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The Relative Effectiveness of Three Methods of Teaching an Elementary Mathematical Task.

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THE RELATIVE EFFECTIVENESS OF THREE METHODS OF TEACHING AN ELEMENTARY MATHEMATICAL TASK

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Abstract

The present study compared the performance of 64 third year students on measures of initial learning, retention, and immediate and delayed transfer under three methods of classroom instruction. Subjects were divided into three experimental groups and one control group (Treatment C), which were balanced for high and low verbal ability. Teaching methods, which included an expository approach (Treatment E), a guided discovery approach (Treatment GD), and a discovery approach (Treatment D), differed primarily in terms of sequence characteristics and presentation of principles to be learned. Experimental subjects were presented 4 days of instruction concerning the area of a rectangle by the experimenter, who adhered to operationalized definitions of teacher behavior under each teaching condition.

Hypotheses included: (a)Treatment E would produce superior results to Treatments GD and D on tests of initial learning. (b) Treatment GD would produce superior results on tests of retention and transfer, followed in order by Treatments E and D. (c)Girls would perform superior to boys under Treatments GD and D. (d) The performance of boys under Treatment E would be superior to that of boys under Treatments D and GD. (e)Treatment GD would produce superior results on measures of attitude toward arithmetic, followed in order by Treatments D and E. Hypothesis (a) was supported at the .01 level for high-verbal-ability subjects. Results failed to support hypothesis (b). Hypothesis (e) was slightly supported on post-test measures. Hypotheses (c) and (d) regarding sex were neither supported nor rejected due to unexpected confounding of results by the verbal ability factor. In addition to consideration of the above findings, interesting results yielded by high-verbal-ability subjects on measures of learning performance, as well as implications for future research and educational practice were discussed.

THE RELATIVE EFFECTIVENESS OF THREE METHODS OF TEACHING AN ELEMENTARY MATHEMATICAL TASK

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Of major concern in the field of education is the development of the most efficient methods of imparting knowledge in the classroom. Educators, such as John Holt (1964), have criticized traditional approaches of educating a child, those employing didactic methods, for their failure to produce learning that is permanent, or relevant, or useful. Holt (1964) emphasizes the importance of a method which allows children to make their own discoveries, using their own procedures, and asking their own questions for producing more lasting and useful understanding. He further stresses the importance of concrete operations in the elementary education curriculum, because the language used in a didactic approach to instruction may not "make sense." Holt's criticisms seem most closely related to the controversial issue of "discovery learning" in educational psychology.

The controversy over the relative effectiveness of "discovery" and "expository" methods of teaching is revealed in the contrasting viewpoints of Gagne, Bruner, Ausubel, and Friedlander. Bruner is perhaps the strongest adherent of the discovery method. He defines discovery as "...a matter of rearranging or transforming evidence in such a way that one is enabled to go beyond the evidence so reassembled to additional new insights (Bruner, 1961, p. 22)". Bruner's first hypothesis is that a person who learns through discovery techniques is better able to organize information, and this information is more readily available in problem solving. Secondly, Bruner (1961) hypothesized that discovering what is to be learned will encourage the child to "...carry out his learning activities with the autonomy of self-reward, or more properly by reward that is discovery itself (p. 26)." His third hypothesis is that through practicing the discovery techniques one will learn the "working heuristics of discovery" and will be able to generalize the discovery approach into a style of problem solving that will serve any task that he encounters. Finally, he hypothesized that material learned through discovery will be "more readily accessible in memory."

Friedlander (1965) views discovery learning less favorably, especially when applied to children. He acknowledges the motivating effects of learning by discovery; the fact that it "...capitalizes on the very strong reward value of bringing order, clarity, and meaning to experiences that were previously disorderly (p. 28)." He further acknowledges the facilitating effect the discovery approach has on retention through involving the student as an active participant in his own instruction; but Friedlander suggests many disadvantages characteristic of discovery learning. These include: (1)the ease with which one can pursue highly unproductive trains of thought through errors of logic and reasoning (2)forgetting what is learned through discovery unless it is successfully assimilated (3)failure to aid the student in synthesizing what he has learned on his own and helping him incorporate this new knowledge into an orderly

> abstract subject matter, particularly prior to adolescence. ...It is also indispensible for testing the meaningfulness of knowledge and for teaching the scientific method and effective problem solving skills. Furthermore, various cognitive, and motivational factors undoubtedly enhance the learning, retention, and transferability of meaningful material learned by discovery (p. 143).

Despite his acknowledgment of the above stated advantages, Ausubel feels the most crucial point at issue is whether, considering the great time required for learning by discovery, it is a feasible method for transmitting knowledge to "cognitively mature" students who have already mastered the "rudiments and basic vocabulary" of a specific discipline. He believes that a meaningful expository approach is more efficient than and as effective as a discovery approach, provided the learner is able to assimilate this knowledge into his existing cognitive structure. Contrary to the hypothesis of Bruner regarding the heuristics of discovery, Ausubel states that "...critical thinking ability can only be enhanced within the context of a specific discipline (Ausubel, 1963, p. 153)." He further questions the feasibility of teaching principles of inquiry to elementary school children due to the level of abstractness involved. Like Friedlander, Ausubel also questions the ability of elementary school children to reason logically, because of their subjectivism, their tendency to jump to conclusions, to overgeneralize, and to consider only one aspect of a problem at a time. In reply to Bruner's first hypothesis, Ausubel feels that learning by discovery will only lead to better organization, transformation, and use of knowledge if "...the learning situation is highly structured, simplified, and skillfully programmed ...(Ausubel, 1963, p. 160)." Regarding Bruner's second hypothesis, Ausubel proposes that "...discovery learning is more often associated with extrinsic motivation than is reception learning (Ausubel, 1963, p. 163)." Lastly, Ausubel questions Bruner's hypothesis of the value of discovery in facilitating retention.

In the opinion of the experimenter of the present study, Ausubel and Gagne view the discovery method in a more practical light than does Bruner. The discovery method has both advantages and disadvantages as an approach to acquiring information. The research literature on discovery learning not only provides support for discovery as a superior method of instruction, but it also provides evidence to the contrary.

Difficulty arises in comparing results of studies on discovery learning because of the failure of most studies to provide an operational definition of the teaching methods employed. The review of the literature to follow will equate discovery approaches with an inductive approach to learning in which examples precede the discovery of a principle, and expository teaching will be equated with a deductive approach, in which the general rule or principle is provided prior to application of the rule. The majority of studies conducted to assess the effectiveness of discovery learning have been short-term experiments. Those involving long-term experimentation include Boeck (1951), Beckland (1968), Worthen (1968), Karle (1960), Michael (1949), Wiesner (1969), and Klechner (1969).

Evidence supporting the discovery method is cited below. The results of a number of studies involving formal subject matter, including mathematics, and employing students as subjects, have supported the hypotheses of superior performance of "discovery" groups on retention and transfer measures (Ray, 1961; Crabtree, 1966; Worthen, 1968; Winch, 1913; Werdelin, 1966a). Bassler (1968) on the basis of a pilot study in which ten second grade subjects were taught positive and negative integers, hypothesized that subjects given maximal guidance would show superior performance on transfer measures when compared to the performance of subjects given intermediate guidance, while subjects given intermediate guidance would show superior performance on achievement measures. He further hypothesized that there would be no significant differences between the performance of intermediate and maximal guidance groups on retention measures if the mathematical concepts were related to physical situations. Using abstract card material and elementary school subjects, Scandura (1964) found that subjects under the discovery condition showed superior performance on transfer measures; this effect was more pronounced with more complex transfer items. Ninth and tenth grade subjects under the discovery condition showed superior performance on early transfer measures in an experiment involving number series problems (Gagne and Brown, 1961). The results of an experiment by Guthrie (1967) involving a coding task and college students as subjects indicated superior performance under the discovery condition on early transfer measures. Haslerud and Meyers (1958) found that subjects in the minimally directed group continued to make gains on transfer items on a coding task with increased periods of time.

Further evidence supporting the discovery method is indicated in the findings of an experiment conducted by Evans (1967) to assess the effect of high and low achievement motivation on the performance of college students on decoding cryptograms. Results indicated that high-achievers and students of high intelligence under the discovery condition show superior performance on measures of initial learning. Price (1967) found that tenth grade subjects taught mathematics under a discovery method showed a significant positive attitude change, while a control group showed a negative change in attitude. In a study by Kersh (1962) using a mathematical task, the discovery group exhibited increased motivation. In a study by Becklund (1968) in which third, fourth, and fifth grade students were taught principles of vectors, results indicated that activity oriented materials presented with teacher guidance produced superior performance on measures of convergent thinking, independent study skills, and the ability to answer questions related to subject matter content; groups using activity oriented materials independent of teacher guidance showed superior performance on measures of divergent thinking at the fourth grade level. Although the results of the above mentioned studies indicate superior performance of subjects using a discovery approach, other studies have found evidence to the contrary.

In a long-term study by Klechner (1969) involving the teaching of mathematics to ninth and tenth grade low achievers, results favored a conventional approach over a discovery method on measures of initial learning, retention, and transfer. Rowell, Simon, and Wiseman (1969) found similar results in an experiment in which college students were taught "stable cognitive schemata." In each study the experimenters credited the results to subjects' extensive experience with reception learning. Initial learning and retention were significantly favored by an expository method in studies by Ter Keurst and Martin (1968) involving number series and fourth grade subjects and Kittell (1957) involving word tasks and sixth grade subjects. Kersh (1958) found superior performance on measures of initial learning produced by a directed approach in teaching an arithmetic task to college students. Grote (1960) found results which favored the expository approach on measures of initial learning by eighth grade students who were taught principles of mechanics. Results of this study indicated that discovery learning was favored by low ability subjects. In an experiment involving word relations, Craig (1953) found results which favored an expository approach on measures of retention and transfer and measures of transfer only (Craig, 1956). An expository approach to teaching number series produced superior performance on measures of initial learning in a study by Scandura, Barksdale, and Durnin (1969). The experimenters suggested that these results may be due to the averse effects of failure on the discovery program. Using Katona's match task, Corman (1957) found that superior performance was produced by an expository approach, but the discovery group made greater gains with increased time. Results of this study also indicated that low ability students tended to learn relatively better using the discovery approach. Wittrock (1962) found that subjects using an expository approach performed in a superior manner when compared to subjects using a discovery approach on a transfer task involving coding.

In addition to studies which support a discovery approach and those whose results favor expository teaching, many studies have found generally non-significant differences between the two different approaches. Included in this category are long-term studies involving regular classroom material (Wiesner, 1969; Michael, 1949; Boeck, 1951; Karle, 1960). Wolfe (1963) found no significant differences between expository and discovery approaches on measures of initial learning when teaching ninth and tenth grade subjects a programmed mathematics task. Yarbroff (1963) found no significant differences between egrule and ruleg approaches in teaching college undergraduates elementary statistics. High ability eighth and ninth grade subjects performed equally well under conditions of expository and discovery approaches when learning a number sequence task (Meconi, 1967). The results of an experiment by Werdelin (1966) indicated no significant differences in the performance of eighth grade subjects on measures of initial learning, transfer, and retention after they had been taught a foreign alphabet under either an expository or discovery method. Similar results were obtained when students were taught principles of geometry by an expository or discovery approach (Nichols, 1956). Tanner (1969) obtained non-significant differences in the performance of ninth grade subjects taught physical science principles under expository-deductive, discovery-inductive, and unsequenced discovery conditions. Results of this study indicated that girls and high intelligence subjects were favored by the discovery treatment, whereas boys and low intelligence subjects were favored by the expository treatment. Results of a study by Hermann (1971) in which fifth and ninth grade subjects were taught tasks involving both principle and concept learning using ruleg and egrule programmed instruction, indicated no significant differences in performance under the two methods. Results of this study did indicate that the egrule approach is more suited to students of high intelligence.

It is quite apparent from the review of the literature on discovery learning that no consistent set of results has emerged. Extensive reviews by Hermann (1969) and Tanner (1969) have credited such equivocal results to failure to control for confounding variables, poor experimental design, and inadequate statistical analysis of data. Confounding variables include: (1) employing two different instructional media with different mediums for each treatment (Kersh, 1962) (2) failure to control the degree of interaction with the teacher (Winch, 1913; Ray, 1961) (3) failure to control for sources of variability introduced in year-long classroom studies (Boeck, 1951; Karle, 1960; Wiesner, 1967; Klechner, 1969) (4) allowing discovery groups a greater amount of time (Gagne and Brown, 1961; Scandura, 1964; Wittrock, 1963) (5)failure to control the number of examples given to each experimental group (Gagne and Brown, 1961) (6)failure to equate the amount learned by different groups (Craig, 1956; Gagne and Brown, 1961; Kersh, 1958; Wittrock, 1963) (7)failure to equate the degree of meaningfulness for each group (Scandura, 1964). Problems in experimental design include: (1)inadequate experimental analysis (Haslerud and Meyers, 1958; Guthrie, 1967) (2)failure to control verbal communication between subjects (Kersh, 1958) (3)including material which is too difficult to be acquired without a given principle (Kittell, 1957) (4)failure to control treatment of prerequisite material for each group (Scandura, 1964) (5)failure to control amount of guidance given (Kersh, 1958, 1962) (6)failure to control time between learning and testing sessions (Kersh, 1958).

In addition to the above mentioned confounding variables and inadequacies in experimental design, few studies on discovery learning have employed operational definitions of teaching methods used. The present study will define the three different methods of teaching to be employed as follows, similar to the definitions employed by Worthen (1968).

Expository Method: Verbalization of the required concept or generalization is the initial step in the instructional sequence by which the concept or generalization is to be taught. The mathematical principle is presented to the student and explained verbally using concrete illustrations. The student works with examples of the principle or generalization only after the initial verbal preparation. Particular attention is given to insure that practice is made meaningful by continual stress being placed upon the relation of the example of the generalization and upon "why" the genralization operates as it does. This is to minimize rote memorization of the principle by the student.

Guided Discovery: Verbalization of each concept or generalization is delayed until the end of the instructional sequence by which the concept or generalization is taught. The student is presented with an ordered, structured series of examples of a generalization. The sequence of presentation maximizes the possibility of the student formulating awareness of the generalization more readily than if the examples are randomly presented. The experimenter presents the students with a series of carefully structured questions to help the student recall relevant concepts and lead the student to the discovery of an underlying principle.

Discovery Method: The concept or generalization is not verbalized. The student is presented with an ordered, structured series of examples of a generalization. The sequence of presentation maximizes the possibility of the student formulating awareness of the generalization more readily than if the examples are randomly presented. No explanation of the examples is given, nor is there any hint that there is an underlying principle to be discovered. The student, merely instructed to solve the problem, is expected to acquire the mathematical concept, principle, or generalization through inference of his own.

Teacher behavior under each condition will be defined by a limited number of specific behaviors to which the teacher must conform.

Of primary interest in research on discovery learning is the exploration of subsets of subjects in an effort to determine for whom discovery learning is most appropriate. Subject variables include age, sex, intelligence, verbal ability, familiarization with discovery techniques, previous experience with subject matter to be taught. Ausubel (1961) suggested that the discovery technique is appropriate for children approximately below the age of 12...in the concrete stage of development...during the elementary school years. Crabtree (1966) further emphasizes the appropriateness of discovery learning in the elementary school. Hermann (1969) on the basis of a study using ninth and fifth graders which produced non-significant results regarding teaching method, proposed an investigation using third or fourth grade subjects, suggesting perhaps Ausubel's approximation was too high. Ausubel (1961) states

> In the absence of prior discovery and non-verbal experience, children approximately below the age of twelve tend to find directly presented verbal constructs of any complexity unrelated to existing cognitive structure and hence devoid of potential meaning. Until they consolidate a sufficiently large working body of key verbal concepts based on appropriate experiences, and until they become capable of directly interrelating abstract propositions without reference to specific instances, children are

closely restricted to basic empirical data in the kinds of operations they can relate to cognitive structure (p. 20).

In light of the above contentions the present study used elementary school children at the third year level as subjects. Concrete manipulation of objects was employed in the tasks design, and verbal ability of the subjects was compared with performance under each teaching condition.

The results of a recent study (Tanner, 1969) indicated that girls performed best after using the discovery treatment while boys performed best after using the expository treatment. The effects of sex of subjects on performance under different teaching methods was also considered in the present study. A number of recent studies (Hermann, 1971; Worthen, 1968; Tanner, 1969) have emphasized the averse effect the lack of experience in discovery techniques may have on the performance of subjects under discovery conditions. Subjects in the present study had experience with discovery techniques as part of their regular school cirriculum, therefore such adverse effects should have been minimized.

To allow for greatest generalizability of results from a study on discovery learning the selection of an appropriate experimental task is important. The task should be meaningful and related to the type of learning which would take place in the school curriculum. It should be at an appropriate level of difficulty for the subjects being tested. As Roughhead and Scandura (1968) pointed out:

... if a person already knows the desired response, then

> he is not likely to discover another rule by which such repsonses may be derived, even if he has all of the prerequisites and is given an opportunity to do so... (p. 288).

Cronbach (1966) believes that discovery learning has more of a place where the body of knowledge is more rational, e. g., mathematics, but has less place in learning situations which do not fit into any system of mutually supporting propositions. In addition to the fact that mathematics lends itself to a discovery approach, it is an area in which many young children experience difficulty. For these reasons, a mathematical task was used in the present experiment.

Another important matter for consideration is the organization of the material to be learned. Both Ausubel (1963) and Gagne (1965) emphasized the importance of structure for meaningful learning to take place. Gagne (1965) in particular, emphasized the importance of heirarchies which define the prerequisite knowledge required to learn a higher-order concept or principle. Discovery learning can take place in a structured framework. It is believed by the experimenter that in a realistic classroom situation a minimal amount of structure is necessary for even the most autonomous learning to take place. Subjects under each teaching condition in the present study received equal amounts of training on prerequisite task and engaged in the same sequence of activities. The structured sequence employed in the present study was not as rigid as programmed instruction and allowed for varying degrees of teacher interaction.

The purpose of the present study was to assess the relative effectiveness of the discovery method of instruction in a normal classroom setting under highly controlled conditions. A mathematical task was administered to third year subjects at a local elementary school. Independent variables included verbal ability, teaching method, and sex of subjects. Dependent variables included measures of attitude, acquisition, transfer, and retention.

Despite the highly inconsistent results obtained by past studies of discovery learning, the following inferences seemed plausable on the basis of results of studies which have employed a relatively high degree of control:

(1) The expository method (E) would produce results superior to the guided discovery (GD) and discovery (D) methods on tests of initial acquisiton.

(2) The GD method would produce superior results on tests of retention and transfer of knowledge, followed in order by D and E methods.

(3) Girls would perform superior to boys under the GD and D treatments.

(4) The performance of boys under the E method would be superior to that of boys under the D and GD methods.

(5) GD and D methods would produce superior results to E method on measures of attitude toward arithmetic. GD would produce superior results to D on measures of attitude toward arithmetic.

Method

Subjects. Sixty-four third year elementary school students attending Rawls Byrd Elementary School in Williamsburg, Virginia served as subjects for the present study. The curriculum at Rawls Byrd is somewhat more progressive than that found at other schools in the area. There is greater emphasis on individualized approaches to teaching, more exploratory activity by the students, team teaching, and other less traditional methods of instruction. It was suggested in an earlier study (Worthen, 1968) that short term studies should favor expository instruction, while long term studies should favor discovery learning. This conclusion was based on the assumption that students have typically been taught using an expository approach and greater time is needed to develop the technique used in the discovery method. Of necessity, the present study involved only four days of learning presentations. It was assumed that students who had experience with exploratory activities as part of their curriculum would be familiar with some discovery techniques and would, therefore, not require additional training to learn such techniques. The use of such students would optimize the possibility of results which favored a discovery approach in a short-term study.

Ausubel (1961) suggested that the discovery approach has its greatest relevance for teaching children "...approximately below the age of twelve;....during the elementary-school years; ...for children who are still functioning at Piaget's level of concrete operations (p. 23)." In a study using example-rule and rule-example teaching methods with fifth and ninth grade students, Hermann (1971) found results which generally favored the rule-example method. He suggested an investigation which used students from grades three and four. The present study used students in the third year of elementary school as subjects because (1)they met with the suggested requirements of Ausubel (1961) and Hermann (1971) and (2)it was assumed that children at this grade level would have had enough experience in the classroom to follow instructions regarding a new teaching method.

Students were divided into four groups: (1)a control group (C) (2)an expository group (E) (3)a guided discovery group (GD) (4) a discovery group (D). All third year students in Team 4 at Rawls Byrd were given the verbal ability section of the SRA Primary Mental Abilities Test and a pretest designed to assess previous knowledge of the experimental tasks. Of those who indicated no knowledge of the principles involved in the experimental tasks, 32 students having the highest verbal scores and 32 students having the lowest verbal scores were selected as subjects. Scores of both High and Low Verbal subjects were ranked and numbered consecutively from 1 to 4. The four groups, including 8 high and 8 low verbal ability subjects per group, were formed by assigning every "High" and "Low" "1" to group 1, every "High" and "Low" "2" to group 2 and so on. The mathematical ability level of each student was assessed by assigning each student to one of four levels according to the level of instruction he was receiving in school. The sex of each subject was also recorded.

<u>Apparatus</u>. Teaching and testing materials directly related to the experimental tasks were developed by the experi-

menter after a thorough investigation of methods of teaching the concept of area and the formula for finding the area of a rectangle (Biggs and MacLean, 1969; Beilin and Franklin, 1962; Dienes, 1966; Esgard, 1969; Grossnickle, Bruekner, and Reckzeh, 1968; Houghton-Mifflin, 1967; Lovell, 1962; Piaget, Inhelder, and Szeminska, 1960; Wertheimer, 1945). Teaching materials consisted of a carefully sequenced set of tasks, similar to the heirarchical structure of mathematical instruction suggested by Gagne (1965). The heirarchical structure used in the present study is illustrated in Figure 1. It is believed that an ordered sequence , including prerequisite tasks, is necessary for effici-

Insert figure 1 about here.

ent instruction by both expository and discovery methods.

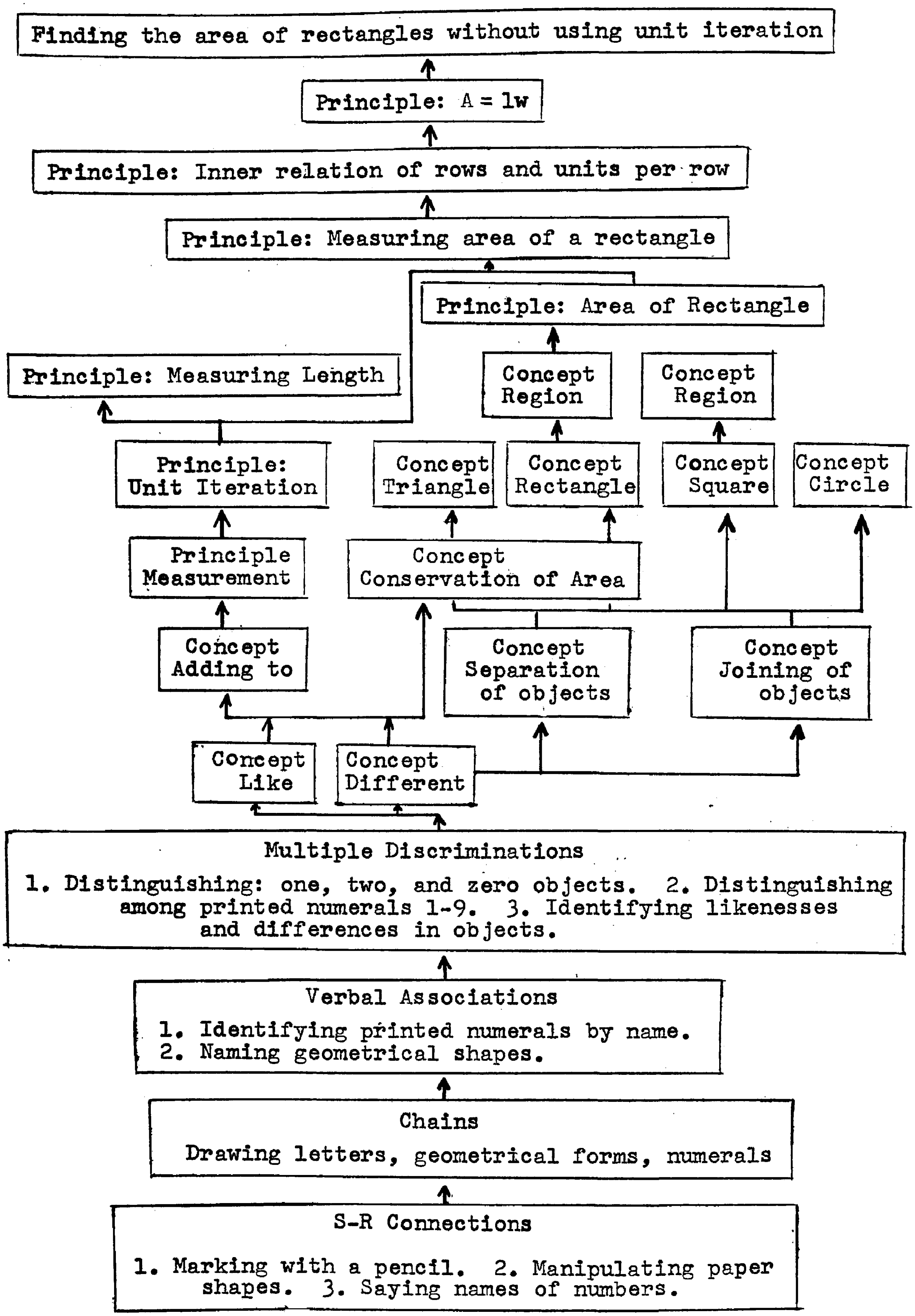
Pretest Materials

The verbal ability section of the SRA Primary Mental Abilities Test was used to assess verbal ability. A ten-item pretest was used to assess the students' knowledge of the material in the experimental tasks. The pretest included two items from Piaget, Inhelder and Szeminska (1960) designed to test conservation of area. Other items tested classification of two-dimensional shapes, including recognition and labeling, labeling the region of a rectangular shape, linear measurement, unit iteration, selection of the most appropriate unit for measuring a rectangular shape, and measuring a rectangular shape without employing unit iteration.

FIGURE 1

Heirarchical Structure of Instruction for

Teaching Area of a Rectangle


Finding the area of rectangles without using unit iteration
Principle: A = lw
Principle: Inner relation of rows and units per row
Principle: Measuring area of a rectangle
Principle: Area of Rectangle
Principle: Measuring Length
Concept Region
Concept Region
Principle: Unit Iteration
Concept Triangle
Concept Rectangle
Concept Square
Concept Circle
Principle Measurement
Concept Conservation of Area
Concept Adding to
Concept Separation of objects
Concept Joining of objects
Concept Like
Concept Different
Multiple Discriminations
1. Distinguishing: one, two, and zero objects. 2. Distinguishing among printed numerals 1-9. 3. Identifying likenesses and differences in objects.
Verbal Associations
1. Identifying printed numerals by name.
2. Naming geometrical shapes.
Chains
Drawing letters, geometrical forms, numerals
S-R Connections
1. Marking with a pencil. 2. Manipulating paper shapes. 3. Saying names of numbers.

Teaching Materials

As indicated above, the experimental tasks concerned finding the rule for the area of a rectangle. The particular mathematical task used in the present study was selected because it lended itself to a discovery approach involving concrete manipulation of objects. Also, students in their third year of elementary school are usually taught this rule during the final lessons of their arithmetic program. As a control measure, teachers of third year students in Team 4 agreed not to teach the unit presenting the area of a rectangle until the present study had been completed.

The learning program included a series of sequenced tasks designed to be presented during a four-day period. All tasks involved manipulation and measurement of rectangles of various sizes which had been cut from colored construction paper. Some tasks included comparison of surface area of objects such as a table top and a chair top and books. Initial measurement was done with units of various shapes, including squares, triangles, rectangles, and circles. In later tasks, only the unit square was used. In the final task, only a ruler marked in inches was used. The tasks were presented to students in each experimental group in "packet" form. Each packet contained a problem sheet and necessary materials, including various colored rectangles.

Posttest Materials

Posttest included a 14-item acquisition test and a 14-item transfer test. The acquisition test contained some items similar to those found on the pretest, measuring knowledge of prerequisite tasks, and other items similar to problems presented during the learning sessions, measuring understanding of principles taught in the learning sessions. The transfer test included items measuring varying degrees of transfer of knowledge from the original tasks. Similar forms of the transfer and acquisition tests were also used to measure retention.

Dutton's Arithmetic Attitude Scale (Dutton, 1956) was used to measure the attitude of students before and after the learning sessions. It was necessary to revise certain statements in the scale using a simpler vocabulary which could be more easily understood by elementary school children.

Observer Rating Scale

The behavior of the experimenter during the learning sessions was rated by four independent observers to assess any biases the experimenter may have exhibited in her interaction with students in a particular group. The rating scale was constructed from selected items of the Classroom Observation Record of the Teacher Characteristics Study (Ryans, 1960). Raters were trained using the description of items found in the scale.

Procedure.

Pretests

The pretests were administered by the experimenter, with the aid of regular classroom teachers, to all third year students in Team 4 approximately five weeks prior to presentation of the learning sessions. The pretest were administered to students in groups of 30 on three consective days, one session each day.

All testing took place in a regular classroom, The experimenter introduced herself explaining:

> I am trying to find out the best way to teach arithmetic to children, and some of you may be selected to help me with my study. I am also trying to find out if how large a vocabulary you have will affect the best way to teach you arithmetic. I will now give you a short test about words. I will then give you a test to see how much you know about arithmetic.

The verbal section of SRA Primary Mental Abilities Test was administered first. The pretest relating to the experimental task was then administered. Students were told that it was very important that they do their own work.

To control for any bias that might have been caused by experimenter knowledge of students' performance on the SRA verbal test and the pretest, each student was assigned a number prior to testing. The number was placed on the front of each test booklet, and the student's name was placed on the back. The experimenter selected students for the four groups previously mentioned on the basis of test scores using the students' numbers only. When the groups were assigned, an assistant matched the names of the students with their respective numbers.

Learning Sessions

Learning sessions were conducted by the experimenter during a four-day period. Each session lasted approximately 40 minutes. All experimental groups were instructed on the

Although the same sequenced instructional material was given to each student in each experimental group, the experimenter's teaching behavior under each teaching condition varied. In an effort to maintain essential differences between each treatment condition, model teaching behaviors under each condition with regard to (a)interjection of teacher knowledge (b) introduction of generalization (c)method of answering questions (d) method of eliminating false concepts was adapted from Worthen (1968). A summary of the prescribed teaching behavior for each treatment on each of the above characteristics in given below.

Interjection of Teacher Knowledge:

Expository: The teacher acts as the primary source of knowledge concerning arithmetic. The students may depend on the teacher when they cannot work a problem. The teacher checks the answers of the students. When an incorrect answer is given, the teacher recognizes it and immediately asks the student if he is certain that his answer is correct. This gives the student an opportunity to correct his own mistakes. The teacher always indicates that he will show the student how to work the problem correctly. If the student is unable to work the problem, the teacher shows him how the correct answer is obtained by use of the principle involved.

Guided Discovery: The teacher does not act as the primary source of knowledge concerning arithmetic. The teacher will not immediately acknowledge an incorrect answer. She may go back a short time later and call the student's attention to it by asking the student if he is sure his answer is correct. If the student cannot work a problem, the teacher will ask leading questions that will aid the student in finding the solution. The teacher will not directly tell the student the correct answer to the problem.

Discovery: The teacher does not act as the primary source of knowledge concerning arithmetic, but seems to depend upon the students to help him work the problems. When a student gives an incorrect answer, the teacher does not acknowledge it immediately. If the student fails to notice the mistake, the teacher goes back a short time later, as if he has just noticed it, and questions the correctness of the earlier response. The teacher checks it by the long method, as if he is not aware of the principle which allows for solution by a "short-cut." The student who gave the response is allowed an opportunity to correct it. If he is unable to do so, the teacher goes through the example, to make sure the student understands the instructions involved, but does not give the answer.

Introduction of Generalization:

Expository: The teacher gives the generalization (rule) before the students are given examples. All examples are then related back to the rule for solution.

Guided Discovery: The teacher delays the verbalization of the generalization until the end of the learning session. He is careful to give no hints that there is a "short cut" to working the problem. He also takes care to avoid the use of vocabulary terms related to the generalization, during the learning sessions.

Discovery: The teacher does not verbalize the generalization.

He is careful to give no hint that there is a "short cut" to working the problem. He also takes care to avoid the use of vocabulary terms related to the generalization, during the learning sessions.

Method of Answering Questions:

Expository: The teacher answers questions by reiterating and explaining the rule and relating it to the question. The teacher then gives examples which will further clarify the way the rule is used in the solution of that type of problem.

Guided Discovery: The teacher answers questions asked by the student by referring to the problem that the student is finding difficult. If the student is still confused, the teacher takes him back through the problem carefully. The teacher may make use of sequenced questions as a clue, but no verbal hint to the rule is given.

Discovery: The teacher answers questions by referring to the problem that the student is finding difficult. If the student is still confused, the teacher takes him back to the problem to make sure the student understands the instructions. The teacher gives no verbal hints regarding method of solving the problem.

Method of Eliminating False Concepts:

Expository: The teacher warns the students of common errors made in applying the principle. She points out specifically the types of problems on which the students are likely to make errors and then gives examples of each kind of error.

Guided Discovery: The teacher includes "trap questions" and gives no verbal warning of any type. If the problem is missed, the teacher does not acknowledge the error immediately. When the teacher acknowledges the error, she checks it to make sure it is wrong. The teacher asks the student questions to aid him in the discovery of why the answer is incorrect.

Discovery: The teacher includes "trap questions" and gives no verbal warning of any type. If the problem is missed, the teacher does not acknowledge the error immediately. When the teacher acknowledges the error, she says nothing about the rule or why the answer is incorrect. She gives no aid in discovering why the answer is incorrect.

Selected portions of each learning session were video-taped for review by independent observers. The experimenter was not aware of the intervals during each session which were taped.

Posttest and Transfer Test

During a three day period immediately following the final learning session, all students in each group, including the three experimental groups and the control group, were given a posttest measuring acquisition and a transfer test. Students were asked to do the best they could on all their work. Four weeks after the final learning session, students were given a similar form of the same posttest again, as a measure of retention. Transfer items were also included in this test.

Attitude Scale

Dutton's Arithmetic Attitude Scale (Dutton, 1956) was given to each student prior to the first learning task presentation. Students were instructed to respond "yes" to each statement that described the way they felt about arithemetic and "no" to each statement that did not describe the way they felt about arithmetic. The Arithmetic Attitude Scale was again administered as part of the posttest and post-posttest series. The scale was presented orally by the experimenter, and, if necessary, unfamiliar terms were explained.

Experimenter Behavior Rating Scale

A 15 minute video-taped presentation of a learning session under each teaching condition (a total of 45 minutes of video tape) was shown to four independent raters. Before viewing the video-taped presentations, the raters were trained using descriptions of each item in the scale. They were then shown the video-taped presentations and asked to rate the experimenter's behavior with regard to the specific items on the scale according to their observations.

Results

Initial learning and retention data, as well as immediate and delayed transfer data, were subjected to 4 x 2 x 2 analyses of variance. Attitude data were subjected to a 4 x 3 x 2 analysis of variance, and data of observers ratings of affective teacher behavior were analyzed by using a simple analysis of variance. In cases where additional analysis was necessary, Tukey's test of multiple comparisons, or orthogonal or simple t-tests were used. The data of subjects who did not complete all tests, including measures of initial learning, retention, and immediate and delayed transfer or who did not complete all attitude measures were not included in the above stated analyses. Results regarding teaching methods will be discussed first, followed by consideration of verbal ability and sex variables. Finally, results of subjects' attitudes toward arithmetic and observers ratings of affective teacher behavior will be presented.

Teaching Method

Data yielded by the test of initial learning did not support the hypothesis that Treatment E would produce results superior to Treatments D and GD on measures of initial learning. Results showed no significant differences among teaching methods, including the control condition, on measures of initial learning. Non-significant results yielded by transfer and retention measures failed to support the hypothesis that Treatment GD would produce superior results on tests of retention and transfer, followed in order by Treatments D and E. As indicated in the analysis of variance summary Table 1, the only significant dif-

- - - - - - - - - - - - - -

Insert Table 1 about here.

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ferences among teaching methods was yielded by combined scores of initial learning and retention ($F = 4.02$, $df = 3/42$, $p < .05$). Further analysis using Tukey's t-test revealed that performance under Treatment D was clearly inferior to performance under Treatment E ($q = 5.08$, $df = 46$, $p < .01$) and under Treatment GD ($q = 4.46$, $df = 46$, $p < .05$) on combined scores of initial learning and retention. Combined mean scores for each treatment condition are presented in Table 2.

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Insert Table 2 about here.

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TABLE 1

Summary of Analysis of Variance of Learning Data

Source	df	MS	F
Between Subjects	49		
Teaching Condition (A)	3	27.65	4.02 *
Verbal Ability (C)	1	123.35	17.92 **
A X C	3	6.22	0.90
Subj. w. Groups	42	6.88	
Within Subjects	50		
Time of Testing (B)	1	14.00	3.16
A X B	3	6.43	1.45
B X C	1	21.32	4.81 *
A X B X C	3	4.28	0.97
B X Subj. w. Groups	42	4.47	
Total	99		

* $p < .05$

** $p < .01$

TABLE 2

Combined $\bar{X}$ Correct Responses on Measures of Initial Learning and Retention

	Teaching Condition			
	E	GD	D	C
$\bar{X}$	8.23	7.91	5.58	6.96

Verbal Ability

Verbal ability exhibited a greater effect than any other factor considered in the present study. As indicated in Table 1, data yielded by combined measures of initial learning and retention showed significantly superior performance by high verbal ability subjects ($F = 17.92$, $df = 1/42$, $p < .01$) across all teaching conditions. Also, there was a significant interaction between verbal ability and initial learning and retention ($F = 4.81$, $df = 1/42$, $p < .05$). Figure 2 shows that high verbal ability subjects performed significantly superior to low verbal ability subjects on measures of retention ($q = 4.88$, $df = 46$, $p < .01$). It is interesting to note that there was a tendency for high

----

Insert Figure 2 about here.

----

verbal ability subjects to show superior performance on measures of retention as compared to performance on measures of initial learning. There was also a tendency for high verbal ability subjects to perform superior to low verbal ability subjects on measures of initial learning. Combined data from measures of immediate and delayed transfer yielded significantly superior performance by high verbal ability subjects ($F = 7.79$, $df = 1/42$, $p < .01$). As indicated in analysis of variance summary Table 3,

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Insert Table 3 about here.

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this was the only significant effect yielded by transfer data.

FIGURE 2

Mean Correct Responses on Measures of Learning Performance for High and Low Verbal Ability Subjects as a Function of Time of Testing

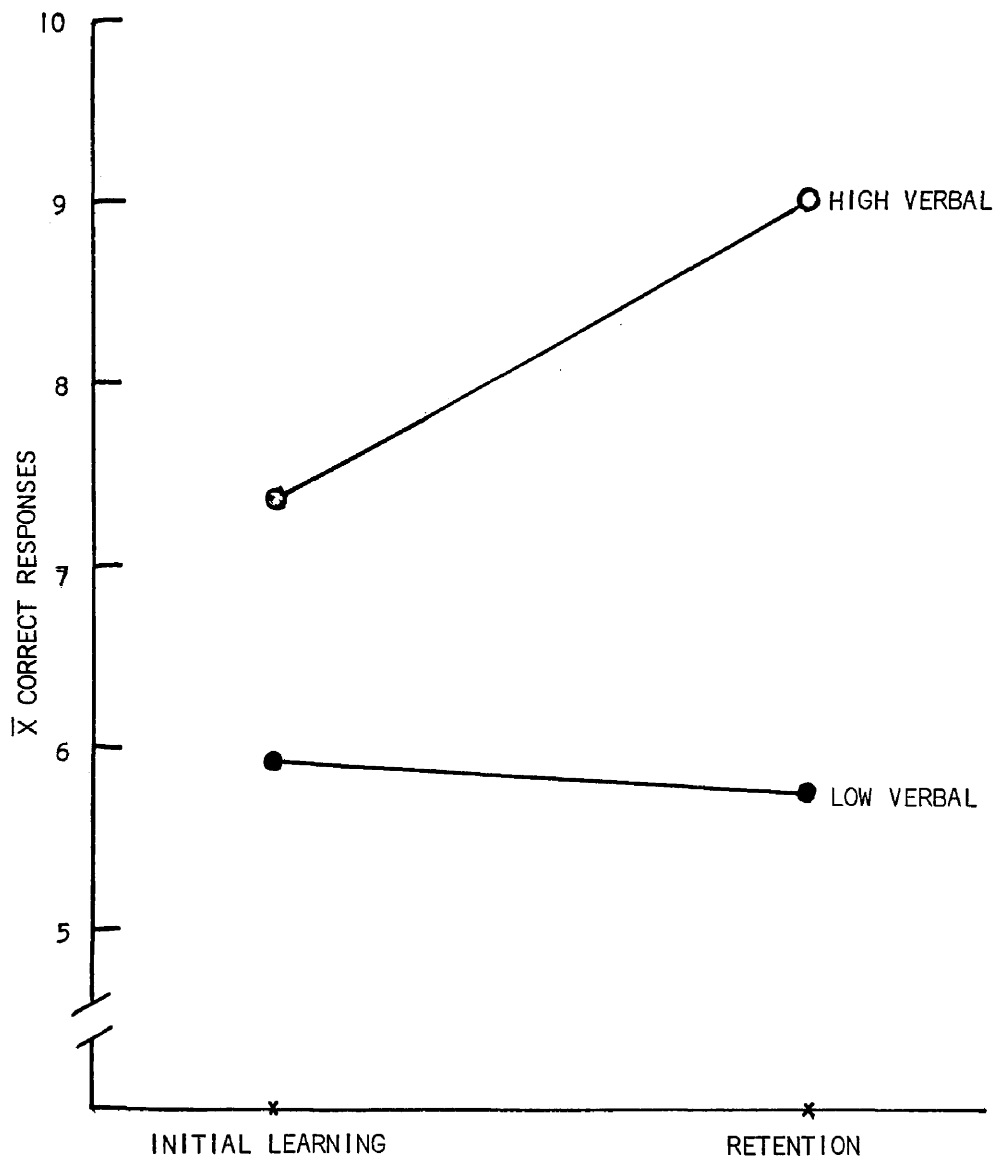

10
9
8
7
6
5
$\bar{X}$ CORRECT RESPONSES
HIGH VERBAL
LOW VERBAL
INITIAL LEARNING
RETENTION

TABLE 3

Summary of Analysis of Variance of Transfer Data

Source	df	MS	F
Between Subjects	49		
Teaching Condition (A)	3	5.36	1.28
Verbal Ability (C)	1	32.57	7.79 **
A X C	3	4.00	0.96
Subj. w. Groups	42	4.18	
Within Subjects	50		
Time of Testing (B)	1	0.01	0.00
A X B	3	2.71	0.98
B X C	1	0.06	0.02
A X B X C	3	0.55	0.20
B X Subj. w. Groups	42	2.77	
Total	99		

** $p < .01$

Because high verbal ability subjects showed significantly superior performance on measures of initial learning and retention, it is of interest to examine their performance on both measures under each teaching condition. These data are presented in Figure 3. Subjects under Treatment E showed significantly

- - - - - - - - - - - - - - -

Insert Figure 3 about here.

- - - - - - - - - - - - - - -

superior performance on measures of initial learning when compared to the performance of subjects under Treatment D ($t = 3.98$, $df = 9$, $p < .01$) and Treatment C ($t = 3.87$, $df = 10$, $p < .01$). There was also a tendency for subjects in the Expository Group to perform superior to subjects under Treatment GD ($p < .20$) on measures of initial learning. Unlike the consistent performance of subjects under Treatments E and D on measures of initial learning and retention, subjects under Treatment C improved significantly on measures of retention ($t = 3.77$, $df = 10$, $p < .01$), and subjects under Treatment GD showed a tendency to improve ($p < .10$). There was also a tendency for subjects under Treatment E to show performance superior to subjects under Treatment D on measures of retention ($p < .10$).

Sex

As indicated in analysis of variance summary Table 4, results produced a significant interaction between sex and per-

- - - - - - - - - - - - - -

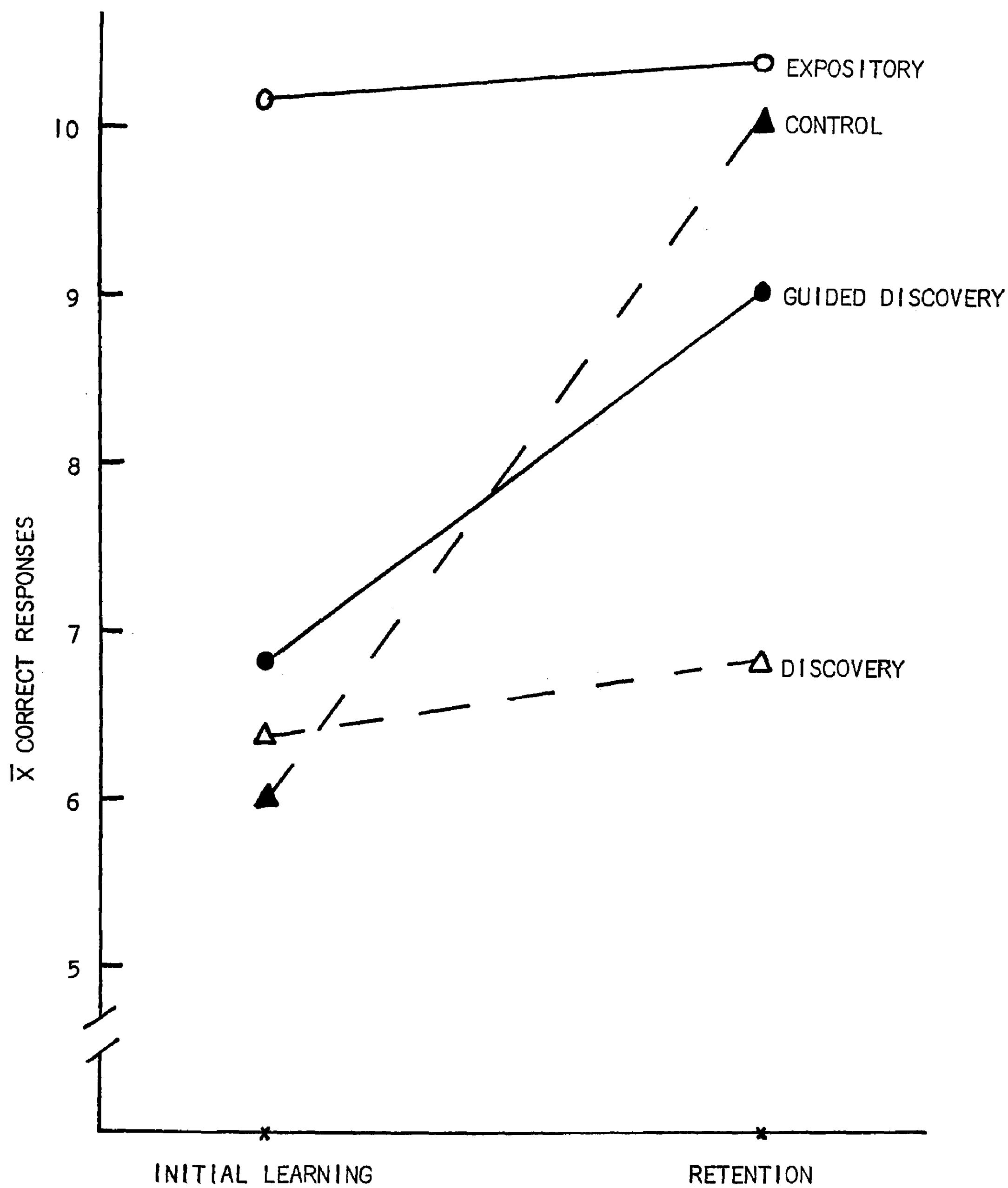
Insert Table 4 about here.

- - - - - - - - - - - - - -

formance on measures of initial learning and retention ($F = 7.82$,

FIGURE 3

Mean Correct Responses on Measures of Learning Performance
for Individual Teaching Conditions
as a Function of Time of Testing



EXPOSITORY
CONTROL
GUIDED DISCOVERY
DISCOVERY
10
9
8
7
6
5
X̄ CORRECT RESPONSES
INITIAL LEARNING
RETENTION

TABLE 4

Summary of Analysis of Variance of Learning Data

Source	df	MS	F
Between Subjects	49		
Teaching Condition (A)	3	30.48	3.52 *
Sex (C)	1	5.46	0.63
A X C	3	22.67	2.62
Subj. w. Groups	42	8.66	
Within Subjects	50		
Time of Testing (B)	1	13.22	3.14
A X B	3	8.84	2.10
B X C	1	32.95	7.82 **
A X B X C	3	3.85	0.91
B X Subj. w. Groups	42	14.21	
Total	99		

* $p < .05$

** $p < .01$

$df = 1/42$, $p < .01$). However, an examination of Figure 4 and a comparison of these results with data presented in Figure 2

- - - - - - - - - - - - - -

Insert Figure 4 about here.

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emphasize the fact that unequal numbers of high and low verbal ability subjects within each group confounded results due to sex differences. Analysis of variance summary Table 5 shows a significant interaction between sex and measures of transfer across

- - - - - - - - - - - - - -

Insert Table 5 about here.

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teaching conditions ($F = 3.35$, $df = 3/42$, $p < .05$). Again, as indicated in Figure 5, it is possible that unbalanced groups according to verbal ability confounded these results. Because

- - - - - - - - - - - - - -

Insert Figure 5 about here.

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results concerning sex as a variable were confounded due to failure to balance groups for verbal ability, further consideration of these data is unwarranted.

Attitude

Measures of attitude toward arithmetic differed significantly across teaching conditions, as indicated in analysis of variance summary Table 6 ($F = 2.87$, $df = 6/80$, $p < .05$). Figure 6 shows

- - - - - - - - - - - - - -

Insert Table 6 about here.

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FIGURE 4

Mean Correct Responses on Measures of Learning Performance for Male and Female Subjects as a Function of Time of Testing

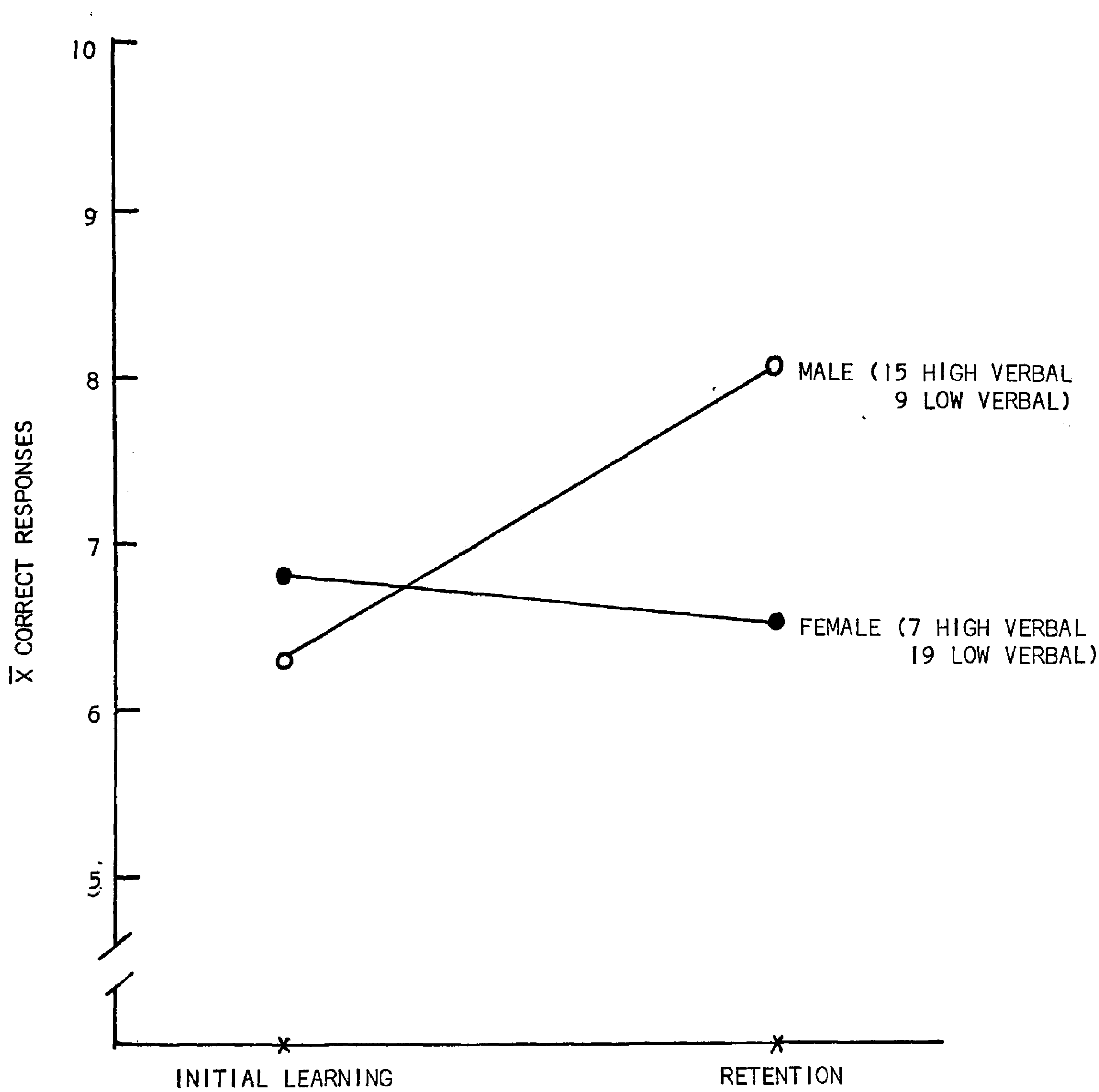

10
9
8
7
6
5
$\overline{X}$ CORRECT RESPONSES
MALE (15 HIGH VERBAL
9 LOW VERBAL)
FEMALE (7 HIGH VERBAL
19 LOW VERBAL)
INITIAL LEARNING
RETENTION

TABLE 5

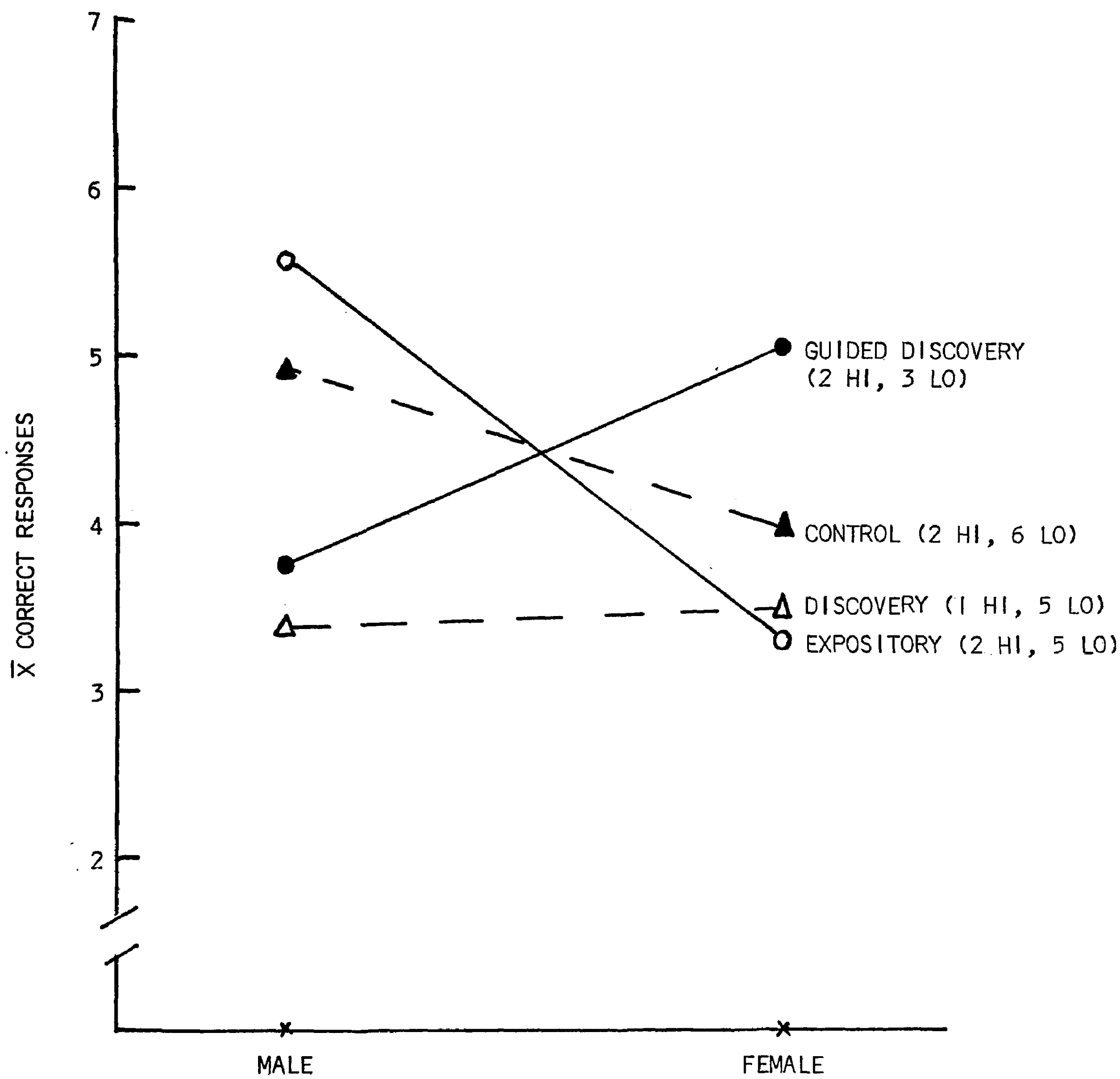
Summary of Analysis of Variance of Transfer Data

Source	df	MS	F
Between Subjects	49		
Teaching Condition (A)	3	5.92	1.42
Sex (C)	1	4.43	1.06
A X C	3	13.97	3.35 *
Subj. w. Groups	42	4.17	
Within Subjects	50		
Time of Testing (B)	1	0.01	0.00
A X B	3	2.09	0.79
B X C	1	4.65	1.76
A X B X C	3	0.96	0.36
B X Subj. w. Groups	42	2.64	
Total	99		

* $p < .05$

FIGURE 5

Mean Correct Responses on Measures of Transfer Performance
for Individual Teaching Conditions
as a Function of Sex of Subject



7
6
5
4
3
2
X̄ CORRECT RESPONSES
GUIDED DISCOVERY
(2 HI, 3 LO)
CONTROL (2 HI, 6 LO)
DISCOVERY (1 HI, 5 LO)
EXPOSITORY (2 HI, 5 LO)
MALE
FEMALE

TABLE 6

Summary of Analysis of Variance of Attitude Data

Source	df	MS	F
Between Subjects	47		
Teaching Condition (A)	3	2.13	1.82
Verbal Ability (C)	1	0.21	0.18
A X C	3	0.49	0.42
Subj. w. Groups	40		
Within Subjects	96		
Time of Testing	2	0.49	1.40
A X B	6	1.00	2.87 *
B X C	2	0.78	2.25
A X B X C	6	0.17	0.47
B X Subj. w. Groups	80	0.35	
Total	143		

* $p < .05$

the mean levels of response on the measure of attitude toward arithmetic for pre-, post-, and post-post-testing periods across all teaching conditions. There were no significant differences

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Insert Figure 6 about here.

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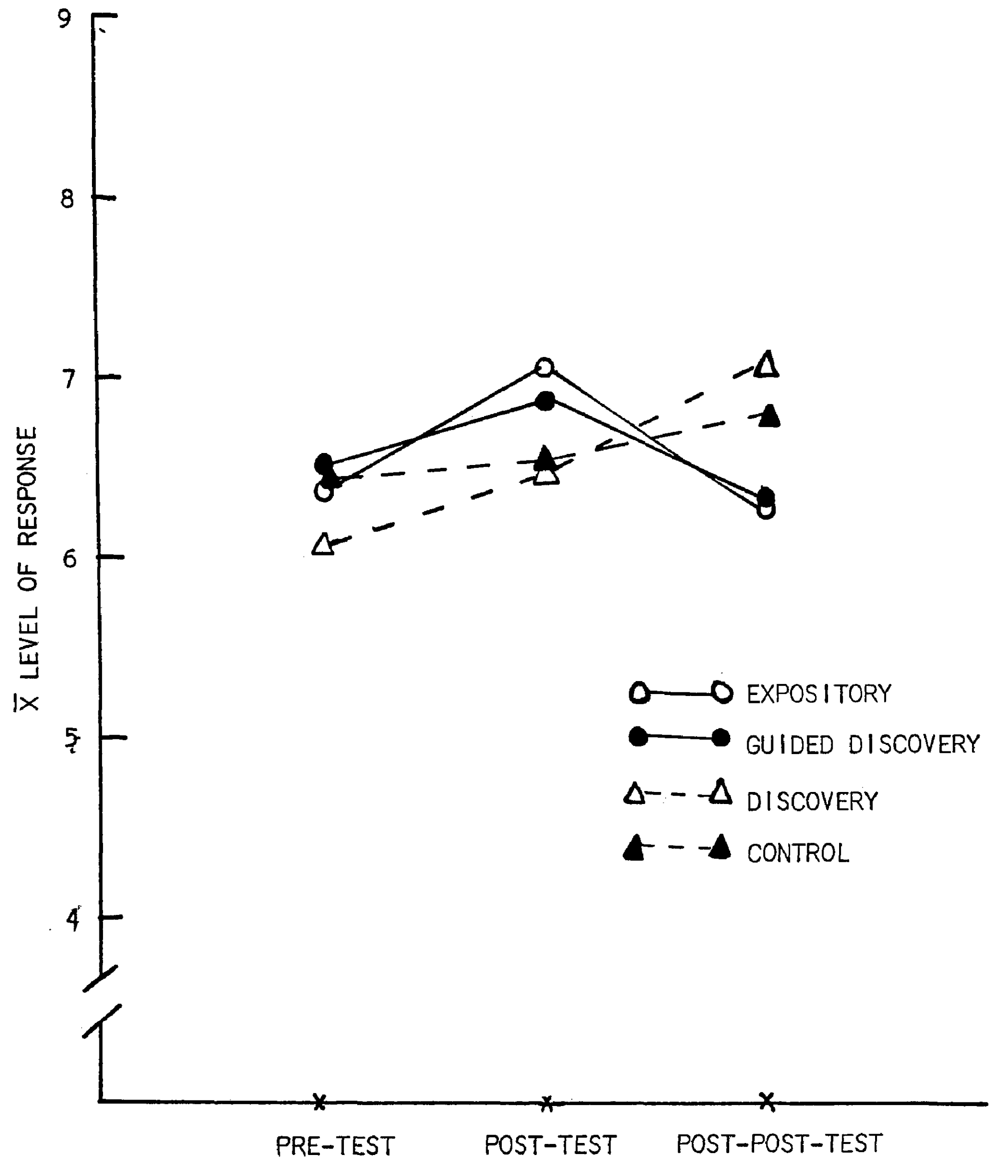
among teaching conditions on pre-attitude measures. Data indicated that subjects under Treatment GD expressed a significantly more favorable attitude toward arithmetic than subjects under Treatment D ($t = 2.34$, $df = 80$, $p < .05$) and Treatment C ($t = 2.09$, $df = 80$, $p < .05$) on a post-test measure. However, on a post-post-test measure, subjects under Treatment D expressed a more favorable attitude toward arithmetic than did subjects under Treatment E ($t = 3.15$, $df = 80$, $p < .01$) and Treatment GD ($t = 3.52$, $df = 80$, $p < .01$). Likewise, subjects under Treatment C expressed a more favorable attitude on post-post-test measures than subjects under Treatments E ($t = 2.07$, $df = 80$, $p < .05$) and GD ($t = 2.41$, $df = 80$, $p < .05$). These findings tend to support the hypothesis that subjects under Treatment GD would produce results superior to subjects under Treatment D on measures of attitude toward arithmetic, but only with regard to post-test measures. Results failed to support the hypothesis that subjects under Treatment GD would produce results superior to subjects under Treatment E on change in attitude. The hypothesis that subjects under Treatment D would produce results superior to subjects under Treatment E on change in attitude received slight support, but only with regard to post-post-test measures of attitude.

FIGURE 6

Mean Levels of Response on Measure of Attitude
for Individual Teaching Conditions
as a Function of Time of Testing


9
8
7
6
5
4
$\overline{X}$ LEVEL OF RESPONSE
EXPOSITORY
GUIDED DISCOVERY
DISCOVERY
CONTROL
PRE-TEST
POST-TEST
POST-POST-TEST

Observers' Ratings of Teacher Behavior

Data comparing affective teacher behavior toward students on seven measures of teacher behavior yielded no significant differences among experimental teaching conditions on any behavior measure. The failure of these ratings to produce significant differences in affective teacher behavior across teaching conditions indicates that experimenter bias, other than adherance to prescribed teaching methods, was not detectable. It can thus be assumed that the affective behavior of the experimenter would not have affected differences among teaching conditions.

Discussion

Results of the present study concerning the effect of teaching condition on performance on measures of initial learning, retention, immediate and delayed transfer will be related to other recent findings. A discussion of the effects of verbal ability and attitude toward arithmetic will then be presented. Finally, problems encountered in experimentation in the classroom, implications for further research, and implications for educational practice will be considered.

The failure of the present study to find significant differences among teaching conditions is similar to the non-significant findings of other recent studies on measures of initial learning (Nichols, 1956), initial learning and transfer (Wiesner, 1969; Michael, 1949), and initial learning, retention, and transfer (Boeck, 1951; Karle, 1960; Yarbroff, 1963; Werdelin, 1966; Meconi, 1967; Tanner, 1969; Hermann, 1971). The significant interaction effect involving teaching condition and initial learning and retention have not been discussed in the literature. However, the performance of subjects under Treatment D was significantly inferior to the performance of subjects under Treatments E and GD on the combined scores of these measures (See Table 2). Consistently superior performance by high-verbal-ability subjects under Treatment E when compared to the performance of high-verbal-ability subjects under Treatment D on measures of both initial learning and retention (See Fig. 3) suggests that replication of the present study using increased sample size may produce significantly superior performance by high-verbal-ability subjects under Treatment E on each measure. The failure of low-verbal-ability subjects to perform significantly different on measures of initial learning and retention across teaching conditions may indicate that materials included in the learning sessions were too difficult to be assimilated in the time alotted.

The significant improvement of high-verbal-ability subjects under Treatment C on measures of retention (See Fig. 3) is an unexpected finding. Subjects in the control group could perhaps be considered as operating under "pure" discovery conditions: they were presented with various problems on the tests of initial learning and retention, and, without any practice on sequenced examples during the learning sessions, virtually had to solve these problems on their own. Unlike a number of subjects in the experimental groups, the majority of subjects in the control group did not express signs of boredom and disinterest while working on their tests. Also, subjects under Treatment C often finished more quickly than subjects under the three experimental conditions, even though they were given the same amount of time to complete their tests. Although the attitude of high-verbal-ability subjects under the control condition was not significantly different from the attitude of high-verbal-ability subjects under the experimental conditions (See Fig. 6), in the opinion of the experimenter, increased motivation may have been a contributing factor toward improvement on retention measures.

Non-significant differences on measures of transfer across teaching conditions may be due to the inclusion of a number of test items which were too difficult for subjects in any group to master. Subsequent researchers should take this factor into consideration.

As might be expected, students of high verbal ability performed significantly better than subjects of low verbal ability on all test measures. Kagan (1966) pointed out in a consideration of the growth of concepts in young children, "American theorists argue that mediation and language are at the heart of reasoning (p. 113)." Indeed, if language is an inherent factor in the reasoning process, subjects with a greater command over their verbal faculties would surpass low-verbal-ability subjects in their ability to reason logically, which is a necessary aptitude for the acquisition of new mathematical concepts and principles. Even under a discovery method of instruction, in which a subject need not cope with extensive verbalization by the instructor, well developed verbal faculties facilitates the assimilation of new concepts and principles, perhaps through their action as mediators in the reasoning process.

Results of attitude change across teaching conditions (See Fig. 6), though significant, are not entirely consistent with performance by various instructional groups on immediate and delayed test measures. The more favorable attitude expressed by subjects under Treatment E when compared to the attitude expressed by subjects under Treatment D immediately following the learning sessions is consistent with the tendency for the expository method to show superior performance on measures of initial learning. However, the tendency for subjects under Treatments E and GD to perform superior to subjects under Treatment D on measures of retention is not consistent with the significantly more positive attitude expressed by Discovery subjects when compared to the attitude expressed by subjects under Treatments E and GD prior to testing for retention and delayed transfer. A positive attitude change expressed by subjects under Treatment D is not inconsistent with other findings (Kersh, 1962; Price, 1967); however, the results of these studies did not favor subjects under Treatment E. The positive change in attitude by subjects in the control group corresponds to their improvement on measures of retention.

Inconsistencies in attitude change and performance are difficult to explain. As suggested by Ventis (1972) the assumption that attitude and performance should be parallel may not necessarily be valid. A child could work very hard on a set of arithmetic problems, become tired and disinterested in arithmetic, yet perform well on subsequent tests. Conversely, a child who was not as conscientious may not have become as tired and bored with his work and, therefore, shown a more positive attitude change. Regular arithmetic activities in which the children engaged between the immediate and delayed tests may also have affected attitude change as measured in the present study. Future studies which use attitude measures as indicators of increased motivation should take the above observations into consideration

Although the high degree of difficulty of a number of tranfer problems may be responsible for the lack of significant differences across teaching conditions on transfer measures, the reasons for differences across teaching conditions on measures of initial learning and retention appear to lie principally in the treatments. Materials presented all subjects were appropriate mathematical tasks for the third year level. Presentations were sequenced to assure greatest understanding. Equal time was alotted subjects in all groups. Only subjects who had not shown prior knowledge of the concepts involved were used as subjects. Attitudes toward arithmetic were not significantly different across treatment groups prior to learning presentations. Differences in affective teacher behavior across treatment groups were not significant. Verbal ability of subjects was equated across groups. The only differential factor among groups, other than operationally defined differences among teaching conditions, was the general behavior of subjects. Subjective reports of both the experimenter and regular classroom teacher confirm the less cooperative nature of the discovery group as a whole. Even before the beginning of the learning sessions, the experimenter was cautioned by the regular classroom teacher that she might experience greater difficulty in handling the students in the discovery group. Frustration and boredom experienced by subjects under Treatment D may have further contributed to their generally uncooperative manner.

Certain problems encountered in experimentation in the classroom must be considered in the interpretation of results of the present study. One such problem was the inability of the experimenter to command the complete attention of subjects during learning and testing sessions. This particular problem may be related to the "substitute teacher" effect. A second problem was the necessary time restrictions placed on each group during learning and testing sessions. Because time restrictions were imposed, a criterion level for attainment of minimum understanding of concepts presented, as suggested by Worthen (1968), could not be established for subjects under all teaching conditions in the present study. If all experimental groups, including subjects of high and low verbal ability, had been alotted a greater amount of time to learn concepts presented during learning sessions, more significant differences may have resulted across teaching conditions on all measures. Although it may be argued on the basis of results of the present study that a tendency for subjects under Treatment E to perform superior to subjects under Treatment D when time is equalized further reinforces Ausubel's (1963) questioning of the impractically of the discovery approach, it must be emphasized that significant differences were obtained only for high-verbal-ability subjects. Perhaps increased time will provide differential results for low ability subjects across teaching conditions. Further research is necessary to examine this possibility.

Although it was assumed that the subjects involved in the present study had more experience with discovery techniques than subjects under a more traditional curriculum, this experience may have been insufficient to have significantly affected the performance of the discovery group. Additional research allowing subjects a greater length of time to develop needed "discovery" skills or research in which subjects are trained in discovery techniques is necessary to supply information regarding the effect of familiarity with discovery techniques.

Findings of the present study neither conclusively support nor reject the viewpoints of Gagne, Bruner, Ausubel, and Friedlander on the relative effectiveness of the discovery method as presented in the introduction. Gagne's contention that discovery learning produces results superior to the expository approach on measures of initial learning, retention, and transfer are not supported by trends in data of the present study. Results indicated slight support for his contention that discovery learning produces a positive attitude change or intrinsic motivation.

Friedlander's and Ausubel's reservations on the effectiveness of the discovery method, cited in the introduction, can be related to the tendency of the subjects under Treatment D in the present study to show inferior performance to subjects under Treatment E. However, results do not conclusively support inferior performance of subjects under the discovery condition.

Gagne's contention that transfer will occur only in situations highly similar to those in which the principle was discovered, was neither supported nor rejected. Use of less difficult transfer items in subsequent studies may produce more conclusive findings regarding the effect of discovery learning on transfer.

In addition to implications for further research on discovery learning previously mentioned, the following suggestions are made: (1) Studies which vary the amount of encouragement given by the teacher to subjects in the discovery group. Increased encouragement may alleviate frustration and boredom. (2) Studies which examine various personality factors of subjects who perform differentially under Treatments E, GD, D, including impulsivity-reflectivity and the experience of anxiety in unguided situations. (3) As suggested by Worthen (1968) studies which vary verbal and non-verbal discovery. This variable could be related to the performance of subjects of high and low verbal ability.

In as much as the results of the present study neither conclusively reject nor support hypotheses of discovery learning as a superior method of imparting knowledge, generalization from these findings to actual instruction in the classroom is difficult. Significantly superior performance on combined measures of initial learning and retention by subjects under Treatment E when compared to the performance of subjects under Treatment D, seems to suggest that the expository approach to instruction is indeed a better method of instructing children. However, such a generalization must be restricted to the teaching of similar subjects and subject matter content. In contrast to the above mentioned conclusion, significantly superior performance on measures of retention by high-verbal-ability subjects under Treatment C suggests that students with relatively well developed verbal faculties may derive greatest benefit from an approach which is highly autonomous. Again, such a generalization must be restricted to teaching similar subjects and concepts.

In closing, a final observation regarding "discovery learning" as a method of classroom instruction is appropriate. As suggested by Ventis (1972), research and discussions on "discovery learning" as presented in the literature is, in reality, research on "discovery teaching." The experimenter presents subjects with materials which he has chosen and requires subjects to "discover" concepts or principles which he has selected. The subjects are not necessarily exploring a phenomena which is of particular interest to them. True "discovery learning" should involve the autonomous selection and exploration of a particular topic by the subjects. Implimentation of "true discovery learning" in the classroom would necessitate a totally individualized approach to instruction, in which each student's particular interests would have to be assessed and appropriate materials provided for exploration in his area of interest. Though highly desirable, the incorporation of such an approach to instruction into current elementary school curriculum would present a vast number of problems in curriculum design and administration.

APPENDIX A

Pretest and Instructions to Subjects

Instructions

I will give you a little test to see what you know about arithmetic. Your book is turned over. Write your name on the back of the book. Turn your books over now and listen carefully while I read Question 1. (Show children a brown and red piece of construction paper, each 9" X 12") If you think the brown piece of paper and the red piece of paper are the same size, write "yes" in the first blank by number 1. If you do not think they are the same size, write "no" in the first blank by number 1. Now I will cut the brown piece of paper like this (Cut on the diagonal, join two triangular pieces to form an isosceles triangle) If you think the red shape is the same amount of paper as the brown shape, write "yes" in the second blank by number 1. If you do not think the red shape and the brown shape have the same amount of paper, write "no" in the second blank by number 1.

Now listen carefully while I tell you about Question 2. (Children are shown two identical sheets of green paper approximately 9" X 12", and told that they are meadows or fields. In the center of each a small toy cow is placed. A small house or hut is then placed in one of the meadows). If you think both cows have the same amount of grass to eat, check "yes" by number 1. If you think the cows do not have the same amount of grass to eat, check "no". (An exactly similar house is placed in the other field and the children are again asked the above question.

Appendix A (con't)

Instructions

Extra houses, all identical with one another are then placed in each field; in the one they are placed tightly side by side as houses in a street, whereas in the other field the houses are spread out. There is always the same number of houses in each field. As extra pairs of houses are added, the children are again asked the above question.)

Now you are ready to do the rest of the questions yourself. Follow as I read, if you do not want to work ahead by yourself. The pieces of paper and the ruler in your envelope will help you answer some of the questions. Do the best job you can on the questions. If a question is too hard for you, don't worry about it. Just go on to the next question. When you finish, sit quietly at your desk and wait until everyone else is done.

Be sure to do your own work, because we want to see how much arithmetic you know. Does anyone have any questions? Let's begin with Question 3.

Appendix A (con't)

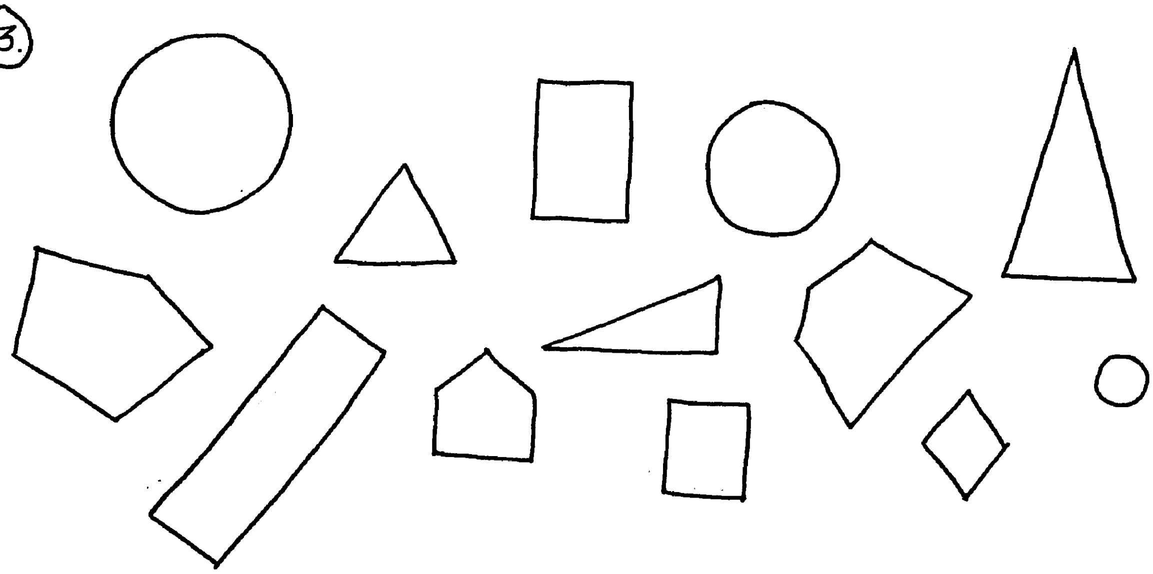
Pretest

(1.) a. ______
b. ______

(2.)

Yes		No	
1.	______	1.	______
2.	______	2.	______
3.	______	3.	______
4.	______	4.	______
5.	______	5.	______
6.	______	6.	______
7.	______	7.	______
8.	______	8.	______

(3.)



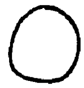
1. Mark an 'A' on all of the circles.
2. Mark a 'B' on all of the shapes with 3 sides.
3. Mark a 'C' on all of the shapes with 4 sides.
4. Mark a 'D' on all of the shapes with 5 sides.

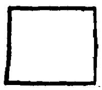
turn the page

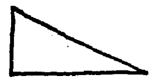
Appendix A (con't)

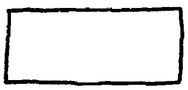
(4) Next to each shapes write 'C' for circles, 't' for triangles, 'r' for rectangles, 's' for squares.

1.  ________.

2.  ________.

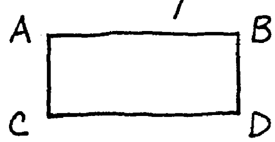
3.  ________.

4.  ________.

5.  ________.

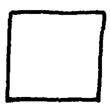
6.  ________.

(5) What do you call figure ABCD?



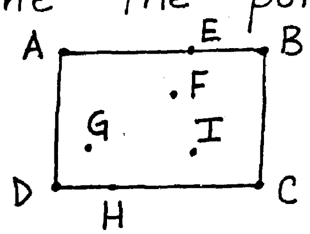
________.

What do you call figure WXYZ?



________.

(6) 1. Name the points <u>on</u> the rectangle.

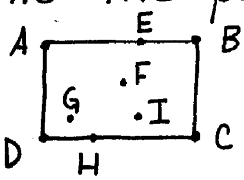


________.

turn the page

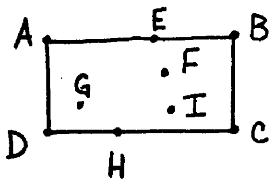
Appendix A (con't)

2. Name the points <u>in</u> the rectangle.



__________.

3. Name the corner points of the rectangle.



__________.

(7.) How long do you think the green strip of paper is?

(Place 7 inch green strip here.)

Write your guess here ________.

Measure the green strip with your yellow ruler. It is in your envelope. It is marked off in inches.

Write your answer here _______ inches.

turn the page

Appendix A (con't)

(8.) Which is larger - the red region or the yellow region? Use the blue squares in your envelope to help you find your answer.

(Place 3 x 4 inch red rectangle here.)

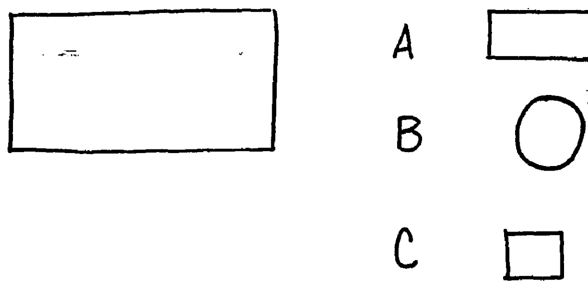
(Place 2×5 inch yellow rectangle here.)

Write your answer here ________.

How do you know which is larger?

turn the page

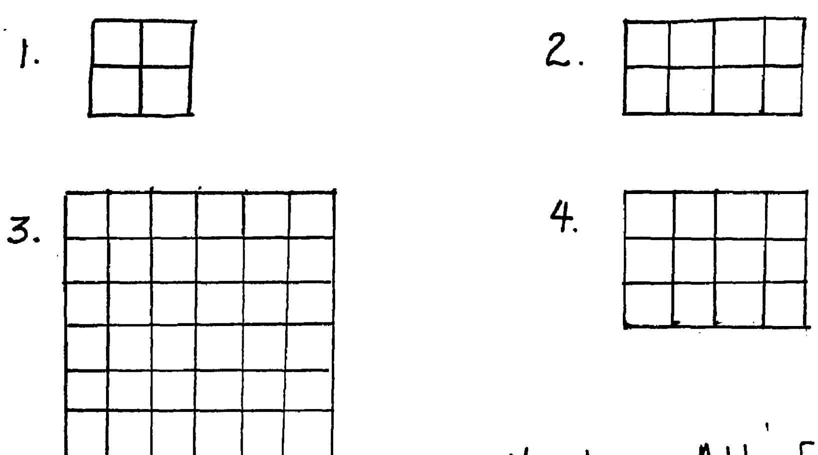
(9.) Should you use A, B, or C to measure this region?



Write your answer here _____.

(10.) Can you find how many small squares make up each of these rectangles without counting each small square? _____.

How?



You're All Finished ☺!

APPENDIX B

Attitude Scale, Instructions, and Answer Sheet

Instructions

Look at the piece of paper that says "Attitude Scale". Write your name in the blank by the word "name". Now listen carefully while I read each sentence. If the sentence tells the way you feel about arithmetic, check "yes" by the number of the sentence. If the sentence does not tell the way you feel about arithmetic, check "no" by the number of the sentence. Only check either "yes" or "no". Are there any questions? Now I'll begin reading the sentences. Listen carefully. If you don't understand the sentence, raise your hand.

Appendix B (con't)

Dutton's Arithmetic Attitude Scale (reworded)

Scale Value	Attitude Statement
1.0	13. I hate arithmetic and try not to use it at any time.
1.5	20. I have never liked arithmetic.
2.0	18. I am afraid of doing word problems.
2.5	11. I have always been afraid of arithmetic.
3.0	22. I can't see much use for arithmetic.
3.2	15. I try never to use arithmetic because I am not very good with numbers.
3.3	9. Arithmetic is something you have to do even though it is not fun.
3.7	2. I don't feel sure of myself in arithmetic.
4.6	6. I don't think arithmetic is fun, but I always want to do well in it.
5.3	7. I am not really excited about arithmetic, but I don't really hate it either.
5.6	4. I like arithmetic, but I like my other subjects just as much.
5.9	8. Arithmetic is as important as any other subject.
6.7	14. I like doing problems when I know how to work them well.
7.0	10. Sometimes I like arithmetic problems that make me think.
7.7	5. I like arithmetic because it is useful.
8.1	19. Arithmetic is very interesting.
8.6	3. I like to see how fast and well I can work arithmetic problems.

Appendix B (con't)

9.0	12. I would like to spend more time in school working arithmetic.
9.5	1. I think about arithmetic problems outside of school and I like to work them out.
9.8	17. I never get tired of working with numbers.
10.4	21. I think arithmetic is the most fun of all of all of my subjects.
10.5	16. Arithmetic thrills me, I like it better than any other subject.

APPENDIX C

Learning Session Units and Instructions to Subjects

Introduction -- First Day

During the rest of this week and part of next you will be learning about finding the size of rectangles. When you leave this room each day, do not tell anyone what you have been doing in class, and do not talk to anyone else about the problems you were working. Other children will be doing the same problems as you, and we want them to have to figure out their answers by themsleves.

Today and during the next three classes we meet, a Cameraman will be taking pictures of me and you while I am teaching. The camera is very quiet and we won't be able to tell when it is on. Do not let the Cameraman bother you. Just do your work and pretend he is not there.

To All Experimental Groups -- First Day

I will now give you each a large brown envelope and a worksheet. Write your name at the top of the first page.

Today you will be learning about the best shape to use to measure a rectangle. This shape is called a rectangle. The corners are squared and the sides across from each other are the same length. All these shapes are called rectangles. When we say we are going to measure a rectangle, we mean we are going to find out what size it is.

Are there any questions?

I will read each problem to you, and then give you time to work on it. If you do not understand the problem after I have

Appendix C (con't)

read it, raise your hand. Please wait until I have read the problem to the whole class before you ask me a question.

To Treatment E -- First Day

Look at Question 3. We have found that the best shape to use to measure rectangles is a little shape like this (hold up inch square). Each side of this square is one inch long. It is called a square inch. You will now measure the colored rectangles with 5 different shapes to find out why the square inch is the best shape to use. When we say "cover" each colored rectangle that means to cover as much of the colored rectangle as you can without the blue shapes lying on top of each other (demonstrate) and without going over the edges of the rectangle (demonstrate).

Look at Question 4. We know that the square inch is best because you are able to cover the whole rectangle without overlapping and without going over the edge of the rectangle.

To Treatment GD -- First Day

Look at Question 3. Define "cover" as for Treatment E.

At completion of Question 4: We have found that this is the best shape to use to measure rectangles (hold up inch square). Each side of this square is one inch long. We see that it is best because we were able to cover the whole rectangle without overlapping onto another blue square and without going over the edge of the colored rectangle.

To Treatment D -- First Day

Look at Question 3. Define "cover" as for Treatments E and GD.

Appendix C (con't)

Learning Unit

1st Day Name ____________________

(1.) Look at the top of your desk.
Look at the seat of your chair.
Which is larger? Check the right answer.

_____ Top of Desk
_____ Seat of chair

(2.) Look at the tops of the two books on your desk. Which is larger - the yellow one or the orange one?

Check the right answer. _____ yellow
_____ orange

(3.) Look in your packet.
Take out the red, yellow, green, orange, purple and white rectangles.
Take out the 5 white envelopes with the blue shapes in them.
We want to find the best blue shape we can use to measure all of the colored rectangles.

1. Use only the circles (O) to cover each colored rectangle.
2. Use only the triangles (Δ) to cover each colored rectangle.
3. Use only the big square (□) to cover each colored rectangle.
4. Use only the little square (□) to cover each colored rectangle.
5. Use only the big rectangle (▭) to cover each colored rectangle.

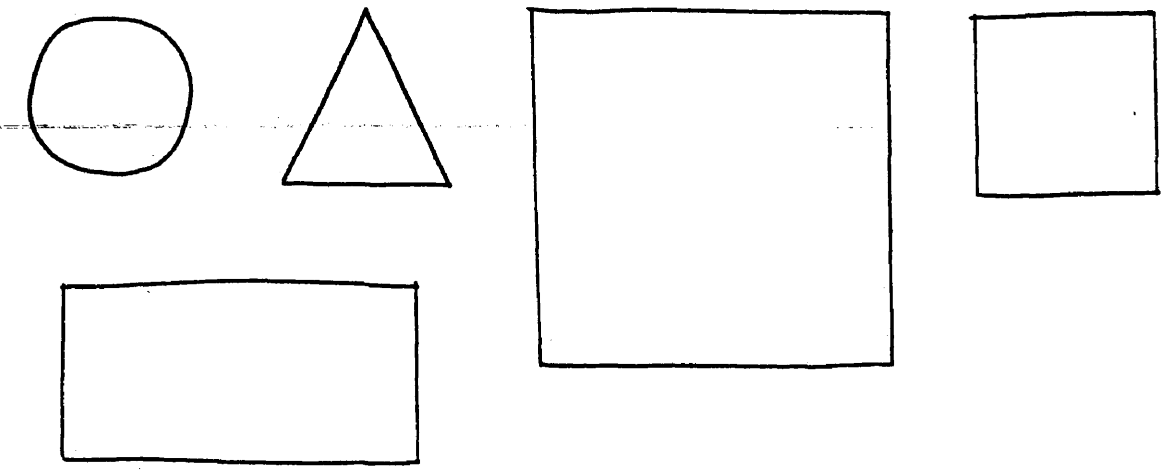
Turn the page ☺

Appendix C (con't)

page 2

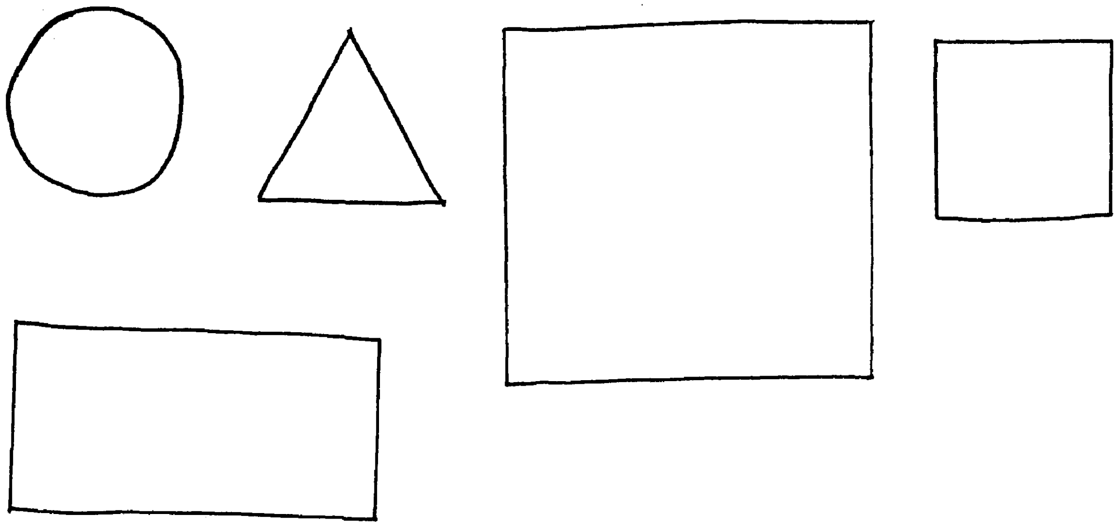
4. Which is the best blue shape to use to measure all of the colored rectangles?

Check the right answer.



5. Which blue shapes did not work well for measuring all of the colored rectangles?

Check all the shapes which did not work well.



Appendix C (con't)

Instructions -- Second Day

To All Experimental Groups

Today you will measure different colored rectangles. You will be using the little blue squares. You will also be drawing your own rectangles with a ruler.

Treatment E

When you want to measure the size of a rectangle, you can use the little blue square-inch shapes, and you can also use a ruler.

Look at this rectangle (hold up black rectangle). You can find the size of this rectangle by covering it with little blue squares and then counting how many you used. This rectangle took 15 blue squares (demonstrate covering rectangle with inch squares). Now look at number 1 and number 2.

Another way to measure a rectangle is to use a ruler and some graph paper. We can draw a rectangle that is 4 inches long and 3 inches high. Draw along the dark green lines on your graph paper (demonstrate). Mark the rectangle off in square inches (demonstrate). The dark green lines make square inches on the graph paper. Be sure to mark the square inches right. Now count the number of square inches. This rectangle has 12 square inches, so we say its size is 12 square inches. Now look at number 3.

Treatment GD

Upon completion of number 5: Now you know two ways to measure the size of a rectangle. You can use the blue square-inch

Appendix C (con't)

shapes or you can use a ruler.

When you use the blue square-inch shapes you cover the rectangle like this (demonstrate) and then you count the number of blue square inches it took. It took 15 blue inch squares to cover this rectangle. Another way to find the size of a rectangle is to use a ruler. On your graph paper you can draw a rectangle (demonstrate). Then mark off the rectangle into square inches and count the number of square inches. This rectangle is marked off into 12 square inches, so we say the size of the rectangle in 12 square inches.

Treatment D

Simply read problems to subjects.

Appendix C (con't)

Learning Unit

2nd Day

Name ______________

(1.) Take the colored rectangles out of your packet.

Guess how many blue squares it will take to cover the big orange rectangle.
Write your guess here ________.

Use your little blue squares to measure the big orange rectangle.
How many squares did you need?
Write your answer here (4 x 5).

(2.) Measure each colored rectangle using your blue squares.

List how many blue squares it took to measure each of the colored rectangles.

Write your answers below:

1. Big red rectangle (5 x 5).
2. Little red rectangle (2 x 3).
3. Big purple rectangle (1 x 8).
4. Little purple rectangle (3 x 4).
5. Big orange rectangle (4 x 5).
6. Little orange rectangle (2 x 8).
7. Big yellow rectangle (3 x 6).
8. Little yellow rectangle (4 x 1).

turn the page ☺

Appendix C (con't)

(3.) How many blue squares did it take to cover the largest rectangle?
Write your answer here ______.

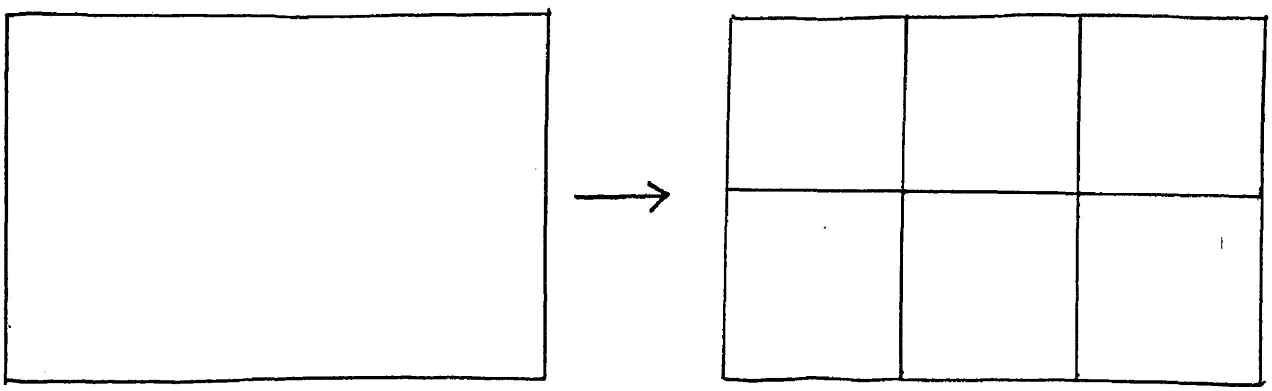
How many blue squares did it take to cover the smallest rectangle?
Write your answer here _____.

(4.) Using your piece of graph paper and your ruler, draw your own rectangles.

1. Make one 2 inches high and 4 inches long.
2. Make one 4 inches high and 4 inches long.
3 Make one 2 inches high and 5 inches long.

* Remember: Your ruler is marked off in inches.

(5.) Mark off nch squares in your rectangle like this.



How many square inches are there in each of your rectangles?

Use your blue squares if you need help

turn the page ☺

Appendix C (con't)

Write your answers below:

1. Rectangle 4 inches long and 2 inches high ______ square inches.

2. Rectangle 4 inches long and 4 inches high ______ square inches.

3. Rectangle 5 inches long and 2 inches high ______ square inches.

Appendix C (con't)

Instructions -- Third Day

To All Experimental Groups

Today you will be measuring rectangles using your ruler and only 9 blue inch squares.

Treatment E

If you know how long a rectangle is (demonstrate), and you know how high it is (demonstrate) you can find how many square inches are the the rectangle without completely covering it with the little blue square inch shapes.

Yesterday I drew a rectangle that was 3" high and 4" long. Then I marked off how many square inches it took to fill the rectangle. It took 12 square inches. But here is an easier way to find how many square inches are in a rectangle. If we know that the rectangle is 3" high and 4" long, them we know that there are 3 rows with 4 square inches in each row (demonstrate). We can multiply 3 rows X 4 squares in each row and we get 12 square inches. The size of the rectangle in 12 square inches.

Here is another rectangle. It is 5" high and 2" long, so we know there are 5 rows with 2 square inches in each row (demonstrate). Five rows X 2 square inches in each row gives us 10 square inches -- the size of the rectangle in 10 square inches. Now look at number 1.

Treatment GD

Upon completion of number 4: Everyone listen carefully to me. Here is a shorter way to measure the size of a rectangle.

Appendix C (con't)

If you know how high a rectangle is and how long a rectangle is, then you can find out how many square inches are in the rectangle. Yesterday I drew a rectangle that was 3" high (demonstrate) and 4" long (demonstrate). Then I marked off how many square inches it took to fill the rectagle. It took 12 square inches. If we know it is 3" high then we know there are 3 rows of inch squares. If we know it is 4" long, we know there are 4 square inches in each row. So we can multiply 3 rows X 4 square inches in each row and get 12 sqare inches.

Here is another rectangle. It is 5" high and 2" long, so we can multiply 5 rows X 2 square inches in each row and we get 10 square inches. The rectangle's size is 12 square inches.

Treatment D

Simply read problems.

Appendix C (con't)

Learning Unit

3rd Day

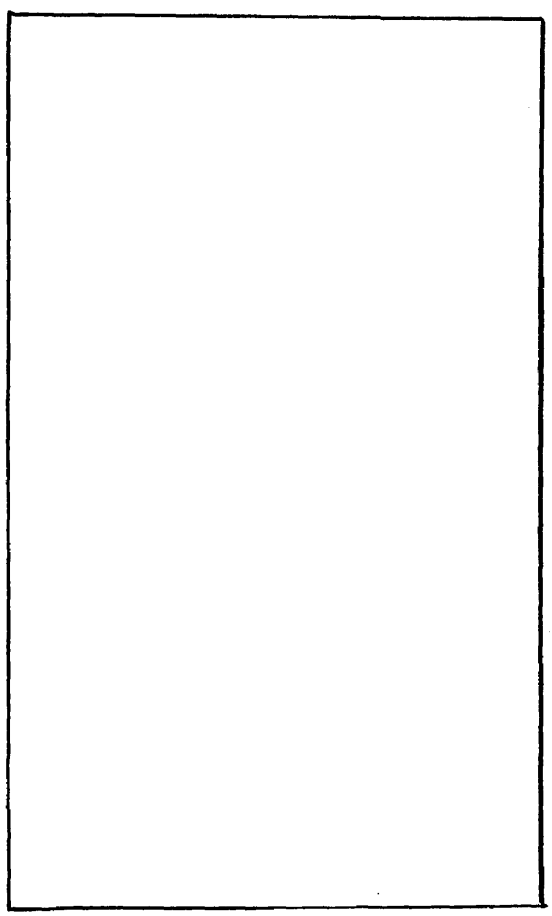
Name ____________________

(1.) There are 10 rectangles on the next 5 pages. Use your yellow ruler and your blue inch squares to find the area of each rectangle.

You have only 9 inch squares to use. Do the best you can with these.

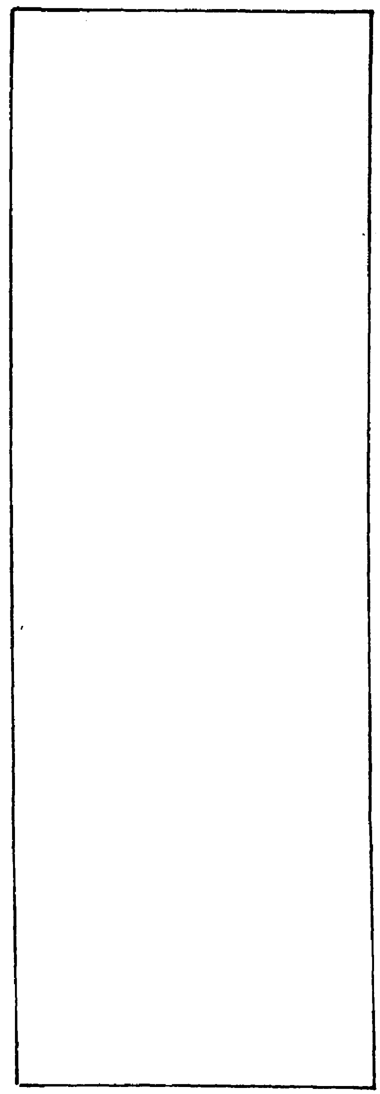
Write how many square inches are in each rectangle under the rectangle, after you measure it.

1.



___ square inches

2.

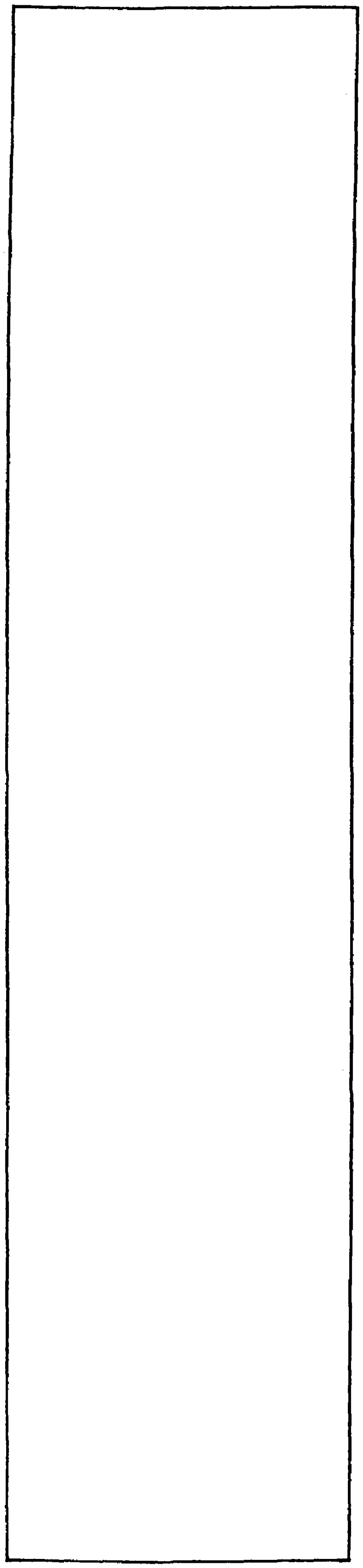


___ square inches

Appendix C (con't)

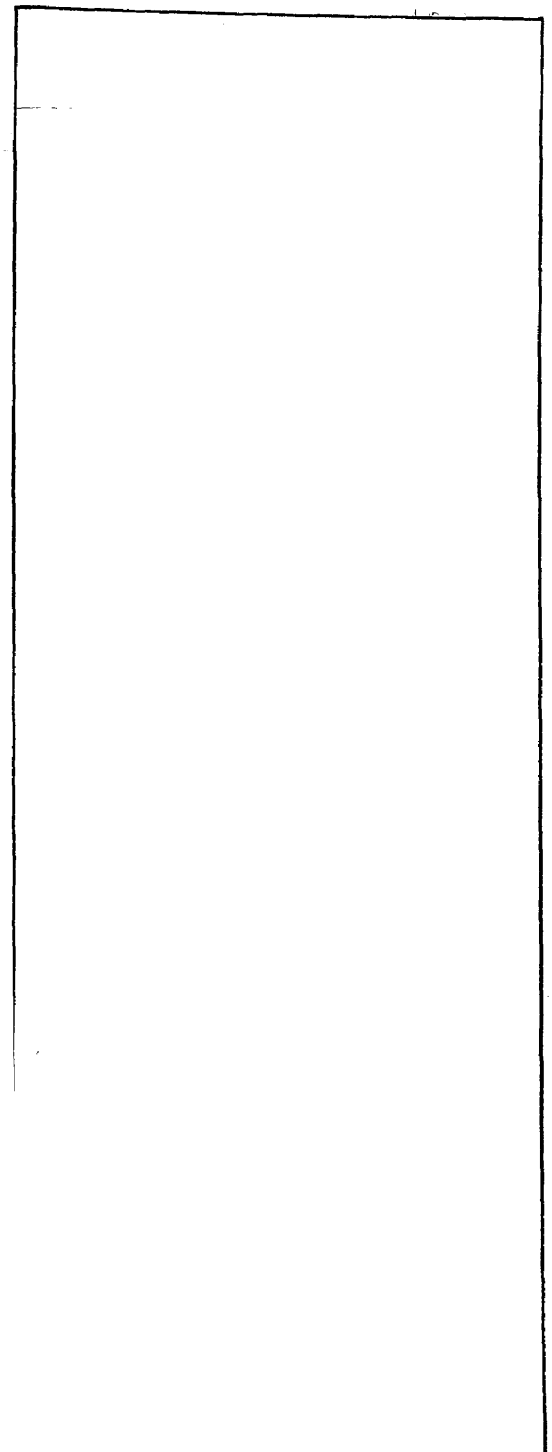
page 2

3.



— square inches

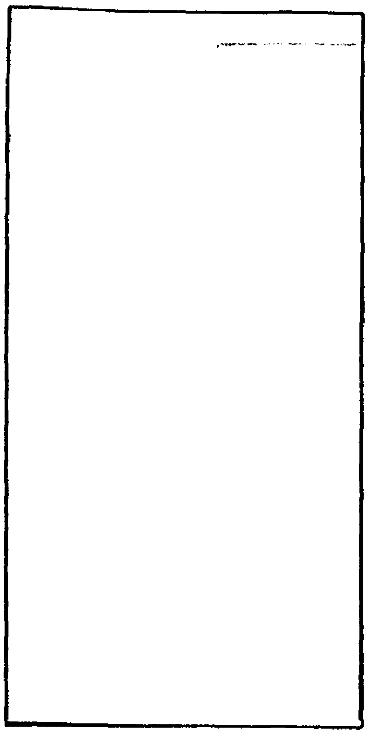
4.



— square inches

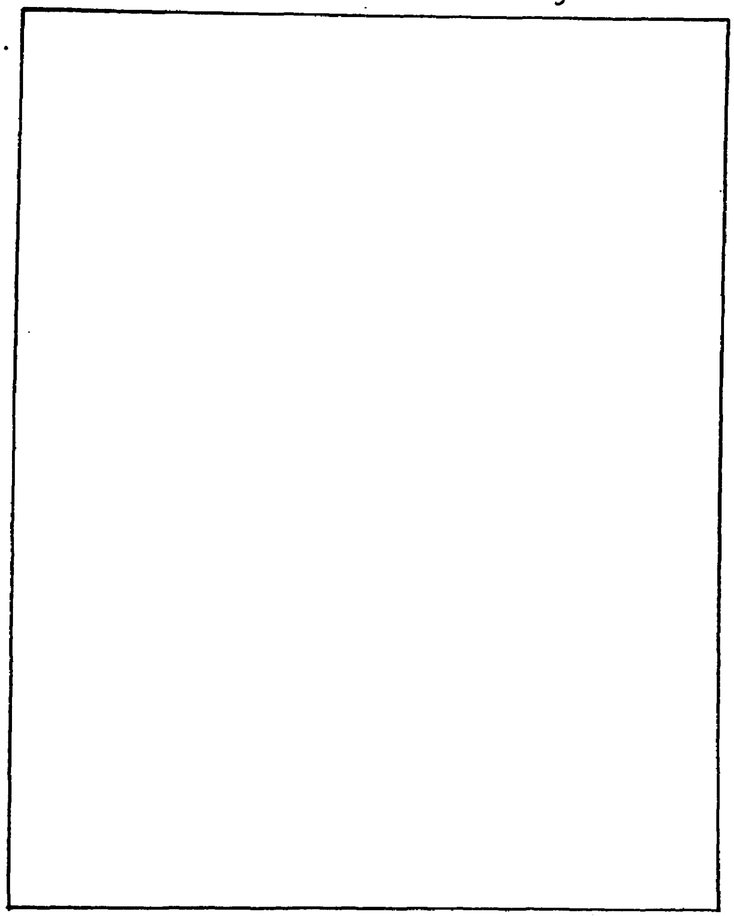
Appendix C (con't)

5.



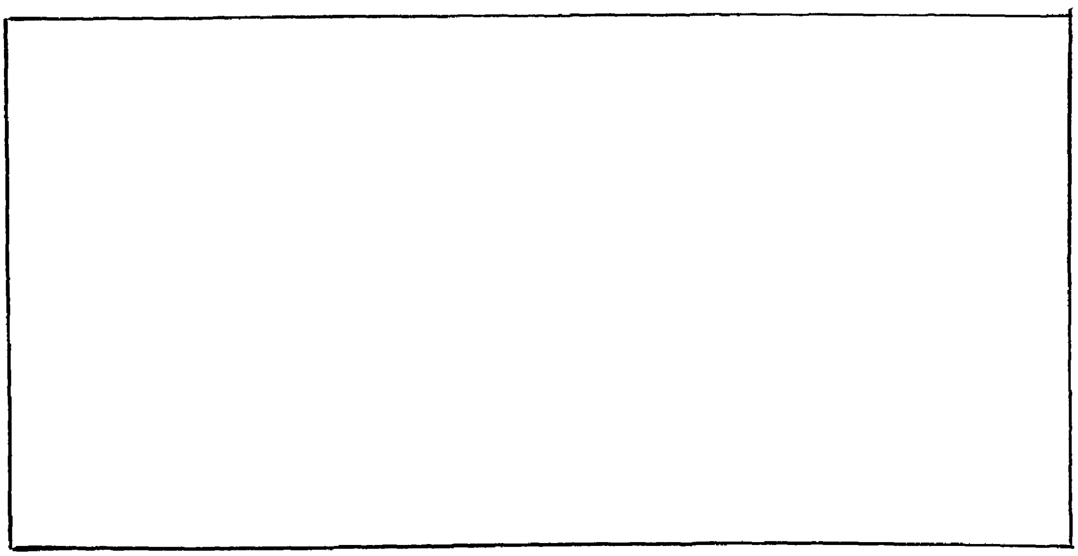
___ square inches

6.



___ square inches

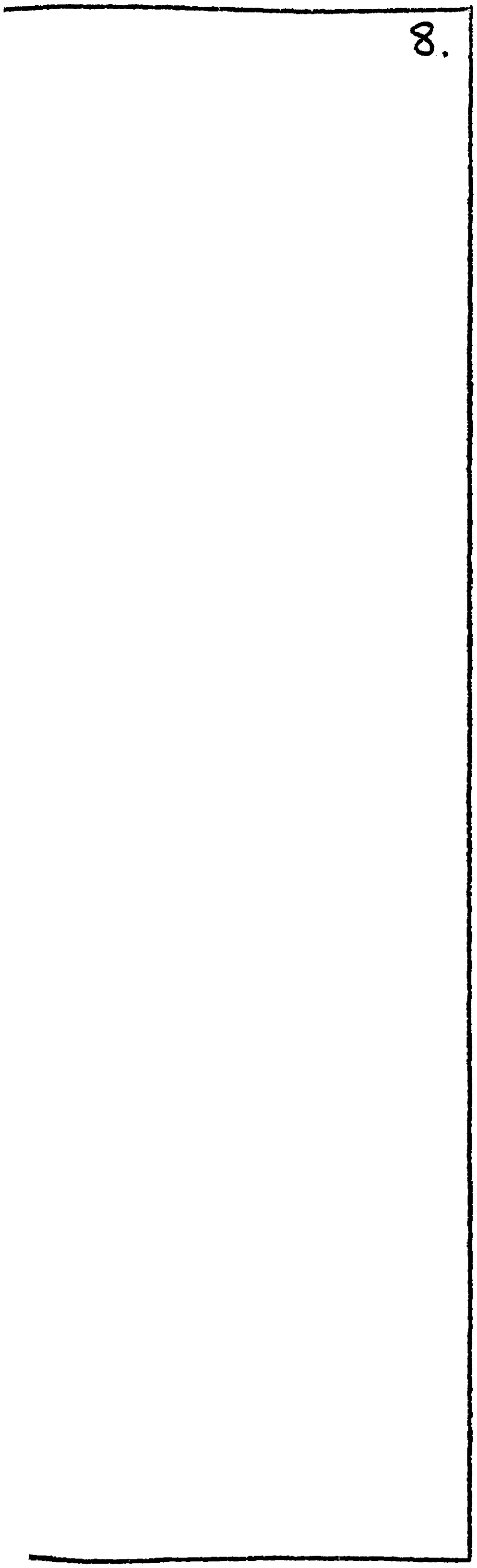
7.



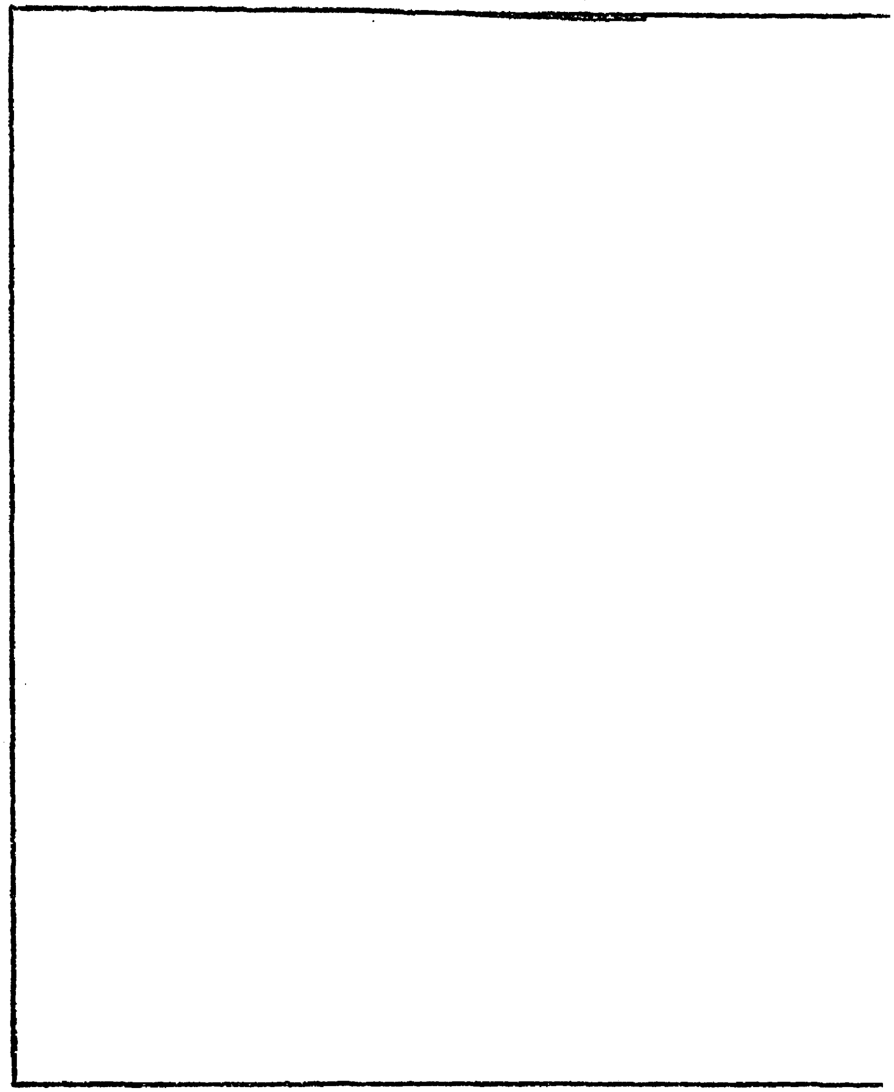
___ square inches

Appendix C (con't)

8.



9.



___ square inches

___ square inches

Appendix C (con't)

10.

____ square inches

turn the page ☺

Appendix C (con't)

(2.) Sally wants to make a blanket for her doll bed. The bed is 6 inches long and 4 inches wide. How many square inches of cloth will Sally need? Draw a picture on your graph-paper to help you find the answer.

Write your answer here: ___ square inches.

(3.) Billy is making a wire cover for the top of his fish bowl. The fish bowl top measures 10 inches long and 7 inches wide. How many square inches of wire will he need? Draw a picture on your graph paper to help you find the answer.

Write your answer here: ___ square inches.

(4.) Mother is covering her kitchen shelf with paper. The shelf is 10 inches long and 12 inches wide. How many square inches of paper will she need?

Write your answer here: ___ square inches

Appendix C (con't)

Instructions -- Fourth Day

To All Experimental Groups

Everyone sit down at a desk which has a brown invelope on it. Write your names at the top of the white worksheet.

Today you will be finding the area of some rectangles by using only your ruler. You will not have any blue square inch shapes to use. You can mark on the colored rectangles, if you want to.

When we say "measure the area of a rectangle" we mean find how many square inches it would take to fill all the space inside the rectangle (demonstrate).

Treatment E

On Friday I told you about a shorter way to find the area of a rectangle. By using this shorter way, you did not need to use the blue square inch shapes. You only needed to use your ruler.

Here is how to find the area of a rectangle by multiplying. First you measure how many inches high the rectangle is. This tells you how many rows of 1" squares there are in the rectangle (demonstrate). This rectangle is 4" high, so there are 4 rows of inch squares. Next, you measure how long the rectangle is. This tells you how many 1" squares are in each row (demonstrate). This rectangle is 5" long, so we know there are 5 square inches in each row. The rectangle is 4" high, so there are 4 rows and the rectangle in 5 " long, so there are 5 square inches in each row. We multiply 4 rows X 5 square inches in each row

Appendix C (con't)

and we get 20 square inches. The area of the rectangle is 20 square inches.

So the short way to find the area of a rectangle is to find how many inches high it is and how many inches long it is, and then multiply the number of inches high X the number of inches long. Remember to measure how high the rectangle is and how long it is. Do not just measure how high it is. Now look at number 1.

Treatment GD

Upon completion of problem 4: On Friday I told you about a shorter way to measure the area of a rectangle. By using the shorter way, you only needed to use your ruler. First you measure how high the rectangle is. This tells you how many rows of 1" squares there are in the rectangle (demonstrate). This rectangle is 4" high, so there are 4 rows of inch squares. Next you measure how long the rectangle is. This tells you how many 1" squares are in each row (demonstrate). This rectangle is 5" long, so we know there are 5 square inches in each row. The rectangle is 4" high, so there are 4 rows, and it is 5" long, so there are 5 square inches in each row. We multiply 4 rows X 5 square inches in each row and we get 20 square inches. The area of the rectangle is 20 square inches.

So the short way to find the area of a rectangle is to measure how many inches long it is and high many inches high it is and multiply the number of inches high X the number of inches long.

Treatment D

Simply read the problems.

Appendix C (con't)

Learning Unit

4th Day Name ______________

(1.) Take the colored rectangles and your ruler out of your packet. Use your ruler to find the area of each of the colored rectangle:

Write your answers below:

1. Big red rectangle ________ square inches.
2. Little red rectangle ______ square inches.
3. Big yellow rectangle ______ square inches.
4. Little yellow rectangle ____ square inches.
5. Big orange rectangle ______ square inches.
6. Little orange rectangle _____ square inches.
7. Big blue rectangle ______ square inches.
8. Little purple rectangle ____ square inches.

(2.) George is helping his Dad build a wall. If he knows that he can fit 6 tiles across the bottom of the wall and 5 tiles up the side, how many tiles will his Dad need altogether?

Draw a picture on your graph paper in your packet, if you need help.

Write your answer here: ___ tiles.

Appendix C (con't)

(3.) Beth has made a pan of fudge. She cut the fudge in 7 pieces from top to bottom. She cut it in 4 pieces from one side to the other. How many pieces will she have altogether?

Draw a picture with your graph paper if you need help.

Write your answer here.____ pieces of fudge.

(4.) Jenny is making placemats for her mother. Each placemat is 10 inches long and 10 inches wide. How many square inches of material will she need for 1 placemat?

Write you answer here: _____ square inches

APPENDIX D

Post-tests* and Instructions to Subjects

Instructions

To All Experimental Groups and Control Group

During the next three days I am going to give you some problems to do to see how much you learned in the last four days. I am also going to read you the sentences that tell how you feel about arithmetic. (Administer the Arithmetic Attitude Scale). Now you ready to work your problems. Some of the problems are like problems you have done before and some are different. I will read the directions for each problem to you. If you do not understand the directions, raise your hand and I will explain them to you. Work as many problems as you can. If a problem seems too hard, skip that problem and do the next one. Do your own work and do not talk aloud while you are working. Are there any questions? Remember, do not talk to anyone about these problems.

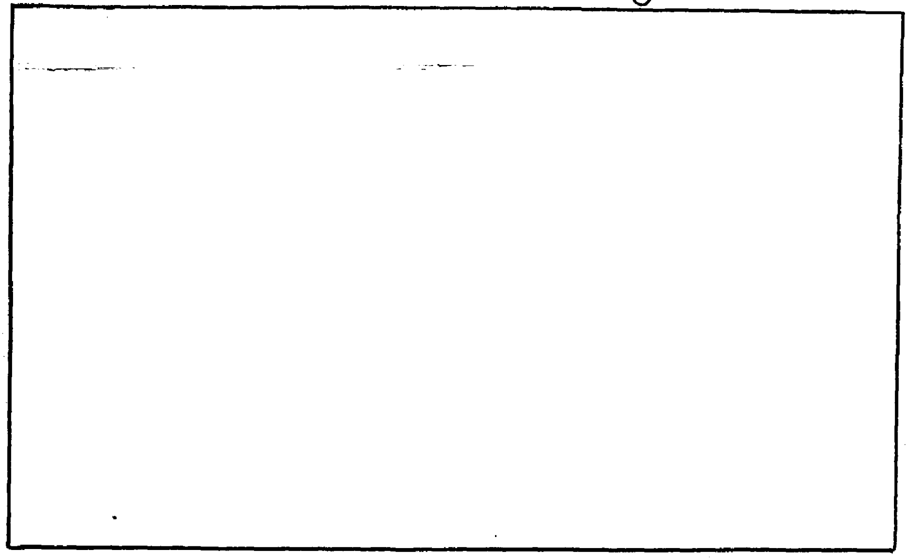
*Numbers in parentheses indicate the order in which test items were presented.

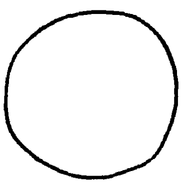
Appendix D (con't)

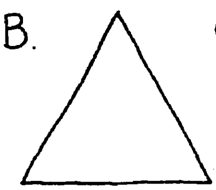
Test of Initial Learning

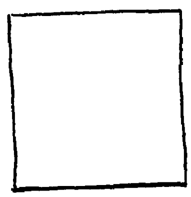
Name ______________

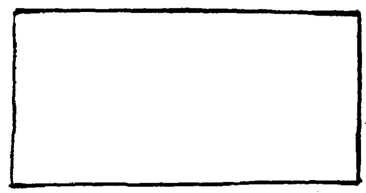
(1.) (1) Should you use shape A, B, C, or D to measure the rectangle?
Put a check on the right shape.



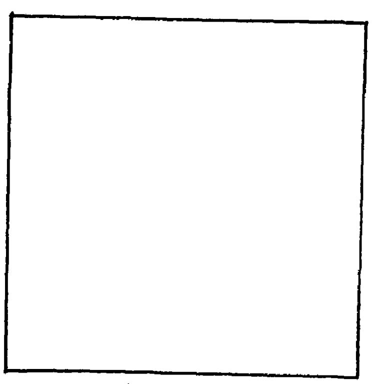
A. 

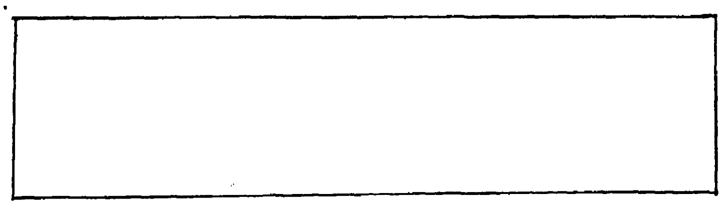
B. 

C. 

D. 

(2.) (2) Use the blue inch squares to find the area of the rectangles on this page and on the next page. Write your answer under each rectangle.





___ square inches

___ square inches

Appendix D (con't)

Name ____________

page 2

3.

____ square inches

4.

____ square inches

5.

square inches.

Appendix D (con't)

Name ____________ page 3

(3.) (13) Beth is making a quilt from square pieces of cloth. She fits 3 square pieces along one side and 5 square pieces along the other side. How many square pieces will she need altogether?

Here is a picture to help you.
Write your answer here. ___ square pieces.

(4.) (14) Leroy wants to cover the top of a box that he uses for his marbles. The top of the box is 10 inches long and 7 inches wide. How many square inches of paper will he need?

Draw a picture on your graph paper to help you find the answer.

Write your answer here. ___ square inches.

Appendix D (con't)

Name __________ page 4

(5.) (12) Use your ruler to find the area of the colored rectangles in your packet.

Write your answers below:

1. Big red rectangle (4x4) square inches.
2. Little red rectangle (2x8) square inches.
3. Big yellow rectangle (6x9) square inches.
4. Little yellow rectangle (1x7) square inches.
5. Big orange rectangle (6x6) square inches.
6. Little orange rectangle (5x2) square inches.

Appendix D (con't)

Test of Immediate Transfer

Name ______________

(1.) (15) The red and yellow shapes on your desk are called cylinders. How many square inches of paper will you need to cover the outside of each cylinder.

Use your ruler to find the answers.

Write your answers below.

1. Red cylinder (5x8) square inches.
2. Yellow cylinder (3x4) square inches.

(2.) (16) Pretend you are going to cover a can with red paper and decorate it for a Valentine's Day mailbox. If your can is 7 inches around the top and bottom and 4 inches high? Will 30 square inches of paper be enough? ____.

How many square inches of paper will you need?

Write your answer here. ______ square inches.

Appendix D (con't)

Name ____________________ page 2

(3.) (3) The blue shape on your desk is called a cube. How many square inches of paper would you need to cover the outside of the cube? (2 x 2 x 2)

Use your blue inch squares to find the answer. Write your answer here: ____ square inches.

(4.) (4) The yellow shape on your desk is a box. How many square inches of paper would you need to cover the outside of this box? (3x4x2)

Use your blue inch squares. Write your answer here: ____ square inches.

(5.) (5) Pretend you have a box. The front and back of the box are 4 inches long and 1 inch high. The top and bottom of the box are 4 inches long and 2 inches wide. The ends of the box are 2 inches long and 1 inch high. What is the area of all the sides of the box added together?

Write your answer here: ______ square inches.

Appendix D (con't)

Name ________________ page 3

(6.) (8) A rectangle has an area of 15 square inches. If it is 3 inches long, how wide must it be?

Write your answer here: ___ inches.

(7.) (9) Mary's bookmarker is made of 12 square inches of paper. It is 6 inches long. How wide must it be?

Write your answer here: ____ inches.

(8.) (10) Look at the red box on your desk. It does not have a top. How many little blocks that are 1 inch on each side would it take to fill up the whole box? (3 × 3 × 2)

Write your answer here: ____ inch boxes

(9.) (11) Sam has a box that he wants to fill up with ice cubes. The box is 5 inches long, 2 inches high, and 1 inch wide.

How many ice cubes that are 1 inch on each side would he need to fill up the whole box?

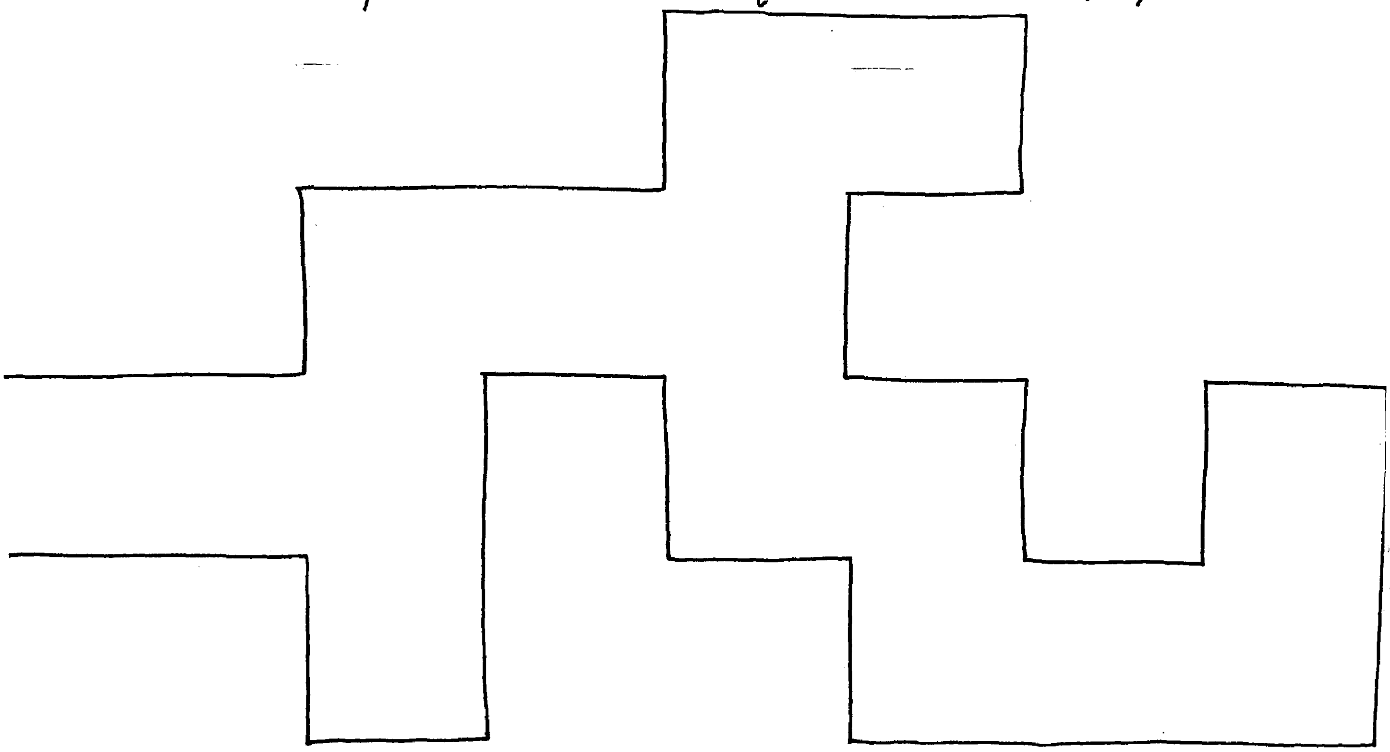
Write your answer here: ____ ice cubes.

Appendix D (con't)

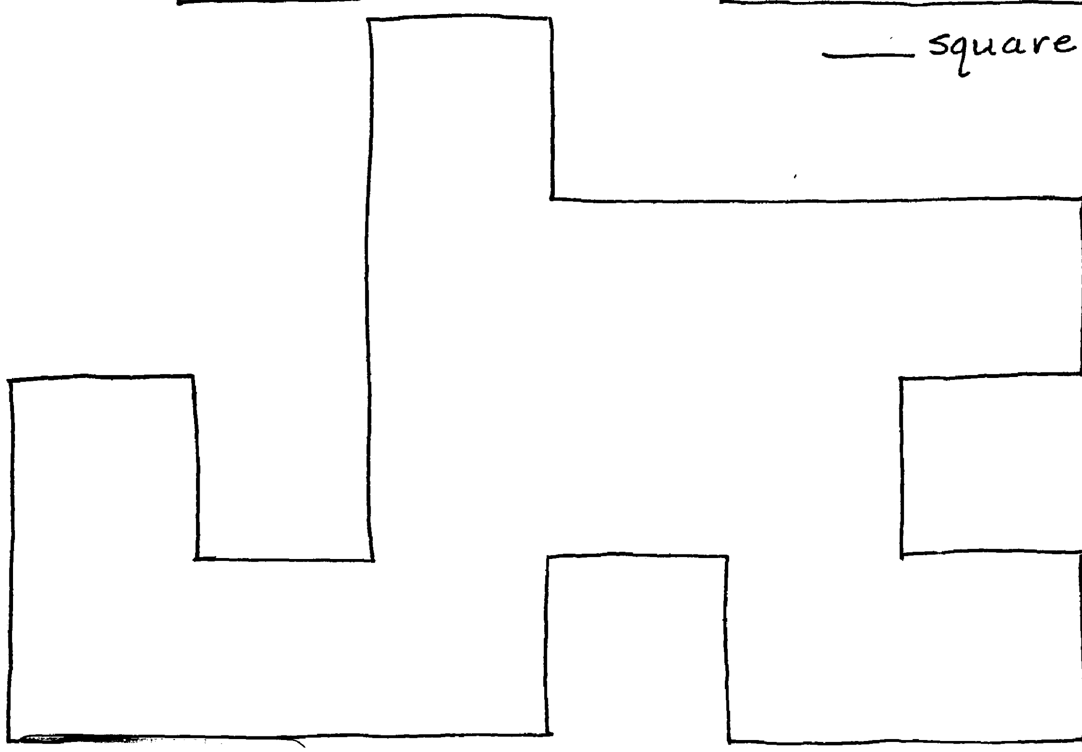
Name ______________ page 4

(10.) (6)

Measure the area of these two shapes. Write the number of square inches in each shape under it. Use your blue inch squares to help you.



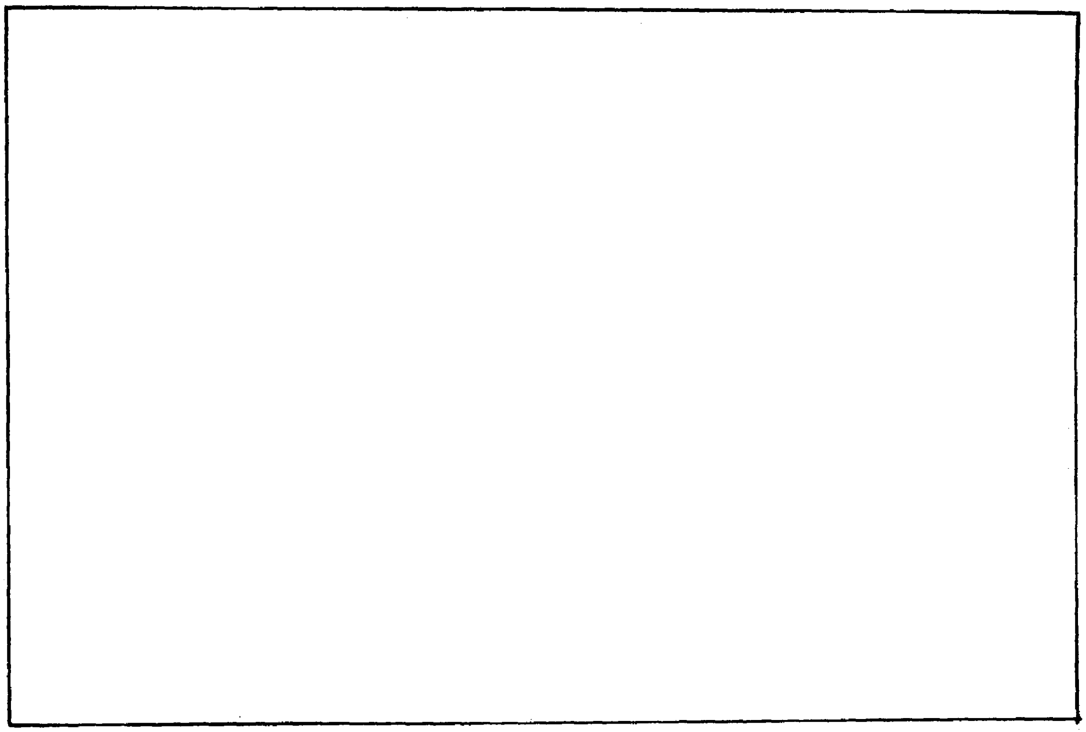
___ square inches



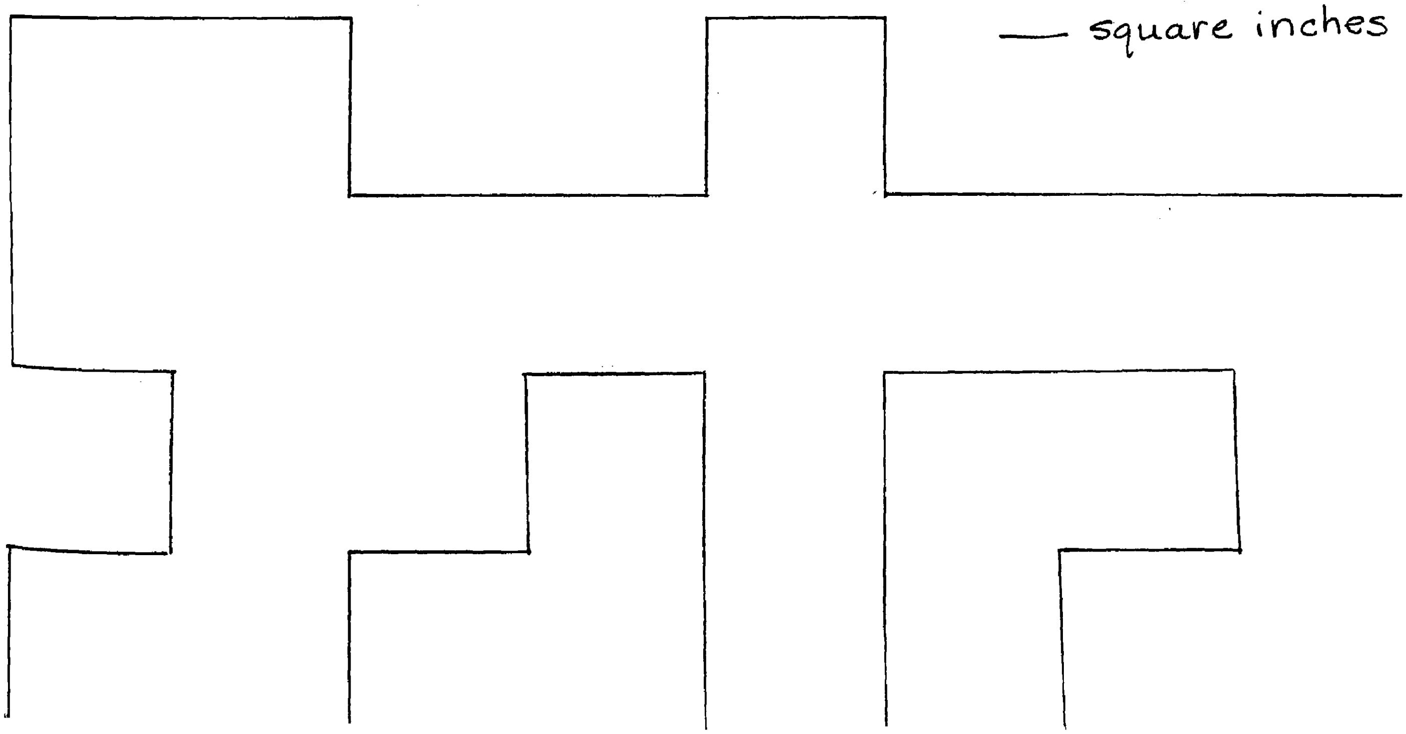
Appendix D (con't)

Name ______________ page 5

(11.) (7) Which shape is largest? Use your blue inch squares to measure the area of each shape. Write the number of square inches in each shape under it.



___ square inches



APPENDIX E

Post-Post-Tests* and Instructions to Subjects

Instructions

To All Experimental Groups and Control Group

During the next three days I am going to give you some problems to see how much you remember about measuring the area of rectangles. The problems will be very much like the problems you worked before.

Before you start your problems, I am going to read the sentences that tell the way you feel about arithmetic to you again. (Administer the Arithmetic Attitude Scale).

Now you are ready to work your problems. Write your name at the top of each page on the blank by "name".

I will read each problem to you. If you don't understand the directions raise your hand, after I have finished reading. I will not help you find the answers to the problems. I want you to do the best job you can by yourself. Remember, do not talk to anyone about the problems.

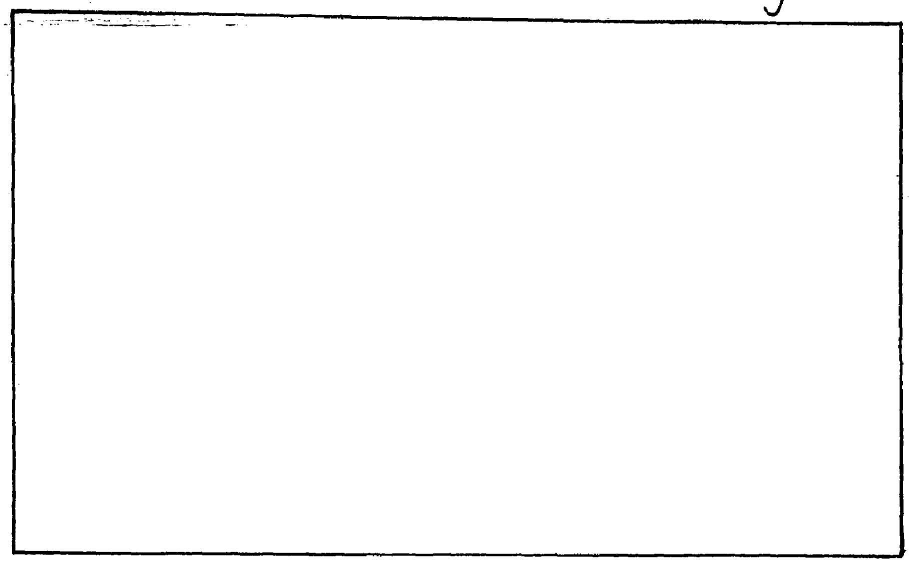
*Numbers in parentheses indicate the order in which test items were presented.

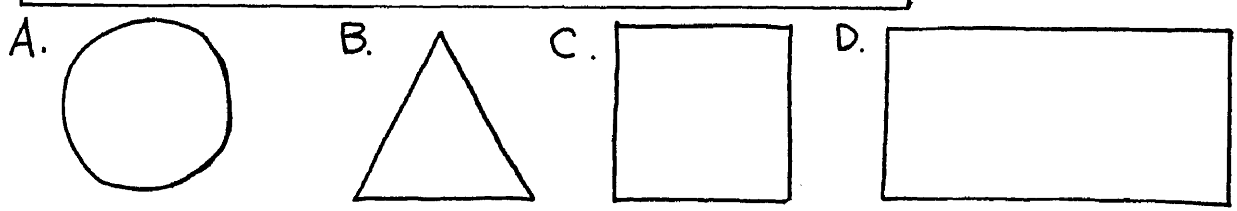
Appendix E (con't)

Retention Test

Name ________________

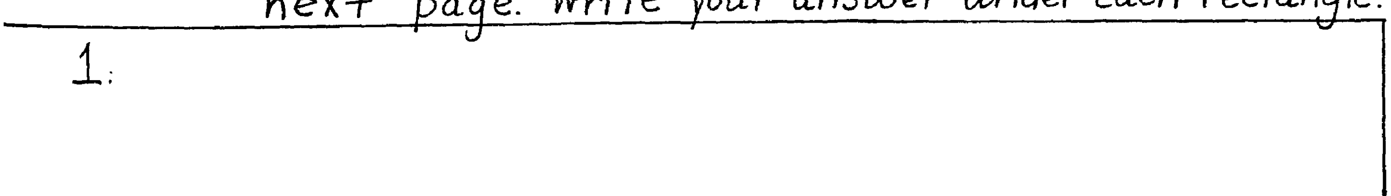
1. (1) Should you use shape A, B, C, or D to measure the rectangle below? Put a check on the right shape.





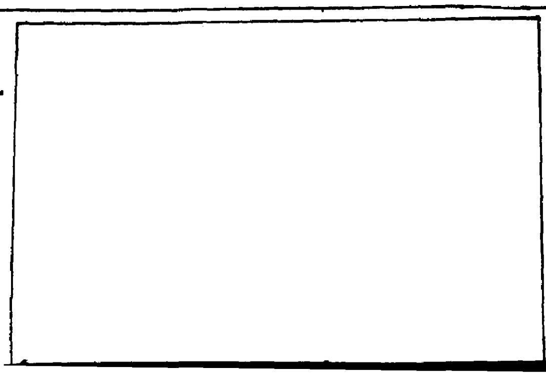
2. (2) Use the blue inch squares to find the area of the rectangles on this page and on the next page. Write your answer under each rectangle.

1.



___ square inches

2.



square inches

Appendix E (con't)

page 2

Name ______________

3.

___ square inches

4.

___ square inches

5.

Appendix E (con't)

page 3

Name ______________

(3.) (13) Beth is making a quilt from square pieces of cloth. She fits 3 square pieces along one side and 6 square pieces along the other side. How many square pieces will she need altogether to make the quilt?

Here is a drawing to help you.

Write your answer here. ___square pieces.

(4.) (14) Leroy wants to cover the top of a box that he uses for his marbles. The top of the box is 10 inches long and 6 inches wide. How many square inches of paper will he need?

Draw a picture on your graph paper to help you find the answer.

Write your answer here. ___ square inches.

Appendix E (con't)

Name ______________

(5.) (12) Use your ruler to find the area of the colored rectangles on your desk.

Write your answers below:

1. Big red rectangle (2 x 7) square inches.
2. Little red rectangle (3 x 3) square inches.
3. Big purple rectangle (4 x 10) square inches.
4. Little purple rectangle (1 x 5) square inches.
5. Big yellow rectangle (5 x 6) square inches.
6. Little yellow rectangle (2 x 4) square inches.

Appendix E (con't)
Test of Delayed Transfer

Name ________________

(1.) (15) The red and yellow shapes on your desk are called cylinders. How many square inches of paper will you need to cover the outside of each cylinder.

Use your ruler to find the answers.

Write your answer below.

1. Red cylinder <u>(5x8)</u> square inches.

2. Yellow cylinder <u>(3x4)</u> square inches.

(2.) (16) Pretend you are going to cover a can with red paper and decorate it for a Valentine's Day mailbox. If your can is 7 inches around the top and bottom and 4 inches high. Will 30 square inches of paper be enough? ____.

How many square inches of paper will you need?

Write your answer here: ____ square inches.

Appendix E (con't)

Name ____________________ page 2

(3.) (3) The blue shape on your desk is called a cube. How many square inches of paper would you need to cover the outside of the cube? (2 x 2 x 2)

Use your blue inch squares to find the answer.

Write your answer here: ___ square inches.

(4.) (4) The yellow shape on your desk is a box. How many square inches of paper would you need to cover the outside of this box? (3 x 4 x 2)

Use your blue inch squares.

Write your answer here: ___ square inches.

(5.) (5) Pretend you have a box. The front and back of the box are 4 inches long and 1 inch high. The top and bottom of the box are 4 inches long and 2 inches wide. The ends of the box are 2 inches long and 1 inch high. What is the area of all the sides of the box added together?

Write your answer here: ___ square inches.

Appendix E (con't)

Name ______________________ page 3

(6.) (8) A rectangle has an area of 15 square inches. If it is 3 inches long, how wide must it be?

Write your answer here: _____ inches.

(7.) (9) Mary's bookmarker is made of 12 square inches of paper. It is 6 inches long. How wide must it be?

Write your answer here: _____ inches

(8.) (10) Look at the red box on your desk. It does not have a top. How many little blocks that are 1 inch on each side would it take to fill up the whole box? (3 × 3 × 2)

Write your answer here: _____ inch boxes.

(9.) (11) Sam has a box that he wants to fill up with ice cubes. The box is 5 inches long, 2 inches high and 1 inch wide.

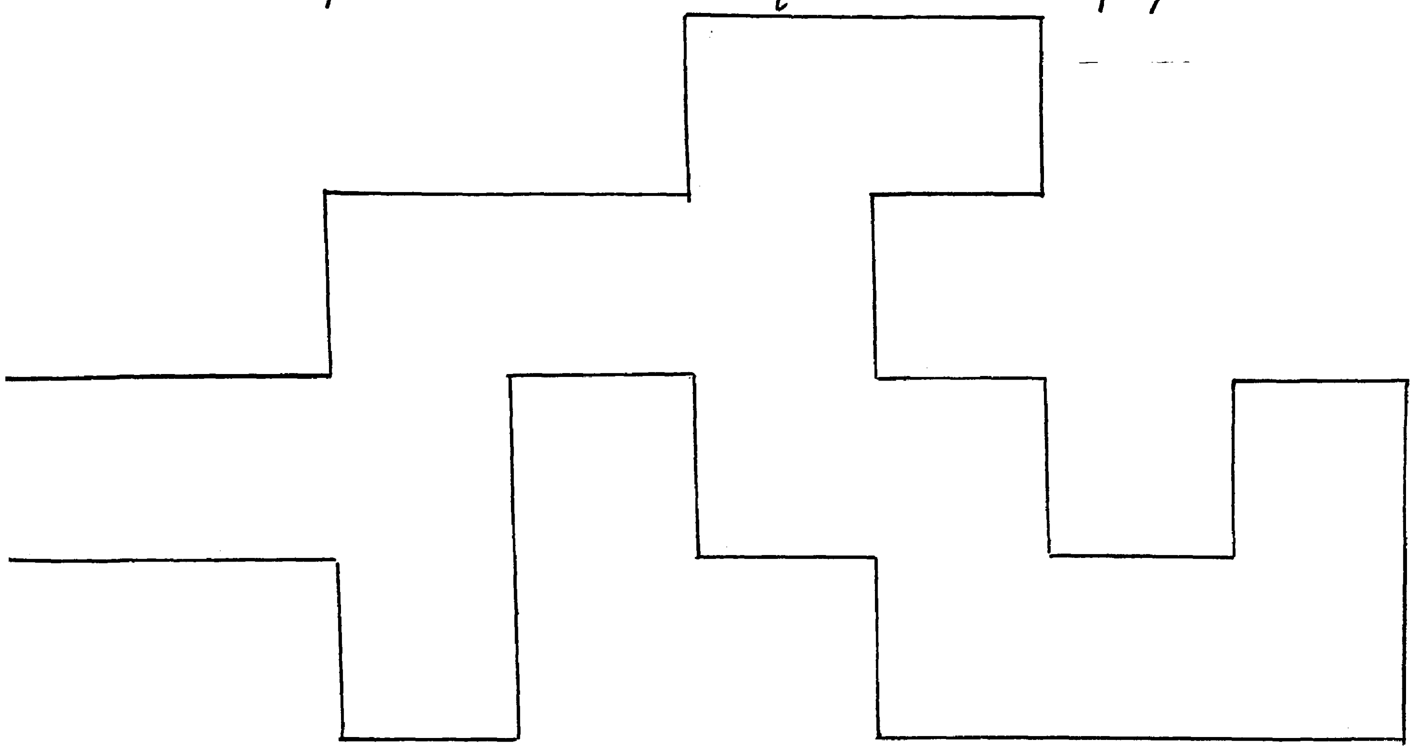
How many ice cubes that are 1 inch on each side would he need to fill up the whole box?

Write your answer here: _____ ice cubes.

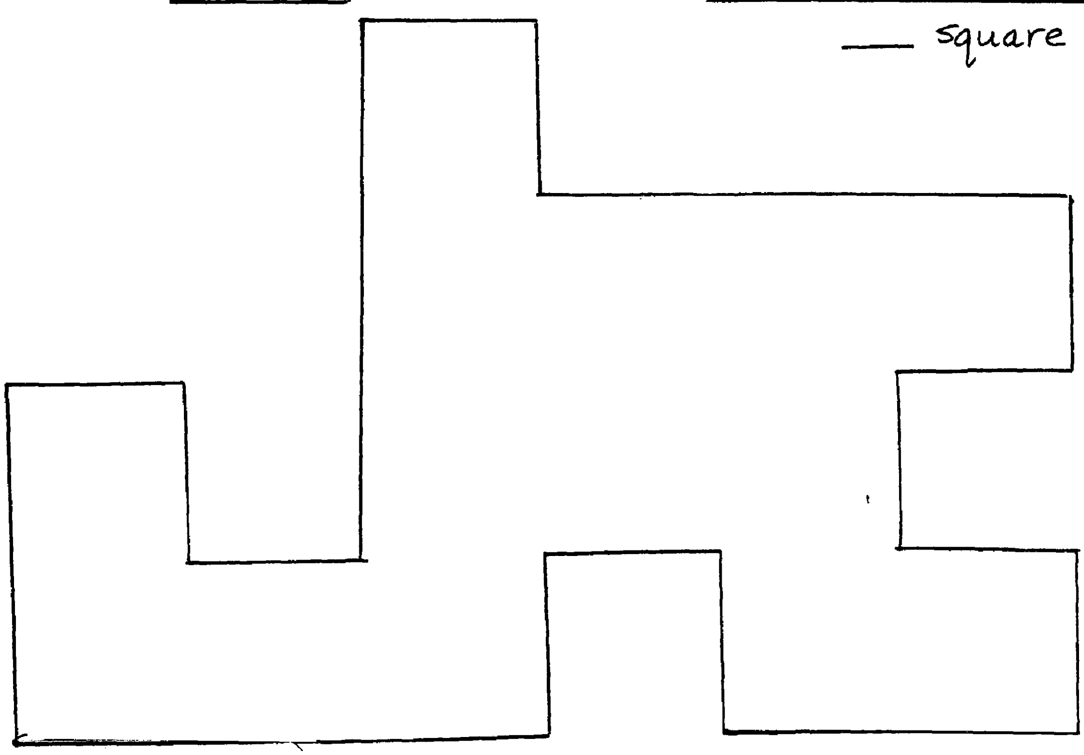
Appendix E (con't)

Name ____________________ page 4

(10.) (6) Measure the area of these two shapes. Write the number of square inches in each shape under it. Use your blue inch squares to help you.



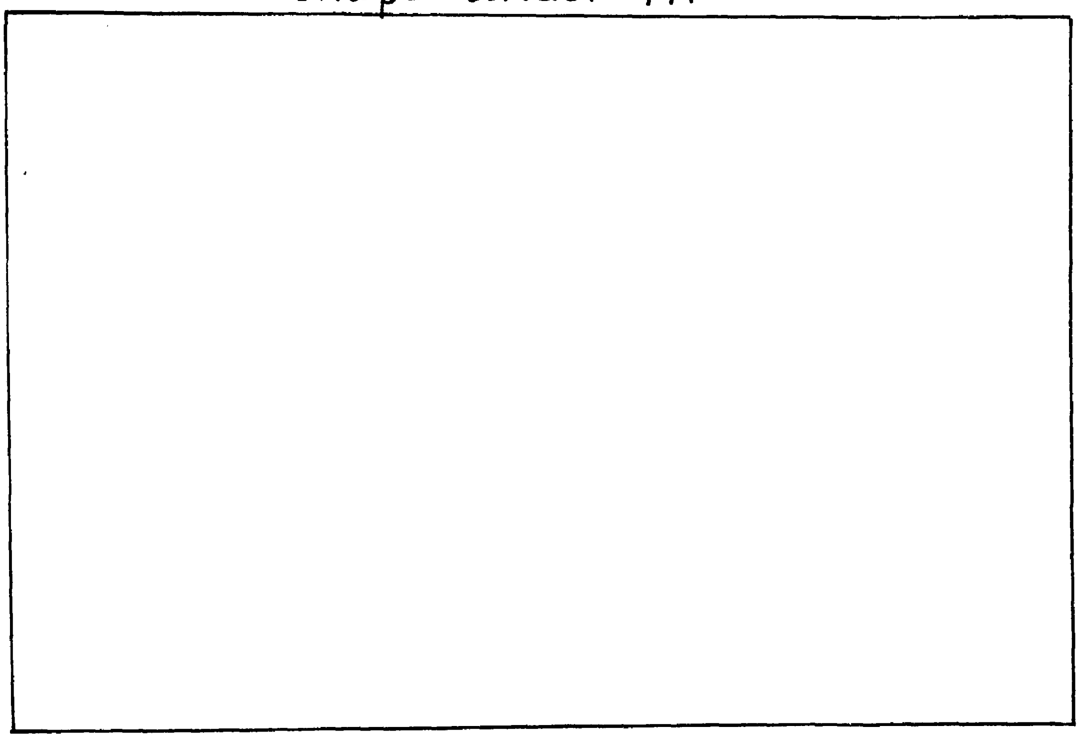
___ square inches



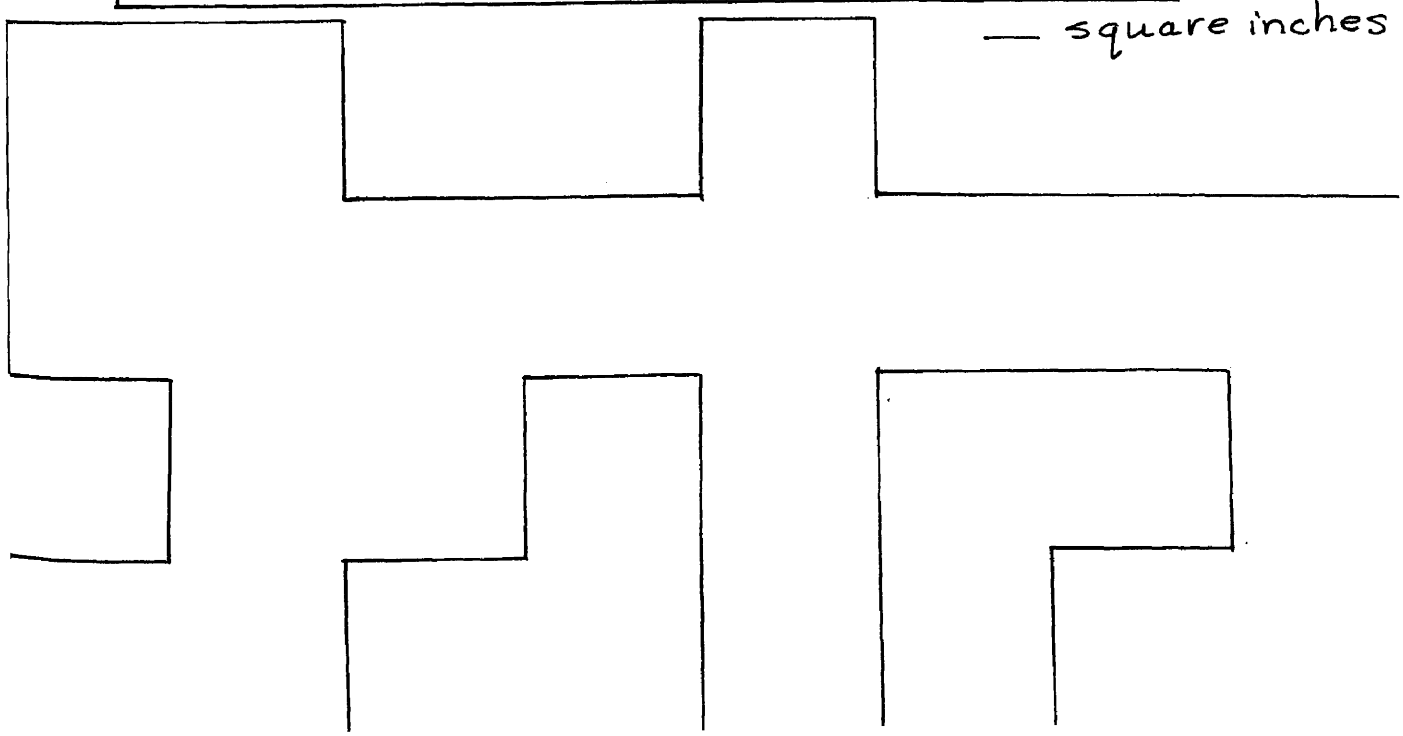
Appendix E (con't)

Name ______________________ page 5

(11.) (7) Which shape is largest? Use your blue inch squares to measure the area of each shape. Write the number of square inches in each shape under it.



___ square inches



APPENDIX F

TEACHER BEHAVIORS

1. Partial-Fair Teacher Behavior

Partial

1. Repeatedly slighted a pupil.
2. Corrected or criticized certain pupils repeatedly.
3. Repeatedly gave a pupil special advantages.
4. Gave most attention to one or a few pupils.
5. Showed prejudice (favorable or unfavorable) toward some social, racial, or religious groups.
6. Expressed suspicion of motives of a pupil.

Fair

1. Treated all pupils approximately equally.
2. In case of controversy pupil allowed to explain his side.
3. Distributed attention to many pupils.
4. Rotated leadership impartially
5. Based criticism on praise on factual evidence, not hearsay.

2. Aloof-Responsive Teacher Behavior

Aloof

1. Stiff and formal in relation with pupils.
2. Apart; removed from class activity.
3. Condescending to pupils.
4. Routine and subject matter only concern; pupils as persons ignored.
5. Referred to pupil as "this child" or "that child."

Responsive

1. Approachable to all pupils.
2. Participated in class activity.
3. Responded to reasonable requests and/or questions.
4. Spoke to pupils as equals.
5. Commended effort.
6. Gave encouragement.
7. Recognized individual differences.

3. Harsh-Kindly Teacher Behavior

Harsh

1. Hypercritical; fault-finding.
2. Cross; curt.
3. Depreciated pupil's efforts; was sarcastic.
4. Scolded a great deal.
5. Lost temper.
6. Used threats.
7. Permitted pupils to laugh at mistakes of others.

Kindly

1. Went out of way to be pleasant and/or to help pupils; friendly.
2. Gave a pupil a deserved compliment.
3. Found good things in pupils to call attention to.
4. Seemed to show sincere concern for a pupil's personal problem.
5. Showed affection without being demonstrative.
6. Disengaged self form a pupil without bluntness.

Appendix F (con't)

4. Apathetic-Alert Teacher Behavior

Apathetic

1. Seemed listless; lanquid; lacked enthusiasm.
2. Seemed bored by pupils.
3. Passive in response to pupils.
4. Seemed preoccupied.
5. Attention seemed to wander.
6. Sat in chair most of time; took no active part in class activities.

Alert

1. Appeared buoyant; wide-awake; enthusiastic about activity of the moment.
2. Kept constructively busy.
3. Gave attention to, and seemed interested in, what was going on in class.
4. Prompt to "pick up" class wher pupils' attention showed signs of lagging.

5. Erratic-Steady Teacher Behavior

Erratic

1. Impulsive; uncontrolled; temperamental; unsteady.
2. Course of action easily swayed by circumstances of the moment.
3. Inconsistent.

Steady

1. Calm; controlled.
2. Maintained progress toward objective.
3. Stable, consistent, predictable.

6. Excitable-Poised Teacher Behavior

Excitable

1. Easily disturbed and upset; flustered by classroom situation.
2. Hurried in class activities; spoke rapidly using many words and gestures.
3. Was "jumpy"; nervous.

Poised

1. Seemed at ease at all times.
2. Unruffled by situation that developed in classroom; dignified without being stiff or formal.
3. Unhurried in class activities; spoke quietly and slowly.
4. Successfully diverted attentic from a stress situation in classroom.

7. Uncertain-Confident Teacher Behavior

Uncertain

1. Seemed unsure of self; faltering, hesitant.
2. Appeared timid and shy.
3. Appeared artificial.
4. Disturbed and embarrassed by mistakes and/or criticism.

Confident

1. Seemed sure of self; self-confident in relations with pupils.
2. Undisturbed and unembarrassed by mistakes and/or criticism.

Appendix F (con't)

CLASSROOM OBSERVATION RECORD

TEACHER BEHAVIOR										REMARKS
1. Partial	1	2	3	4	5	6	7	N	Fair	
2. Aloof	1	2	3	4	5	6	7	N	Responsive	
3. Harsh	1	2	3	4	5	6	7	N	Kindly	
4. Apathetic	1	2	3	4	5	6	7	N	Alert	
5. Erratic	1	2	3	4	5	6	7	N	Steady	
6. Excitable	1	2	3	4	5	6	7	N	Poised	
7. Uncertain	1	2	3	4	5	6	7	N	Confident	

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