talented students view and approach mathematics differently than typical students, it is important that we modify their classroom experience to enhance their learning. Before we can determine the best way to adjust their instruction, it is important to understand that these students have a need for an appropriate pace, challenging tasks, and a supportive environment. Failure to address the needs of mathematically gifted/talented students may cause them to stagnate. A case in point may be seen in Matthews and Farmer’s (2008) study of over 3,600 middle school students which found that gifted students’ abilities were not necessarily reflected in their Algebra I achievement. Unfortunately, many teachers do not view middle school as a crucial period in a student’s academic life. A national survey of 949 middle school principals and teachers (Moon, Tomlinson, & Callahan, 1995) found that almost half of them believed that middle school students are “in a plateau learning period – a theory which supports the idea that basic skills instruction, low level thinking, and small assignments are appropriate” (p. vi).

Need for appropriate pace. The National Math Panel (USDOE, 2008c) pointed out the need for mathematically promising students to move through curricular material at a quicker pace. They stated that “mathematically gifted students with sufficient motivation appear to be able to learn mathematics much faster than students proceeding through the curriculum at a normal pace, with no harm to their learning, and should be allowed to do so” (p. 53). Studies of mathematically gifted students have consistently shown that they
succeed in faster paced classes and tend to retain more when they are in accelerated situations (Rogers, 2007). In fact, Rogers pointed out that “if bright children are to retain what they have learned in mathematics and science, it must be presented at their actual learning rate, not considerably slower than that rate” (p. 390). Moreover, learning math at a faster pace or earlier than normal allows students to be better prepared for college science classes and more advanced in their math education (Sadler & Tai, 2007).

Because gifted students learn more rapidly than non-gifted peers and need less practice to achieve mastery, a teacher needs to ensure she maintains an appropriate pace for these students (Diezmann, 2005). What may be a learning task for the majority of the class may be a practice task for gifted/talented students who already know the material. Just as unchallenging material may cause gifted/talented students to become unmotivated and bored, practicing an already-known skill and moving at too slow of a pace may have the same effect.

Although mathematically gifted/talented students are able to move through the content of the course at a quicker pace, it is important to keep in mind that they may actually need more time to solve certain problems. Sriraman (2004a, 2004b) found that like professional mathematicians, gifted students took longer to plan and execute their strategies than the average students, and so it slowed the time it took them to solve the problem. Steiner (2006) had similar findings, noting that because gifted students took into account the outcomes of their previous attempts to solve problems, they took more time to plan their next attempt. In addition, for students to develop mathematical creativity, they need to be given complex problems which require persistence and reflection (Sriraman, 2004b). These types of complex problems require careful analysis,
planning, and flexible thinking, which cannot be rushed (Garofalo, 1993). Furthermore, gifted students oftentimes prefer to learn all they can about a mathematical topic before moving on to new concepts (Kim, 2006), and by allowing them time to explore ideas in more depth, the teacher may help them avoid the frustration they feel when they are told to move on to another topic.

Overall, flexible pacing is a key component of a math program for gifted students (Miller, 1990) as it allows them to move through the content quicker than their peers, yet provides them time to delve into the focused, in-depth kind of work that is essential to keep them engaged (VanTassel-Baska, 2003). Acceleration and curriculum compacting are effective methods of adjusting the pace for gifted/talented students. In addition, grouping by ability level assists teachers in adjusting the pace to be more appropriate for gifted students (Kim, 2006).

**Need for challenging tasks.** Because mathematically gifted/talented children may think about and solve problems differently than average students, they need to be challenged with greater depth and breadth, complexity, and abstraction (Aussouline & Lupkowski-Shoplik, 2005; Johnson, 2000). Students should “have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort” (NCTM, 2000, p. 52). Such problems should focus on concepts rather than just procedures, and encourage students to analyze a situation, apply or adapt various strategies, and incorporate various skills and processes to discover a solution.

Challenging these students is important because if gifted/talented children are merely asked to solve problems they already understand, they will not be engaged in higher-level cognition. Diezmann and Watters (2002b) found that gifted students who worked on
optimized math tasks clearly showed more cognitive engagement than when working on regular math tasks. Optimized math tasks have been “problematized” (Hiebert et al., 1996, p. 12) to increase their cognitive challenge by requiring students to engage in novel solution processes or by increasing the obstacles to solution-finding. These types of problems encourage students’ metacognitive skills, facilitate cognitive development by providing opportunities for high-level thinking and reasoning, enhance motivation, and help students to develop their “mathematical power” (Diezmann & Watters, 2000, p. 2). In fact, students who have an opportunity to regularly engage in high-level reasoning and problem solving outperform students who do not have this opportunity (Silver & Stein, 1996). In addition, gifted students enjoy being able to work out challenging problems with which they had originally struggled (Diezmann & Watters, 2000; Waxman et al., 1996). In fact, Garofalo (1993) found that mathematically gifted students preferred more complex and challenging problems over simpler ones because of the sense of accomplishment they felt when successfully solving them. Such intrinsically motivated students tend to exhibit many pedagogically desirable behaviors such as persistence, creativity, and greater risk taking (Middleton & Spanias, 1999). Although all students benefit from appropriate challenge, teachers need to ensure that the level of material presented in a standardized curriculum offers the complexity gifted/talented students require. The differentiation strategies presented below may help to address this.

Challenging material is also important because it causes the learner to “exert more attentional effort and to actively process information, leading to superior retention” (USDOE, 2008b, p. 4-7). In fact, encouraging students to solve challenging problems can promote growth in various parts of the brain, which makes the brain even more
capable of solving problems (Sheffield, 1999). On the other hand, if the instructional material is redundant and beneath a student’s readiness level, their brain does not become engaged and therefore does not release the levels of neurochemicals such as serotonin, noradrenalin, and dopamine, required for optimal learning (Schultz, Dayan, & Montague, 1997; Tomlinson & Kalbfleisch, 1998). Moreover, gifted students who are not challenged may become bored, frustrated, or disruptive (McNabb, 2003; VanTassel-Baska, 2003). There are several differentiation strategies including enrichment, open-ended activities, and tiered assignments and questions, that can help address gifted/talented students’ need for challenging material.

**Need for supportive environment.** Whether in a heterogeneous class or a homogeneous group, mathematically gifted/talented students need to have a supportive learning environment. Rayneri, Gerber, and Wiley’s (2006) study of gifted middle school students found that teachers who provided an “appropriately stimulating and flexible learning environment” (p. 116) made a positive difference in student performance. Henningsen and Stein (1997) found that such an optimized environment for mathematics included support factors such as appropriate scaffolding and suitable amounts of time to work on problems, as well as sustained pressure by the teacher for explanation and meaning and teacher modeling of high-level performance. When gifted students encounter optimized math tasks in optimized environments, they demonstrate persistence, collaboration, flexibility in their thinking, metacognition, and inventions of new strategies (Diezmann & Watters, 2002b).

One of the most essential elements in a supportive learning environment is appropriate scaffolding. The fact that mathematically gifted/talented students may need
assistance is often overlooked as is evidenced by the fact that on the previously-mentioned national survey (Farkas & Duffet, 2008), only 5% of teachers said they gave one-on-one attention to advanced students whereas 81% gave it to academically struggling students. Just like average students, however, gifted students need the “support necessary to achieve the new levels of proficiency” (NAGC, 2008, para. 7).

While many gifted/talented students are able to understand complex material with less help than typical students, scaffolding remains a factor that supports high-level cognition. However, while scaffolding should be strategically used for challenging tasks, it should be avoided for unchallenging tasks. In fact, scaffolding for a whole class creates “a paradoxical situation” (Diezmann & Watters, 2002b, p. 3). When students are given a problem which presents a high degree of challenge, but is within their zone of proximal development (Vygotsky, 1978), the task has a high cognitive value which enhances their potential for learning. However, when teachers provide scaffolding to the entire class, it lowers the cognitive value of the task for gifted students who do not require such assistance, thereby limiting their potential for learning (Stein, Grover, & Henningsen, 1996).

Another key aspect of a supportive learning environment is the teacher actively engaging the students for an explanation of how they solved the problem. Krutetskii (1976) found that gifted students oftentimes unconsciously determined the approach to a problem based on previous methods of operation. Because they seem to “skip steps,” they may have difficulty explaining how they arrived at their answer, and because they solve problems rapidly, they oftentimes do not reflect on their solution strategy.
(Diezmann & Watters, 2001). Asking them to explain their thought process helps improve their metacognitive skills.

Several of the differentiation strategies mentioned below help to create a supportive learning environment for mathematically gifted/talented students. Acceleration and enrichment can help a teacher adjust the pace and complexity of the material which may keep gifted/talented students engaged. Similarly, various grouping strategies and providing alternatives and choices are also useful differentiation techniques to help teachers provide a more optimal learning environment.

The research matrix at Table 4 identifies the literature that contributes to the knowledge base on the needs of mathematically gifted/talented students. A detailed description of these studies is at Appendix C.
Table 4

Needs of Mathematically Gifted/Talented Students Research Matrix

<table>
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<tr>
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**Differentiation strategies.** Differentiation offers a way to help meet these students’ needs. Unfortunately, teachers typically do very little curricular or instructional
differentiation for high-ability students (Archambault et al., 1993). This is alarming, especially considering the fact that a child’s motivation toward mathematics generally crystallizes into its adult form in around the seventh grade (Middleton & Spanias, 1999). In fact, Westberg, Archambault, Dobyns, and Slavin (1993) observed 46 classrooms across the United States, and found that there was no curricular or instructional differentiation for high-ability students in 84% of their learning activities. Moreover, they found only 11% of the mathematical activities for gifted students contained advanced content. Although their study was conducted in elementary schools, findings in middle schools are unlikely to be much different. This is concerning in light of the National Math Panel’s Final Report (USDOE, 2008c), that points out the benefits of differentiated instruction for high-ability students, especially when it involves adjusting the pace and level of instruction for these students.

Burns, Purcell, and Hertberg (2006) pointed out that one of the traits of a high-quality curriculum for all students is differentiation. For instance, Gamoran and Weinstein (1998) conducted a yearlong study of 24 “detracked” schools elementary, middle, and high schools observing math and social studies classrooms, interviewing teachers, and evaluating student work for authenticity. They found that high-quality heterogeneous classrooms were most effective for the students when differentiation was used. Differentiation allows students to gain access to the curriculum via different entry points, learning tasks, and outcomes tailored to their individual needs (Access Center, 2005). Specific differentiation strategies such as acceleration, enrichment, flexible grouping arrangements, and individualization have all been shown to be effective approaches to meet the needs of mathematically gifted students (USDOE, 2008b). These
strategies can address their need for appropriate pace, challenging tasks, and a supportive learning environment.

**Acceleration.** Research has repeatedly shown the benefit of acceleration for gifted students (USDOE, 2008b). For example, Kulik (1992) found that gifted students outperformed their peers of equivalent age and IQ by almost a year on achievement tests when enrolled in an accelerated class. Similarly, Kulik and Kulik (1992)’s meta-analysis of the literature concerning grouping found that in all of the 11 studies that compared gifted same-age students who were initially equivalent in aptitude, the students who were placed into an accelerated class showed greater achievement. The average effect size was .87. Although 85% of teachers favor subject acceleration for advanced students (Farkas & Duffet, 2008), schools may neglect to consider this as they increase the enrollment of nongifted students in Algebra I. This is unfortunate as there are at least 18 different types of acceleration (Colangelo, Assouline, & Gross, 2004a) for schools to consider, including such practices as subject-matter acceleration/partial acceleration, where advanced students are placed with older age students in a particular subject area; extracurricular programs, which involve students taking advanced coursework in the summer or after school; and telescoping the curriculum, which enables students to complete a course in a shortened period of time. Continuous progress acceleration is another option which allows students to progressively receive material as they complete and master previous material. Providing sequenced material may either be at the discretion of the teacher or within the student’s control. Self-paced instruction is a subset of continuous progress acceleration, but differs in the fact that the student controls pacing decisions (Colangelo, Assouline, & Gross, 2004b). Ysseldyke, Tardew, Betts, Thill, & Hannigan (2004) found
that individualized, self-paced instruction that was matched to a student’s skill level improved gifted students’ performance in the areas of math concepts, skills, applications, and computation.

There are many viable acceleration options for mathematically gifted/talented students enrolled in Algebra I which may be offered to them as individuals or in groups; however, educators need to consider individual student needs to determine which strategy is most appropriate for them (Rogers, 1991, 2002). Table 5 identifies the literature that contributes to the knowledge base on acceleration. A detailed description of the studies may be found at Appendix D.
Table 5

*Acceleration Research Matrix*

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<tr>
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<td>Ysseldyke et al. (2004)</td>
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*Curriculum compacting.* Curriculum compacting is an effective method of acceleration that addresses a gifted/talented student’s readiness by adjusting instruction to account for the learning objectives they have already mastered. It involves both
diagnosis and prescription (Rogers, 2002). Teachers must pre-assess students to determine what they already know, determine what they still need to know, and come up with a plan (usually involving enrichment or acceleration) for the time that is freed up due to elimination of the material that has already been mastered. This can account for a significant amount of material. Reis and Renzulli (1992) and Reis, Westberg, Kulikowich, and Purcell (1998) found that teachers could eliminate as much as 50% of the regular curriculum without an impact on student math achievement scores.

Eliminating repetitive learning is crucial for these students as repetition “often lead[s] to boredom, underdeveloped study skills, and disenchantment with school in general” (Reis & Renzulli, 1992, p. 51). In fact, Reis et al. (1993) found that students with a compacted math curriculum scored significantly higher on the math concepts portion of the Iowa Test of Basic Skills than did their peers who did not have a compacted curriculum. An example of curriculum compacting can be found at Johns Hopkins University’s Center for Talented Youth (n.d., Honors Algebra I section, para. 2), which routinely offers a three month Algebra I course for gifted students. Even more impressive, Brody and Benbow (1990) found that 13-year old students in the top 1% of academic ability can cover a whole year of mathematics course material in as little as three intensive weeks.

In fact, the Study of Mathematically Precocious Youth (SMPY) found that:

half of all 7th graders who score 500-800 on the SAT-M . . . know more algebra, as measured by the standardized algebra test, before they study the subject in school than do half of the students after completing a school year of it. (Stanley, 2000, p. 217)
Overall, many mathematically gifted/talented students in Algebra I could benefit from curriculum compacting as it is an effective method by which the teacher can adjust the pace of the course. By allowing students to move through the basic material more rapidly, they end up with extra time to investigate some of the more complex material they need to stimulate their thought processes. Table 6 lists resources related to curriculum compacting. A detailed description of these studies is at Appendix D.

Table 6

Curriculum Compacting Research Matrix

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Enrichment. One method by which teachers can provide challenging tasks for gifted/talented students is through enrichment. Enrichment allows students to be exposed
to a variety of topics that are related to the regular classroom material and to explore them in greater detail (Rotigel & Fello, 2004). Sheffield (1999) pointed out the importance of providing more challenging material to these advanced students stating, “services for our most promising students should look not only at changing the rate or the number of mathematical offerings but also at changing the depth or complexities of the mathematical investigation” (p. 45). Likewise, Gavin et al. (2007) talked about how enriched units may help to fill a curriculum void for meeting the needs of mathematically advanced students.

Mathematically gifted students think about mathematics in ways similar to that of professional mathematicians (Sriraman, 2004a), and thus need to be provided with opportunities to engage in the role of a practicing professional. Renzulli and Reis’s (1997) Schoolwide Enrichment Model emphasizes this role of practicing professional in their Type III enrichment activities. These activities are focused on real-world problem solving or creation of an original product to acquire “an advanced-level understanding of the knowledge (content) and methodology (process) that are used within particular disciplines” (Renzulli & Reis, 1997, p. 15). Students who participate in these independent study projects have been found to initiate their own creative products both in and out of school more frequently than other students (Starko, 1986) and these activities have been shown to serve as preparation for later productivity (Delcourt, 1993; Herbert, 1993).

Tieso (2002) found that math units with enrichment improved the academic achievement of high-ability students. Similarly, Kulik (1992) found that gifted students in enriched classes outperformed their peers of equivalent age and IQ on grade equivalent
scales by four to five months. Rogers (2002), however, warned that in many schools enrichment is provided infrequently and it is often “busy work” (p. 259), rather than a true learning experience for the gifted students. In fact, Assouline and Lupkowski-Shoplik (2005) identified four types of enrichment typically found in mathematics classrooms: busywork, irrelevant academic enrichment, cultural enrichment, and relevant academic enrichment. Only the latter type of enrichment serves to provide mathematically gifted/talented students with exposure to special topics that truly enrich their mathematics learning.

Enrichment provides mathematically gifted/talented students in Algebra I the opportunity to strengthen their higher-order thinking skills and to further explore challenging topics that may be of interest to them. This, in turn, may help them to develop a more complete understanding of principles, concepts, and generalizations (Rogers, 1991). Table 7 contains literature that contributes to the knowledge base on enrichment. A detailed description of these studies is at Appendix D.
Table 7

*Enrichment Research Matrix*

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*Open-ended activities.* Open-ended problems and activities are another way to challenge gifted/talented students. Rotigel and Fello (2004) pointed out that mathematically gifted students need to undertake complex tasks in the form of inquiry-based or discovery problems. These problems should not only require the students to use higher-order thinking skills, but should also provide them opportunities for open-ended responses. Because complex problems may have more than one solution or more than one way to find the answer, they require students to use extended reasoning (Johnson,
2000). The importance of selecting worthwhile problems that facilitate these types of activities for high-ability students cannot be overemphasized. Problem solving is an "integral part of all mathematics learning" (NCTM, 2000, p. 52) and Grouws and Cebulla (2000a, 2000b) pointed out that problem solving can help students develop more sophisticated mathematical skills while also helping them to understand concepts. In addition, problems that are open-ended frequently evoke students to pose their own problems which expand their mathematical thought processes and allow the teacher to assess their "intellectual agendas" (Waxman, Robinson, & Mukhopadhyay, 1996, p. 31).

Similarly, Sheffield (2000, 2003) proposed an open-approach heuristic to develop mathematic potential in gifted students. By allowing them to creatively investigate problems without set answers, students are encouraged to create, relate, investigate, evaluate, and communicate.

Open-ended problems and activities not only add a level of abstractness to the task, but they also provide students with an authentic learning experience. Professional mathematicians are frequently faced with open-ended issues to solve, and by allowing mathematically gifted/talented students the opportunity to grapple with these ill-defined tasks, we help them to develop the persistence and confidence to tackle these types of real-world problems. Table 8 identifies literature related to open-ended activities. A detailed description of these studies is at Appendix D.
Table 8

*Open-Ended Activities Research Matrix*

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<tr>
<th>References</th>
<th>Empirical Study</th>
<th>Think Piece</th>
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<td>Ball &amp; Bass (2003)</td>
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*Tiered assignments and questions.* A third way to challenge students is to adjust the depth of a lesson via tiered assignments. Tiered assignments allow the teacher to vary the problem-solving process or product to account for the student’s readiness level, enabling them to grapple with different levels of abstractness, complexity, or open-endedness. These strategies encourage students to think at deeper levels (Stepanek, 1999). By varying the level of difficulty, the teacher increases the chance that each student will be appropriately challenged and that all students will gain the essential skills and understandings for that particular topic (Tomlinson, 1999).

Just as a teacher can adjust the depth and complexity of a problem, he or she may also adjust these elements within questions. The use of multiple levels of questions encourages students to build off the ideas of others and allows the teacher to address connections between ideas that students might not necessarily make (Access Center,
2005). By adjusting the types of questions and the way in which they are asked, a teacher can easily encourage deeper thinking from the students. Open-ended questions are also an important tool for eliciting higher-level mathematical reasoning (Ball & Bass, 2003). In addition to questions that probe the depth of a student’s understanding, questions may also help a teacher elicit responses aimed at demonstrating a student’s fluency, flexibility, elaboration, originality, and reasoning (Sheffield, 2000, 2003), key aspects of creative and critical thinking.

Tiered assignments should be a strategy of choice for teachers of mathematically gifted/talented students in Algebra I. Rather than having these students practice problems that they already know how to solve, tiered assignments allow these students to work with algebraic concepts at a level that is appropriately challenging for them. Similarly, when a teacher asks different levels of questions to students based on their level of understanding, he or she provides the higher-ability students with an opportunity to become cognitively engaged by thinking about the material in a way that they may not have otherwise considered. Table 9 provides references related to tiered assignments and questions. A detailed description of these studies is at Appendix D.
Table 9

Tiered Assignments and Questions Research Matrix

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<tr>
<th>References</th>
<th>Empirical Study</th>
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<tr>
<td>Access Center (2005)</td>
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<td>Archambault et al. (1993)</td>
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<tr>
<td>Ball &amp; Bass (2003)</td>
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<tr>
<td>Diezmann &amp; Watters (2000)</td>
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<td>Johnson (2000)</td>
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<td>Sheffield (2000)</td>
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<td>Tomlinson (2000)</td>
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**Grouping strategies.** In the past, many gifted/talented students took Algebra I in homogeneous middle school classes or in classes comprised of mostly academically-advanced students, but because of the recent push for increased enrollment in the course, their classes may now be more heterogeneous. Gamoran and Weinstein (1998) found that heterogeneous grouping is more problematic for math teachers than for teachers of other subjects because the sequential nature of mathematic requires students to master certain concepts before moving on to others. To reach students at these various ability levels requires extra effort on the part of the teacher.

**Ability grouping.** Farkas and Duffet (2008) found that 74% of teachers believed “mathematics was the one subject where students could really benefit from homogeneous grouping” (p. 64) by allowing them to learn faster and in more depth, but 59% of them indicated there was little or no homogeneous grouping for advanced students in their
schools. This is unfortunate, as Gamoran and Weinstein (1998) found that grouping by ability encourages higher-quality instruction for academically-advanced students. In fact, the teachers in Gentry and Owen’s (1999) study found that ability grouping in math made it easier to challenge students at the appropriate levels. Rogers (1991) pointed out that ability grouping for enrichment produces substantial academic gains for gifted students in creativity, general achievement, and critical thinking. Furthermore, Niehart’s (2007) review of the literature on peer ability grouping identified several socioaffective benefits for gifted students including a more positive attitude toward the subject matter, increased development of career interests, increased motivation, and healthy social relationships. This is important because it is oftentimes difficult for these “atypical children to find like-minded peers” (Winner, 2000b, p. 163). In fact, Benbow, Lubinski, Shea, and Eftekhari-Sanjani (2000) found on their 20-year follow-up of 1,975 mathematically gifted adolescents who were involved in the Study for Mathematically Precocious Youth that 80% of them did not support the idea of eliminating homogeneous ability grouping for instruction in school. Clearly they saw the benefits of such an arrangement.

Delcourt, Cornell, and Goldberg (2007) conducted a two-year study of gifted elementary students and found that ability grouping was an effective educational practice for gifted children although it was least effective for gifted students when done in a within-class setting. They discovered that pullout classes, separate classes, or separate schools had substantially more impact on gifted students’ mathematics achievement than did within-class grouping. They pointed out however, that the other three types of grouping arrangements in their study had an academic focus, while the within-class program had lesser focus on academic skills. This is not to say, however, that within-
class grouping does not provide academic benefits. Kulik and Kulik’s (1992) meta-
analysis of the literature on grouping found a .30 effect size for high-ability students
when grouped within the class by ability. This arrangement was not detrimental for the
other students as was noted by the .18 and .16 effect sizes for average and low-ability
students, respectively. They noted that one of the benefits of within-class grouping is that
the teacher normally provides differentiated instruction for each group. Rogers’ (2007)
more recent synthesis of gifted research had similar findings, noting a .34 academic effect
size for within-class grouping. Rogers (2002, 2007) likewise noted the importance of
teachers differentiating curricular materials and tasks for the different ability groups.

Grouping is an important consideration because high-ability math students’
exceptionally strong short-term memory allows them to handle complex, unstructured,
and abstract math problems that are typically not found in regular classrooms (Dark &
four-fifths of a year additional academic achievement in math when grouped within a
class by performance level and provided with a compacted, fast-paced, beyond-grade-
level curriculum. Grouping by ability must be done with care, however. Westberg,
Archambault, Dobyns, and Slavin (1993) found that when gifted students were grouped
within classes by ability, their learning activities were not differentiated 84% of the time.

Flexible grouping. Mills, Ablard, and Gustin (1994), however, found that grouping
is not sufficient without differentiation to accommodate the specific abilities of gifted
students. Flexible grouping is a way of organizing students in ways that target these
individual differences in readiness, interest, and learning style (Tomlinson, 1995b).
Depending on the content or task, students may be assigned to various groups for
differing periods of time. Group assignments may be made by the teacher, students, or random assignment depending on the reason for grouping. Slavin (1987) pointed out that one of the biggest benefits of flexible grouping is the fact that the groups are temporary in nature. This strategy is helpful because it allows students to work with a variety of classmates and may avoid the issue of group labels that full-time ability grouping oftentimes provokes (Access Center, 2005). In addition, it allows the teacher to see the students in a variety of settings so he or she is better able to assess the strengths and weaknesses of each student in different group environments (Tomlinson, 2000). Tieso (2002) conducted a quasi-experimental study of mathematics instruction involving 31 fourth and fifth grade teachers and their 645 students from four diverse New England school districts. She found that students enjoy working in a variety of grouping arrangements and that placing them into different groups does not damage their self-efficacy or self-esteem. She also found that students who had a mid- or high-level of prior mathematical knowledge had an increased level of academic achievement over their like-ability peers when flexible grouping was used with a differentiated math unit.

Overall, grouping strategies are a part of a high-quality, comprehensive curriculum (Burns, Purcell, & Hertberg, 2006). There are times when Algebra I teachers should place the high-ability students together for various activities, but there are also times when these students should be grouped with other members of the class. The teacher needs to ensure that he or she takes these grouping strategies into consideration when determining how to best meet the needs of the students in the class. Table 10 identifies literature that contributes to the knowledge base on grouping strategies. A detailed description of these studies is at Appendix D.
Table 10

*Grouping Strategies Research Matrix*

<table>
<thead>
<tr>
<th>References</th>
<th>Empirical Study</th>
<th>Think Piece</th>
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<td>Dark &amp; Benbow (1990)</td>
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<td>Dark &amp; Benbow (1991)</td>
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<tr>
<td>Delcourt, Cornell, &amp; Goldberg (2007)</td>
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<td>Farkas &amp; Duffett (2008)</td>
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<td>Gentry &amp; Owen (1999)</td>
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<td>Mills, Ablard, &amp; Gustin (1994)</td>
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<td>Neihart (2007)</td>
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<td>Renzulli &amp; Reis (1997)</td>
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<td>Rogers (1991)</td>
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<td>Slavin (1987)</td>
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<td>Westberg, Archambault, Dobyns, &amp; Slavin (1993)</td>
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*Alternatives and choices.* Alternatives and choices offer another way to provide a supportive learning environment for gifted/talented students. VanTassel-Baska and
Stambaugh (2005) pointed out the importance of educators’ flexibility in content, process, and products when providing for the needs of the gifted students in their classroom. Such flexibility entails providing students with choices and alternatives in these areas. For example, students should occasionally be allowed to select the content they would like to investigate in greater detail, the modalities for gaining that knowledge, and the methods by which they will demonstrate what they have learned. Similarly, Stepanek (1999) pointed out that students should be allowed choice in deciding when and how they work in at least some of their activities while Johnson (2000) said that gifted students should be allowed choice in both individual and group activities. Choice plays a major role in the enrichment clusters that are a part of Renzulli and Reis’s (1997) Schoolwide Enrichment Model. Providing students with a choice helps to keep them engaged and encourages independence. By allowing high-achieving students to have a choice in activities, a teacher can provide them with a balance of accelerated and enriched activities that provide an appropriate challenge (Gentry & Owen, 1999). Table 11 lists literature related to providing students with alternatives and choices. A detailed description of these studies is at Appendix D.
Table 11

*Alternatives and Choices Research Matrix*

<table>
<thead>
<tr>
<th>References</th>
<th>Empirical Study</th>
<th>Think Piece</th>
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<td>Access Center (2005)</td>
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<td>Archambault et al. (1993)</td>
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<td>Gentry &amp; Owen (1999)</td>
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<td>VanTassel-Baska &amp; Stambaugh (2005)</td>
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**Summary.** Overall, because of their cognitive differences, mathematically gifted/talented students need complex, challenging tasks to facilitate cognitive development. Knowing and controlling one’s cognitive function is essential to high achievement (Schraw & Graham, 1997), in part because it helps students to realize when to modify their strategies. Without appropriate pacing and challenge, gifted students may become unmotivated and at risk for underachievement (Mills, Ablard, & Gustin, 1994). In fact, research consistently reveals that for these students to reach their full potential in mathematics, they need accelerated, demanding instruction (USDOE, 2008b). Gifted/talented students also need to have a supportive environment where the teacher provides them the attention, assistance, and modeling they need to achieve higher levels of learning. Differentiation strategies offer a way for teachers to address these needs so that gifted/talented students can reach their potential.
Although there is a large amount of research pertaining to differentiation strategies for gifted/talented students, relatively little of it addresses mathematically gifted/talented students during their middle school years, and none was found specifically addressing Algebra I. In addition, there has been limited research on whether regular classroom teachers provide adequate challenge for the gifted students in their class (Westberg et al., 1993) or the nature of mathematically gifted students' collaboration on complex tasks (Diezmann & Watters, 2001). Likewise, there have been few studies examining the effects of whole group instruction and curricular enhancements or ability grouping linked to the curricular enhancements and differentiation based on the prior knowledge of gifted students (Tieso, 2002). There is also a need for additional research comparing the effects of different grouping profiles within gifted programs (Delcourt et al., 2007).

Furthermore, the National Math Panel (USDOE, 2008b) pointed out the need for more high-quality experimental and quasi-experimental studies to determine the overall effectiveness of gifted interventions, particularly pertaining to the mathematical content of academically rigorous enrichment programs. In fact, there were so few high-quality studies available on the teaching of mathematically gifted students that the National Math Panel felt it necessary to relax the rigorous criteria by which they included studies (USDOE, 2008c).

**Conclusion**

Overall, the literature related to the topic of teachers differentiating their middle school Algebra I classes to meet the needs of their mathematically gifted/talented students reveals several things. First, research supports the value of having more students take Algebra I in middle school than are at enrolled in the course at the present time. We
know that algebra is beneficial for all students and that students who take the course prior to high school take more math courses and have higher mathematical achievement. This suggests that middle schools will continue to see their Algebra I course enrollments grow and their classes become increasingly heterogeneous.

Second, research from both the cognitive neuroscience and educational arenas supports the fact that mathematically gifted/talented students are different from their non-gifted peers. They process information differently, have better working memories, have more insightful thinking, develop metacognitive strategies and control earlier, view mathematical problems differently and use higher-level strategies to solve them, and have a conception of mathematics that focuses more on the underlying concepts than on the surface features. The literature is clear that mathematically gifted/talented students need to be exposed to more challenging material to be cognitively engaged, work at a quicker pace than the average student, and have a supportive learning environment that includes appropriate scaffolding, teacher modeling, and a focused effort by the teacher to pressure these students for meaning.

Finally, the extant literature supports the effectiveness of various differentiation strategies especially acceleration, enrichment, and homogeneous grouping in addressing the needs of gifted/talented students. Unfortunately, research also shows that teachers do not necessarily know how to address the academic diversity in their classrooms and oftentimes do not see the need to change their behaviors in order to do so (Tomlinson, 1995a). This attitude is damaging to gifted/talented students, especially if the teacher does not have a thorough understanding of the math content or an understanding of gifted students. A combination of these factors may prove to be detrimental to enabling these
advanced students to reach their potential. In fact, “without clear definitions of what constitutes appropriately differentiated classes, teachers may believe that making occasional minor modifications in lessons is adequate to address academic diversity” (Tomlinson, 1995a, p. 86). This study sought to discover what modifications teachers were actually making to meet the needs of their mathematically gifted/talented students.

In conclusion, just like every other student, mathematically gifted/talented students are unique individuals with educational needs that should be addressed. The fact that they process and understand mathematical material differently than typical students does not make them any less deserving of the best educational experience we can offer. As Winner (2000b) so accurately stated:

These children are our national resources, and we should cultivate them so they can become our future leaders and innovators. . . . Schools cannot be truly egalitarian unless they acknowledge learning differences, including those differences possessed by students of high ability. (p. 166)
Chapter 3: Methodology

Chapter three details the study’s methodology including the research design, research strategy, research sample, instrumentation, data collection, data analysis, trustworthiness and authenticity factors, and ethical considerations. The research design section includes a discussion of the type of study and the paradigm and theoretical perspective through which the researcher approached the study. The research strategy section outlines the methods by which the researcher gained access to relevant information. The sample section contains a description of the study participants, while the instrumentation section discusses the observation tool and its validity and reliability evidence as well as the interview question design. The data collection section describes the pilot study and the procedures used for the classroom observations and interviews, while the data analysis section outlines the methods by which the observation and interview data were analyzed. The next section addresses the validity and reliability of the study by describing the methods by which trustworthiness and authenticity were achieved. Finally, the ethical consideration section discusses safeguards to study participants.

Research Questions

1. To what extent do middle school Algebra I teachers modify the pace of instruction for their gifted/talented students?
2. In what ways do middle school Algebra I teachers increase the level of challenge for their gifted/talented students?

3. What differentiation strategies do middle school Algebra I teachers use to meet the needs of their gifted/talented students?

4. In what ways do middle school Algebra I teachers provide a supportive environment for their gifted/talented students?

Research Design

This descriptive study sought to determine the ways in which Algebra I teachers modified their instructional practices for the gifted/talented students in their classes. The purpose of a descriptive study is to enlighten and display deep insight by "depict[ing] complex social processes and understanding through detailed descriptions" (Rossman & Rallis, 2003, p. 18). By using a mixed methods approach, the researcher sought to quantify observed teacher behaviors toward gifted/talented students as well as to obtain the teachers' own perspectives related to their interactions with these students. The nature of the researcher's relationship with the teachers depended on both the rapport the researcher built with them, as well as a clear awareness of the effect of the researcher's own biases toward the topic of investigation (Glesne, 2006).

The study used an interpretivist paradigm which "tries to understand the social world as it is (the status quo) from the perspective of individual experience" (Rossman & Rallis, 2003, p. 46). Interpretivists "consider that every human situation is novel, emergent, and filled with multiple, often conflicting, meanings and interpretations" (Glesne, 2006, pp. 27-28). The researcher sought to co-construct knowledge with the teachers by building a thick description and rich explanation of how they modified the pace of instruction,
increased the level of challenge, and provided a supportive environment for their gifted/talented students, as well as any other ways in which they differentiated to meet the needs of these students. Comparing the teachers’ perspectives to their observed behavior allowed the researcher to interpret the extent to which the teachers met the needs of the high-ability students.

The researcher interpreted these elements using the theoretical perspective laid out in Gagne’s (2003) Differentiated Model of Giftedness. This model identifies giftedness as “the possession and use of untrained and spontaneously expressed natural abilities (called aptitudes or gifts) in at least one ability domain” (p. 60) and talent as “the superior mastery of systematically developed abilities (or skills) and knowledge in at least one field of human activity” (p. 60). To turn a gift into a talent requires a developmental process involving learning and practicing, with the biggest impact coming from formal institutional learning (Gagne, 2003). In other words, a child’s natural ability may be turned into an achievement in a domain through proper nurturing. Intrapersonal catalysts, environmental catalysts, and chance may interact with the learning and practicing process to facilitate or hinder talent development. For example, a child may be gifted, but that aptitude may never fully develop into a talent if he is unmotivated (an intrapersonal catalyst) because of a lack of challenge or appropriate pace in class, his teacher does not provide a supportive environment (an environmental catalyst), or if he is assigned to a teacher who does not understand differentiated instruction (chance).

Research Strategy

This research used a case study design to investigate the ways in which teachers modified their Algebra I course for their gifted/talented students. The purpose of a case
study is to “gather comprehensive, systematic, and in-depth information” (Patton, 2002, p. 447) about the topic of interest. The researcher looked at seven separate cases, thus creating a “collective case study” (Glesne, 2006, p. 13). By interviewing teachers and recording their behaviors during classroom observations using both a standardized observation tool and field notes, the researcher gained insight into the ways in which teachers addressed the needs of their gifted/talented students. By triangulating the data, the research sought to corroborate the findings (Gall, Gall, & Borg, 2007). Triangulation is important as it is “a mode of improving the probability that findings and interpretations will be found credible” (Lincoln & Guba, 1985, p. 305). Using the triangulated data, the researcher conducted a cross-case analysis to look for patterns across the cases (Glesne, 2006).

Prior to beginning the study, the researcher wrote a “researcher as instrument” statement (Appendix E) to discuss her experience with mathematically gifted/talented students, expectations of the study, values and beliefs related to the education of gifted/talented students, what she was willing and not willing to discover through her research, and to whom the results of the research may be useful. Throughout the study, the researcher kept a reflexive journal to reflect on self and to record various observations and insights pertaining to the study. A reflexive journal “is analogous to the anthropologists’ field journals and is the major means for an inquirer to perform a running check on the biases which he carried with him into the context” (Lincoln & Guba, 1982, p. 11). It should include logs of evolving perceptions, day-to-day procedures and personal introspections, methodological decision points, and developing hypotheses and insights (Lincoln & Guba, 1982). In other words, the journal records an
“ongoing examination of what I know and how I know it” (Patton, 2002, p. 64). A sample entry from the reflexive journal is at Appendix F. The researcher also used a community of practice and peer debriefer to critically analyze and review the research process and findings. Eliciting scrutiny from peers helps the researcher to obtain a deeper understanding of the material and can produce a more robust study (Rossman & Rallis, 2003).

**Research Sample**

This study used criterion sampling to determine the research participants (Patton, 2002). The teachers selected for this study taught Algebra I at the middle school level because most gifted/talented students take that course prior to their entrance into high school. There are 281 public middle schools containing grades 6 through 8 in Virginia (Virginia Department of Education, 2010). A medium-sized middle school (approximately 800 students) typically has two Algebra I teachers, indicating that the accessible population for this study was approximately 562 teachers. The researcher selected seven volunteer Algebra I teachers from middle schools within southeastern Virginia. The selected teachers taught heterogeneous Algebra I classes which included at least one student who had been identified as gifted. The table below shows the criteria by which the students were identified as gifted by the school district.
Table 12

Criteria for Gifted Identification

<table>
<thead>
<tr>
<th>Standardized/Norm-Referenced Ability Test</th>
<th>Standardized/Norm-Referenced Achievement Test</th>
<th>Teacher Observation Report</th>
<th>Parent Questionnaire</th>
<th>Scholastic Records</th>
<th>Other</th>
<th>Identify Students as Gifted by Domain?</th>
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<tbody>
<tr>
<td>95th-99th percentile (overall score)</td>
<td>95th-99th percentile (overall score)</td>
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<td>Yes</td>
<td>Yes</td>
<td>Interview; Samples of student work</td>
<td>No</td>
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The researcher checked the Virginia Department of Education website to ensure that the schools from which the teachers were selected listed all teachers as “highly qualified,” which indicated that the teachers were not teaching core academic subjects outside their area of endorsement. Because Algebra I requires a specific add-on endorsement, this ensured that the teachers selected for the study were legitimate Algebra I teachers. Teachers had a variety of overall years of teaching experience as well as different experience levels teaching Algebra I.

Instrumentation

Observations. The researcher used the Classroom Observation Scale-Revised (COS-R) created by the College of William and Mary’s Center for Gifted Education (VanTassel-Baska et al., 2005a). This was a public domain instrument which did not require permission for use or modification (B. Bracken, personal communication, April 27, 2010). Specific teaching behaviors from five of the six clusters of teaching behaviors listed in the Observable Evidence of Classroom Behaviors--Mathematics appendix of the COS-R were observed including Curriculum Planning and Delivery, Accommodations for Individual Differences, Problem Solving, Critical Thinking Strategies, and Creative Thinking Strategies. Four additional behaviors within these clusters as well as the
Research Strategies cluster were not observed because they did not directly pertain to the research questions. In fact, the COS-R mathematics appendix specified that the Research cluster of behaviors may not be applicable to mathematics classes.

The COS-R provided a quantitative mechanism by which to assess teachers’ behavior in relation to the high-ability students in their classes. The observation instrument was developed by a team of experts in gifted education using the extant literature pertaining to educational reform, effective teaching methods, differentiated instruction for gifted students, and professional development (VanTassel-Baska et al., 2005b). The tool may be used in all types of classrooms and in all subject areas. A specific appendix for each core subject area was developed to help observers identify potential behaviors related to each area. The full mathematics appendix is at Appendix G.

**Teacher behaviors.** Teacher behaviors related to mathematics that the researcher observed included the following 16 areas listed in the COS-R. The brackets indicate the aspect of the study the behavior addressed. The teacher:

1. Set high expectations for student performance [challenge, supportive environment].
2. Incorporated activities for students to apply new knowledge [supportive environment].
3. Engaged students in planning, monitoring, or assessing their learning [supportive environment, differentiation].
4. Encouraged students to express their thoughts [supportive environment].
5. Had students reflect on what they had learned [challenge].
6. Provided opportunities for independent or group learning to promote depth in understanding content [differentiation, challenge, supportive environment].

7. Accommodated individual or subgroup differences [differentiation].

8. Encouraged multiple interpretations of events and situations [supportive environment].

9. Allowed students to discover key ideas individually through structured activities and/or questions [differentiation].

10. Engaged students in problem identification and definition [challenge].

11. Engaged students in solution-finding activities and comprehensive solution articulation [challenge, supportive environment].

12. Encouraged students to judge or evaluate situations, problems, or issues [challenge].

13. Engaged students in comparing and contrasting ideas [challenge].

14. Provided opportunities for students to generalize from concrete data [challenge].

15. Solicited many diverse thoughts about issues or ideas [supportive environment].

16. Provided opportunities for students to develop and elaborate on their ideas [supportive environment]. (VanTassel-Baska et al., 2005a, Appendix A, pp. 1-4)

In addition to the 16 areas identified above, the following teaching behaviors related to the issue of pacing and the environment were added to the COS-R for use in this study.

The teacher:

17. Allowed the students to move through material at an individual pace [pace].

18. Allowed students sufficient time to thoroughly explore complex problems [pace].
19. Provided a reasonable amount of attention (as appropriate to the situation) to the gifted/talented students in the class compared to other students [supportive environment].

This modified observational tool was referred to as the COS-R (modified). A copy is located at Appendix H.

**Reliability and validity.** The COS-R was piloted on 50 teachers participating in William and Mary’s Saturday Enrichment Program for gifted students and was replicated during a later session with 17 additional teachers. The teachers in the replication study taught classes that were related to the development of problem-solving skills (two classes), math (three classes), science (five classes), and humanities (five classes). In addition, the COS-R was used twice during Project Athena, a Javits-sponsored program which implemented a language arts program to disadvantaged high-ability students in grades 3 through 5. Reliability from the pilot study and the two Project Athena observations were .92, .91, and .93, respectively. In addition, the subscale reliability for the six clusters from the Project Athena observations had an average above .70 (VanTassel-Baska et al., 2005b). For scales intended to be used for educational research, Wasserman and Bracken (2003) suggest reliability should be at least at a level of .70.

Six experts in gifted education reviewed the COS-R for content validity. They were asked to rate the importance of each behavioral item and the clarity of the language used in each item. The intra-class coefficient alpha was .86 for the importance and .99 for the clarity of language. The overall content validity was .98.

For this study, a panel of five gifted education experts likewise rated the clarity and relevance of the three items that were added to the COS-R (modified) to establish their
content validity. These experts all had their doctorates and were faculty and staff members of a gifted education center at a college located in the Mid-Atlantic. If there was less than an 80% agreement on the clarity and relevance of a behavior, it was carefully reviewed for modification or elimination.

Although the panel members found the first additional behavior (originally phrased as: *The teacher allowed the students to move through material at a pace appropriate to their level of understanding*) to be both clear and relevant to characterizing the pace of instruction, one member pointed out that since the researcher did not personally know the students, there would really be no way to determine each student’s level of understanding. This behavior was therefore rephrased as: *The teacher allowed the students to move through material at an individual pace.* All panel members agreed that the second behavior (*The teacher allowed students sufficient time to thoroughly explore complex problems*) was both clear and relevant, and thus it remained unchanged. The third behavior received the most comment. Although all members felt this behavior (originally written as: *The teacher provided a reasonable amount of attention to the gifted/talented students in the class compared to the other students*) was relevant to characterizing a supportive environment, three members felt that the wording should be clarified since different situations could call for a teacher to provide different levels of attention. This behavior was therefore reworded to read: *The teacher provided a reasonable amount of attention (as appropriate to the situation) to the gifted/talented students in the class compared to the other students.* The original and revised behaviors are listed in Table 13.
Table 13

*Changes to Teacher Behaviors Based on Panel of Experts’ Feedback*

<table>
<thead>
<tr>
<th>Original Behavior</th>
<th>The behavior was worded clearly</th>
<th>The behavior was relevant</th>
<th>Revised Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher allowed the students to move through material at a pace appropriate to their level of understanding.</td>
<td>100%</td>
<td>100%</td>
<td>The teacher allowed the students to move through material at an individual pace.</td>
</tr>
<tr>
<td>The teacher allowed students sufficient time to thoroughly explore complex problems.</td>
<td>100%</td>
<td>100%</td>
<td>No change</td>
</tr>
<tr>
<td>The teacher provided a reasonable amount of attention to the gifted/talented students in the class compared to the other students.</td>
<td>40%</td>
<td>100%</td>
<td>The teacher provided a reasonable amount of attention (as appropriate to the situation) to the gifted/talented students in the class compared to the other students.</td>
</tr>
</tbody>
</table>

**Interviews.** The researcher created an interview guide including standardized open-ended interview questions to guide the interview process. These questions were precisely worded and were provided to all the teachers in a particular order to ensure that the researcher asked the key questions in the same way. Because of the potential for the teachers to bring up unanticipated topics, it was important that the researcher be open to examining these new ideas. The interview guide provided the researcher with the flexibility to probe into areas of interest and to craft follow-up questions to pursue topics and themes that emerged (Rossman & Rallis, 2003).

The interview questions were submitted to the panel of experts mentioned above to gain feedback on their clarity and relevance as well as the order in which they were presented. Similar to the panel’s review of teaching behaviors, if there was less than an
80% agreement on the clarity and relevance of a question, it was carefully reviewed for modification or elimination. Of the 12 questions in the interview guide, the panel members reached this level of agreement on all questions except for question 5 (What strategies do you use to modify mathematical tasks to make solution-finding more challenging?). Two panel members thought the term solution-finding was confusing, and so this question was reworded to read: What strategies do you use to modify mathematical tasks to make them more challenging? The panel members also had several minor suggestions to improve the wording of the questions which the researcher took into consideration when creating the final version of the interview guide. In addition, all panel members suggested moving the final question relating to mathematical giftedness to the very beginning of the interview guide to lend focus to the interview. Furthermore, an additional question was added related to pre-assessment and question 10 was eliminated as it was viewed as redundant with the next question. A detailed explanation of these suggestions and the resulting modifications to the questions can be found in Appendix I. The final version of the interview guide is at Appendix J. The original and modified questions and numbering are in Table 14.
Table 14

*Panel of Experts’ Feedback on the Interview Guide*

<table>
<thead>
<tr>
<th>Original Question</th>
<th>The question was worded clearly</th>
<th>The question was relevant</th>
<th>Revised Question Wording and Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Question</td>
<td>N/A</td>
<td>N/A</td>
<td>1. In your opinion, what are indicators of mathematical giftedness?</td>
</tr>
<tr>
<td>1. How do you decide the pace for the class?</td>
<td>100%</td>
<td>100%</td>
<td>2. No wording changes</td>
</tr>
<tr>
<td>New question</td>
<td>N/A</td>
<td>N/A</td>
<td>3. How do you determine what your students already know?</td>
</tr>
<tr>
<td>2. How do you handle a student who has already mastered the material you plan to cover during a lesson?</td>
<td>80%</td>
<td>100%</td>
<td>4. What modifications do you make for a student who has already mastered the material you plan to cover during a lesson?</td>
</tr>
<tr>
<td>3. How do you balance the time spent in practice versus learning tasks for your gifted students?</td>
<td>80%</td>
<td>80%</td>
<td>5. How do you balance the time spent on practice of known concepts versus learning new concepts for your gifted students?</td>
</tr>
<tr>
<td>5. What strategies do you use to modify mathematical tasks to make solution-finding more challenging?</td>
<td>60%</td>
<td>100%</td>
<td>6. What strategies do you use to modify mathematical tasks to make them more challenging?</td>
</tr>
<tr>
<td>4. What happens when a high-ability student wants to spend more time working on a problem?</td>
<td>100%</td>
<td>100%</td>
<td>7. When you raise the level of complexity for your advanced students, how do you deal with the additional time they may need to work on such problems?</td>
</tr>
<tr>
<td>6. How do you differentiate your instruction?</td>
<td>100%</td>
<td>100%</td>
<td>8. No wording changes</td>
</tr>
<tr>
<td>Original Question</td>
<td>The question was worded clearly</td>
<td>The question was relevant</td>
<td>Revised Question Wording and Number</td>
</tr>
<tr>
<td>----------------------------------------------------------------------------------</td>
<td>---------------------------------</td>
<td>--------------------------</td>
<td>--------------------------------------</td>
</tr>
<tr>
<td>7. What factors play into how you group your students during classroom activities?</td>
<td>100%</td>
<td>80%</td>
<td>9. What criteria do you use to group students during classroom activities?</td>
</tr>
<tr>
<td>8. What opportunities are high-ability student given to extend their mathematical learning outside the classroom?</td>
<td>80%</td>
<td>100%</td>
<td>10. What enrichment opportunities are high-ability students given to extend their mathematics learning outside the classroom?</td>
</tr>
<tr>
<td>9. What are some of the ways you provide a supportive learning environment for your high-ability students?</td>
<td>100%</td>
<td>100%</td>
<td>11. No wording changes</td>
</tr>
<tr>
<td>10. What do you do to scaffold instruction?</td>
<td>80%</td>
<td>100%</td>
<td>Question deleted</td>
</tr>
<tr>
<td>11. How do you ensure high-ability students have an appropriate level of scaffolding?</td>
<td>80%</td>
<td>80%</td>
<td>12. No wording changes</td>
</tr>
<tr>
<td>12. In what ways do you model high-level performance for your advanced students?</td>
<td>100%</td>
<td>100%</td>
<td>13. No wording changes</td>
</tr>
<tr>
<td>13. Which children in your class do believe are mathematically gifted?</td>
<td>100%</td>
<td>100%</td>
<td>Moved to question 1.</td>
</tr>
</tbody>
</table>

**Data Collection**

The researcher conducted a pilot study during the summer of 2010. After modifying the instruments and techniques, she conducted the observations and interviews between September and November, 2010.

**Pilot study.** The researcher conducted a pilot study prior to the actual data collection to ensure she understood the types of behaviors to look for and the mechanics
of recording the information. The pilot study took place during a summer enrichment program for gifted/talented students run by the aforementioned gifted education center. As part of the pilot study, the researcher was trained on the use of the COS-R by an observer who had previously used the instrument. To establish inter-rater reliability, the experienced observer and researcher jointly coded observations until an approximately 80% consistency was achieved. The researcher then observed four hours of an algebraic thinking class for gifted students going into the fifth and sixth grade. The researcher took field notes using the Field Notes Form (Appendix K), allowing her to practice the mechanics of taking field notes in a mathematics setting. She found that by periodically glancing at the COS-R (modified) throughout the observation, she was better able to ensure her field notes were capturing the types of questions and comments that would help to evaluate teachers more accurately on the Teacher Observation Form. She discovered that she needed more space to write down questions, comments, and other observations and thus she made minor revisions to the form.

Upon the completion of each hour-long observation, the researcher evaluated the instructor using the COS-R (modified). The researcher discovered it was important to use the Observable Evidence of Classroom Behaviors - Mathematics listing (Appendix H) as she scored each behavior to ensure she was considering how each behavior might be manifested in a mathematics classroom. She also learned the importance of distinguishing between rating a behavior as ineffective versus not observed. This nuance was important because in the former instance, the observed teacher made an attempt – although ineffective – at the behavior, while in the latter case, no attempt was made and the behavior was not demonstrated at all during the period of the observation.
There were three behaviors that were not observed during this period. Behavior 14 (The teacher provided opportunities for students to generalize from concrete data) was not seen, although there seemed to be several opportunities when the instructor could have allowed the students to do so. Likewise, behavior 15 (The teacher solicited many diverse thoughts about issues or ideas) was not observed. As the instructor walked around, he occasionally mentioned that someone appeared to have an interesting way of solving a particular problem, but he never told the rest of the class what the unique method was, nor did he ask the student to elaborate on their different methods of finding the solution. Finally, although the instructor did make an effort to pay attention to each student, behavior 19 (The teacher provided a reasonable amount of attention (as appropriate to the situation) to the gifted/talented students in class compared to other students) was not observed because the class was made up entirely of gifted/talented students. This was not an issue in the actual research study because all classes selected for observation were heterogeneous. Despite the fact that these three behaviors were not observed in the pilot study, the researcher did not modify the COS-R (modified) based on the pilot study since she anticipated seeing these behaviors in a heterogeneous classroom and the panel of experts likewise believed these behaviors were relevant to the study.

The researcher then interviewed the teacher of the pilot study class using the interview guide. He commented that it was a good idea to start off with the question about mathematical giftedness because it helped him to frame his answers. He did not have any suggestions on the wording or order of the questions, although he did think that the questions about differentiation were somewhat redundant. For example, he talked about grouping students and providing enrichment opportunities for them when
answering question 8 (How do you differentiate your instruction?). However, because the researcher thought it was important to ask all the questions during this practice interview, the next two questions (What criteria do you use to group students during classroom activities? and What enrichment opportunities are high-ability students given to extend their mathematics learning outside the classroom?) queried about issues he had already talked about. During the interviews in the actual study, the researcher did not repeat questions that had been previously addressed.

Research study. The researcher contacted the district-level math specialists in five school districts to explain the study. She then applied through each district’s research committee to seek approval to conduct the study in their middle schools. She received approval to conduct the study from two school districts. Upon approval, the researcher contacted the applicable school principals to explain the study and to seek volunteer teachers. She provided the principals with a written description of the study to forward to their respective Algebra I teachers (Appendix L). At the end of the description, there was a statement asking teachers to forward their name, years of Algebra I teaching experience, total years of teaching experience, number of gifted students in their class, and a description of any students with disabilities to the researcher. In this way, the researcher ensured that the classes were heterogeneous. One of the school districts consisted of seven middle schools; however, only one of the principals responded to the researcher’s repeated requests to conduct the study. There were no volunteer teachers from that school. The other school district consisted of four middle schools, and all four principals agreed to allow their teachers to participate. There were a total of seven Algebra I teachers in the school district that had gifted students in their classes, and all
seven volunteered to participate in the study. For this reason, the study focused on a single school district. The fact that the researcher was able to interview and observe all of these Algebra I teachers within one district allowed her to see variations in the teachers' interactions with the gifted students while dealing with a consistent Algebra I curriculum.

The researcher contacted the volunteers to further explain the study and schedule observation and interview times. The teachers were informed that the study involved approximately four hours of classroom observation followed by an approximately one hour interview, with the potential for a follow-up interview. The researcher kept in touch with the teachers via email and informed them that they were free to stop their participation in the study at any time without negative consequences. They received a $10 Barnes and Nobles gift card as a token of appreciation and they were provided with a copy of the completed study per their request. Prior to initiation of the study, the teachers signed a form consenting to their participation (Appendix M). The teachers' characteristics are shown in Table 15.
Table 15

Teacher Characteristics

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Total Years Teaching</th>
<th>Years Teaching Algebra I</th>
<th>Route to Teaching Certif.</th>
<th>Degree</th>
<th>Gifted Endorse.</th>
<th>Gifted Prof. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lila</td>
<td>14</td>
<td>13</td>
<td>Provisional</td>
<td>BS (Math)</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Casey</td>
<td>9</td>
<td>9</td>
<td>Provisional</td>
<td>BS (Math and Psychology)</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Hillary</td>
<td>8</td>
<td>8</td>
<td>Traditional</td>
<td>BS (Math) MS (Education)</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Melinda</td>
<td>7</td>
<td>7</td>
<td>Career Switcher</td>
<td>BS (Economics) MS (Economics)</td>
<td>Yes (in another school district)</td>
<td></td>
</tr>
<tr>
<td>Sam</td>
<td>5</td>
<td>3</td>
<td>Traditional</td>
<td>BS (Math) MS (Math)</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Rachel</td>
<td>4</td>
<td>2</td>
<td>Career Switcher</td>
<td>BA (Accounting)</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Kelly</td>
<td>&lt;1</td>
<td>&lt;1</td>
<td>Traditional</td>
<td>BS (Math) MA (Education)</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Observations. Each teacher was observed for approximately four hours. Spending this amount of time in the classroom helped the researcher to “identify those characteristics and elements in the situation that are most relevant to the problem or issue being pursued and focusing on them in detail” (Lincoln & Guba, 1985, p. 304). The school district had a block schedule of approximately 90 minute classes, and so the researcher observed three blocks of instruction per teacher. Algebra I classes met every other day. If a teacher had only one class that included gifted students, the researcher observed the same class on three separate occasions. This meant that she saw the teacher instruct three different lessons. In contrast, if the teacher had multiple classes with gifted students, the researcher was able to observe the teacher instructing the same topic to different classes. Two schools had mixed 7th/8th grade Algebra I classes, while two schools kept the two grades separate. By observing teachers in these various situations,
the researcher was able to get a more diversified view of how the teachers interacted with their gifted students. None of the observed classes contained students with disabilities. It was important for the researcher to identify whether a teacher had such students in the class as it may have caused the teacher to focus on attending to their special needs.

Prior to the actual observation, the researcher discussed the procedure with the teacher to ensure he or she understood the process. The researcher also requested a seating chart so that she was aware of where the gifted/talented students sat. The researcher showed up a few minutes prior to class to answer any last minute questions and to sit wherever the teacher deemed appropriate.

During the class, the researcher merely observed; she did not interfere in any way. She kept track of the mathematical observable evidence related to the teacher behaviors on the COS-R by tallying the number of times each behavior was observed. She also wrote field notes to capture the interactions between the teacher and gifted/talented students as well as significant quotations, interpretations of events, and insights. This helped the researcher to write a thick description of how the teachers engaged the gifted/talented students during the class and the quotations assisted in providing an emic perspective (Patton, 2002). A sample of field notes from an observation is at Appendix N. Immediately following the observation, the researcher asked the teacher if he or she had any comments about the lesson that needed to be addressed. This brief discussion was meant only to clarify the lesson itself, and should not be confused with the teacher interview described below.

The tally of observable evidence and field notes assisted the researcher in her evaluation of teacher behaviors. As soon as possible after the observation, the researcher
recorded these evaluations on the Teacher Observation Form (Appendix O). A sample of a completed Teacher Observation Form is at Appendix P.

Table 16 describes the classes in which the observations were performed.
Table 16

Classroom Observations

<table>
<thead>
<tr>
<th>Teacher/Observation Number</th>
<th>Block - Grade</th>
<th>Time of Day (Minutes)</th>
<th>Total Number of Students</th>
<th>Number of Gifted Students</th>
<th>Grade of Gifted Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lila 1</td>
<td>4B - 7th/8th</td>
<td>1:03-2:30 (87 min)</td>
<td>29</td>
<td>1</td>
<td>1-8th</td>
</tr>
<tr>
<td>Lila 2</td>
<td>1B - 7th/8th</td>
<td>7:57-9:26 (89 min)</td>
<td>26</td>
<td>3</td>
<td>2-7th; 1-8th</td>
</tr>
<tr>
<td>Lila 3</td>
<td>1B - 7th/8th</td>
<td>7:57-9:26 (89 min)</td>
<td>26</td>
<td>3</td>
<td>2-7th; 1-8th</td>
</tr>
<tr>
<td>Casey 1</td>
<td>1B - 7th/8th</td>
<td>7:57-9:27 (90 min)</td>
<td>23</td>
<td>3</td>
<td>3-7th</td>
</tr>
<tr>
<td>Casey 2</td>
<td>3B - 7th/8th</td>
<td>11:03-12:15 (72 min)</td>
<td>23</td>
<td>3</td>
<td>3-7th</td>
</tr>
<tr>
<td>Casey 3</td>
<td>1A - 7th/8th</td>
<td>7:57-9:27 (90 min)</td>
<td>23</td>
<td>7</td>
<td>7-7th</td>
</tr>
<tr>
<td>Hillary 1</td>
<td>4A - 7th/8th</td>
<td>1:03-2:30 (87 min)</td>
<td>27</td>
<td>2</td>
<td>1-7th; 1-8th</td>
</tr>
<tr>
<td>Hillary 2</td>
<td>4A - 7th/8th</td>
<td>1:03-2:30 (87 min)</td>
<td>27</td>
<td>2</td>
<td>1-7th; 1-8th</td>
</tr>
<tr>
<td>Hillary 3</td>
<td>4A - 7th/8th</td>
<td>1:03-2:30 (87 min)</td>
<td>27</td>
<td>2</td>
<td>1-7th; 1-8th</td>
</tr>
<tr>
<td>Melinda 1</td>
<td>2B - 7th/8th</td>
<td>9:30-10:57 (87 min)</td>
<td>27</td>
<td>5</td>
<td>5-7th</td>
</tr>
<tr>
<td>Melinda 2</td>
<td>2A - 7th/8th</td>
<td>9:30-10:57 (87 min)</td>
<td>27</td>
<td>1</td>
<td>1-7th</td>
</tr>
<tr>
<td>Melinda 3</td>
<td>2B - 7th/8th</td>
<td>9:30-10:57 (87 min)</td>
<td>27</td>
<td>5</td>
<td>5-7th</td>
</tr>
<tr>
<td>Sam 1</td>
<td>1B - 7th</td>
<td>7:57-9:31 (94 min)</td>
<td>21</td>
<td>5</td>
<td>5-7th</td>
</tr>
<tr>
<td>Sam 2</td>
<td>3A - 7th</td>
<td>11:36-1:00 (84 min)</td>
<td>31</td>
<td>5</td>
<td>5-7th</td>
</tr>
<tr>
<td>Sam 3</td>
<td>1B - 7th</td>
<td>7:57-9:31 (94 min)</td>
<td>21</td>
<td>5</td>
<td>5-7th</td>
</tr>
<tr>
<td>Rachel 1</td>
<td>3B - 7th</td>
<td>11:45-1:06 (81 min)</td>
<td>26</td>
<td>8</td>
<td>8-7th</td>
</tr>
<tr>
<td>Rachel 2</td>
<td>4B - 7th</td>
<td>1:10-2:35 (85 min)</td>
<td>26</td>
<td>4</td>
<td>4-7th</td>
</tr>
<tr>
<td>Rachel 3</td>
<td>3B - 7th</td>
<td>11:45-1:06 (81 min)</td>
<td>26</td>
<td>8</td>
<td>8-7th</td>
</tr>
<tr>
<td>Kelly 1</td>
<td>1B - 8th</td>
<td>8:04-9:38 (94 min)</td>
<td>23</td>
<td>1</td>
<td>1-8th</td>
</tr>
<tr>
<td>Kelly 2</td>
<td>1B - 8th</td>
<td>8:04-9:38 (94 min)</td>
<td>23</td>
<td>1</td>
<td>1-8th</td>
</tr>
<tr>
<td>Kelly 3</td>
<td>1B - 8th</td>
<td>8:04-9:38 (94 min)</td>
<td>23</td>
<td>1</td>
<td>1-8th</td>
</tr>
</tbody>
</table>

*Interviews.* Each interview lasted approximately an hour and was scheduled at the convenience of the teacher. The interview was conducted after the observation portion of
the study was completed to avoid influencing the teacher’s behavior during the observed class periods. Using the interview guide, the researcher began by asking the prearranged set of interview questions. She also asked follow-up and clarifying questions as needed to help facilitate her understanding. Questions and responses were recorded using a tape recorder. The researcher wrote field notes including ideas for follow-up questions and observations concerning the teachers’ demeanor and any unusual antics. Throughout the interview, the researcher member checked. Member checking ensures the researcher is accurately representing the participant’s views (Glesne, 2006) and “is the most crucial technique for establishing credibility” (Lincoln & Guba, 1985, p. 314).

As soon as possible following the interview, the researcher transcribed the tape recording. Using the transcription and field notes, she summarized the findings and sent a copy of the summary to the teacher via email. A sample of an interview summary is at Appendix Q. This member checking allowed the teacher to verify that the researcher accurately captured the ideas from the interview and also allowed the subject to add any additional thoughts he or she might have.

Data Analysis

Data from the observation form, field notes, and interviews were analyzed to form a triangulated portrayal of how teachers modified the course for their gifted/talented students. The researcher also took note of any relevant ideas recorded in her reflexive journal and had a peer debriefer and community of practice review the overall findings. The collective data were organized using graphical displays and tables to assist in interpreting whether teachers were meeting the needs of their gifted/talented students.
Table 17 depicts the methods by which the researcher collected and analyzed material to answer the research questions.

Table 17

Data Analysis

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Data Sources</th>
<th>Data Analysis</th>
</tr>
</thead>
</table>
| To what extent do middle school Algebra I teachers modify the pace of instruction for their gifted/talented students? | Observed teaching behaviors: 17, 18
Interview questions: 2, 3, 4, 5, 7
Field notes | Observations: Descriptive statistics
Interviews: Thematic analysis using constant comparative method
Field notes: Holistic coding |
| In what ways do middle school Algebra I teachers increase the level of challenge for their gifted/talented students? | Observed teaching behaviors: 1, 5, 6, 10, 11, 12, 13, 14
Interview question: 3, 4, 6
Field notes | Observations: Descriptive statistics
Interviews: Thematic analysis using constant comparative method
Field notes: Holistic coding |
| What differentiation strategies do middle school Algebra I teachers use to meet the needs of their gifted/talented students? | Observed teaching behaviors: 3, 6, 7, 9
Interview questions: 8, 9, 10
Field notes | Observations: Descriptive statistics
Interviews: Thematic analysis using constant comparative method
Field notes: Holistic coding |
| In what ways do middle school Algebra I teachers provide a supportive environment for their gifted/talented students? | Observed teaching behaviors: 1, 2, 3, 4, 6, 8, 11, 15, 16, 19
Interview question: 11, 12, 13
Field notes | Observations: Descriptive statistics
Interviews: Thematic analysis using constant comparative method
Field notes: Holistic coding |

**Observations.** The COS-R (modified) allowed the observer to record how well each teaching behavior was demonstrated using a scale of 3 (*effective*), 2 (*somewhat effective*), 1 (*ineffective*), and N/O (*not observed*). The following descriptions from the COS-R User’s Manual were used to determine the rating:
• **Effective:** The teacher evidenced careful planning and classroom flexibility in implementation of the behavior, eliciting many appropriate students responses. The teacher was clear, and sustained focus on the purposes of learning.

• **Somewhat Effective:** The teacher evidenced some planning and/or classroom flexibility in implementation of the behavior, eliciting some appropriate student responses. The teacher was sometimes clear and focused on the purposes of learning.

• **Ineffective:** The teacher evidenced little or no planning and/or classroom flexibility in implementation of the behavior, eliciting minimal appropriate student responses. The teacher was unclear and unfocused regarding the purpose of learning.

• **Not Observed:** The listed behavior was not demonstrated during the time of the observation. (VanTassel-Baska et al., 2005b, p. 13)

Descriptive statistics were used to determine the mean, standard deviation, and range of these behaviors.

The field notes taken during the observation were analyzed holistically. Holistic coding is appropriate when the researcher’s focus is on descriptions (Rossman & Rallis, 2003). The researcher wrote notes in the columns of the Field Note Forms. These notes were developed into common codes, and then the codes were related to each other to develop broader categories and overall themes. A sample of holistically-coded field notes is provided in Appendix R.

**Interviews.** The researcher transcribed the material and then conducted a thematic analysis using the constant comparative method beginning with the a priori categories of
pace, challenge, differentiation, and supportive learning environment. Miles and Huberman (1994) explained that such a “start list” (p. 58) of codes oftentimes comes from the research questions as these a priori categories did. Stake (1995) pointed out that in collective case study, it is important to commit to common topics early to facilitate cross-case analysis, and by beginning with these a priori categories, “it forces the analysts to tie research questions or conceptual interests directly to the data” (Miles & Huberman, 1994, p. 65), which enabled the researcher to more easily compare and contrast how the research questions were addressed in the various cases. Table 18 provides a description and the literature support for these categories.
Table 18

*Table of Specifications*

<table>
<thead>
<tr>
<th>A priori Category</th>
<th>Description</th>
<th>Literature Support</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A priori Category</strong></td>
<td><strong>Description</strong></td>
<td><strong>Literature Support</strong></td>
</tr>
<tr>
<td>-----------------------</td>
<td>-----------------</td>
<td>-----------------------</td>
</tr>
</tbody>
</table>
| Differentiation       | Differentiation refers to the teacher using a variety of instructional approaches to modify the content of the material presented to the students, the process by which the students gain access to the material, or the product that the students produce to demonstrate mastery of the information. | Access Center (2005)  
Archambault et al. (1993)  
Assouline & Lupkowski-Shoplik (2005)  
Ball & Bass (2003)  
Benbow, Lubinski, Shea, & Eftekhari-Sanjani (2000)  
Brody & Benbow (1990)  
Colangelo, Assouline, & Gross (2004a, 2004b)  
Dark & Benbow (1990, 1991)  
Delcourt (1993)  
Delcourt, Cornell, & Goldberg (2007)  
Diezmann & Watters (2000)  
Farkas & Duffett (2008)  
Gamoran & Weinstein (1998)  
Gavin et al. (2007)  
Gentry & Owen (1999)  
Herbert (1993)  
Johnson (2000)  
Kim (2006)  
Kulik (1992)  
Kulik & Kulik (1992)  
Mills, Ablard, & Gustin (1994)  
Neihart (2007)  
Reis & Renzulli (1992)  
Reis et al. (1993)  
Reis, Westberg, Kulikowich, & Purcell (1998)  
Renzulli & Reis (1997)  
Rotigel & Fello (2004)  
Sadler & Tai (2007)  
Slavin (1987)  
Stanley (2000)  
Starko (1986)  
Stepanek (1999)  
Tieso (2002)  
USDOE (2008b)  
VanTassel-Baska & Stambaugh (2005)  
Waxman, Robinson, & Mukhopadhyay (1996)  
Westberg, Archambault, Dobyns, Salvin (1993)  
Ysseldyke et al. (2004) |
In addition to these general codes – representing an etic perspective – more specific subcategories – representing a more emic perspective (Miles & Huberman, 1994) – emerged as the researcher used the constant comparison method of open coding. This process involved dividing the transcript into separate complete thoughts of a few words each and then determining the category that best described the essence of a particular segment. After categorizing the first thought, the researcher compared the second phrase to the first one and either assigned it the same category or a new one. In this way, the researcher proceeded through the transcript, comparing the categories assigned to segments and revising them as necessary. Categories were written in the margins of the
transcript next to each thought. This method helped the researcher to not only refine and clarify the meaning of each category, but also to determine the categories that appeared to be the most important to study (Gall, Gall, & Borg, 2007). A sample of open coding from an interview segment is at Appendix S.

Once the initial coding was complete, the researcher used axial coding to link the categories and subcategories together and selective coding to combine the subcategories into themes. This method allowed the researcher to stay close to the data which helped to build a thick description. Thick description “makes analysis and interpretation possible” (Rossman & Rallis, 2003, p. 275). A list of codes and definitions is at Appendix T.

Trustworthiness and Authenticity

To address validity and reliability issues, the researcher demonstrated trustworthiness and authenticity of the study.

Trustworthiness. Trustworthiness relates to research validity (Glesne, 2006) and should be considered both prior to and during data collection. Lincoln and Guba (1985) pointed out that the basic issue related to trustworthiness is how the researcher can “persuade his or her audiences (including self) that the findings of an inquiry are worth paying attention to” (p. 290). There are four different dimensions of trustworthiness – credibility, transferability, dependability, and confirmability.

Credibility. Credibility relates to how representative the findings are. Lincoln and Guba (1985) related credibility to internal validity. To establish credibility, the researcher used member checks during the interview process and maintained a reflexive journal to record reflections on self and various observations and insights pertaining to the study. She also used a community of practice to engage in critical discussion and to
comment on referential adequacy. Furthermore, she used a peer debriefer to serve as an intellectual watchdog and triangulated the data from the interviews, observations, and field notes to corroborate the findings.

Transferability. Transferability relates to the applicability of the findings as far as the extent to which the findings can be applied to other individuals or contexts. Transferability is similar to external validity (Lincoln & Guba, 1985). Transferability was established through the purposeful sampling of middle school Algebra I teachers, a thick description and rich explanation of how the teachers differentiated for the gifted/talented students in their class, and the maintenance of a reflexive journal.

Dependability. Dependability or reliability (Lincoln & Guba, 1985) relates to the consistency of the findings. Dependability was established by the researcher maintaining a reflexive journal and by creating an audit trail whereby another individual would be able to easily understand the steps of the investigation and the details of the findings.

Confirmability. Confirmability relates to the extent to which the findings report the teachers’ perspectives. Put another way, confirmability relates to objectivity (Lincoln & Guba, 1985). Similar to dependability, confirmability was established via a reflexive journal and audit trail.

Authenticity. Authenticity relates to the “reflexive consciousness about one’s own perspective, appreciation for the perspectives of others, and fairness in depicting constructions in the values that undergird them” (Patton, 2002, p. 546). Authenticity has five separate dimensions – fairness, and ontological, educative, catalytic, and tactical authenticity.
**Fairness.** Fairness relates to whether all the teachers’ voices are heard in the findings. Patton (2002) pointed out that fairness has the following features:

- It assumes multiple realities or truth . . . .
- It is adversarial rather than one-perspective in nature . . . .
- It is assumed that the subject’s reaction to the reporter and interactions between them heavily determines what the reporter perceives . . . .
- It is a relative criterion that is measured by *balance* rather than by isomorphism to enduring truth. (p. 575)

Fairness was established by member checking and peer debriefing.

**Ontological authenticity.** Ontological authenticity relates to whether the teachers’ understanding of themselves and their context increases based on their participation in the study. This aspect of authenticity was addressed by asking the teachers follow-up questions during the interview.

**Educative authenticity.** Educative authenticity refers to whether the teachers’ understanding of other teachers’ perspectives increases based on their participation. The researcher provided copies of the research results and discussed the results upon completion of the study with the participants if they desired.

**Catalytic authenticity.** Catalytic authenticity addresses the issue of whether the teachers’ actions and decisions are facilitated by their participation in the study. This aspect of authenticity was addressed through member checking and asking follow-up questions throughout the interviews, and providing results and a discussion of the study upon its completion as requested.
**Tactical authenticity.** Tactical authenticity relates to whether the teachers feel empowered to act as a result of their participation. Like catalytic authenticity, it was addressed through member checking, asking follow-up questions throughout the interviews, and providing results and a discussion of the study upon its completion.

**Ethical Considerations**

Prior to initiating the study, the researcher submitted the study proposal to the William and Mary School of Education Internal Review Committee (EDIRC) for approval. Submission to the EDIRC is required because doctoral dissertations constitute generalizable knowledge in the fact that the abstract will be published in *Dissertation Abstracts International* (Gall, Gall, & Borg, 2007). The EDIRC submission included a statement on the protection of human subjects. This study posed minimal psychological discomfort to the subjects and subjects were free to withdraw from the study without negative consequences. Confidentiality was maintained by assigning pseudonyms to the school district, schools, and individual study participants. These pseudonyms were used in all published material related to this study. Following approval from the EDIRC, the researcher contacted the applicable school districts to seek approval to conduct the study in their middle schools.
Chapter 4: Results

Background

This descriptive study sought to determine the ways in which Algebra I teachers from a suburban school district in southeastern Virginia modified their instructional practices for the gifted/talented students in their classes. The researcher observed heterogeneous Algebra I classes that consisted of seventh or eighth grade students, or a mix of both. The average class size was approximately 25 students; anywhere from one to eight of the students had been identified as gifted. The teachers had a variety of teaching experience, ranging from less than a year to over 14 years, and all had taught Algebra I for the majority of their careers. Only one of the teachers had a gifted endorsement, and none of the others had any professional development related to gifted education. By conducting a case study of each teacher and then by using a cross-case analysis, the researcher discovered common themes in how these teachers addressed the needs of their gifted/talented students.

Restatement of Research Questions

This study used observations and interviews to answer the following research questions:

1. To what extent do middle school Algebra I teachers modify the pace of instruction for their gifted/talented students?
2. In what ways do middle school Algebra I teachers increase the level of challenge for their gifted/talented students?

3. What differentiation strategies do middle school Algebra I teachers use to meet the needs of their gifted/talented students?

4. In what ways do middle school Algebra I teachers provide a supportive environment for their gifted/talented students?

Data Presentation

The results of the study are presented, first, by discussing the quantitative findings from the observations. Next, teachers who were generally rated as “effective,” are discussed individually, using findings from the field notes, observation forms, and interviews. This step is especially important because, according to Patton (2002), “The analyst’s first and foremost responsibility consists of doing justice to each individual case. All else depends on that” (p. 449). The teachers who were generally rated as “somewhat effective” are discussed next, and so on. The findings are then examined using a cross-case analysis to present the emergent themes. Stake (1995) pointed out the importance of examining the relative frequencies of behaviors within cases to help find the common relationships among the cases. He also noted that in a collective case study, the variety and redundancy noted across cases is important because it can lead to better understanding about a larger collection of cases (Stake, 2000). Such a cross-case analysis helps to deepen the explanation and understanding of the phenomena (Miles & Huberman, 1994), and this facilitated the researcher’s interpretation of the way in which the teachers were addressing the needs of their gifted/talented students. These emergent themes are presented as they relate to the specific research questions.
COS-R Results

The researcher evaluated each teacher using a separate COS-R Observation Form during each of the three class periods. Teachers were given ratings of 3 (effective), 2 (somewhat effective), 1 (ineffective), or N/O (not observed). A rating of not observed was not considered negative and did not reflect the effectiveness of the teacher in any way (VanTassel-Baska et al., 2005b). The ratings for each teaching behavior were averaged over the three observations. If a specific behavior was not observed, it was not considered in calculating the means of the ratings. The results are shown in Table 19.
Table 19

**Teacher Behaviors**

<table>
<thead>
<tr>
<th>Teacher Behaviors</th>
<th>Lila</th>
<th>Sam</th>
<th>Casey</th>
<th>Melinda</th>
<th>Rachel</th>
<th>Hillary</th>
<th>Kelly</th>
<th>∑ Rating of Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Set high expectations for student performance</td>
<td>3.00</td>
<td>3.00</td>
<td>2.33</td>
<td>3.00</td>
<td>2.00</td>
<td>3.00</td>
<td>1.67</td>
<td>2.57</td>
</tr>
<tr>
<td>2. Incorporated activities for students to apply new knowledge</td>
<td>2.67</td>
<td>3.00</td>
<td>3.00</td>
<td>2.33</td>
<td>2.67</td>
<td>2.67</td>
<td>2.00</td>
<td>2.62</td>
</tr>
<tr>
<td>3. Engaged students in planning, monitoring or assessing their learning</td>
<td>3.00</td>
<td>2.50</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>1.00</td>
<td>2.07</td>
</tr>
<tr>
<td>4. Encouraged students to express their thoughts</td>
<td>3.00</td>
<td>3.00</td>
<td>2.67</td>
<td>3.00</td>
<td>2.00</td>
<td>2.67</td>
<td>2.00</td>
<td>2.62</td>
</tr>
<tr>
<td>5. Had students reflect on what they had learned</td>
<td>2.67</td>
<td>2.50</td>
<td>2.67</td>
<td>2.00</td>
<td>1.67</td>
<td>2.50</td>
<td>2.00</td>
<td>2.29</td>
</tr>
<tr>
<td>6. Provided opportunities for independent or group learning to promote depth in understanding content</td>
<td>2.67</td>
<td>2.33</td>
<td>3.00</td>
<td>2.00</td>
<td>3.00</td>
<td>2.50</td>
<td>2.00</td>
<td>2.50</td>
</tr>
<tr>
<td>7. Accommodated individual or subgroup differences</td>
<td>2.00</td>
<td>3.00</td>
<td>3.00</td>
<td>1.33</td>
<td>2.33</td>
<td>1.67</td>
<td>2.00</td>
<td>2.19</td>
</tr>
<tr>
<td>8. Encouraged multiple interpretations of events and situations</td>
<td>2.67</td>
<td>3.00</td>
<td>2.00</td>
<td>2.67</td>
<td>2.00</td>
<td>1.67</td>
<td>1.00</td>
<td>2.14</td>
</tr>
<tr>
<td>9. Allowed students to discover key ideas individually through structured activities and/or questions</td>
<td>2.67</td>
<td>2.33</td>
<td>2.67</td>
<td>2.33</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.29</td>
</tr>
<tr>
<td>10. Engaged students in problem identification and definition</td>
<td>2.33</td>
<td>2.33</td>
<td>2.00</td>
<td>2.33</td>
<td>2.67</td>
<td>1.67</td>
<td>1.67</td>
<td>2.14</td>
</tr>
<tr>
<td>11. Engaged students in solution-finding activities and comprehensive solution articulation</td>
<td>3.00</td>
<td>2.67</td>
<td>2.67</td>
<td>2.67</td>
<td>2.67</td>
<td>2.33</td>
<td>2.00</td>
<td>2.57</td>
</tr>
<tr>
<td>12. Encouraged students to judge or evaluate situations, problems, or issues</td>
<td>2.33</td>
<td>2.33</td>
<td>2.67</td>
<td>3.00</td>
<td>3.00</td>
<td>2.33</td>
<td>2.00</td>
<td>2.52</td>
</tr>
<tr>
<td>13. Engaged students in comparing and contrasting ideas</td>
<td>2.67</td>
<td>3.00</td>
<td>2.00</td>
<td>2.33</td>
<td>1.67</td>
<td>2.50</td>
<td>2.00</td>
<td>2.31</td>
</tr>
<tr>
<td>14. Provided opportunities for students to generalize from concrete data</td>
<td>2.67</td>
<td>2.00</td>
<td>2.50</td>
<td>2.00</td>
<td>1.50</td>
<td>2.00</td>
<td>1.50</td>
<td>2.02</td>
</tr>
<tr>
<td>15. Solicited many diverse thoughts about issues or ideas</td>
<td>2.67</td>
<td>2.33</td>
<td>2.00</td>
<td>3.00</td>
<td>2.00</td>
<td>1.33</td>
<td>2.00</td>
<td>2.19</td>
</tr>
<tr>
<td>16. Provided opportunities for students to develop and elaborate on their ideas</td>
<td>2.67</td>
<td>2.67</td>
<td>2.33</td>
<td>2.67</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.33</td>
</tr>
<tr>
<td>17. Allowed the students to move through material at an individual pace</td>
<td>1.33</td>
<td>1.33</td>
<td>2.00</td>
<td>1.00</td>
<td>1.67</td>
<td>1.00</td>
<td>1.67</td>
<td>1.43</td>
</tr>
<tr>
<td>18. Allowed students sufficient time to thoroughly explore complex problems</td>
<td>2.33</td>
<td>3.00</td>
<td>2.33</td>
<td>2.00</td>
<td>2.67</td>
<td>2.00</td>
<td>2.00</td>
<td>2.33</td>
</tr>
<tr>
<td>19. Provided a reasonable amount of attention (as appropriate to the situation) to the gifted/talented students in the class compared to other students</td>
<td>3.00</td>
<td>3.00</td>
<td>2.67</td>
<td>2.67</td>
<td>3.00</td>
<td>2.33</td>
<td>2.33</td>
<td>2.71</td>
</tr>
<tr>
<td>∑ Rating of Individual Teacher</td>
<td>2.60</td>
<td>2.59</td>
<td>2.45</td>
<td>2.33</td>
<td>2.24</td>
<td>2.11</td>
<td>1.83</td>
<td>2.31</td>
</tr>
</tbody>
</table>
The results show that, overall, the Algebra I teachers observed were somewhat effective in their teaching behaviors ($\bar{x}=2.31$). Teachers were most effective at providing a reasonable amount of attention to the gifted/talented students ($\bar{x}=2.71$), followed by incorporating activities for students to apply new knowledge ($\bar{x}=2.62$), encouraging students to express their thoughts ($\bar{x}=2.62$), setting high expectations for student performance ($\bar{x}=2.57$), engaging students in solution-finding activities and comprehensive solution articulation ($\bar{x}=2.57$), and encouraging students to judge or evaluate situations, problems, or issues ($\bar{x}=2.52$).

The observed teachers were least effective at allowing students to move through material at an individual pace ($\bar{x}=1.43$), followed by providing students opportunities to generalize from concrete data ($\bar{x}=2.02$), engaging students in planning, monitoring, or assessing their learning ($\bar{x}=2.07$), encouraging multiple interpretation of events and situations ($\bar{x}=2.14$), and engaging students in problem identification and definition ($\bar{x}=2.14$).

Looking at it another way, of the 133 individual behaviors observed (19 behaviors for 7 teachers), 18% were rated as effective, almost 40% were between somewhat effective and effective, and nearly 28% were somewhat effective. The remaining 14% of behaviors were rated as below somewhat effective. Table 20 shows a summary of the effectiveness ratings for individual behaviors.
The most effective teachers were Lila and Sam with mean scores of 2.60 and 2.59, respectively, although both were rated only slightly higher than ineffective (1.33) at allowing students to move through material at an individual pace. On the other hand, Casey’s mean score (2.45) was slightly lower, although all of her behaviors were rated as somewhat effective or higher. Melinda, Rachel, and Hillary also had overall means in the somewhat effective range. Kelly was the only teacher to have an overall rating of less than somewhat effective. Her mean score of 1.83 was almost 1 SD below the mean of the group. While selected general tendencies do appear when reviewing the teachers collectively, the data make clear that rating teachers as “effective” or otherwise is not a straight-forward proposition. To provide additional analysis, descriptive statistics for each teacher are provided in Table 21.
Table 21

*Descriptive Statistics of Teaching Behaviors of Individual Teachers*

<table>
<thead>
<tr>
<th></th>
<th>Lila</th>
<th>Sam</th>
<th>Casey</th>
<th>Melinda</th>
<th>Rachel</th>
<th>Hillary</th>
<th>Kelly</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{X})</td>
<td>2.60</td>
<td>2.59</td>
<td>2.45</td>
<td>2.33</td>
<td>2.24</td>
<td>2.11</td>
<td>1.83</td>
<td>2.31</td>
</tr>
<tr>
<td>SD</td>
<td>0.41</td>
<td>0.45</td>
<td>0.37</td>
<td>0.56</td>
<td>0.49</td>
<td>0.50</td>
<td>0.35</td>
<td>0.51</td>
</tr>
<tr>
<td>Range</td>
<td>1.33-3.00</td>
<td>1.33-3.00</td>
<td>2.00-3.00</td>
<td>1.00-3.00</td>
<td>1.50-3.00</td>
<td>1.00-3.00</td>
<td>1.00-3.00</td>
<td>1.00-3.00</td>
</tr>
</tbody>
</table>

**Observable Evidence of Selected Mathematics Instructional Attributes**

The COS-R ratings were supported by the researcher’s field notes and her observation of whether the specific mathematical observable evidence associated with each teaching behavior was observed during each classroom observation. It should be emphasized, however, that the mathematical behavioral indicators (observable evidence) were intended to be illustrative, not exhaustive, and not all indicators needed to be present to rate a teacher as *effective* or *somewhat effective* (VanTassel-Baska et al., 2005b). In fact, just because a teacher evidenced a particular mathematical behavior, did not mean that he or she did so effectively. It should also be noted that not all behaviors were expected to be observed in each lesson.

These behaviors were examined in light of the four research questions focusing on pace, challenge, differentiation, and supportive environment. Because these four areas have many overlapping elements, the mathematical behaviors were identified with the category to which they were most closely associated. Each of the researcher’s observations presented a single “observation opportunity” for each of the pieces of mathematical observable evidence. In other words, the researcher either saw the evidence during the lesson or she did not. For example, there were five pieces of mathematical observable evidence related to differentiation. Because the researcher
conducted three classroom observations per teacher, she had 15 observation opportunities to see differentiation-related evidence per teacher. A discussion of each individual teacher’s observable evidence is included in the detailed case studies found in Appendices U through AA.

**Pace.** Mathematical behaviors related to pace were observed during 59% of the observation opportunities as shown in Table 22. In other words, behaviors related to pace were demonstrated in 62 of the 105 observation opportunities (62/105 = 59%). The number of observation opportunities was determined by the fact that there were five behaviors related to pace that could have been observed during each of the seven teachers’ three observations (5 pace-related behaviors x 7 teachers x 3 observations = 105 observation opportunities).
Looking at specific mathematical observable evidence, the behavior most often seen was teachers providing a mix of learning and practice activities. This was observed in 81% of the lessons (1 behavior x 7 teachers x 3 observations = 21 observation opportunities; the behavior was seen during 17 of the 21 observations). Teachers also adjusted the pace for the class as a whole in 71% of the observed lessons (15 of 21 observation opportunities); however, as was shown in Table 19, teachers were ineffective
at allowing students to move through the material at an individual pace. The teachers checked for prior knowledge and allowed time for the students to figure out and discuss solutions about half the time. On the other hand, only 38% of the lessons provided time for students to persist in investigations of challenging topics (8 of 21 observation opportunities).

There was substantial variation among the teachers as far as how often they displayed pace-related behaviors. Although some of the teachers who were rated as more effective displayed the observable evidence related to pace more frequently than did the other teachers, this was not the case with all teachers. For example, Lila and Sam – the most effective teachers – demonstrated pace-related evidence in 87% and 73% of the observation opportunities, respectively, but Casey – also one of the more effective teachers – only demonstrated pace-related evidence during 33% of the opportunities to do so. In contrast, Kelly – the least effective teacher – showed such evidence in 67% of the observation opportunities. This suggests that adjusting pace, in and of itself, does not necessarily make a teacher effective or ineffective in addressing the needs of his or her gifted/talented students.

**Challenge.** The teachers demonstrated observable evidence related to challenge in 54% of the observation opportunities (11 challenge-related behaviors x 7 teachers x 3 observations = 231; the behaviors were seen in 125 of the 231 observation opportunities) as shown in Table 23.
Table 23

**Mathematical Observable Evidence Related to Challenge**

<table>
<thead>
<tr>
<th>Mathematical Observable Evidence</th>
<th>Lila</th>
<th>Sam</th>
<th>Casey</th>
<th>Melinda</th>
<th>Rachel</th>
<th>Hillary</th>
<th>Kelly</th>
<th>Total times behavior observed</th>
<th>Total observation opportunities</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analyzed concrete examples</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>13</td>
<td>21</td>
<td>62%</td>
</tr>
<tr>
<td>Provided appropriate/advanced level of challenge</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>16</td>
<td>21</td>
<td>76%</td>
</tr>
<tr>
<td>Emphasized fluency and depth of understanding</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>13</td>
<td>21</td>
<td>62%</td>
</tr>
<tr>
<td>Higher-level questions to make connections</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>11</td>
<td>21</td>
<td>52%</td>
</tr>
<tr>
<td>Pattern recognition</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>21</td>
<td>14%</td>
</tr>
<tr>
<td>Connect real-world problems to math</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>15</td>
<td>21</td>
<td>71%</td>
</tr>
<tr>
<td>Challenging questions to meet student readiness</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>11</td>
<td>21</td>
<td>52%</td>
</tr>
<tr>
<td>Identify and define real-world problems</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>21</td>
<td>5%</td>
</tr>
<tr>
<td>Analyze/compare methods of solution</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>14</td>
<td>21</td>
<td>67%</td>
</tr>
<tr>
<td>Connections between old and new learning</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>21</td>
<td>21</td>
<td>100%</td>
</tr>
<tr>
<td>Make generalizations</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>21</td>
<td>33%</td>
</tr>
<tr>
<td><strong>Total times behaviors observed per teacher</strong></td>
<td><strong>23</strong></td>
<td><strong>23</strong></td>
<td><strong>18</strong></td>
<td><strong>21</strong></td>
<td><strong>20</strong></td>
<td><strong>14</strong></td>
<td><strong>6</strong></td>
<td><strong>125</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total times behaviors possible per teacher</strong></td>
<td><strong>33</strong></td>
<td><strong>33</strong></td>
<td><strong>33</strong></td>
<td><strong>33</strong></td>
<td><strong>33</strong></td>
<td><strong>33</strong></td>
<td><strong>33</strong></td>
<td><strong>231</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Percent of times behaviors observed versus number of possible times</strong></td>
<td><strong>70%</strong></td>
<td><strong>70%</strong></td>
<td><strong>55%</strong></td>
<td><strong>64%</strong></td>
<td><strong>61%</strong></td>
<td><strong>42%</strong></td>
<td><strong>18%</strong></td>
<td><strong>54%</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Teachers made connections between old and new learning in 100% of the observed lessons. This held true for even the most ineffective teacher, Kelly. On the other hand, out of the 21 lessons observed, only one (5%) challenged the students to identify and define real-world problems, although almost three quarters (71%) made some sort of connection between real-world problems and math. Several other challenge-related behaviors were observed in over half the lessons such as providing an advanced level of challenge, emphasizing fluency and depth of understanding, analyzing and comparing methods of solution, and posing challenging questions to the students. Teachers rarely had the students recognize patterns or make generalizations, both strategies in mathematics that encourage higher-level thinking.

Similar to what was seen with pace-related behaviors, there was substantial variation between the teachers in their challenge-related behaviors, although there was a more consistent trend of teachers who were rated as more effective also demonstrating behavior related to challenge more often. For example, Lila and Sam both demonstrated challenge-related evidence during 70% of their opportunities to do so, while Kelly only did so during 18% of her opportunities.

**Differentiation.** Observable evidence related to differentiation was seen in only 28% of the observation opportunities (5 differentiation-related behaviors x 7 teachers x 3 observations = 105; the behaviors were seen in 29 of the 105 observation opportunities) as shown in Table 24.
Tables 24

Mathematical Observable Evidence Related to Differentiation

<table>
<thead>
<tr>
<th>Mathematical Observable Evidence</th>
<th>Lila</th>
<th>Sam</th>
<th>Casey</th>
<th>Melinda</th>
<th>Rachel</th>
<th>Hillary</th>
<th>Kelly</th>
<th>Total times behavior observed</th>
<th>Total observation opportunities</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group work to deepen understanding</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>21</td>
<td>24%</td>
</tr>
<tr>
<td>Collaboration/group work to problem solve/discuss solutions</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>11</td>
<td>21</td>
<td>21</td>
<td>52%</td>
</tr>
<tr>
<td>Provided choices</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>21</td>
<td>29%</td>
</tr>
<tr>
<td>Grouped via interest or ability</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>21</td>
<td>10%</td>
</tr>
<tr>
<td>Students create own problems</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>21</td>
<td>24%</td>
</tr>
<tr>
<td>Total times behaviors observed per teacher</td>
<td>8</td>
<td>2</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total times behaviors possible per teacher</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>105</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent of times behaviors observed versus number of possible times</td>
<td>53%</td>
<td>13%</td>
<td>67%</td>
<td>13%</td>
<td>20%</td>
<td>7%</td>
<td>20%</td>
<td></td>
<td></td>
<td>28%</td>
</tr>
</tbody>
</table>

Teachers had students use collaboration or group work to solve problems and discuss solutions in over half of the lessons (52%). They provided choice, allowed students to create their own problems, and had students work in groups to deepen their understanding in approximately one-fourth of the lessons observed. The groups were formed by interest or ability, however, in less than 10% of the lessons.

The teachers demonstrated evidence related to differentiation much less frequently than they did with pace, challenge, or supportive environment; however, there was
greater variation among the teachers in how often they demonstrated behaviors related to
differentiation than with any of the other themes. In addition, their use of differentiation
strategies was not directly related to their overall effectiveness ratings. For example,
although Lila demonstrated more evidence related to differentiation than all but one
teacher, Sam – who was rated just slightly below Lila in overall effectiveness – very
rarely used differentiation in his lessons.

**Supportive environment.** The teachers demonstrated observable evidence of a
supportive environment in 74% of the observation opportunities (17 supportive
environment-related behaviors x 7 teachers x 3 observations = 357; the behaviors were
seen in 265 of the 357 observation opportunities) as shown in Table 25.
Table 25

**Mathematical Observable Evidence Related to a Supportive Environment**

<table>
<thead>
<tr>
<th>Mathematical Observable Evidence</th>
<th>Lila</th>
<th>Sam</th>
<th>Casey</th>
<th>Melinda</th>
<th>Rachel</th>
<th>Hillary</th>
<th>Kelly</th>
<th>Total times behavior observed</th>
<th>Total observation opportunities</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Questions to solicit responses rather than tell answer</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>21</td>
<td>21</td>
<td>100%</td>
</tr>
<tr>
<td>Allowed time to practice</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>20</td>
<td>21</td>
<td>95%</td>
<td></td>
</tr>
<tr>
<td>Application activities for new concepts</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>19</td>
<td>21</td>
<td>90%</td>
</tr>
<tr>
<td>Built new math knowledge through simulated or real-world problem solving</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>21</td>
<td>10%</td>
</tr>
<tr>
<td>Reflect on own reasoning</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>11</td>
<td>21</td>
<td>52%</td>
</tr>
<tr>
<td>Reflect on reasons leading to inappropriate solutions</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>12</td>
<td>21</td>
<td>57%</td>
</tr>
<tr>
<td>Input from multiple students</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>19</td>
<td>21</td>
<td>90%</td>
</tr>
<tr>
<td>Follow-up questions to probe student reasoning</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>19</td>
<td>21</td>
<td>90%</td>
</tr>
<tr>
<td>Discuss new info via journal or think/pair/share</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>6</td>
<td>21</td>
<td>29%</td>
</tr>
<tr>
<td>Solicited varied solutions/methods/rationale</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>18</td>
<td>21</td>
<td>86%</td>
</tr>
<tr>
<td>Solicited conjectures</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>17</td>
<td>21</td>
<td>81%</td>
</tr>
<tr>
<td>Encouraged number of ways of thinking</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>17</td>
<td>21</td>
<td>81%</td>
</tr>
<tr>
<td>Questions to define problem</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>15</td>
<td>21</td>
<td>71%</td>
</tr>
<tr>
<td>Questions to facilitate problem solving</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>19</td>
<td>21</td>
<td>90%</td>
</tr>
<tr>
<td>Questions to help students elaborate their thinking</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>14</td>
<td>21</td>
<td>67%</td>
</tr>
<tr>
<td>Provided scaffolding</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>18</td>
<td>21</td>
<td>86%</td>
</tr>
<tr>
<td>Pressed for explanation</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>18</td>
<td>21</td>
<td>86%</td>
</tr>
<tr>
<td><strong>Total times behaviors observed per teacher</strong></td>
<td>46</td>
<td>43</td>
<td>35</td>
<td>40</td>
<td>43</td>
<td>28</td>
<td>30</td>
<td>265</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total times behaviors possible per teacher</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>357</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent of times behaviors observed versus number of possible times</td>
<td>90%</td>
<td>84%</td>
<td>69%</td>
<td>78%</td>
<td>84%</td>
<td>55%</td>
<td>59%</td>
<td>74%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
All teachers used questioning to solicit responses from the students rather than to tell them the answer in every lesson. Similarly, there were several other behaviors that were observed in almost all of the lessons such as allowing the students time to practice (95%), having the students do application activities for new concepts (90%), soliciting input from multiple students (90%), asking follow-up questions to probe student reasoning and facilitate problem solving (both 90%), soliciting varied solution methods (86%), providing scaffolding (86%), and pressing students for explanation (86%). All of the other observable evidence related to a supportive learning environment was observed in one-half to three-quarters of the lessons, with one notable exception. There was only one teacher who built new math knowledge through simulated or real-world problem solving. This happened to be only the teacher (Rachel) who taught Algebra I as part of a middle school version of the International Baccalaureate (IB) program. In addition to the Algebra I curriculum, she was also required to provide interdisciplinary activities for her students.

The more effective teachers generally had more observable evidence related to a supportive learning environment than did the less effective teachers, although there was less variability within the group than with pace, challenge, or differentiation. In addition, mathematical observable evidence related to a supportive environment was seen much more often than the evidence related to pace (seen in 59% of the observation opportunities), challenge (seen in 54% of the observation opportunities), or differentiation (seen in 28% of the observation opportunities). In fact, Hillary – the teacher who demonstrated the least supportive environment – was still relatively supportive. She demonstrated supportive behaviors in 55% of the observation
opportunities, which was a similar frequency to the averages of the entire group for pace (59%) and challenge (54%).

**Summary.** Overall, the teachers who were rated as being more effective generally demonstrated more mathematical observable evidence related to pace, challenge, differentiation, and a supportive environment than did the less effective teachers, although as noted above, there were specific exceptions to this. The two most effective teachers – Lila and Sam – demonstrated evidence related to these areas in 75% and 60% of their overall observation opportunities, respectively. In contrast, Hillary and Kelly – the two least effective teachers – demonstrated mathematical observable evidence in only 36% and 41% of their overall observation opportunities, respectively. While this supports the idea that more effective teachers show mathematical behavioral indicators related to the areas of pace, challenge, differentiation, and a supportive environment more frequently than less effective teachers, the fact that Hillary demonstrated observable evidence less often than did the lowest rated teacher (Kelly) suggests that tying specific mathematical behaviors to teacher effectiveness is not a simple or clear-cut proposition.

**Case Studies**

Each teacher was individually interviewed by the researcher. The interview was transcribed, summarized, and sent back to the teacher to member check. The researcher then conducted a thematic analysis of each interview transcription using the constant comparative method of open coding. The researcher compared the resulting categories to those identified from the holistic coding of her field notes and to the categories identified in the mathematical observable evidence associated with the COS-R teaching behaviors. This resulted in the four a priori categories of pace, challenge, differentiation, and
supportive environment and the emergent categories of mathematical giftedness and meeting gifted students' needs. The detailed case study on each teacher – starting with the most effective teacher and progressing to the least effective teacher based on their average scores on the COS-R – can be found in Appendices U through AA. The results were then analyzed by the cross-case themes that emerged from viewing the individual cases as a whole.

**Research Question Findings**

By using a cross-case analysis to view the data presented in the seven cases, the researcher was able to answer the research questions. Several cross-case themes emerged related to pace, challenge, differentiation, and a supportive environment. These will be presented under the applicable research questions. Two additional findings related to the teachers' perceptions of mathematical giftedness and meeting gifted students' needs will then be discussed.

**Research question 1. To what extent do middle school Algebra I teachers modify the pace of instruction for their gifted/talented students?**

The teachers were eager to talk about the pace of the course and of the lessons, themselves. As was noted previously, teachers demonstrated pace-related evidence during 59% of their opportunities to do so during the observations. Two main themes emerged from the interviews and observations – the overall pace of the course was driven from the district level, and modifications to the pace within the classroom were largely done for the class as a whole. Table 26 provides a listing of the cross-case themes related to pace and the teachers who expressed ideas related to the theme.
Table 26

Cross-case Themes Related to Pace

<table>
<thead>
<tr>
<th>Cross-case Themes</th>
<th>Lila</th>
<th>Sam</th>
<th>Casey</th>
<th>Melinda</th>
<th>Rachel</th>
<th>Hillary</th>
<th>Kelly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pace driven from outside the classroom</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X X X X X X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• District curriculum framework/quarterly assessments</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X X X X X X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Block scheduling impacts</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X X X X X X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modifications to Pace</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X X X X X X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Determine prior knowledge</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X X X X X X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Adjust for class as a whole</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X X X X X X</td>
<td></td>
<td></td>
</tr>
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<td>• Individual pace within activities</td>
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**Pace driven from outside the classroom.** Every teacher indicated that the overall pace of the Algebra I course was driven from outside the classroom. Teachers felt they had little control over the overall pace of the course, describing the pace as “pretty much laid out for me,” “little flexibility,” and “my hands are tied.” Hillary’s response to the researcher’s question about how she decided the pace for the class was typical of the other teachers:

I don’t decide it. The state decides it and the county decides. Unfortunately, we have a certain amount to cover and we have just enough time to cover that material, so the pace is pre-determined. We don’t have much flexibility at all.

The teachers specifically pointed out that the district’s planning documents and block scheduling had a significant impact on their pace.

**District curriculum framework/quarterly assessments.** All of the teachers except Casey pointed out that the district’s curriculum framework or planning guide indicated the amount of time allocated for each topic. Rachel explained:
The curriculum guide gives me the pace that I have to take. I have to teach certain topics . . . it’s pretty much laid out for me by the quarter and by the week, and how many blocks I should spend on a certain subject to get the whole course done in the right amount of time.

She further pointed out that this guide was developed in a way that allocated more time to the historically difficult topics; however, there was a significant amount of material to get through, so the overall pace was rapid. Rather than citing the curriculum framework, Casey acknowledged that the pace was driven by the school district’s quarterly assessments.

**Block scheduling impacts.** Six of the teachers pointed out that the block schedule also impacted the pace. Because Algebra I met every other day, the teachers oftentimes had to cover more than one concept during a lesson to maintain a pace that allowed them to get through the material required by the district planning guide. Sam pointed out that “it’s a real challenge to cover all the material,” while Melinda expressed that the amount of material made it “difficult to slow down,” and at times, she just had “to move on” to get through everything. Hillary noted that because of the amount of material they had to cover in one lesson, “Time is a critical factor, it really is. There’s like no time to even breathe. If you come and watch me in Algebra I versus Pre-Algebra, it’s crazy. Try putting a week into two or three lessons.” In addition, both Hillary and Casey pointed out that middle school students were not able to handle multiple concepts in one day. Casey explained that:

The block schedule has killed it for the kids. I taught block with high school students and they were able to do it and they were lower, not gifted students. They
were able to handle the block. The middle school students can’t handle that many concepts in a day and keep them straight. I try not to cover the foundation and then the building material on the same day. So I might do two foundations of two different topics on the same day and then build on those two topics on the same day. We go so fast.

Sam, Kelly, Hillary, and Rachel also talked about how this was the first math course that met every other day and how difficult this was for the students. Sam explained:

The curriculum sets them up for failure in one sense because they do get this great whole year of math – 90 minutes every day [the year prior to Algebra I] – but the pace is slower, they don’t do as many skills all at once, then they come here and it’s multiple skills all at once and it’s every other day. It’s more complex – it’s a tough adjustment – and of course they’re seventh graders so they’re going through other adjustments in life.

Similarly, Kelly did not think the students had figured out how to allocate their time when they had a day between classes. She said:

I really wish this class was every day and not every other day because there’s a lot, especially with algebra. With algebra they’re still getting used to the change from last year when they had an hour and a half [of math] every day, so they are not doing enough on their own yet. They’re just not comfortable with that yet. I guess they don’t really know how much they need to do on their own.

It is interesting to note that Lila, rated as the most effective teacher, was the only teacher who did not seem to feel that the block schedule caused an issue with pace. This suggests
that her lengthy experience in teaching algebra may have allowed her to more easily adapt her instruction to accommodate the schedule.

**Modifications to pace.** Despite the fact that the teachers felt the overall pace of the course was beyond their control, they did feel like they were able to modify the pace somewhat within their individual classes. To make adjustments, several teachers determined prior knowledge and then modified the pace of the class as a whole, although students were sometimes allowed to move through various activities at an individual pace.

**Determine prior knowledge.** To decide how to adjust the pace, five of the teachers determined the students’ prior knowledge, generally by informally walking around to see how they solved various problems or based on the responses to questions they asked. The two most effective teachers determined prior knowledge during each observation, while the less effective teachers did it less frequently. The method was the same, regardless of whether the teacher had been rated as one of the more effective or less effective teachers. Neither Rachel nor Casey was observed assessing the students, although Casey did indicate that she tried to pre-assess the students prior to the end of the previous unit so that she could determine “what direction to take them.” She explained why she felt it was important to pre-assess the students:

> I know what has been covered in the previous courses. Unfortunately, the students are all coming from different places so they have all had different experiences . . . so I know that there are a few students who haven’t seen the baby steps and then I know there are students in here that solved equations with variables on both sides last year.
None of the teachers, however, pre-assessed their students with an eye toward compacting the material or accelerating individual students. Melinda explained that “it would be ideal if we could pre-assess the students and differentiate the classroom, but it’s just impossible because of the time and the curriculum we have to cover.” Over half of the teachers pointed out that Algebra I was mostly new material to which the students had not been previously exposed, and so they did not often run into a situation where a student had already mastered the material. Rachel explained: “None of them are going to have mastered it. It’s all going to be brand new. It’s going to be a matter of who catches on faster at that point.”

**Adjust for class as a whole.** The teachers only used their awareness of their students’ prior knowledge to modify the pace for the class as a whole. Furthermore, when the researcher asked how they balanced their practice versus learning tasks specifically for their gifted/talented students, every teacher responded by talking about adjustments for the entire class. Several teachers talked about how they taught the same lesson to different classes and adjusted the pace within the lesson for each particular class. In this way, they were able to balance the practice and learning activities in a way that met the needs of the class as a whole. For example, Lila explained:

> Once I start getting into the notes and as I question – you’ll notice a lot of questioning throughout the notes – and based on that questioning, I will skip. If you took a set of each of the algebra notes out of all three classes, none of them would look the same in terms of what problems we do and how much highlighting or emphasis and such. I sort of gear it toward each class.
The teachers pointed out, however, that they attempted to keep all of their classes together as far as covering the same lesson during the same day. In addition, Lila, Melinda, and Hillary taught in the same school and because they planned collaboratively, they all tried to teach the same lesson during the same day. If some event caused a class to fall behind, Lila explained that:

Very rarely do we go separate other than the fact of... let’s say on the schedule there would be an assembly or a snow day or something that puts us out of sequence, then we’ll do things like make copies of the notes available on-line. If we see them for only 15-20 minutes versus the rest of the class for an hour and a half, then we would provide after- or before-school tutoring for that particular group if they so elect.

Similarly, most teachers pointed out that if an individual could not keep up with the pace, they needed to come by for additional help or use the additional resources they had made available.

Casey admitted that because she adjusted the pace for the class as a whole, she tried to “hit the average” with the amount of guided classwork she did in a lesson, which probably gave her high-ability students more practice than they needed. Although five of the teachers commented that they thought the pace of the course was already quick, when the teachers determined that most of the class understood the material, they moved ahead, generally by skimming through notes or skipping problems. This allowed them to allocate time for the students to work on the more complex problems.

*Individual pace within activities.* Despite the focus on adjusting the pace for the class as a whole, students were allowed to progress through individual activities at their
own pace. In four of the teachers’ classes, this entailed allowing the students to move through guided practice problems at an individual pace, while the remaining three teachers allowed students to work on station activities at their own pace. However, once the gifted students had finished with the activity, they were not provided with any enrichment or acceleration opportunities. Most teachers simply indicated the students should find something to do. In the case of Lila, Casey, and Rachel, this entailed having the students complete a worksheet, work on homework, or finish a project, while Sam had his gifted students put answers on the board. Similarly, Kelly explained:

Some kids will always just get things quicker than others . . . like today, I gave them a Sudoku to work on . . . that’ll keep them going for a while because it’s hard. There are different difficulty levels of those . . . or I’ll give them a few more problems to do. Sometimes I’ll see them helping students around them.

Similarly, Melinda talked about how her advanced students helped others when they were done with their work. In three of the classrooms, these activities were not enough to occupy the students’ time, and they ended up talking. The researcher did not see a single case where the students were allowed to proceed beyond the topic at hand.

**Summary.** In summary, the teachers believed the overall pace for the course was set by the district. Within a particular lesson, they only modified the pace of instruction for their gifted/talented students as part of their overall modification for the entire class. The majority of teachers informally checked their students for prior knowledge and were able to adjust their lessons accordingly, although those adjustments were based on looking at what the average student understood. On the other hand, because the classes were on a block schedule, they only met every other day which sometimes required the teachers to
cover two different concepts per day. This resulted in lessons that were rather quickly paced to begin with. Most teachers were fairly adept at moving quickly through the basic material and on to more complex problems, although the less effective teachers tended to stay on the more basic material longer. The average COS-R rating of 2.33 on the behavior, the teacher allowed students sufficient time to thoroughly explore complex problems, reflected that they were a little higher than somewhat effective in this regard.

When the teachers allowed the students to move at their own pace, it was within a constrained activity, such as guided practice problems or a station. This was true regardless of whether a teacher had been rated as more or less effective than the others. Once the students completed that activity, they were not provided with any enrichment material and so they generally worked on homework, helped other students, read a book, or talked. It should be noted, however, that because the pace of the observed classes was fairly quick, the amount of “dead time” the gifted/talented students encountered only amounted to five to ten minutes per lesson. When looked at throughout an entire school year, however, this amount of extra time adds up. This was reflected in the mean COS-R rating of 1.42 (between ineffective and somewhat effective) for the behavior, the teacher allowed the student to move through the material at an individual pace.

Research question 2. In what ways do middle school Algebra I teachers increase the level of challenge for their gifted/talented students?

Although the researcher noted varying levels of challenge during the observed lessons, the general consensus from the teachers was that the Algebra I material was already challenging for the gifted/talented students. As such, the researcher only saw teachers display challenge-related behaviors in a little over half of the observation
opportunities (54%). As discussed in the previous section, they indicated that none of the students had already mastered the material since it was new. Sam explained:

Some of my brightest students comment to me that they are actually rejuvenated and energized by the algebra curriculum because it was probably too boring to them prior to now, but at least now they are kind of getting something new every day that is going to challenge them a little bit. They may get it right away, but it’s not something they’ve had before.

In fact, Sam indicated that the level of complexity of the course would increase since the new state standards were “even more complicated” than the old and Hillary pointed out that the material was “already rigorous enough.” Although Lila also thought the course was challenging, she explained that she constantly adjusted the level of challenge because student readiness varied each day.

Two main themes emerged from the observations and interviews. First, when teachers raised the level of challenge, they did so for the entire class, and second, the teachers raised the level of challenge in three main ways – by adding complexity into the problems, the processes, and various concepts. Table 27 provides a listing of the cross-case themes related to challenge and the teachers who expressed ideas related to the theme.
Table 27

Cross-case Themes Related to Challenge

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<thead>
<tr>
<th>Cross-case Themes</th>
<th>Teachers</th>
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<tr>
<td></td>
<td>Lila</td>
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<tr>
<td>Increase Challenge as a Whole Class</td>
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<tr>
<td>Increased Complexity/Abstraction</td>
<td>X</td>
</tr>
<tr>
<td>• Complexity within problems</td>
<td>X</td>
</tr>
<tr>
<td>• Complexity with process</td>
<td>X</td>
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<tr>
<td>• Complexity with concepts</td>
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**Increase challenge as a whole class.** Although Lila mentioned that she individually provided a gifted/talented student with more challenging material the previous year, during the classroom observations conducted during this study, none of the teachers provided an increased the level of challenge specifically for their gifted students. All of the students worked on the same material during class and had the same homework. As with pace, the teachers increased the level of challenge for the entire class or directed the students who had finished the current activity to proceed to the more challenging problems that were contained within the particular worksheet or notes. Lila, Melinda, and Hillary collaborated on the typed notes they provided the students, and they indicated that they always included more challenging problems at the end so that even if the entire class did not have time to go through all of the problems, those students who were able to move at a quicker pace would have more complex problems to tackle. Lila explained:

We have a diverse class and there are kids who get it and they can move on independently and I encourage them to go on with the other problems and they do,
but they know to do that . . . if they are showing proficiency, they move, they move themselves on . . . and there is plenty built into the notes so that they can.

While the four other teachers did not provide written notes, the problems they presented to the class during guided practice became increasingly challenging. In these classes, students were only able to move on to more challenging problems on their own if they happened to have a worksheet with such problems on them. None of the teachers provided the students with enrichment activities that might have increased the level of challenge, nor did they accelerate the students into more challenging material. In fact, Rachel illustrated the prevalent mindset that the class should stay together when she pointed out that although there were students in her class who could handle more complex problems, there was not time during the class period. She said she would tell her students, “I get that you get it and we could go further with that, but I don’t have time to go further. Not in the classroom.”

The teachers also noted some issues with increasing the level of challenge for the class as a whole. Lila pointed out that when she asked challenging questions, she sometimes lost the part of the class. She explained:

There are many times with my gifted kids with my questioning and answering that we lose the rest of the class. Not that I want to lose them, but meaning they are getting the full impact of my questioning because the gifted child stays with me.

Rachel pointed out that if the material became too challenging, it was a waste of time because the students just became confused. Kelly expressed a similar sentiment, pointing out:
If you do something that is way too complex, you’re going to lose those one or two at the [lower] end... I don’t want to lose anyone... once you lose them, they won’t have any confidence at all.

On the other hand, Kelly pointed out that even if the students did not fully understand the more complex material, they were encouraged to think at a higher level when they considered it. Lila noted that it was difficult to challenge the gifted students in class because of the “dilemma” it posed as far as drawing attention to them. Both Melinda and Hillary talked about how more challenging material required time tradeoffs because tackling such material gave them less time to practice other concepts the students needed to know. The researcher noted the frustration in students when Melinda provided a challenging problem, but did not give them adequate time to solve it. On the other hand, Rachel was able to move through the material in her IB class at a quick enough pace that they had plenty of time to work on the challenging problem she presented.

Increase complexity/abstraction. Although the challenge was provided to the class as a whole, the teachers had many methods by which they were able to increase the level of complexity and abstraction of the material. They did this by modifying the actual problems, making the solution process more difficult, or making the concepts more complex. To facilitate all three methods of increasing the complexity of the material, the teachers used open-ended and higher-level questions to help their students think at deeper levels.

Complexity with problems. To make the actual problems more complex, the teachers used a variety of methods, such as using larger numbers, increasing the distribution involved in the problems, and using negative numbers. They also used fractions,
especially those with different denominators, or incorporated order of operations into the problems. Rachel explained how she increased the difficulty of problems:

Instead of something basic, I throw in some things that might throw them for a loop, things that could stump them or trip them, something that if they don’t do it in order, or something that makes them think a little more.

Lila, Sam, Melinda, and Hillary did not allow the students to use calculators when concepts were first introduced, a step that not only made the problem more difficult to solve, but also one that helped ensure they truly knew how to do the skills involved. In addition, Sam, Rachel, and Kelly gave the students word problems or multistep problems to increase the level of challenge. Every teacher talked about several methods of making the actual problems more complex, and this was the technique of adding challenge most often seen by the researcher.

**Complexity with process.** To increase the complexity of the problem-solving process, Lila occasionally had the students use flowcharts, while Kelly had her students turn number problems into word problems or write an explanation of why they used a particular process to solve a problem. Sam, Rachel, and Melinda encouraged their students to find alternative methods of solving problems and then had them compare and contrast the solution methods with others. Several teachers had their students create their own problems, determine what would make a problem false, reflect on what the next step of a new concept might be, or analyze an incorrect solution to determine where the error was. Furthermore, Hillary thought that cooperative learning added a level of complexity to the process since the students depended on each other to solve the problems.
Complexity with concepts. To increase the level of challenge even further, the teachers added complexity to various mathematical concepts. Casey explained one way to do this:

I think it’s much more challenging for a student to come up with their own rule than to repeat a rule and repeat the process – which we do a lot. We give them a rule and they repeat the process instead of them investigating and finding their own rule.

When I do want to challenge, they come up with their own rule first.

Other teachers asked students to link two mathematical concepts together or even to link mathematics to another content area. Lila demonstrated this when she asked her class to relate an English phrase to a mathematical expression. In addition, Sam and Casey encouraged their students to recognize patterns, an activity that encouraged higher-level thinking. Similarly, although every teacher related mathematical concepts to the real world, Rachel actually had her students use mathematics to identify and define real-world problems through her very challenging design cycle activity. This design cycle activity was part of a larger water conservation project the IB students were addressing in several of their classes. For this lesson, the students were asked to figure out how the technology design cycle (investigate, plan, create, and evaluate) related to mathematics. Once they correlated their mathematical problem-solving technique to the design cycle, they worked with a partner to use this design cycle to actually solve a math problem. Furthermore, two teachers encouraged their students to explore mathematical tools and concepts which they had not previously seen. Casey let her students explore algebra tiles and figure out how they might be used, while Rachel asked her students to figure out why a number was imaginary. In both cases, the gifted/talented students seem to relish the challenge.
Hillary was the only teacher that did not demonstrate or discuss ways in which she might increase the complexity of concepts for her students.

**Summary.** Overall, the level of challenge presented in Algebra I was fairly high to begin with. Most of the Algebra I concepts were not covered in previous mathematics coursework, and so the teachers indicated that none of the students came to the class already knowing the material. Since Algebra I is a foundational course for higher level mathematics and science courses, the teachers thought it was important to ensure that all of the students learned about the various concepts and the problem-solving processes in the same way to ensure they built a solid base for these students. The consensus among the algebra teachers was that the course was generally challenging enough for their gifted/talented students without much modification. Hillary explained:

I think the class itself – the way that it’s set up – is challenging no matter if you’re working two years above grade level or not. I believe that this is the first class for a lot of students to understand what true math is and really get challenged for the first time. I do believe that the seventh graders, the gifted students do get challenged. Do they get challenged as much as the others? Maybe not, but they do get challenged.

Several other teachers agreed that Algebra I was the first mathematics class in which their gifted students were actually challenged. Consequently, the teachers in this study never specifically raised the level of challenge for their gifted/talented students, although these students were provided with more complex problems, processes, and concepts as part of the entire class. The teachers were generally successful in providing a challenging environment as was reflected in the fact that the overall mean for every COS-R behavior related to challenge was between a 2.0 (*somewhat effective*) and 3.0 (*effective*). It should
be noted, however, that the gifted/talented students were not provided any enrichment that included more challenging material, nor were they allowed the opportunity to accelerate into more complex material. Instead, they gained access to the more challenging material via guided practice, homework problems, and challenging questions posed by the teachers.

Of interest, although all of the teachers increased the level of challenge for the students to some extent, the less effective teachers – Hillary and Kelly – spent far more time working through routine problems, oftentimes in the form of homework reviews, than did the more effective teachers. The result was that the more effective teachers had more time to move their students to higher complexity levels than did the less effective teachers. This points out how pace and challenge have a distinct bearing on each other.

**Research question 3. What differentiation strategies do middle school Algebra I teachers use to meet the needs of their gifted/talented students?**

The teachers portrayed a rather limited knowledge of differentiation strategies. The teachers were familiar with grouping and enrichment as differentiation strategies, but only a single teacher mentioned tiered assignments and curriculum compacting, and none mentioned open-ended activities or choice. They also expressed different ideas about the term, *differentiation*, with Hillary saying it was “vague, since it could mean many things.” Similarly, Sam pointed out:

> In a sense, I think of two definitions of differentiate – getting to the material in different ways versus challenging some students more than others – but the curriculum is what it is, so there’s not a whole lot of ways to deviate from that.

Rachel further explained:
True differentiation is when you have different levels of instruction going on in the classroom at the same time with the students. Honestly, in algebra, I really don’t have to do that. I don’t have to teach that way because we have a specific curriculum that I have to get through.

The overall theme that emerged when looking across the cases was the fact that differentiation was limited in the observed classrooms. In fact, teachers demonstrated observable evidence related to differentiation less often than any other mathematical behaviors. The more effective teachers did not use any more types of differentiation strategies than did the less effective teachers, although Lila – the most effective teacher – did use groupwork more frequently than the other teachers. Table 28 provides a listing of the cross-case themes related to differentiation and the teachers who demonstrated the different strategies.

Table 28

Cross-case Themes Related to Differentiation

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<td>Limited Differentiation Practices</td>
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<td>• Acceleration beyond classroom material</td>
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<td>• Curriculum compacting</td>
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<tr>
<td>• Enrichment</td>
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<td>• Open-ended activities</td>
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<td>• Tiered assignments/questions</td>
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<tr>
<td>• Flexible Grouping</td>
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<td>• Alternatives and choices</td>
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**Limited differentiation practices.** None of the teachers provided acceleration or curriculum compacting for their students, noting that that the Algebra I material was new to the students and the pace was already quick enough. Likewise, no teacher provided tiered assignments or tiered questions for their students, although several teachers did use effective questioning techniques to help students elaborate their thinking. Melinda explained the prevailing opinion, pointing out:

> With algebra, we are really teaching the same things. Once in a while we will have activities that will differentiate, but those are rare, those are spaced out. . . . We spend a lot of time planning, anyway. I think if we could see more room for implementing differentiated instruction, we would. It’s hard.

Several teachers also mentioned time constraints as a reason why they did not differentiate more. Lila explained:

> We always talk about differentiation within our lesson plans, our learner plan, and the template . . . we try to incorporate some ideas for all levels of small group instruction or strategies related to differentiation . . . but that time factor always seems to be my biggest downfall, unfortunately.

**Enrichment.** Rachel was the only teacher to provide an enrichment activity during one of the observed lessons. Her class did the design cycle activity as part of the IB program, which she characterized as allowing “the gifted students opportunities to shine.” However, despite doing enrichment activities with the IB class, she pointed out that she did not think the Algebra I curriculum needed enrichment because of the challenge and rigor already built in. Sam reflected a similar viewpoint. When asked about enrichment, Hillary mentioned that she – along with Lila and Melinda – planned to teach an
additional unit on word problems, but based on her description, the unit seemed to be focused more on remediation than on broadening or deepening the students' understanding. Casey mentioned that her school was about to initiate an on-line program where she planned to post challenge problems, which she hoped would enrich the students. In addition, she indicated that she sometimes provided extension activities within the topical area for students who understood the homework while she went over the homework with the struggling students.

Time constraints were a factor cited by four of the teachers as to why they did not provide enrichment for their students. Casey characterized this by explaining that “because of the [Algebra I] requirements, there’s not enough seat time any more. There’s not enough hours.” Despite this, Hillary admitted that she “probably could have done something a little more enriching for the high-ability students.” Furthermore, Lila pointed out her concern about gifted/talented students being provided with enrichment within the classroom. She explained:

We have the classroom resources . . . you hesitate mentioning too many things because sometimes they like to get ahead and a little too much beyond . . . you want to make sure they’re still staying with you.

Five teachers were aware of outside enrichment opportunities such as MathCounts, Odyssey of the Mind, the SAT question of the day, a NASA technology program and mentorships, and a local university’s math competition; however, many of the teachers were unsure of what these programs entailed or which of their students participated. Hillary explained: “I don’t know what the options are . . . maybe that’s what it is. What are my options to give them more of a chance to get enriched in the subject?” Sam and
Casey were not aware of any enrichment opportunities, with Sam mentioning that although the students could seek something out, “everyone tends to be challenged somewhat by the material every day.” Although Melinda was aware of many of the above opportunities, she cautioned that the curriculum was tough enough and the students had sufficient disruptions to their schedules without these programs.

Open-ended activities. Three teachers provided open-ended activities for their students during the observed lessons. Rachel’s students tied the technology design cycle to the mathematical problem-solving process in a way that made sense to them. Casey provided materials related to particular mathematical concepts – such as order of operations, equations, and properties – at various stations and allowed the students to determine how they wanted to use them. Finally, Hillary talked about the video project her class would do later in the semester where they would select a topic, create a storyboard, and then present it to the class. While several of the other teachers had specific activities or games in which their students participated, these were aimed at reviewing mathematical concepts, and thus had specific right and wrong answers.

Flexible grouping. Grouping was the differentiation strategy most frequently noted by the researcher, seen on at least one occasion during the observation of each teacher. Grouping was used for the purpose of allowing students to collaborate on problem solving, deepen their understanding, and discuss solutions. Lila also liked to group students because:

As you can see, I like to generative conversation within the group. I like them to come together and look after each other. They’re normally pretty good when it’s a mix of abilities like that. They’ll be changed throughout the year.
While every teacher allowed the students to check answers with a partner sitting next to them, Sam was the only teacher who used this partnering strategy exclusively. He explained that he simply did not do much group work in Algebra I. When asked why, he explained:

A reason for that is . . . I would say . . . it’s almost every day when they come in here, it’s a new topic. There’s no time to spend two days on that topic. I wish there was more time when they could kind of do some group exploratory type of stuff and see where they went with it, but I just don’t find the time in the curriculum to do it.

The other teachers used several different grouping strategies based on ability, choice, or interest. They pointed out that they changed groups often throughout the year. Rachel occasionally created groups randomly, while Lila, Casey, Melinda, and Kelly said they sometimes allowed students to select their own groups. Kelly explained that when the students worked with someone they liked, they stayed on task better. She also pointed out her perception that students thought “when I don’t understand something, I want to talk to somebody I know and get along with.”

The most common grouping strategy was to form mixed groups of higher- and lower-ability students. These groups typically contained four students, although they ranged in size from two to six people. Teachers assigned these ability groups in various ways. Heather paired the student who had the highest grade with the lowest student, while Lila took into consideration their grades as well as their personalities and work ethics, pointing out that if those attributes did not complement each other, the group would not work. This mixed-ability arrangement enabled the high-ability students to
serve as peer helpers. All of the teachers, except for Sam and Melinda, specifically mentioned using gifted students in this role. Hillary explained this practice as follows:

The highest person in the class is with the lowest, and so forth . . . and so when they are working with partners, they are working with someone who can pull, and hopefully help them, and the high student can pull the low student up.

Lila explained that peer coaching was a “good reinforcement” for the high-ability students. The researcher also observed that the gifted/talented students were frequently the self-selected or assigned spokesperson or leader of the group when it consisted of students with mixed abilities.

Several teachers commented that they avoided putting two lower-ability students together; however, Lila and Casey were the only teachers who occasionally made high-ability groups. Lila sometimes put two gifted students together so that they could challenge each other, but said this was not her normal practice. She explained:

I don’t tend to put all the gifted together. They just pull so much attention toward themselves because they’re zooming . . . but sometimes I will put two of them together to just allow them to have that extra peer challenge.

Alternatives and choices. Although all the teachers allowed their students to select and use the solution method they understood the best, only three teachers provided a choice related to specific activities in the observed lessons. Kelly and Lila provided students a choice in selecting the materials and problems they wanted to solve, while Casey allowed her students to choose how they wanted to use the various materials during a station activity. These three teachers were also the only three to allow their students the opportunities to create their own problems.
Summary. Overall, the middle school teachers were rather limited in the differentiation strategies they used to meet the needs of their gifted/talented students, focusing on grouping, and to a very limited extent, choice and open-ended activities. Acceleration, curriculum compacting, and tiered questions and assignments were never observed. All of the students worked on the same material during the class period and they all had the same homework. Likewise, these topics were rarely mentioned during the interviews, suggesting that they were not the strategies that were in the forefront of the teachers’ minds when asked about differentiation. Some of the teachers felt the Algebra I curriculum did not need to be enriched because of the challenge and rigor built into it, while others wished they had more time to provide enrichment activities for the class. Like their view concerning challenge and pace, the teachers viewed enrichment within the classroom as a whole class activity.

The differentiation strategy most commonly noted was flexible grouping based on ability, interest, and choice. Grouping by ability was almost always done as mixed-ability groups consisting of both high- and low-ability students. Teachers largely supported the idea of using the high-ability students as peer tutors for the lower-ability members as it tended to pull them up.

The limited use of differentiation strategies was reflected in the mean rating of 2.19 for the COS-R behavior, the teacher accommodated individual or subgroup differences. This was the fifth lowest rated behavior. This is in contrast to another differentiation-related behavior, the teacher provided opportunities for independent or group learning to promote depth in understanding content, which received a mean rating of 2.50.
Research question 4. In what ways do middle school Algebra I teachers provide a supportive environment for their gifted/talented students?

All of the teachers provided a supportive environment for their students, although the more effective teachers demonstrated evidence related to a supportive environment more often than did the less effective teachers. Four major themes emerged in this area including the teacher providing a conducive learning atmosphere, scaffolding, high expectations, and modeling high-level performance. Table 29 provides a listing of these cross-case themes related to a supportive environment and the teachers who demonstrated the different themes.

Table 29

Cross-case Themes Related to Supportive Environment

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<tr>
<th>Cross-case Themes</th>
<th>Teachers</th>
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<td></td>
<td>Lila</td>
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<tr>
<td>Conducive learning atmosphere</td>
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<tr>
<td>Scaffolding</td>
<td>X</td>
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<td>High expectations</td>
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<td>Models high-level performance</td>
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Conducive learning atmosphere. Five teachers provided an atmosphere that was conducive to learning for the gifted/talented students in the class. These teachers provided a warm, positive environment where the students felt at ease asking questions and taking risks. Lila, Sam, Casey, Melinda, and Hillary were complimentary and encouraging, saying things such as “that’s great understanding,” “those are excellent questions,” and “I know you can handle this.” Lila explained:
Regardless of whether they are gifted or not, I just always make sure they know I am approachable and that I’m going to provide additional help for them. . . . They’re always welcome to make an appointment for early morning help or after-school help.

In addition, Casey made a special point to encourage the girls in her class because she felt that they were not as confident in their abilities. She elaborated:

The boys have been told over and over again how smart they are in math, and someone forgot to tell the girls because they really are as gifted as the boys. The boys are just more confident and that's why they are more willing to say, “I've got this. I need something harder.”

Similarly, Lila talked to her students about mathematics opportunities to expose them to potential career venues. She said:

I think I also just try to encourage them . . . I try to reflect on my experiences as a computer programmer and systems analyst, you know all the different opportunities that they might have, and for them to really become knowledgeable about it because they obviously are gifted in math, and I try to give them exposure to what’s available.

Casey also stressed that she tried to let the gifted students “be themselves in class,” something she felt they struggled with in other classrooms. She explained:

I frown on the bullying. I think a lot of times they get in their other classes, because their other classes are not by ability, they end up with students they don’t interact well with and they are not able to express themselves as much as they normally do.
So I try to make sure that in here they feel safe and they are able to express themselves the way they want to.

The teachers specifically pointed out that making errors was essential to the learning process. Several mentioned that this was the first course in which the high-ability students may have struggled, and so it was important to let them know that making mistakes was part of the learning process. Sam stated:

I realize for a lot of these kids it will be the first time they struggle with math. I know that I’ll have that conversation with parents very early in the year with some of these kids that hit the math wall for the first time. So I tell them, “If you already knew it, you wouldn’t be in this class. It’s okay to miss stuff.” In fact, we try to model how that’s more powerful learning, how we can learn from our mistakes.

Sam also pointed out how there were “great wrong answers,” while Lila told them, “don’t be embarrassed if your answer is not right; that’s how we learn.” Hillary encouraged the students to examine their errors by saying, “Describe to me what’s happening,” rather than making the students uncomfortable in front of their classmates.

Rachel and Kelly did not provide an atmosphere that was necessarily as conducive to learning. While Rachel occasionally complimented her students, her classroom did not feel as inviting as the others in the study in that her overall manner was rather brusque, somewhat limiting the conversational atmosphere seen in the other classrooms. For example, she became visibly upset with the students who did not do their homework, speaking in a rather harsh tone that made even the researcher feel uncomfortable. During the interview, she described her frustration, stating:
You heard me yesterday with the homework. It’s ridiculous! I’ve been doing it [reminding the students about homework] for four weeks now. How come you don’t understand that you go to the internet and there’s all the stuff on there [homework help]?

Similarly, at times Rachel appeared to be talking at the students, rather than talking with them about a particular concept. On the other hand, although Kelly was positive and it was apparent that the students enjoyed having her as a teacher, her struggles with classroom management made the atmosphere less conducive to learning than it might have been. Kelly constantly had to ask the students to be quiet, and consequently, she had to repeat herself numerous times because students either could not hear her or were not paying attention.

**Scaffolding.** All of the teachers provided scaffolding for the gifted/talented students as part of the whole class. The most common ways they did this was by building on prior knowledge and making connections between old and new learning. To associate new concepts with ones the students had previously learned, the teachers asked questions such as, “What does this look like?” or “What does this remind you of?” Lila, Sam, Casey, Melinda, and Hillary encouraged the students to use their metacognition by reflecting on what they knew and how they might use that to think about other problems. For example, Sam asked the students, “Using the skills you already have, how might you solve this?” The teachers also built from basic skills to more complex ones. For example, Rachel said:

> When I go to teach a lesson, I really try to put myself in the students’ position. I say, what at this point do they know that they can use for this, and what is it that they
have never done before? I try to make it so that what they know fits into what we’re
doing new and go that way and build up to it gradually.

Casey pointed out that it was difficult to provide the appropriate level of scaffolding for
her high-ability students because they came from different courses and some were
missing foundational material. Furthermore, several teachers mentioned that to increase
the level of challenge, they expanded the difficulty of known concepts, rather than
challenging students with totally new concepts.

To ensure the students had the same basic understanding of the material, Lila,
Melinda, and Hillary provided typed notes for the students on each topic, leaving an
occasional blank space for the students to fill in. Sam wrote notes on the board for the
students to copy into their mathematics notebooks, and the remainder of the teachers
provided occasional handouts related to the concept at hand. The four teachers that
provided notes for the students made it a point to encourage them to highlight particularly
challenging areas and to annotate typical errors made with various concepts. In addition,
every teacher walked around and individually assisted students during the guided practice
portion of the lessons and they all attempted make algebra more relevant to the students
by relating mathematical concepts to the real world. Sam, Melinda, and Hillary also
provided tips for better ways to approach problems on homework and quizzes, such as
drawing a number line on the paper. Furthermore, all of the teachers pressed their
gifted/talented students for explanation, asking such questions as “Why does that rule
work all the time?” or “How can you prove your method is correct?” Lila pointed out
that when she questioned her gifted students to encourage them to elaborate their
answers, the conversation was sometimes lost on the rest of the class, but she believed
that by pressing them for an explanation, the gifted students were able to gain a deeper meaning from the lesson.

In addition to providing scaffolding to the students, all of the teachers encouraged the students who were confused or needed help to stay after school. Many did so in a general manner, just stating that they would be available; however, Sam further extended the invitation by individually encouraging students to come to after-school help as he walked around and saw that they were having difficulty. In addition to providing help after school, Lila, Melinda, and Hillary also indicated that they helped students before school and during their lunch hour. Furthermore, all of the teachers had resources posted on-line to assist the students. Several teachers indicated that they posted the homework answers on-line as well, so that the students could check their own work and come to class with questions. This was also intended to shorten the amount of time spent going over homework in class.

**High expectations.** The teachers relayed their high expectations in many ways, such as verbalizing their expectations that the students complete their homework and study algebra every night. Various teachers also talked about performance expectations, making comments such as “We’re all going to get 100s on this quiz,” or “A good student may want to practice that.” Casey further elaborated:

Performance-wise, for me there are two types of performance. There’s the effort and there’s the outcome of the effort. It is as important to me that the effort is as important as the outcome. So I try to make sure that they know that even if they haven’t quite got the concept, their effort is going to help them later in life as well.
If they can learn to be resilient and keep trying, they’re eventually going to get where they need to go, which has nothing to do with math.

In addition, Lila and Casey talked to their students about what they expected to see in their math notebooks, and Hillary and Sam both expressed their expectation that the students should start to become independent thinkers. Sam explained that he expected his students to check their notes, rather than immediately asking him questions about previous concepts, while Hillary told her students, “I want you to start thinking on your own.” In addition, Sam expected them to memorize certain formulas they would use throughout their mathematics careers. Furthermore, Casey and Rachel talked about behavior expectations when the students worked in groups, with Rachel commenting, “This is disappointing,” when students became off-task.

While Rachel demonstrated her high expectations in several ways, she gave conflicting messages about her high expectations when she indicated she would be satisfied if the students could do all but the most challenging problems on their homework. Kelly sent a similar message when she told the students to not even bother trying a challenging homework problem. Furthermore, although Kelly talked to her students about how she expected them to make an effort at home and stay after school for help if they needed it, she failed to convey her classroom behavior expectations to her students. The result was that they constantly ignored her pleas for them to be quiet.

**Models high-level performance.** Another way in which the teachers provided a supportive environment was in their modeling of high-level performance. Every teacher in the study consistently demonstrated the proper way to write out each step of the
algebra problems, while explaining the steps as they went. Hillary talked about this, saying:

I emphasize showing all your work and make sure I do it. Anything I expect from the kids, I do myself. I try to explain that everything I do is for a reason, and I try to explain why I do what I do and why it works so that the students not only understand the concept, but understand the reason behind it.

In addition, all of the teachers demonstrated multiple ways of solving the problems, allowing the students to use the method that made the most sense to them. They encouraged this by saying, “Excellent, that’s one way,” or “Can anyone think of another way?” They all demonstrated a thorough understanding of the material, and all of them used proper mathematical terminology as they taught. All but Kelly made it a point to stress that their students use this terminology as well. In fact, Hillary had a vocabulary wall for her students’ reference, and helped her students to understand the proper use of the terms by asking them, “What do you really mean by that?” when they used a term incorrectly. Furthermore, Melinda pointed out that “we are getting closer to being mathematicians when we use these terms,” and Lila talked about the correct use of mathematical symbols, telling her students, “A true mathematician will write variables in lower case.”

Both Casey and Melinda said they modeled high-level performance by keeping the level of the material high and ensuring that it was rigorous enough to keep the students challenged. Several teachers also mentioned the importance of modeling organizational skills. In addition, Casey pointed out that she did not “just leave things,” meaning that
she was not satisfied with her teaching until the students understood the concepts. She cited the example of teaching mathematical properties:

Some of my classes have properties until May, some finish in December, and sometimes they [properties] go away after the first test. But properties will stay until the students get them, because I know they can do it; they are just being stubborn.

She also thought it was important to let her students know that she was accountable for their performance, telling the students “We didn’t do well,” rather than just telling them that they performed poorly. Furthermore, Casey, Melinda, and Hillary believed they also modeled high level performance by demonstrating that they had high expectations for themselves as teachers. These three teachers also thought it was important to convey to the students the importance of students being helpful to each other. Finally, Lila, Melinda, Hillary, and Sam mentioned the fact that they modeled how to study effectively by providing the students with study tips, such as demonstrating how to highlight their notes or what types of material they might want to keep to review for the end-of-year standards test. These teachers stressed the fact that it was important for the students to learn good study skills now so they would have them when the material became more difficult.

Summary. Overall, the middle school Algebra I teachers provided a supportive environment in several ways. They provided an atmosphere that was conducive to learning by being approachable and creating an environment where the students felt comfortable taking risks. By pointing out that making mistakes was an important part of learning, the teachers encouraged the students, for whom this may have been their first challenging mathematics class, to not feel embarrassed when they made an error. This
was especially important for the gifted/talented students in the class who may have had issues with perfectionism. The teachers also provided scaffolding for their students, most frequently in the form of building on prior knowledge. Several of the teachers solicited conjectures and had their students reflect on how they might think about new concepts. In this way, they encouraged metacognition. They also solicited different solution methods, which helped students think in a divergent way. All of the teachers pressed their students for explanation, which encouraged them to think at a deeper level. Furthermore, several of the teachers routinely provided notes for the students and helped model how they might use them for study purposes. In addition, they all provided the students with additional resources on-line and were available for help outside of class time.

All of the teachers except for Kelly, the most inexperienced and ineffective teacher, expressed their high expectations of their students, not only in reference to their academic performance, but also in regard to their classroom behavior, study habits, and work ethics. They also modeled high-level performance in several ways. They had a thorough knowledge of the material, they demonstrated the correct problem-solving procedures, and modeled the correct use of mathematical terminology. The teachers also connected several problems to the real world in an effort to help the students see the relevance of certain concepts.

The teachers' overall ability to provide a supportive environment and their willingness to pay attention to their gifted students is reflected in the fact that the five COS-R behaviors receiving the highest mean ratings were all related to a supportive environment. For example, the mean rating for the behavior, the teacher provided a
reasonable amount of attention (as appropriate to the situation) to the gifted/talented students in the class compared to other students, was a 2.71, the highest of all the behaviors rated. Similarly, the behavior, the teacher incorporated activities for students to apply new knowledge, received a rating of 2.62, as did the behavior, the teacher encouraged students to express their thoughts.

Additional Findings

In addition to discovering several themes related to the research questions, the researcher also found that the teachers had definite views about mathematical giftedness and whether they were able to meet the needs of their gifted/talented students. These findings are presented below.

Mathematical giftedness. Although all seven teachers taught gifted/talented students, only Melinda had ever had any training in gifted education. It was important to examine the teachers’ basic understanding of giftedness, because it impacted how they viewed and responded to their gifted students. Two major themes emerged in this area including the fact that the teachers’ view of mathematical giftedness was decidedly practitioner-oriented, and the fact that prior to the study, they were unaware of who their gifted students were. Table 30 provides a listing of the cross-case themes related to mathematical giftedness and the teachers who expressed ideas related to the theme.
Table 30

Cross-case Themes Related to Mathematical Giftedness

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<th>Cross-case Themes</th>
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<td>Lila</td>
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<tr>
<td>Practitioner’s View of Mathematical Giftedness</td>
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<tr>
<td>Gifted characteristics</td>
<td>X</td>
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<tr>
<td>Gifted in domains</td>
<td>X</td>
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<tr>
<td>Inaccurate identification</td>
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<tr>
<td>Lack of Awareness of Gifted Students</td>
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Practitioners’ view of mathematically gifted students. Due to their lack of gifted education training, the understandings the teachers had about their gifted students were largely gained in the classroom, and in the case of Lila and Melinda, as parents of gifted children. The descriptions provided by the teachers to characterize their mathematically gifted students were decidedly practitioner-oriented, reflecting the expressions of giftedness they saw in their students within a mathematics classroom setting. Appendix T provides a listing of these descriptions.

Gifted Characteristics. Teachers frequently used phrases like, “They just get it,” “They zoom through material,” and “They’re go getters” to describe their gifted/talented students. For example, Robin described mathematically gifted/talented students as follows:

They have the ability to catch onto what you’re teaching them fairly quickly; it registers with them fairly quickly. They don’t have a lot of struggling with the concept. It kind of clicks; they see it a couple of times and it clicks. They also
analyze and think about what you’re doing and then they get it quickly . . . a thorough understanding and it sticks with them better.

Six of the teachers provided similar descriptions of the personal characteristics that identified mathematically gifted students. These included the fact that these students typically had a strong work ethic, demonstrated more initiative, were more focused, and that they could think more abstractly and at higher levels than typical students. Kelly explained that “I don’t see advanced kids so much as they’re smarter than anyone else; it’s that they have the motivation to learn and they do more than they’re supposed to and they actually try.” Melinda added:

They are able to see relationships that other kids are not able to see, maybe higher; their thinking is more abstract. They can grasp in a way and can see things in a way that it takes other kids a long time to get.

The six teachers also pointed out that these students were oftentimes distinguished by their actions in class, such as coming up with answers, performing well on assessments, completing their homework, and getting high grades. Sam, however, used none of these terms, pointing out that he did not think mathematically gifted students were any different from any of the nongifted students in his Algebra I class since both types of students worked at a high mathematical level.

In a similar vein, although Rachel described mathematically gifted students using the terms above, she also conveyed that all of her students were of high ability, originally telling the researcher that her entire IB class was gifted. Melinda articulated this confusion when she pointed out that sometimes the students who were typically
considered to be gifted were simply the high-achieving, internally-driven students in the classroom. She explained:

I think what we consider gifted are the high-performing kids in the classroom. Sometimes we don’t see, we don’t have the ability to see really how far they can go, to see how deep they are into the material, so it’s more like they are high-performing, high-achievers, self-driven. They have a network somewhere that has instilled that ethic in them which is a big contributor. I really don’t know if these kids are exceptionally creative or high-performers.

It should be noted that Sam and Rachel were the only teachers with classes made up entirely of seventh graders. Since these students were all taking Algebra I two years ahead of their peers, it stands to reason that the students in these classes, whether they had been identified as gifted or not, represented a more homogeneous high-ability group than did the classes with mixed grades.

While four of the teachers only focused on the positive attributes of mathematically gifted students, three teachers also pointed out other aspects of their giftedness. Both Sam and Kelly indicated that these students might not put forth the effort, with Sam further expressing that these students might think they “know it all,” and thus might not be as willing to learn from the teacher as other students were. Lila, in particular, provided a more sophisticated portrayal of gifted students, pointing out that these students might not perform up to their abilities, might become bored and frustrated if not challenged, and might not want to be viewed as being different from the other students. She talked at length about “differentiating discretely” for them so as not to make them uncomfortable in front of their classmates.
Gifted in domains. All seven teachers believed students could be gifted in a specific domain. Several of them specified that a student could be verbally gifted and not mathematically gifted or vice versa. Only Lila went beyond that viewpoint, pointing out that students might also be gifted in specific areas within the content area. She gave the example of being gifted in spatial thinking but not algebraic reasoning.

Inaccurate identification. Every teacher thought that some of their students identified as gifted were not necessarily mathematically gifted, based on the fact that they did not exemplify the characteristics noted above. Likewise, they all believed they had mathematically gifted students in their Algebra I classes who had not been identified. They based this on the students' performance on assessments, the type of questions they asked, and the speed and ease with which they learned new material. Some teachers expressed ideas as to why their students were not identified. Hillary pointed out that poor verbal abilities might have played a role, while Casey indicated that a student's lack of exposure to mathematical concepts at an early age might have hampered a student's identification as gifted.

Lack of awareness of gifted students. Despite the teachers' awareness about mathematical giftedness, none of them knew who their gifted students were prior to this study. They indicated that the schools' guidance counselors were willing to share that information when asked, but none of the teachers had requested the list prior to the initiation of this study. Several teachers explained that the only time they were routinely notified about their gifted students was when they were provided with a list of seventh grade students who would miss their class to attend the weekly gifted program. Since this program did not include eighth grade students and because in some schools this
program did not interfere with the Algebra I class – or their seventh grade students had chosen not to participate in the program – the teachers had never received a complete listing of their gifted students.

**Summary.** Overall, the teachers expressed their ideas about mathematical giftedness in a very common-sense way, describing the attributes they saw the students display in the classroom. Most talked about the fact that these students understood complex material easily and were able to figure things out on their own, although a few teachers pointed out that these students might also have characteristics that were not as positive. They understood the idea of students being gifted in domains, and realized that a gifted student might not necessarily be the top student in their mathematics class. They also expressed concern with the identification process, pointing out that they had students they perceived to be mathematically gifted who had not been identified. Despite their accurate observations about gifted students, none of the teachers had thought to ask about which of their students had been identified as gifted until the researcher asked about conducting the study.

**Meeting gifted students' needs.** The teachers had various opinions on how schools should decide which students should take Algebra I and the impact the increased number of students had on the gifted/talented students in the class. Four major themes emerged when looking at the seven cases – placement issues, potential course modifications, pushing struggling students up versus lowering the level of the course, and gifted needs being met. Table 31 provides a listing of the cross-case themes related to meeting gifted students’ needs and the teachers who demonstrated the different themes.
Table 31

Cross-case Themes Related to Meeting Gifted Students’ Needs

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<th>Cross-case Themes</th>
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<td>Lila</td>
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<tr>
<td>Placement issues</td>
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<tr>
<td>Potential course modifications</td>
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<tr>
<td>Push students up</td>
<td>X</td>
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<tr>
<td>Gifted needs being met</td>
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**Placement issues.** Over the past few years, six of the seven teachers had noticed an increase in the number of students that were placed into Algebra I in middle school. As a first year teacher, Kelly was not able to make any observation. While the school district did not have an official quota for the number of students that should be enrolled in the course, Lila, Melinda, and Hillary mentioned that their school felt pressure from the district to increase their numbers since they had the lowest percentage of students in the course. Melinda indicated that the expectation was that there would only be one eighth grade class that was not at the Algebra I level or higher. Hillary explained that over the past few years, the school district had lowered the minimum requirements a student needed to meet in order to be enrolled in Algebra I. She explained:

We apparently always have fewer students in Algebra I than everybody else. So we’re being compared to other schools. Why do you have so many Pre-algebra classes? Why aren’t more students in Algebra? We base it off of what we truly believe and others are going strictly by the [school district] criteria, and we think it’s more important for them to have a good foundation. This is the foundation class for the rest of their career.
While Sam had mixed feelings about moving students into algebra so soon, he pointed out that if the students were willing to work, he was willing to help them get through the course. Sam, Casey, Hillary, and Kelly all noted that they currently had students in Algebra I who were not adequately prepared to be in the course. Hillary talked about the increasing number of unprepared students:

My after-school [help] has increase significantly, especially at the beginning of the year. Two years ago, I would have nobody staying after because this first quarter is basically a review with a few new concepts thrown in there. I had ten students stay after on Tuesday, and I have more kids coming during lunch. The other algebra teachers come in the morning and they have kids coming in left and right. We have an abundance of meetings with parents. It’s time-consuming to be an algebra teacher!

Several teachers pointed out that the district had revised the mathematics curriculum leading up to Algebra I to better prepare the students, but Melinda and Rachel pointed that some students were simply not ready for algebra by eighth grade. The teachers had the option to move a student back into a lower level mathematics course if they felt he or she had been misplaced, but Sam noted that this caused “a scheduling nightmare.”

Although the teachers played a significant role in recommending where a student should be placed, several mentioned that the parents sometimes pushed the students into Algebra I before they were ready. Sam felt conflicted over whether students who had no interest in pursuing math- or science-related fields should be pushed into taking algebra so early. He stated:
It has always been a tug-of-war in my mind. Why are parents doing this to them? They don’t really need to be doing this now [taking Algebra I]. As long as they get to Calculus by their senior year, these kids are going to be competitive for anything. These kids are on track to possibly take Calculus their junior year and there’s a lot of kids that have no interest in going into anything math- or science-related, so what are they doing here?

Melinda expressed a similar sentiment, explaining that Algebra I was currently geared toward college-bound students with a long career ahead of them in mathematics, and not all the students being put into the course necessarily had that focus. Because of the large number of students taking Algebra I in middle school, Casey was concerned that parents might now view a student as being “advanced” only if their child took Advanced Geometry, the next course in the mathematics sequence, in eighth grade.

**Potential course modifications.** To accommodate the increased number of lower-ability students the teachers foresaw moving into Algebra I in the future, the teachers had various suggestions. All of the teachers who taught mixed-grade classes (Lila, Casey, Melinda, and Hillary) suggested separating the classes in some way. Hillary advocated creating separate seventh and eighth grade classes with the eighth grade students meeting for 90 minutes every day, while Lila and Melinda advocated separate classes for higher- and lower-ability students, with the lower-ability class meeting daily. This would allow the teacher to slow the pace down for these students. Melinda pointed out that academic ability might not be the only issues with these students; some did not have the internal motivation or self-discipline required of a course at that level. Casey went so far as to
advocate a single class for the gifted/talented students with just their intellectual peers.

She elaborated:

I do think that for the most part, if the gifted students were allowed to be on a single
track where they weren’t in mixed classes . . . I think there’s a need for time to be in
inclusion just like all of our special education students to be included to work
together, but I also think they need a time not to be. They should have a gifted class
with a challenge on a daily basis where they can move forward. Mathematically
speaking, they should be able to go faster than what they’re doing. They are being
held back by the other students.

On the other hand, Sam’s school already taught Algebra I by grade level. He thought the
increased number of students being moved into algebra would impact the eighth grade
classes more than the seventh. Rather than suggesting the eighth grade class meet more
frequently, he pointed out that the teachers might need to take out some of the more
challenging material and focus just on the basic algebra concepts for them.

*Push students up.* When discussing their current Algebra I classes, all of the
teachers indicated they had students who struggled. However, all but Kelly indicated that
they pushed the struggling students up, rather than lowering the level of the course to
accommodate them. Hillary pointed out that the increased number of students in algebra
actually hurt the struggling students more than the high-ability ones because the level of
the course had not been lowered. Similarly, Casey stated that she “did not dumb down
material” for her struggling students. Several teachers explained that because they had so
much material to cover in so short of time, they could not afford to stay on a concept for a
great deal of extra time. Robin described having struggling students in her class:
It tends to make me want to slow down, but I can’t slow down. It’s like, for these students to get it, I would have to slow down. I have to keep moving and they have to come after school for help. I don’t have the opportunity to go to them and talk to them one-on-one a lot . . . a little bit here and there. The level of the course has not lowered. I’m riding them [to come after school for help] or else they are deciding to move out [of the class].

As Melinda explained, there came a point where they just had “to move on” in order to cover all of the material. As discussed in the Supportive environment section, the teachers provided additional resources and offered after-school help for these students, but it was really up to them to make the effort to learn the material. Many of the teachers also talked about involving the parents in their attempts to help the struggling students. Kelly, on the other hand, seemed to cater more to the struggling students, pointing out that she needed to “make sure everyone gets it,” and that she could not let these students “fall through the cracks.” She explained:

I think it hurts the gifted kids a little bit because if you have a class where say five people aren’t up to where everyone else it, you can’t just let them fail. You have to do what you can so that those kids can succeed too. If that means taking extra time to go over topics that you know the more gifted kids already know – it’s just a review for them – but you have to make sure you go over it so everyone gets it. I think it holds back a little bit to what the gifted kids could do.

**Gifted needs being met.** Despite the fact that they had struggling students in their classes, six of the seven teachers thought the needs of the gifted students in their Algebra I classes were being met to some extent. Rachel did not think the move toward putting
more students into algebra had impacted the gifted students. Similarly, Hillary and Sam
pointed out how these students were challenged and “energized,” while Melinda pointed
to the fact that she had not received any feedback from parents or students that would
lead her to believe they were not being adequately served. Lila and Kelly also believed
their gifted students’ needs were being met, although Lila pointed out that it was not to
the level she would like to see. She said:

I do, at times, make myself feel guilty over whether I am giving to my gifted
children as well as I should. I think that’s some of my own motherly type thing
coming in. From a teacher viewpoint, yes, but from a motherly viewpoint, I’d say I
could probably do more with them, knowing they can do so much more.

Similarly, Kelly thought that the move to place more students into algebra had hurt the
gifted students somewhat because of the teacher’s need to focus on the struggling
students. Casey, the lone teacher who did not believe the needs of the gifted/talented
students were being met, expressed her concern that these students were being “held
back” by teachers who had to focus on the whole class.

**Summary.** Overall, the teachers had noticed an increase in the number of students
who were placed into Algebra I in middle school and several teachers felt pressure from
the division to increase enrollment. They had different philosophies about whether
students who only met the minimum criteria should be placed into the course, and several
questioned the rationale for moving students into algebra a year or two early when they
had neither the prerequisite skills nor the interest in mathematics. Similarly, several
teachers expressed a concern with parents pushing students into the course before they
were ready. The teachers foresaw the necessity of modifying the course in the future as
eighth grade Algebra I became the norm, rather than the exception. Despite this, the majority of teachers thought that at the present time the gifted/talented students in Algebra I were being adequately – but not ideally – served. All but one of the teachers kept the level of the course high and expected the struggling students to get extra help if they were not able to keep up with the rest of the class, rather than lowering the level of the course to accommodate them.
Chapter 5: Discussion

The purpose of this study was to determine the ways in which middle school Algebra I teachers modified their Algebra I course for their gifted/talented students in light of the national move to place more students into Algebra I prior to high school. To find this out, the researcher conducted a mixed methods study concerning how teachers modified the pace of instruction, increased the level of challenge, used various differentiation strategies, and provided a supportive environment for the gifted/talented students in their heterogeneous Algebra I classes. Specifically, the researcher sought to answer the following research questions:

1. To what extent do middle school Algebra I teachers modify the pace of instruction for their gifted/talented students?
2. In what ways do middle school Algebra I teachers increase the level of challenge for their gifted/talented students?
3. What differentiation strategies do middle school Algebra I teachers use to meet the needs of their gifted/talented students?
4. In what ways do middle school Algebra I teachers provide a supportive environment for their gifted/talented students?

To conduct the study, the researcher observed all of the middle school Algebra I teachers in a Virginia school district who had gifted students in their heterogeneous classes. Students in these classes were either one or two years above grade level. She
conducted approximately four hours of observations in the Algebra I classrooms of each of the seven teachers, took field notes, and evaluated their effectiveness using the COS-R. She then interviewed each teacher to determine how he or she modified instruction in relation to the areas listed above.

Chapter five contains a discussion of the results of the study in light of the relevant literature. The chapter includes the interpretation of the results, a summary of findings, implications for practice, and suggestions for future research.

**Interpretation of Results**

**Pace.** Although there was substantial variation among the teachers as far as how often they displayed pace-related behaviors, the pace-related themes that emerged were very consistent among them. These findings were not surprising. Because this was a course for which students were required to pass the state standards test, the pace was carefully laid out by the school district to ensure all the standards were covered within the allocated time. In addition, the state was undergoing a transition between two sets of standards, so during the year of this study, teachers were actually instructing toward both sets of standards, which entailed teaching additional material. Furthermore, because Algebra I actually consisted of two semester courses – Algebra IA and Algebra IB – the teachers had to ensure that the Part A material had been completed prior to the end of the first semester.

Compounding this issue was the fact Algebra I was the first mathematics course offered in the school district that met every other day, and so the teachers also had to deal with the issue of showing the students how to adjust their own study pace. As far as the teachers, themselves, were concerned, rather than being able to introduce a concept,
practice the concept, and reflect on the concept prior to introducing the next new idea, the block schedule frequently necessitated that they essentially cover two lessons in one class period. This meant that they did not have time to reinforce or dwell on topics, even when some of the students did not fully understand the material. All of the teachers made a point of reminding their struggling students that additional help was available outside of class, and as will be further discussed later in the chapter, all but one of the teachers felt they were able to preserve the high level of the course by insisting that the struggling students not be allowed to significantly slow the class down.

Despite this, it was apparent through the observations that several of the gifted/talented students could have moved at a somewhat quicker pace. Rogers (2007) pointed out that gifted students retain more when they are in accelerated situations, and although in one sense, the fact that they were in a high school-level course one or two years ahead of their peers did provide them with acceleration, Rogers also pointed out that these students need to be presented material at their actual learning rate. The fact that most of these students finished the material at hand before their classmates suggests that they were picking up on (i.e., understanding) the material at a quicker rate. Diezmann (2005) also pointed out that these students need the proper balance of practice and learning tasks, and Miller (1990) talked about the importance of flexible pacing to keep gifted students engaged. As was discussed in Chapter 4, this was not always the case. Most of the teachers aimed for the average student when determining how much practice was needed and the resulting pace for class.

Furthermore, Sriraman (2004a, 2004b) and Steiner (2006) discussed the fact that gifted/talented students took longer to think through their strategies, thus slowing down
the time it took to solve complex problems. Only one of the observed lessons – the design cycle problem with the IB class – could truly be classified as a complex problem of the type to which the researchers were referring. This activity took far more planning and reflection to solve than did the typical algebra problem because it required the students to engage in high-level, abstract thinking to tie together three seemingly unrelated topics – algebra, the technology design cycle, and water conservation. The teacher allowed most of the block for the students to persist in their investigation. The researcher was not able to determine whether the gifted students actually took longer to think through their strategies in this problem than did the other students because as the leader of each group, the gifted students essentially drove the pace of work on the problem.

**Challenge.** Similar to what was seen with pace, there was substantial variation between the teachers in the frequency of their challenge-related behaviors, and yet the cross-case themes related to challenge were very consistent. As anticipated, the researcher observed the level of challenge within the Algebra I course to already be at a fairly high level without any specific modifications for the gifted/talented students. The NCTM (2000) advocated providing problems that focused on concepts rather than just procedures, and problems that allowed the students to apply or adapt various strategies. The teachers in this study were generally successful in doing that. With the exception of Kelly, each teacher demonstrated an advanced level of challenge in at least two of his or her observed lessons. Although she had some excellent original suggestions on the ways in which she could increase the level of challenge for her students, Kelly’s class was taught at a much lower level of challenge than the other Algebra I classes the researcher
observed. She did not ask students higher-level questions to challenge them, she did not emphasize a depth of understanding of the material, and she never encouraged the students to find patterns or make generalizations. Even though Kelly struggled with classroom management issues and had a class made up solely of eighth graders, the overall level of her class simply did not compare to that of the other teachers.

Despite the prevailing notion among the teachers that the level of challenge was already high enough for the entire class, the researcher noted that there was an occasion in each teacher’s class where a gifted/talented student either completed the work prior to the other students, or appeared to be bored, suggesting that the level of challenge could have been increased somewhat (McNabb, 2003; VanTassel-Baska, 2003). She also noted occasions where the teachers lowered the level of challenge when they answered their own higher-level questions, told the class how to do a procedure rather than allowing them to discover it, or indicated that they did not need to attempt the more challenging homework problems. In fact, even Lila – evaluated as the most effective teacher overall and the teacher who demonstrated the most mathematical behaviors related to challenge – told the students during one lesson, “We’re going to do this together because it worries me that you don’t know how.”

Assouline and Lupkowski-Shoplik (2005) and Johnson (2000) pointed out that mathematically gifted students need to be challenged with greater depth and breadth, abstraction, and complexity than typical students. In this study, while the gifted students were provided with greater abstraction and complexity, the pace of the class did not allow time to go into depth in any of the observed classes. As noted previously, because of the block scheduling and the rapid pace of the course, the teachers oftentimes had to teach
two different concepts during one class period which limited the opportunity to go into depth on any one topic. Likewise, the teachers did not have time to expand the topic to provide the students exposure to a greater breadth of mathematical material.

In addition, the NCTM (2000) advocated that gifted students should be provided with frequent opportunities to grapple with problems that required “a significant amount of effort” (p. 25). The researcher only observed one problem – the design cycle problem in the IB class – that really provided the students with such an opportunity. This represented the type of optimized math task advocated by Diezmann and Watters (2002b) to cognitively engage the students. On the other hand, the fact that the researcher noted several gifted students in other classes who finished their work prior to the other students suggests that they were not struggling with problems that required a great deal of effort on their part. This was unfortunate as mathematically gifted students enjoy the sense of accomplishment that comes with solving complex problems with which they had originally struggled (Garofalo, 1993; Waxman et al., 1996). Furthermore, such challenge helps students to develop their “mathematical power” (Diezmann & Watters, 2000, p. 2).

**Differentiation.** While there was very little variability among the cases on the themes related to pace, challenge, and supportive environment, differentiation was somewhat different. While all of the teachers had very limited differentiation practices, with the exception of flexible grouping, they differed substantially in their use of other differentiation strategies. While two teachers demonstrated three different strategies, one teacher did not demonstrate any and another teacher only demonstrated one. This corresponds with the previous observation that there was greater variation between the
teachers in how often they demonstrated behaviors related to differentiation than with pace-, challenge-, or supportive environment-related behaviors.

One of the traits of a high-quality curriculum is differentiation (Burns, Purcell, & Hertberg, 2006). This study found very limited differentiation, consisting of grouping, and infrequent examples of open-ended activities and choice. This was not surprising, considering the fact that teachers in heterogeneous classes typically do very little curricular or instructional differentiation for their high-ability students (Archambault et al., 1993). On two occasions, the researcher was surprised by the way in which teachers talked about differentiation. Although Sam explained it could mean challenging students at different levels or getting at the material in different ways, when asked to give examples of how he differentiated the material, he revealed a very literal interpretation, explaining how he had the students highlight material in their notes using different color markers. Similarly, his comment that "algebra is what it is" and thus there were not many ways to deviate from the material reflected a rather narrow understanding of differentiation. Tomlinson's (1995a) definition of differentiation as "the consistent use of a variety of instructional approaches to modify content, process, and/or products in response to the learning readiness and interest in academically diverse students" (p. 80) highlights the rather surface-level notion Sam had of the concept.

**Acceleration.** None of the students in this study were provided an opportunity for acceleration beyond the worksheet or notes on which the whole class was currently working. The National Math Panel (USDOE, 2008c) specifically pointed out the benefits of adjusting pace when providing differentiated instruction for high-ability students. Although all of the students in the Algebra I classes in this study were considered to be a
year or two above grade level, the material and pace for the class were very similar to what was offered to students who took the course in high school. In other words, although the students were accelerated in the fact that they took the course early, they did not move through the course any quicker than did students on grade level. Assouline and Lupkowski-Shoplik (2005) pointed out that even when subject-matter acceleration was employed for students in mathematics, the pace may still be too slow. Although the pace of the Algebra I classes in this study was relatively quick, there were still occasions where it was apparent that the gifted/talented students could have moved through the material more rapidly. Lila’s comment about her hesitancy to put gifted students together because they were able to “zoom ahead” indicated she knew they had the ability to be accelerated in some manner. However, her hesitancy to mention all of the resources available to the gifted students because they “like to get ahead and sometimes go a little too much beyond the class,” demonstrated the prevailing attitude of the teachers in this study that the whole class should say together.

Curriculum compacting. Curriculum compacting was not observed with any of the teachers, although it might have provided a viable option for accelerating those students who understood more of the material than the teachers realized. While all of the teachers except Rachel determined the prior knowledge of their students in one way or another, they all did it very informally and as a class. While the teachers pointed out that the majority of the algebra material was new to the students, they never conducted a more rigorous pre-assessment to really determine areas that the students may have already mastered. Only Casey indicated that she sometimes pre-assessed the students prior to the end of the previous unit, but that was so she could modify the material for the class as a
whole. Had the teachers done a more thorough pre-assessment, they might have discovered that some of the students were able to have the curriculum compacted. This could have freed up a significant amount of time (Reis & Renzulli, 1992; Reis, Westberg, Kulikowich, & Purcell, 1998), thus enabling these students to move on to more challenging material. It was interesting to note the unique view Rachel had about curriculum compacting. Rather than considering enrichment opportunities as a potential outgrowth of curriculum compacting, she took the opposite viewpoint, expressing that the reason why she did not do enrichment activities with her non-IB students was because it would require her to compact the lessons, which she did not believe the students could afford.

**Enrichment.** The fact that only one teacher demonstrated any enrichment activities was rather disappointing since enrichment could have provided students with an opportunity to be exposed to a broader array of topics and to explore them in greater detail (Rotigel & Fello, 2004). By allowing students to investigate challenging topics and to use their higher-order thinking skills, they may have developed a more complete understanding of concepts, principles and generalizations (Rogers, 1991). Rachel pointed out that the IB curriculum provided enrichment for the students in the program, and the design cycle activity the researcher observed bore this out. This activity was only a portion of a larger water project the students were undertaking. Renzulli and Reis (1997) pointed out how such Type III enrichment activities allow students to be creative and gain a more advanced understanding of the content and process used in various disciplines. By using their mathematics skills to focus on a real-world problem, the students in Rachel’s class were doing just that. The fact that time was the most
frequently cited factor as to why teachers did not do enrichment activities reflected the whole class mentality also seen in discussions about **pace** and **challenge**. Rather than thinking of enrichment for the gifted individuals in their class, the teachers' comments suggested that if the entire class would not have time to do an activity, nobody should be offered the opportunity.

*Open-ended activities.* Open-ended activities allow mathematically gifted students to undertake complex tasks in the form of discovery problems (Rotigel & Fello, 2004). Rachel’s design cycle problem was the most comprehensive open-ended activity the researcher observed, although Hillary talked about the video project her class would do later in the semester. In addition to allowing the students to use higher-order thinking skills, both of these projects allowed open-ended responses, important because they require students to use extended reasoning (Johnson, 2000). In particular, Rachel’s activity emphasized problem-solving, an “integral part of all mathematics learning” (NCTM, 2000, p. 52), and a process that allows the students to develop more sophisticated mathematical skills (Grouws & Cebulla, 2000a, 2000b).

*Tiered assignments and questions.* Aside from Melinda’s mention of a tiered assignment she had given her Algebra I students the previous year, none of the teachers used this strategy to differentiate for their students. Stepanek (1999) pointed out that by challenging the students using tiered assignments, students are able to grapple with different levels of open-endedness, abstractness, and complexity, which encourages them to think at deeper levels. Melinda pointed out that tiered lessons took up a significant amount of time in class; however, providing tiered homework assignments would have been a relatively easy way for teachers to provide an appropriate level of challenge for
their gifted/talented students. Casey mentioned that she wished she could give the students different homework assignments, but she did not do so because of perceived parental pressure. On the other hand, some teachers did use open-ended questions which encouraged higher-level mathematical reasoning (Ball & Bass, 2003). Sheffield (2000, 2003) pointed out that teachers can use these questions to elicit responses that demonstrate not only a student's critical thinking skills, but also their creative ones.

**Grouping strategies.** Grouping strategies are a part of a high-quality, comprehensive curriculum (Burns, Purcell, & Hertberg, 2006), and they were by far the most common differentiation strategy observed. The teachers' use of a flexible grouping strategy which allowed students to work with different groups for various activities was beneficial because these temporary groups kept the gifted/talented students from being labeled as such (Slavin, 1987; Access Center, 2005). It also allowed the teachers an opportunity to assess the strengths and weaknesses of students in different group environments (Tomlinson, 2000). Lila also pointed out a practical utility of placing students in groups in that it allowed her student to practice “helping interactions” with various classmates.

The teachers often formed mixed-ability groups. The general belief seemed to be that this benefited all students because it pulled the lower-ability students up while providing reinforcement to the higher-ability ones. This strategy, however, actually “works against the educational needs of gifted students” (Davis & Rimm, 2004, p. 15) because gifted/talented students oftentimes end up doing most of the work, may assume the role of a teacher, and miss chances for accelerated or enriched work. The fact that five of the teachers advocated having gifted students serve in a peer tutoring role suggests
the teachers may have been more focused on the educational development of the lower-achieving students than the higher-achieving ones.

The only teachers who allowed high-ability students to work together were Lila and Casey, although Lila seemed reluctant to do that because the students moved ahead of the class when she did so. High-ability students benefit from being grouped together (Kulick & Kulick, 1992) and ability grouping makes it easier to challenge students at the appropriate level (Gentry & Owens, 1999). Rogers (2002, 2007) pointed out the importance of teachers differentiating the curricular material for the different ability groups; however, Westberg, Archambault, Dobyns, and Slavin (1993) found that usually was not the case for gifted students. As was seen in all the classrooms, the students covered the same material regardless of their level of understanding.

**Alternatives and choices.** Three of the seven teachers allowed the students a choice in various activities, a practice supported by Stepanek (1999) and Johnson (2000). VanTassel-Baska and Stambaugh (2005) pointed out the importance of educators being flexible in content, process, and product by allowing students choices and alternatives. This allows them to pursue problems and activities that pique their interest and engagement, which ultimately may encourage them to investigate the topic at a deeper level. The fact that all seven of the teachers allowed the students to essentially choose the solution method that worked best for them is also worth noting because it demonstrated their flexibility in allowing the students to use the process that made the most sense to them.

**Supportive environment.** As with the themes of pace and challenge, the cross-case themes that emerged related to a supportive environment showed little variability among
the teachers. This corresponds with the fact that behaviors related to a supportive environment showed the least variability among teachers as well. In general, the teachers in this study provided a supportive environment for the gifted/talented students in their classrooms. All of the teachers provided scaffolding, pressed their students for explanation, and modeled high-level performance, measures Henningsen and Stein (1997) found to be part of an optimized environment for mathematics learning. Unfortunately, these students did not necessarily encounter optimized math tasks, because it was apparent that there were occasions when the teachers could have done a better job of adjusting the pace and making the material more challenging for these students. Diezmann and Watters (2002b) pointed out that when gifted students are provided optimized math tasks in optimized environments, they demonstrate collaboration, persistence, the invention of new strategies, metacognition, and flexibility in their thinking.

Contrary to Farkas and Duffet’s findings (2008), every teacher in this study paid an adequate amount of attention to the gifted/talented students, giving them one-on-one attention when they needed it. This type of support helps them to achieve higher levels of proficiency (NACG, 2008). All of the teachers had the students reflect on their solution strategies and their reasoning, although there was only one teacher who did so during all three observed lessons. Reflection is important because gifted/talented students sometimes unconsciously skip steps, and have a difficult time explaining how they arrived at their answer (Krutetskii, 1976; Diezmann & Waters, 2001). By having the students consciously talk or write about how and why they came to their answer, teachers can help them to develop their metacognitive skills.
In addition, most of the teachers made a specific effort to let the students know that making a mistake in class was not of major concern. This is important because one of the affective characteristics of gifted/talented students is a tendency toward perfectionism (VanTassel-Baska, 2003). Several teachers also made it a point to relate what they were teaching to the real world, not just to make a connection to how one might use the skill, but also to make a connection to professional mathematicians and how they used various terms and symbols. Because mathematically gifted students think about mathematics in ways that are similar to professional mathematicians (Sriraman, 2004a), they need to be given opportunities to engage in the role of a practicing professional. Similarly, Casey’s emphasis on building confidence in the females in her class was especially important as gifted/talented girls may not receive the type of encouragement they need to achieve in mathematics (Reis & Gavin, 1999). Every teacher also had the students come up with different ways of solving problems. This encouraged divergent thinking, a process skill that should be incorporated into gifted students’ learning activities (VanTassel-Baska, 2003).

Finally, several teachers did a commendable job of verbalizing their high expectations concerning both academic and behavioral issues. By doing so, their classes ran smoothly and the students were able to focus on learning. The notable exception was Kelly, who as a first-year teacher, was still refining her classroom management skills, and was unable to provide a learning environment free from distractions.

Mathematical giftedness. One of the most interesting findings of this study was also the first that came to the attention of the researcher. Despite the fact that each teacher had gifted students in his or her Algebra I classes, not a single one knew which
students had been identified as gifted prior to volunteering for this study. The researcher initially assumed this might be because the study was conducted near the beginning of the school year, but upon further investigation, she discovered that the teachers were never routinely given the complete list of their gifted students, and none had thought to ask.

This was of concern, especially in light of the fact that only one of the teachers had ever received training in gifted education. This finding is not unusual. Various studies have found that over 60% of teachers neither had training in gifted education during their teacher preparation program (Farkas & Duffet, 2008) or as professional development (Archambault et al., 1993). Few of the teachers seemed aware of some of the issues facing these students, most notably underachievement. Several teachers informally told the researcher they were surprised when they discovered certain students had been identified as gifted. Rather than recognizing that a gifted student could be a poor performer, these teachers portrayed the attitude expressed by Kelly when she told the researcher that she wondered how the student was identified as gifted in the first place. In other words, rather than recognizing that the student might have been a gifted underachiever or gifted just in the verbal domain, she questioned whether the student was gifted at all.

As expected, the teachers’ responses to questions pertaining to mathematical giftedness generally reflected a practitioner’s view, with many of the characterizations referring to surface-level observations or even speculation. The teachers varied somewhat in the focus of the characterizations. For instance, Hillary’s description centered more on the outcomes of the students’ abilities (good scores on tests, good grades), while Kelly’s focus was more on their effort (motivated, try hard). Only Lila
provided a more sophisticated description of these students, describing them as “able to analyze and apply the material in ways beyond the normal classroom instruction.” Despite the rather straight-forward descriptions, the teachers hit upon many of the characteristics of mathematically gifted students reflected in the literature. Table 32 lists some of the characteristics research has shown mathematically gifted students to have and the corresponding descriptions provided by the teachers in this study.
<table>
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<th>Research Descriptions</th>
<th>Teacher Descriptions</th>
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<tbody>
<tr>
<td><strong>Cognitive Processing</strong></td>
<td>• Quick cognitive processing &lt;br&gt;<strong>Geary &amp; Brown, 1991; Krutetskii, 1976; O'Boyle, 2005a, 2008; Swanson, 2006</strong></td>
<td>• “Zooms through material”&lt;br&gt;• Does not struggle</td>
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<tr>
<td><strong>Memory</strong></td>
<td>• More efficient memory system &lt;br&gt;• Excellent long-term memory organization &lt;br&gt;• Better at manipulating material in working memory&lt;br&gt;&lt;br&gt;<strong>Dark &amp; Benbow, 1990, 1991; Geary &amp; Brown, 1991; Robinson et al., 1996</strong></td>
<td>• New concepts “stick”</td>
</tr>
<tr>
<td><strong>Insightful Thinking</strong></td>
<td>• Better at selective encoding, comparison, and combination &lt;br&gt;• Perform selective comparison spontaneously &lt;br&gt;• Omit seemingly essential links in a logical train of thought&lt;br&gt;&lt;br&gt;<strong>Davidson &amp; Sternberg, 1984; Gorodetsky &amp; Klavir, 2003</strong></td>
<td>• Understands extensions</td>
</tr>
<tr>
<td><strong>Metacognition</strong></td>
<td>• Early development of metacognitive knowledge and control &lt;br&gt;• More likely to use metacognitive skills to find unknown solutions &lt;br&gt;• Observe own metacognitive behaviors more accurately&lt;br&gt;&lt;br&gt;<strong>Gorodetsky &amp; Klavir, 2003; Schraw &amp; Dennison, 1997</strong></td>
<td>• Focused</td>
</tr>
<tr>
<td><strong>Strategy Knowledge and Use</strong></td>
<td>• Use higher-level strategies &lt;br&gt;• Use strategies more consistently and flexibly &lt;br&gt;• Can inhibit task irrelevant information&lt;br&gt;&lt;br&gt;<strong>Coyle et al., 1998; Geary &amp; Brown, 1991; Steiner, 2006; Swanson, 2006</strong></td>
<td>• Higher-level thinking skills &lt;br&gt;• Abstract thinker</td>
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Research Descriptions | Teacher Descriptions
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**Problem Solving** | • Focus on conceptual underpinnings of problems  
• Consider several problem-solving techniques  
• Meaning-oriented approach  
• Abstract similarities in structures of problems  
• Better able to make generalizations  

*(Garofalo, 1993; Grouws, Howald, & Colangelo, 1996; Siraman, 2003)*  

**Conception of Mathematics** | • Want full understanding  
• Patient  
• Ask the right questions  
• Strong work ethic  
• Independent worker  
• Motivated to learn  

*(Grouws, Howald, & Colangelo, 1996; Sriraman, 2004a, 2004b)*

Research also has pointed out that gifted students are not all alike. Not only did the teachers talk about the students being gifted in domains (e.g., Dark & Benbow, 1990, 1991; Gardner, 2005), but Lila even discussed the fact that these students may be gifted in different areas of mathematics (e.g., Diezmann & Watters, 2002a; Krutetskii, 1976; Sak, 2009). Similarly, the descriptions the teachers provided pointed out that they had seen these students in different lights. For example, Lila and Rachel said gifted students had a strong work ethic, but Sam pointed out they might lack effort; Rachel said the students did not struggle, but Lila and Sam said they might struggle; and Lila, herself, said that mathematically gifted students might strive for attention or not want attention at all. These observations dovetail with the extant literature. As discussed in Chapter 2, gifted students can vary in their abilities and prior knowledge (Armstrong, 1992; Davis & Rimm, 2004); interests, motivation, and preferred modes of learning (NACG, 2008); their
computational and conceptual understanding (Krutetskii, 1976; Rotigel & Fello, 2004); and so forth.

**Meeting gifted students’ needs.** The most surprising finding of this study related to the teachers’ mindset that they needed to push the struggling students up, rather than lowering the level of the course to accommodate them. The researcher anticipated that the teachers would spend the majority of their time focused on their struggling students, thus lowering the level of challenge; however, her observations showed that they also paid a suitable amount of attention to the gifted/talented students. Each class had so much material to cover in so short of time, that even if the struggling students did not fully understand the material, the teacher eventually had to press ahead. This benefitted the gifted students, for although the pace was not necessarily optimal for them, the teacher was not able to belabor a point for too much time either.

It was interesting to note that although Kelly and Casey both thought that the teacher had to focus more on the struggling students, Kelly still believed the gifted students’ needs were being met while Casey did not. This corresponded with the researcher’s observations that Kelly, more than any of the other teachers, was focused on the struggling students in her class. The researcher noted this both in the classroom observations and in her responses to the interview questions. In fact, in sharp contrast to the other teachers in the study, Kelly seemed to accept the idea that the level of Algebra I should be lowered to accommodate these struggling students, rather than staying at a higher level and requiring the struggling students to come up to that level. It was also interesting to note Casey’s suggestion to have a gifted-only class in light of the National Opinion Research Center study (USDOE, 2008B) which found that over half of Algebra I
teachers they surveyed considered mixed-ability classes to be a serious or moderate
problem. In fact, the teachers who suggested having leveled classes were suggesting a
similar idea: by creating classes more similar in ability, the better all students’ needs can
be met.

Summary of Findings

In summary, major findings of the study can be characterized as:

1. The pace of Algebra I was driven from outside the classroom. The school
division’s curriculum framework and block scheduling played major roles in
determining the pace.

2. Teachers modified the pace of instruction for the class as a whole. Most
teachers determined the prior knowledge of the students and aimed the pace of
instruction toward the average students. Students were only allowed to move at
an individual pace within constrained activities.

3. Teachers increased the level of challenge for the class as a whole. Although the
level of challenge was already fairly high, the only access individual students
had to more challenging material was through the more difficult problems at the
end of the worksheet, guided practice, or homework. They were not accelerated
or provided enrichment with more challenging material.

4. Teachers increased the level of challenge by increasing the level of complexity
and abstraction of the material. They used multiple methods to increase the
complexity of individual problems, the problem-solving process, and the
mathematical concepts, themselves.
5. *Teachers practiced limited differentiation strategies.* The most common strategy used was flexible grouping, oftentimes made up of mixed-ability groups. The teachers did not provide acceleration, curriculum compacting, or tiered assignments and questions, and only a few provided open-ended activities, enrichment, or choice.

6. *Most teachers provided an environment conducive to student learning.* They created a positive, encouraging atmosphere where mistakes were viewed as learning experiences and students were willing to take risks. It is important to note that actual measures of student achievement were not included in the study.

7. *Teachers provided scaffolding for gifted students as part of the whole class.* They built on prior knowledge and made explicit connections between old and new learning.

8. *Teachers generally had high expectations for their students.* They talked to students about both performance and behavior expectations.

9. *Teachers consistently modeled high-level performance.* They did this by demonstrating proper problem-solving techniques, using mathematical terminology, modeling organizational and study skills, and demonstrating high expectations of themselves.

10. *Teachers had a practitioner's view of mathematical giftedness.* Teacher understandings about giftedness were largely gained in the classroom since only one teacher had any gifted education training. They believed students could be gifted in domains, and that some of their students were mathematically gifted, but had not been identified.
11. *Teachers were unaware of their gifted students.* None of the teachers knew who their gifted/talented students were prior to the initiation of this study.

12. *Teachers believed there were placement issues concerning putting students into Algebra I.* Teachers expressed concern with district pressure to increase enrollment and parental pressure to put students in the course when they were not prepared for it.

13. *Teachers believed Algebra I needed to be modified to accommodate the increasing number of lower-ability students.* Teacher suggested creating separate Algebra I courses with the lower-ability students meeting twice as often as the higher-ability students or modifying the course material so the lower-ability students could focus only on basic algebra concepts.

14. *Most teachers pushed the struggling students up, rather than lowering the level of the course.* Teachers insisted that the struggling students get help outside of class, rather than lowering the challenge of the material to accommodate them.

15. *Most teachers believed the needs of their gifted/talented students were being met to some extent.* Teachers talked about Algebra I being the first class to challenge their gifted students, but some pointed out that the gifted students were held back somewhat because of the teacher’s focus on struggling students.

**Implications for Practice**

This study considered the different ways in which teachers modified their instruction to meet the needs of the gifted/talented students in their heterogeneous Algebra I classes. The researcher found that the teachers did very little aimed specifically at the gifted student segment of the class. Nonetheless, it was found that most of the gifted students
generally were engaged. This level of engagement may be attributed to the fact that the course already had challenge and rigor built in, the pace was fairly quick, and the teachers provided a supportive environment where the students felt free to take risks. Despite their concerns about the increasing number of students taking Algebra I in middle school, the teachers kept the rigor of the course high and provided the gifted/talented students with adequate attention. Because of this, the needs of the gifted/talented students appeared to be met to some degree. This study did, however, reveal several areas where administrators and teachers could adjust their practices to more fully address the needs of the gifted/talented students.

**Provide gifted education for teachers.** One of the most significant, but easily remedied, shortcomings identified through this study was the fact that the school district had not provided the teachers with any professional development in gifted education. Teachers need to be aware that the differences between mathematically gifted/talented and average students go far beyond simply being able to solve problems more quickly and accurately. Not only do they process material more rapidly than average students, but gifted students quickly develop and rely on higher-level strategies, approach problems solving by looking at the structures of problems, and have unique abilities to generalize. They can manipulate mathematical material better in their working memories, have more insightful thinking, and generally have better metacognitive skills than average students. They view mathematics through a lens of conceptual connections rather than as numbers and procedures, and they approach complex problems in ways similar to professional mathematicians, sometimes requiring additional time to plan their solution approach. In other words, a gifted/talented student may think about an algebra
problem in a substantially different way than the other students in the class. If the teacher is not aware of this possibility, he or she may stifle the gifted students’ creativity and discourage them from being as cognitively engaged as they might be. Furthermore, without an understanding of giftedness, a teacher may mistake a gifted student’s boredom as a lack of understanding or his or her underachievement as a poor attitude. Schools spend a significant amount of time educating teachers on how to meet the special needs of students with disabilities; they should likewise educate teachers on the ways to meet the needs of their gifted/talented population.

**Provide professional development concerning differentiation.** Most of the teachers in this study did not seem to be aware of the many different ways in which they could differentiate for specific students within their classrooms. The teachers only modified the pace and level of challenge for the class as a whole and students were not provided an opportunity to move beyond the material at hand. Likewise, when the teachers talked about enrichment, it was for all members of the class, rather than allowing certain individuals to pursue an activity to broaden or deepen their understanding. Furthermore, the differentiation strategy the teachers most frequently used was mixed-ability grouping in which the gifted students frequently took on the role of peer tutor, a function that provides more benefit to the lower-ability students than the higher-ability ones (Davis & Rimm, 2004). Similarly, the options of acceleration and tiered assignments were not actively pursued. Professional development – specifically aimed toward ways to differentiate in mathematics – would not only help teachers to provide more challenge for their gifted/talented students, but it would also make them aware of strategies they could use to better address the needs of all the students in their classroom.
As noted by many of the teachers in this study, there is an increasing population of struggling students being placed into Algebra I. A thorough understanding of various differentiation strategies would help teachers to more effectively target this population of students, while potentially decreasing the before- and after-school tutoring many of these students require.

**Make the gifted population known.** The schools in this study failed to provide the teachers with a listing of the gifted/talented students in their class. All of the teachers knew who their students with special needs were, but none knew who their gifted students were. Consequently, even if the teachers were trained in gifted education, they still could not address the specific needs of these students since they did not know who they were. Again, this situation could be easily remedied, but it would require administrators to consciously focus on gifted/talented students as a segment of the population in need of attention. One of the most gratifying outcomes of this research was the comment made by Casey, who said that because of this study, she planned to work with a fellow teacher to head up a “we have another special population in our room and we have misplaced them” committee at her school.

**Consider identifying students as gifted within domains.** The school division in this study only identified gifted/talented students as gifted overall. They did not make any distinction between students who had high abilities in the mathematical domain, the verbal domain, or both. Several teachers noted that they had students who had been identified as gifted, but did not appear to be mathematically gifted, or that appeared to be mathematically gifted, but had not been identified at all. Identifying the areas in which students are gifted would help teachers to be more aware of their specific needs. This
would help with the situation noted with Kelly where she assumed that a struggling gifted student was misidentified, not realizing that he or she may only have been gifted in the verbal domain. Furthermore, the school district in this study simply used an arbitrary cutoff score on two norm-referenced tests as part of the identification process. School districts should investigate more sophisticated ways in which to identify their gifted population so that students, teachers, and parents understand the specific areas in which the student is gifted. In this way, teachers will be better equipped to identify the instructional modifications that will help them to more effectively target the student’s specific needs.

Selectively put students into Algebra I in middle school. The school division in this study did not have quotas for the number of students who should be enrolled in Algebra I during middle school, and yet the teachers who had a lower percentage of students in the course felt pressure to increase enrollment. We know from research that 1) all students benefit from taking algebra at some point in their mathematics career; 2) students who take algebra prior to ninth grade take more math courses; and 3) overall mathematical achievement is related to the number of math classes a student takes from the Algebra I level and beyond. However, this does not mean that all students are ready to take the course in middle school. Despite the fact that the school division had recently revised its curriculum to better prepare students for Algebra I, several teachers in this study commented on the growing number of students who sought outside help from the teachers as they tried to keep up with the material. It does little good to move students into a course for which they are simply not cognitively ready, especially considering that Algebra I is a foundational course for higher-level mathematics.
Emphasize the importance of assessing students for prior knowledge. In this study, teachers only informally pre-assessed their students, oftentimes by simply asking questions or checking warm-up problems. Teachers then used the results of this pre-assessment to modify instruction for the class as a whole. Teachers need to understand the important role assessment has in providing effective instruction. Without a thorough knowledge of what a student understands, a teacher is unable to modify instruction in a way that truly meets the needs of individual students. In the case of gifted/talented students, a thorough pre-assessment is necessary for a teacher to effectively compact the curriculum or to provide meaningful enrichment. While many teachers think of assessment of learning, they also need to understand the importance of assessment for learning. By coming up with an assessment plan and modifying instruction based on the individual results, teachers will have a critical tool to help them more effectively meet the needs of all of their students.

Provide leveled Algebra I classes. Administrators may also want to consider the suggestions made by several teachers as far as providing two levels of Algebra I classes – one for the gifted and higher-ability students, and another for the average and lower-ability students. The teachers in this study were able to keep the rigor of the course high and to proceed at a fairly rapid pace, but their concerns about being able to continue to do so as more students were placed into algebra prior to high school should not go unnoticed. Although “leveling” has gained a negative connotation in recent years, if we are serious about raising student achievement in mathematics, it only makes sense to teach the students in the setting in which the teacher can best address their needs. The teachers in this study were more than willing to provide additional help for their
struggling students, but to place these students into an advanced class for which they are not prepared is neither fair to them nor is it fair to the higher-ability students who are ready for a greater challenge and a quicker pace.

**Stress the importance of a supportive environment.** School officials sometimes become so focused on the importance of teaching the content required by the standards that they neglect to stress the importance of the classroom environment. One of the reasons the teachers in this study were able to at least partially meet the needs of their gifted/talented students was simply because of the atmosphere they created in their classrooms. As several teachers pointed out, Algebra I was the first math class in which some of the gifted students felt challenged by the material, and by creating an atmosphere where students felt free to take risks and to make mistakes, they helped the gifted students who may have had issues with perfectionism. The supportive environment was further enhanced by the fact that the teachers expected a high level of performance from the students and they modeled what it should look like. They pressed students to explain their thinking, provided scaffolding when needed, and encouraged metacognition and higher-level thinking. The students were encouraged to talk and work as true mathematicians, and by having an atmosphere that encouraged a “give and take” type of relationship with the teacher, the students felt free to ask questions and explore different solution methods. In short, creating a supportive environment helps the students to focus less on peripheral issues and more on learning the material at hand.

**Suggestions for Future Research**

There are several extensions to this research that would be worthwhile to pursue:

1. This study was conducted in a single school district with only seven teachers. A
larger sample with a more diverse group of schools might provide a wider variety of teacher behaviors.

2. In this study, “average” Algebra I students were, in fact, still considered a year or two above grade level since ninth grade was considered the normal year for a students to take Algebra I. A study conducted in a school district that has a mandated “algebra for all” policy in middle school would provide more truly “average” students and might provide additional insight into the impact struggling students have on the gifted population.

3. The teachers in this study were exceptionally well-qualified to teach Algebra I. A larger study consisting of a more diverse group of teachers would provide insight into various types of learning environments and the impact they have on gifted/talented students.

4. Since the teachers in this study were quite concerned with the fact that their Algebra I class only met every other day, a study that investigated whether the frequency of class meetings really had an impact on student learning in Algebra I would be worthwhile.

5. Most teachers in this study had no gifted education and many seemed unaware of various differentiation strategies. It would be interesting to note whether professional development in gifted education and differentiation would impact the instructional modifications for the gifted/talented students in heterogeneous Algebra I classes.

6. Finally, a study involving the gifted students’ perceptions on whether the teachers were able to meet their needs in heterogeneous Algebra I classes might provide
valuable insight into how teachers could better serve that segment of the student population.
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# Appendix A

## Studies Related to Algebra in the Curriculum

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Sample</th>
<th>Description</th>
<th>Major Findings</th>
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<tbody>
<tr>
<td>Fratt (2006)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Pushing students into eighth grade algebra may not be in their best interests. Students should be exposed to pre-algebra concepts starting in elementary school, but many elementary teachers lack sufficient understanding of math concepts.</td>
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<tr>
<td>Gamoran &amp; Hannigan (2000)</td>
<td>12,506 students Grades 8 -10</td>
<td>The study compared the effects of taking algebra by using data from the first two waves of the National Educational Longitudinal Study (NELS). Achievement was measured in 10th grade by a 40 item multiple choice test.</td>
<td>All students benefit from taking algebra, even those with low prior achievement. Low achievement students gain less, but they still gain. Differences in achievement growth were far greater between students who did and did not take algebra than the differences between students who took algebra at different periods of time.</td>
</tr>
<tr>
<td>Jones et al. (1986)</td>
<td>9,627 students Grade 10</td>
<td>The study used High School and Beyond data to compare mathematic achievement test scores to the number of advanced math courses completed (Algebra I and beyond).</td>
<td>There is a strong relationship between the number of math courses a student takes from Algebra I on and their Grade 12 mathematics achievement. The number of advanced math courses taken accounted for 53% of the variance in the Grade 12 math test scores.</td>
</tr>
<tr>
<td>Lee et al. (1998)</td>
<td>3,430 students Grade 8</td>
<td>The study used data from the High School Effectiveness Supplement of the NELS to compare mathematics achievement in grade 12 with the highest math course completed.</td>
<td>Students who took courses lower than Algebra I scored lower overall on mathematics achievement in grade 12 than did students who took Algebra I or higher level courses. When schools offer many low-end math courses, students tend not to move into the upper level courses; when they offer fewer low-end courses, student tend to take more advanced courses.</td>
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<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
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<td>Loveless (2008a)</td>
<td>160,000 students Grades 4 &amp; 8</td>
<td>This study used national, state, and student-level restricted-use NAEP files from 1990 - 2007 to analyze math and reading scores of students in the 10th and 90th percentiles to determine trends since the implementation of NCLB.</td>
<td>Gains by low-achieving students are outpacing those of high-achieving students by a factor of two or three to one. Internationally, U.S. top achievers fall short.</td>
</tr>
<tr>
<td>Loveless (2008b)</td>
<td>160,000 students Grade 8</td>
<td>This study used restricted-use file from the 2005 NAEP to match student course-taking with NAEP scores.</td>
<td>More U.S. eighth graders take algebra than any other math course. In 2000 26.7% of the eighth graders taking the NAEP were in Algebra I, Geometry, or Algebra II, but by 2005, that number had increased to 36.6%. This indicates the campaign for more students to take tougher math classes in middle school has worked. There are 120,000 students misplaced into higher level courses (Algebra I and beyond)--as evidenced by their scores at the 10th percentile or below on the NAEP. This equates to the amount of math a typical second grader knows. An algebra teacher can expect to have 2 of 26 students several years behind grade level.</td>
</tr>
<tr>
<td>Ma (2000)</td>
<td>3,116 students Grades 7-12</td>
<td>This study used data from the Longitudinal Study of American Youth (LSAY)--a 6-year panel study of math and science education in public middle schools and high schools--to compare mathematics course work, math achievement, and attitudes toward mathematics.</td>
<td>Students who took Pre-algebra or higher-level courses in grade 7 had higher achievement in grade 8 than those students not enrolled in the advanced courses (after controlling for prior mathematical achievement, socioeconomic status, gender, and age). Similarly, students who were enrolled in Algebra I or higher math courses in grade 8 scored higher on grade 9 achievement tests than those students not enrolled in the advanced courses.</td>
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<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
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<td>Ma (2005)</td>
<td>276 gifted students Grade 7</td>
<td>This study used a subset of data from the LSAY to examine whether early acceleration of students into Algebra I in grade 7 or 8 resulted in growth in certain mathematical areas (basic skills, quantitative literacy, algebra, and geometry) as measured by items adopted from the NAEP.</td>
<td>Low-achieving students who were accelerated into algebra performed higher than low-achieving and high achieving students who were not accelerated. Accelerated low-achievers had a rate of growth similar to accelerated high-achievers. Early acceleration promoted stability of growth across mathematical areas.</td>
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<tr>
<td>Moses (1995)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Moses called algebra “the new civil right.” He founded the Algebra Project in the 1980s to help low income and students of color develop the mathematical literacy required for college. The project develops curricular materials, trains teachers, and involves the community in efforts to change mathematics education.</td>
</tr>
<tr>
<td>Smith (1996)</td>
<td>6,894 students Grade 10</td>
<td>This study used High School and Beyond data to explore the impact of early access to algebra based on the number of years of advanced courses taken and math achievement in grade 12.</td>
<td>Algebra is a “gatekeeper” course for advanced mathematics and science. For a student to grasp the complexities advanced courses require, an understanding of algebraic concepts is essential. Access to algebra prior to ninth grade increased the amount of math the students and their teachers expected them to take in high school, and so it socialized them into actually taking more mathematics courses. These additional math courses resulted in higher math achievement and attainment in high school. Math achievement in early high school is the strongest predictor of later achievement in high school and whether students will continue to take advanced courses.</td>
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<td>Researchers</td>
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<td>Spielhagen (2006a)</td>
<td>2,634 students Grade 12</td>
<td>This mixed methods study looked at two pools of students--those who took algebra in eighth grade, and those that took it later. It used student transcripts, the state standardized Algebra I test, the Stanford 9 mathematics score, and teacher interviews to determine the potential benefits and policy implications of providing Algebra I to all students.</td>
<td>Enrollment in algebra in Grade 8 provided students with access to higher level courses. Students who took algebra early stayed in the math pipeline longer. Schools with higher-SES populations gave more opportunities for mathematics enrichment experiences that provided the students with the knowledge they needed to gain access into the early algebra group.</td>
</tr>
<tr>
<td>Spielhagen (2006b)</td>
<td>2,634 students Grade 12</td>
<td>This was a follow-up to Spielhagen (2006a). It used student transcripts, SAT-M scores in grade 11, and college attendance to look at the long-term effects of taking algebra in grade 8.</td>
<td>The strongest variable in predicting selection for algebra in eighth grade was identification as gifted. Students who had early access to algebra took a greater number and more advanced math courses and attended college at a greater rate than students who did not take the course until later.</td>
</tr>
<tr>
<td>Steen (1999)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Algebra is the language of mathematics and is key to accessing a technological society. Algebra is &quot;an invaluable engine of equity&quot; (p. 6). Algebra for all is an appropriate educational goal, but algebra for all in eighth grade is not the way to go about it.</td>
</tr>
<tr>
<td>USDOE (1997)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>This white paper was prepared for the U.S. Secretary of Education and looked at how mathematics provides opportunity. Students who take rigorous math courses are more likely to go to college and rigorous courses are especially important for low-income students. Algebra is a gateway course and math achievement depends on the courses a student takes rather than the type of school a student attends. When parents are involved in students' schoolwork, they are more likely to take challenging math courses like algebra and geometry early.</td>
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<td>Researchers</td>
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<td>USDOE (1998)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>President Clinton called on schools to offer more challenging math courses in middle and high school and for more students to take them. Only a fourth of U.S. students take algebra before high school.</td>
</tr>
<tr>
<td>USDOE (2008b, 2008c)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>The National Mathematics Advisory Panel had a clear emphasis on algebra. Although panel members reviewed over 16,000 studies, there were only six that met their criteria for high-quality research and addressed the long-term benefits for taking Algebra I prior to ninth grade. Almost 40% of our nation's middle school students are currently enrolled in an algebra course compared to only 25% a decade ago. Research evidence, as well as the experience of other countries, supports the value of preparing a higher percentage of students than the U.S. does at present to complete an Algebra I course or its equivalent by Grade 7 or 8, but students need to be prepared. K-8 teachers should focus on the critical foundations of algebra. There are no research studies that identify a sequence of math topics across grades that assures algebra success, nor are there studies pertaining to the effectiveness of a single-subject versus an integrated approach to algebra.</td>
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<tr>
<td>Usiskin (1995)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Algebra as the language of mathematics. It provides an easy way to describe patterns. Without knowledge of algebra, a person will have difficulty understanding many ideas in business, economics, psychology, physics, chemistry, and the earth sciences.</td>
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<td>Researchers</td>
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<tr>
<td>Wilkins &amp; Ma</td>
<td>3,116 students</td>
<td>This study used of data from the LSAY to determine factors related</td>
<td>Students who took Algebra I prior to ninth grade had significantly higher rates of growth in their</td>
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<td>(2002)</td>
<td>Grades 7-12</td>
<td>to student learning and in algebra geometry, and statistics in middle school</td>
<td>mathematical content knowledge than did their peers. A 1 standard deviation (SD) difference in parent</td>
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<td>and high school. It used achievement tests composed of NAEP items.</td>
<td>push predicted a 1% of a SD increase in a student's algebra growth per year. A 1SD difference in</td>
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<td>teacher push predicted a 2% of a SD increase in student growth per year. Math self-concept is a</td>
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<td>strong predictor of growth in middle school, but not in high school.</td>
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## Appendix B

### Studies Related to Mathematical Giftedness

<table>
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<tr>
<th>Researchers</th>
<th>Sample</th>
<th>Description</th>
<th>Major Findings</th>
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<tr>
<td>Achter, Lubinski, &amp; Benbow (1996)</td>
<td>1,975 students Age 12-13 years</td>
<td>Participants took Strong-Campbell Interest Inventory, the RIASEC (vocational interests), Study of Values, Bennett Mechanical Comprehension Test, Vandenberg Mental Rotation Test</td>
<td>Multipotentiality was prevalent in less than 5% of intellectually talented adolescents. Use of developmentally inappropriate assessment tools with low ceilings may have contributed to the pervasive notion of multipotentiality among these students.</td>
</tr>
<tr>
<td>Alexander, Carr, &amp; Schwanenflugel (1995)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Declarative metacognitive knowledge, far transfer abilities, and spontaneous use of strategies show a giftedness effect (which may be domain specific). Cognitive monitoring shows a developmental, but not giftedness effect.</td>
</tr>
<tr>
<td>Armstrong (1992)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Gifted students demonstrate common learning characteristics. There are individual differences between gifted children.</td>
</tr>
<tr>
<td>Benbow &amp; Minor (1990)</td>
<td>144 students Age 13 years &quot;700Ms&quot;=106 &quot;630Vs&quot;=20 &quot;Doubles&quot;=18 Gifted Criteria: SAT-M ≥ 700 SAT-V ≥ 630 Represented top 1 in 10,000 in age group</td>
<td>Used various tests to measure spatial ability, nonverbal reasoning, mechanical comprehension, vocabulary, general information knowledge, memory, speed, and mechanics of English expression.</td>
<td>Gifted student scores were generally equivalent to individuals at least five years older. At least two types of giftedness exist--mathematical and verbal. Mathematically gifted students scored higher on tests of memory, speed, spatial ability, mechanical comprehension, and nonverbal reasoning while verbally gifted students scored higher on general knowledge and verbal tests. The highest mean scores were by &quot;doubles&quot; (verbally and mathematically gifted).</td>
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<tr>
<td>Brody &amp; Stanley (2005)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Many educators equate a high general ability in students as &quot;giftedness,&quot; but each student has an individual cognitive profile. Some may have exceptional mathematical reasoning, but poor verbal abilities or vice versa. The measurement of a specific aptitude is much more useful than simply measuring a general IQ.</td>
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<tr>
<td>Researchers</td>
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<tr>
<td>Carr, Alexander, &amp; Schwanenflugel (1996)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Metacognition is important to the development of high achievement in a domain. Gifted children demonstrate better declarative metacognitive knowledge and far transfer than average students, but not better cognitive monitoring.</td>
</tr>
<tr>
<td>Costa (2003)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>To skillfully generate knowledge and engage in problem solving or decision making, an individual needs to be persistent, inquisitive, demonstrate empathy, and decrease impulsivity. Such habits of mind provide the dispositions necessary for strategic thinking.</td>
</tr>
<tr>
<td>Coyle, Read, Gautney, Bjorklund (1998)</td>
<td>166 students Grades 2-4 Gifted=85 Nongifted=81</td>
<td>Students completed five recall trials of 18 words each. The experimenter spoke the words, the child studied the words for 2 min using any strategy, and was then required to recall the words. Strategies assessed were sorting (physically moving word cards), rehearsal (saying name aloud), category naming (&quot;fruit&quot;), and clustering (recalling words by category).</td>
<td>Gifted students most frequently used a two-strategy combination of sorting and clustering while average students most often used a four-strategy combination of sorting, rehearsal, category naming, and clustering. Gifted students showed lower levels of variability (higher levels of stability) in strategy use and they showed a higher level of recall than non-gifted students. Cognitive stability is a prominent characteristic of gifted cognition.</td>
</tr>
<tr>
<td>Dark &amp; Benbow (1990)</td>
<td>80 students Grade 7 &amp; undergrad Math gifted=20 Verbal gifted=20 Avg. ability=20 Undergrads=20</td>
<td>Experiment 1: Students were presented with sentences and asked to rewrite them as equations. Students were then given seven story problems and given 2 min to recall information from them. Experiment 2: Students were given a series of up to 10 digits and asked to recall them in order. They were also given characters spaced apart and asked to recall their spatial locations. There were 60 stimuli of each type.</td>
<td>Mathematically gifted students had enhanced problem-translation skills and were better than verbally gifted or college students at writing equations to express complex relationships. Mathematically gifted students had an enhanced ability to represent and manipulate information in their working memory. They outperformed verbally gifted and college students in these areas.</td>
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<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<tr>
<td>Dark &amp; Benbow (1991)</td>
<td>64 students</td>
<td>The span task included 20 lists of stimuli (letters, digits, words, and locations). Students were asked to recall the lists. The paired association task included 24 paired associations (e.g. F=6). Students were asked to recall the associations.</td>
<td>Mathematically gifted students had an enhanced capacity for location and digit stimuli. They outperformed verbally gifted students in working memory manipulations, but verbally gifted students performed better than mathematically gifted on the retrieval of a representation from long-term memory into working memory.</td>
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<td>Grades 7-8</td>
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<td></td>
<td>&quot;MV&quot; group=20</td>
<td></td>
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<tr>
<td></td>
<td>&quot;Mv&quot; group=22</td>
<td></td>
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<tr>
<td></td>
<td>&quot;mV&quot; group=22</td>
<td></td>
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<tr>
<td>Gifted Criteria:</td>
<td>SAT-M≥ 500</td>
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<tr>
<td></td>
<td>SAT-V ≥ 430</td>
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<tr>
<td>Davidson &amp; Sternberg (1984)</td>
<td>86 students</td>
<td>Experiment 1: Selective encoding--students were given math insight problems and asked what information was relevant. Experiment 2: Selective combination--students were given math insight problems with and without cues. Experiment 3: Selective comparison--students were given example problems and told how to solve them. They were later given other problems which could be facilitated by recognizing similarities to the example.</td>
<td>Selective encoding, selective combination, and selective comparison were important in solving insight problems. Selective encoding ability was a key factor distinguishing gifted from non-gifted students. Selective combination was easy for gifted children and cueing did not provide them additional benefits, but it was difficult for non-gifted students and cueing helped them. Gifted children performed selective comparison spontaneously while non-gifted students only performed it when prompted. The researchers suggested that some aspects of insight can be improved with training.</td>
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<tr>
<td></td>
<td>Grades 4-6</td>
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<tr>
<td></td>
<td>Gifted=43</td>
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<tr>
<td></td>
<td>Non-gifted=43</td>
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<tr>
<td>Gifted Criteria:</td>
<td>School district identification using IQ scores, Torrance creativity test scores, teacher recommendations, and achievement test scores</td>
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</tr>
<tr>
<td>Diezmann (2005)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Mathematically gifted students learn more rapidly than average students, need less time to achieve mastery, and need challenging environments. This may entail problematizing tasks by inserting obstacles to the solution.</td>
</tr>
<tr>
<td>Diezmann &amp; Watters (2002a)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Mathematically gifted students may be analytically or spatially gifted. The most significant factor impacting teachers' attitudes toward the gifted is whether they have studied gifted education.</td>
</tr>
<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<tr>
<td>Diezmann &amp; Watters</td>
<td>20 math gifted students</td>
<td>Students were exposed to three conditions: regular tasks in a regular environment (RT/RE); regular tasks in an optimized environment (RT/OE); and optimized tasks in an optimized environment (OT/OE).</td>
<td>Gifted students in the RT/RE condition were bored, disinterested, and were engaged in undesirable behaviors. Gifted students in the RT/OE condition completed tasks, but in unimaginative ways. Gifted students in the OT/OE condition demonstrated greater persistence, flexibility in thinking, collaboration, metacognition, and invented new strategies. Unnecessary scaffolding inhibited learning. Problematizing tasks and increasing the pace were necessary to engage students and provide worthwhile learning experiences.</td>
</tr>
<tr>
<td>Fernandez-Duque,</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Metacognitive regulation includes inhibitory control, error detection and correction, planning, and resource allocation. Metacognition is closely related to executive function. Executive function involves the ability to control and monitor the information processing required to control voluntary action.</td>
</tr>
<tr>
<td>Baird, &amp; Posner</td>
<td></td>
<td>Think piece</td>
<td>There are eight different types of intelligences: logical-mathematical, linguistic, interpersonal, intrapersonal, musical, spatial, bodily-kinesthetic, and naturalistic. There may be enough evidence for an &quot;existential&quot; intelligence, but Gardner is not quite ready to call it the ninth.</td>
</tr>
<tr>
<td>Garofalo (1993)</td>
<td>8 students</td>
<td>Students solved routine, multistep and non-routine verbal problems in a one-on-one setting with the interviewer. Students described their problem-solving strategies, beliefs, metacognition, affects, and preferences for problem type.</td>
<td>Average students were number-oriented while gifted students were meaning-oriented. Number-oriented students preferred easy, one-step problems, and their goal was to get enough answers correct to satisfy authority figures. Meaning-oriented students wanted to come up with optimal solution approaches. They enjoyed challenging problems for the feeling of accomplishment they received when solving them.</td>
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<td>Grade 7</td>
<td>Gifted=3</td>
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<td>Average=5</td>
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<td></td>
<td>Scored in the 99th</td>
<td>Iowa Test of Basic Skills (ITBS)-math</td>
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<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
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<tr>
<td>Geary &amp; Brown (1991)</td>
<td>41 students Grades 3-4 Gifted=14 Normal=12 Mental disability=15</td>
<td>Students solved 40 single-digit integer addition problems and described the strategy they used.</td>
<td>Strategies fell into: counting fingers, fingers (without verbally counting), verbal counting, or memory retrieval. Gifted students used memory retrieval most often with less retrieval errors. Normal and mentally disabled groups relied heavily on counting strategies. Gifted students showed an almost adult-like long-term memory organization of basic facts. They had a verbal counting range within adult ranges and less than 50% of the counting range for the other two groups.</td>
</tr>
<tr>
<td>Gorodetsky &amp; Klavir (2003)</td>
<td>121 students Grades 7-8 Gifted=60 Average=61</td>
<td>Students solved insight problems—some with no prior learning and others after seeing examples. The students then reflected on their solution process via questionnaires and interviews.</td>
<td>Gifted and average students used different sub-processes to solve insight problems. The gifted used selective combination and selective encoding in problem-solving processes both before and after learning while average students used selective retrieval and selective combination prior to learning, but switched to selective encoding, selective retrieval, and selective comparison after learning.</td>
</tr>
<tr>
<td>Grouws, Howald, &amp; Colangelo (1996)</td>
<td>167 students Grades 9-11 Gifted=55 Average=112</td>
<td>Students filled out the Conceptions of Mathematics Inventory which included questions on the nature of mathematical knowledge, the character of mathematical activity, and the essence of learning mathematics. Nineteen students were interviewed in depth.</td>
<td>Gifted students thought the power of math was in underlying concepts, principals, and generalizations. They made sense of math through personal reflection and could learn by independently trying to solve problems. Average students viewed problem-solving techniques as tied to certain problems and focused on the numbers and surface features rather than conceptual underpinnings.</td>
</tr>
<tr>
<td>House (1999)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Mathematically promising students have a quick mastery of new learning, analytical and original thinking, and the ability to concentrate and work independently. Their mental processes are flexible and they are able to make rapid and broad generalizations of mathematical relations and operations.</td>
</tr>
<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<tr>
<td>Kalbfleisch (2008a)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>The main limitation of cognitive neuroscience is that it takes students out of the context where they normally display their precocity. The challenge is to determine how what we know about the brain translates into the context of learning and performance. This article also provides a tutorial on different types of neuroimaging techniques.</td>
</tr>
<tr>
<td>Kalbfleisch (2008b)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>The amount of brain activation for mathematically gifted students is several times greater than for average ability peers. Mathematically gifted students have enhanced right hemisphere processing and both hemispheres are equally effective in processing what would normally be considered a predominantly left-hemisphere task. Enhanced brain connectivity contributes most to math giftedness.</td>
</tr>
<tr>
<td>Krutetskii (1976)</td>
<td>Over 1,000 students Ages 6-17 years</td>
<td>Krutetskii collected data on over 1,000 students, conducted various comparison studies on over 200 students, and did a longitudinal study of nine extremely gifted students. He studied students at home and in class. He conducted interviews and gave questionnaires to parents, teachers, and mathematicians.</td>
<td>Mathematically gifted students view the world through a mathematical lens, paying attention to spatial, quantitative, and functional relationships. There are three types of &quot;mathematical casts of mind&quot; (p. 302)—analytical, geometric, and harmonic. Gifted students can make generalizations on the spot, come up with an inspired solution, and yet may be unaware of the process by which they arrived at the answer. Computational abilities are not obligatory for mathematical giftedness.</td>
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<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<tr>
<td>Lohman (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Selective encoding involves sorting out and encoding the information that is relevant to solving a problem; selective comparison involves comparing new information to material that was previously learned and retrieving from memory only the material that is relevant; and selective combination involves assembling the relevant pieces of information together in the working memory to come up with a solution. On reasoning tasks, a major source of individual differences can be attributed to how much information can be maintained and transformed in the working memory. Working memory seems to operate with different effectiveness on verbal content and numerical-spatial content.</td>
</tr>
<tr>
<td>Lubinski &amp; Benbow (2006)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>After 35 years of longitudinal research, the Study of Mathematically Precocious Youth (SMPY) found that special educational opportunities markedly enhanced talent development; and more males than females entered math-science fields, although females had proportionally similar advanced degrees and high-level positions in areas of their preference.</td>
</tr>
<tr>
<td>Lubinski, Webb, Morelock, &amp;</td>
<td>320 students Age &lt; 13 years</td>
<td></td>
<td>The correlation between SAT-M and SAT-V for students who participate in talent searches is ( r = 0.55 ). Differences in the participant's intellectual strengths (verbal or qualitative reasoning) predicted differences in their occupational pursuits. Over 95% used a form of acceleration to individualize their education.</td>
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<tr>
<td>Benbow (2001)</td>
<td>Gifted Criteria: Either SAT-M ( \geq 700 ) or SAT-V ( \geq 630 )</td>
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<tr>
<td>Researchers</td>
<td>Sample</td>
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<td>Major Findings</td>
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<tr>
<td>Mills, Ablard, &amp; Gustin (1994)</td>
<td>306 math gifted students Grades 3-6</td>
<td>Students attended a flexibly-paced math course at Johns Hopkins Center for Talented Youth (3 hrs on weekends for seven months).</td>
<td>A flexibly paced program with continuous learning progress provided a source of motivation for gifted students. Students as young as grade 4 successfully completed pre-algebra. A grade-appropriate curriculum may put gifted students at risk for a decline in achievement and motivation. The prior knowledge and range of abilities in the top three percentiles on a grade appropriate test is as great as that found within a general population of students; it is only through above-level testing that the actual variability in their ability becomes evident. Teachers may ask too little of gifted students.</td>
</tr>
<tr>
<td>NAGC (2008)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Every learner needs opportunities and support to reach their potential. High ability adolescents may differ from their peers in modes of learning, motivation, interests, and cognitive skills. This is a joint position statement with the National Middle School Association.</td>
</tr>
<tr>
<td>O'Boyle (2008)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Compared to average children, mathematically gifted children have enhanced right hemisphere development and greater interaction between the two hemispheres of the brain. They tend to rely on mental images. When mentally rotating a 3-D shape, they have much greater brain activation and the activation is in different locations from average children. The activated areas are known to mediate working memory, spatial information, and executive functions.</td>
</tr>
<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<tr>
<td>O'Boyle &amp; Benbow (1990)</td>
<td>Experiment 1: 67 students Grades 7-8 Gifted=47 Average=20</td>
<td>Experiment 1: Dichotic listening task--students listened through headphones for two different syllables presented simultaneously, one to each ear. The students identified which syllables they heard. There were 60 trials.</td>
<td>Experiment 1: Average students showed right ear/left hemisphere superiority indicating left hemisphere dominance for language processing. Gifted students failed to show any ear/hemisphere advantage indicating the enhanced involvement of the right hemisphere during processing.</td>
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<tr>
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<td>Experiment 2: 80 students Grades 7-8 Gifted=60 Average=20</td>
<td>Experiment 2: Students identified which of two chimeric faces and were asked to judge which was happier. This was repeated for 36 pairs.</td>
<td>Experiment 2: Gifted students chose the happier brain had a larger number of regions of the brain activated during the mental rotation task. The regions that were activated were more bilateral than for average students. Gifted students had significantly greater activation in areas related to working memory, spatial attention, and executive functions.</td>
</tr>
<tr>
<td></td>
<td>Gifted Criteria: Top .05% of the SAT</td>
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<tr>
<td>O'Boyle, Benbow, &amp; Alexander (1995)</td>
<td>Experiment 1: 35 students Ages 12-14 years Gifted=19 Average=16</td>
<td>Experiment 1: Students chose which of two words conveyed a happier sentiment (e.g., vomit/smile).</td>
<td>Gifted students had enhanced right hemisphere involvement during basic information processing and better coordination and allocation of cortical resources between the two hemispheres.</td>
</tr>
<tr>
<td></td>
<td>Experiment 2: 43 students Age 12-14 years Gifted=22 Average=21</td>
<td>Experiment 2: Students chose which of two chimeric faces was happier. EEG activity was monitored during both experiments.</td>
<td>Experiment 1: During word processing, gifted students' brains were more active and they relied on the frontal regions, whereas average student brains were activated in the temporal regions.</td>
</tr>
<tr>
<td></td>
<td>Gifted Criteria: Top 1/2% of the SAT</td>
<td></td>
<td>Experiment 2: During the chimeric face tasks, the gifted students showed left hemisphere inhibition, which allowed the right hemisphere to play a predominant role.</td>
</tr>
<tr>
<td>O'Boyle et al. (2005)</td>
<td>12 students Age 14 years Gifted=6 Average=6</td>
<td>Images of a 3-D cube structure were depicted with four choices of how the structure would look if rotated. FMRI measured the blood oxygen-level-dependent brain activation of the students.</td>
<td>Mathematically gifted students had a larger number of regions of the brain activated during the mental rotation task. The regions that were activated were more bilateral than for average students. Gifted students had significantly greater activation in areas related to working memory, spatial attention, and executive functions.</td>
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<tr>
<td></td>
<td>Gifted Criteria: 99th percentile on the School and College Abilities Test III</td>
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<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<tr>
<td>Perleth, Schatz &amp; Monks (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Gifted children's &quot;superiority&quot; can be attributed to higher cognitive efficiency (i.e., a higher basic speed of information processing and higher level of automation). The main cause of the differences between retarded, average, and gifted children is the efficiency of their memory systems.</td>
</tr>
<tr>
<td>Robinson (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Gifted students tend to observe their own metacognitive behaviors more accurately than average students. They master tasks efficiently and are able to easily transfer their insights to new problems.</td>
</tr>
<tr>
<td>Robinson, Abbott, Berninger, &amp; Busse (1996)</td>
<td>310 math gifted students Grades K-1</td>
<td>Students were administered 15 separate measures which were selected to tap the working memory in quantitative, visual-spatial, and verbal domains (e.g. Stanford-Binet IV quantitative, vocabulary, pattern analysis). Precocity in mathematical ability was identified at an early age. Working memory was domain specific for quantitative tasks. The strongest correlations were between visual-spatial and mathematical skills; however, correlation was significantly lower in the older children, suggesting increased differentiation.</td>
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<tr>
<td>Rotigel &amp; Fello (2004)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Mathematically gifted students often are much stronger in concept development than computation and show an uneven pattern of mathematical development and understanding. Differentiation is important and should match content and pace with the learner's needs.</td>
</tr>
<tr>
<td>Sak (2009)</td>
<td>291 math gifted students Grades 6-8 4 schools</td>
<td>Students were administered 27 math problems which had been rated on cognitive complexity and mathematical giftedness factors of knowledge, analytical ability, and creativity. The three mathematical-minds model (M3) (based on Sternberg) suggests forms of mathematical giftedness include: analyst, creator, knowledge expert, creative analyst, creative expert, expert analyst, and master. Findings showed partial support for reliability and validity of the M3 test, but the test needs revision.</td>
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<tr>
<td>Schneider (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Gifted students have high-capacity memories. Differences in performance can be attributed to qualitative and quantitative information processing differences.</td>
</tr>
<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
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<tr>
<td>Schraw &amp; Dennison</td>
<td>Experiment 1: 197 undergrads</td>
<td>Experiment 1: Subjects took the Metacognitive Awareness Inventory (MAI) (52 item self-report questionnaire).</td>
<td>Experiments showed strong support for a two-component view of metacognition. Metacognitive knowledge includes awareness of one's strengths and weakness, knowledge about strategies and when and why to use strategies. Metacognitive regulation includes knowledge about strategy planning, implementing, monitoring, and evaluating. Metacognitive knowledge corresponds to better metacognitive control; better metacognitive control leads to acquisition of new metacognitive knowledge.</td>
</tr>
<tr>
<td>(1994)</td>
<td>Experiment 2: 110 undergrads</td>
<td>Experiment 2: Students took the MAI and Nelson-Denny Reading Comprehension Test and then rated their confidence for each question and their perceived monitoring ability.</td>
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<tr>
<td></td>
<td>Gifted Criteria: Not applicable</td>
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<tr>
<td>Schraw &amp; Graham</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Metacognition consists of metacognitive knowledge and metacognitive control. Both develop earlier in gifted students. Metacognitive control improves performance by better use of existing strategies, better use of attentional resources, and awareness of breakdowns in comprehension. Gifted students have more strategies and are more motivated to use, learn, and transfer strategies. Metacognitive knowledge contributes to successful problem solving over and above the contributions of task-relevant strategies and IQ.</td>
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<td>(1997)</td>
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<tr>
<td>Shaw et al.</td>
<td>307 individuals Ages 5-19</td>
<td>Subjects received MRI scans at two-year intervals in this longitudinal study. Most were scanned two or more times.</td>
<td>More intelligent children had more plasticity in their cerebral cortex with a prolonged phase of increase in thickness, and then a rapid thinning of the cortex by early adolescence. The rate of change in thickness was most closely related to level of intelligence.</td>
</tr>
<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<tr>
<td>Shore (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Some gifted elementary students use metacognitive and problem solving strategies (such as working with a plan and organizing their knowledge in a hierarchical manner) commonly found in adult experts. There are clear differences in the speed and fluency with which gifted students invoke and use different strategies.</td>
</tr>
<tr>
<td>Shore &amp; Kanevsky</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Gifted students show flexibility in their thinking. This means they are able to see alternate representations and adopt alternate strategies to successfully complete a task. Gifted students have a preference for complexity. Gifted students spend more time on planning during problem solving than average students.</td>
</tr>
<tr>
<td>Silverman (1993)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Gifted students have intellectual curiosity, rapid learning rates, complex thought processes, exceptional reasoning abilities, perception, insight, perseverance, and high-capacity memories. Gifted students have an intense need for intellectual stimulation.</td>
</tr>
<tr>
<td>Singh &amp; O'Boyle (2004)</td>
<td>60 individuals</td>
<td>Subjects were shown pairs of figures made up of letters and were asked whether the figures matched based on the overall configuration or the elements making them up. Figures were shown in the right vision field (left hemisphere), left vision field (right hemisphere) or in a cooperative aspect (bilateral). Reaction times were calculated.</td>
<td>Mathematically gifted students were equally able to perform letter matches with either hemisphere, suggesting both hemispheres were actively engaged and equally able to process the information. Average students and college students relied more heavily on their left hemisphere.</td>
</tr>
</tbody>
</table>

**Gifted Criteria:** Qualified for Challenges for Youth-Talented program at Iowa State University (Math gifted had mean SAT-M scores of 620)
<table>
<thead>
<tr>
<th>Researchers</th>
<th>Sample</th>
<th>Description</th>
<th>Major Findings</th>
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</thead>
<tbody>
<tr>
<td>Sowell, Zeigler, Bergwall, &amp; Cartwright (1990)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>A review of 13 empirical studies from the 1970s-1980s related to the identification and description of mathematically gifted students showed there are at least two types of mathematically gifted students--those who use qualitatively different thinking processes and those who are precocious and are able to do the mathematics typically done by older students.</td>
</tr>
<tr>
<td>Sriraman (2003)</td>
<td>9 students, Grade 9, Gifted=4, Average=5</td>
<td>Five increasingly complex problems were assigned over three months. Students kept journals revealing their thought processes and each student was interviewed separately for each of the five problems.</td>
<td>Gifted students used consistent problem-solving strategies, using simpler cases to model the way to solve more complex cases. They could abstract similarities in the structure of problems and verbalize common principles. They viewed math as a way of thinking and persevered on difficult aspects of the problems. Average students approached the problems inconsistently, had difficulty articulating generalities, and viewed math as operations on numbers.</td>
</tr>
<tr>
<td>Sriraman (2004a)</td>
<td>4 gifted students, Grade 9</td>
<td>Students were asked to prove a relationship between circles and triangles. Each student was separately interviewed to determine their thought process for creating their proof.</td>
<td>Mathematically gifted students established a &quot;proof&quot; although they had never been exposed to that process. They demonstrated flexibility and reversibility in thinking, tenacity and perseverance, used their intuition, made generalizations, and came up with examples and non-examples. They used many of the same processes seen in professional mathematicians.</td>
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<td>Sriraman (2004b)</td>
<td>5 Ph.D. mathematicians</td>
<td>Professional mathematicians were interviewed to determine how they normally went about solving problems.</td>
<td>Professional mathematicians spent much time researching the context of the problem and they tended to work on more than one problem at a time. They looked for examples and nonexamples to gain insight, they used mental imagery, and they put the problem aside if the answer was not forthcoming. The transition from incubation to illumination was often unexpected, and illumination was followed by verification of the result.</td>
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<tr>
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<td>Description</td>
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<tr>
<td>Steiner (2006)</td>
<td>50 students Grade 2</td>
<td>Students were videotaped solving computer strategy game problems.</td>
<td>Gifted children took more time to plan and carry out strategies. They learned more from their mistakes, they developed and relied on higher-level strategies, and they had more sophisticated planning, which resulted in a slower time to solve problems. The average students relied on lower-level strategies.</td>
</tr>
<tr>
<td></td>
<td>Gifted=25</td>
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<td></td>
<td>Average=25</td>
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<tr>
<td></td>
<td>Gifted Criteria:</td>
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<tr>
<td></td>
<td>State identification</td>
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<tr>
<td></td>
<td>using three of four</td>
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<td></td>
<td>criteria: 1) 96th</td>
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<td>percentile on test of</td>
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<td></td>
<td>mental ability; 2)</td>
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<td></td>
<td>90th percentile on</td>
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<td></td>
<td>achievement test; 3)</td>
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<td>90th percentile on</td>
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<td></td>
<td>motivation scale; 4)</td>
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<tr>
<td></td>
<td>90th percentile of</td>
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<tr>
<td></td>
<td>measure of creativity</td>
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<tr>
<td>Steiner &amp; Carr</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>A review of 32 studies related to gifted cognitive development showed that gifted students process information more rapidly than average students. The cumulative effect of faster processing yields a vastly increased knowledge base, greater cognitive proficiency, and more sophisticated intellectual skills. Gifted students take more time on problem exploration and planning than average students. They have better metacognitive abilities at all ages and this contributes to the high performance of gifted students.</td>
</tr>
<tr>
<td>(2003)</td>
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<tr>
<td>Sternberg</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Within the gifted field, there is an inconsistency in research and identification of gifted individuals without understanding what it means to be gifted. Giftedness has cognitive and motivational aspects and there are multiple forms of giftedness. The environment is an important element as to whether the potential for gifted performance is realized.</td>
</tr>
<tr>
<td>(2004)</td>
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<tr>
<td>Researchers</td>
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</tbody>
</table>
| Swanson (2006)         | 127 students Grades 1-3  
Gifted=50  
Average=77  
Gifted Criteria:  
1.5 SD above the average on the Wechsler Individual Achievement Test and the math subset of the Wide Range Achievement Test III | Students were tested on 20 various measures to determine calculation skills, speed, short-term memory, working memory, and inhibition. | Mathematically precocious children performed better on executive processing, inhibition, and naming speed than average children. Students with higher math ability were better able to inhibit unnecessary information from entering their working memory. |
| Threlfall & Hargreaves (2008) | 705 students Gifted=47, age 9  
Avg=230, age 13  
Gifted Criteria: Teacher identified as being "in the top 10% ability range in mathematics" | Students were administered questions from the World Class Tests for 9-year-olds. Questions were related to concepts that were not directly taught. | Gifted 9-year-olds and average 13-year-olds had comparable abilities and used similar strategies. Because the problems were novel, this suggests the gifted students developed the same strategies at a younger age that the average students developed later. |
<p>| USDOE (2008b, 2008c)  | Not applicable      | Think piece                                                                                                                                  | The National Mathematics Advisory Panel considered students at or above the 90th percentile on standardized achievement tests as gifted. There were very few studies that examined the cognitive processes underlying mathematically gifted students' accelerated learning. Mathematically gifted students appear to have an enhanced ability to retrieve numerical and spatial—but not verbal information—from long-term memory and an enhanced ability to manipulate it in their working memory. |
| USHHS (2006)           | Not applicable      | Think piece                                                                                                                                  | This NIH press release on Shaw et al.'s (2006) findings explains that the brain's outer layer (cortex) reaches its peak thickness in gifted children later than in average children, suggesting a longer development window for &quot;high-level circuitry&quot; (para 1). It also thins faster in gifted teens due to withering of unused neural connections as the brain streamlines its operations. |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Waxman, Robinson, &amp; Mukhopadhyay (1996)</td>
<td>284 gifted students, Grades K-2</td>
<td>This two-year study looked at the cognitive development of young mathematically talented children using classroom observations.</td>
<td>Within mathematically gifted children, spatial reasoning was related more closely to math reasoning than was verbal reasoning even though the students were ahead in all three domains. A gifted weekend course proved effective in enhancing mathematical reasoning. Mathematically gifted students have a rapid and intuitive understanding of math, long periods of absorption, persistence, and enjoy math challenges.</td>
</tr>
<tr>
<td>Wieczerkowski, Cropley, &amp; Prado (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Divergent thinking is essential, but not sufficient for true giftedness. Mathematical giftedness also includes accuracy, speed, mastery of basic facts, rapid recall of material from memory, and other similar factors.</td>
</tr>
<tr>
<td>Winner (2000a)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Gifted children are qualitatively and quantitatively different from average children. Gifted children think differently and develop more rapidly than others, but more research is needed.</td>
</tr>
<tr>
<td>Winner (2000b)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Gifted children have enhanced right-hemisphere development and atypical brain organization. Gifted children may have uneven cognitive profiles. Some students are gifted with both words and numbers; others may tend more toward verbal or math precociousness.</td>
</tr>
</tbody>
</table>
## Appendix C

### Studies Related to Mathematically Gifted/Talented Students' Needs

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Sample</th>
<th>Description</th>
<th>Major Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assouline &amp; Lupkowski-Shoplik (2005)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Mathematically gifted children require a different path to develop their unique talents. Because mathematically gifted children may think about and solve problems differently than average students, they need to be challenged with greater depth and breadth, complexity, and abstraction. Mathematically gifted children have more energy, time, and need for further exploration than average students.</td>
</tr>
<tr>
<td>Diezmann (2005)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Mathematically gifted students learn more rapidly than average students, need less time to achieve mastery, and need challenging environments. This may entail problematizing tasks by inserting obstacles to the solution.</td>
</tr>
<tr>
<td>Diezmann &amp; Watters (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Challenging tasks enhance motivation and help mathematically gifted students develop mathematical reasoning and metacognitive skills. Time is required for incubation of ideas.</td>
</tr>
</tbody>
</table>
| Diezmann & Watters (2001) | 6 math gifted students Ages 11-12 years **Gifted Criteria:**
1) Top 10% on researcher-administered tests; 2) teacher identified as one of top performers; and 3) peers identified as one of top three math performers | Students were presented with four problems of varying levels of difficulty and were allowed to solve them in the: 1) Quiet Zone; 2) Work Zone; 3) Chat Zone; or 4) Teacher Zone. Students were video- and audiotaped. Student work was collected and students were asked about task and problem preferences. | Mathematically gifted students preferred to work independently on tasks they considered easy such as those at grade level; therefore, requiring students to collaborate on unchallenging tasks is not an authentic learning experience and supports socialization rather than cognitive engagement. When the tasks were challenging, students benefited from collaboration affectively, cognitively, and metacognitively. |
<table>
<thead>
<tr>
<th>Researchers</th>
<th>Sample Description</th>
<th>Major Findings</th>
</tr>
</thead>
</table>
| Diezmann & Watters (2002b) | 20 math gifted students Ages 11-12 years  
**Gifted Criteria:**  
Test scores, classroom performance, and peer nominations | Students were exposed to three conditions: regular tasks in a regular environment (RT/RE); regular tasks in an optimized environment (RT/OE); and optimized tasks in an optimized environment (OT/OE).  
Gifted students in the RT/RE condition were bored, disinterested, and were engaged in undesirable behaviors. Gifted students in the RT/OE condition completed tasks, but in unimaginative ways. Gifted students in the OT/OE condition demonstrated greater persistence, flexibility in thinking, collaboration, metacognition, and invented new strategies. Unnecessary scaffolding inhibited learning. Problematizing tasks and increasing the pace were necessary to engage students and provide worthwhile learning experiences. |
| Farkas & Duffet (2008)  | 900 teachers Grades 3-12  
**Gifted Criteria:** Characterized as "advanced students" | This survey was administered to a nationally representative sample of public school teachers. The results were combined with qualitative data from five focus groups.  
The survey found: 65% of teachers had little or no gifted training in their teacher preparation program; 81% gave one-on-one attention to struggling students, but only 5% gave it to advanced students; 60% said struggling students were a top priority, only 23% said advanced students were; teachers felt pressure to focus on struggling students, but believed that all students deserved attention and challenge. |
| Garofalo (1993)       | 8 students Grade 7  
**Gifted**=3  
**Average**=5  
**Gifted Criteria:** Scored in the 99th percentile of the ITBS-math | Students solved routine, multistep, and non-routine verbal problems in a one-on-one setting with the interviewer. Students described their problem-solving strategies, beliefs, metacognition, affects, and preferences for problem type.  
Average students were number-oriented while gifted students were meaning-oriented. Number-oriented students preferred easy, one-step problems, and their goal was to get enough answers correct to satisfy authority figures. Meaning-oriented students wanted to come up with optimal solution approaches. They enjoyed challenging problems for the feeling of accomplishment they received when solving them. |
<table>
<thead>
<tr>
<th>Researchers</th>
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<th>Major Findings</th>
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</thead>
<tbody>
<tr>
<td>Henningsen &amp; Stein</td>
<td>12 middle school mathematics</td>
<td>Three 3-day observations took place in 12 mathematics classrooms over a three-year period. Researchers used the Classroom Observation Instrument to evaluate factors that encouraged or inhibited student engagement in 58 math tasks.</td>
<td>Factors that encouraged high-level cognition included tasks that built on students' prior knowledge, an appropriate amount of time, sustained pressure by the teacher for explanation and meaning, scaffolding, and modeling of high-level performance. Factors that inhibited high-level cognition included whether the task was inappropriate for students, an inappropriate amount of time allotted to the task, removal of challenging aspects of the task, lack of accountability for high-level processes or products, focus on completion rather than understanding, and classroom management problems.</td>
</tr>
<tr>
<td>(1997)</td>
<td>classrooms</td>
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<tr>
<td></td>
<td>Gifted Criteria: None specified</td>
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<tr>
<td>Hiebert et al.</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Mathematics should be problematized to enable students to search for solutions and resolve incongruities. This leads to the construction of understanding. Tasks, themselves, are neither routine nor problematic--how the teachers and students treat them will determine whether they are routine or not.</td>
</tr>
<tr>
<td>(1996)</td>
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<tr>
<td>Johnson (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Mathematically gifted students differ in pace of learning, depth of understanding, and interest from average students. They need differentiated instruction including a curriculum that is broader, faster, and deeper. Problems should be complex, open-ended, and require higher-level thinking.</td>
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<tr>
<td>Matthews &amp; Farmer (2008)</td>
<td>3,622 students</td>
<td>Math ability was measured by SAT-M scores. Math achievement was measured by</td>
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<td></td>
<td>Grades 7-9</td>
<td>standardized end-of-course Algebra I exam scores.</td>
<td>Identification as gifted had a moderate effect on mathematical reasoning; math reasoning had a moderate effect on Algebra I achievement. Gifted status was not significantly related to Algebra I achievement after controlling for SAT-M test (the entire impact of giftedness could be explained by the SAT-M test). A student's giftedness influenced math ability, but was not necessarily reflected in their math achievement.</td>
</tr>
<tr>
<td></td>
<td>Gifted=2,925</td>
<td></td>
<td>giftMRI</td>
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<td></td>
<td>Average=697</td>
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<td></td>
<td>Gifted Criteria:</td>
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<td></td>
<td>School district criteria</td>
<td></td>
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<tr>
<td>McNabb (2003)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Gifted students who are not challenged may express their boredom through misbehavior which may result in a negative reaction from the teacher. This, in turn, may reinforce the students' perception that their needs are unreasonable.</td>
</tr>
<tr>
<td>Middleton &amp; Spanias (1999)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>When students are intrinsically motivated, they demonstrate pedagogically-desirable behaviors such as persistence, creativity, increased time on task, choice of harder tasks, greater risk taking, more efficient strategies.</td>
</tr>
<tr>
<td>Miller (1990)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Math programs for gifted students should be flexibly paced. Students should be placed at an appropriate instructional level based on their knowledge and skills and the pace should be limited only by the student's ability and motivation. Appropriate pace can be achieved through continuous progress, compacted courses, advanced-level courses, grade skipping, early entrance, dual enrollment, and credit by examination.</td>
</tr>
<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<tr>
<td>Moon, Tomlinson, &amp; Callahan (1995)</td>
<td>500 middle school principals 449 middle school teachers</td>
<td>A separate survey was administered to administrators and teachers asking about their beliefs, policies, and practices related to serving their students.</td>
<td>Almost half of the teachers and administrators believed middle school is a plateau learning period, therefore, they underchallenge advanced middle school students. There is a lack of responsiveness to student differences. Student choice and modifying the curriculum to meet diverse talents was ranked lowest by teachers and administrators on factors that shape the curriculum. Advanced students receive less attention than remedial or special needs students.</td>
</tr>
<tr>
<td>NAGC (2008)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Every learner needs opportunities and support to reach their potential. High-ability adolescents may differ from their peers in modes of learning, motivation, interests, and cognitive skills. This is a joint position statement with the National Middle School Association.</td>
</tr>
<tr>
<td>NCTM (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td><em>Principles and Standards</em> explains the importance of problem solving, pointing out that problem solving involves engaging in a task where the method of solution is not known ahead of time. Students should have many opportunities to come up with and grapple with complex problems which require significant effort to solve.</td>
</tr>
<tr>
<td>Rayneri, Gerber, &amp; Wiley (2006)</td>
<td>80 gifted middle school students 6th grade=26 7th grade=34 8th grade=20</td>
<td>Students took the Learning Style Inventory and the Student Perception Inventory to assess the compatibility between their classroom environment and learning style.</td>
<td>Gifted middle school students have higher achievement if taught by an informed teacher who is aware of their needs. Teachers need to modify the environment to support student engagement in the learning process.</td>
</tr>
<tr>
<td>Schultz, Dayan, &amp; Montague (1997)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>If learning is redundant, the brain does not release the levels of noradrenalin, serotonin, dopamine, and other neurochemicals needed for optimal learning.</td>
</tr>
<tr>
<td>Researchers</td>
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<td>Description</td>
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<tr>
<td>Sheffield (1999)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>The needs of mathematically promising students can be met through acceleration, enrichment, depth, and complexity. Brain research tells us that solving challenging problems can promote growth in various parts of the brain which makes the brain even more capable of solving problems.</td>
</tr>
<tr>
<td>Silver &amp; Stein (1996)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>A five year study as part of the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project found that students who have opportunities to engage in high-level reasoning, multiple strategies, multiple representations, and mathematical explanations outperform students who do not have that opportunity.</td>
</tr>
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<td>Sriraman (2004a)</td>
<td>4 gifted students Grade 9</td>
<td>Students were asked to prove a relationship between circles and triangles. Each student was separately interviewed to determine their thought process for creating their proof.</td>
<td>Mathematically gifted students established a &quot;proof&quot; although they had never been exposed to that process. They demonstrated flexibility and reversibility in thinking, tenacity and perseverance, used their intuition, made generalizations, and came up with examples and non-examples. They used many of the same processes seen in professional mathematicians.</td>
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<td>Professional mathematicians spent much time researching the context of the problem and they tended to work on more than one problem at a time. They looked for examples and nonexamples to gain insight, they used mental imagery, and they put the problem aside if the answer was not forthcoming. The transition from incubation to illumination was often unexpected, and illumination was followed by verification of the result.</td>
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<tr>
<td>Stein, Grover, &amp; Henningsen (1996)</td>
<td>12 middle school classes</td>
<td>Three 3-day observations took place in 12 mathematics classrooms over a three-year period. Researchers used the Classroom Observation Instrument to evaluate factors that encouraged or inhibited student engagement in 144 mathematical tasks.</td>
<td>Tasks that were set up to require high levels of cognition, sustained thinking, and reasoning declined during implementation so that they were much less cognitively demanding. The factor most often found for reducing the challenge of the task was either the teacher telling the students how to do the task or taking over the challenging aspects of the task for the students, or through students pressuring the teacher to reduce the complexity and ambiguity by specifying steps. Providing scaffolding to the entire class in this way took away student opportunities for discovery.</td>
</tr>
<tr>
<td>USDOE (2008b)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Challenging material is important because it causes the learner to actively process the information which leads to better retention. Demanding instruction and acceleration is needed for gifted students to reach their full potential in mathematics.</td>
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</table>
## Appendix D

### Studies Related to Differentiation Strategies

<table>
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<tr>
<th>Researchers</th>
<th>Sample</th>
<th>Description</th>
<th>Major Findings</th>
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</thead>
<tbody>
<tr>
<td>Access Center (2005)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Teachers differentiate in response to students' learning profile, interests, and/or their readiness. Tiered assignments may have different levels of abstractness, open-endedness, and complexity. Compacting involves assessing what a student knows, making a plan for what they need to know, and figuring out what to do with freed-up time. Flexible grouping can use groups that are assigned purposefully or randomly.</td>
</tr>
<tr>
<td>Archambault et al. (1993)</td>
<td>7,314 teachers Grades 3-4</td>
<td>Administered Classroom Practices Questionnaire to determine classroom practices teachers used with gifted and average students.</td>
<td>Teachers made only minor modifications to the regular curriculum to meet the needs of gifted students and 61% had never had staff development in gifted education. The teachers that differentiated did so by independent projects, enrichment, and advanced readings. There were few differences in instruction between schools with and without formal gifted programs.</td>
</tr>
<tr>
<td>Assouline &amp; Lupkowski-Shoplik (2005)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Mathematically gifted children require a different path to develop their unique talents. Because mathematically gifted children may think about and solve problems differently than average students, they need to be challenged with greater depth and breadth, complexity, and abstraction. Mathematically gifted children have more energy, time, and need for further exploration than average students.</td>
</tr>
<tr>
<td>Ball &amp; Bass (2003)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Open-ended questions are an important tool for eliciting higher-level mathematical reasoning. Mathematical tasks may be enhanced to create a demand for mathematical reasoning.</td>
</tr>
<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
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<tr>
<td>Benbow, Lubinski, Shea, &amp; Eftekhari-Sanjani (2000)</td>
<td>1,975 math gifted adults from two cohorts of SMPY</td>
<td>Gifted Criteria: At least SAT-M &gt; 390 before age 13 (top 1% in math reasoning)</td>
<td>This 20-year follow-up of mathematically gifted adolescents found they had become exceptionally accomplished and were very satisfied with their successes. Gender differences were apparent in career selection and in life priorities. They were overwhelmingly proponents of tailoring the curriculum to meet the needs of gifted students.</td>
</tr>
<tr>
<td>Brody &amp; Benbow (1990)</td>
<td>244 gifted students, Grade 7</td>
<td>Gifted Criteria: SAT-M ≥ 500 SAT-V ≥ 430 and Test of Standard Written English score ≥ 35.</td>
<td>Thirteen-year-olds in the top 1% of academic ability can cover mathematics coursework that would normally take one or more years in school in as little as three intensive weeks. In-depth instruction over a short time did not increase SAT scores.</td>
</tr>
<tr>
<td>Colangelo, Assouline, &amp; Gross (2004a, 2004b)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>There are 18 different types of acceleration. For gifted students, acceleration is the single most effective intervention. Acceleration has long-term benefits socially and academically.</td>
</tr>
<tr>
<td>Dark &amp; Benbow (1990)</td>
<td>80 students Grade 7 &amp; undergrad Math gifted=20 Verbal gifted=20 Avg. ability=20 Undergrads=20</td>
<td>Gifted Criteria: SAT-M ≥ 500 Math gifted means: SAT-M=651 SAT-V=452 Verbal gifted means: SAT M=499 SAT-V=533</td>
<td>Mathematically gifted students had enhanced problem-translation skills and were better than verbally gifted or college students at writing equations to express complex relationships. Mathematically gifted students had an enhanced ability to represent and manipulate information in their working memory. They outperformed verbally gifted and college students in these areas.</td>
</tr>
<tr>
<td>Researchers</td>
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<td>Description</td>
<td>Major Findings</td>
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</tr>
<tr>
<td>Dark &amp; Benbow (1991)</td>
<td>64 students Grades 7-8</td>
<td>The span task included 20 lists of stimuli (letters, digits, words, and locations). Students were asked to recall the lists. The paired association task included 24 paired associations (e.g. F=6). Students were asked to recall the associations.</td>
<td>Mathematically gifted students had an enhanced capacity for location and digit stimuli. They outperformed verbally gifted students in working memory manipulations, but verbally gifted students performed better than mathematically gifted on the retrieval of a representation from long-term memory into working memory.</td>
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<tr>
<td></td>
<td>&quot;MV&quot; group=20 &quot;Mv&quot; group=22</td>
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<tr>
<td></td>
<td>Gifted Criteria: SAT-M ≥ 500 SAT-V ≥ 430</td>
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<tr>
<td>Delcourt (1993)</td>
<td>18 students Grades 9-12</td>
<td>Students went to schools that used the Enrichment Trial Model (Renzulli, 1977). Parents and students each completed two questionnaires concerning family background and the quality and quantity of student projects. Students were also interviewed.</td>
<td>Students who participate in and out of school in Type III projects maintain their career aspirations and interests in college. Adolescents can be producers and consumers of information. Persistence was the most salient characteristic of the students.</td>
</tr>
<tr>
<td></td>
<td>Gifted Criteria: 4+ years in school gifted and talent program; students selected based on creative/productive potential</td>
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<tr>
<td>Delcourt, Cornell, &amp; Goldberg (2007)</td>
<td>460 students Grades 2-3</td>
<td>This study used achievement tests, a motivation inventory, and a self-perception survey to determine the impact of gifted programs.</td>
<td>Ability grouping is effective for gifted students. Students in a within-class grouping setting had the lowest achievement scores when compared to gifted peers in pullout classes, separate classes, or special schools.</td>
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<tr>
<td></td>
<td>Gifted=290 High-achieving, but not identified=50 Non-gifted=120</td>
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<tr>
<td></td>
<td>Gifted Criteria: School district criteria</td>
<td></td>
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<tr>
<td>Diezmann &amp; Watters (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Challenging tasks enhance motivation and help mathematically gifted students develop mathematical reasoning and metacognitive skills. Time is required for incubation of ideas.</td>
</tr>
<tr>
<td>Farkas &amp; Duffet (2008)</td>
<td>900 teachers Grades 3-12</td>
<td>The survey was administered to a nationally representative sample of public school teachers. The results were combined with qualitative data from five focus groups.</td>
<td>The survey found: 65% of teachers had little or no gifted training in their teacher preparation program; 74% said students would benefit from homogeneous grouping in math, but 59% said there was little/no homogeneous grouping in their schools. Forty-six percent said their schools did not allow grade skipping and 63% opposed the practice, but 85% favored more subject acceleration. An overwhelming 96% favored enrichment opportunities outside of schools.</td>
</tr>
<tr>
<td></td>
<td>Gifted Criteria: Characterized as &quot;advanced students&quot;</td>
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<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<tr>
<td>Gamoran &amp; Weinstein (1998)</td>
<td>20,292 students 24 schools (8 elementary, 8 middle, 8 high school)</td>
<td>Observers from the Center on the Restructuring of Schools (CORS) conducted site visits and classroom observations, and interviewed teachers, administrators, parents, district personnel, and agencies that influence innovations at the schools to look at differentiation and opportunities.</td>
<td>Heterogeneous grouping is more problematic to teachers of math because they believe math should be taught sequentially and that students should master certain concepts before moving to others. Teaching mixed-ability classes requires extra effort on the part of teacher. Grouping by ability encourages higher-quality instruction in high-ranked groups. High-quality instruction is possible in heterogeneous math classes, but heterogeneous classes are most effective when the teacher uses differentiated instruction.</td>
</tr>
<tr>
<td>Gavin et al. (2007)</td>
<td>200 math talented students Grade 3-5 in 11 schools</td>
<td>Students were instructed using 12 Project M3 (Mentoring Mathematical Minds) units. Teachers had a two week training session and 4-6 professional development sessions prior to teaching each unit.</td>
<td>Using accelerated and enriched units, students showed a significant increase in their understanding of all mathematical concepts tested. Effect sizes for Project M3 units ranged from 1.55 to 3.49 (.80 is considered large).</td>
</tr>
<tr>
<td>Gentry &amp; Owen (1999)</td>
<td>334 students Grades 2-5 14 teachers 3 administrators</td>
<td>This mixed-methods study used standardized achievement measures (ITBS and California Achievement Test) and semi-structured interviews with teachers and administrators to investigate the impacts of flexible grouping.</td>
<td>The use of ability and other forms of flexible grouping provides academic gains for all students. Flexible achievement grouping in conjunction with challenging curriculum benefits all. Teachers found achievement grouping in math made it easier for them to appropriately challenge the students.</td>
</tr>
<tr>
<td>Herbert (1993)</td>
<td>9 students Grade 11-12</td>
<td>Students went to schools that used the Enrichment Triad Model (Renzulli, 1977). Students were extensively interviewed about their educational experiences 10 years after their involvement.</td>
<td>Five major themes emerged: Type III interests have an impact on post-secondary plans; high school students need creative outlets; there is a decrease in the number of Type IIIs pursued in middle school; Type IIIs served as training for later productivity; and non-intellectual traits such as creativity and task commitment remained consistent throughout the students’ school years.</td>
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<td>Researchers</td>
<td>Sample</td>
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<tr>
<td>Johnson (2000)</td>
<td>Not applicable</td>
<td>Mathematically gifted students differ in pace of learning, depth of understanding, and interest from average students. They need differentiated instruction including a curriculum that is broader, faster, and deeper. Problems should be complex, open-ended, and require higher-level thinking.</td>
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<tr>
<td>Kim (2006)</td>
<td>Not applicable</td>
<td>Mathematically gifted students benefit from being grouped by ability because it is conducive to higher math achievement, more positive attitudes, and allows the teacher to cover concepts at a more appropriate pace.</td>
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<tr>
<td>Kulik (1992)</td>
<td>Not applicable</td>
<td>Talented students from accelerated classes outperform peers by almost a year on achievement tests while talented students from enriched classes outperform peers by 4-5 months on grade equivalent scales. The value of grouping depends on the students’ achievement level: grouping gifted by ability with no curricular adjustment increases achievement over peers by 1 month on grade equivalent scales, but there is no difference for medium- or low-ability students.</td>
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<tr>
<td>Kulik &amp; Kulik (1992)</td>
<td>Not applicable</td>
<td>The effect of grouping depends on the program type. Multi-level classes have little impact on student achievement. Cross-grade and within-class grouping have positive effects. Enriched and accelerated classes involve the most curricular adjustment and produce the largest impact on student learning.</td>
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<td>Researchers</td>
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<tr>
<td>Mills, Ablard, &amp; Gustin (1994)</td>
<td>306 math gifted students Grades 3-6</td>
<td>Students attended a flexibly-paced math course at Johns Hopkins Center for Talented Youth (3 hrs on weekends for seven months).</td>
<td>A flexibly paced program with continuous learning progress provided a source of motivation for gifted students. Students as young as grade 4 successfully completed pre-algebra. A grade-appropriate curriculum may put gifted students at risk for a decline in achievement and motivation. The prior knowledge and range of abilities in the top three percentiles on a grade appropriate test is as great as that found within a general population of students; it is only through above-level testing that the actual variability in their ability becomes evident. Teachers may ask too little of gifted students.</td>
</tr>
<tr>
<td>Neihart (2007)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Acceleration should be routine for highly gifted students. Students considered for acceleration should be screened for motivation, emotional maturity, and social readiness. Every gifted student is not a good candidate for grade skipping. Peer ability grouping is associated with strong benefits in achievement.</td>
</tr>
<tr>
<td>Reis &amp; Renzulli (1992)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Teachers can eliminate up to 50% of the regular curriculum without an impact on student math achievement scores. Eliminating repetitive learning is crucial for gifted students because repetition can lead to boredom, disenchantment toward school, and underdeveloped study skills.</td>
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<tr>
<td>Researchers</td>
<td>Sample</td>
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<tr>
<td>Reis et al. (1993)</td>
<td>436 teachers</td>
<td>This study assessed the effects of curriculum compacting on student achievement, preferences, and attitudes, as well as how well the teachers were able to implement curriculum compacting. It used student achievement tests, attitude surveys, teacher questionnaires, and data forms.</td>
<td>Math was the content area most frequently compacted. Teachers can eliminate up to 50% of the regular curriculum in math with no difference on out-of-level post achievement tests (ITBS). Students with compacted math curriculum scored significantly higher on the math concepts posttest (ITBS) than the control group who did not have compacting in math. Staff development and peer coaching can improve teachers' use of compacting.</td>
</tr>
<tr>
<td></td>
<td>783 high-ability students Grades 2-6</td>
<td>Median ITBS math concepts = 93rd percentile Median ITBS math computation = 90th percentile</td>
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<tr>
<td></td>
<td>Gifted Criteria: Identified by school; if no gifted program, teacher selected top students.</td>
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<tr>
<td>Reis, Westberg, Kulikowich, &amp; Purcell, (1998)</td>
<td>336 high-ability students Grades 2-6</td>
<td>The researchers used data from the Curriculum Compacting Study (Reis et al., 2003) to further examine the impact of compacting on achievement test scores.</td>
<td>Teachers can eliminate 40-50% of the regular curriculum content without a decline in student's national test scores (ITBS). Results are based on out-of-level test scores. It is crucial for teachers to identify high-ability students and provide appropriate instruction for them.</td>
</tr>
<tr>
<td>Renzulli &amp; Reis (1997)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>The Schoolwide Enrichment Model (SEM) helps develop creative productivity through three types of enrichment activities: Type I are general exploratory activities; Type II are group training activities; Type III are individual and small group investigations of real-world problems. The model includes curricular modification techniques and a total talent portfolio. Type I and II activities benefit gifted and average students. Students show improved self-efficacy and creative productivity.</td>
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<td>Researchers</td>
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<tr>
<td>Rogers (1991)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Full-time ability grouping in special gifted/talented programs produces substantial academic benefits for the gifted, but there is no difference for average or low students. Ability grouping for enrichment produces substantial academic gains for gifted in creativity, general achievement, and critical thinking. Ability grouping has little impact on gifted students' self-esteem. Within class grouping for specific instructional purposes produces substantial academic gains if the instruction is differentiated. Cross-grade grouping and cluster grouping also have substantial beneficial academic effects.</td>
</tr>
<tr>
<td>Rogers (2002)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Curriculum compacting involves diagnosis and prescription. To determine which acceleration option is best, one must look at the students' learning strengths and preferences, cognitive functioning, interests, and developmental age or grade. Within-class grouping requires that the teacher is committed to differentiate in expectation, coverage, pacing, and difficulty.</td>
</tr>
<tr>
<td>Rogers (2007)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>This is a synthesis of the research on educating gifted and talented students. These students need a daily challenge in their area of talent; opportunities to work independently in their area of talent and passion; acceleration; opportunities to learn and socialize with peers of like-ability; differentiation by pace, amount of practice and review, and content presentation and organization.</td>
</tr>
<tr>
<td>Rotigel &amp; Fello (2004)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Mathematically gifted students often are much stronger in concept development than computation and show an uneven pattern of mathematical development and understanding. Differentiation is important and should match content and pace with the learner's needs.</td>
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<tr>
<td>Researchers</td>
<td>Sample Description</td>
<td>Major Findings</td>
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<tr>
<td>Sadler &amp; Tai</td>
<td>8,474 undergrad</td>
<td>Learning math at a faster pace or earlier than normal allows students to be better prepared for college science classes and more advanced in their math</td>
<td></td>
</tr>
<tr>
<td>(2007)</td>
<td>students tailored to physics, chemistry, and biology and interviews with high school teachers, college students, and professors.</td>
<td>education. The number of years of math instruction was a significant predictor of performance in all college science subjects.</td>
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<td></td>
<td>77 colleges &amp;</td>
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<td>universities</td>
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<tr>
<td></td>
<td><strong>Gifted Criteria:</strong> Not applicable</td>
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<tr>
<td>Sheffield</td>
<td>Not applicable</td>
<td>The needs of mathematically promising students can be met through acceleration, enrichment, depth, and complexity. Brain research tells us that solving challenging problems can promote growth in various parts of the brain which makes the brain even more capable of solving problems.</td>
<td></td>
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<tr>
<td>(1999)</td>
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<tr>
<td>Sheffield</td>
<td>Not applicable</td>
<td>A heuristic model for problem exploration that encourages students to investigate, relate, communicate, evaluate, and create helps to inspire them to think like mathematicians.</td>
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<tr>
<td>(2000)</td>
<td></td>
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<tr>
<td>Sheffield</td>
<td>Not applicable</td>
<td>To develop mathematical promise, teachers should ask a variety of questions to help explore problems in depth. Students should be assessed using criteria to include depth of understanding, fluency, flexibility, originality, elaboration, generalization, and extensions.</td>
<td></td>
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<tr>
<td>(2003)</td>
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<tr>
<td>Slavin (1987)</td>
<td>Not applicable</td>
<td>Cross-grade grouping is instructionally effective (ES = .45). Within-class ability grouping in mathematics is instructionally effective (ES = .34). One of the biggest benefits of flexible grouping is the fact that the groups are temporary in nature.</td>
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<td>Researchers</td>
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<td>Major Findings</td>
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<td>Stanley (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>50% of 7th graders who score 500-800 on the SAT-M know more algebra prior to taking the course than 50% of students know after taking the course. Algebra I can be taught in one day to the highly gifted. Stanley proposes homogeneous grouping with longitudinal teaching teams so students only learn what they don't know.</td>
</tr>
<tr>
<td>Starko (1986)</td>
<td>102 students&lt;br&gt;Grades 7-8&lt;br&gt;RDIM=58&lt;br&gt;Non-RDIM=44</td>
<td>This study compared gifted students who participated in the Revolving Door Identification Model (RDIM) of gifted education with gifted students who were identified, but had not received services. Questionnaires were used to determine the Type III activities students had pursued, students were interviewed, and administered a self-efficacy scale.</td>
<td>Participation in the RDIM program and the number of projects completed in school were predictors of creative productivity outside of school. Students reported that Type III activities improved their attitude toward school, impacted their awareness of their weaknesses and strengths, and improved their research skills.</td>
</tr>
<tr>
<td>Stepanek (1999)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Many strategies that work for instructing gifted students in math work for the whole class. Students should be provided with problems at different levels of abstractness, complexity, and open-endedness to encourage deeper thinking. Students should be allowed choice in deciding when and how they work in at least some of their activities.</td>
</tr>
<tr>
<td>Tieso (2002)</td>
<td>645 students&lt;br&gt;Grades 4-5&lt;br&gt;31 teachers</td>
<td>Teachers implemented three different grouping practices (whole class, flexible small groups, and Joplin Plan) and either a modified or differentiated mathematics curriculum. Students were given pre- and post-mathematics tests based on the curriculum.</td>
<td>An enrichment/enhanced curriculum improves student achievement; the highest-ability students gain the most. A differentiated curriculum in conjunction with flexible grouping improves achievement; the highest ability students gain the most.</td>
</tr>
<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<tr>
<td>Tomlinson (1995a)</td>
<td>1 middle school</td>
<td>The researchers conducted interviews with teachers, administrators, parents, and students (28 hours); classroom observation (30 hours); attended teacher team meetings (11 hours), staff development sessions (34 hours), faculty meetings (4 hours); and analyzed school documents (district memos, lesson plans, notes to parents, student handouts)</td>
<td>Understanding differentiated instruction is essential if teachers in heterogeneous classrooms are to become viable for &quot;academic outliers&quot; (p. 77). Intense, sustained staff development is needed to train teachers on how to differentiate for academically diverse learners.</td>
</tr>
<tr>
<td>Tomlinson (1995b)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Instruction for gifted should be concept focused, have on-going student assessment, and consistently use flexible grouping. There are many types of readiness-based adjustments such as simple to complex, quicker to slower, concrete to abstract, and less independence to more independence.</td>
</tr>
<tr>
<td>Tomlinson (1999)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>By varying the level of difficulty, the teacher increases the chance that each student will be appropriately challenged and that all students will gain essential skills and understandings. Key principles of differentiated classrooms include: the teacher understands and addresses student differences; the goal is to maximize growth and individual success; instruction and assessment are inseparable.</td>
</tr>
<tr>
<td>Tomlinson (2000)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Teachers should differentiate the content, product, and process. Assessment should be ongoing and linked to teaching. Students should participate in activities that are respectful. Flexible grouping should be used extensively.</td>
</tr>
<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<td>USDOE (2008b, 2008c)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>Acceleration, enrichment, flexible grouping arrangements, and individualization are effective differentiation approaches to meet the needs of mathematically gifted students. Challenging material is important because it causes the learner to actively process the information which leads to better retention. Demanding instruction and acceleration is needed for gifted students to reach their full potential in mathematics.</td>
</tr>
<tr>
<td>VanTassel-Baska &amp; Stambaugh (2005)</td>
<td>Not applicable</td>
<td>Think piece</td>
<td>The many obstacles to differentiating include a lack of knowledge of the subject matter or for modifying the curriculum, classroom management skills, and a lack of planning time, administrative supports and pedagogical skills.</td>
</tr>
<tr>
<td>Waxman, Robinson, &amp; Mukhopadhyay (1996)</td>
<td>284 gifted students, Grades K-2</td>
<td>This two-year study looked at cognitive development of young mathematically talented children.</td>
<td>Within mathematically gifted children, spatial reasoning was related more closely to math reasoning than was verbal reasoning even though the students were ahead in all three domains. A gifted weekend course proved effective in enhancing mathematical reasoning. Mathematically gifted students have a rapid and intuitive understanding of math, long periods of absorption, persistence, and enjoy math challenges.</td>
</tr>
<tr>
<td>Westberg, Archambault, Dobyns, Slavin (1993)</td>
<td>96 students Grades 3-4 46 teachers</td>
<td>Using the Classroom Practices Record observation instrument, researchers observed heterogeneous classrooms and selected one gifted/high-ability and one average student each day as “target students.” They observed a total of 92 target gifted and 92 target average students.</td>
<td>In 92 days of observation in all content areas, gifted students were only homogeneously grouped for instruction 21% of time. Gifted students received no curricular or instructional differentiation in 84% of their instructional activities. There was advanced content instruction (above-grade level or material several units ahead of classmates) in 11% of math activities.</td>
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<tr>
<td>Researchers</td>
<td>Sample</td>
<td>Description</td>
<td>Major Findings</td>
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<tr>
<td>Ysseldyke, Tardew, Getts, Thill, &amp; Hannigan (2004)</td>
<td>100 gifted students Grades 3-6</td>
<td>This study used a four group pretest, posttest control group design. Students worked on mathematics material at their individual skill level.</td>
<td>Individualized, self-paced instruction matched to skill level improved gifted student performance in areas of math concepts, skills, applications, and computation.</td>
</tr>
</tbody>
</table>
Appendix E

Researcher as Instrument Statement

To help ensure the study’s findings are representative of the participants’ experiences, as a researcher it is important to reveal my thoughts about the teaching of gifted/talented students. By discussing my personal experience with mathematically gifted/talented students, values and beliefs related to the education of gifted/talented students, expectations of the study, what I am willing and not willing to discover through the research, and to whom the results of the research may be useful, I will offer the reader a better understanding of the lens through which I view this aspect of education.

Experiences with Gifted/Talented Students

There were no official gifted/talented programs in the schools I attended when growing up, so my first experience with a gifted/talented program was when my own children were identified as gifted. Both of them participated in gifted programs in elementary school and then took accelerated classes in middle and high school. When they were in heterogeneous classes, they sometimes mentioned being frustrated when a teacher did not challenge them or simply used them as a peer tutor. They were likewise frustrated when the teacher gave them extra independent work just to keep them busy. Although I shared their discouragement from a parent’s perspective, it was not until I became a teacher, myself, that I really understood the challenge educators face in meeting the needs of gifted/talented students.

My first experience teaching gifted/talented students was at the Air Force Academy, where I was responsible for teaching occasional classes on military-related topics. Because of the stringent academic requirements to get accepted into the Academy, all of
my students were high-ability, and I was not faced with the situation where I had to spend an excessive amount of time with struggling students. It was, in many ways, an ideal teaching situation because I was not faced with discipline or motivation problems, and everyone understood the material. This experience showed me how much gifted/talented students can accomplish when properly challenged.

I was faced with an entirely different situation when I retired from the Air Force and became a middle school math teacher. I taught several math courses including Mathematics 7, Pre-Algebra 7, Pre-Algebra 8, and Mathworks 7 (remedial math). The only course that normally included gifted/talented students was Pre-Algebra 7. Students in this course were a year ahead of their average peers in math. Typically, about four or five students in a class of 30 had been identified as gifted, but not all participated in the school’s gifted pull-out program. This program required students to do their gifted assignments as well as the regular assignments for the classes from which they were pulled, so several students actually declined to participate in the gifted program because they viewed the workload as too heavy. I felt like the gifted students were put into a lose-lose situation. Either they participated in the gifted program and did twice the work of other students, or they did not participate and thus did not receive the academic, social, and emotional benefits such a program offered. In my mind, the program did a disservice to the gifted/talented students. This experience showed me how important it is for a regular classroom teacher to endeavor to meet the needs of her gifted/talented students because these students may not be participating in a gifted program. Furthermore, even if these students do participate in a gifted program, their needs do not disappear when they are in a regular classroom.
For the most part, I found the gifted/talented students in my heterogeneous classes to be a real pleasure to teach. They understood the material quickly, most were motivated to put forth their best effort, and they were generally well-behaved. Unfortunately, because of the overwhelming emphasis placed on student performance on the standards tests, I found myself more focused on ensuring the struggling students could understand the material than on challenging the advanced students who were ready for more complex material. I also found that I did not have a very thorough understanding of giftedness. I was not aware of the affective issues that faced some of these students, nor did I understand that students might be gifted only in a certain domain. For example, one of my students had been identified as gifted, but was the lowest performer in the entire pre-algebra class. I could not understand why he seemed to struggle with concepts more than the rest of the class. I assumed that his identification as gifted meant he was mathematically gifted, but in retrospect, I realize that he was only gifted in the verbal domain. I had another gifted student – again, the lowest performer in class – who never did homework and did not seem to care about his grades. In retrospect, I realize he was a classic gifted underachiever. Unfortunately, it was not until I took a graduate-level gifted education course that I was able to recognize what was going on with these two students – too late to address the issues as I should have. I believe many teachers in the classroom today have a similar lack of knowledge about gifted students.

I also have experience teaching gifted/talented students at the other end of the age spectrum. I currently teach math to gifted prekindergarten through first grade students as part of a college’s gifted enrichment program. These classes are made up entirely of students who have been identified as gifted, and so I do not have to address the kinds of
struggling-student issues I faced in a heterogeneous class. Through teaching these
classes, I have learned that students who are gifted in mathematics can accomplish
unbelievable things if given the proper encouragement and support.

Values and Beliefs about the Education of Gifted/Talented Students

As a researcher, it is important that I acknowledge my thoughts about gifted/talented
students and how they should be taught. I believe that every student deserves an
equitable amount of the teacher’s attention in class. I realize that in today’s standards-
driven environment, a teacher is pressured to focus on her struggling students to ensure
they can pass the end-of-course standards test; however, failing to pay adequate attention
to the brightest students in the class does a disservice to them.

When gifted/talented students are placed in heterogeneous classes, I believe it is
important for the teacher to behave in a way that demonstrates that no single child is
more important than another. All children are different, and all bring unique aspects of
their home environment, life experiences, and personality to the classroom. Every
student has needs, and classroom teachers should attempt to meet those needs that are
within their power to address. In essence, we are paying a teacher for her time and talent.
If she spends the majority of her time and ability working only with struggling students,
are we not, in fact, saying that these students are more worthy of the teacher than the
other students? Just because the gifted/talented students can easily understand the
material being presented to the class does not mean that their needs have been addressed.

I believe the development of a student’s gifts is a responsibility of our educational
system. Some schools spend a significant amount of money on special programs for
struggling students, but yet do not offer a legitimate gifted program to meet the needs of
the gifted/talented students. One might argue that gifted students are more likely to receive support at home and that they will do just fine without additional resources at school. I believe this argument fails to account for a student’s potential. A school district can spend an inordinate amount of time and money to help a less cognitively capable student reach a minimal level of competency and yet completely ignore the fact that there are gifted students who have not come anywhere near reaching their own potential. This sends the message that one student’s potential is more important than another’s. Students do not all start at the same point; our goal should not be for them to all finish at the same point. Our goal should be for each child to go as far as they are able. I liken it to someone with lots of natural athletic ability. They may be able to become proficient at a sport very easily, but if we want them to develop into Olympic athletes, they need dedicated practice, coaching, and support. The same holds true with gifted/talented students. They may have lots of natural mathematics ability, but if we want them to develop their ability into true mathematical talent, it requires a similar level of dedicated practice, instructional coaching, and support from the teacher. The mathematical and scientific minds that will move our country ahead in the future are far more likely to come from the high-ability students than from a student who has minimal competency in math. These intelligent students need support and encouragement just like any other student. Teachers need to nurture these bright minds so that these students can reach their potential.

To help them achieve their potential, teachers need to challenge gifted/talented students and not allow them to float through the curriculum with minimal effort. Simply providing them more of the same type of work that the rest of the class is doing does not
cognitively engage them. Putting gifted students in groups with slower students, although important for their social development, does not challenge them. In fact, in many cases the high-ability students end up doing the majority of the work and feel resentful because of it. To challenge gifted/talented students, a teacher must first have an understanding of what these children already know. The curriculum should then be compacted for them so that they can work on legitimate enrichment activities or move on to more complex material. Gifted/talented students should be pressured to explain their answers, make connections between ideas, and come up with generalizations from abstract ideas. All of these things require engagement with the teacher. In addition, the teacher needs to ensure these students have a positive, supportive environment where they are free to share their ideas. Gifted students can be perfectionists, so it is also important that they learn it is okay to make mistakes.

**Expectations of the Study**

I expect to find that the teachers in heterogeneous Algebra I classes spend the majority of their time and attention focused on the struggling students. They may differentiate to help these students, but I do not anticipate seeing differentiation specifically aimed at the gifted/talented students in the class. Instead, I expect to see gifted students covering the same material at the same pace as the rest of the class. When they are done with their classwork, I anticipate these students being called upon to serve as peer tutors. While I may see teachers give high-ability students who have finished their work additional problems of a similar nature to solve, I would be very surprised to see them modify the level of complexity of either their classwork or homework or to provide enrichment
activities. Furthermore, I do not expect to see much in the way of pre-assessment to find out what the gifted/talented students already know.

Teachers may group students in different ways during the observed classes, and I expect that the teacher will either allow the students to choose their own groups or to group the students by ability level. While allowing gifted students to work together is beneficial, the teacher also needs to provide them material that challenges them, which I do not expect to see. The teacher may ask students how they got their answers; however, I anticipate seeing this as a whole class, rather than specifically pressing gifted students to explain their thought processes. Likewise, I do not anticipate teachers asking students to solve problems in different ways or to use many real-life problems to show how algebra is used outside the classroom.

Overall, I do not expect the teachers to be very familiar with issues of giftedness. In fact, I anticipate teachers will initially be surprised when I ask them about scaffolding for gifted students, because I think many teachers assume that such students just “get it.” I expect them to feel pressure to help the struggling students perform, but to not have the time or energy to focus as much on the gifted/talented students as they might. In short, I anticipate that they will be very similar in their attitudes and behaviors as I was as a classroom teacher.

Willingness to Discover

I am willing to discover whatever my observations reveal or the teachers tell me during their interviews, regardless of whether my expectations are correct or incorrect. I may discover that the teachers have totally different values and beliefs toward gifted education than I have. I am willing to find out that they believe struggling students in
heterogeneous classrooms are more worthy of their time and attention than are
gifted/talented students. I am also willing to discover that the teachers are very informed
or uninformed about gifted education. Throughout the course of this study, I may find
out that my observation techniques or tool are flawed or that my interview questions are
not as thorough as I thought and that I need to improve as an interviewer. I also
anticipate discovering that I have significantly more to learn about quantitative and
qualitative research. I cannot think of anything I am unwilling to discover.

Usefulness of the Results

The results of this research should be useful in making teachers and administrators
more aware of the needs of gifted/talented students. While I am not approaching this
study from a critical perspective, I anticipate that when educators look at the type of
teacher behaviors I observed and the types of questions I asked in the interviews, they
will begin to reflect on whether their own practice is truly helping their own
gifted/talented students reach their potential. I also hope that it will inspire them to learn
more about giftedness and some of the issues that face gifted/talented students. Likewise,
this study may make teachers and administrators more aware of mathematical giftedness
and how it may manifest itself in a middle school Algebra I classroom.
Appendix F

Sample Entry from Reflexive Journal

November 8, 2010: Today I conducted my third observation of Kelly and her students. She covered literal equations. This was the final observation of my study and was definitely the worst behaved class I have observed. In fact, they are really the only poorly behaved class I have seen. My experience with teaching different levels of math suggests that the more advanced classes tend to behave a little better; however, since this class is made up solely of eighth grade students, it may not be as advanced as all of the other classes I have observed which have either been seventh grade only or a mix of seventh and eighth grade. Today, the students were even more talkative than during my first two observations of them. Although Kelly projects a maturity and confidence in working with the subject matter, it becomes obvious that she is a new teacher when watching her classroom management skills. Unfortunately, this detracts from her teaching of the material as she constantly has to tell the students to quiet down. Kelly was very patient with the class – almost to a fault. It seemed to me that if she would have portrayed that she was more serious about the talking, the noise level might have gone down.

As in my other two observations, the gifted student sat in the very back corner of the room and did not pay much attention. He seemed rather oblivious to the lesson. Kelly has previously mentioned to me that she was surprised when she found out he had been identified as gifted since he is not one of her top students. He was not talkative, but neither did he participate in the lesson. Kelly never called on him to answer a question. I will have to ask Kelly when I interview her about how she found out the student was
gifted and whether she thinks any of her other students are gifted, but have not been identified.

I was hoping Kelly would vary her pace when covering the material in the lesson, but she kept the whole class together which seems to happen a lot in Algebra. Rather than letting the students who understood the material proceed, she slowed the entire class down to help the struggling students. I was rather surprised when she gave them their homework and told them to not even attempt the last problem because they had not specifically covered it yet. I suspect that in most of the other classes I have observed, the teacher would have told them to at least try it. In fact, this was in stark contrast to the same literal equation lesson I observed Hillary teaching last week. Hillary challenged the students to try the hardest problem, which actually seemed to motivate them. In fact, Hillary taught the entire lesson at a much higher level than did Kelly. She moved through the material much quicker and her students seemed much more engaged. I suspect this may be because of the grade level (Hillary had a mixed seventh/eighth grade class) as well as teaching experience. In fact, in the other two observations of Kelly’s class, I also noticed a distinct difference in the level of challenge presented to the students compared to the other classes I have observed. I will have to bring that up in the interview. Frankly, it did not appear that the gifted student desired much more of a challenge since he raised his hand with the “no idea” group when Kelly asked whether an answer to a review problem was correct.

On the positive side, Kelly provided good scaffolding for the class. She also related literal equations to the formulas for perimeter and area so the students could see a real-world application for the concept. She modeled how to show all the steps of the problem
and explained as she went along. She also related the problem at hand to other problems they had done in the past. Despite this, it almost seemed that she was teaching to the lower end of the spectrum. I will talk to her about this in her interview since all of the other teachers have indicated that rather than lowering the level of challenge of the course to accommodate the struggling students, they have expected the struggling students to make the extra effort to get help so that they can rise up to the level of the rest of the class.
Appendix G

The William and Mary Classroom Observation Scale-Revised

Observable Evidence of Classroom Behaviors - Mathematics

The following examples serve as an indicator of potential classroom practices that might be observed. The examples are not inclusive but included only to help clarify the listed behavior as it pertains to mathematics.

<table>
<thead>
<tr>
<th>Teacher Behavior</th>
<th>Observable Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CURRICULUM PLANNING AND DELIVERY</strong></td>
<td>The teacher…</td>
</tr>
<tr>
<td>1. Set high expectations for student performance</td>
<td>✓ Analyzed concrete examples of appropriate and/or inappropriate solutions to mathematical problems</td>
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<tr>
<td></td>
<td>✓ Provided appropriate/advanced level of challenge in lecture/materials/classroom exercise/worksheets/homework extension</td>
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<td></td>
<td>✓ Emphasize both fluency and depth of understanding of concepts</td>
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<tr>
<td>2. Incorporated activities for students to apply new knowledge</td>
<td>✓ Allowed time for students to practice a skill or concept (problem-solving exercises in class, discussion and finding solutions to real-world problems with assistance of mathematical knowledge)</td>
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<td></td>
<td>✓ Structured an application activity to illustrate a math concept, proof and logic of a theorem being studied</td>
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<tr>
<td></td>
<td>✓ Built new mathematical knowledge through problem-solving (simulated and real world)</td>
</tr>
<tr>
<td>3. Engaged students in planning, monitoring, or assessing their learning</td>
<td>✓ Encouraged and facilitated students to discuss and reflect on the reasoning and methods of proof they employed in solving a math problem (through whole class instruction, journal writing, small group discussion)</td>
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<tr>
<td></td>
<td>✓ Encouraged students to analyze wrong proofs or arguments and reflected upon reasons led to inappropriate solutions</td>
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<tr>
<td></td>
<td>✓ Required students to complete a self-evaluation form prior to submitting projects</td>
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</tbody>
</table>
| 4. Encouraged students to express their thoughts | ✓ Solicited input from multiple students  
✓ Allowed wait time for students to be able to express why they did or did not reach a solution/proof  
✓ Asked follow-up questions to probe student reasoning and methods of thinking in reaching a solution |
| 5. Had students reflect on what they had learned | ✓ Required journal writing or think/pair/share to discuss new information  
✓ Asked higher-level questions that help students make connections to previous learning and consider new learning  
✓ Encouraged pattern recognition in learning mathematics  
✓ Encouraged students to connect real-world problem and real-life context to mathematical knowledge/skills/concepts |
| **ACCOMMODATIONS FOR INDIVIDUAL DIFFERENCES** | **The teacher...** |
| 6. Provided opportunities for independent or group learning to promote depth in understanding content | ✓ Allowed time for a variety of options that allowed students to pursue personal study  
✓ Assigned group work that deepened understanding of a concept/skill/proof/theorem  
✓ Encouraged collaboration in solving a problem (simulated or real-world problem) by assigning group project |
| 7. Accommodated individual or subgroup differences | ✓ Provided choices in assigning problems/assignments of different levels of difficulty  
✓ Asked challenging questions to accommodate individual or subgroup differences  
✓ Adjusted pacing for varied students  
✓ Grouped according to interest or ability |
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</table>
| **8.** Encouraged multiple interpretations of events and situations, and/or multiple ways of thinking and solutions to a problem | ✓ Solicited varied solutions and proofs to a problem  
✓ Asked students to work in small groups to discuss their personal solutions to a problem and the reasoning behind of it  
✓ Encouraged students to make conjectures and use a number of ways of representation of their solutions/proofs |
| **9.** Allowed students to discover key ideas individually through structured activities and/or questions | ✓ Encouraged and nurtured a number of ways of thinking (deductive thinking, inductive thinking, analytical thinking, and synthesis)  
✓ Encouraged pattern recognition in class lecture and assignments  
✓ Used questions to solicit responses instead of giving an answer away |

**PROBLEM-SOLVING STRATEGIES**

<table>
<thead>
<tr>
<th></th>
<th>The teacher...</th>
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</table>
| **10.** Employed brainstorming techniques | ✓ Solicited a variety of solutions to a mathematical problem from each individual student  
✓ Asked students to work in groups to come up with as many solutions/proofs as possible on a problem for a specified length of time |
| **11.** Engaged students in problem identification and definition | ✓ Asked students to come up with problems of their own  
✓ Encouraged students to identify and define real-world problems  
✓ Asked questions such as “What is the problem? Where does this problem come from? What is the impact of this problem? What is the mathematical form of the problem?” |
| **12.** Engaged students in solution-finding activities and comprehensive solution articulation | ✓ Facilitated students to develop the problem-solving skills by asking questions such as “What do you already know? What do you need to solve the problem? How can you satisfy these conditions to reach the solution?”  
✓ Encouraged students to express their solutions by sharing with the class in small groups  
✓ Asked questions such as “What if I did it another way?” |
<table>
<thead>
<tr>
<th>CRITICAL THINKING STRATEGIES</th>
<th>The teacher...</th>
</tr>
</thead>
</table>
| 13. Encouraged students to judge or evaluate situations, problems, or issues | ✓ Asked students to think of alternative and/or better solutions to a problem  
✓ Asked students to assess and analyze their own solutions/proofs and synthesize methods of solution and reasoning process  
✓ Asked boundary/condition questions about the proof/theorem such as “Under what condition will this proof hold up and under what conditions it will not?” |
| 14. Engaged students in comparing and contrasting ideas/solutions/methods | ✓ Asked students to analyze and compare different solutions/solutions/proofs to a problem and the rationale behind each  
✓ Asked students to compare, connect, and contrast mathematical concepts learned previously and that of the current learning to see the connections |
| 15. Provided opportunities for students to generalize from concrete data | ✓ Encouraged pattern recognition in different stage of learning of mathematics  
✓ Asked questions such as “What patterns do you see in this data? What generalization can you make from this pattern? What proof do you have?” |
| 16. Encouraged student synthesis or summary of information within or across disciplines | ✓ Asked students questions such as “What is the reasoning behind each of these solutions?”  
✓ Asked questions such as “What is the algebra representation in geometry?” or “Where can we find these mathematical solutions/theorems in the real world?”  
✓ Asked students to make connections of mathematical problems/concepts to other subject areas (economics, physics, chemistry, etc.) |
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<tr>
<th>CREATIVE THINKING STRATEGIES</th>
<th>The teacher...</th>
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</thead>
</table>
| 17. Solicited many diverse thoughts about issues or ideas | ✓ Asked questions such as “Did anyone have a different idea?” or “How else would we think about this question?”  
✓ Encouraged students to provide varied solutions/proofs/questions/rationale/process  
✓ Valued multiple solutions to a single problem |
| 18. Engaged students in the exploration of diverse points of view to reframe ideas | ✓ Selected and used various types of reasoning and methods of proofs  
✓ Developed and evaluated mathematical arguments in groups or whole class discussion  
✓ Solved mathematical problems arising in many different contexts |
| 19. Encouraged students to demonstrate open-mindedness and tolerance of imaginative, sometimes playful solutions to problems | ✓ Made positive comments when given an unusual solution/proof  
✓ Was open to students’ non-routine method of solution (even though they might be wrong); listened carefully the reasoning behind it and provided positive feedback and follow-up questions where appropriate  
✓ Allowed students to present ideas in multiple modes |
| 20. Provided opportunities for students to develop and elaborate on their ideas | ✓ Allowed time for students to write extended responses to present their solutions/proofs  
✓ Asked “why,” “how,” and “what if” questions to help students elaborate their thinking  
✓ Provided opportunities to write, reflect, analyze and synthesize their reasoning process in solving mathematical problems |

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<tr>
<th>RESEARCH STRATEGIES *</th>
<th>The teacher...</th>
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</table>
| 21. Required students to gather evidence from multiple sources through research-based techniques | ✓ Asked students to read multiple sources (print, non-print) on a specific issue  
✓ Asked students to come up with questions for research, create surveys or interview questions, and gather empirical evidence |
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| 22. Provided opportunities for students to analyze data and represent it in appropriate charts, graphs, or tables | ✓ Asked students to create a meaningful way to represent findings from research  
✓ Provided lessons in graphing results, chart construction, etc. |   |
| 23. Asked questions to assist students in making inferences from data and drawing conclusions | ✓ Required answers to questions such as “What are your findings . . .?”  
✓ Asked students to write up conclusions based on a dataset. |   |
| 24. Encouraged students to determine implications and consequences of findings | ✓ Required answers to questions such as “How will your findings affect . . .?” or “What are the consequences of . . .?”  
✓ Asked students to determine short and long term effects of a character’s action |   |
| 25. Provided time for students to communicate research study findings to relevant audiences in a formal report and/or presentation | ✓ Provided time for students to give a PowerPoint (or other formal) presentation on findings of gathered evidence  
✓ Required a written research report of findings |   |

* This cluster of behaviors may not apply to mathematics classrooms.

Appendix H

The William and Mary Classroom Observation Scale-Revised (Modified)

Observable Evidence of Classroom Behaviors - Mathematics

*The following examples serve as an indicator of potential classroom practices that might be observed. The examples are not inclusive but included only to help clarify the listed behavior as it pertains to mathematics.*

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<td><strong>The teacher…</strong></td>
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| 1. Set high expectations for student performance | ✓ Analyzed concrete examples of appropriate and/or inappropriate solutions to mathematical problems  
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   ✓ Emphasize both fluency and depth of understanding of concepts |
| 2. Incorporated activities for students to apply new knowledge | ✓ Allowed time for students to practice a skill or concept (problem-solving exercises in class, discussion and finding solutions to real-world problems with assistance of mathematical knowledge)  
   ✓ Structured an application activity to illustrate a math concept, proof and logic of a theorem being studied  
   ✓ Built new mathematical knowledge through problem-solving (simulated and real world) |
| 3. Engaged students in planning, monitoring, or assessing their learning | ✓ Encouraged and facilitated students to discuss and reflect on the reasoning and methods of proof they employed in solving a math problem (through whole class instruction, journal writing, small group discussion)  
   ✓ Encouraged students to analyze wrong proofs or arguments and reflected upon reasons led to inappropriate solutions  
   ✓ Required students to complete a self-evaluation form prior to submitting projects |
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✓ Allowed wait time for students to be able to express why they did or did not reach a solution/proof  
✓ Asked follow-up questions to probe student reasoning and methods of thinking in reaching a solution |
| **5. Had students reflect on what they had learned** | ✓ Required journal writing or think/pair/share to discuss new information  
✓ Asked higher-level questions that help students make connections to previous learning and consider new learning  
✓ Encouraged pattern recognition in learning mathematics  
✓ Encouraged students to connect real-world problem and real-life context to mathematical knowledge/skills/concepts |
| **ACCOMMODATIONS FOR INDIVIDUAL DIFFERENCES** | **The teacher...** |
| **6. Provided opportunities for independent or group learning to promote depth in understanding content** | ✓ Allowed time for a variety of options that allowed students to pursue personal study  
✓ Assigned group work that deepened understanding of a concept/skill-proof/theorem  
✓ Encouraged collaboration in solving a problem (simulated or real-world problem) by assigning group project |
| **7. Accommodated individual or subgroup differences** | ✓ Provided choices in assigning problems/assignments of different levels of difficulty  
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<td>✓ Facilitated students to develop the problem-solving skills by asking questions such as “What do you already know? What do you need to solve the problem? How can you satisfy these conditions to reach the solution?”</td>
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✓ Asked “why” “how” “what if” questions to help students elaborate their thinking  
✓ Provided opportunities to write, reflect, analyze and synthesize their reasoning process in solving mathematical problems |
<table>
<thead>
<tr>
<th>ADDITIONAL STRATEGIES</th>
<th>The teacher...</th>
</tr>
</thead>
</table>
| 17. Allowed the students to move through material at an individual pace | ✓ Determined students’ prior knowledge and adjusted the lesson accordingly  
✓ Provided an appropriate mix of learning and practice activities  
✓ Encouraged students to pursue more difficult material once mastery was achieved |
| 18. Allowed students sufficient time to thoroughly explore complex problems | ✓ Provided students with opportunities to persist in their investigations of challenging topics  
✓ Allowed students time to create and explain algorithms |
| 19. Provided a reasonable amount of attention (as appropriate to the situation) to the gifted/talented students in the class compared to other students | ✓ Provided scaffolding to gifted/talented students on difficult topics  
✓ Pressed gifted/talented students for explanation of their solution methods and meaning |

Adapted from VanTassel-Baska, J. et al., (2005a).
Appendix I

Panel of Expert’s Review of the Interview Guide

The following suggestions from the panel of experts were incorporated into the final version of the interview guide.

- Question 1 (*How do you decide the pace for the class?*): All members thought the question was clear and relevant so no changes were made.

- New question: One member thought it would be worthwhile to add a question near the beginning of the interview pertaining to pre-assessments. This made sense because it directly led into question 2. A new question (*How do you determine what your students already know?*) was added to the interview guide.

- Question 2 (*How do you handle a student who has already mastered the material you plan to cover during a lesson?*): One member thought the word “handle” was unclear. To clarify the wording, this question was rephrased as: *What modifications do you make for a student who has already mastered the material you plan to cover during a lesson?*

- Question 3 (*How do you balance the time spent in practice versus learning tasks for your gifted students?*): One member thought the wording was unclear because she thought some teachers might view practice as a learning task. To clarify the intent of the question, it was reworded to read: *How do you balance the time spent on practice of known concepts versus learning new concepts for your gifted students?*

- Question 4 (*What happens when a high-ability student wants to spend more time working on a problem?*): All members found this question to be clear and
relevant; however, one member was concerned that the way it was worded might lead a teacher to think that students who need more time are less ready for a challenge. To avoid this issue and to focus more on the intent of the question, it was reworded as: *When you raise the level of complexity for your advanced students, how do you deal with the additional time they may need to work on such problems?*

- **Question 5 (What strategies do you use to modify mathematical tasks to make solution-finding more challenging?):** Two members thought the phrase “solution-finding” was confusing. To clarify the question, it was reworded to read: *What strategies do you use to modify mathematical tasks to make them more challenging?* This question was then moved to so that it was asked immediately prior to the previous question.

- **Question 6 (How do you differentiate your instruction?):** All members thought the question was clear and relevant so no changes were made.

- **Question 7 (What factors play into how you group students during classroom activities?):** One panel member thought this question was not relevant to the concept of differentiation because a teacher might use factors such as behavior to group students. While this is a valid point, discovering that a teacher grouped only by behavior would still provide the researcher with valuable information, and so the question was not changed for this particular reason. However, another member suggested phrasing the question a little more formally, and so it was reworded as: *What criteria do you use to group students during classroom activities?*
• Question 8 (*What opportunities are high-ability students given to extend their mathematics learning outside the classroom?*): One panel member thought this question was unclear because she did not know whether the question referred to classroom-associated activities, such as homework, or those activities not necessarily associated with the classroom, such as a math club. The intent of the question was to focus on all enrichment activities whether they were directly or indirectly related to classroom assignments. For example, a student might extend his learning by working on a project with a mentor as part of a class assignment, or he might extend his learning by joining an after-school math club. While teachers may not necessarily control the types of activities in which gifted/talented students participate outside the classroom, by making these students aware of various math-related enrichment opportunities, they may be helping to address these students' needs. This question was clarified to read: *What enrichment opportunities are high-ability students given to extend their mathematics learning outside the classroom?*

• Question 9 (*What are some of the ways in which you provide a supportive learning environment for your high-ability students?*): All members thought the question was clear and relevant so no changes were made.

• Question 10 (*What do you do to scaffold instruction?*): One panel member thought this question was unclear because it did not specify for which group of students the question was aimed. This question was intentionally written without specifying a group of students to see whether the teachers mentioned scaffolding for gifted/talented students. However, because question 13 (*Which children in...*
your class do you believe are mathematically gifted?) was moved to the very front of the interview, the interviewees would have already known that the focus of the interview was on gifted/talented students, and thus this question became redundant with the next question. For that reason, this question was eliminated.

- Question 11 (How do you ensure high-ability students have an appropriate level of scaffolding?): One panel member thought the question was unclear because it did not specify a particular type of activity. This question was intentionally written to be broad so as to not lead the teacher in a certain direction and therefore, it was not reworded. Another member thought the question was more relevant to characterizing differentiation than to characterizing a supportive environment; however, since differentiation is a part of providing gifted/talented students with a supportive environment, the question remained unchanged.

- Question 12 (In what ways do you model high-level performance for your advanced students?): All members thought the question was clear and relevant so no changes were made.

- Question 13 (Which children in your class do you believe are mathematically gifted?): All panel members thought this question was clear and relevant; however, one member pointed out that some teachers might focus on naming specific children (whom the researcher would not know) rather than on focusing on the characteristics of these children. The question was therefore reworded as: In your opinion, what are indicators of mathematical giftedness? Every panel member suggested starting out the interview with this question to lend focus to
the interview. They also believed it might help teachers provide more examples specifically related to *mathematically* gifted students throughout the interview.
Appendix J

Interview Questions

1. In your opinion, what are indicators of mathematical giftedness?

2. How do you decide the pace for the class? [pace]

3. How do you determine what your students already know? [pace, challenge]

4. What modifications do you make for a student who has already mastered the material you plan to cover during a lesson? [pace, challenge]

5. How do you balance the time spent on practice of known concepts versus learning new concepts for your gifted students? [pace]

6. What strategies do you use to modify mathematical tasks to make them more challenging? [challenge]

7. When you raise the level of complexity for your advanced students, how do you deal with the additional time they may need to work on such problems? [pace]

8. How do you differentiate your instruction? [differentiation]

9. What criteria do you use to group students during classroom activities? [differentiation]

10. What enrichment opportunities are high-ability students given to extend their mathematics learning outside the classroom? [differentiation]

11. What are some of the ways in which you provide a supportive learning environment for your high-ability students? [supportive environment]

12. How do you ensure high-ability students have an appropriate level of scaffolding? [supportive environment]

13. In what ways do you model high-level performance for your advanced students? [supportive environment]

These questions may be revised based on the classroom observations.
Appendix K

Field Notes Form

<table>
<thead>
<tr>
<th>Activity</th>
<th># of Mins</th>
<th>Questions or Comments</th>
<th>Other Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Teacher</td>
<td>Student</td>
</tr>
</tbody>
</table>


Appendix L

Letter to Algebra I Teachers

August 30, 2010

Dear Algebra I teachers,

I am a graduate student at the College of William and Mary pursuing my Ph.D. in the Educational Policy, Planning and Leadership program. As part of my dissertation study, I am investigating how gifted/talented middle school students engage with the instruction in their heterogeneous Algebra I classes. This study will involve approximately four hours of classroom observations per teacher, where I will simply observe the class and take notes. I can observe any Algebra I class as long as the class contains at least one student who has been identified as gifted. Following each class, I will fill out an observation form which will assist me in comparing the interactions in the various classrooms. I will then conduct an approximately one-hour interview with each observed teacher to get their perceptions on teaching gifted students in heterogeneous Algebra I classes.

The study will begin at the end of September and I am looking for ten teachers who would be willing to participate. To maintain the confidentiality of the participants, I will assign pseudonyms to all individuals, schools, and divisions that are a part of this study. Should you decide to participate, you will receive a $10 Barnes and Nobles gift card as a token of my appreciation. If you would be willing to assist me in this study, please fill out the information at the bottom of this form and email it to me at: vctonneson@email.wm.edu.

Thank you for your consideration and I hope to talk with you soon!

Sincerely,

Ginny Tonneson

Name: ___________________________________________ School: ________________________________

Total years teaching experience: _______

Total years teaching Algebra I: _______

Number of gifted students out of the total students in your Algebra I class (Example: 7 gifted out of 28 students). If you teach more than one Algebra I class, please list each class separately. ________________________________________________

Students with disabilities (please explain): ________________________________
Appendix M

Consent for Participation Form

Teacher Instructional Practices Designed to Meet the Individual Learning Needs of Mathematically Gifted/Talented Students in Middle School Algebra I

I, ________________________________, agree to participate in a descriptive study involving middle school Algebra I teachers. The purpose of this study is to determine how gifted/talented students engage in Algebra I instruction in heterogeneous classes. My participation will help contribute to the limited body of knowledge related to this topic. The researcher is conducting this dissertation research to complete her doctoral studies in the Educational Policy, Planning and Leadership Program at the College of William and Mary.

I understand that the researcher has purposefully selected four middle schools in southeastern Virginia to be involved in this study. As a participant, I understand that I will allow the researcher to observe my Algebra I classes for approximately four hours. Depending on my teaching schedule, the observations may need to be broken into several sessions. I understand that the researcher will take field notes and will fill out an observation form at the completion of each observation; however, she will not interfere in the classroom instruction in any way. I also understand that the researcher will conduct an approximately one hour interview with me to discuss my perceptions pertaining to the gifted/talented students in my class and that she will tape record the session. The researcher may request a follow-up interview if necessary.

The researcher will maintain my confidentiality by assigning fictitious names to all individuals, schools, and divisions and these entities will only be referred to by these fictitious names in all published material. I will receive a copy of the results of the study via email upon request.

I understand that there may be minor or minimal psychological discomfort directly involved with this research. Further, I understand that I do not have to answer every question asked of me, and I am free to withdraw my consent and discontinue participation in this study at any time by informing the researcher by telephone (757-218-2154) or email (vctonneson@email.wm.edu). My decision to participate or not participate will not affect my relationships with my school or school division in general. If I have any questions or problems that arise in connection with my participation in this study, I should contact Dr. James Stronge, the study’s principal investigator, at 757-221-2339 or jhstro@wm.edu. If I have ethical concerns related to this study, I should contact Dr. Thomas Ward, the chair of the School of Education Internal Review Committee at the College of William and Mary, at 757-221-2358 or tjward@wm.edu.

My signature below signifies that I am at least 18 years of age, that I have received a copy of this consent form, and that I consent to participating in this descriptive study and the tasks outlined above or herein.
Signature of Participant

Investigator

THIS PROJECT WAS FOUND TO COMPLY WITH APPROPRIATE ETHICAL STANDARDS AND WAS EXEMPTED FROM THE NEED FOR FORMAL REVIEW BY THE COLLEGE OF WILLIAM AND MARY PROTECTION OF HUMAN SUBJECTS COMMITTEE (Phone: 757-221-3941) ON 2010-07-12 AND EXPIRES ON 2011-07-12.
## Appendix N

### Sample of Field Notes

**Field Notes Form**

<table>
<thead>
<tr>
<th>Activity</th>
<th># of Mins</th>
<th>Questions or Comments</th>
<th>Other Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Teacher</td>
<td>Student</td>
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</tbody>
</table>

*G1* gave answer
*G2* gave answer
*G3* said, "Excellent- that one wasn’t..."
Appendix O

Classroom Observation Scale-Revised (Modified) Teacher Observation Form

The William and Mary Classroom Observation Scales, Revised Teacher Observation

Directions: Please employ the following scale as you rate each of the checklist items. Rate each item according to how well the teacher characteristic or behavior was demonstrated during the observed instructional activity. Each item is judged on an individual, self-contained basis, regardless of its relationship to an overall set of behaviors relevant to the cluster heading.

<table>
<thead>
<tr>
<th>3=Effective</th>
<th>2=Somewhat Effective</th>
<th>1=Ineffective</th>
<th>N/O = Not Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher evidenced careful planning and classroom flexibility in implementation of the behavior, eliciting many appropriate student responses. The teacher was clear, and sustained focus on the purposes of learning.</td>
<td>The teacher evidenced some planning and/or classroom flexibility in implementation of the behavior, eliciting some appropriate student responses. The teacher was sometimes clear and focused on the purposes of learning.</td>
<td>The teacher evidenced little or no planning and/or classroom flexibility in implementation of the behavior, eliciting minimal appropriate student responses. The teacher was unclear and unfocused regarding the purpose of learning.</td>
<td>The listed behavior was not demonstrated during the time of the observation.</td>
</tr>
</tbody>
</table>

General Teaching Behaviors

Curriculum Planning and Delivery

<table>
<thead>
<tr>
<th>The teacher...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. set high expectations for student performance.</td>
</tr>
<tr>
<td>2. incorporated activities for students to apply new knowledge.</td>
</tr>
<tr>
<td>3. engaged students in planning, monitoring or assessing their learning.</td>
</tr>
<tr>
<td>4. encouraged students to express their thoughts.</td>
</tr>
<tr>
<td>5. had students reflect on what they had learned.</td>
</tr>
</tbody>
</table>

Comments:

Differentiated Teaching Behaviors

Accommodations for Individual Differences

<table>
<thead>
<tr>
<th>The teacher...</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. provided opportunities for independent or group learning to promote depth in understanding content.</td>
</tr>
<tr>
<td>7. accommodated individual or subgroup differences (e.g., through individual conferencing, student or teacher choice in material selection and task assignments).</td>
</tr>
<tr>
<td>8. encouraged multiple interpretations of events and situations.</td>
</tr>
<tr>
<td>9. allowed students to discover key ideas individually through structured activities and/or questions.</td>
</tr>
</tbody>
</table>

Comments:
<table>
<thead>
<tr>
<th><strong>Problem Solving</strong></th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>N/O</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher…</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. engaged students in problem identification and definition.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>11. engaged students in solution-finding activities and comprehensive solution articulation.</td>
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<tr>
<td><strong>Comments:</strong></td>
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<td></td>
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</table>

<table>
<thead>
<tr>
<th><strong>Critical Thinking Strategies</strong></th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>N/O</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher…</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>12. encouraged students to judge or evaluate situations, problems, or issues.</td>
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<tr>
<td>13. engaged students in comparing and contrasting ideas (e.g., analyze generated ideas).</td>
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<tr>
<td>14. provided opportunities for students to generalize from concrete data or information to the abstract.</td>
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<tr>
<td><strong>Comments:</strong></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Creative Thinking Strategies</strong></th>
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<th>2</th>
<th>1</th>
<th>N/O</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher…</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15. solicited many diverse thoughts about issues or ideas.</td>
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<td></td>
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<tr>
<td>16. provided opportunities for students to develop and elaborate on their ideas.</td>
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</tr>
<tr>
<td><strong>Comments:</strong></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Additional Strategies</strong></th>
<th>3</th>
<th>2</th>
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<th>N/O</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher…</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>17. allowed the students to move through material at an individual pace</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18. allowed students sufficient time to thoroughly explore complex problems</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19. provided a reasonable amount of attention (as appropriate to the situation) to the gifted/talented students in the class compared to other students.</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Comments:</strong></td>
<td></td>
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</tbody>
</table>
Appendix P

Sample of Completed Teacher Observation Form

The William and Mary Classroom Observation Scales, Revised
Teacher Observation
Bruce Bracken, Ph.D. Dianne Drummond, M.Ed. Tamra Stambaugh, M.Ed. Chwee Quek, M. Ed.

Directions: Please employ the following scale as you rate each of the checklist items. Rate each item according to how well the teacher characteristic or behavior was demonstrated during the observed instructional activity. Each item is judged on an individual, self-contained basis, regardless of its relationship to an overall set of behaviors relevant to the cluster heading.

<table>
<thead>
<tr>
<th>3=Effective</th>
<th>2=Somewhat Effective</th>
<th>1=Ineffective</th>
<th>N/O = Not Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher evidenced careful planning and classroom flexibility in implementation of the behavior, eliciting many appropriate student responses. The teacher was clear, and sustained focus on the purposes of learning.</td>
<td>The teacher evidenced some planning and/or classroom flexibility in implementation of the behavior, eliciting some appropriate student responses. The teacher was sometimes clear and focused on the purposes of learning.</td>
<td>The teacher evidenced little or no planning and/or classroom flexibility in implementation of the behavior, eliciting minimal appropriate student responses. The teacher was unclear and unfocused regarding the purpose of learning.</td>
<td>The listed behavior was not demonstrated during the time of the observation. (NOTE: There must be an obvious attempt made for the certain behavior to be rated “ineffective” instead of “not observed”).</td>
</tr>
</tbody>
</table>

General Teaching Behaviors

Curriculum Planning and Delivery

<table>
<thead>
<tr>
<th>3</th>
<th>2</th>
<th>1</th>
<th>N/O</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher...</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. set high expectations for student performance.</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. incorporated activities for students to apply new knowledge.</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>3. engaged students in planning, monitoring or assessing their learning.</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>4. encouraged students to express their thoughts.</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>5. had students reflect on what they had learned.</td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Comments: Talked about expectations for behavior when working in groups, several instances of self-reflection.

Differentiated Teaching Behaviors

Accommodations for Individual Differences

<table>
<thead>
<tr>
<th>3</th>
<th>2</th>
<th>1</th>
<th>N/O</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher...</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. provided opportunities for independent or group learning to promote depth in understanding content.</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>7. accommodated individual or subgroup differences (e.g., through individual conferencing, student or teacher choice in material selection and task assignments).</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>8. encouraged multiple interpretations of events and situations.</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>9. allowed students to discover key ideas individually through structured activities and/or questions.</td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Comments: Group activities, individual conferencing, but not individual pacing, excellent questioning.
### Problem Solving

<table>
<thead>
<tr>
<th>The teacher...</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>N/O</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. engaged students in problem identification and definition.</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>11. engaged students in solution-finding activities and comprehensive solution articulation.</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

**Comments:** Discussed algebraic vocabulary, evaluated variable expressions, identified properties that allowed one to take certain steps.

### Critical Thinking Strategies

<table>
<thead>
<tr>
<th>The teacher...</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>N/O</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. encouraged students to judge or evaluate situations, problems, or issues.</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>13. engaged students in comparing and contrasting ideas (e.g., analyze generated ideas).</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>14. provided opportunities for students to generalize from concrete data or information to the abstract.</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
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</table>

**Comments:** “How could you relate a phrase in your English class to an expression in algebra?” Several questions along the lines of “How is this like...?” and “How is this different from...?”

### Creative Thinking Strategies

<table>
<thead>
<tr>
<th>The teacher...</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>N/O</th>
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<tbody>
<tr>
<td>15. solicited many diverse thoughts about issues or ideas.</td>
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<td>X</td>
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</tr>
<tr>
<td>16. provided opportunities for students to develop and elaborate on their ideas.</td>
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<td>X</td>
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</tbody>
</table>

**Comments:** Students reflected on rationale and different ways to solve problems.

### Additional Strategies

<table>
<thead>
<tr>
<th>The teacher...</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>N/O</th>
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</thead>
<tbody>
<tr>
<td>17. allowed the students to move through material at an individual pace</td>
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<td>X</td>
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</tr>
<tr>
<td>18. allowed students sufficient time to thoroughly explore complex problems</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>19. provided a reasonable amount of attention (as appropriate to the situation) to the gifted/talented students in the class compared to other students.</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

**Comments:** Tried to draw in gifted students, used gifted work as an example, redirected gifted student when he was done with problems and reading a book.
Appendix Q

Sample Interview Summary

Lila believes that children who are mathematically gifted are easily able to keep up with her instruction and are able to analyze and apply the material in ways beyond the normal classroom instruction. She feels that students can be gifted in different domains and even in different aspects of mathematics. For example, she has had students who were gifted in algebraic thinking, but not spatial reasoning, and vice versa. Because of the limited amount of time she has been teaching her gifted students this year, she is unsure whether all of the students who have been identified as “gifted” are actually mathematically gifted. On the other hand, she believes that there are probably three or four other students within her algebra classes who are mathematically gifted, but have not been officially identified as gifted. She bases this not only on their performance on assessments, but also on their focus, the ease with which they pick up new concepts, and the fact that they seem to want a full understanding of the material as evidenced through the type of questions they ask.

Lila indicated that the pace of the class is determined by the quarterly planning document. The three algebra teachers get together to look at the district’s curriculum framework and the amount of time they have to cover each topic. They make adjustments based on whether the students seem to understand the material. The three teachers try to cover the material at the same time. If there is an assembly or some other event that causes a class to fall behind, the students can catch up via before- or after-school tutoring, reading the notes and textbook on-line, or going to the classzone website to get extra help.
Lila frequently uses self-assessments – based on the students’ opinions or their performance on a particular task – to help her determine what her students already know. She also uses worksheets or other problems to check for prior knowledge. She oftentimes “previews” the lesson by having a question and answer session where the students throw different ideas around. She finds this to be extremely effective. In fact, she tells the students that listening to these other ideas is sometimes even more valuable than the notes or instructions because they are getting ideas and concepts clarified which helps them to connect the new ideas to their previous knowledge. She does a lot of questioning as she goes through the notes and when she discovers that some of the students already know the material, she will either just skim the notes or pick selective problems embedded within the notes. She adjusts the pace of the individual classes by spending more or less time on the notes and problems therein. In this way, she is able to provide scaffolding to the classes that need it and more independent practice to those classes that are comfortable with the material. She would like to try to adjust the pace more within the class. She indicated that the set of notes from each of her three algebra classes looks different in terms of which problems they did or what areas were emphasized. She characterizes her constant evaluation as “tap dancing,” indicating that she is constantly adjusting the lesson. Some classes and some students need her to tell them every step, while others are ready to move ahead. She constantly tries to question them to get a higher level of thinking. Sometimes she feels like the higher-level questions are the only benefit she gives the gifted students. She believes that even if the students do not understand the point she is trying to make, they start to pick up on the terminology and use the vocabulary of real mathematicians in their math conversations.
Currently, Lila’s class is alphabetically arranged into pods of four students each. As she gets to know the students better, she will adjust the groups in different ways. Although she does not generally put all the gifted students together because they tend to pull so much attention toward themselves as they “zoom” ahead, sometimes she will put two of them together to challenge each other as peers and build off each other. She prefers not to put one gifted student with three low students although she frequently puts gifted students with the weaker students as peer helpers. This tends to pull the other students up. She prefers to have a blend of student abilities in the groups. She likes for conversations to be generated in the groups and for the students to look out for each other. She has tried different grouping configurations – ability, choice, and interest – but she feels it is important to consider the students’ personalities and work ethic when she creates the groups. If the students’ personalities and work ethics do not match each other, the group is not going to be successful. She strives to have at least one person in each group who will be on task and have the ability to follow her and pick up the concepts easily, but her groups are constantly changing.

Lila has various activities she may do when students are grouped by ability. For example, she sometimes does a “four corners” activity based on how students did on an assessment. At the various locations there might be a folder that explains the problem-solving process step-by-step to provide scaffolding for the groups that are having difficulty or a folder that just has a sheet of challenge problems for those groups that understand the material. She likes to use the “rally coach” activity where one student coaches another student through a problem because it provides good reinforcement for the gifted (and non-gifted) students and models the type of helping interaction she wants
the students to have with each other. She also occasionally uses Inspiration for flowcharting to have the class work on problems.

Although she does not believe her differentiation for gifted students is a weak area, Lila feels it sometimes poses a dilemma. She is constantly looking at ways to make math tasks more challenging for her gifted students. She monitors their work and then tells them to move ahead in the notes and work on the problems which are more challenging. Because the class might not get to these problems during the class period, she may ask a gifted student to explain how they solved it to the group. Her learner plans try to incorporate ideas for all levels of small group instruction or differentiation strategies; however, the time factor is always an issue. She hesitates asking the gifted students to do extra problems and occasionally uses her gifted students – whether labeled or not – as peer teachers within the groups. She raises the level of complexity for her gifted students through her questioning and having them apply the concepts in a more advanced way. She had an extremely gifted student last year, and she provided him with special questions and problems, sometimes accelerating him into Algebra II subject material. She would ask him to explain what he thought would happen in particular problems and tried to get him to reflect on what the next step in the process would be, even beyond the topic they were covering. Lila realized that the student was sometimes bored and thus tried to challenge him and push him to the next level; however, she did not feel like she could direct as much attention to him as he deserved, but considering the time constraints, the number of students in the class, and the SOLs that needed to be covered, it was the best she could do.
Lila identified *MathCounts* as a program that provides an enrichment opportunity for gifted and other students. This program is a math competition which involves weekly practice of basic math skills, problem solving, and reasoning. The coaches – one of whom is a gifted student from a previous year – develop a tool box of skills the students need for the various math concepts. Lila pointed out that at the high school level they do math testing, but they have not introduced that at the middle school level yet. She encourages her gifted students to do the on-line SAT question of the day at home because she thinks it is important for them to have exposure to that type of question. Lila pointed out that she has classroom resources the gifted students can use, but that she hesitates mentioning too many things because they like to get ahead and sometimes go a little too much beyond the class, and she wants to make sure they are still staying with what is going on in the class. She encourages the students to get involved in internships in high school. She tries to expose them to what is available by reflecting on her experiences as a computer programmer and systems analyst, and all the different opportunities that they might have.

One way that Lila provides a supportive learning environment is by her questioning. In fact, she indicated that sometimes her conversations with her gifted students lose the rest of the class, but they enable the gifted student to get the full impact of the lesson. She also ensures that all her students know she is approachable and will provide them additional help – before or after school – regardless of whether it is for enrichment or not. She feels that after school is sometimes the best avenue for enrichment because she can bring in laptops, have students investigate specific websites, and expose them to higher-level and other types of problems.
When she increases the level of challenge for her gifted students, Lila does not give them something totally new that they have not had instruction on. She does not give them a new concept; rather she will take the concept that has been taught to them and make it more difficult by involving more distribution, changing basic expressions to rational expressions, and so forth. This is difficult, however, because the gifted students do not want to be viewed as different. She has to be careful how she approaches the issue of adding complexity. Sometimes after school help is the best avenue because it is more discrete. She pointed out that the highly gifted student last year did not like the other students knowing he was so advanced, and so she pointed out that the other students were actually complimenting and looking up to him.

Lila models high-level performance for her advanced students through her own organization and by making sure the students have a good system of organization that works for them (via their notebook). Since their future math classes are going to move at an increasingly rapid pace, she feels it is important that they are organized to help relieve some of the frustration they feel by wanting to achieve and be the best. She stresses the specific process they should use to show their solution method, neatness and checking of their work, and making sure they understand why they missed problems. She provides the students with personal hints to help with mathematical concepts and exposes them to advanced mathematical symbols, such as using set notations when solving functions and using three dots to mean therefore. She tries to push things that she definitely knows they are going to encounter in the future. Since her husband teaches math in high school, she is very aware of what they will see in future math courses.
Lila thinks her gifted Algebra I students’ needs are being met, but not to the level she would like to see. As the parent of gifted children, she understands that gifted students are oftentimes asked to do things on their own, rather than garnering the teacher’s attention. She is an advocate for enrichment. She feels like she challenges her gifted students and that she gives them everything they need to move on to the next level, but she would love to do much more in terms of differentiation and being able to take them to levels beyond what they are currently capable of. She is constantly challenged by time constraints, but every year she tries to focus on how she can enrich the gifted students and provide some additional instruction or support for them.

Although there is no official quota for the percentage of students who should be enrolled in Algebra I, as the math department chair, Lila has been in meetings where it has been suggested that they need to increase the number of students enrolled in Algebra I because they have the lowest number of students in Algebra I ratio-wise in the district. The algebra teachers at Lila’s school spend a lot of time looking at the criteria and determining the appropriate level of student placement. The teacher’s recommendation is generally the ruling factor. She believes they have been able to keep the level of student placed in the course relatively high. She feels they are adequately placing students at the right level as evidenced by the fact that relatively few students are put back into Algebra I when they get to high school. After the first unit, she normally sees the class begin to stratify, but most students remain above the D level. It is a challenge however. She has oftentimes wished that algebra was leveled within the middle school. For example, the gifted and high-ability algebra students would go to algebra classes on alternate blocks (either “A” or “B” days), whereas the lower students would have a double block,
meaning they would have algebra on both “A” and “B” days and give up an elective. That would allow the teachers to see the struggling students every day. In essence, there would be two levels of algebra to accomplish the same thing. She is unsure of whether they would be allowed to adopt this as the new curriculum at the school-level or whether it would have to be at the district-level.

Lila indicated that scheduling has caused the Algebra I classes to be a mix of seventh and eighth grade students. She does not like to know which grade students are in. She views the class as an algebra class, not a seventh grade or eighth grade algebra class. She suspects, however, that if the leveling issue was approved, they might go back to having separate seventh and eighth grade Algebra I classes because there tends to be a big difference between the students. One year she taught a special pre-algebra class for students who had been successful in pre-algebra, but were not quite ready for algebra. She found this “advanced pre-algebra” class to be very successful. In heterogeneous classes, she finds it difficult to balance the time spent with struggling students and tries not to spend an inordinate amount of time working with them in class. She tells them they need to come in before or after school for tutoring. She talks to the parents and tries to identify the reason why the student is struggling.

Lila believes her three Algebra I classes are very different. On some days the students seem very needy and she cannot accomplish half of what she intended. Sometimes she feels like she almost needs to re-teach the previous day’s concepts, which can be frustrating. She is constantly trying to balance teaching new concepts, while making sure the students really understand the previous concepts. She does, at times, feel guilty about whether she is giving to her gifted students as much as she should.
teacher’s viewpoint, she is doing the best she can, but from a mother’s viewpoint, she could probably do more with them, knowing they are capable of so much more. Some of her gifted students have built walls up and it takes several months of encouragement to really get them going. Because Lila also teaches Advanced Geometry, she is able to work with her gifted students for two or three years in a row. This allows her to know their abilities coming into the class and to better meet their needs.
Appendix R

Sample of Holistically Coded Observation

Lila Observation 2   24 Sep 7:57-9:26   Block 1B Grades 7/8   3 gifted/26 total

This was a mixed class of seventh and eighth grade students. Of the 26 students, three had been identified as gifted. Two of these students were in seventh grade (one boy and one girl) and one boy was in eighth grade. They each sat with a different pod of four students.

The class began with Lila explaining her expectations for the day. She reminded them of a website that had algebra vocabulary cards and exercises for extra practice. She explained that although they had finished their first unit, she expected them to keep their notebook to review prior to the district assessments and SOL test. Later on, she referred to a worksheet and commented that, “A good student may want to practice that.”

Lila told the students that much of the lesson should be a review for them with the exception of matrices, which she characterized by saying, “but that’s easy.” She then reviewed the homework and had the students do a self-evaluation. All of the gifted students indicated they did well and understood the material. Lila then checked for prior knowledge by having the students work on three problems individually. They then played the “spy” game where the students were allowed to circulate around the classroom and look at other students’ answers. Several students rushed to the gifted eighth grade student’s desk, suggesting that they recognized his ability in math. Lila then had a group present the answer and another group explain the “rule” they had derived from doing the problem. The three gifted students were the spokespersons for each of their groups. She then proceeded to go through the notes for the lesson, showing examples and letting the students work problems on their own. On several occasions, Lila had the students raise their hands if they successfully solved the problems. She then talked about matrices. She did not do a pre-assessment for this portion of the lesson. All of the gifted students took notes, but by the time they practiced the matrix problems, the gifted eighth grader had become disengaged. The final activity was a game called “rally coach” where the students worked with their “face partner.” One student coached the other student through problems and then they switched roles. At this point, all the gifted students seem engaged. Lila closed the lesson with another student self-evaluation. All of the gifted students raised five fingers, indicating they understood the material well.

PACE: Because of the success most students had on the pre-assessment, Lila moved quickly through her review of integers, only showing and having the students work through a few examples. She did not, however, allow the student who understood the other material to move ahead at a different pace. The class worked on the same material throughout the lesson.

CHALLENGE: When doing the three pre-assessment problems, one of the gifted seventh graders finished well before the others and then sat quietly at his desk. It was
apparent that he was easily able to solve the problems. In addition, as they were going through example problems, Lila stated, “We’re going to do these together because it worries me that you don’t know how.” It seemed as though this was a missed opportunity to challenge the students. As they were going through the notes, the gifted seventh graders added their own annotations to the note sheet, but the gifted eighth grader just sat there. He eventually started talking to the girls in his pod, requiring Lila to call his name and then say, “I’m waiting . . . focus.” This successfully redirected him toward the lesson. Later when she asked how many had gotten the practice problems correct, none of the gifted students raised their hands. Because they had previously indicated they were doing well on the problems, it seems as though they may have been tiring of doing problems that were not challenging for them. Once she moved to matrices, the two gifted seventh graders became engaged and took notes; however, the gifted eighth grader did not. He tapped on his desk and seemed rather bored. He became re-engaged, however, when the class played “rally coach.” Lila explained that she had selected the most challenging problems from the previous year for this game.

SUPPORTIVE ENVIRONMENT: Lila encouraged different methods of solving the problems, telling one gifted student, “Excellent, that’s one way.” Later in the lesson, the gifted eighth grade student asked whether he could “change adding a negative into subtracting a positive.” Lila told him that was fine, allowing him to solve the problem using a method he understood, but that most of the class did not. In fact, while one of the other gifted students understood what he meant, the third gifted student did not, so she asked what Lila meant. Later, when Lila was talking about changing a mixed number into an improper fraction, she asked, “Who would like to help me teach this?” The gifted eighth grader immediately volunteered. The gifted seventh grade boy then explained to the class how he used a different method to do the same thing. Lila was very complimentary to both students and explained that there were different ways to do some procedures in math and that the students should use the method with which they were most comfortable. In fact, later in the lesson, she referred back to the gifted eighth grader’s method saying that this is “where his rule comes in.”

DIFFERENTIATION: As in the previous lesson, Lila had the desks set up into groups of four with “face partners” and “shoulder partners.” She used “face partners” for the “rally coach” activity. These groups, however, were not based on readiness or any other criteria; they were simply based on the random seating assignment from the beginning of the year. All students did the same work in class and had the same homework, so there was no differentiation of content or product. In fact, the only differentiation observed was when the gifted students “self-differentiated” by using a different process by which to solve the problems.
Appendix S

Sample of Open Coding of Interview Segment

(abbreviated for illustrative purposes)

When you discover that some of the kids already know the material, how do you adjust your class?

I will just skim the notes or just pick selective problems embedded within the notes.

I’ll skip some, we’ll move on to others.

For instance, I felt that this class was a little bit stronger than yesterday’s class so we went to the “showdown” much faster than my previous class.

In fact, my other class needed more scaffolding and we didn’t get . . . I’m not even sure we got to the “showdown,” but this class was really ready for it so I wanted them to be working independently and then with my monitoring of how they were doing on those answers.

So do you generally keep it as a class or do you find that you need to make adjustments even within the class as far as kids that know and kids that don’t know?

I would like to try to adjust more within the class, and what I’ll do sometimes . . . this is the initial alphabetical order grouping to form my group, the little pods as I call them.

As I get to know them, and I will rearrange seats and sometimes I will purposefully put some of my gifted children together

Adjust pace: skim notes Adjust pace: do selective problems

Adjust pace: skip problems Adjust pace: by class ability

Adjust pace: when ready to work independently Adjust pace: teacher monitoring

Adjust pace: wants to do in class Group: alphabetical order

Group: as pods Group: adjusts when familiar

Group: gifted together
Appendix T

Codes and Definitions

Pace
(Speed at which course material is covered)

<table>
<thead>
<tr>
<th>Pace Driven from Outside the Classroom</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>District curriculum framework/quarterly assessments</strong></td>
</tr>
<tr>
<td>• Driven by district</td>
</tr>
<tr>
<td>• According to curriculum framework</td>
</tr>
<tr>
<td>• Driven by quarterly assessments</td>
</tr>
<tr>
<td>• Common planning by course teachers</td>
</tr>
<tr>
<td>• Fits within schedule</td>
</tr>
<tr>
<td>• Complete course by semester</td>
</tr>
<tr>
<td>• Pre-determined</td>
</tr>
<tr>
<td>• Follow textbook suggestions</td>
</tr>
<tr>
<td>• Laid out</td>
</tr>
<tr>
<td>• Little flexibility</td>
</tr>
<tr>
<td>• “Hands are tied”</td>
</tr>
<tr>
<td>• Look at timeframe</td>
</tr>
<tr>
<td>• Class moves too fast</td>
</tr>
<tr>
<td>• New standards = quicker pace</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Block Scheduling Impacts</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Plan to cover too much</td>
</tr>
<tr>
<td>• Difficult to slow down</td>
</tr>
<tr>
<td>• Must understand material since math builds on it</td>
</tr>
<tr>
<td>• Struggling students suffer because of pace</td>
</tr>
<tr>
<td>• Simply move on</td>
</tr>
<tr>
<td>• “No time to breathe”</td>
</tr>
<tr>
<td>• Pace for one class drives another</td>
</tr>
<tr>
<td>• Challenge to cover material</td>
</tr>
<tr>
<td>• Tough schedule adjustment for students</td>
</tr>
<tr>
<td>• Sets students up for failure</td>
</tr>
<tr>
<td>• Can’t handle multiple concepts</td>
</tr>
<tr>
<td>• Won’t cover foundational and building topics in same lesson</td>
</tr>
<tr>
<td>• Can’t keep concepts straight</td>
</tr>
<tr>
<td>• Moves too fast</td>
</tr>
<tr>
<td>• Time is critical factor</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Modifications to Pace</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Determine Prior Knowledge</strong></td>
</tr>
<tr>
<td>• Throw ideas around</td>
</tr>
<tr>
<td>• Give warm-up problems</td>
</tr>
<tr>
<td>• Informal assessment</td>
</tr>
<tr>
<td>• “Tell me what you know”</td>
</tr>
<tr>
<td>• Requires teacher monitoring</td>
</tr>
<tr>
<td>• Ask questions</td>
</tr>
<tr>
<td>• Check problem solutions</td>
</tr>
<tr>
<td>• Done via lesson preview</td>
</tr>
<tr>
<td>• Pre-assess prior to unit</td>
</tr>
<tr>
<td>• Self-evaluation</td>
</tr>
<tr>
<td>• Student opinion</td>
</tr>
<tr>
<td>• No formal pre-assessment</td>
</tr>
</tbody>
</table>
### Adjust for Class as a Whole

<table>
<thead>
<tr>
<th>Action</th>
<th>Reason</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skim notes</td>
<td>Extra help after class</td>
<td>Adjust if students lost</td>
</tr>
<tr>
<td>Do selective problems</td>
<td>Requires teacher “tap dancing”</td>
<td>Teachers know what’s in previous courses</td>
</tr>
<tr>
<td>Skip problems</td>
<td>Spend more or less time going over homework</td>
<td>Sometimes can’t do challenge problems</td>
</tr>
<tr>
<td>Post homework answers on-line</td>
<td>Ask student where to start instruction</td>
<td>Based on student understanding</td>
</tr>
<tr>
<td>Adjust within class</td>
<td>Hit main ideas and move on</td>
<td>Based on material needed to cover</td>
</tr>
<tr>
<td>Varies by class</td>
<td>Limit note-taking time</td>
<td>Students’ needs override lesson plans</td>
</tr>
<tr>
<td>Teachers adjust/stay together</td>
<td>Students from different courses = different understanding</td>
<td>“Hit the average”</td>
</tr>
<tr>
<td>Provide notes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difficult to balance practice vs. learning</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Individual Pace within Activities

<table>
<thead>
<tr>
<th>Action</th>
<th>Reason</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra material is new</td>
<td>Acceleration is real struggle</td>
<td>Individual pace on guided practice</td>
</tr>
<tr>
<td>No one has mastered</td>
<td>When students ready to work independently</td>
<td>Peer tutoring when done</td>
</tr>
<tr>
<td>Wants to do more within class</td>
<td>Individual adjustment to try challenge problems</td>
<td>Worksheet/homework when done</td>
</tr>
<tr>
<td>Individual pace at stations</td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
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</tbody>
</table>
### Challenge

*(Methods a teacher may use to provide more challenge to mathematically gifted students)*

**Increase Challenge as a Whole Class**

<table>
<thead>
<tr>
<th>Material is challenging</th>
<th>New standards are more complicated</th>
</tr>
</thead>
<tbody>
<tr>
<td>No need for more</td>
<td>No students have already mastered</td>
</tr>
<tr>
<td>challenge</td>
<td>material</td>
</tr>
<tr>
<td>Difficult to challenge</td>
<td>Students may not know</td>
</tr>
<tr>
<td>gifted</td>
<td>all of concept's pitfalls</td>
</tr>
<tr>
<td>Student readiness varies</td>
<td>Provide challenge</td>
</tr>
<tr>
<td>by day</td>
<td>problems to do at home</td>
</tr>
<tr>
<td>Must constantly adjust</td>
<td>Gifted energized by</td>
</tr>
<tr>
<td>level of challenge</td>
<td>course</td>
</tr>
</tbody>
</table>

**Increased Complexity/Abstraction**

#### Complexity within Problems

| No calculators          | Multistep problems               |
| Use negatives           | Increase distribution            |
| Use larger numbers     | Increase rational numbers        |
| Multiple decimal places | Solve for unknowns              |
| Reversal of inequality  | Use things that may              |
| sign                   | "trip them up"                   |
| Use word problems      | Incorporate order of operations  |
| Make longer problems   | Increase number of fractions     |
|                        | Use fractions with different    |
|                        | denominators                    |

#### Complexity with Process

| Use flowcharts          | Reflect on process              |
| Solve it in a different | Find mistakes in example        |
| way                    | Give advanced course material   |
| Compare solution       | Write explanations              |
| methods                | Reflect on what next step might |
| Explain own solution   | be                       |
| method                 |                          |
| Find what would make   | Students create problems        |
| problem false          | Use cooperative learning        |
|                        | Discuss needed problem-solving  |
|                        | information                     |
|                        | Turn number problems into word  |
|                        | problems                       |
|                        | Determine solution process      |

#### Complexity with Concepts

| Explain concept to group | Tie two math concepts together |
| Generalizations          | Tie math concepts with other   |
| Pattern finding          | content areas                  |
| Come up with rules       | Analyze situations             |
| Explain why rules work   | Explore future concepts        |
| Compare and contrast     | Solve real-world problems      |
| concepts                | Use high-level questions       |

- Apply knowledge to advanced area within topic
- Use open-ended questions
- Connect to real world
- Put concepts into words
Differentiation
(Ways in which the teacher modifies content, process, or product for gifted students)

<table>
<thead>
<tr>
<th>Limited Differentiation Practices</th>
<th>Curriculum Compacting</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Enrichment causes compacting</td>
<td>• Students cannot afford to miss instructional time</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Enrichment</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Wants to improve</td>
</tr>
<tr>
<td>• Wants to do more</td>
</tr>
<tr>
<td>• “Algebra is what it is”</td>
</tr>
<tr>
<td>• Nothing specific for gifted</td>
</tr>
<tr>
<td>• On-line opportunities</td>
</tr>
<tr>
<td>• New SOLs = more time</td>
</tr>
<tr>
<td>• Gifted could get too far ahead</td>
</tr>
<tr>
<td>• Pace limits opportunities</td>
</tr>
<tr>
<td>• Not needed</td>
</tr>
<tr>
<td>• After school better venue</td>
</tr>
<tr>
<td>• Keep within topical areas</td>
</tr>
<tr>
<td>• Built into IB program</td>
</tr>
<tr>
<td>• Time constraints biggest downfall</td>
</tr>
<tr>
<td>• Challenge and rigor in curriculum</td>
</tr>
<tr>
<td>• Material allows self-differentiation</td>
</tr>
<tr>
<td>• Curriculum drives students to cover same material</td>
</tr>
<tr>
<td>• Follow same routine</td>
</tr>
<tr>
<td>• Not aware of outside opportunities</td>
</tr>
<tr>
<td>• Students already have enough disruption</td>
</tr>
<tr>
<td>• Students can seek outside opportunities</td>
</tr>
<tr>
<td>• MathCounts</td>
</tr>
<tr>
<td>• Odyssey of the Mind</td>
</tr>
<tr>
<td>• SAT question of day</td>
</tr>
<tr>
<td>• Math test program</td>
</tr>
<tr>
<td>• Classroom resources</td>
</tr>
<tr>
<td>• After school exploration of websites</td>
</tr>
<tr>
<td>• NASA mentorships</td>
</tr>
<tr>
<td>• University math competition</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tiered assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Basic level</td>
</tr>
<tr>
<td>• Real-world problem</td>
</tr>
<tr>
<td>• Original problem</td>
</tr>
<tr>
<td>• Wants to do more</td>
</tr>
<tr>
<td>• No time</td>
</tr>
<tr>
<td>• Teachers plan, but can’t implement</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flexible grouping</th>
</tr>
</thead>
<tbody>
<tr>
<td>• By ability level</td>
</tr>
<tr>
<td>• By interest</td>
</tr>
<tr>
<td>• By choice</td>
</tr>
<tr>
<td>• By on-task behavior</td>
</tr>
<tr>
<td>• By assessment results</td>
</tr>
<tr>
<td>• By level of engagement</td>
</tr>
<tr>
<td>• Alphabetical order</td>
</tr>
<tr>
<td>• No choice</td>
</tr>
<tr>
<td>• None</td>
</tr>
<tr>
<td>• As pods/natural quartet</td>
</tr>
<tr>
<td>• Random</td>
</tr>
<tr>
<td>• Gifted together</td>
</tr>
<tr>
<td>• Gifted with weaker students</td>
</tr>
<tr>
<td>• Person sitting next to them</td>
</tr>
<tr>
<td>• Study buddy</td>
</tr>
<tr>
<td>• Shoulder partner</td>
</tr>
<tr>
<td>• Face partner</td>
</tr>
<tr>
<td>• Pair highest with lowest</td>
</tr>
<tr>
<td>• Groups change</td>
</tr>
<tr>
<td>• Gifted together pull attention</td>
</tr>
<tr>
<td>• Personality and work ethic important</td>
</tr>
<tr>
<td>• Rallies students</td>
</tr>
<tr>
<td>• Peer coaching is good reinforcement</td>
</tr>
<tr>
<td>• Gifted together gives peer challenge</td>
</tr>
<tr>
<td>• Gifted pull others up</td>
</tr>
<tr>
<td>• Friends stay on task</td>
</tr>
<tr>
<td>• To achieve a blend of abilities</td>
</tr>
<tr>
<td>• To generate conversation</td>
</tr>
<tr>
<td>• To look after each other</td>
</tr>
<tr>
<td>• To share answers</td>
</tr>
</tbody>
</table>
Supportive Environment
(Describes the type of environment in which gifted students learn)

<table>
<thead>
<tr>
<th>Conducive Learning Atmosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Compliment students</td>
</tr>
<tr>
<td>• Encourage gifted</td>
</tr>
<tr>
<td>• Provide tools to move to higher level</td>
</tr>
<tr>
<td>• Question gifted students’ understanding</td>
</tr>
<tr>
<td>• Challenge students</td>
</tr>
<tr>
<td>• Push level of course up</td>
</tr>
<tr>
<td>• Encourage resiliency</td>
</tr>
<tr>
<td>• Encourage effort and outcome</td>
</tr>
<tr>
<td>• Encourage involvement</td>
</tr>
<tr>
<td>• Use effective questioning</td>
</tr>
<tr>
<td>• Be approachable</td>
</tr>
<tr>
<td>• Encourage student understanding</td>
</tr>
<tr>
<td>• Discuss issues with students and parents</td>
</tr>
<tr>
<td>• Respond to gifted students’ personalities</td>
</tr>
<tr>
<td>• Encourage gifted girls</td>
</tr>
<tr>
<td>• Differentiate discretely</td>
</tr>
<tr>
<td>• Eliminate embarrassment</td>
</tr>
<tr>
<td>• Create positive atmosphere</td>
</tr>
<tr>
<td>• Be motivational</td>
</tr>
<tr>
<td>• Mistakes are learning opportunities</td>
</tr>
<tr>
<td>• Let them know they can do it</td>
</tr>
<tr>
<td>• Individual conferences</td>
</tr>
<tr>
<td>• Let gifted be themselves</td>
</tr>
<tr>
<td>• Expose to different opportunities</td>
</tr>
<tr>
<td>• Reflect on real-world experiences</td>
</tr>
<tr>
<td>• Help students become knowledgeable</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scaffolding</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Connect to prior knowledge</td>
</tr>
<tr>
<td>• Build from basic to challenging</td>
</tr>
<tr>
<td>• Scaffolding is difficult for gifted</td>
</tr>
<tr>
<td>• Challenge within known concepts</td>
</tr>
<tr>
<td>• Discuss anticipated questions</td>
</tr>
<tr>
<td>• Keep within topical area</td>
</tr>
<tr>
<td>• Provide outside resources</td>
</tr>
<tr>
<td>• Provide additional help</td>
</tr>
<tr>
<td>• Show areas of typical difficulty</td>
</tr>
<tr>
<td>• Tie old and new concepts together</td>
</tr>
<tr>
<td>• Provide on-line resources</td>
</tr>
<tr>
<td>• Point out trick words</td>
</tr>
<tr>
<td>• Point out common mistakes</td>
</tr>
<tr>
<td>• Tips for homework and quizzes</td>
</tr>
<tr>
<td>• Students scaffold for each other</td>
</tr>
<tr>
<td>• Coach’s tool box for MathCounts</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Express expectations</td>
</tr>
<tr>
<td>• Notebook organization</td>
</tr>
<tr>
<td>• Show steps</td>
</tr>
<tr>
<td>• Think independently</td>
</tr>
<tr>
<td>• Homework</td>
</tr>
<tr>
<td>• Study each day</td>
</tr>
<tr>
<td>• Practice on off-math days</td>
</tr>
<tr>
<td>• Don’t do challenge problem</td>
</tr>
<tr>
<td>• Memorize formulas</td>
</tr>
<tr>
<td>• Get help if needed</td>
</tr>
<tr>
<td>• Group interactions</td>
</tr>
<tr>
<td>Models High-Level Performance</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>• Model steps of problems</td>
</tr>
<tr>
<td>• Model organization</td>
</tr>
<tr>
<td>• Model mathematical</td>
</tr>
<tr>
<td>process</td>
</tr>
<tr>
<td>• Model concepts for</td>
</tr>
<tr>
<td>future classes</td>
</tr>
<tr>
<td>• Model neatness</td>
</tr>
<tr>
<td>• Model checking work</td>
</tr>
<tr>
<td>• Model desired</td>
</tr>
<tr>
<td>interactions</td>
</tr>
<tr>
<td>• Show all steps</td>
</tr>
<tr>
<td>• Model helping interactions</td>
</tr>
<tr>
<td>• Expose to SAT question</td>
</tr>
<tr>
<td>• Expose to advanced math</td>
</tr>
<tr>
<td>• Model knowledge of</td>
</tr>
<tr>
<td>content</td>
</tr>
<tr>
<td>• Use math rules</td>
</tr>
<tr>
<td>• Include “self” in class</td>
</tr>
<tr>
<td>• Explain rationale for</td>
</tr>
<tr>
<td>steps</td>
</tr>
<tr>
<td>• Discuss study techniques</td>
</tr>
<tr>
<td>• Keep course level high</td>
</tr>
<tr>
<td>• Provide personal hints</td>
</tr>
<tr>
<td>• Use math symbolism</td>
</tr>
<tr>
<td>• Use math terminology</td>
</tr>
<tr>
<td>• Show quicker way to solve</td>
</tr>
<tr>
<td>• Encourage different</td>
</tr>
<tr>
<td>solution methods</td>
</tr>
<tr>
<td>• Encourage reflection</td>
</tr>
<tr>
<td>• Model highlighting</td>
</tr>
<tr>
<td>notes</td>
</tr>
<tr>
<td>• Demonstrate</td>
</tr>
<tr>
<td>organization</td>
</tr>
<tr>
<td>• Label answers</td>
</tr>
</tbody>
</table>
Mathematical Giftedness

(Attributes that may identify a student as mathematically gifted)

<table>
<thead>
<tr>
<th>Practitioners’ View of Mathematical Giftedness</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gifted Characteristics</strong></td>
</tr>
<tr>
<td>• Good work habits</td>
</tr>
<tr>
<td>• Strong work ethic</td>
</tr>
<tr>
<td>• Higher-level thinker</td>
</tr>
<tr>
<td>• Abstract thinker</td>
</tr>
<tr>
<td>• Easily understands concepts</td>
</tr>
<tr>
<td>• New concepts “stick”</td>
</tr>
<tr>
<td>• Can see relationships</td>
</tr>
<tr>
<td>• More responsible/mature</td>
</tr>
<tr>
<td>• Natural math talent</td>
</tr>
<tr>
<td>• Focused</td>
</tr>
<tr>
<td>• Patient</td>
</tr>
<tr>
<td>• Independent worker</td>
</tr>
<tr>
<td>• Motivated to learn</td>
</tr>
<tr>
<td>• Initiative/“go getter”</td>
</tr>
<tr>
<td>• Wants to be the best</td>
</tr>
<tr>
<td>• Does not struggle</td>
</tr>
<tr>
<td>• Problem-solver</td>
</tr>
<tr>
<td>• “Zooms through material”</td>
</tr>
<tr>
<td>• Want full understanding</td>
</tr>
<tr>
<td>• Understands extensions</td>
</tr>
<tr>
<td>• Responds to instruction</td>
</tr>
<tr>
<td>• Follows directions</td>
</tr>
<tr>
<td>• Participates in class</td>
</tr>
<tr>
<td>• Comes up with answers</td>
</tr>
<tr>
<td>• Performs well on assessments</td>
</tr>
<tr>
<td>• Gets good grades</td>
</tr>
<tr>
<td>• Completes homework</td>
</tr>
<tr>
<td>• Knows how to study</td>
</tr>
<tr>
<td>• Frequently asks questions</td>
</tr>
<tr>
<td>• Asks the right questions</td>
</tr>
<tr>
<td>• Can handle any work</td>
</tr>
<tr>
<td>• Other factors besides intelligence</td>
</tr>
<tr>
<td>• Doesn’t want to be different</td>
</tr>
<tr>
<td>• Same as high-ability student</td>
</tr>
<tr>
<td>• “Know it all”</td>
</tr>
<tr>
<td>• Not willing to learn from teacher</td>
</tr>
<tr>
<td>• “Pulls attention toward self”</td>
</tr>
<tr>
<td>• May not like attention</td>
</tr>
<tr>
<td>• May struggle</td>
</tr>
<tr>
<td>• May start out slowly</td>
</tr>
<tr>
<td>• May not challenge self</td>
</tr>
<tr>
<td>• May build walls up</td>
</tr>
<tr>
<td>• May lack effort</td>
</tr>
<tr>
<td>• May not try to please teachers</td>
</tr>
<tr>
<td>• May not like math</td>
</tr>
<tr>
<td>• Focus may not match abilities</td>
</tr>
<tr>
<td>• May be bored</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gifted in Domains</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Gifted may not be gifted in all areas</td>
</tr>
<tr>
<td>• Gifted in content areas (verbal vs. math)</td>
</tr>
<tr>
<td>• Gifted w/in content area (spatial vs. reasoning)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inaccurate Identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>• High achieving may be mistaken as gifted</td>
</tr>
<tr>
<td>• Self-driven may be mistaken as gifted</td>
</tr>
<tr>
<td>• Poor verbal skills hampers identification</td>
</tr>
<tr>
<td>• Lack of math exposure hampers identification</td>
</tr>
<tr>
<td>• Not all “gifted” are mathematically gifted</td>
</tr>
<tr>
<td>• Not all mathematically gifted have been identified</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lack of Awareness of Gifted Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Did not know who gifted were</td>
</tr>
<tr>
<td>• Not routinely provided names of all gifted</td>
</tr>
<tr>
<td>• Do not routinely ask for gifted names</td>
</tr>
<tr>
<td>• Only provided gifted names if missing class</td>
</tr>
<tr>
<td>• Not all gifted choose to be in gifted program</td>
</tr>
<tr>
<td>• Gifted program not for eighth grade</td>
</tr>
<tr>
<td>• Guidance provides names if asked</td>
</tr>
</tbody>
</table>
Meeting Gifted Students’ Needs
(How well do teachers perceive they are meeting the needs of their gifted students?)

<table>
<thead>
<tr>
<th>Placement Issues</th>
</tr>
</thead>
<tbody>
<tr>
<td>More students in course</td>
</tr>
<tr>
<td>Told to increase numbers</td>
</tr>
<tr>
<td>Pressure from district</td>
</tr>
<tr>
<td>Teachers spend lots of time placing students</td>
</tr>
<tr>
<td>Teachers focus on correct level</td>
</tr>
<tr>
<td>Students generally placed well</td>
</tr>
<tr>
<td>Questions why students are pushed</td>
</tr>
<tr>
<td>Placement is a challenge</td>
</tr>
<tr>
<td>Teachers are ruling factor</td>
</tr>
<tr>
<td>Set district criteria for placement</td>
</tr>
<tr>
<td>Minimum requirements lowered</td>
</tr>
<tr>
<td>Properly placed students rally</td>
</tr>
<tr>
<td>All classes different</td>
</tr>
<tr>
<td>Move out of Algebra I is scheduling nightmare</td>
</tr>
<tr>
<td>Parental involvement</td>
</tr>
<tr>
<td>On track since Grade 5</td>
</tr>
<tr>
<td>Few non-Algebra I classes in future</td>
</tr>
<tr>
<td>Current Algebra I for college-bound</td>
</tr>
<tr>
<td>Future Algebra I for different audience</td>
</tr>
<tr>
<td>Some students drop out of Algebra I</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Potential Course Modifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking/leveled class</td>
</tr>
<tr>
<td>Double block - Grade 8</td>
</tr>
<tr>
<td>Stratified levels of abilities</td>
</tr>
<tr>
<td>Big difference in grade level classes</td>
</tr>
<tr>
<td>Gifted-only class</td>
</tr>
<tr>
<td>Slower pace for lower-ability</td>
</tr>
<tr>
<td>Gaps in student knowledge</td>
</tr>
<tr>
<td>Gifted held back</td>
</tr>
<tr>
<td>May have issues beyond academic</td>
</tr>
<tr>
<td>Some students needy</td>
</tr>
<tr>
<td>Cannot do arithmetic</td>
</tr>
<tr>
<td>Gifted program ineffective</td>
</tr>
<tr>
<td>Not encouraged to become mathematicians</td>
</tr>
<tr>
<td>Teach basic material</td>
</tr>
<tr>
<td>Re-teach concepts</td>
</tr>
<tr>
<td>Offer help outside of class</td>
</tr>
<tr>
<td>Remove some challenge</td>
</tr>
<tr>
<td>May be developmentally unready</td>
</tr>
<tr>
<td>May not have work skills and study habits</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Push Students Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher pushes up</td>
</tr>
<tr>
<td>Does not dumb down material</td>
</tr>
<tr>
<td>Hard to find time to help in class</td>
</tr>
<tr>
<td>May not be socially ready for HS course</td>
</tr>
<tr>
<td>Tutoring</td>
</tr>
<tr>
<td>Discusses with parents</td>
</tr>
<tr>
<td>Slows lesson down</td>
</tr>
<tr>
<td>Takes teacher’s time</td>
</tr>
<tr>
<td>Frustrates teacher</td>
</tr>
<tr>
<td>Constantly dealing with struggling</td>
</tr>
<tr>
<td>Impacts Grade 8 gifted more than Grade 7</td>
</tr>
<tr>
<td>May have poor readers in course</td>
</tr>
<tr>
<td>Need additional work outside class</td>
</tr>
<tr>
<td>Suffer because of pace and rigor</td>
</tr>
<tr>
<td>Everyone worries about other end</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gifted Needs being Met</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing gifted helps meet needs</td>
</tr>
<tr>
<td>Only benefit to gifted is questioning</td>
</tr>
<tr>
<td>Want to move them on</td>
</tr>
<tr>
<td>Meeting needs, but not to level desired</td>
</tr>
<tr>
<td>Adequately served</td>
</tr>
<tr>
<td>Treat as high school students</td>
</tr>
<tr>
<td>Want to give additional support to gifted</td>
</tr>
<tr>
<td>Desire to expand capabilities for gifted</td>
</tr>
</tbody>
</table>
Appendix U

Case Study - Lila

Lila was evaluated as being the most effective Algebra I teacher observed, with an average rating of 2.60 on the COS-R. Lila had been teaching for 14 years, 13 of which were in an Algebra I classroom. She had a Bachelor’s degree in Mathematics and was in the process of completing her Master’s degree in Education. Her experience also included serving as the Mathematics Department Chair at her middle school. She did not have a gifted endorsement or any professional development in gifted education, although she seemed quite knowledgeable about giftedness since two of her children had been identified as gifted. The researcher observed her teaching two separate classes of algebra students. Both had a mix of seventh and eighth grade students.

Mathematical giftedness. Lila believed students who were mathematically gifted were easily able to keep up with her instruction and were “able to analyze and apply the material in ways beyond the normal classroom instruction.” She believed that students could be gifted in different domains, such as verbal and mathematical thinking, and even in different aspects of mathematics. For example, she had taught students who were gifted in algebraic thinking, but not in spatial reasoning, and vice versa. She was not convinced that all of her algebra students who had been identified as “gifted” were actually mathematically gifted. On the other hand, she believed that there were probably three or four other students within her algebra classes who were mathematically gifted, but had not been officially identified as gifted. She based this not only on their performance on assessments, but also on their focus, the ease with which they picked up
new concepts, and the fact that they seemed to “want a full understanding of the material” as evidenced through the type of questions they asked.

Of interest, Lila indicated that she was not routinely told which of her students had been identified as gifted. Her school normally sent out a list of seventh grade students who participated in the weekly gifted program, but since not all of the gifted students chose to participate, the list did not include all the seventh grade students who had been identified as gifted. The gifted program was not available to eighth grade students. Lila indicated that she had to actively seek out the listing of gifted students from the guidance office.

**Pace.** Lila demonstrated observable behaviors related to pace more often than any other teacher. She indicated that the pace of her Algebra I classes was determined by the quarterly planning document. The three algebra teachers within the school – Lila, Melinda, and Hillary – would get together to look at the district’s curriculum framework and the amount of time they had to cover each topic. They made adjustments to the pace at which they covered various concepts based on whether the students seemed to understand it, but the three teachers tried to cover the material at the same time. If there was an assembly or some other event that caused a class to fall behind, the students were able to catch up via before- or after-school tutoring, reading the notes and textbook online, or going to the class website to get extra help.

Lila determined the students’ prior knowledge in all three of the observed lessons, and she adjusted the pace of the class as a whole accordingly. She indicated that the set of notes from each of her three Algebra I classes looked different in terms of which problems they did or what areas were emphasized. She spoke of constantly adjusting the
lessons, characterizing her frequent evaluations as "tap dancing." Some classes and some students needed her to tell them every step, while others were ready to move ahead. She indicated that she frequently used self-assessments – based on the students’ opinions or their performance on a particular task – to help her determine what her students already knew. She also used worksheets or other problems to check for prior knowledge. One of the most effective ways in which she felt she was able to assess students’ understanding was when she previewed the lesson by having a question and answer session where the students tossed different ideas around. She indicated she asked many questions as she went through the notes on a particular topic and when she discovered that some of the students already knew the material, she would either just skim the notes or pick selective problems embedded within the notes. By spending more or less time on the notes and problems therein, she was able to adjust the pace of the individual Algebra I classes, which she did during all three observations. This allowed her to provide scaffolding to the classes that needed it and more independent practice to those classes that were comfortable with the material. In this way, she was able to balance practice and learning activities for the class as a whole.

Lila indicated that she “would like to adjust the pace more within the class.” In the three observed lessons, while she let students work through problems within their notes at an individual pace, they were not given the opportunity to move on to different topics. For example, on two occasions, Lila noticed one of the gifted students was done with the worksheet the rest of the class was working on. In one instance, the student finished his work and had pulled out a book to read. Lila redirected him by giving him a choice of either working on a worksheet the class previously failed to complete or beginning his
math homework for that night. In the other instance, Lila told him to work on the next section of a review pamphlet each student was creating in preparation for a test. The spare time was not used to investigate more challenging topics, and the activities provided neither enrichment nor acceleration.

**Challenge.** Lila demonstrated behaviors related to challenge more frequently than the other teachers. In all three of the observed lessons, Lila provided all the students with an advanced level of challenge which emphasized fluency and depth of understanding. In fact, Lila cited an example where she had the class use *Inspiration* software to create flowcharts to model their problems. She also made connections between old and new learning and connected real-world problems to math. During all three lessons, Lila also challenged the students to see if they could come up with different ways to solve the problems. During two of the lessons, students analyzed and compared these different methods of solution. She also encouraged them to synthesize the information to come up with generalizations. She indicated that for one of the games the class played, she selected the most challenging problems from the previous year.

Lila indicated that she was constantly looking at ways to make math tasks more challenging specifically for her gifted students. She monitored their work and encouraged them move ahead in the notes and work on the problems that were more challenging. Because the class might not get to these problems during the class periods, she said she sometimes had a gifted student explain how they solved a particular problem to their group. She indicated, however, that she was hesitant about giving the gifted students extra problems to do. Instead, she occasionally used them as peer teachers within a group.
One of the ways Lila raised the level of complexity for her students was through her questioning. In fact, she said that she sometimes felt like “the only benefit [she] gives the gifted students is higher-level questioning.” She said she constantly tried to question the students to get a higher-level of thinking. During two of the lessons, she asked higher-level questions to challenge the students to make connections between various mathematical concepts and other content areas. Many of her questions were open-ended, such as, “How could you relate a phrase in your English class to an expression in algebra?” She also challenged the students to compare and contrast different ideas, using such phrases as, “How is this like . . .” and “How is this different from . . .” She frequently used questions to elicit responses, rather than telling the students the answer. She also said she sometimes asked the students to explain what they thought would happen in a particular problem that extended the topic at hand and to reflect on what the next step in the process might be.

In addition, she spoke of raising the level of challenge by having students apply concepts in a more advanced way. Lila said she did not give her gifted students advanced problems dealing with a totally new concept; rather she would take a concept previously taught to them and make it more difficult by involving more distribution, changing basic expressions to rational expressions, and so forth. Lila pointed out that she had to be careful about how she approached the issue of adding complexity because some gifted students did not want to be viewed as being different. She said that challenging these students sometimes “posed a dilemma” and that in some cases it was better to have the students stay after school to work on challenging problems because it was more discrete for them.
Lila’s class was taught at a relatively high level compared to some of the other teachers and her gifted students were generally very engaged. There were times during the observations, however, when one or two of the gifted students looked bored. On one occasion, a gifted student finished his pre-assessment problems well before the rest of the class, but Lila did not provide him any more challenging material; he just sat there. Later, although this student had already demonstrated the ease with which he was able to solve the problems, Lila told the class, “We’re going to do these together because it worries me that you don’t know how.” This was a missed opportunity to challenge the student to move ahead. On another occasion, one of the gifted students appeared bored and eventually started talking to the other members of his group, requiring Lila to redirect him toward the lesson. Later, when she asked how many students got the practice problems correct, none of the gifted students raised their hands. Because the students previously indicated they were doing well on the problems, it appeared as though they may have tired of doing problems that were not challenging and stopped working on them.

**Differentiation.** Although Lila showed the second highest level of differentiation compared to the other teachers observed, her differentiation strategies were somewhat limited. For example, although her gifted students may have been able to progress through the given set of problems more rapidly than the other students, they still worked on the same set of problems during class, rather than being accelerated. They had the same homework as the other students, rather than having tiered assignments according to their level of understanding. They were, however, provided a choice in selecting problems during two activities. In addition, some students “self-differentiated” when
they decided to solve problems using a different process. Lila said that she tried to have her learner plans “incorporate ideas for all levels of small group instruction or differentiation strategies; however, the time factor is always an issue.” She pointed out that this sometimes posed a dilemma for her as far as specifically differentiating for gifted students, although she did not consider differentiation to be a weak area for her.

The majority of the differentiation strategies observed and discussed during the interview focused on grouping. Lila allowed the students to work in groups during all three lessons to deepen their understanding or to allow collaboration in problem solving. For example, she had students work in groups of four on a set of problems. One group presented the answer and another group explained the rule they had derived from doing the problem. The three gifted students were the student-selected spokespersons for each of their groups.

Lila had the desks set up into “pods” or groups of four. She used a grouping strategy of “face partners,” which involved the two people sitting opposite of each other in the pod, and “shoulder partners,” which included the two people sitting next to each other. She used both sets of partners during the observed lessons. For example, during one of the observations, the students played “rally coach,” where one student coached another student through a problem. She also used partners in two think/pair/share activities. Lila indicated that working with partners in that way provided good reinforcement for both gifted and non-gifted students and modeled the type of positive helping interaction she wanted the students to have with each other.

Lila said that she did not generally put all the gifted students together because “they tend to pull so much attention toward themselves as they zoom ahead.” She preferred to
have a blend of student abilities in the groups, although she sometimes put two gifted students together “to challenge each other as peers and to build off each other.” She preferred not to put one gifted student with three low students although she frequently used gifted students as peer helpers. This practice tended “to pull the other students up.” She found that groups helped to generate conversation and to teach students to look out for each other. Lila indicated that she wanted to have at least one person in each group who would be on task and have the ability to follow her and pick up the concepts easily.

Lila occasionally grouped by ability based on how students did on an assessment. She described a “four corners” activity with different folders at each location. One folder might explain the problem-solving process step-by-step to provide scaffolding for the students who were having difficulty, while another folder might have a sheet of challenge problems for those students who understood the material. She used the same “four corners” activity during one of the observed lessons, allowing the students to stand in the corner associated with where they would like to vacation, and then giving each group a different type of problem to solve.

Lila indicated that during the year, the groups would be “constantly changing.” Over the years she has tried different grouping configurations – ability, choice, and interest – and found it was important to consider the students’ personalities and work ethic when she created the groups. If these attributes did not complement each other, the group would not be successful.

The classroom observations did not reveal any enrichment activities for the gifted students. Lila pointed out that she had classroom resources the gifted students could use, but that she hesitated mentioning too many of the resources because the gifted students
“like to get ahead and sometimes go a little too much beyond the class.” She indicated that she wanted to make sure that these students “are still staying with what’s going on in the class.” She pointed out that after school was sometimes the best avenue for enrichment because she could bring in laptops, have the students investigate specific websites, and expose them to higher-level thinking and other types of problems.

When asked what type of enrichment opportunities were available to these students outside of class, Lila discussed MathCounts, a math competition which involved weekly practice of math skills, problem solving, and reasoning. Both gifted and non-gifted students were allowed to participate. The coaches – one of whom was a gifted student from a previous year – developed “a tool box of skills” to help the students with various math concepts. In addition, Lila encouraged her gifted students to do the on-line SAT question of the day at home because she thought it was important for them to have exposure to that type of question.

**Supportive environment:** Lila had more observable evidence related to a supportive learning environment than all of the other teachers observed. She had a warm, positive atmosphere in her classroom and was very approachable. She indicated that all her students knew she was available to give them help before or after school. She provided equitable attention to the gifted students in the class and was complimentary toward all of her students. It was apparent that the students enjoyed being in the class and they felt comfortable asking questions and expressing their ideas. During each of the three observed lessons, she solicited conjectures on how one might go about addressing a new type of problem, and the students did not hesitate to respond. In fact, during one lesson, she told a gifted student, “Don’t feel embarrassed if your answer is not quite right; that’s
how we learn.” Lila encouraged students to reflect on their own reasoning and why solutions might be incorrect.

Lila also had high-level expectations for her students which she clearly expressed. For example, she told the students that she expected them to keep their notebook to review prior to the district assessments and standards test. She reminded them of a website that had algebra vocabulary cards and exercises for extra practice, and when referring to extra problems on a worksheet, she commented that, “a good student may want to practice that.” She also stressed the importance of all students making the class notes meaningful, saying “don’t just copy what I write; make it important for you.”

To facilitate learning, Lila allowed students the time to practice problems, provided the necessary scaffolding, and ensured there were application activities for each new concept. She called on multiple students and encouraged students to think on their own by using questions to solicit responses rather than to tell the answers. Lila also did frequent self-assessments so that the students could reflect on what they understood and she could see where her lesson needed to be adjusted. She did this before and after the homework reviews, during the lesson preview, after guided practice, and at the end of the lesson. She was effective at using questions to help student define the problem, figure out what step might come next in the problem solving process, and to help students elaborate on their thinking. She also pressed students to explain their rationale, at one point, asking a gifted student for an explanation as to why the exponent in a problem only referred to a particular number, and on another occasion, asking another gifted student to elaborate on his answer having to do with absolute value. Lila indicated that sometimes
her conversations with her gifted students lost the rest of the class, but they enabled the gifted student "to get the full impact of the lesson."

One of Lila’s strongest areas was in encouraging students to solve problems using various methods. By using such phrases as, "excellent, that’s one way," she let students know that there were different ways to do some procedures in math and that the students should use the method with which they were most comfortable. For example, on one occasion a gifted student asked whether he could "change adding a negative into subtracting a positive." Lila agreed and encouraged him to solve the problem using the method he understood, but that most of the class did not. The gifted students seemed to thrive on coming up with different methods of solution. Lila was complimentary toward each student, and in fact, referred back to one of their methods saying, "This is where Matt’s rule comes in." She also provided opportunities for students to showcase their methods, asking, "Who would like to help me teach this?" One of the gifted students immediately volunteered.

Lila modeled high-level performance in several ways. She stressed the specific process the students should use to show their solution method, consistently demonstrating how to write every step of the problem and how to check their work, something she expected the students to do. She made sure students understood why they missed problems and gave them personal hints to help them with difficult mathematical concepts. She also used proper mathematical terminology, and insisted that her students do so as well. She pointed out that even if all the students did not understand the point she was trying to make, they started to pick up on the terminology and use the vocabulary of real mathematicians in their math conversations. She also exposed her students to
advanced mathematical symbols, such as using set notation when solving functions and using three dots to mean *therefore*. During one lesson, she related her mathematical notations to that of a real mathematician saying, “a true mathematician will write variables in lower case.” Lila also revealed that she tried to push ideas and concepts that the students would encounter in the future. Since her husband was a high school mathematics teacher, she was very cognizant of the vertical articulation in the mathematics curriculum. Lila also commented that she modeled high-level performance for her students through her own neatness and organization and by making sure the students had a good system of organization that worked for them. She thought this was an especially important tool for students “to help relieve some of the frustration they feel by wanting to achieve and be the best,” especially as they progressed into advanced mathematics which was more challenging and moved at an increasingly rapid pace. Furthermore, she encouraged students to expand their horizons by getting involved in internships in high school and by reflecting on her own experiences as a computer programmer and systems analyst. In this way, she hoped to expose the students to the different opportunities that they might have.

*Meeting gifted students’ needs.* Lila thought her gifted Algebra I students’ needs were being met, but not to the level she would like to see. As the parent of gifted children, she indicated that gifted students were oftentimes asked to do things on their own, rather than garnering the teacher’s attention. For this reason, she said she was an advocate for enrichment. She thought she challenged her gifted students and that she gave them everything they needed to move to the next level, but she wanted to do much more in terms of differentiation and being able to take them to beyond their current level.
She said she was constantly challenged by time constraints, but every year she tried to focus on how she could enrich the gifted students and provide some additional instruction or support for them.

This issue was compounded, however, by the push to move more students into Algebra I within her school district. Although the district had no official quota, as the Mathematics Department Chair, Lila had been in meetings where it was suggested that her school needed to increase the number of students enrolled in Algebra I because they had the lowest percentage in the district. This was disturbing to her because the algebra teachers at Lila’s school spent a significant amount of time determining the appropriate level of student placement. She indicated that the teacher’s recommendation was generally the ruling factor for where a student was placed, although parents could override that decision. She believed that the algebra teachers in her school had been able to keep the level of students placed in the course relatively high. She cited the fact that few of the school’s students were put back into Algebra I when they reach high school as evidence that their students had been properly placed.

She indicated, however, that in heterogeneous classes, she found it difficult to balance the time spent with struggling students versus the other students. She tried not to spend an inordinate amount of time working with the lower students in class, and instead, told them they needed to come in before or after school for tutoring. In essence, she pushed the struggling students up, rather than lowering the level of her course. On some days, however, her students were “very needy,” and she could not accomplish half of what she intended. Sometimes she felt like she almost needed to re-teach the previous day’s concepts, which was frustrating. She indicated she was constantly trying to balance
teaching new concepts, while making sure the students really understood the previous concepts. For this reason, Lila advocated providing two different levels of Algebra I in the middle school to better meet the needs of all students: Algebra I on alternate days for the gifted and high-ability students, and every day for the struggling students.

She said she sometimes felt guilty about whether she was giving to her gifted students as much as she should. From a teacher’s viewpoint, she believed she was doing the best she could, but from a gifted parent’s viewpoint, she thought she could probably do more with them, knowing they were capable of so much more. She cited the example of a gifted student from her class the previous year, saying that although she tried to challenge and push him, she did not feel like she could direct as much attention to him as he deserved, “but considering the time constraints, the number of students in the class, and the standards that need to be covered, it was the best, I could do.”
Appendix V

Case Study - Sam

Sam was evaluated as being the second most effective Algebra I teacher observed, with an average rating of 2.59 on the COS-R. Sam had taught for five years, three of which were in an Algebra I classroom. He had a Bachelor’s and Master’s degree in Mathematics, but did not have a gifted endorsement or any professional development in gifted education. The researcher observed him teaching two separate classes of seventh grade algebra students.

Mathematical giftedness. Sam did not think that there was really any quality that was different between students who had been identified as gifted and the other students taking Algebra I in seventh grade because both groups of students were working at a very high mathematical level. Prior to this study, he was not aware of who his gifted students were because they did not get pulled out of algebra for the weekly gifted program. In previous years, the gifted students were pulled from his class for the gifted program, and he recalled being concerned because some of those students were struggling and could not afford to miss any lessons. Because of this, he thought there were probably students identified as gifted, who were not mathematically gifted. He also believed that some gifted students did not necessarily care for mathematics and did not try to please the teacher or put effort into his class. He indicated that gifted students sometimes thought they “knew it all” and were not as willing to learn from the teacher as were other students. On the other hand, he thought that there were as many as eight other students in his Algebra I classes who were probably mathematically gifted, but had not been identified as such. He based this on the fact that they did not seem to be particularly
challenged even though he taught algebra “at a pretty high level.” They were able to handle whatever work he gave them.

**Pace.** Sam believed that the pace for the class was essentially determined by the school district’s curriculum guide. He pointed out that the Algebra I class was actually two separate semester courses – Algebra I-A and Algebra I-B. The students were required to finish the material for Algebra I-A by the end of January in order to take the end-of-course exam. He found it very difficult to fit all the material into the allotted time. Sam stated that the algebra teachers in his school created a quarterly plan to determine how they were going to fit in that quarter’s worth of material, but it caused the class to move faster than what he would have liked. He anticipated that this would become even more of an issue because the material required in the state’s new Algebra standards was “more complicated than what they have taught in the past.” In previous years, the teachers covered the advanced material after the standards test in May because it was not considered testable material. With the new standards, the material needed to be taught prior to the end-of-year standards test which would add more material into an already packed curriculum. Of interest, Sam thought that “the curriculum actually set the students up for failure in a sense” because they were used to having 90 minutes of math a day with a slower pace and fewer skills, and when they came to Algebra I, they had to use multiple skills at once and they only met every other day. He perceived this as “a tough adjustment for the students, especially considering the other adjustments in life that seventh graders typically experience.” Despite his concerns, Sam revealed that his school had a 100% passing rate for seventh graders on the Algebra standards test every year since his arrival. He pointed out, however, that the students only needed to get 50% of
the problems right to pass the test, possibly because students have to pass the test to graduate.

All of the observed classes moved at a fairly rapid, but steady pace. During two of the three observed lessons, the students were given enough time to work out the problems related to the new material, but not so much that they became bored or disengaged. There was not enough time during the third lesson for the students to come up with solutions and talk about them. During all three lessons, the students worked through their guided practice problems at an individual pace, but because Sam did not give them other problems to work on when they were finished, the class stayed together. He adjusted the pace of the class as a whole during two of the observed lessons. For example, during one lesson, several students had questions on homework, so he spent almost 20 minutes addressing their concerns. He then spent quite a bit of time explaining the rationale behind cross product problems and quicker ways to solve them. This left only 15 minutes in class to cover percent equations, and so Sam adjusted the pace of the lesson to get through the material. The students seemed to keep up with him. The researcher observed Sam teaching the same lesson to a different class the following day, and although the material was the same, the pace was somewhat different to accommodate the different levels of understanding between the two classes. Sam did not, however, adjust the pace for individuals in either of the lessons. There was only one occasion where a gifted student finished well before the others, and rather than providing any acceleration or enrichment, Sam asked him to put the answer on the board.

To determine what the students knew about a particular topic, Sam walked around to see how the students were doing as they worked on problems. By using this informal
method, he picked up on things the students may have forgotten, and he was able to cover some of that material in the lesson. He was seen determining students’ prior knowledge in this way during all three of the observed lessons. Sam indicated that he had not run into the situation where the students had already mastered the material he planned to cover in a lesson because the curriculum prior to Algebra I did not address many of the algebraic topics. He pointed out that the material was new to the students even if they were able to understand it right away.

To balance the time the class spent on practicing known concepts versus learning new concepts, Sam had the students work on more complex, multi-step problems with the basic skills built in. He provided resources for the basic skills if students needed to practice, but he felt they needed to see the more complex problems in class, so that they could learn how to focus on the various parts of the problem. He had a mix of practice and learning activities during all three of the observed lessons.

Sam indicated out that although he was able to adjust the pace of the class somewhat based on his interaction with the students, he still had a certain amount of material to cover each day. To maximize instructional time, he posted solutions to the homework for the students to check online. He did, however, try to spend time going over the homework because the students needed to understand that there were different ways to solve the problems, but at some point, he had to say that “they have hit the main ideas and it’s time to move on.” Sam was also aware of pace when the students copied notes from the board. He pointed out the importance of comprehensive notes, but said he realized that when they were taking notes, the students were not actually working through the math. He wanted to students to be able to write such thing as which property justified
each step of a problem, but he purposely did not have them write it down because "some people would be constantly copying, but never really thinking." He summed up the pace issue by stating that "it's a real challenge to cover all the material in the curriculum."

**Challenge.** Sam exhibited mathematical behaviors related to challenge more often than any other teacher except Lila. He emphasized fluency and depth of understanding, and in two of the observed lessons, he specifically encouraged the students to find the pattern in the material. This was a way to encourage higher-level thinking and the researcher only observed this technique in one other class. Similarly, he had the students analyze and compare different methods of solution and asked challenging questions to meet student readiness more frequently than any other teacher. Furthermore, he made specific connections between old and new learning in all the observed lessons.

Sam's lessons built from basic to increasingly challenging problems, and he ended the observed lessons having students solve problems with a level of complexity not observed with most other teachers. By doing this, it appeared he was able to engage every student in the class. In addition to presenting the material in a way that encouraged the students to see whether they could solve the problems, he occasionally passed out candy to the correct solvers, which undoubtedly inspired them. He also mentioned that he sometimes gave the students challenging problems to work at home to see what they could do. He did not hold it against the students who could not figure out the problems, but he wanted to see who could rise up to the challenge and come up with the process to solve the problem. During the next class, he gave them hints to how to solve the challenge problems or he would use them as a warm-up.
Sam emphasized that the Algebra I material was much more challenging than the material in previous math courses and it involved many different skills. He commented that the algebra curriculum was “pretty challenging as it is,” but stated that to modify the problems to make them even more challenging, he used fractions with different denominators and decimals with multiple decimal places. He also used several techniques during the observed lessons to increase the level of challenge. For example, he created problems that involved the distribution of negative numbers and the reversal of the inequality sign. He also probed the students on their understanding of inequalities by asking which inequality symbols would make a problem false. This identified whether they really understood the “or equals to” portion of an inequality ($\geq$, $\leq$). In addition, he put an incorrectly-solved problem on the board and challenged the students to find the mistake. Furthermore, he challenged students to solve the problems in their own way and then to explain their method to the class. At one point in the lesson, he asked “Are you ready for a challenge?” and a large portion of the class shouted, “Yes!” He then gave them a word problem to solve. Sam also commented that he did not let the students use calculators when they first learned a skill, a practice that was “very shocking to students from other districts.” They were eventually able to use calculators once they had the concepts “really down,” but he thought it was very important for the students to be able to do all of the skills without the assistance of a calculator. Sam thought his school was doing a favor for the students by making them do the skills required within the complex problems by hand because “it gets them working at a higher level.”

The gifted students were very involved in the observed classes. Although one of them was usually the first to solve the guided practice problems, not all of the gifted
students correctly solved the problems each time. This indicated that the material was challenging to these students as well. In fact, Sam commented that occasionally one of his brightest students would approach him to comment that “they were actually rejuvenated and energized by the algebra curriculum because math was too boring to them prior to that point.” Sam pointed out that even if students had worked with their parents over the summer and had mastered some of the material, he did not think they had “picked up on all of the pitfalls,” so he was still able to challenge them.

**Differentiation.** Sam demonstrated mathematical behaviors related to differentiation much less frequently than his behaviors related to challenge. Sam indicated that he thought of differentiation in two different ways: “to get at the material in different ways” versus “challenging some students more than others.” He thought that because “the curriculum is what it is,” there were not many ways to deviate from the material at hand. He cited his modeling of note-taking and having the students use different colors to highlight their notes as examples of how he differentiated the material. He mentioned that he occasionally provided graphic organizers for the students to paste in their notebooks, but that “algebra is what it is.” Sam also pointed out that he did not have the students do peer teaching because “the curriculum drives everyone to essentially be on the same page.” He stated that they covered a new topic almost every lesson, and although he wished they had more time to do exploratory activities and “see where they went with it,” there simply was not time. He explained that the classes essentially followed the same routine: he showed them how to do the concept, talked about the areas that might confuse the students, and then let them practice.
This limited view of differentiation was borne out in the three lessons the researcher observed. Differentiation methods such as acceleration, enrichment, tiered assignments, open-ended activities, or choice—other than allowing the students to set up and solve problems using a method that made sense to them—were not observed. In fact, all of Sam’s instruction was as a whole group and all students had the same homework. Furthermore, because he did not provide a set of practice problems ahead of time, the students were only able to work on the problem at hand, and were not able to work ahead.

When asked specifically about grouping, Sam revealed that although he grouped students in other classes, he did not do it much in Algebra I. In fact, the researcher observed only very limited cases of partner work to discuss solutions and to deepen the students’ understanding. These occasions essentially involved the students sharing their ideas or solutions with the person sitting next to them. Sam pointed out that although four of the gifted students sat next to each other (two pairs of students), it was just a coincidence.

Looking specifically at enrichment, Sam stated that there was “no time for enrichment within the classroom because of time constraints.” In fact, they sometimes had to cover two concepts in a single lesson because there was so much material to cover in a short period of time. Sam was not aware of any enrichment opportunities for the students to extend their mathematics learning outside the classroom. He indicated that students could “seek something out, but everyone tends to be challenged somewhat by the material every day.”

**Supportive environment.** Sam demonstrated mathematical behaviors associated with a supportive environment more frequently than all the teachers except Lila. He
solicited responses rather than telling answers, he encouraged students to use different ways of thinking, and to come up with different methods of solving problems. He also effectively used questioning to help students elaborate their thinking, to probe their reasoning, and to press them for explanation.

To provide a supportive environment, Sam talked to his students about his expectations. He stated that he wanted a positive atmosphere in the class, and this was evidenced during the observations. He was very upbeat and motivational, encouraged the students to ask questions, and made sure to include all the students when he called on them. He had an excellent rapport with the students, and was able to joke around with them while easily maintaining control of the classroom. He made comments such as, “that’s great understanding,” “that’s a good ‘lesson to learn’ question,” and “fabulous – did everyone hear what he just said?” The students seemed motivated by his positive words.

Sam pointed out that Algebra I was the first math course with which some of the students had struggled, causing them to wonder whether something was wrong with them. Since these students were afraid of solving problems incorrectly, he told them that more powerful learning could come from making mistakes. During one observation when a student expressed concern about getting the wrong answer, Sam told him, “Now is the time to not be afraid!” When students made mistakes, he pointed out that other students made the same errors and he then had the class analyze what caused the mistake. He told the students that if they already knew the material, they would not be in the class. He indicated that he liked pointing out “great wrong answers” and tried to eliminate
embarrassment by telling the students to celebrate their wrong answers so they could learn from it in class.

Throughout the observed lessons, Sam provided scaffolding by referring back to previous lessons. In fact, during one observation, he demonstrated how to multiply and divide decimals even though this was a skill most students typically master prior to algebra. In this way, he cleared up common mistakes without making the students feeling bad about forgetting the process. To help them with review skills, early in the year Sam taught a lesson covering all the basic skills. The students wrote this in their notebook so that when they got stuck, they could refresh their memories by looking at the notes. While he sometimes modeled basic skills on the board, as the year progresses, he expected the students to use their notes and figure out those skills individually. He also encouraged the students to use what they already knew when they approached a new concept. During one lesson, he said, "Using the skills you already have, how might you solve this?" He further encouraged such reflection prior to solving problems by saying, "think about where you might want to start," "there are many places to begin," and "think to yourself – what is the process you go through to solve an equation?" In this way, he tried to build up the students’ confidence that they could solve more complex problems. Sam indicated that he tried to ensure an appropriate level of scaffolding by tying the big picture of where they had been to where they were going in that day’s lesson. He attempted to connect the learning and explain how the material built upon previous concepts. He said he told the students this so they knew they could not afford to miss out on learning a skill, but also to let them know that the material was not going to be too
overwhelming. By showing them how to use the things they had already learned, more complex problems did not tend to overwhelm or faze them as much.

Sam modeled how to write the step of each problem and explained why he was using the method he did. He oftentimes used colored markers to highlight various points, which modeled how the students might highlight their own notes. Sam also talked to the students about how to show their work and the reasons for doing so. He pointed out trick words and common student mistakes, and told the students to annotate the typical errors in their notes. Sam also modeled high level performance by requiring the students to show all their work and to properly label their answers. He required the students to memorize the “percent of” formula (is/of = %/100) and then had them shout it out with their eyes closed, an activity they thoroughly enjoyed. He also encouraged the students to speak using proper mathematical terms. Moreover, he gave the students simple tips for improving performance on homework and quizzes such as drawing a number line on their paper. In addition, on two occasions, he reminded the students they had certain information in their notes, highlighting the importance of being organized.

Sam encouraged solving the problems in different ways, sometimes asking, “Can anyone think of a different way to do this?” When a student suggested one solution method, he carried it all the way through to show the students how it would turn out. He then solved the same problem in another way to show that the answer was the same. He had the students compare and contrast solution methods, which caused them to think at a higher level. Although they talked about the benefits and pitfalls of each method, Sam allowed the students to use the method with which they were most comfortable. He showed that he valued the different techniques when he referred back to different
students’ solution methods later in the lesson. At one point, he also tied the lesson to the real world, asking them, “When would you use that skill?”

As the class went through practice problems, Sam walked around and held individual informal conferences with the students. He indicated that he tried to make himself available as much as possible after school, giving personal invitations to the students who needed extra help. He did not always have time to help the struggling students one-on-one in class because of the number of students and the pace with which he needed to cover the material. He indicated that he tried to give his students—especially the struggling ones—the message that they needed to practice math every day. He encouraged them to take it upon themselves to do a little additional work. The students had been provided with online resources as well as a textbook at home where they could find additional problems and the answers.

Meeting gifted students’ needs. Sam pointed out that his school had the highest percentage of seventh grade students in Algebra I within the district. He attributed much of that to parental involvement since the students did not have a formal pre-assessment to place them into the course. In fact, Sam indicated they “have essentially been on the track to put them into the course since fifth grade.” He sometimes felt “a tug-of-war in [his] mind about whether certain students need to be pushed into algebra so soon,” especially students who had no interest in going into anything math- or science-related since the course put them on track to possibly take calculus in their junior year.

He pointed out that “more kids than ever” were going to be moved into Algebra I next year and he thought it would have an impact on eighth grade gifted students more so than the seventh graders. He said that if he taught eighth grade Algebra I, he might have
to modify the assessments because some students could hold their own, but there might be others who were reading at a third grade level in the same class. He would probably have to take out some of the more challenging aspects of the course to ensure that the students understood just the basic algebra concepts.

In fact, he indicated that one of his biggest challenges with his current seventh grade class was that the students were not particularly strong in the prerequisite skills. The students understood the algebraic concepts, but they performed the actual arithmetic calculations incorrectly which slowed them down. When his students were struggling, rather than lowering the level of the lesson, it was up to them to come up to the level of the rest of the class. If students were struggling, he talked to the parents and told them that as long as the student was willing to work, he was happy to help them. The teachers had the option to move the students back into a lower-level mathematics course at any time, but that caused “a scheduling nightmare.” Sam pointed out that the only students who had moved out of Algebra I during the current year had moved into the district from elsewhere, suggesting that the current algebra placement within the school was satisfactory.

Overall, Sam believed the gifted students were challenged and he did not need to bring the course down to a lower level for the struggling students. He believed the gifted students in his class were being served. He pointed out that they were very engaged in the class, and frequently raised their hands to provide answers or ask questions. In addition, they seemed to be energized by the class and the curriculum because “it challenged them for the first time at a level that makes them happy.”
Appendix W

Case Study - Casey

Casey was third most effective teacher observed, with an average rating of 2.45 on the COS-R. She was the only teacher to be evaluated as somewhat effective or effective in all areas of the instrument, a finding which will be discussed further in the Pace section below. Casey was the second most experienced teacher observed with nine years in the classroom. She had been an algebra teacher her entire career. Casey had a Bachelor’s degree in Mathematics and Psychology. She did not have a gifted endorsement, nor had she received any gifted professional development from her school district. She was taking graduate classes in Education with the goal of eventually moving into administration. The researcher observed three of Casey’s Algebra I classes, each with a mix of seventh and eighth grade students.

Mathematical giftedness. Casey believed that mathematically gifted students had the ability to ask the right question and the initiative to problem-solve to find the answer. She suspected that there were at least three or four students in her Algebra I classes who were mathematically gifted, but not formally identified. She assumed that some of them had “gone under the wire because of behavior or some other issue.”

Casey believed there were different types of giftedness, but thought that schools sometimes tested the students too early to determine if they were mathematically gifted. She thought that in American culture, students were not exposed to math at an early enough age, so when they answered questions on tests dealing with new mathematical concepts, it was a brand new experience for them; therefore, they did not test as well and
were not identified as gifted. On the other hand, she believed there were students identified as gifted who were not necessarily mathematically gifted.

Casey indicated that the teachers in her school did not normally find out who the gifted students were in their classes. Although she was informed about the sixth and seventh grade students in the weekly gifted program, not all gifted students chose to participate, and eighth grade gifted students were not included in the program at all. To get a list of all 13 gifted students in the three observed classes, Casey had to contact her school’s guidance counselor.

Pace. Casey demonstrated the lowest number of mathematical behaviors related to pace compared to the other teachers in the study. Two of the three observed lessons focused entirely on practicing known concepts, as opposed to learning new concepts. In addition, the researcher did not observe Casey pre-assessing the students to determine what they already knew and adjusting the pace in response. Despite this, Casey did adjust the pace of the class as a whole during two of the lessons, but this appeared to be based on the amount of material that needed to be covered in the period rather than on what the students knew. Of note, during two of the observed lessons, the students rotated through various stations where they were able to work individually as quickly or slowly as they wanted. For the most part, the gifted students worked diligently and were able to progress through the material at their own speed. This was in contrast to other teachers’ classes the researcher observed where the pace was adjusted for the class only as a whole and the gifted students did not have the opportunity to move through problems at an individual pace. The gifted students seemed fairly engaged throughout these lessons and
were able to not only finish all the assigned work at each station, but complete their homework as well.

To decide the pace for the class, Casey had a long-range plan for the quarter, the semester, and the year driven by the district’s quarterly assessments. She also had weekly plans, but was willing to change them in the middle of class based whether the students seemed to understand the material. When she saw that about half of the students were giving her the “I haven’t seen this look,” she indicated she would adjust the lesson to address the particular concept with which they were having difficulty. She was normally able to cover the applicable material for the quarterly assessments, but if her classes had not gotten to a particular topic, she simply told the district’s math specialist that her class was behind and they could be expected to miss the particular questions related to the material they had not yet covered.

To determine what the students already knew, Casey indicated she informally pre-assessed them on the subject matter to determine “what direction to take them.” She generally tried to pre-assess the students prior to the unit test so she had a week or so to sort out where to begin the next unit. If she was unable to pre-assess each class, she would try to do so with at least one class and that would drive the pace for her other classes. She also knew what was covered in the previous math courses and adjusted her lessons accordingly; however, her school district was in the middle of transitioning to a new middle school mathematics curriculum, so her Algebra I students came from three different courses where they had different experiences. That meant that some students may not have seen even the initial ideas of a particular concept, while others may have had experience with the concept at a much more advanced level. The result was that
Casey found it difficult to balance the time she spent practicing known concepts versus learning new concepts because her students were at these different levels. To determine the number of problems assigned or the amount of guided classwork she would do in a lesson, she tried to hit the average. She explained that this was probably a little more practice than what a high-ability student needed, but about the right amount for the other students.

Similarly, Casey said she found it difficult to adjust the time allocated for students to do more complex tasks because sometimes the students understood the concept quickly, and at other times they struggled, so she had to have a backup plan to allow her to move on or slow down. She believed that many teachers were overly concerned when they were not doing specifically what they said they would do in the lesson plan. She preferred to adjust her lesson plan to her students’ needs. If the students needed more or less time on a topic, she would adjust her lesson accordingly. She felt this was important, because unlike other subjects, math required a student to understand a concept in order to move on to another. The block schedule made this difficult – even for gifted students – because in Casey’s opinion, middle school students were not able to handle that many concepts in a day and keep them straight, especially when the overall pace of the course was so rapid. Consequently, she tried not to cover the foundational and the building material on the same day. Instead, she said she might cover the foundations of two different mathematical topics on the same day and then build on those two topics on subsequent days.

**Challenge.** Casey scored in the lower half of teachers in exhibiting behaviors associated with challenge, although it appeared that the level of challenge in the observed
lessons was appropriate for the majority of the students in her classes, including the
gifted ones. During all of the lessons, she helped students make connections with
previously-taught concepts and connected real-world problems to mathematics. She did
not, however, emphasize fluency, pattern recognition, or generalizations.

During two of the lessons, the researcher observed Casey using high-level
questioning to make connections between different mathematical concepts. Casey
indicated that she oftentimes made mathematical tasks more challenging by using
different questioning techniques. In addition, to increase the level of challenge, she asked
students to come up with their own mathematical rules related to a particular concept
rather than simply asking them to repeat a rule or a mathematical process. Casey
indicated that she gave the students problems with “tricky” negative terms to add
difficulty to specific mathematical concepts. She also had the students tie together
different concepts such as combining “percent off” and “percent of” problems with scalar
multiplication (multiplying a number by entries in a matrix). Furthermore, Casey
provided the students with an opportunity to explore concepts they would cover in the
future to see what they could figure out. She demonstrated this by having the students try
to determine what algebra tiles represented and how they might set up an equation with
them even though the students had never been exposed to the tiles. She also encouraged
the gifted students to create more difficult problems when the ones they were working on
were too easy. The researcher observed this when Casey told one gifted student who had
figured out the purpose of the algebra tiles, “Let’s make it harder for you. You draw your
own example and then explain what it is.” Similarly, she told a gifted student to make up
his own order of operations problems that were more difficult than the ones his group had
been attempting. Both students relished these tasks. It appeared that they were ready for greater challenge, although not all of the gifted students appeared to need more difficulty. In fact, over 30% of one of the classes had been identified as gifted, and the two students mentioned above were the only ones who gave an indication that the material was too simple.

Differentiation. Casey demonstrated the most mathematical behaviors related to differentiation of all the teachers observed. As previously mentioned, she gave her students an opportunity to create their own problems to increase the level of challenge. She also allowed them to work in various grouping configurations. For example, each student had a “study buddy,” a partner of his or her own choosing. Not surprisingly, in one observed class, four of the gifted students had chosen each other as study buddies. On three occasions during the observed lessons, Casey allowed the students to collaborate on problem solving or to check their answers with their study buddies. Each student also had a “shoulder partner,” assigned by ability. Two sets of shoulder partners were assigned as a group of four or a “natural quartet.” These groups were theoretically made up of two stronger students and two weaker students, although during the observed lessons near the beginning of the year, the quartets were arranged alphabetically because Casey was still getting to know her students. She indicated that the gifted students frequently served as peer tutors within their groups, even though she did not specifically assign them this task. The researcher observed the gifted students in this role as they rotated through several activity stations using their natural quartet.

Casey also differentiated for the students by allowing them choice as far as how they wanted to proceed during the stations’ activities. For example, at one station, the
students were given cards with mathematical properties and various mathematical equations. Most of the students simply decided to play a matching game to associate the example with a property. Rather than playing a game, however, two of the gifted students simply used the cards to update their notes. Throughout the lessons, Casey allowed the students choice as far as how they wanted to set problems up and solve them as long as they could support their reasons for using a particular method.

Although the researcher did not observe any acceleration or enrichment activities for the students, Casey indicated that when she discovered that there were students who already knew the material she planned to cover, she tried “to have something else available for them so they were not just practicing the concepts they already knew.” She gave them the work they were going to do later in that class or gave them a challenge problem from the same topical area. She preferred to do this rather than giving the students extra work. Casey also talked about a new on-line program her school was about to begin using which would allow her to post challenge exercises for the students to investigate. She noted that she was not aware of any other enrichment or extension activities available to the students outside the classroom and was hopeful that the new program would allow the students an opportunity to extend their mathematics learning.

She pointed out, however, that she needed “to give all the students the same homework assignments because the parents got upset if they found out some students had different assignments.” She felt that if she did not have this outside pressure and “could figure out how not to cause a parental uproar,” she would not assign as many problems to the gifted students. She would like to be able to tell all the students that once they demonstrated the knowledge, they were finished. She explained that one of the
challenges with math, however, was that there were multiple variations of the same type of problems, so a student might be able to demonstrate understanding on a certain type of question, but might miss other variations that he had not seen yet. Furthermore, she said she was concerned with the maturity level of her gifted students and whether they would be able to handle the independent assignments. Casey had found that her gifted seventh grade students were “very immature” compared to the eighth graders in the same class. They played differently, they interacted differently, and they seemed “more like children than nearly high school students.”

Supportive environment. Casey demonstrated a supportive learning environment in several ways. She paid an appropriate amount of attention to the gifted students, encouraging them to create new problems and then pressing them for an explanation of their problem-solving methodology. She also had students reflect on their learning by writing about various mathematical activities and she used questioning to help them elaborate their answers. When students put incorrect answers on the board, she pointed out that “the answers they are putting up may or may not be correct, but they may help us see common mistakes. We learn from our mistakes.” She also conveyed her support for the students, telling them, “I want you to meet with success.” Furthermore, Casey demonstrated high expectations for her students several times during the observed lessons. She required them to precisely show their steps in solving mathematical problems, ensured they used appropriate mathematical terms, made clear what each group should accomplish during the class period, and talked about the type of material she expected to see in their notebooks. Casey also explained that she had high expectations for herself as the teacher.
Casey said she modeled high-level performance for her advanced students by keeping the level of material high. She specifically pointed out that she “did not dumb down material” for the struggling students. She believed there were two kinds of performance – effort and the outcome of that effort. The effort was as important as the outcome and she ensured that the students knew that even if they did not quite understand the concept, their effort would help them. She further explained that “if they learned to be resilient and keep trying, they’re eventually going to get where they need to go – a lesson that goes far beyond math.” She also talked to the students about their performance as a class and thought it was important to include herself in the discussion. For example, if the class did not do well on an assessment, she talked to them about how “we didn’t do well.” She said she remediated and the students tried again; she did not “just leave things.” To illustrate the point, she explained that some of her classes stopped talking about properties after the first test, some finished the topic by December, and sometimes they talked about mathematical properties until May, but she persisted with covering properties until the students demonstrated that they understood them. She also modeled high-level performance by helping the students understand that performance was not just about how they did on the test; it also included learning the material, participating in class, and helping other students. On a more concrete level, not only did Casey model how to write the applicable steps for solving different kinds of problems, but she also modeled what the students should do at various stations in the classroom.

When asked about things she might do to provide her gifted students with a supportive learning environment, Casey indicated she was willing to accommodate her
students when they were ready to move on, although she did require that the students be able to demonstrate they could solve the problems with the applicable algebraic steps. She indicated that she sometimes had to encourage the girls to tell her when they were ready for a greater challenge because they seemed to be more hesitant than the boys. She said that she thought that “someone told the boys how smart they were in math growing up, but neglected to tell the girls.” She observed that the gifted boys seemed to be more confident than the girls even though they had comparable abilities. Casey also pointed out the importance she placed on allowing the gifted students “to be themselves in class, so that they feel safe and are able to express themselves in the way they want to.”

Casey thought it was difficult to provide the appropriate level of scaffolding for high-ability students because some of them were missing the foundations of algebra. As mentioned earlier, students in Casey’s class came from three different courses. She monitored the students informally via their homework and classwork, and tried to make up for their deficiencies by working with them after school. She said she always answered questions on homework and she sometimes created a section of the class for those students who did not understand the homework and those who did. When she provided extension activities to the latter group, she made sure it was within the same topical area because the students “had already had the scaffolding to allow them to tackle the more challenging problems.”

Meeting gifted students’ needs. Over the past few years, Casey had seen an increase in the number of students in Algebra I. She pointed out that the district did not have a quota and that placement was “really still based on the needs of the students, although some parents do push their children into Algebra I.” By giving all students to opportunity
to take Algebra I by eighth grade, Casey was concerned that parents might now view a student as being “advanced” only if they took Algebra I in seventh grade and Advanced Geometry in eighth grade, thus pushing the student into higher-level high school mathematics classes before they were socially ready.

She also indicated that the increased number of students in Algebra I impacted the gifted students because some of the students were not well-prepared for the subject. Despite the fact that the district’s revised curriculum seemed to be improving the preparation of students for Algebra I, Casey thought that the gifted students “should be able to go faster than they currently are.” She further explained that these students were “held back by the other students since the focus is on everyone, not just the gifted students.” She explained that she understood “the need for inclusion just like when special education students are included to work together,” but she thought that gifted students should be allowed to be on a single track so they could also have time just with their intellectual peers. She proposed that “they should have a gifted class during the day similar to how the special education population has a special life skills class.” She indicated she “would feel better if there was something else in place for them since they are not being served the way they need to be served.” Although she thought the school district’s gifted program was “good in theory,” she did not think it had “been as effective as it was meant to be” because it was very general. She thought that gifted students should be allowed to choose what area they wanted to work on within the gifted program because they could be gifted in different domains. She explained that the various sides of their creative personality needed to be encouraged, but if they had never been exposed to math in the gifted program, they would not “be able to decide to go in that direction.”
Casey indicated that because of this study, she planned to work with an English teacher to head up a “we have another special population in our room and we have misplaced them” committee. She indicated that “everyone worried about the other end of the population rather than the gifted students.”
Appendix X

Case Study - Melinda

Melinda had an average COS-R rating of 2.33, which placed her in the middle of teachers as far as overall effectiveness. She was a career switcher who had been teaching for a total of seven years, all of it within algebra classrooms. Her Bachelor’s and Master’s degrees were both in Economics. She was the only teacher in the study to have a gifted endorsement. In addition to taking gifted education coursework from a local university, she had also received gifted professional development from a school district other than the one to which she was currently assigned. The researcher observed two of Melinda’s Algebra I classes, each containing a mix of seventh and eighth grade students.

**Mathematical giftedness.** Melinda believed that students who were mathematically gifted were able to grasp a concept or an idea that was beyond the mechanics of simply solving the problem. They were able to see relationships that other students were not able to see. Their thinking was more abstract and at a higher level, and they could easily understand concepts that it took other students a long time to comprehend. She believed that she had students who were mathematically gifted but had not been identified as gifted. On the other hand, she did not think that all of her students who had been identified as gifted were necessarily mathematically gifted, because some of the errors they made were “not congruent with the fact that they were gifted.” She pointed out that a child who was identified as gifted in kindergarten might not necessarily be mathematically gifted by the time she saw that student in eighth grade. Similarly, as the parent of gifted children, she believed that sometimes the students typically considered to be gifted were simply the high achieving, self-driven students in the classroom. She
thought that somewhere along the line, those students had a network of people who had
instilled a high work ethic in them. She explained that it was difficult to tell whether
such students were exceptionally creative or simply high performers since sometimes
"the teacher did not have the ability to really see how far they could go and how deeply
they could think."

Pace. Melinda said that the pace for the class was "pretty much dictated by the
curriculum guide," which the teachers in her school followed very closely. When the
three Algebra I teachers in her school planned together, they divided the curriculum up so
that they could get through all the material. This made the pace quick and it was
"difficult to slow down if [they] want[ed] to get through all of the material." Despite
moving through the material quickly, Melinda pointed out that that most of the students
were hard workers who did well in the course. There were some students, however, that
were simply not ready for the material and who did not have the needed work skills and
habits. These students "suffer because of the pace and the rigor."

To "help move things along," Melinda provided notes to the students and had the
students fill in the blanks, rather than taking notes from scratch. The algebra teachers
"stayed in synch" and there was little flexibility in the pace. Melinda thought that, in
general, the teachers planned to get through too much material in a lesson, and when they
did not get to an activity, they simply had to move on. If the students did not understand
the lesson, it was up to them to see Melinda outside of class for additional help. She
indicated that she provided them the tools to understand the material, but because of the
pace, she had to move on. This was challenging for both the students and the teachers.
To determine what her students already knew, Melinda gave them various exercises and walked around to see how they were responding to the problems. She did not base her lesson plan on a formal pre-assessment because that “would be impractical to do time-wise.” She pointed out that ideally the teachers should differentiate the classroom based on a pre-assessment, but that it was “impossible because of the amount of material” they had to cover in such a limited amount of time. During one of the observed lessons, she decided to skip a portion of the notes for that day based on her informal pre-assessment. She asked the students to take two minutes and browse through the notes, highlighting the material they thought was important. She pointed out that she was going to skip some material since the class already knew it and asked the students where they wanted her to begin her instruction. By giving the students a choice, she acknowledged that she valued their time and did not want to waste it going over material they had previously mastered. The researcher observed Melinda teaching the same lesson to another class. She adjusted the pace for the whole class, spending more time on the homework and warm-up problems and less time on the example problems. When the class started the guided practice portion of the lesson, however, Melinda told the students, “If you know you’ve got it, you can move on.” This allowed them to move through the practice problems at their own pace, but she did not provide any enrichment or acceleration for the students once they had finished. She indicated that normally when her gifted students finished early, they helped other students.

**Challenge.** Melinda was ranked third in number of mathematical behaviors related to challenge. She emphasized fluency and depth of understanding, made strong connections between old and new learning, and was one of the four teachers who had
their students make generalizations about their learning. She also had the students compare methods of solution and used questioning to challenge her students. She did not, however, ask the students to recognize patterns or to define real-world problems in a way that involved mathematics.

Melinda pointed out that not only was Algebra I fast-paced, but it was already quite rigorous. During the observed lessons, Melinda did not dwell on concepts the class understood, but instead, moved to more complex problems. In several instances, she merely set up the problem so that the students could see how to do it, and did not waste time demonstrating how to actually do the math to solve it since they already understood that portion of the process. She indicated, however, that it was rare to find a student who had already mastered all the material she planned to cover, although if they understood the concepts, she encouraged them to move on independently to the more complex problems that were already built into the notes. She did not want them to spend time on things they had already mastered; instead, if they could demonstrate proficiency, they “knew to move ahead by themselves.” She pointed out that there was so much rigor and challenge built into their notes that she had not seen the need to make other modifications for the students who caught on quickly. The exception was when they started using the graphing calculator. For those students who already knew how to use that technology, she had extensions built into the lesson so that they were able to move on to other material.

Melinda indicated she also increased the level of challenge by having the students think through why a rule worked. She also explained that she tried to go deeper into various concepts, but this involved a time tradeoff, because the students had less time to
practice all the other concepts they were required to know. The researcher saw evidence of this dilemma when Melinda put a unique way to solve multiplication problems on the board. She encouraged the students to figure out why the procedure worked, but after only a few minutes, she moved on with the lesson, and many students who were enthusiastic about this challenge appeared to be frustrated that they did not have time to really think about what she had done. In addition, despite her efforts to increase the challenge, one of the gifted students appeared bored and disengaged through most of the lesson and another appeared to be working ahead on homework while Melinda explained something to the class. She later indicated that one of these students had the lowest grade in the class and that she was trying to find a way to reach him.

**Differentiation.** Melinda demonstrated very few of the mathematical behaviors related to differentiation during the observed lessons. The researcher did not note any differentiation directed specifically toward the gifted students in class. They worked on the same material, had the same homework, and were not provided with any opportunities for acceleration other than Melinda encouraging them to move to the more challenging problems at the end of the notes. Furthermore, she did not allow students an opportunity to create their own problems or provide them with a choice of activities. While none of the activities were open-ended, she did use open-ended questions to help the students think more deeply about the subject. Melinda indicated that she was occasionally able to differentiate her instruction “but those types of activities are rare and spaced out.” As an example, she said that last year she had taught a tiered lesson with three different levels which was based on the results of a previous assessment. One group worked at a basic level, one group worked on a real-world problem, and a third
group came up with an original problem. She said that she was not able to do those kinds of lessons as often as she would like because of the limited time they had in class. Although the teachers planned together for differentiated lessons, when it came time to implement them in class, there simply was not time.

On two occasions, the researcher observed activities involving groups – one with a partner and another with a group of four students. These “pods” had been assigned alphabetically. Melinda indicated that as she became more familiar with the students, she would adjust their seating based on their level of engagement or lack of engagement. As the year progressed, she would be better able to tell whether the students were adjusting to the pace and curriculum and whether they need help or not. Once she figured that out, she would pair high performers with the lower performing students. Despite going through the effort to create this arrangement, Melinda said that the class simply did not have much time for group exploration.

Likewise, Melinda did not think that there was enough time for the students to do enrichment activities during class, although she was aware of a few enrichment opportunities outside of the classroom. She indicated that some of her gifted students participated in *Odyssey of the Mind* and others were part of the *MathCounts* program. She talked about a university-hosted math competition that she thought would be ideal for her gifted students, but her school had not yet decided to participate. She was also aware of a NASA offer to mentor students, but her school had not become involved with that yet. Melinda cautioned that the curriculum was already tough and the students already had enough disruptions to their schedules without these programs.
Supportive environment. Melinda demonstrated several mathematical behaviors related to a supportive environment, particularly in relation to her questioning. She used questions to solicit responses, facilitate problem solving, to help students elaborate on their thinking, and to probe student reasoning. For example, she asked questions such as, “Why does that rule work all the time?” and “How can you prove that your method is correct?” In contrast, one of the students she believed to be gifted wanted to understand the material so deeply and asked her so many “what if” questions that she eventually gave him a pad of sticky notes on which to write his questions. In this way, she could move on with the lesson for the rest of the class and could respond to his in-depth questions when the timing was more appropriate.

She also solicited conjectures, encouraged students to solve problems in different ways, and to reflect on reasons why a solution was incorrect. Melinda provided a positive atmosphere where students felt comfortable asking questions and providing responses. In fact, on separate occasions, two gifted students corrected Margarita when she neglected to write a negative sign, and she was very complimentary toward them, noting that it was easy to forget a sign if one was not paying attention. She was also very encouraging, telling the students, “I know you can handle this,” and “Those are excellent explanations.” She was also very motivational, complimenting the class on their overall improvement in grades from the quiz to a subsequent test.

Melinda indicated she created a supportive environment for her gifted students by providing a curriculum that was rigorous enough to keep her students challenged. She explained that she allowed students the opportunity to explore various calculator functions on their own, but other than that, she had not seen a need to build in extensions
because the high-performing students seemed to be challenged enough already. She also created a supportive environment by providing scaffolding to help the students with challenging problems and by providing tips on how to study effectively.

To model high-level performance for her gifted students, Melinda said she “manifest[ed] rigor and understanding of the content” as she taught. She was extremely organized in her instruction and she had high expectations, saying such things as, “We want everyone to get an A on the quiz” and “The key to your success if how organized your work is.” She demonstrated the proper way to show her work when solving problems and she posted many resources on the school’s on-line system. By doing this, she believed the students could see that her expectations of herself were just as high as her expectations of them. The researcher also noted several instances where Melinda modeled high-level performance by making sure that the students used proper mathematical terms. For example, she pointed out that the students could only solve for unknowns, while everything else was simplifying or evaluating. She also stressed using mathematical terms such as absolute value, dividend, reciprocal, and complex fractions. She encouraged the use of such terms by telling students “we are getting closer to being mathematicians when we use these terms,” and saying “we are more sophisticated” when they used common language rather than proper mathematical terminology. She also encouraged the students to state mathematical rules properly. When talking about dividing fractions, one of the gifted students said the rule was, “Don’t cry, flip and multiply.” Margarita humorously told him “That is diaper talk. To divide fractions, we multiply the dividend by the reciprocal of the divisor.” She then explained what each
term meant. Her rapport with the students was such that they laughed at her expression and learned how to properly state the rule.

*Meeting gifted students' needs.* Melinda believed the gifted students in her classes were being adequately served. She had not received any feedback to the contrary from parents or students, nor had she sensed that the students were bored. She believed she treated them as high school students and she taught at a similarly high level.

Despite this, Melinda anticipated seeing a detrimental impact from the increased number of students being pushed to take algebra in middle school. She pointed out that there was “always an issue of how to correctly place algebra students.” Although she had not seen a large impact from increased numbers in her school district yet, she anticipated that within a couple of years there would be so many students taking algebra in her middle school that there would only be one eighth grade class that was *not* of at least an Algebra I level. She thought this might necessitate tracking the students so that there was a more rigorous class for the higher-level students and a less rigorous class for the lower-ability students.

Melinda pointed out that in her current Algebra I classes, she expected struggling students to seek extra help to come up to the current high level of the course, rather than lowering the level of the course to accommodate them. While she worked with the students and the parents of struggling students and provided resources such as homework solutions and notes via their on-line system, she was concerned that this might not be enough assistance if the level of student going into Algebra I was significantly lowered. She explained that the current Algebra I curriculum was geared toward college-bound students with a long career ahead of them in mathematics. As the number of overall
students in Algebra I increased, Melinda believed Algebra I teachers would need to adapt their teaching somewhat to cater to the students who might be focused on taking the minimum number of mathematics courses in high school. She thought that some of those students would not be ready for the rigor of algebra in eighth grade and thus might need to have a separate class. Melinda found that there was a significant jump from pre-algebra to Algebra I and that some students could not adjust. She did not necessarily think ability was the only issue. She believed it to be more the fact that some students were not independent learners, did not apply themselves, and did not have the self-discipline. In other words, some students were developmentally just not ready for algebra.
Appendix Y

Case Study - Rachel

Rachel was the second least experienced teacher observed. Her average rating on the COS-R was a 2.24, slightly below that of Melinda. She was a career switcher with a previous job in accounting, the field of her Bachelor's degree. She did not have a gifted endorsement, nor had she received any gifted professional development. Rachel taught two different types of Algebra I classes. One was for the district’s middle school IB program, which was housed at her school. The other was a traditional Algebra I class for those students not enrolled in the program. The researcher observed both types of classes. The curriculum of the IB program involved interdisciplinary study, which was evidenced in the observed lessons for this class. While students did not need to be identified as gifted to enroll in the IB program, there was a selection process involved, and so these students were generally of a higher ability than typical students. In fact, almost one-third of the IB students had been identified as gifted, but only half that number had been identified in the non-IB class.

Mathematical giftedness. Rachel believed there were several indicators of mathematical giftedness. She characterized her gifted students as “go-getters.” She thought they were generally more mature and more responsible than the other students, “but not always so, since they are still seventh graders.” She pointed out that her mathematically gifted students caught on fairly quickly and they did not struggle much with new concepts. They only needed to “see new ideas a couple of times and it clicks.” New concepts seemed to stick with them better. In contrast, she explained that students who were not mathematically gifted might appear to understand a concept when it was
taught in class, but when they tried to do the problem themselves, they did not understand it. Rachel believed there were two of her IB students who were probably mathematically gifted, but had not been identified. She pointed out that to be accepted into the IB program, the students had “to be able to think outside the box and be creative,” although that did not necessarily mean they were gifted in mathematics. She believed that students could be gifted in certain domains, and so her students who had been identified as gifted were not necessarily gifted in mathematics.

Prior to this study, Rachel had not been told which of her students had been formally identified as gifted. In fact, when the researcher originally contacted her, she referred to her entire IB class as “gifted.” She did not find out which of her students had formally been identified as such until she queried the school’s guidance counselor.

**Pace.** Rachel demonstrated several mathematical behaviors related to pace. She provided a mix of learning and practice activities and gave the class time to talk about their solutions. In both of her IB lessons, she also allowed the students a significantly greater amount of time to investigate challenging topics than did any other teacher.

To decide the pace for the class, Rachel used the district’s curriculum guide. She said the pace was “pretty much laid out” for her as far as how many blocks she should spend on a certain topic to get the whole course covered. She explained that when the curriculum guide was created, the curriculum committee sometimes worked backwards, looking at the topics with which the students typically struggled, and allocating three or four weeks for those areas. They then allocated the leftover time to the remaining topics. Rachel indicated that much of the first quarter was dedicated to review material. Although it was covered at a quicker pace than the rest of the year would be, Rachel
thought the pace should be even quicker to allow extra time for the more difficult topics
that would be taught later.

To balance the time she spent on having the students practice known topics versus
learning new topics, Rachel said she determined what the students already knew by using
questions as she taught. She asked the students how they got their answers and then
listened to their explanations. She said she could also “tell by looking at their faces”
whether they already understood the material. If they already knew the material, she
would do four or five problems and then move on. She said she had not run into the
situation where a student had already mastered the material because much of the
information in Algebra I was new to the students. She pointed out that rather than
students already knowing the material, it was “more a matter of who catches on faster.”

To adjust the pace, Rachel indicated that if the class was “doing something really
well in one area,” she would move on to the next topic, because she knew that eventually
the class would get to a topic where she would need extra time. She said she used the
students’ abilities and how well they were doing overall to help guide her pacing. She
knew that if most of the students did not seem to understand something, she had to go
back and reteach it. She emphasized to the struggling students that she had many
resources available to them through her web page and the district’s on-line system. She
pointed out that Algebra I was an adjustment for all the students because it was the first
time they had not had math every day.

Rachel found that she was able to use a quicker pace with her IB class as a whole
because they had a higher ability than her other Algebra I class. The researcher noted the
quicker pace when observing the same lesson for the IB class versus Rachel’s normal
Algebra I class. The quicker pace for this class was also driven by the fact that there were certain IB projects the students needed to complete in addition to the standard Algebra I curriculum. Rachel indicated that most of the IB students were able to keep up with the quicker pace, but some of them needed to stay after school for extra help. She said she sometimes felt like her “hands are tied.” She had to keep moving along because of the amount of material she had to cover, which meant that a few of the students might be left behind. Rachel felt it was hard to individually adjust the pace within the class because “if one student is going backward and everyone else is moving forward, the gap gets bigger.”

Although she allowed students the opportunity to move through various activities at an individual pace, any changes to the overall pace were made for the class as a whole. For example, during one observed lesson, several students finished a problem before the rest of the class, and Rachel encouraged them to move on to the more challenging problems at the end of the worksheet. However, when they finished the worksheet, rather than providing them with any extensions or enrichment, she simply allowed them to talk while the rest of the class completed the assignment.

Rachel indicated that she thought the pace was quick enough for the gifted students because they did not appear to be bored; however, the researcher noted two occasions where three of the gifted students had finished the material and had pulled out books to read. When Rachel noticed this, she redirected the students to work on homework problems. Later in the same class period, the same three students finished another activity early and again began reading books, suggesting that they may have benefitted from a quicker pace.
**Challenge.** Rachel exhibited challenge-related mathematical behaviors in several ways. She emphasized fluency and depth of understanding and she used higher-level questions to make connections between both old and new learning as well as between different content areas. In fact, she was the only teacher to have her students identify and define a real-world problem in a way related to mathematics. She told the students they were going to do an activity to answer the question, “How does my ability to solve algebraic equations help communities in another country?” This activity was part of a larger water conservation project the IB students were addressing in several of their classes. For this lesson, the students were asked to figure out how the technology design cycle (investigate, plan, create, and evaluate) related to mathematics. Once they correlated their mathematical problem-solving technique to the design cycle, they worked with a partner to use this design cycle to actually solve a math problem. They then shared their answers with the class. This activity required the students to think abstractly and at a high level. Rachel pointed out that she had not run into the problem of needing to give the IB students additional time for these complex problems because one of the reasons she raised the complexity in the first place was because the students had moved quickly through the other material.

Rachel indicated that to make standard mathematical tasks more challenging, she included “things that might trip the students up,” such as incorporating order of operations into the problem. She challenged students to put into words what various algebraic solutions meant and what type of information they needed before they could solve various problems. She also solicited various solution methods, asking, “Can anyone do it a different way?” Furthermore, Rachel also raised the level of challenge by
asking students to solve problems that required tying together two different mathematical concepts. For example, rather than just having the students simplify the expression as required on the homework (which entailed leaving $x$ in the answer), she had the students actually solve for the variable (determine what $x$ stood for) which significantly increased the level of difficulty of the problem because the students had to use their knowledge of geometry and create an equation that would allow them to solve for the unknown quantity. Similarly, during a discussion of real numbers, one of the gifted students asked what an “unreal number” would be. Rachel gave the example of the square root of -1, and then asked the students to figure out why that number was imaginary (you can’t multiply anything by itself to end up with a -1). By challenging the students to reason out a concept to which they had never been exposed, she significantly raised their level of thinking. Although Rachel used these techniques to increase the level of challenge, she sent a mixed message when she did not address the most challenging problems on the students’ homework, telling the class that she was satisfied if they could do the easier ones.

**Differentiation.** Other than grouping and providing her IB students with the open-ended design cycle activity, Rachel did not demonstrate any mathematical behaviors related to differentiation. The students were not provided with any choices, they all worked on the same material, they all had the same homework, and they were not accelerated beyond the problems contained within the lesson. She explained that “true differentiation is when you have different levels of instruction going on in the classroom at the same time.” She stated that she did not have to do that in Algebra I because they had a specific curriculum they needed to get through. Rather than teaching at different
levels, Rachel explained that if there was a student who was struggling, he or she needed to get help from her outside of class and to make use of the resources she had made available. On the other hand, if one of the students was catching on significantly quicker than the rest of the class, she would find something to engage him or her, although the researcher never saw this during the observed lessons. In fact, Rachel indicated that she was considering asking the principal to allow her to use her higher-ability students to help students in another seventh grade class who need remediation.

Rachel believed that the material she used in her IB class allowed the students to “self-differentiate.” She pointed out that the IB class, which contained the majority of her gifted students, “could take a project like the design cycle and really spend a lot of time on it and take it as far as they want.” This was evident in the fact that some groups delved much deeper into the design cycle activity than did others. Because of the structure in the IB class, she felt she was able to give everyone “opportunities to run with the material and engage in higher-order thinking,” although she admitted that she was sometimes unsure of what higher-order thinking really was. She expressed the fact that she liked the IB program because “it gives the gifted students the opportunity to shine.”

Rachel said that she used various criteria to group her students. She did not let them choose groups because she was afraid they would just choose their friends and not work. Sometimes she assigned groups at random by counting off and sometimes she simply mixed the students up. She tried to make sure there was a mix of higher- and lower-ability students, although she was not sure whether that was always optimal. She wanted to try grouping two middle-ability students with two lower-ability students, but she was concerned that they would get behind. She said she looked for the gifted students to
work with their peers to help explain concepts that they do not understand. During the
observed lessons, the students worked with assigned partners and later in groups of three
or four. Rachel selected the gifted students to be the leaders of each group and most of
them could be seen explaining their ideas to the group. They appeared to relish the role.

Rachel did not believe there was a need for enrichment within the class based on the
level of challenge and rigor that was already built into the Algebra I curriculum. She
pointed out that the students in the IB Algebra I class had enrichment activities built into
the program, but she did not think her non-IB class could do similar projects due to time
constraints. She indicated such activities would require her to compact her other lessons,
and she did not think the students could afford to miss out on the instructional time.
Rachel pointed out that MathCounts provided “a perfect opportunity” for enrichment
outside the classroom. She worked with the math honor society president at the local
high school to help coach the program. Rachel also indicated the students could
participate in the weekly gifted program, although she was not aware of what they did
that was specifically related to mathematics.

Supportive environment. Rachel demonstrated many mathematical behaviors
related to providing a supportive environment for her gifted students. She effectively
used questioning to solicit different solution methods, probe student understanding, press
the students for explanation, and facilitate problem solving. For example, when students
became confused on the design cycle problem, Rachel asked them, “What is it that you
are really trying to solve?” This helped them to put the problem into the proper context.
During all three observed lessons, she also solicited conjectures and had the students
reflect on the reasoning behind incorrect solutions. Of note, she was the only teacher to
build new mathematical knowledge through simulated or real-world problem solving as was evidenced through her design cycle activity.

When asked about how she thought she provided a supportive environment for her high-ability students, Rachel said that she instructed at a pace that that was comfortable for these students. Rather than slow the pace of instruction down, Rachel indicated that if the lower-ability students found the pace harried, they needed to use the outside resources she provided, do their homework, and come in for tutoring.

Rachel modeled high level performance by encouraging the students to use mathematical terminology, such as *identity solution*, when explaining their answers. She also insisted they use the use correct mathematical phrasing, such as “five times the quantity of $x$ plus 2,” pointing out that “five times $x$ plus 2” would give a totally different answer. Furthermore, Rachel explained the rationale for each step of a problem by explaining the property that allowed one to conduct that operation. She also gave students other ways to approach a problem to encourage their higher-order thinking. As she demonstrated the different solution methods, she explained why each procedure worked. She said she realized that by demonstrating alternative methods, the average students might become confused, but she encouraged those students who understood the alternative methods to go ahead and use whatever method made the most sense to them. She pointed out, however, that it was important that the students were able to solve the problems in a way that demonstrated the basic skills that would be tested on the end-of-course standards test. For example, the students needed to be able to identify the equation associated with a word problem even though they might be able to reason out the solution to the word problem without creating an equation.
To scaffold her instruction for her high-ability students, Rachel indicated that she started with the basics of a concept and told the students they were going to build on that knowledge, encouraging them to not make it more difficult than it really was. When she taught a lesson, she tried to put herself into her students’ position, asking herself what they already knew at that particular point that might help them with the new concepts. She would then gradually build to more complex problems. She indicated that she “never start[ed] from the top and work[ed] down.” Rachel further explained that she encouraged the students by telling them they were capable of doing more complex problems or else she would not have asked them to even attempt them. She explained that if a concept was too far above the students’ level, it was a waste of time to try it because the students just became confused. She admitted that there were a few students who could handle more complex concepts, but that she “could not go further with them in the classroom because there just isn’t time.” Although Rachel said she was available to stay after school to give students more challenging problems, she indicated that she had never had a student take her up on her offer.

Rachel also stated that she had high expectations of the students, indicating that she knew her students were “capable of moving to a higher level, and so [she] takes them there.” When a group of students got off-task, she told them, “This is disappointing,” and when several students did not do their homework, she became somewhat exasperated since she had repeatedly shown them where she had posted resources to help them. She once again showed the students where the material was located on the computer, while admonishing them for not using these additional sources of help. While Rachel demonstrated high expectations of her students in several ways, she diluted that message
when she told the class that she would be satisfied if they could do all but the most challenging problems on their homework.

Rachel also presented a rather confusing message about her approachability. On one hand, there were several occasions when she was complimentary toward the students, saying, “That was a good question!” or “What you said was crucial.” On the other hand, her somewhat brusque demeanor appeared to be rather intimidating to some of the students. While this did not preclude students from asking questions, the researcher did not observe the “give-and-take” dynamic seen in some of the other teachers’ classrooms.

Meeting gifted students’ needs. Rachel believed that, in general, the gifted students in the IB program were being adequately served, although she did not feel like she could speak for the gifted students who tried, but did not get into the program. She pointed out that these students were eligible to attend the weekly gifted program although that program was very small at her school.

Looking specifically at mathematics, Rachel did not believe the move toward placing more students into middle school Algebra I had impacted the gifted students, although she indicated that she tended to want to slow down to make sure all the students understood the material. Since she did not have much time to give struggling students one-on-one help during class time, they needed to come after school for help. She pointed out that she had not lowered the level of her Algebra I class to accommodate these struggling students. Instead she pushed them up to the level of the rest of the class or else they decided to move out of the course on their own. A teacher could also recommend students be moved into a lower-level mathematics course and the school supported their decision. Prior to doing so, Rachel indicated she would contact their
former mathematics teachers and the guidance counselor to learn about their background. She would also contact the parents and encourage the students to make use of the available resources. She pointed out that even though her school district was in the process of revising the curriculum to better prepare students to take Algebra I in middle school, “realistically, some students will not be ready for algebra at that point.”
Appendix Z

Case Study - Hillary

Hillary had been a teacher for eight years and had taught algebra for the same amount of time. Her Bachelor’s degree was in Mathematics, while her Master’s degree was in Education. She did not have a gifted endorsement, nor had she experienced any professional development related to gifted education. She had recently been selected as the Mathematics Department Chair at her school. Although Hillary was evaluated as the second least effective teacher in the study, her average rating of a 2.11 on the COS-R indicated that she was somewhat effective in her instruction. Because Hillary only had gifted students in one class, the researcher observed the same class on three separate occasions.

Mathematical giftedness. Hillary believed that good grades, good study habits, good scores on the standards test, and good scores on the district’s quarter test were all signs of mathematical giftedness. She also thought that mathematically gifted students knew how to study and completed their homework. She believed that some students were gifted only in a certain domain such as English or mathematics. She pointed out that she had students in her class who were particularly strong in mathematics, but were weaker in English, and because of their poor verbal skills, they were not identified as gifted. She suspected that there were five or six of her students who were mathematically gifted, but had not been identified as such.

Pace. Hillary had the fewest mathematical behaviors related to pace of all the teachers observed. She adjusted the pace for the class as a whole during two lessons, and although she allowed the students to work through the guided practice problems on their
notes at their own pace, once they were finished, they just sat there. It was especially
apparent that the pace was too slow for her two gifted students during one of the observed
lessons. While they both initially paid attention to the subject matter, they quickly
completed the practice problems and became bored. As the lesson wore on, Hillary had
to redirect them several times for disturbing the students around them. Eventually Hillary
ended up giving one of the gifted students lunch detention due to his excessive talking.

Hillary indicated that the state and county decided the pace for her class. There was
a certain amount of material she had to cover, and just enough time to cover that material,
so “the pace was pre-determined.” She did not feel like she had much flexibility. She
generally kept the overall pace of her Algebra I classes the same so that they covered the
same material on the same day, although she did adjust the pace within each class as a
whole based on student needs. If one class had more questions on homework or needed
more time to go through the notes than another class, they might spend less time on the
activity for the day. She had found that her Algebra I classes were generally at different
levels. In the higher level class, she was able to adjust the pace to get through the
activities because the students did not have as many questions. In the lower class, she
covered as much as she could. She pointed out that she was always able to cover the
objectives; it was just a matter of how much practice each class received. She indicated
that she taught the material “as a whole class” and did not adjust the pace for individual
students. She also pointed out that she liked having mixed seventh and eighth grade
classes because it brought the eighth graders “up to a different level,” but it also made the
class more difficult because the eighth grade students had more questions on material
they did not understand and it occasionally held the seventh grade students back. She cited the fact that it sometimes took up to 30 minutes to go over homework in class.

To determine what the students already knew, Hillary said that she was aware of what material was covered prior to Algebra I, although because students came into her class from different courses, some students were more familiar with certain topics than others. This meant that some of the material was a review for some students, but Hillary thought it was important that they all be taught the material in the same way so they could build off it later in the course. The researcher only observed Hillary checking for prior knowledge and adjusting the lesson on one occasion.

To balance the time she spent between practicing known concepts versus learning new concepts, Hillary quickly went through the material she thought the students already knew. She estimated that she spent about 30% of the class reviewing and practicing old material, and approximately 70% of the time covering new material. She pointed out, however, that “time is a critical factor.” She characterized her time constraints as having “no time to even breathe.” She pointed out the fact that Algebra I was the first mathematics class the students had that met every other day. The course contained new material and was “a big jump from their previous mathematics.” She felt that it was “ridiculous trying to cover a week’s worth of material in two or three days,” especially for the eighth grade students. Hillary also thought that the block schedule forced her to cover too many concepts within one lesson. For that reason, the teachers in Hillary’s school planned to suggest that eighth grade Algebra I be double blocked the following year, meaning that it would meet every day and they could cover the material at a slower
pace. They did not think a double block should be an option for seventh grade Algebra I students since those students were working two years above grade level.

**Challenge.** Hillary was ranked as the second lowest teacher as far as her demonstration of mathematical behaviors related to challenge. Although she made connections between old and new learning in each of her lessons, there was only one lesson in which she emphasized fluency and depth of understanding. She also only asked challenging questions on one occasion although she did challenge the students to come up with different methods of solution during two of the lessons.

Hillary pointed out that time was always an issue. Sometimes the students ran out of time when working on a challenging problem in class and so she would add the problem to their homework. She explained that she simply did “not have the time to allow students to ponder a single question for 30 minutes.” She expressed the importance of rigor and relevance and indicated she “would rather have one good question than 10 that are insignificant,” and although questioning was an area she wanted to improve, she did not have time to “tweak the lessons or think about doing that.”

When asked how she made mathematical tasks more challenging, Hillary said that she used cooperative learning. She also did not let the students use calculators at the beginning of the year because she wanted to ensure they understood the procedures. She thought the content was “already rigorous enough without the teachers really doing anything to make it more difficult.” She further explained that she did not think she needed to make the problems more challenging for her gifted students because they were “challenged right along with the rest of the class.” She indicated that she could take the problems to a higher level by making the problems longer, increasing the number of
fractions, increasing the amount of distribution that needed to be done, increasing the
number of negatives, or having students create their own problems, but she had not run
into the situation where she felt like the students were at such a high level that they
needed that.

The researcher, however, saw evidence that the class may not have been as
challenged as Hillary portrayed. As mentioned in the previous section, during one of the
observed lessons, it was apparent that the gifted students were bored. In fact, the
researcher heard one of them mumbling, “It’s easy.” During that lesson several students
quickly finished the problems, but Hillary did nothing to increase the level of challenge
for those students who easily understood the material. In fact, she told the students, “If
you’re done, I need your lips sealed and your patience,” something that appeared to be
difficult for the gifted students to do. Furthermore, there were several instances where
she simply told the class how to do something, rather than allowing them to discover it on
their own. At one point, she told the students she was “going to start you off because I
know you are going to make a mistake” when referring to a problem with negative
fractions. By doing this, she missed an opportunity to allow the students to demonstrate
whether they already understood the material and were ready for an increased challenge.
In fact, in a later lesson, the students demonstrated their ability to rise to a challenge
when Hillary introduced them to literal equations (equations with several variables). In
contrast to the previous lessons, all of the students were engaged and because they did not
have time to do very many problems, Hillary selected the most difficult problems for the
students to solve with her.
**Differentiation.** Hillary demonstrated the fewest mathematical behaviors related to differentiation of all the teachers. She found the term *differentiation* "to be vague since it could mean many things." The only behavior related to differentiation that the researcher observed was when Hillary had the students work within a group to discuss their answers to a set of problems. During all three observed lessons, the students worked on the same material in class and had the same homework. Hillary indicated that she did not do tiered lessons because she "did not have the time." In addition, the observed lessons did not contain any choice or open-ended activities, although she did ask open-ended questions to the entire class during one of the lessons.

When asked what type of enrichment opportunities were available for her high-ability students, Hillary talked about a video project she had the entire class do for a quiz grade. The project required the students to pick a topic, create a storyboard, and then teach the concept. Hillary also indicated that the three Algebra I teachers provided enrichment for all the students by teaching a word problem unit enriched with geometry concepts. This was not part of the material that was required under the state standards, but because the students had so much difficulty with word problems, the teachers thought it was important to focus on them. Hillary was not aware of any other enrichment options outside the classroom, but expressed a desire to learn more about what might be available.

Hillary also discussed the fact that because most of the concepts in Algebra I were new to the students, there had been very few times when she felt she needed to make modifications for students who already understood the concept. She stated that she "probably could have done something a little more enriching for the high-ability
students” during the first unit since it was mostly review material, but after that, the material was new, even to the gifted students. She did not often have time to enrich the material she was covering in class, and she did not feel a need to do so because “the high-ability students are challenged by the concepts as they are.” She also pointed out the fact that the algebra teachers needed to teach extra material because they had to address both the old and new sets of state standards as the new standards were phased in. The following year, they would only have to address the new standards, which would give the teachers “more breathing room to provide additional enrichment.”

Hillary talked about several different grouping arrangements. She indicated that she put the students into “pods” of four students for cooperative learning. Each student had a partner within the group. To determine the partners, Hillary listed the students by the highest to lowest grade and then matched the highest person in the class with the lowest and so forth, so the two average students ended up together. She grouped them in this way so that the higher students could “pull the lower students up.” She pointed out that she liked to give the students opportunities to teach each other. When the group of four worked together for cooperative learning, there were two higher and two lower students in one pod. She indicated that she changed their groups every four to six weeks.

**Supportive environment.** Hillary demonstrated more behaviors related to a supportive environment than she did with pace, challenge, or differentiation. When compared to the other teachers, however, she ranked last, largely because she did not use questioning techniques, solicit conjectures, or press students for explanation as often as the other teachers. As mentioned in the **Challenge** section, Hillary acknowledged that her questioning was an area that needed work.
Despite these areas of weakness, Hillary did provide a supportive environment in several ways. For example, she modeled high-level performance for her students, explaining that anything she expected from her students, she did herself. She emphasized the importance of the students showing all their work and she modeled this each time she solved a problem. As she worked through the problems, she explained why she took certain steps and why the process worked so the students understood both the concept and the reasoning behind it. Hillary also modeled using proper mathematical terminology in class and had a vocabulary wall in her classroom for the students’ reference. She indicated that she would stop students in the middle of a sentence to ask, “What do you really mean by that?” to ensure they really understood what they were saying. She found than emphasizing vocabulary made a difference in student performance. In addition, she gave them tips on how to highlight their notes and on how to create good study habits so they would have those skills when the material became more difficult.

Hillary was very approachable and it was apparent that the students enjoyed having her as a teacher. The students seemed very willing to ask and answer questions, and she was complimentary, saying things such as “It looks like you are going a good job of helping each other.” The students did not appear to be embarrassed when they made a mistake, and Hillary made a point to explain that mistakes were learning experiences. For example, one of the gifted students gave an incorrect answer, and rather than simply telling him he was wrong, she prompted him to examine his response by saying, “Describe to me what’s happening.” In this way, she was able to hear him verbalize his reasoning and correct his misperceptions. She also encouraged the students to rework the incorrect problems on their test so they could reflect on the thought process that led them
to the incorrect answer. She was available to help students both during lunch and after school.

Hillary provided scaffolding for her students by helping them to build on their prior knowledge. When introducing literal equations, she asked the students, “What does this look like?” and then compared the steps to what they had previously learned about two-step equations. She encouraged them to use their logic and what they had already learned to help them solve the new type of problems, and had them reflect on why they had just done a certain step. Hillary also showed the students different methods to solve various types of problems and allowed them to use whatever solution process worked best for them. Furthermore, she demonstrated areas where students typically made mistakes so that the students were aware of what to look for.

Hillary also expressed her high expectations for her class. In one of the observed lessons, she told the students that they should be studying algebra for an hour a night, and when only a few students raised their hands to indicate they were doing that, she said “All your hands should be up.” She also talked to them about how she expected them to turn in their homework on time, and made comments such as, “we’re all going to get 100s on this quiz,” and “I want you to start thinking on your own,” to relay her expectations to the students. Furthermore, Hillary had the students reflect in writing on how they planned to improve different portions of their grades.

Meeting gifted students’ needs. Hillary believed the gifted students were being served in Algebra I. She thought the class was challenging in and of itself, whether a student was working two years above grade level or not. She pointed out that Algebra I was the first class for many of the students “to understand what true math is and really
get challenged for the first time.” She explained that while the gifted students might not be as challenged as the other students, they still were challenged because there was a very significant leap between pre-algebra work and Algebra I.

She was, however, concerned with the recent push to place more students into Algebra I in middle school. She indicated that over the past three or four years the school district had lowered its standards for a student to be enrolled in the course. The teachers were told that if the students met the minimum requirements, they should be allowed to try Algebra I, although the teachers did have a significant input into the process. Hillary’s school was compared to other schools in the district because they had a lower percentage of students in Algebra I than the other schools. The school district had told her school they wanted them to have only two eighth grade classes below Algebra I the following year, which essentially “would turn Algebra I into the new eighth grade math class.”

Hillary pointed out that the teachers in her school made their recommendation for advancement into Algebra I based on whether they truly believed a student was ready for the course while the other schools in the district went strictly by whether a student met the minimum requirements. The criteria was such that a student only needed a C in the previous year’s Foundations of Algebra 2 course and a passing grade on the standards test to meet the requirements for Algebra I even though they were taking it a year early. Hillary did not think it made sense to move ill-prepared students into Algebra I since “it is the foundation class for the rest of their career.” She commented that if they wanted students who met the minimum requirements to be successful in the course, they would
need to double block it so that they met twice as often as students in Algebra I currently did.

Despite the more rigorous stance Hillary’s school took in placing students into Algebra I, Hillary noted that there was still a growing number of students in the course who were not prepared. The number of students who attended her after-school help sessions had “increased significantly” over the past year. She indicated that the other Algebra I teachers had noted the same thing. She had “an abundance of meetings with parents” and believed that those students who could not keep up with Algebra I in middle school should be filtered out of the course. In fact, she noted that between 10 and 15 Algebra I students had dropped out of the course the previous year.

Despite the increase in the number of struggling students, Hillary did not let these students hold back the other students during class. She said that in essence, putting more students into Algebra I in middle school hurt the struggling students rather than the high-ability students because she required the lower students to come up to a certain standard, rather than lowering the standard for the rest of the class. Hillary pointed out that all of the Algebra I students in her school had passed the Algebra I end-of-course standards test each year.
Appendix AA

Case Study - Kelly

Kelly was a first year teacher, although she already had her Master’s degree in Education. Her Bachelor’s degree was in Mathematics. She did not have a gifted endorsement or any gifted professional development. As a new teacher, Kelly was still learning classroom management techniques and honing her pedagogical skills. She was evaluated as the least effective Algebra I teacher observed, with an average rating of 1.83 on the COS-R. She was the only teacher whose COS-R rating was lower than somewhat effective. Because Kelly only taught one gifted student, the researcher observed the same class on three separate occasions. The class consisted solely of eighth grade students. All of the other Algebra I classes the researcher observed contained a mix of seventh and eighth grade students or only seventh graders. Because of Kelly’s limited teaching experience and the fact that the students in her class were slightly older, she provided the researcher with a useful contrast to the other teachers.

**Mathematical giftedness.** Kelly believed that mathematically gifted students were “motivated to learn, try hard, and to do what they were supposed to do,” pointing out that it was “not so much that they are smarter than everyone else.” She thought that some students were gifted in general as opposed to being mathematically gifted. Kelly indicated that there were four other students in her class that had not been identified who were probably mathematically gifted. She pointed out that “they definitely have the drive and that natural math talent, and they seem to just naturally get things better.”

Kelly had not been told which of her students were gifted; in fact, it was not until Robin showed her the list of gifted students from the school’s guidance office, that she
realized she even had a gifted student in class. She was rather surprised by this because the gifted student was not very motivated and frequently did not do his homework. She indicated that she did not know the criteria by which he was identified as gifted, but if he put for more effort, he could do much better.

**Pace.** Kelly demonstrated the third highest number of pace-related mathematical behaviors of the observed teachers. She had a mix of practice and learning activities, she gave the students time to figure out and present solutions to problems, and she adjusted the overall pace of the class. For example, during one lesson, she had originally planned to do a group activity, but since the class had many questions on the previous night's homework, she skipped the activity and focused on reviewing the concepts covered in the homework.

Although she adjusted the pace for the class as a whole, Kelly generally kept the students within the class at the same pace, stating that “because it's an advanced class, if they can't keep up with it, they really shouldn't be in there.” She pointed out that there were a few students who were a little slower than the rest of the class, and although she was willing to give the other students things to work on while the slower ones completed their work, she did not allow them an inordinate amount of extra time. On the other hand, she said there were consistently a few students who seemed to understand concepts quicker than everyone else, and when they finished their work, she gave them additional problems or activities such as Sudoku to work on. She also pointed out that they sometimes helped the other students who were still working. During one lesson, the researcher observed the students working through problems at an individual pace during a station activity, but once they finished the work, Kelly told them that they “should work
Most students interpreted this to mean they could start talking. The gifted student did not finish his work and he did not appear to be engaged in the lesson.

To decide the pace for the class, Kelly looked at the amount of time the textbook suggested spending on each topic. She pointed out that although this was her first year of teaching, she had completed a field experience in an algebra class, and so she could sense the areas with which the students were likely to have difficulty. She felt she knew “what may trip them up,” which gave her an indication of which type of problems she would need to spend more time covering. She also indicated that the district’s curriculum guide drove her pacing in the fact that it showed what she needed to accomplish during each quarter.

To determine what her students already knew, Kelly looked at past quizzes and tests to identify where the majority of the class struggled. She made sure that she continued to stress that topic until they understood it. She stated that even if the majority of the class understood the material, she still tried to go over the problems with which only a few students struggled. She also pointed out that her warm-up problems were generally a review of the concepts they had covered in the previous class. These concepts eventually led to the material for the present lesson. In that way, she pre-assessed the students. She indicated that she had not yet been faced with the situation where she had needed to modify the lesson to accommodate students who had already mastered the material she planned to cover during a lesson because the material was new to most students.

When asked how she balanced the time she spent on practicing known concepts with learning new concepts, Kelly said that she looked at how well the students understood the material. If they were able to do a certain type of problem, she moved on to more
difficult problems or provided them with examples of problems they were likely to solve incorrectly. If the students struggled with a concept that was essential to understand in the long run, she would do her best to find time to review the skills. However, she pointed out that if the skill was something they were never going to see again, she would not spend too much time going over it.

Kelly expressed the opinion that she wished the class met every day rather than every other day, because the students did not seem to be comfortable with the schedule. She pointed out that last year, the students had an hour and a half of mathematics every day, but because Algebra I only met every other day, the students had “not figured out how much they really need to do on their own.” Kelly also said she wished she had the time to take a week out of the curriculum just to deal with fractions because they were an essential building block and many students were not comfortable with them.

**Challenge.** Kelly demonstrated a significantly lower number of mathematical behaviors related to challenge than any of the other teachers, and in fact, the overall level at which the class was taught was much lower than that of the other classes the researcher observed. As was previously noted, this was the only class made up entirely of eighth grade students, so they may not have been as advanced as the classes with seventh graders or a mix of seventh and eighth grade students. It should be noted, however, that this class was still considered a year ahead of grade level in that particular school district. The gifted student was disengaged during most of the three observed lessons.

Kelly said that to modify mathematical tasks to make them more challenging, she turned numerical problems into word problems or she gave the students writing assignments where they actually had to explain what they did. She gave the example that
students might be able to determine that a problem had an identity solution or no solution, but they struggled to explain in words what that really meant. In addition, during one of the observed lessons, Kelly increased the level of challenge by tying two different concepts — geometry and equations — together. To deal with the additional time that was required to solve more complex problems, Kelly either had the students finish the problems for homework, or she modified her plan for the next lesson so they could finish the problem in class. Kelly said she generally made problems more challenging for the class as a whole, rather than making them more challenging for some students and not others. By challenging the students who did not understand the material as well, she believed that she encouraged them to think at a higher level. Despite her attempts to increase the level of student thinking, she sent a mixed message to the students. During another observed lesson, rather than encouraging students to challenge themselves, Kelly told the students to not even attempt one of the homework problems because she had not yet covered the material.

**Differentiation.** Kelly demonstrated few of the mathematical behaviors related to differentiation, although during one of the observed activities, she allowed the students a choice of rectangles for an area equation activity. She also allowed students to create their own problem during another lesson. Nonetheless, none of the students were provided with acceleration opportunities during the lessons the researcher observed; all of the students worked on the same material and had the same homework. When asked about enrichment activities, Kelly could not think of anything pertaining to enrichment in her classroom, but did mention that students could participate in *MathCounts* after school. She also indicated that some of her students had applied to a technology institute
sponsored by NASA. The program involved Saturday workshops where students could see how mathematics was applied in the real world.

When asked about how she differentiated for her high-ability students, Kelly indicated that she could not think of anything in particular that she did to differentiate specifically for the gifted student in her class. She did, however, talk about how she constantly balanced and adjusted what she taught in class. She made modifications to her lessons based on what happened when she taught the material to another class. She determined what she could have done differently, and she thought the classes she taught later in the day benefited from the modifications. Kelly said she differentiated “by coming up with more than one activity to support a concept.” If there was a concept that certain students did not understand, she came up with another activity to help support them, pointing out that “if they don’t get it one way, they might get it another way.”

During one of the observed lessons, Kelly allowed the students to work in groups of five or six students based on their current seating. She indicated that she tried to arrange the students so that at least one person in each group could “take control of things” and was motivated to do the various activities. She found that she did not need to tell the more advanced students to help the others; they just did it on their own. She pointed out, however, that if the students did not act mature enough, she did not do group activities. Kelly also said she frequently allowed the students to work with a partner of their own choosing. She found that when students selected someone they got along with, they generally stayed on task and helped each other.

*Supportive environment.* Kelly demonstrated several behaviors related to providing a supportive environment to her students. She was very approachable and it was obvious
that the students felt comfortable asking and answering questions. She encouraged the
students to participate, saying, "I want everyone to get this," and when the students
worked in a group she encouraged their collaboration. She spent a portion of each lesson
walking around and individually helping students and she remained very patient when
she repeatedly had to explain the same thing to various students. However, because
Kelly did not have much experience in the classroom, her classroom management skills
were still developing, so much of her time was also spent trying to quiet the students
down and focus them on the material at hand. Because of that, the students who wanted
to learn may have been somewhat distracted. While she did a commendable job of
demonstrating supportive behaviors such as soliciting various solution methods and
questioning the students to help them solve problems, it was apparent that she, too,
became distracted by the disruptive students.

When asked how she provided a supportive environment for her high-ability
students, Kelly said she gave her students a chance to work on problems individually in
class so if they needed extra help, they had the opportunity to ask her. She also stayed
after school to help the students who were having difficulty. Kelly talked about the
importance of "starting off with the basics to make sure everyone understands them and
has the skills to move on to the next point." She said she made it a point to cover the
more difficult problems in class as well as any questions she anticipated the students
would have. The homework she gave the students was not as difficult as the problems
they did in class. Kelly also believed that if the students could discover concepts on their
own, they would understand them a little better. Consequently, she frequently asked
them what the next step in a problem might be so that they started thinking, "rather than
just regurgitating" what she said. She also provided scaffolding by relating the problem at hand to those they had done in the past.

When asked how she modeled high-level performance for her advanced students, Kelly focused instead on what she had the students do to improve their own performance. For example, she said she gave the students assignments where the problems had been worked out incorrectly, and then required them to determine what was wrong, explain the mistake, and then work out the problem correctly. In this way, the students could see the type of things she was looking for when she graded their problems. She also pointed out that she took off points if the students did not show their work because she wanted to see how they derived their answers. She pointed out that this was especially a problem with the more advanced students.

Despite the fact that Kelly could not think of examples about how she, individually, modeled high level performance, the researcher noted that she consistently demonstrated to the students how to properly write each step of a problem and she explained each step of the process. She also solicited various ways to solve problems and encouraged the students to use the method that made the most sense to them. She was, however, inconsistent in her expectations of her students. During one of the observed classes, she spent several minutes talking about how she expected the students to make a better effort at home, and if they did not understand something, she expected them to stay after school. She told them if they were not willing to put in the effort, “they had no business being in the class.” She also pointed out the fact that Algebra I was a high school class, and that whatever grade they received from her would be on their high school transcript. In contrast, on a homework worksheet, she told the students to not even try the last
problem since they had not covered it, rather than encouraging them to give it a try. In
addition, Kelly’s use of questioning techniques was inconsistent. At times, the questions
she asked were very basic, and on several occasions, when she asked higher-level
questions, she answered them herself.

Meeting gifted students’ needs. Kelly believed the needs of the gifted students were
being met although she felt that there were definitely students in her Algebra I classes
who did not belong. She pointed out that some teachers recommend placing students into
Algebra I if they passed the standards test, “even though the students had not really
mastered the skills they needed to be successful in algebra.” She thought there was no
reason for the struggling students to be in Algebra I since it was an advanced class. She
indicated that there was “nothing wrong with being on grade level, but the parents had a
big impact on pushing their students into algebra.”

She thought that moving more students into Algebra I in middle school hurt the
gifted students somewhat “because if a class has several students who are not up to the
level of the rest of the class, the teachers can’t just let them fail.” The teacher needed to
do whatever she could so that the struggling students were successful too. Kelly stressed
that the teacher could not let these students “fall through the cracks.” She pointed out
that this might mean that the teacher had to spend extra time to go over topics that she
knew the gifted students already understood, but it was up to the teacher to “make sure
that everyone gets it.” Because of this focus on the struggling students, Kelly thought the
gifted students were “held back a little bit as to what they could really do.” She pointed
out that if the material became too complex, she would completely lose a few of the
students, which necessitated her changing what she had originally wanted to do in the
class. She was concerned for these students because if they became "lost and frustrated, they won’t have any confidence at all.”
Vita

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