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Partial Wave Analysis of the $\omega \pi^-$ Final State Photoproduced at GlueX

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Partial Wave Analysis of the $\omega\pi^-$ Final State Photoproduced at GlueX

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A Dissertation presented to the Graduate Faculty
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ABSTRACT

The meson has traditionally been understood as the bound state of a quark and an antiquark, but there is experimental evidence for exotic meson states, which have properties that cannot be produced by this quark-antiquark model. One candidate for such exotic systems is the hybrid meson, predicted by lattice QCD, which includes gluonic excitation in its wavefunction. One of the prominent decay modes of the lightest predicted exotic hybrid is expected to be $b_1\pi$. This dissertation presents an analysis of the $\gamma p \rightarrow \omega\pi^-\Delta^{++}$ channel produced at the GlueX experiment, in the mass range where the $b_1(1235)$ meson is prominent. An amplitude analysis is performed on the decay of the $\omega\pi^-$ system, in order to extract the resonance parameters of the $b_1^-$, as well as analyze the resonant and non-resonant background.
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For my grandparents, Keith and Jeanette Schertz.
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PARTIAL WAVE ANALYSIS OF THE $\omega\pi^-$ FINAL STATE PHOTOPRODUCED AT GLUEX
CHAPTER 1

Introduction

This dissertation describes a partial wave analysis of the $\omega\pi^-$ meson system in the photoproduction reaction $\gamma p \rightarrow \omega\pi^- \Delta^{++}$. The data were collected at the GlueX experiment, where we aim to study the spectrum of light hadrons and compare to theoretical calculations based on the theory of the strong nuclear force. In this channel, we observe a strong signal for the axial vector $b_1^- \rightarrow \omega\pi^-$ decay, which has not been studied previously in photoproduction.

While most mesons can be described by the constituent quark model, a class of hadrons known as spin-exotic mesons cannot. As discussed in Section 1.2.5, one candidate for a spin-exotic meson is the hybrid meson, which includes gluonic excitation in its wavefunction. Theoretical calculations predict the spectrum of light quark mesons and that the lightest spin-exotic is the $\pi_1(1600)$, which is predicted to decay dominantly to $b_1\pi$. The $\gamma p \rightarrow \omega\pi^- \Delta^{++}$ channel, described in this thesis, is also the largest background contributor to the $b_1^\mp\pi^\mp p$ final state, predicted to be the dominant decay mode of the spin-exotic $\pi_1(1600)$ meson. Understanding this background is crucial for future analyses of the $b_1^\mp\pi^\mp p$ channel in the search for spin-exotic mesons at GlueX.
In this analysis, we extract the ratio of $D$-wave to $S$-wave in $b^- \rightarrow \omega \pi^-$ decay, and determine the dominant reflectivity of $b^-\pi$ photoproduction with a charged exchange mechanism. Chapter 1 introduces the Standard Model of particle physics, with an emphasis on the strong nuclear force, and provides motivation for this study. Chapter 2 outlines the formalism used to model the $b^- \rightarrow \omega \pi^-$ decay, and Chapter 3 gives an overview of previous studies, experimental and theoretical, of the $\omega \pi$ decay channel. Chapter 4 describes the GlueX experiment, where the data for this study was collected, and Chapter 5 describes my service work to the experiment, calculation of how charged pions are tracked within the GlueX detector system [1], critical to this analysis. Chapter 6 describes how I selected the events used in this analysis from background events [2, 3]. Chapter 7 discusses the mechanics behind the partial wave analysis, Chapter 8 provides an analysis of the results, and Chapter 9 summarizes this work and discusses what further steps should be taken.

1.1 The Standard Model

There are four fundamental forces in the known universe: gravity, the weak nuclear force, electromagnetism, and the strong nuclear force. The Standard Model of particle physics is able to describe all but gravity, and classify the known elementary particles: quarks, leptons, gauge bosons, and the Higgs boson, illustrated in Fig. 1.1. The Standard Model is a quantum field theory, meaning that fundamental forces are mediated by virtual particles called gauge bosons. Gravity, which governs the attraction of masses to each other, is described by the theory of general relativity and is the one force not described by the Standard Model. Fortunately, at the mass scales discussed in this dissertation, the effects of gravity are negligible.

Electromagnetism, which governs the attraction and repulsion of electrically charged particles and currents, as well as electric and magnetic fields, is carried by the photon. The
interactions between photons and electrically charged particles are described by quantum electrodynamics (QED), which is based on the gauge group $U(1)$.

The weak nuclear force governs nuclear decay, among other things, and can be unified with the electromagnetic interactions using the gauge group $SU(2) \times U(1)$. The weak force is carried by the $W^\pm$ and the $Z$ bosons, and it affects particles with weak charge, such as quarks and leptons. It is notably the only force in the Standard Model that includes interactions with neutrinos.

The strong nuclear force is what binds quarks and gluons into composite particles.
called hadrons. It is described by quantum chromodynamics (QCD) using the group $SU(3)$, and the force is carried by gluons. Quarks carry a “color” charge, which is unlike the electric charge in that it can take on any one of three values, which are denoted red ($R$), blue ($B$), and green ($G$). Only color-neutral states, such as red-antired ($R\bar{R}$) or red-green-blue ($RGB$) are observed in nature, a property unique to QCD called confinement, to be further discussed in Section 1.2.

The quarks, leptons, and force carrier bosons are shown in Fig 1.1. Every particle described by the Standard Model has an associated antiparticle. A particle and its antiparticle have the same mass, spin, and lifetime, but opposite additive quantum numbers, such as electric charge, color charge, and magnetic moments. For example, an up quark’s antiparticle is an anti-up antiquark, with a mass equal to that of the up quark, but an electric charge of $-2/3$. If that up quark’s color charge were red, its antiquark would have a color charge of anti-red. Some neutral particles, such as photons, are their own antiparticles.

### 1.2 Quantum Chromodynamics

Quantum chromodynamics describes strong interactions using a quantum field theory of quarks and gluons, based on the $SU(3)$ color gauge symmetry group. $SU(3)$ is an exact symmetry of QCD, since the quark mass is independent of color. Color charges in QCD are analogous to electric charges in QED, though there are three distinct color charges as opposed to one electric charge, and the force carrier gluons carry color charge, unlike the electrically neutral photon. Since gluons carry color charge, they can in principle be constituents of hadrons. States involving gluonic excitation are not forbidden in the hadron spectrum, and some experimental evidence for such states will be described in Sec. 1.2.5. One candidate model for such a state is the hybrid meson ($q\bar{q}g$), which consists
of a quark-antiquark pair and an excited gluonic field in its wavefunction.

The QCD Lagrangian is given by

\[ \mathcal{L}_{QCD} = \bar{\psi}_i \left[ \delta_{ij} \left( i \partial_\mu \gamma^\mu - m_q \right) - g_s \left( \gamma^\mu \lambda^a_{ij} A^a_\mu \right) \right] \psi^j - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}, \tag{1.1} \]

for a single quark flavor (to include more, we would sum the quark term over the relevant flavors). The field strength tensor \( F^a_{\mu\nu} \) is

\[ F^a_{\mu\nu} = \partial_\nu A^a_\mu - \partial_\mu A^a_\nu + g_s f^{abc} A^b_\mu A^c_\nu, \tag{1.2} \]

where \( \bar{\psi}_i \) are the quark fields, \( A^a_\mu \) are the vector potentials of the gluon fields, \( \lambda^a \) are the Gell-Mann \( SU(3) \) generators, which satisfy the commutation relation \( [\lambda^a, \lambda^b] = 2i f^{abc} \lambda^c \), where \( f^{abc} \) are the structure constants of the non-Abelian group. The QCD coupling “constant,” \( \alpha_s = \frac{g^2}{4\pi} \), where \( g_s \) is the bare coupling, is actually a function of the energy scale of the process, which is inversely related to the distance between the quarks. As the distance between a quark and an antiquark increases, so does the strength of the color attraction between them, due to self-interaction between gluons. This “running” of the QCD coupling constant is illustrated in Fig. 1.2, where the coupling is plotted as a function of \( Q \), the energy involved in the QCD process. At high energies (short distances), \( \alpha_s \) approaches zero, which is a property called asymptotic freedom, and allows high-energy QCD processes to be calculated perturbatively. Conversely, at lower energies (large distances), such as the study described in this dissertation, the coupling constant is large and forbids the use of perturbation theory. If we want to make predictions at this hadronic scale, we must rely on techniques such as lattice QCD, in which the four-dimensional space-time continuum is discretized onto a lattice of points, giving a numerical solution to the QCD Lagrangian.
1.2.1 The Quark Model

Despite the challenges of direct QCD calculations, models can be used to describe much of what we observe in nature. The quark model, published by Gell-Mann in 1964 [5], describes hadrons as particles composed of members of a flavor symmetry $SU(3)$ triplet, called “quarks” ($q$). The three known quarks at the time were up ($u$), down ($d$), and strange ($s$). In this model, baryons, which have half-integer spin, can be formed by the combinations ($qqq$), ($qqqqar{q}$), and so on, where $\bar{q}$ denotes the antiquark. Mesons, which have integer spin, are made by ($q\bar{q}$), ($qq\bar{q}\bar{q}$), and so on. The only configurations observed at
the time were the lowest in each category, with the \((qqq)\) configuration giving the baryon singlet, octet, and decuplet representations (see Section 1.2.4), and the \((q\bar{q})\) configuration giving the meson singlet and octet (see Section 1.2.3 and Fig. 1.3).

\[ \text{FIG. 1.3: Pseudoscalar } (J^P = 0^-) \text{ meson octet. The horizontal axis indicates the third component of isospin } (I_3, \text{ Section 1.2.2}), \text{ the vertical axis indicates the strangeness quantum number } (S), \text{ and the diagonal axis indicates electric charge } (Q). \text{ At the center lie the neutral } \pi^0 \text{ and } \eta \text{ mesons. Not shown is the } \eta' \text{ meson, which mixes with the } \eta, \text{ but is closer to the pseudoscalar singlet. Source: Trassiorf, Public domain, via Wikimedia Commons.} \]

Mesons, which have integer spin, can be divided into categories based on the combination of their total angular momentum, \(J\), and the behavior of their wavefunction under a parity transformation, \(P\). This nomenclature is outlined in Table 1.1. Another quantity we consider is the “naturality,” \(\tau = P(-1)^J\), of the particle. Mesons with \(\tau = 1\) are said to be natural, or to have natural parity, and mesons with \(\tau = -1\) are said to be unnatural, or have unnatural parity. This naturality is useful in describing hadron production mechanisms, as we’ll see in Section 1.3.

1.2.2 Flavor Symmetry of Meson Nonet

Analogous to the intrinsic spin of many elementary particles, hadrons possess a quantum number called isospin (see Chapter 2.1 of Ref. [6] for the historical reasons behind
the name). In the same way that a two-electron system can exist in a spin triplet or singlet, a quark-antiquark system can exist in an isospin triplet, with total isospin 1, or an isospin singlet, with total isospin 0, based on the “flavor” of the quarks in its wavefunction. Illustrated on the horizontal axis of Fig. 1.3, the pions exist in an isospin triplet. Each has total isospin 1, with $I_3$ ranging from +1 for the $\pi^+$ to −1 for the $\pi^-$. If we consider the strange quark, isospin symmetry can be extended to an $SU(3)$ flavor symmetry, but this symmetry is only approximate, due to the unequal mass of the up, down, and strange quarks. The $\eta$ meson is in an isospin singlet, with total isospin 0, but it mixes with the $\eta'$, as neither is a true $SU(3)$ eigenstate. Four pseudoscalar kaons, which each have one strange and one non-strange quark or antiquark in their wavefunction, compose the rest of the pseudoscalar nonet.

Meson octets (sometimes called nonets when the $SU(3)$ singlet state is considered) like the one illustrated in Fig. 1.3 exist for all accessible $J^{PC}$ states. For example, the vector mesons, with $J^{PC} = 1^{--}$, exist in a nonet with the $\rho^{\pm,0}$ mesons analogous to the pseudoscalar pions, with the $\omega$ and $\phi$ mesons roughly analogous to the $\eta$ and $\eta'$. Unlike the pseudoscalar mesons, there is almost no mixing between the $\omega$ and $\phi$, with the neutral $\rho$ and $\omega$ containing exclusively up and down quark content, while the $\phi$ is nearly completely an $s\bar{s}$ state. The vector kaons are denoted as $K^*$.

Another meson nonet is the axial vectors, with $J^{PC} = 1^{+-}$, and the $b_{1}^{\pm,0}$ mesons

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>Name</th>
<th>Naturality</th>
</tr>
</thead>
<tbody>
<tr>
<td>0+</td>
<td>Scalar</td>
<td>Natural</td>
</tr>
<tr>
<td>0−</td>
<td>Pseudoscalar</td>
<td>Unnatural</td>
</tr>
<tr>
<td>1+</td>
<td>Axial Vector</td>
<td>Unnatural</td>
</tr>
<tr>
<td>1−</td>
<td>Vector</td>
<td>Natural</td>
</tr>
<tr>
<td>2+</td>
<td>Tensor</td>
<td>Natural</td>
</tr>
<tr>
<td>2−</td>
<td>Pseudotensor</td>
<td>Unnatural</td>
</tr>
</tbody>
</table>

TABLE 1.1: Nomenclature of integer-spin hadrons up to $J = 2$. 
taking the place of the pions, the $h_1$ taking the place of the $\eta$, and the $K_1$ mesons taking the place of the pseudoscalar kaons. More on the various meson nonets can be found in Table 1.2. Note that the pseudotensor $^3D_2$ mesons with $J^{PC} = 2^{--}$ have not been observed experimentally.

The $b_1^-$ meson is a member of the axial vector isospin triplet, and has a PDG-reported mass and decay width of 1235 MeV and 142 MeV, respectively. This analysis is focused on the decay channel $\omega \pi^-$, to which the $b_1^-$ has been previously shown to be a strong contributor. The $b_1^-$ is the isospin partner of the neutral $b_1$ studied by our colleagues at University of Regina, which in principle has the same mass, width, and decay properties, such as the ratio of $D$-wave to $S$-wave in its decay. As we will discuss in Section 2.1, the $b_1$ meson can decay to $\omega \pi$ only for even values of relative orbital angular momentum, meaning $S$, $D$, $G$-waves, and so on. The ratio of $D$-wave to $S$-wave has previously been measured by the E852 collaboration to be $D/S = 0.269 \pm (0.009)_{\text{stat}} \pm (0.01)_{\text{sys}}$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
$L$ & $S$ & $2S+1L_J$ & $J^{PC}$ & Nonet & $I = 0$ & $I = 1$ & $I = \frac{1}{2}$ \\
\hline
0 & 0 & $^1S_0$ & 0$^{++}$ & Pseudoscalar & $\eta, \eta'$ & $\pi$ & $K$ \\
0 & 1 & $^3S_1$ & 1$^{--}$ & Vector & $\omega, \phi$ & $\rho$ & $K^*$ \\
1 & 0 & $^1P_1$ & 1$^{+-}$ & Axial Vector & $h_1, h'_1$ & $b_1$ & $K_1$ \\
1 & 1 & $^3P_0$ & 0$^{++}$ & Scalar & $f_0, f'_0$ & $a_0$ & $K^*_1$ \\
1 & 1 & $^3P_1$ & 1$^{--}$ & Axial Vector & $f_1, f'_1$ & $a_1$ & $K_1$ \\
1 & 1 & $^3P_2$ & 2$^{++}$ & Tensor & $f_2, f'_2$ & $a_2$ & $K^*_2$ \\
2 & 0 & $^1D_2$ & 2$^{+-}$ & Pseudotensor & $\eta_2, \eta'_2$ & $\pi_2$ & $K_2$ \\
2 & 1 & $^3D_1$ & 1$^{--}$ & Vector & $\omega, \phi$ & $\rho$ & $K^*_1$ \\
2 & 1 & $^3D_2$ & 2$^{--}$ & Pseudotensor & $\omega_2, \phi_2$ & $\rho_2$ & $K_2$ \\
2 & 1 & $^3D_3$ & 3$^{--}$ & & $\omega_3, \phi_3$ & $\rho_3$ & $K_3^*$ \\
\hline
\end{tabular}
\caption{Quark model meson nonets up to $L = 2$ and $S = 1$. Source: Table 1.3 of Ref. [8].}
\end{table}

1.2.3 Angular Momentum Addition Rules for Mesons

If mesons can only exist as the bound state of a quark and an antiquark, the quantum numbers of that meson state, $I^G(J^{PC})$, must abide by certain rules, where two spin-1/2
fermions combine with relative angular momentum $L$. The total angular momentum, $J$, is a function of the total spin of the quark-antiquark system, $S$, and the relative angular momentum between them, $L$.

$$J = L \oplus S = L + S, L, |L - S|.$$  \hfill (1.3)

The parity transformation, $P$, takes the spatial coordinates of a wavefunction to their negatives,

$$P[\psi(\vec{r})] = \psi(-\vec{r}) = \eta_P \psi(\vec{r}),$$  \hfill (1.4)

where $\eta_P$ is the eigenvalue of the parity transformation, $\eta_P = \pm 1$. The total parity of a quark-antiquark system is a function of the relative angular momentum,

$$P(q\bar{q}) = -(-1)^L = (-1)^{L+1}.$$  \hfill (1.5)

Charge conjugation, or $C$-parity, is a transformation that takes a particle to its antiparticle. For a $q\bar{q}$ system, the eigenvalue is $\eta_C = (-1)^{L+S}$. However, charge conjugation is only a good quantum number for electrically neutral states, where the particle is its own antiparticle and thus an eigenstate of charge conjugation. An extension that can apply to electrically charged states is $G$-parity, which involves a rotation in isospin as well as electrical charge. For a state whose electrically neutral member has $C$-parity $C$, and whose total isospin is $I$,

$$G(q\bar{q}) = C \cdot (-1)^I = (-1)^{L+S+I}.$$  \hfill (1.6)

These addition rules allow certain $J^{PC}$ combinations for $q\bar{q}$ systems

$$J^{PC} = 0^{++}, 0^{-+}, 1^{++}, 1^{+-}, 1^{--}, 2^{++}, 2^{-+}, 2^{--}, 2^{++}, ...$$  \hfill (1.7)
while excluding others

\[ J^{PC} = 0^{--}, 0^{--}, 1^{-+}, 2^{+-}, ... \]  \hspace{1cm} (1.8)

Meson states with these excluded quantum numbers are called spin-exotic, or often just exotic, and cannot be quark-antiquark systems. When states with these quantum numbers are observed, it is a clear indication that the traditional model of a meson as a quark-antiquark pair is incomplete. The properties of the exotic states that we may observe will provide insight into the role of gluonic excitation within hadrons, which will deepen our understanding of QCD.

1.2.4 Baryons

Hadrons containing three quarks have half-integer spins and are called baryons. A similar quark model description with \( SU(3) \) flavor symmetry can be used to compute the ground-state spin-1/2 baryon octet, containing the proton and neutron, which is shown in Fig. 1.4, and the ground-state spin-3/2 baryon decuplet, which contains the \( \Delta \) baryons, and is shown in Fig. 1.5. The \( \Delta^{++} \) baryon, indicated in the upper right corner of Fig. 1.5, is a contributor to the final state studied in this dissertation. It consists of three up quarks, has a resonance mass and width of 1232 MeV and 117 MeV, respectively, and decays nearly 100\% of the time through \( \Delta^{++} \rightarrow p\pi^+ \).
FIG. 1.4: Spin-1/2 baryon octet. The horizontal axis indicates the third component of isospin ($I_3$, Section 1.2.2), the vertical axis indicates the strangeness quantum number ($S$), and the diagonal axis indicates electric charge ($Q$). Source: Trassiorf, Public domain, via Wikimedia Commons

FIG. 1.5: Spin-3/2 baryon decuplet. The horizontal axis indicates the third component of isospin ($I_3$, Section 1.2.2), the vertical axis indicates the strangeness quantum number ($S$), and the diagonal axis indicates electric charge ($Q$). Source: Trassiorf, Public domain, via Wikimedia Commons
1.2.5 Exotic Mesons

There are several models for meson states that could produce exotic quantum numbers. One candidate is the hybrid meson, which consists of a quark and antiquark held together by an excited gluon. This gluonic excitation gives access to the exotic quantum numbers forbidden to a quark-antiquark pair. The Hadron Spectrum Collaboration (Had-Spec) has used lattice QCD calculations to predict the isovector and isoscalar meson spectrum of states without open strangeness, shown in Fig. 1.6. These calculations, as is normal for lattice calculations, were performed with unphysically heavy quark masses, corresponding to a pion mass of roughly 391 MeV. Thus, the mass scale on the vertical axis of Fig. 1.6 is presumed to be heavier than what we would observe in experiment, but the relative positions on the mass scale should approximately correspond to what we can measure. The exotic quantum number systems are shown in the rightmost three columns. The isovector ($I = 1$) states are indicated in blue, and the isoscalar ($I = 0$) states with light (up, down) and strange quark content are indicated in black and green, respectively. The states highlighted in orange are ones that both involve gluonic excitation in their wavefunctions and are at an energy scale comparable to the lightest exotic $J^{PC} = 1^{-+}$ states. These states are postulated as the lightest hybrid meson supermultiplet, which corresponds to a chromomagnetic excitation coupled to a color octet $q\bar{q}$ through an $S$-wave.
FIG. 1.6: Isoscalar and isovector meson spectrum on the lattice. The vertical height of each box indicates the statistical uncertainty on the mass determination. Source: Fig. 11 of Ref. [9].
**π_1(1600) and π_1(1400) in η^{0,\prime}π**

The lightest predicted exotic meson, which is one of the most well-studied, is the π_1, which has quantum numbers \( J^{PC} = 1^{-+} \). In 2021, HadSpec published the first study [10] of the branching fractions of this state, predicting that the \( b_1π \) decay mode of this system was dominant by at least an order of magnitude over every other decay mode. Therefore, understanding the \( b_1 \) meson is crucial to the analysis of the \( π_1 \). Since the dominant contributor to the \( ωπ \) system in the mass range just above threshold is the \( b_1 \), the analysis of the \( ωπ^- \) system described in this dissertation is an important stepping stone in the search for the exotic \( π_1 \).

Despite the predicted dominance of the \( b_1π \) decay mode, many searches for the \( π_1 \) have been performed in the \( ηπ \) and \( η'π \) channels. Since any state in either of these systems with odd angular momentum \( L \) will have exotic quantum numbers \( J^{PC} = 1^{-+}, 3^{-+}, 5^{-+},... \), the observation of a \( P- \) or \( F- \) wave in \( η^{0,\prime}π \) is, in principle, clear evidence of an exotic state. In 2015, the COMPASS collaboration [11] published a partial wave analysis of \( ηπ^- \) and \( η'π^- \) produced by a 191 GeV π^- beam in the four-momentum transfer region \( 0.1 < −t < 1.0 \text{ GeV}^2 \). As illustrated in Fig. 1.7, COMPASS found that the even partial waves are very similar between the two channels, while the odd partial waves are suppressed in \( ηπ^- \) when compared to \( η'π^- \). A clear peak in the \( η'π^- P- \) wave around 1.6 GeV is seen, but the \( ηπ^- P- \) wave peaks closer to 1.4 GeV. These results were consistent with previous observations of exotic isovector states near 1.4 and 1.6 GeV, which were named the \( π_1(1400) \) and \( π_1(1600) \), respectively.

**JPAC Unification of \( π_1(1400) \) and \( π_1(1600) \)**

The \( π_1(1400) \) and \( π_1(1600) \) were controversial for several years, since the existence of both states was inconsistent with phenomenological models and lattice QCD predictions.
In 2019, the Joint Physics Analysis Center (JPAC) \cite{jpac2019} published a fit of the intensities and phases extracted by COMPASS in Ref. \cite{compass2018}. This fit was done with a coupled-channel amplitude enforcing unitarity and analyticity of the $S$-matrix, and resulted in a single exotic $\pi_1$ resonant pole, with mass $1564 \pm 24 \pm 86$ MeV and width $492 \pm 54 \pm 102$ MeV, which couples to both $\eta\pi$ and $\eta'\pi$. The $\eta\pi$ coupling peaks on the real energy axis near 1400 MeV, which explains the observations previously attributed to an exotic $\pi_1(1400)$. JPAC reported no evidence for a second exotic state.
\( \pi_1(1600) \) in \( \omega \pi \pi \) at E852

As described earlier, the most dominant decay mode of the \( \pi_1 \) is \( b_1 \pi \). In 2005, the E852 collaboration [13] published a partial wave analysis of the \( \omega \pi^0 \pi^- \) channel produced by an 18 GeV \( \pi^- \) beam at BNL, where they observed signals for \( J^{PC} = 1^{--} \) in \( b_1 \pi^- \) decay near 1.6 GeV and 2 GeV.

![Graph](image)

**FIG. 1.8**: E852 extraction of wave intensities for (a) \( 1^{--}(b_1 \pi)^S_1 \), (b) \( 1^{--}(b_1 \pi)^S_0 \), (c) \( 2^{++}(\omega \rho)^S_2 \), and (d) \( 4^{++}(\omega \rho)^D_2 \). The solid line is the Breit-Wigner result for two \( 1^{--} \) poles and the dashed line is for one. Source: Fig. 2 of Ref. [13].

### 1.3 Photoproduction

Most of the previous exotic searches have been conducted using pion production, \( p \bar{p} \) annihilations, and \( e^+ e^- \) collisions. This analysis uses linearly polarized photoproduction, where the photon acts as an electromagnetic probe and can be thought of as a virtual vector meson or a \( q \bar{q} \) fluctuation. The linear polarization adds new degrees of freedom and allows extraction of quantities like a beam asymmetry, \( \Sigma = \frac{\sigma_{\perp} - \sigma_{\|}}{\sigma_{\perp} + \sigma_{\|}} \), as measured in Ref. [14] for the reaction \( \gamma p \rightarrow \pi^- \Delta^{++} \), shown in Fig. 1.9. This beam asymmetry is calculated using
the cross-sections collected from two datasets, one with the beam polarization oriented parallel to a given reference plane, $\sigma_{\parallel}$, and the other one perpendicular to that reference plane, $\sigma_{\perp}$.

Beam asymmetries give insight into production mechanisms, such as the possible quantum numbers exchanged in a $t$-channel process. A negative beam asymmetry indicates a $t$-channel process (see Sec. 1.3.1) with an unnatural (see Table 1.1) exchange particle, and positive beam asymmetry indicates a $t$-channel process proceeding through natural exchange.

FIG. 1.9: Beam asymmetry for the reaction $\gamma p \rightarrow \pi^- \Delta^{++}$. Source: Fig. 5 of Ref. [14].

Photoproduction is expected to provide complementary data to existing spectroscopic experiments. The GlueX experiment, described in Chapter 4, located in Hall D of Jefferson Lab in Newport News, Virginia, has already collected an unprecedented level of statistics. Its goal is to map the light and strange-quark meson spectrum, producing an experimental
version of Fig. 1.6. This analysis focuses on the $\gamma p \rightarrow \omega \pi^- \Delta^{++}$ channel produced at GlueX, where the properties of the $b_1^-$ meson, the lightest entry in the $J^{PC} = 1^{+-}$ column of Fig. 1.6, can be extracted.

### 1.3.1 Mandelstam Variables and $t$-Channel Processes

Consider a photoproduction reaction

$$\gamma p \rightarrow X^- \Delta^{++},$$

where $X^-$ is some intermediate meson state. At the energies used by GlueX, the reaction is expected to proceed predominantly through the $t$-channel. There are three possible

![Diagram of kinematic variables for a two-to-two scattering process. Source: Fig. 49.6 of Ref. [4].](image)

“channels” for two-to-two reactions when there is only one virtual particle in the Feynman
These channels are based on the Mandelstam variables

\[ s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \] (1.10)

\[ t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \] (1.11)

\[ u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \] (1.12)

(NB: since \( t \) and \( u \) can appear interchangeable, it is customary to define \( t \) as the squared difference in momenta of the most similar initial and final particles. In the case of our reaction, this would be the beam photon and the meson \( X^- \), or the target proton and recoil \( \Delta^{++} \)). In \( t \)-channel meson production, the produced mesons tend to have forward momenta in the center of mass frame, and in \( u \)-channel meson production, they tend to go backwards. A sketch of \( t \)-channel photoproduction of the \( b_1^- \) recoiling off a \( \Delta^{++} \) is shown in Fig. 1.11, where the reaction can proceed through unnatural exchange such as \( \pi^- \) or natural exchange such as \( \rho^- \). The four-momentum transfer squared, \(-t\), between

![Diagram](image)

FIG. 1.11: Sketch of \( t \)-channel photoproduction of the \( b_1^- \) recoiling off a \( \Delta^{++} \)
the incoming photon and the meson $X^-$ is written as

$$-t = (p_\gamma - p_{X^-})^2 = (p_p - p_{\Delta^{++}})^2. \tag{1.13}$$

The dependence of the intensity for the production of meson $X^-$ on $-t$ is expected to be proportional to

$$I_{X^-} \propto e^{-b|t|}, \tag{1.14}$$

where $b$ is called the $t$-slope, and has units GeV$^{-2}$. We will make cuts on $-t$ to isolate the production mechanism, to be further discussed in Section 6.2.

### 1.3.2 Future Searches for $\pi_1(1600) \rightarrow b_1 \pi$ at GlueX

The analysis discussed in this dissertation will be a precursor to analysis of the $b_1 \pi$ decay channel in the search for the exotic $\pi_1(1600)$. Not only do we need to understand the decay of the $b_1$, but, as shown in Fig. 1.12, the $b_1^- \Delta^{++}$ is the most significant background contribution to the $\gamma p \rightarrow \pi_1 p \rightarrow b_1 \pi p$ decay channel at the energies used by GlueX (excited baryonic backgrounds contribute much less at higher energy experiments like COMPASS), so an understanding of the whole system is critical if we want our eventual $b_1 \pi$ analysis to be meaningful.
FIG. 1.12: Comparison of $t$-channel photoproduction of $\pi_1 \rightarrow b_1 \pi$ recoiling off a proton with $b_1^i \rightarrow \omega \pi^-$ recoiling off a $\Delta^{++}$. Both share the same final state of $\omega \pi^+ \pi^- p$. 
CHAPTER 2

Formalism for Photoproduction of Vector-Pseudoscalar Systems

Partial Wave Analysis (PWA) is a technique that involves decomposing an intensity into its orthogonal angular momentum components. We will discuss its use in Chapters 7 and 8. We use a maximum likelihood method to determine the set of amplitudes, defined by their quantum numbers, that can describe the production and decay of the intermediate state $X^-$ in the reaction

$$\gamma p \rightarrow X^- \Delta^{++} \rightarrow \omega \pi^- \Delta^{++}. \quad (2.1)$$

This analysis is focused on the properties of the system where $X^-$ is the axial vector $b_1^-$, with a vector background. The formalism involving a general intermediate state decaying to $\omega \pi^-$ is outlined in this chapter.
2.1 $\omega\pi^-$ Photoproduction

The photoproduction of an $\omega\pi^-$ system recoiling off a $\Delta^{++}$ baryon is written as

$$\gamma (\lambda, p_\gamma)p(\lambda_1, p_p) \rightarrow \pi^- (p_\pi) \omega (\lambda_\omega, p_\omega) \Delta^{++} (\lambda_2, p_{\Delta^{++}}),$$

(2.2)

where the helicities ($\lambda$) of each particle with spin are defined in the helicity reference frame, which is the rest frame of the $\omega\pi^-$ system, where the $x$-axis is defined to be the direction opposite the recoil system, in this case the $\Delta^{++}$. The amplitude for this reaction is described by $A_{\lambda_\omega;\lambda_1,\lambda_2} (\Omega, \Omega_H)$, where $\Omega$ is the solid angle describing the direction of the $\omega$ meson in the $\omega\pi^-$ helicity frame, and $\Omega_H$ is the solid angle that describes the direction of the normal vector to the $\omega \rightarrow \pi^+\pi^-\pi^0$ decay plane. These angles will be discussed in detail in Section 2.2, and are illustrated in Fig. 2.2.

In Chapter 7, we discuss how we fit the data with a model for the intensity, $I (\Omega, \Omega_H, \Phi)$, which can be described, up to an overall phasespace factor discussed in Appendix A, Eqn. A.5, in terms of the differential cross-section

$$I (\Omega, \Omega_H, \Phi) \equiv \frac{d\sigma}{dt dm_{\omega\pi^-} d\Omega d\Omega_H d\Phi} = \sum_{\lambda,\lambda';\lambda_1,\lambda_2} A_{\lambda;\lambda_1,\lambda_2} (\Omega, \Omega_H) \rho_{\lambda\lambda'} (\Phi) A^*_{\lambda';\lambda_1,\lambda_2} (\Omega, \Omega_H) = I^0 (\Omega, \Omega_H) - P_\gamma I^1 (\Omega, \Omega_H) \cos 2\Phi - P_\gamma I^2 (\Omega, \Omega_H) \sin 2\Phi,$$

(2.3)

where $\rho_{\lambda\lambda'} (\Phi)$, the photon spin density matrix, describes the dependence on $\Phi$, the angle of linear polarization of the incoming photon beam measured with respect to the $\omega\pi^-$ production plane. The Mandelstam variables $t = (p_{\Delta^{++}} - p_p)^2$ and $m_{\omega\pi^-}^2 = (p_\omega + p_\pi)^2$ are the square of the four-momentum transferred between the target and recoil baryons, and the $\omega\pi^-$ invariant mass squared, respectively. The other Mandelstam variable that will
come up later is \( s = (p_\gamma + p_p)^2 \), the center of mass energy squared. The components of the intensity function used in Eqn. 2.3 are defined in Eqn. 4 of Ref. 15 and are given here as Eqns 2.4-2.6. Note that the baryon helicity indices are suppressed here and will be reintroduced in Eqn. 2.26.

\[
I^0 (\Omega, \Omega_H) = \frac{1}{2} \sum_{\lambda} A_{\lambda} (\Omega, \Omega_H) A^*_{\lambda} (\Omega, \Omega_H), \quad (2.4)
\]

\[
I^1 (\Omega, \Omega_H) = \frac{1}{2} \sum_{\lambda} A_{-\lambda} (\Omega, \Omega_H) A^*_{\lambda} (\Omega, \Omega_H), \quad (2.5)
\]

\[
I^2 (\Omega, \Omega_H) = \frac{i}{2} \sum_{\lambda} \lambda A_{-\lambda} (\Omega, \Omega_H) A^*_{\lambda} (\Omega, \Omega_H). \quad (2.6)
\]

If we plug these definitions into Eqn. 2.3, we can write the intensity in terms of the amplitudes \( A_{\lambda} (\Omega, \Omega_H) \)

\[
I (\Omega, \Omega_H, \Phi) = \frac{1}{2} \sum_{\lambda} \left[ A_{\lambda} (\Omega, \Omega_H) A^*_{\lambda} (\Omega, \Omega_H) \\
- P_\gamma \cos 2\Phi A_{-\lambda} (\Omega, \Omega_H) A^*_{\lambda} (\Omega, \Omega_H) \\
- i\lambda P_\gamma \sin 2\Phi A_{-\lambda} (\Omega, \Omega_H) A^*_{\lambda} (\Omega, \Omega_H) \right]. \quad (2.7)
\]

The beam photon helicity, \( \lambda \), can take on the values of \( \pm 1 \). Making the sum over \( \lambda \) explicit, we obtain

\[
I (\Omega, \Omega_H, \Phi) = \frac{1}{2} \left[ A_+ (\Omega, \Omega_H) A^*_+ (\Omega, \Omega_H) + A_- (\Omega, \Omega_H) A^*_- (\Omega, \Omega_H) \\
- P_\gamma e^{2i\Phi} A_- (\Omega, \Omega_H) A^*_+ (\Omega, \Omega_H) - P_\gamma e^{-2i\Phi} A_+ (\Omega, \Omega_H) A^*_- (\Omega, \Omega_H) \right]. \quad (2.8)
\]

If we fold the phase-dependence into the helicity-basis amplitudes, the resulting phase-
rotated amplitudes are
\[
\tilde{A}_\pm (\Omega, \Omega_H, \Phi) \equiv e^{\mp i\Phi} A_\pm (\Omega, \Omega_H). \tag{2.9}
\]

We can then write the intensity in terms of the sum and difference of opposite-beam-helicity phase-rotated amplitudes
\[
I (\Omega, \Omega_H, \Phi) = \frac{1}{2} \left[ \tilde{A}_+ (\Omega, \Omega_H, \Phi) \tilde{A}_+^* (\Omega, \Omega_H, \Phi) + \tilde{A}_- (\Omega, \Omega_H, \Phi) \tilde{A}_-^* (\Omega, \Omega_H, \Phi) - P_\gamma \tilde{A}_- (\Omega, \Omega_H, \Phi) \tilde{A}_+^* (\Omega, \Omega_H, \Phi) - P_\gamma \tilde{A}_+ (\Omega, \Omega_H, \Phi) \tilde{A}_-^* (\Omega, \Omega_H, \Phi) \right] \tag{2.10}
\]
\[
= \frac{1}{4} \left[ (1 - P_\gamma) \left| \tilde{A}_+ (\Omega, \Omega_H, \Phi) + \tilde{A}_- (\Omega, \Omega_H, \Phi) \right|^2 + (1 + P_\gamma) \left| \tilde{A}_+ (\Omega, \Omega_H, \Phi) - \tilde{A}_- (\Omega, \Omega_H, \Phi) \right|^2 \right]. \tag{2.11}
\]

The beam helicity amplitudes themselves are written as sums of the partial-wave and decay amplitudes, \( T_{\lambda, m}^i \) and \( X^i_m (\Omega, \Omega_H) \), respectively. The sums are over the possible \( J^P \) states and spin projections of the \( \omega\pi^- \) system
\[
A_{\lambda_i} (\Omega, \Omega_H) = \sum_{i=1,2,\ldots} \sum_{m=-J_i\cdots J_i} T_{\lambda_i, m}^i X^i_m (\Omega, \Omega_H), \tag{2.12}
\]
where the decay amplitude of the resonance \( i \) is defined as
\[
X^i_m (\Omega, \Omega_H) = \sum_{\lambda_{\omega} = -1,0,1} D_{m, \lambda_{\omega}}^{J_i^*} (\Omega) F^i_{\lambda_{\omega}, 0} (\Omega_H) D_{\lambda_{\omega}, 0}^{1*} (\Omega_H) G_{Dalitz}. \tag{2.13}
\]

Here \( D_{m, \lambda_{\omega}}^{J_i^*} (\Omega) \) and \( D_{\lambda_{\omega}, 0}^{1*} (\Omega_H) \) are the complex conjugates of the Wigner D-functions, \( G_{Dalitz} \) describes the \( \omega \rightarrow \pi^+\pi^-\pi^0 \) decay as discussed in Section 2.3, and \( F^i_{\lambda_{\omega}} \) describes the
decay of a resonance of spin $J_i$ to $\omega \pi^-$, when the $\omega$ has a helicity of $\lambda_\omega$

$$F_{\lambda_\omega}^{J_i} = \sum_l \langle J_i \lambda_\omega | l0, 1 \lambda_\omega \rangle C_{\lambda_\omega}^l.$$  \hfill (2.14)

In the case of the $b_1$, where $J_i = 1$, the coefficient $C_l$ is nonzero only for even values of orbital angular momentum $l$, and Eqn. 2.14 simplifies to

$$F_{\lambda_\omega}^{b_1} = \langle 1 \lambda_\omega | 00, 1 \lambda_\omega \rangle C_0^{b_1} + \langle 1 \lambda_\omega | 20, 1 \lambda_\omega \rangle C_2^{b_1},$$  \hfill (2.15)

where the ratio $C_2^{b_1}/C_0^{b_1}$ describes the ratio of $D$-wave to $S$-wave for the $b_1$ meson, which will be further discussed in Sections 3.1.2, 3.2.1, and 1.2.2. Note that $G$-waves and above are neglected in this analysis.

**2.1.1 The Reflectivity Basis**

We opt to work in the reflectivity basis to avoid interference between waves of opposite reflectivity. At the energies used by GlueX, the naturality of the exchange particle ($\tau_\epsilon$) and the reflectivity of the reaction ($\epsilon$) are directly correlated - only positive (negative) reflectivity waves result from natural (unnatural) exchange. This makes extraction of the naturality of the exchange particle convenient. For example, in Section 1.3.1, we discussed the $t$-channel photoproduction of a $b_1^-$ meson, which could proceed by exchanging a virtual $\rho^-$, which is a natural exchange particle, or by exchanging a virtual $\pi^-$, which is unnatural. Since both are allowed, extraction of the reflectivity gives us insight into which one is favored in nature.

The reflectivity operator, $\Pi_y$, is defined as a rotation about the $y$-axis by $\pi$ radians, followed by a parity operation

$$\Pi_y = PR_y(\pi),$$  \hfill (2.16)
where the $y$-axis is defined as the direction of the normal vector to the production plane. We can then express our production amplitudes as eigenstates of the reflectivity operator.

The partial wave amplitudes in the reflectivity basis, $^{(e)}V^l_m$, are defined in analogy with the $\eta\pi$ formalism in Ref. [15] using a linear combination of the partial wave amplitudes in the helicity basis, where $\tau_i$ is the naturality of the resonance that decayed to the $\omega\pi^-$ system

$$^{(e)}V^l_m = \frac{1}{2} \left[ T^l_{+1,m} - \tau_i \epsilon (-1)^m T^l_{-1-m} \right]. \quad (2.17)$$

Since the $\omega\pi$ reflectivity amplitudes depend on the naturality of the resonance, we require extra terms when writing the partial wave amplitudes in the helicity basis, which can be written in terms of the reflectivity amplitudes as

$$T^i_{+1,m} = (+) V^i_m + (-) V^i_m \quad (2.18)$$

and

$$T^i_{+1,m} = \tau_i (-1)^m (-) V^i_{-m} (+) V^i_{-m} \quad (2.19)$$

These extra terms are not necessary in the $\eta\pi$ formalism used in Ref. [15], which is discussed in Appendix A.

Now we can write the full phase-rotated helicity amplitudes in terms of the reflectivity amplitudes

$$\tilde{A}_+ = e^{-i\Phi} \sum_{i,m} \left[ (+) V^i_m + (-) V^i_m \right] X^i_m (\Omega, \Omega_H) \quad (2.20)$$

$$= \sum_{i,m} \left[ (+) V^i_m + (-) V^i_m \right] Z^i_m (\Phi, \Omega, \Omega_H) \quad (2.21)$$
\[ \tilde{A}_- = e^{i \Phi} \sum_{i,m} (-)^i V_{m}^{i} - (-)^m V_{-m}^{i} \tau_i (-1)^m X_{-m}^{i} (\Omega, \Omega_H) \]  
\[ = \sum_{i,m} (+)^i V_{m}^{i} - (-)^m V_{-m}^{i} Z_{m}^{i} (\Phi, \Omega, \Omega_H), \]  
(2.22)

where \( Z_{m}^{i} (\Phi, \Omega, \Omega_H) = e^{-i \Phi} X_{m}^{i} (\Omega, \Omega_H) \) is the phase-rotated decay amplitude for a given resonance \( i \).

The sum and difference of the full phase-rotated amplitudes can be written in terms of the real and imaginary parts of the phase-rotated decay amplitudes

\[ \tilde{A}_+ + \tilde{A}_- = 2 \sum_{i,m} i (-)^i V_{m}^{i} \text{Im}(Z_{m}^{i}) + (+)^i V_{m}^{i} \text{Re}(Z_{m}^{i}) \]  
and

\[ \tilde{A}_+ - \tilde{A}_- = 2 \sum_{i,m} i (+)^i V_{m}^{i} \text{Im}(Z_{m}^{i}) + (-)^i V_{m}^{i} \text{Re}(Z_{m}^{i}). \]  
(2.23)

These can be plugged into the intensity definition

\[ I(\Phi, \Omega, \Omega_H) = \sum_{\lambda_1 \lambda_2} \left\{ (1 - P_\gamma) \left| \sum_{i,m} i (-)^i V_{m}^{i} \text{Im}(Z_{m}^{i}) + (+)^i V_{m}^{i} \text{Re}(Z_{m}^{i}) \right|^2 \right. \]  
\[ + \left. (1 + P_\gamma) \left| \sum_{i,m} i (+)^i V_{m}^{i} \text{Im}(Z_{m}^{i}) + (-)^i V_{m}^{i} \text{Re}(Z_{m}^{i}) \right|^2 \right\}. \]  
(2.24)

We reintroduce the sum over baryon helicities \( \lambda_1, \lambda_2 \), which can be translated into a sum over baryon spin flip and non-flip, \( k \), again using parity invariance

\[ (\epsilon)^{i \lambda_1, -\lambda_1 - \lambda_2} = \epsilon(-1)^{\lambda_1 - \lambda_2} (\epsilon)^{i \lambda_1, \lambda_1, \lambda_2}. \]  
(2.25)
and we define the complex parameters for the baryon spin flip and non-flip amplitudes, respectively,

\[
[J_i]^{(\epsilon)}_{m,0} = (\epsilon)V_{m;+}^i \quad [J_i]^{(\epsilon)}_{m,1} = (\epsilon)V_{m;+}^i.
\] (2.28)

We can relate the sum over baryon helicities to the sum over baryon spin (non-)flip using

\[
\sum_{\lambda_1\lambda_2}(\epsilon)V_{m;\lambda_1\lambda_2}^{i}(\epsilon')V_{m';\lambda_1\lambda_2}^{i'} = (1 + \epsilon\epsilon')\sum_k [J_i]^{(\epsilon)}_{m,k}[J_{i'}]^{(\epsilon')}_{m',k}.
\] (2.29)

then

\[
I(\Phi, \Omega, \Omega_H) = 2 \sum_k \left\{ (1 - P_\gamma) \left[ \sum_{i,m} [J_i]^{(-)}_{m,k} Im(Z_m^i) \right]^2 + \left[ \sum_{i,m} [J_i]^{(\epsilon)}_{m,k} Re(Z_m^i) \right]^2 \right\} + \right.
\]

\[
(1 + P_\gamma) \left[ \sum_{i,m} [J_i]^{(\epsilon)}_{m,k} Im(Z_m^i) \right]^2 + \left[ \sum_{i,m} [J_i]^{(-)}_{m,k} Re(Z_m^i) \right]^2 \right\}
\] (2.30)

where the \([J_i]^{(\epsilon)}_{m,k}\) are the free complex parameters in the model describing the production amplitude for a given reflectivity amplitude, and the angular dependence is contained in the real and imaginary components of \(Z_m^i(\Phi, \Omega, \Omega_H)\). We should note here that the \(\Delta^{++}\) baryon is a spin-3/2 particle, meaning that there are four possible amplitudes, instead of just spin flip and non-flip. We are making an assumption that one of these baryon spin exchanges is dominant, and we perform our fits without the sum over \(k\). This assumption should be revisited in future work.
FIG. 2.1: Illustration of the decay angles of an $\omega\pi^-$ system.

2.2 $\omega\pi^-$ Decay Angles

This section is based on Appendix A of Ref. [16], and outlines the angles that describe the decay of a resonance $X \rightarrow \omega\pi^-$, where the $\omega$ then decays to $\pi^+\pi^-\pi^0$. The solid angle $\Omega = (\cos \theta, \phi)$ describes the direction of the $\omega$ meson in the $\omega\pi^-$ helicity frame. Starting in the $\gamma p$ center of mass frame, we let $\hat{z}$ be a unit vector along the direction of the $\omega\pi^-$ system, and $\hat{k}$ be a unit vector in the beam ($\gamma$) direction. The normal to the production plane is defined as $\hat{y} = \frac{\hat{k} \times \hat{z}}{|\hat{k} \times \hat{z}|}$, and the third unit vector to describe this frame is $\hat{x} = \hat{y} \times \hat{z}$, from the right-hand rule. We perform a Lorentz transformation along the $z$ direction to
find the momentum vector of the \( \omega \) in the \( \omega \pi^- \) rest frame. The angle \( \Omega = (\cos \theta, \phi) \) is then

\[
\cos \theta = \frac{\vec{\omega} \cdot \hat{z}}{|\vec{\omega}|}, \quad \phi = \arctan \left( \frac{\vec{\omega} \cdot \hat{y}}{\vec{\omega} \cdot \hat{x}} \right). \tag{2.31}
\]

The solid angle \( \Omega_H = (\cos \theta_H, \phi_H) \) describes the direction of the normal vector to the \( \omega \rightarrow \pi^+ \pi^- \pi^0 \) decay plane in the helicity system of the \( \omega \) meson. We let \( \hat{z}_H = \frac{\vec{\omega}}{|\vec{\omega}|} \) be the unit vector in the \( \omega \) direction in the \( \omega \pi^- \) center of mass frame. The other unit vectors for this frame are then

\[
\hat{y}_H = \frac{\hat{z} \times \hat{z}_H}{|\hat{z} \times \hat{z}_H|}, \quad \hat{x}_H = \hat{y}_H \times \hat{z}_H. \tag{2.32}
\]

We perform a Lorentz transformation along the \( \hat{z}_H \) direction to find the momentum vectors \( (\vec{\pi}_+, \vec{\pi}_-) \) for the charged daughter pions in the \( \omega \) rest frame. The normal vector to the decay plane is

\[
\hat{n} = \frac{\vec{\pi}_+ \times \vec{\pi}_-}{|\vec{\pi}_+ \times \vec{\pi}_-|}, \tag{2.33}
\]

and the solid angle \( \Omega_H \) is

\[
\cos \theta_H = \hat{n} \cdot \hat{z}_H, \quad \phi_H = \arctan \left( \frac{\hat{n} \cdot \hat{y}_H}{\hat{n} \cdot \hat{x}_H} \right). \tag{2.34}
\]

All four of these decay angles are illustrated in Fig. 2.1.

### 2.3 Decay of the \( \omega \) Meson

The factor \( G_{Dalitz} \) in Eqn 2.13 describes the Dalitz distribution of the pions in the \( \omega \rightarrow \pi^+ \pi^- \pi^0 \) decay, and is a scalar function of the Mandelstam variables \( u = (p_- + p_0)^2 \), \( t = (p_+ + p_0)^2 \), and \( s = (p_+ + p_-)^2 \). The Dalitz formalism is briefly sketched here and

33
discussed in detail in Section V of Ref. [17].

\[ |G_{Dalitz}(z, \vartheta)|^2 = |N|^2 \left(1 + 2\alpha z + 2\beta z^{3/2} \sin(3\vartheta) + 2\gamma z^2 + \mathcal{O}(z^{5/2})\right). \] (2.35)

Here \(N\) is a normalization constant, and \(\alpha, \beta, \) and \(\gamma\) are parameters taken from a fit performed by JPAC and are shown in Table 2.1. The polar variables \(z\) and \(\vartheta\) parametrize the Dalitz plot distribution of the \(\omega \rightarrow \pi^+\pi^-\pi^0\) decay,

\[
z = \frac{3(t-u)^2 + 9(s_c-s)^2}{4M^2(M-3m_{\pi})^2}, \quad \tan \vartheta = \frac{\sqrt{3}(s_c-s)}{(t-u)}, \tag{2.36}
\]

where \(M\) is the invariant mass of the 3\(\pi\) system, \(m_{\pi}\) is the isospin-averaged pion mass, and \(s_c = \frac{1}{3}(M^2 + 3m_{\pi}^2)\) is the location of the center of the Mandelstam triangle.

<table>
<thead>
<tr>
<th></th>
<th>(\alpha \times 10^3)</th>
<th>(\beta \times 10^3)</th>
<th>(\gamma \times 10^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (\phi_{\omega\pi^0}(0))</td>
<td>121.2(7.7)(0.8)</td>
<td>25.7(3.3)(3.3)</td>
<td>-</td>
</tr>
<tr>
<td>High (\phi_{\omega\pi^0}(0))</td>
<td>120.1(7.7)(0.7)</td>
<td>30.2(4.3)(2.5)</td>
<td>-</td>
</tr>
<tr>
<td>Low (\phi_{\omega\pi^0}(0))</td>
<td>112(15)(2)</td>
<td>23(6)(2)</td>
<td>29(6)(8)</td>
</tr>
<tr>
<td>High (\phi_{\omega\pi^0}(0))</td>
<td>109(14)(2)</td>
<td>26(6)(2)</td>
<td>19(5)(4)</td>
</tr>
</tbody>
</table>

TABLE 2.1: JPAC calculation of Dalitz parameters for \(\omega\) decay. The upper (lower) rows are the results when 2 (3) Dalitz plot parameters are determined. The term \(\phi_{\omega\pi^0}(0)\) refers to the relative phase involved in the dispersion relation used by JPAC. The top row contains the values used in this analysis, but the other values should be used in future systematic studies. Source: Table 1 of Ref. [18].
CHAPTER 3

Current Experimental and Theoretical Situation

There exist several experimental and theoretical studies of the $\omega\pi$ system. The axial vector $b_1 (J^P = 1^+)$ state was discovered in this system by early pion beam experiments in the 1970s and 80s, which were limited by the statistical precision of these relatively low intensity experiments. Later measurements in $e^+e^-$ offer high precision, but the only accessible states are the vectors, with $J^{PC} = 1^{--}$, formed in $e^+e^-$ annihilation. Photoproduction can in principle study both $J^P = 1^+$ and $1^-$, as well as other quantum number states. A previous photoproduction study, discussed in Section 3.1.1, was performed by the Omega Photon collaboration in the 1980s, however the analysis described in Chapters 6, 7, and 8 of this dissertation will outpace its statistical precision by several orders of magnitude, and is the first to study a charged exchange mechanism in photoproduction. Theoretical calculations discussed in Section 3.2.1 have found that the $\omega\pi$ channel couples strongly to an axial vector resonance analogous to the $b_1$. 
3.1 Previous Measurements

3.1.1 $\omega\pi^0$ in Photoproduction (1983)

The Omega Photon Collaboration [16] studied the neutral channel in photoproduction, $\gamma p \rightarrow \omega\pi^0 p$, gathering 2,304 weighted $\omega\pi^0$ events from the Super Proton Synchrotron (SPS) at CERN. They performed a moment analysis that showed the $\omega\pi^0$ system is consistent with a high percentage of $b_1(1235)$ ($J^P = 1^+$), with a background of non-resonant $J^P = 1^-$. This study was performed in search of radial recurrences of the $\rho$ meson, which are also referred to as excited $\rho$ isovector states. The lightest of these was predicted at the time to have a mass of $\sim1.3$ GeV [19].

The angular distributions plotted in Fig. 3.1 are the same ones discussed in Section 2.2 and are not acceptance corrected, but with the exception of $\theta$, the angles are not strongly biased by detector acceptance. The $\cos \theta_H$ distribution shows a strong $\sin^2 \theta_H$ component, which corresponds to $\omega$ helicity $\lambda_\omega = \pm 1$, indicating that the dominant spin-parity of the $\omega\pi^0$ system is not $0^-$. The $\phi_H$ distribution shows a strong $\cos 2\phi_H$ component, which indicates either a $1^-$ or $1^+$ state. The $\phi$ distribution shows a $\sin \phi$ component, which indicates interference between the $\omega\pi^0$ helicities $\pm 1$ and 0. These angular analyses indicate that the $\omega\pi^0$ system should be dominated by either a $1^+$ or a $1^-$ state, which should interfere with a less prominent $0^-$ state.

The Omega Photon Collaboration performed a moment analysis to determine whether the dominant spin-parity of the $\omega\pi^0$ enhancement was $1^-$ or $1^+$, and concluded that the $1^+$ intensity is dominant and peaks at $\sim1.2$ GeV, with a full width of $\sim200$ MeV. The $D/S$ ratio in the peak region is $\sim0.25$. The $1^+$ state was found to be consistent with the $b_1(1235)$ meson. There was no evidence for a resonant $1^-$ enhancement, such as an excited $\rho$ meson.
FIG. 3.1: Angular decay distributions for $\gamma p \rightarrow \omega \pi^0 p$ (not acceptance corrected). The smooth curves are the result of a simulation. Source: Fig. 8 of Ref. [16]
3.1.2 $\omega\pi^\pm$ in Pion Production (1975, 2002)

Chung et. al. (1975)

In 1975, Chung et. al. [20] published a spin-parity analysis on the pion production experiment, $\pi^+ p \rightarrow \omega\pi^+ p$, done by Berkeley Group A at SLAC, which collected roughly 7,000 $\omega\pi^+$ events. This number was gathered by performing an integration by eye of Figure 2 in Ref. [20]. This study was one of the earliest to consider the interference effects between different spin-parity states, rather than assuming non-interference, and found that the best solution was that of the then-named $B$ meson as a $1^+$ state over a smooth background consisting of a $1^-$ state along with possibly a small amount of $0^-$. They also investigated the $t'$-dependence of each state, finding that the $1^+$ state is peaked at low $t'$. The variable $t'$ is related to the Mandelstam variable $t$ by the following relation

$$t' = t - t_{\text{min}}$$

(3.1)

The density matrices of the $1^+$ state indicated that it is produced mainly via natural parity exchange, with the authors naming $\omega$ exchange as the most likely candidate.

E852 at BNL (2002)

In 2002, the E852 Collaboration [7] published a study of the negatively charged channel in pion production, $\pi^- p \rightarrow \omega\pi^- p$, using 168,000 $\omega\pi^-$ events obtained from the Multi-Particle Spectrometer facility at BNL. E852 performed a partial wave analysis (PWA), which involves decomposing an intensity into its orthogonal angular momentum components, finding that the $b_1^-(1235)$ is dominant in the $\omega\pi^-$ mass spectrum, and that $b_1^-$ production is dominated by natural parity exchange, in the neutral exchange mechanism. They also measured the $S$-wave and $D$-wave amplitudes for $b_1 \rightarrow \omega\pi$ and
found the amplitude ratio $D/S = 0.269 \pm (0.009)_{\text{stat}} \pm (0.01)_{\text{sys}}$ and phase difference 
$\varphi_D = 10.54^\circ \pm (2.4^\circ)_{\text{stat}} \pm (3.9^\circ)_{\text{sys}}$.

The main contributors to the total intensity in the PWA performed by E852 are $J^{PC} = 1^{+-}$, which is dominated by $b_1(1235)$, and $J^{PC} = 3^{--}$, which is consistent with the $\rho_3(1690)$. An enhancement in the potentially exotic $J^{PC} = 2^{+-}$ intensity around 1.6 GeV was also observed, but a detailed study of this enhancement was left for future work. The PWA fit results are shown in Fig. 3.2.
FIG. 3.2: Acceptance-corrected intensities from the results of a PWA fit performed by E852. Source: Fig. 5 of Ref. [7]
3.1.3 $\omega \pi^0$ in $e^+e^-$ Annihilation

The neutral $\omega \pi^0$ channel can also be studied in electron-positron annihilation, though due to the production mechanism, only the vector states ($J^{PC} = 1^{--}$) are accessible. Notable studies of this channel include the one by BESIII in the center-of-mass energy range 2.00-3.08 GeV [21], and the one by SND in the center-of-mass energy range 1.05-2.00 GeV [22].

**BESIII Study (Above 2 GeV)**

The BESIII collaboration [21] obtained the Born cross section for the process $e^+e^- \rightarrow \omega \pi^0$ in the center-of-mass energy range 2.00-3.08 GeV at the BEPCII collider. A resonance denoted $Y(2040)$ was observed in the cross section with a reported significance of more than 10$\sigma$. The mass and width of this resonance were measured to be $m = 2034 \pm (13)_{\text{stat}} \pm (9)_{\text{sys}}$ MeV and $\Gamma = 234 \pm (30)_{\text{stat}} \pm (25)_{\text{sys}}$ MeV. This structure is consistent with an excited $\rho$ meson, either the $\rho(2000)$ or the $\rho(2150)$.

**SND Study (Below 2 GeV)**

The SND collaboration [22] analyzed a 37 pb$^{-1}$ data sample collected at the VEPP-2000 collider in the center-of-mass energy range 1.05-2.00 GeV, and measured the $e^+e^- \rightarrow \omega \pi^0 \rightarrow \pi^0\pi^0\gamma$ cross section, shown in Fig 3.3. They fit this measured cross section with a vector meson dominance (VMD) model using the $\rho(770)$, $\rho(1450)$, and $\rho(1700)$ resonances, which described the data well. However, this model was not able to describe the data on the $\gamma^* \rightarrow \omega \pi^0$ transition form factor obtained from the $\omega \rightarrow \pi^0\mu^+\mu^-$ decay.
3.2 Theoretical Calculations

3.2.1 HadSpec (2019)

In 2019, HadSpec published the first lattice QCD calculation of coupled $\omega \pi$ and $\phi \pi$ scattering, incorporating coupled $S$ and $D$-wave $\omega \pi$ in $J^P = 1^+$. At light quark masses corresponding to a pion mass of $m_\pi \approx 391$ MeV, HadSpec found a narrow axial vector resonance analogous to the $b_1$ meson, with mass $m_R \approx 1380$ MeV and width $\Gamma_R \approx 91$ MeV. This $b_1$ analogue was found to couple dominantly to $S$-wave $\omega \pi$, with a much weaker coupling to $D$-wave $\omega \pi$, and a negligible coupling to $\phi \pi$. These couplings were extrapolated to the physical value of the light quark masses by assuming them to be independent of the light quark masses once the threshold behavior is removed. This independence is not
guaranteed, but has been observed in other lattice calculations at various values of $m_\pi$. Upon this extrapolation to the physical light quark masses, the $D/S$ ratio of the $1^+$ state is

$$\left| \frac{\epsilon_{\pi\omega}^{\text{phys.}}(\bar{3}D_1)}{\epsilon_{\pi\omega}^{\text{phys.}}(\bar{3}S_1)} \right| = 0.27 \pm 0.20.$$  

(3.2)

This value was calculated at the complex pole position, not on the real energy axis, and is therefore not the same value as the one measured by E852 in Ref. [7]. The error is high due to the extrapolation to the physical light quark masses.
CHAPTER 4

The GlueX Experiment at CEBAF

This chapter provides a brief overview of the GlueX experiment in Jefferson Lab’s Hall D, which was used to collect the data for this analysis. A more detailed description is given in Ref. [24], cited throughout this chapter.

4.1 CEBAF at Jefferson Lab

The Continuous Electron Beam Accelerator Facility (CEBAF) at Jefferson Lab consists of a mile-long racetrack configuration, with two parallel linear accelerators, illustrated in Fig. 4.1. It is able to accelerate electrons up to a maximum energy of 12 GeV in 5.5 passes around the racetrack accelerator, with energy increases of 1090 MeV per linear accelerator. The GlueX experiment utilizes a relatively low current electron beam with an average of 150 nA, provided to Hall D in bunches at a rate of 249.5 MHz, or one bunch every 4.008 ns. The GlueX-I data was collected with an electron beam energy of $E_e = 11.6$ GeV.
4.2 Linearly Polarized Photon Beam

4.2.1 Diamond Radiator

In Hall D, the electron beam from CEBAF is incident on a 50 μm thick diamond crystal, or radiator. When the radiator is properly aligned, a linearly polarized photon beam is produced through coherent bremsstrahlung. The diamond radiator is used in four standard orientations which set the linear polarization at four different angles: 0°, 45°, 90°, and 135° to the horizontal. The diamond is rotated every few hours to collect independent data sets with different polarization angles. The four polarization orientations allow us to measure polarization-dependent quantities like beam asymmetries, as well as cancel out apparatus effects. An aluminum radiator is also used to produce a control beam without linear polarization. The GlueX experiment nominally uses a 9 GeV photon beam, which
corresponds to a peak linear polarization of approximately 40%. This analysis focuses on events for which the beam energy falls in the coherent peak range of 8.2-8.8 GeV, where the degree of photon beam polarization is maximal, as shown in Fig. 4.2(b). The average photon beam polarization in the coherent peak range is given in Table 4.1 for each of the four orientations. The polarization fraction is treated as a constant 0.35 in this analysis, since that approximates the $0^\circ$ orientation, but effects of the 1.5% systematic uncertainty will need to be the subject of future systematic studies, and the correct values will need to be used for the other three orientations, when they are analyzed.
TABLE 4.1: Average photon beam polarization fraction in the coherent peak energy region. There is a 1.5% systematic uncertainty and a 3% statistical uncertainty associated with each value.

<table>
<thead>
<tr>
<th>Angle</th>
<th>( P_\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.3519</td>
</tr>
<tr>
<td>90°</td>
<td>0.3303</td>
</tr>
<tr>
<td>45°</td>
<td>0.3374</td>
</tr>
<tr>
<td>135°</td>
<td>0.3375</td>
</tr>
</tbody>
</table>

4.2.2 Photon Tagging Spectrometer

After the radiator produces a photon beam, the photons need to be “tagged,” so we know their energies. This is done by sending the combined photon and electron beams through the photon tagging spectrometer, which uses a dipole magnet to sweep out the electrons. The full-energy electrons are sent to a shielded beam dump, while the post-bremsstrahlung electrons are deflected to larger angles, having lower energy. These electrons are detected by the Tagger Hodoscope (TAGH), which is an array of scintillators which span an energy range from \( 3.0 - 11.6 \) GeV. In the coherent peak energy region, the Tagger Microscope (TAGM) utilizes smaller spatial separation to achieve better energy resolution. The TAGH and TAGM both use the relation \( E_\gamma = E_0 - E_c \), where \( E_0 \) is the electron beam energy before the interaction with the radiator, and \( E_c \) is the energy of the post-bremsstrahlung electron, which is determined by measuring its angular deflection by the dipole magnet. Many candidate beam photons are measured in each event, and must be matched in time and energy/momentum with the final state particles observed in the main spectrometer (see Chapter 6 for details).

4.2.3 Triplet Polarimeter

A Triplet Polarimeter (TPOL) is used to continuously measure the degree of polarization of the photon beam, through \( \gamma e^- \rightarrow e^+e^-e'^- \) pair production on atomic electrons
in a 75 μm thick beryllium target foil. The azimuthal distribution of the recoil electrons, \( e^{-'} \), from triplet photoproduction gives information on the degree of polarization of the photon beam using

\[
\sigma_t = \sigma_0 [1 - P_{\gamma} \Sigma \cos(2\varphi)]
\]  

(4.1)

where \( \sigma_t \) and \( \sigma_0 \) are the polarized and unpolarized triplet cross sections, respectively, \( P_{\gamma} \) is the degree of photon beam polarization, \( \Sigma \) is the beam asymmetry for this process, and \( \varphi \) is the azimuthal angle of the recoil electron trajectory. The degree of photon beam polarization \( P_{\gamma} \) is determined by fitting the \( \varphi \) distribution to this function, where all but \( P_{\gamma} \) are known quantities. This is a destructive measurement for the small fraction of the photon beam intensity which undergoes triplet production on the beryllium converter, but this fraction is small enough that the measurement can be performed while the experiment is collecting data.

4.2.4 Pair Spectrometer

The Pair Spectrometer (PS) measures the spectrum of the collimated photon beam and the photon beam flux. It reconstructs the energy of a beam photon by converting it to an \( e^+e^- \) pair on the same beryllium foil as discussed for the TPOL, which gets deflected by a magnetic field into two detector arms containing scintillator counters. The PS is able to reconstruct photon energies from 6 GeV, well below the coherent peak, to 12.4 GeV, past the beam endpoint energy. This is also a destructive measurement, but again, only a small fraction of the intensity is converted into \( e^+e^- \) pairs.
4.3 The GlueX Detector

The GlueX detector is located roughly 75 m downstream of the photon beam production at the radiator described in Sec. 4.2.

The polarized photon beam is incident on a liquid hydrogen cryotarget, illustrated in white in Fig. 4.3, at a fluid density of $71.2 \pm 0.3$ mg/cm$^3$, cooled to a temperature of 20.1 K, which is 1 K below the saturation temperature of liquid H$_2$, to keep the hydrogen in the cell from boiling.

Surrounding the GlueX target and central detector system is a superconducting solenoid, illustrated in green in Fig. 4.3, approximately 4.8 m long with a bore radius of approximately 1 m. A nominal current of 1350 A produces a magnetic field of approximately 2 T along the $z$-axis.

The coordinate system used by GlueX is right-handed, with the $z$-axis defined along the beamline pointing downstream, the $y$-axis pointing vertically up, and the $x$-axis pointing horizontally to the left, looking downstream.
4.3.1 Charged Particle Tracking

The Central and Forward Drift Chambers, illustrated in orange in Fig. 4.3, are used to track charged particles by measuring their position, timing, and energy loss, yielding the candidate $\pi^+$, $\pi^-$, and proton tracks required for the analysis presented in this dissertation.

Central Drift Chamber

The Central Drift Chamber (CDC) surrounds the cryotarget and has an active volume that covers particles ejected from the target at polar angles between $6^\circ$ and $168^\circ$, with optimal coverage between $29^\circ$ and $132^\circ$. The CDC consists of 28 layers of Mylar® straw tubes containing gold-plated tungsten anode wires, in a 1.5 m long cylindrical volume. Twelve of these straw tube layers are axial, and 16 are at stereo angles of $\pm 6^\circ$. This enables measurement of position along the beam direction. The straws are filled with a 50/50 mixture of Ar and CO$_2$ gas at 1 atm, which becomes ionized when charged particles pass through it, sending a signal along the anode wires.

Forward Drift Chamber

The Forward Drift Chamber (FDC) is made of 24 disc-shaped planar drift chambers in 4 packages along the inner radius of the solenoid magnet. Each FDC chamber is made of a wire plane with cathode planes 5 mm from the wires on either side. These cathodes allow for better multi-track separation. All chambers of the FDC can detect particles from polar angles of $1^\circ$ to $10^\circ$, with partial coverage up to $20^\circ$.

Performance

To reconstruct a charged particle track, we first look for hits in adjacent layers of the FDC. These hits are formed into track segments, and the segments that get linked
together from different packages form track candidates. Similarly, hits in adjacent CDC rings are also formed into segments and linked together from different axial layers to form track candidates. Track candidates that pass through both the FDC and the CDC at a polar angle between $5^\circ$ and $20^\circ$ are linked together. Next, a Kalman filter is used to obtain an initial guess for the fitted track parameters. Each fitted track is assumed to be a pion unless its momentum is below 0.8 GeV, in which case it is assumed to be a proton. The drift information is not used at this stage. Finally, each fitted track is matched to a hit from either the Start Counter, the Time-of-Flight, the Barrel Calorimeter, or the Forward Calorimeter. This determines a start time, $t_0$, which allows each track to be refitted with the more precise drift time information. This is done for each track separately for several values of mass, corresponding to assumptions that the track could be any one of $\pm$, $\pi^\pm$, $K^\pm$, or $p^\pm$. Further details on performance of the tracking system can be found in Chapter 5.

4.3.2 Electromagnetic Calorimeters

The calorimetry system, illustrated in blue in Fig. 4.3, collects energy deposits from photon showers and charged particles. It is especially useful for measuring the energies of photons decaying from hadrons, such as $\pi^0 \rightarrow \gamma\gamma$ in the reaction studied in this dissertation.

Barrel Calorimeter

The Barrel Calorimeter (BCAL) [25] is an electromagnetic sampling calorimeter in the shape of an open cylinder surrounding the CDC. It can detect photon showers with energies from 0.05 GeV to several GeV, with polar angles between $11^\circ$ and $126^\circ$. It has 360$^\circ$ azimuthal coverage.
**Forward Calorimeter**

The Forward Calorimeter (FCAL) is located 5.6 m downstream from the center of the GlueX target. It consists of 2800 lead glass blocks in a circular array with a diameter of 2.4 m, and can detect photon showers with a minimum energy of 0.1 GeV, between $1^\circ$ and $11^\circ$ in polar angle.

**Performance**

The calorimetry system’s performance is described by how well it measures the energy, position, and timing of electromagnetic showers. The energy resolutions of the FCAL and BCAL are both extracted from the measured $\pi^0$ and $\eta$ distributions, and give consistent results.

### 4.3.3 Scintillation Detectors

The scintillators, illustrated in yellow in Fig. 4.3, collect timing and energy-loss information used for particle identification.

**Start Counter**

The Start Counter (ST) is a barrel-shaped scintillation detector surrounding the liquid hydrogen target. It covers about 90% of the solid angle for particles that originate at the center of the target. Since the ST is so close to the target, it is able to provide a timing signal that is relatively independent of particle type and trajectory. It uses the energy deposited into it ($dE/dx$) in conjunction with the flight time from the Time-of-Flight counters for particle identification.
**Time-of-Flight**

The Time-of-Flight (TOF) system is a 2.5\(\times\)2.5 m\(^2\) wall consisting of two planes of scintillator paddles in the horizontal and vertical directions, in front of the FCAL (about 5.5 m downstream from the target). It covers a polar angular region from 0\(^\circ\) to 13\(^\circ\), and provides fast timing signals from charged particles to assist with particle identification. The TOF has a 12\(\times\)12 cm\(^2\) hole in the center to allow the photon beam to pass through.

**Performance**

The two scintillator systems are used to measure the flight times of particles, with the ST selecting the electron beam bunch that generated the tagged photon that caused a reaction in the target, and using that in conjunction with a signal from the CEBAF accelerator, synchronized with the Radio Frequency (RF) time structure of the machine, to determine the event start time. The TOF is used to measure the event stop time. Both scintillators are also used in charged particle identification (PID). The ST uses energy deposition, \(dE/dx\), to reliably separate protons from other charged particles (electrons, pions, and kaons) up to a momentum of \(p = 0.9\) GeV. The TOF uses the velocity, \(\beta\), of the charged track as a function of momentum to further identify charged particles. It can reliably separate kaons from pions up to a momentum of 2 GeV, and can separate protons up to a momentum of 4 GeV.

### 4.4 Event Recording and Trigger

The trigger, which signals when to record an event, is built for high acceptance of high-energy hadronic interactions while minimizing the background rate from electromagnetic and low-energy hadronic interactions. Two main trigger types are used, the first is the
pair spectrometer trigger, which measures the flux of beam photons and requires a time coincidence of hits in both arms of the PS detector. The other is the physics trigger, which is based on energy depositions in the BCAL and FCAL. This trigger is generated when the energies satisfy one of two conditions: \(2 \times E_{FCAL} + E_{BCAL} > 1 \text{ GeV AND } E_{FCAL} > 0 \) GeV, which uses the fact that most events produce forward-going energy, or \(E_{BCAL} > 1.2 \) GeV, which is used to accept events with large transverse energy, such as \(J/\psi \rightarrow e^+e^-\) decays. For the reaction studied in this dissertation, \(\gamma p \rightarrow \pi^+\pi^+\pi^+\pi^0p\), the \(\pi^0 \rightarrow \gamma\gamma\) energy deposits and five minimum ionizing charged particles provide more than enough energy to satisfy the first trigger condition.

### 4.4.1 Event Reconstruction

Phase I of GlueX running resulted in approximately 3 PB of collected physics-quality raw data, which was reconstructed into over 500 TB of Reconstructed Events Storage (REST) files. Since this is too much data for anyone to individually analyze, we have a system for individuals to extract reaction-specific ROOT trees, which are described in section 6.1.1 and contain reconstructed beam photon candidates, photon showers, and charged particle tracks to be used in studying any final state reaction.
CHAPTER 5

Charged Pion Tracking and Efficiency Studies

Determination of particle tracking efficiencies is crucial for confirming that our simulation is correctly modeling what’s happening in the detector. If we want to trust our partial wave analysis results, we need to know that we’ve modeled the detector acceptance correctly. The study outlined in this chapter constitutes my service work to the GlueX Collaboration, and is documented in an internal GlueX analysis note [1].

The charged pion tracking efficiencies in the GlueX detector are determined using the reaction

\[ \gamma p \rightarrow \pi^+ \pi^- \pi^0 p, \]  

which is dominated by exclusive \( \omega \rightarrow \pi^+ \pi^- \pi^0 \) events. One of the charged pions in this reaction is treated as a missing particle in the reconstruction of particle combinations by the GlueX software, which produces the data files utilized here. The yield of \( \omega \) events is obtained from mass spectra for two cases: (1) where the pion treated as missing is “found” and (2) where it is “missing.” More details on the efficiency calculation are given
5.1 Data and Simulation Samples

The ROOT files for this analysis were produced by selecting events with one charged pion track, two neutral showers from a decaying \( \pi^0 \), one proton track, and one “additional” track in each event, which in this case would be the candidate pion for which we will determine the efficiency. We also store the unused charged track and neutral shower in the ROOT tree to be used in the following efficiency analysis.

The data files for this analysis are from the Spring 2017 GlueX run period and encompass the full run range (approximately 6.6B entries before cuts). The simulation sample was generated on the Open Science Grid for a shorter range of runs (approximately 40M entries before cuts).

5.2 Event Selection

We applied cuts to the events in both data and Monte Carlo:

Unused Shower Energy Cut

We cut away all events with unused shower energy greater than 1.0 GeV, as shown in Fig. 5.1.

Missing Mass Cut

The missing mass in this reaction,

\[
\text{MM} = \sqrt{(p_\gamma + p_p - p_{\pi^\pm} - p_{\pi^0} - p_{\nu'})^2},
\] (5.2)
plotted in Fig. 5.2 should peak around the charged pion mass, since the conservation of four-momentum is required and all final-state particles except one of the charged pions are detected.

We cut away all events with missing mass greater than 0.25 GeV, to ensure that the events we examine are truly missing a single charged pion, and not some misidentified heavier particle.

**Kinematic Fit Confidence Level Cut**

To further ensure energy-momentum conservation, we cut away all events where the kinematic fit (see Section 6.1.5) confidence level was less than 0.1, or a 90% confidence level, illustrated in Fig. 5.3.
Two Photon Mass Cut

To ensure that the two photons in our final state came from the decay of a neutral pion, we cut away all events where the invariant mass of the two photon showers was greater than 15 MeV away from the mass of the $\pi^0$, illustrated in Fig. 5.4.

Kinematic Variable Difference Cut

Reconstructed tracks in Monte Carlo were required to be loosely consistent with the kinematic variables $\phi$, $\theta$, and $p$ of the true values generated in the simulations, as listed in Table 5.1. These cuts are very loose, and are performed mainly to remove obviously junk events.

Reconstructed tracks in data and Monte Carlo were also required to be loosely consistent to the same degree with the kinematic variables $\phi$, $\theta$, and $p$ of the predicted missing mass.
FIG. 5.3: Cut on kinematic fit confidence level for $\pi^\pm$.

$\pi^\pm$.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\Delta p</td>
</tr>
<tr>
<td>$</td>
<td>\Delta \phi</td>
</tr>
<tr>
<td>$</td>
<td>\Delta \theta</td>
</tr>
</tbody>
</table>

TABLE 5.1: Values used for kinematic variable difference cuts in data and Monte Carlo.
FIG. 5.4: Cut on two photon invariant mass for $\pi^+$

$0.12 < M_{\gamma\gamma} < 0.15$ GeV
5.3 Procedure

We define the efficiency for reconstructing a charged pion as

\[ \varepsilon = \frac{F}{F + M} = \frac{“\text{found”}}{“\text{found”} + “\text{missing”}} = \frac{“\text{found”}}{“\text{produced”}} \]  

(5.3)

where a “missing” pion is a track that was assumed to be missing in the combination formation, and no candidate charged track matching those kinematics was reconstructed in the event. The \( \omega \) yield for these events is determined by fitting the missing mass off the proton (MMOP) when there are no “unused” tracks in the ROOT tree. The MMOP is defined as

\[ \text{MMOP} = \sqrt{(p_\gamma + p_p - p_{p'})^2}, \]  

(5.4)

where \( p_\gamma, p_p, p_{p'} \) are beam, target, and recoil proton four-momenta, respectively. The MMOP corresponds to the mass of the meson system decaying to \( \pi^+\pi^-\pi^0 \) for the case of exclusive meson production. The details of the fits to these spectra to obtain the \( \omega \) yields are described in Sec. 5.3.1.

A “found” pion is a track that was assumed to be missing in the combination formation, and a candidate track matching those kinematics was reconstructed. This \( \omega \) yield for these events can be determined with two different methods:

- **Method 1**: Fitting the MMOP for events where the missing track candidate was reconstructed to determine the \( \omega \) yield.

- **Method 2**: Measuring the invariant mass of the \( \pi^+\pi^-\pi^0 \) using the reconstructed missing track candidate to determine the \( \omega \) yield.

After the event selection cuts, we plotted the \( \pi^+\pi^-\pi^0 \) invariant mass and the missing
mass off the proton versus the kinematic variables $\phi_{\text{lab}}$, $\theta_{\text{lab}}$, and $p$. The lab angles $\phi_{\text{lab}}$ and $\theta_{\text{lab}}$ describe the azimuthal and polar angles in reference to the beamline, and will be referred to in this chapter as $\phi$ and $\theta$. The “found” and “missing” $\omega$ yields are found by fitting the mass distributions in these kinematic bins as described in the sections below. The two definitions for found allow for two methods of defining efficiency, which allows us to check if they agree (they do, within $\sim 5\%$).

5.3.1 Fit Functions

The two mass distributions have very different resolutions and require different fit functions in order to extract the $\omega$ yields. The fits were performed in 20 equal bins of $\phi$ from $-180^\circ$ to $180^\circ$, 20 equal bins of $\theta$ from $0^\circ$ to $30^\circ$, and 12 equal bins of $p$ from 0 to 6 GeV.

Missing Mass off the Proton

The MMOP was fit with an asymmetric Gaussian function with a cubic polynomial background

$$f(x) = g_{\text{asymm}}(x) + p_4 + p_5 x + p_6 x^2 + p_7 x^3, \quad (5.5)$$

where

$$g_{\text{asymm}}(x) = \begin{cases} 
    p_0 \exp \left( -0.5 \left( \frac{x-p_1}{p_2} \right)^2 \right) & \text{if } x > p_1 \\
    p_0 \exp \left( -0.5 \left( \frac{x-p_1}{p_2-p_3(x-p_1)} \right)^2 \right) & \text{if } x < p_1.
\end{cases} \quad (5.6)$$

The cubic polynomial was able to approximate the background for the dataset used in this study, but future studies of the full GlueX-I dataset, with roughly six times the statistics, may need to make a more refined choice. The $\eta$ and $\phi$ peaks were not pronounced enough to require their own Gaussian fits, making the fit function simpler than the one required
for the $\pi^+\pi^-\pi^0$ invariant mass.

In Eqn. 5.6, $p_1$ is the mean of the Gaussian, sometimes called $\mu$, and $p_2$ is the width, sometimes called $\sigma$. The “skew” of the Gaussian is given by $p_3$, where large negative or positive values indicate a high degree of asymmetricity, and a skew of zero gives a symmetric Gaussian distribution.

The $\omega$ yield in each bin is given by the integral under the asymmetric Gaussian function $g_{asym}(x)$. Example fits are shown in Figs 5.5 and 5.6.

![FIG. 5.5: Example fit to MMOP spectrum in GlueX data. The blue curve represents the asymmetric Gaussian function in Eqn. 5.6, and the purple curve is the cubic polynomial background.](image-url)
FIG. 5.6: Example fit to MMOP spectrum in Monte Carlo. The blue curve represents the asymmetric Gaussian function in Eqn. 5.6, and the purple curve is the cubic polynomial background.
Three Pion Invariant Mass

The $\pi^+\pi^-\pi^0$ invariant mass distribution had distinct $\eta$ and $\phi$ peaks, which required Gaussian fits. The $\omega$ peak required a double Gaussian function (two Gaussians with the same centroid but different widths and heights), as its change in width from the peak region to the edges of the peak region could not be correctly modeled by a single Gaussian. The background was still modeled with a cubic polynomial

$$f(x) = g_{\text{double}}(x) + g_\eta(x) + g_\phi(x) + p_4 + p_5 x + p_6 x^2 + p_7 x^3,$$

(5.7)

where

$$g_{\text{double}}(x) = p_0 \exp\left(-0.5 \left(\frac{x - \mu}{\sigma_0}\right)^2\right) + p_1 \exp\left(-0.5 \left(\frac{x - \mu}{\sigma_1}\right)^2\right).$$

(5.8)

The $\omega$ yield in each bin is given by the integral under the double Gaussian function $g_{\text{double}}(x)$. Example fits are shown in Figs 5.7 and 5.8.

FIG. 5.7: Example fit to the $\pi^+\pi^-\pi^0$ mass spectrum in GlueX data. The dark and light blue curves each represent one term of the double Gaussian function in Eqn. 5.8. The green curve is the sum of the two Gaussian functions used to model the $\eta$ and $\phi$ background, and the purple curve is the cubic polynomial background.
FIG. 5.8: Example fit to the $\pi^+\pi^-\pi^0$ mass spectrum in Monte Carlo. The dark and light blue curves each represent one term of the double Gaussian function in Eqn. 5.8. The green curve is the sum of the two Gaussian functions used to model the $\eta$ and $\phi$ background, and the purple curve is the cubic polynomial background.

### 5.3.2 Error Propagation

The uncertainty on the efficiency in Eqn. 5.3 can be written as

$$\sigma_\varepsilon = \sqrt{\left( \frac{\partial \varepsilon}{\partial F} \right)^2 \sigma_F^2 + \left( \frac{\partial \varepsilon}{\partial M} \right)^2 \sigma_M^2},$$

(5.9)

where $F$ and $M$ are independent samples of events and a fit to a histogram of these event yields in either case can determine the integral and uncertainty on the integral independently. The uncertainties on $F$ and $M$ are treated as uncorrelated. Taking the derivatives and simplifying, we find that

$$\sigma_\varepsilon = \sqrt{\left( \frac{M}{(F+M)^2} \right)^2 \sigma_F^2 + \left( \frac{F}{(F+M)^2} \right)^2 \sigma_M^2},$$

(5.10)

which is implemented in our efficiency calculation macro.
5.4 Efficiencies

5.4.1 Integrated Efficiencies

The charged pion efficiencies are shown in Figs 5.9 and 5.10 as functions of the missing track’s $\phi$, $\theta$, and 3-momentum. Note that the error bars are statistical only.

![Efficiencies graph](image)

FIG. 5.9: $\pi^+$ efficiencies as functions of $\phi$, $\theta$, and 3-momentum (note that datapoints are offset for clarity)

![Efficiencies graph](image)

FIG. 5.10: $\pi^-$ efficiencies as functions of $\phi$, $\theta$, and 3-momentum (note that datapoints are offset for clarity)

The efficiencies in $\phi$ are integrated over all $\theta$ and all $p$. The efficiencies in $\theta$ are integrated over all $\phi$ and all $p > 0.5$ GeV. The efficiencies in $p$ are integrated over all $\phi$ and all $\theta > 1.15^\circ$.

The ratios between the efficiencies for data/MC can be found in Figs 5.11 and 5.12. The difference between efficiencies for data and MC is mostly within 5%. The data/MC ratio for both pion charges seems to fall around $\theta = 15^\circ$, possibly due to an inaccurate accounting for the drop in efficiency in the region where the CDC and FDC overlap.
FIG. 5.11: $\pi^+$ efficiency ratios (data/MC) as functions of $\phi$, $\theta$, and 3-momentum

FIG. 5.12: $\pi^-$ efficiency ratios (data/MC) as functions of $\phi$, $\theta$, and 3-momentum
5.4.2 Efficiencies in $p$ vs $\theta$

We were able to implement finer binning in 3-momentum and the angle $\theta$ of the missing track, which allows for a better view of efficiency in that phase space. This can be seen in Figs 5.13 and 5.14

FIG. 5.13: $\pi^+$ efficiency as a function of $\theta$, binned in 3-momentum
FIG. 5.14: $\pi^-$ efficiency as a function of $\theta$, binned in 3-momentum
CHAPTER 6

\( \omega \pi^- \Delta^{++} \) Event Selection

This analysis was performed using the GlueX Phase I data set, which was collected using a linearly polarized photon beam on a liquid hydrogen target in three run periods over two years described in Table 6.1. The electron beam used to produce the photon beam was delivered at an average intensity of 150 nA.

<table>
<thead>
<tr>
<th>Data set</th>
<th>Spring 2017</th>
<th>Spring 2018</th>
<th>Fall 2018</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>2017.01</td>
<td>2018.01</td>
<td>2018.08</td>
<td>GlueX-I</td>
</tr>
<tr>
<td>Runtime (days)</td>
<td>20.4</td>
<td>43.7</td>
<td>31.0</td>
<td>95.1</td>
</tr>
<tr>
<td>Luminosity (pb(^{-1}))</td>
<td>21.8</td>
<td>63.0</td>
<td>40.1</td>
<td>125.0</td>
</tr>
</tbody>
</table>

TABLE 6.1: GlueX Phase I properties. Luminosity here refers to photon flux measured in the coherent peak region of the beam energy spectrum, discussed in Section 6.1.4. Runtime refers to the number of calendar days spent running times the fraction of Acceptable Beam Used (ABU) time.
6.1 Event Selection

6.1.1 Analysis Launch Cuts

As discussed in section 4.4.1, the GlueX Phase I data exist in over 500 TB of Reconstructed Events Storage (REST) files. Individual reactions require only a fraction of this data, which is filtered for the final state of interest. This “reaction filter” process is run in parallel for many reactions where users provide their reaction’s configuration through a web interface which is periodically downloaded to a configuration file for an “analysis launch.” This analysis uses the exclusive (all final state particles are detected) reaction $\gamma p \rightarrow \omega \pi^+ \pi^- p$, where $\omega \rightarrow \pi^+ \pi^- \pi^0$. During an analysis launch, for each submitted reaction, the GlueX analysis library creates possible particle combinations from the reconstructed tracks and showers saved in the REST format. In this case, we need a beam photon detected by the tagger described in Section 4.2.2, and in the final state we need three positive charged tracks and two negative charged tracks measured by the drift chambers, and two neutral showers measured by the calorimeters. Energy showers in the calorimeters are assumed to be neutral if they are not associated with any charged tracks in the drift chambers. Selection criteria are applied for exclusivity and particle identification, then a kinematic fit is performed. If the kinematic fit converges for a combination of tracks and showers, that event is stored into a reaction-specific ROOT tree for our analysis. The selection criteria for data are compared to simulated $\gamma p \rightarrow \omega \pi^- \Delta^{++}$ events based on a model described in Section 7.3.1. These simulated Monte Carlo (MC) events are run through a GEANT4 model of the GlueX detector and reconstructed with the same analysis software as the data, as described in Section 14 of Ref. [24].
Preselection Cuts

Any given charged track must have at least one mass hypothesis with a hit in the BCAL, the FCAL, the TOF, and/or the SC, or it will not be considered a candidate for one of our final state charged particles. This is done to remove pileup or out of time tracks that don’t properly coincide with another detector. A neutral shower will be removed from the list of candidate photons if it does not have a shower energy of at least 100 MeV, and if the neutral shower has hits in the BCAL, it must have hits in at least two cells or it will be removed, in order to reject hits from beam background and to minimize “noise” from electronics.

Timing Cuts

Timing cuts are applied to make sure that the tracks we see in our detectors came from the correct RF bunch (see Section 6.1.3). If a certain track is too far temporally from when we would expect to see it in our detector, it is cut away. The timing cuts shown in Table 6.2 are applied to the system with the best timing information available, which is, in order, BCAL, TOF, FCAL, SC. This means that if, for example, a track has a hit in the TOF, no timing cuts will be applied on the FCAL or SC. Since the detectors occupy different angular regions, a track with hits in the BCAL will probably not hit the FCAL or TOF. There is considerable overlap between the FCAL and TOF, however. Two caveats: i) some hypotheses might not have a hit in any system, and therefore will survive all timing cuts, and ii) the SC timing isn’t good enough for PID over the short flight distance in question, so if the track is matched to more than one SC hit, there is no timing cut, unless it is the only track in the combination. These timing cuts are redone after the kinematic fit has updated the momentum and vertex information.
<table>
<thead>
<tr>
<th>PID</th>
<th>BCAL/RF $\Delta t$ (ns)</th>
<th>TOF/RF $\Delta t$ (ns)</th>
<th>FCAL/RF $\Delta t$ (ns)</th>
<th>SC/RF $\Delta t$ (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>$\pm 1.5$</td>
<td>NA</td>
<td>$\pm 2.5$</td>
<td>NA</td>
</tr>
<tr>
<td>$\pi^\pm$</td>
<td>$\pm 1.0$</td>
<td>$\pm 0.5$</td>
<td>$\pm 2.0$</td>
<td>$\pm 2.5$</td>
</tr>
<tr>
<td>$p$</td>
<td>$\pm 1.0$</td>
<td>$\pm 0.6$</td>
<td>$\pm 2.0$</td>
<td>$\pm 2.5$</td>
</tr>
</tbody>
</table>

**TABLE 6.2: Rough Timing Cuts**

**Track Energy Loss Cuts**

Cuts are also placed on the energy loss of each charged track. Different types of particles lose energy at different rates, making energy loss cuts useful for PID. For the charged particles in this reaction (protons and pions), the only cuts we place are on tracks in the CDC, which are outlined in Table 6.3. An example plot of energy loss vs three-momentum for the proton is shown in Fig. 6.1.

<table>
<thead>
<tr>
<th>PID</th>
<th>CDC $dE/dx$ (keV/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$dE/dx &gt; e^{-4.0p+2.25} + 1.0$</td>
</tr>
<tr>
<td>$\pi^\pm$</td>
<td>$dE/dx &lt; e^{-7.0p+3.0} + 6.2$</td>
</tr>
</tbody>
</table>

**TABLE 6.3: Cuts on energy loss in the CDC for protons and charged pions.**

**FIG. 6.1:** Energy loss vs three-momentum for protons in the CDC, after analysis launch cuts, in GlueX data (left) and MC (right). Clear bands from the proton are evident in both, along with some background from the $\pi^+$ in the data.
6.1.2 Particle ID

To better identify particles, we apply timing cuts to charged particle tracks. An example shown in 6.2 is for proton tracks, where we see that the majority of the proton tracks land between polar angles 20° and 60°, which is the region covered by the BCAL. We place a cut on the timing of the track in the BCAL to eliminate spurious tracks. If a proton is temporally removed by more than 1.0 ns from when we would expect it, that event is removed.

![Graphs showing three-momentum vs θ for proton tracks in GlueX data (left) and MC (right). Bottom row: ΔT in the BCAL vs three-momentum for the proton hypotheses, in GlueX data (left) and MC (right). A cut is made on the vertical axis of the bottom two plots at ΔT = ±1.0 ns.](image)

FIG. 6.2: Top row: Three-momentum vs θ for the proton tracks in GlueX data (left) and MC (right). Bottom row: ΔT in the BCAL vs three-momentum for the proton hypotheses, in GlueX data (left) and MC (right). A cut is made on the vertical axis of the bottom two plots at ΔT = ±1.0 ns.
**Missing Energy Cut**

GlueX is designed to detect all final state particles, in exclusive reactions. Since no particles are “missing,” we place a loose cut on the missing energy

$$-3.0 < E_{\text{Miss}} < 3.0 \text{ GeV}$$ \hspace{1cm} (6.1)

where

$$E_{\text{Miss}} = E_{\gamma} + E_{p} - \sum E_{f},$$ \hspace{1cm} (6.2)

where $E_{\gamma}$ is the beam energy, $E_{p}$ is the energy of the target proton, and $E_{f}$ are the energies of each final state particle. If a particle were treated as missing, such as in the charged pion tracking analysis in Chapter 5, we would place a cut on the missing mass instead, corresponding to the nominal mass of the missing particle. This missing energy cut is very loose, and is applied mainly to keep the file size down by removing obviously useless events.

**6.1.3 Accidental Subtraction**

CEBAF’s electron beam arrives in bunches at a frequency of 249.5 MHz, corresponding to 4.008 ns between bunches. There are approximately 70 hits on the tagger system for every recorded event, even though only one of these hits is from the photon that produced the final state particles in question. Even if we applied all other event selection cuts, multiple tagger hits per event would still exist, and are called tagger accidentals. Early in our list of cuts, we perform a background subtraction by taking the four beam bunches before and after the “on time,” or coincidence bunch, called the sideband bunches, illustrated in Fig. 6.3, and assuming that a similar background structure exists under the on time peak. We give each event combination a weight depending on its tagger photon
bunch, with combinations from the sideband bunches getting a weight of $-1/8$, and the on time bunch getting a weight of $+1$.

![Graph](image.png)

FIG. 6.3: Coincidence RF bunch with the eight sideband RF bunches in GlueX data (left) and MC (right), before accidental subtraction.

Out of time hits are included in the Monte Carlo by combining signal Monte Carlo events with data events collected with a random trigger. These events are added at the detector hit level and reconstructed with the same software as the data.

### 6.1.4 Photon Beam Energy

In order to extract the polarization dependence of our physics observables, we place a cut (Eqn. 6.3) on the photon beam energy such that only the events in the coherent peak, which is the region of maximum polarization illustrated in Fig. 6.4, are kept. Note that the signal Monte Carlo was generated only in the coherent peak region, because we only are interested in fitting that region, and thus contains no events outside of it.

$$8.2 < E_\gamma < 8.8 \text{ GeV}$$  \hspace{1cm} (6.3)
FIG. 6.4: Photon beam energy spectrum in GlueX data (left) and MC (right). The coherent peak region, where polarization is maximal, is shaded.
6.1.5 Missing Mass Squared

The missing mass squared

\[
MM^2 = (p_\gamma + p_p - \sum p_f)^2,
\]

(6.4)

where \( p_\gamma \) is the four-momentum of the photon beam, \( p_p \) is that of the target proton, and \( p_f \) are the four-momenta of the measured final state particles, should peak around zero, since this is an exclusive reaction. To ensure that this is truly the final state we want, we cut the missing mass squared

\[-0.05 < MM^2 < 0.05 \text{ GeV},\]

(6.5)

as shown in Fig. 6.5

![Graph showing missing mass squared for this reaction in GlueX data (left) and MC (right). The shaded regions indicate combinations kept by the missing mass squared cut.](image-url)
Kinematic Fitting

We use kinematic fitting (see Section 15.3 of Ref. [24]) to improve the resolution of our measured tracks and photon showers and distinguish between different reactions. We can exploit the fact that in GlueX, the initial state is very well known, with the target proton at rest and the incident photon energy measured to a high precision of roughly $10^{-25}$ MeV resolution, depending on the energy. The kinematic fits used here impose energy-momentum conservation between the initial and final-state particles

$$p_{\gamma} + p_{p} - \sum p_{f} = 0,$$

(6.6)

where $p_{\gamma}$ is the four-momentum of the photon beam, $p_{p}$ is that of the target proton, and $p_{f}$ are the four-momenta of the measured final state particles. We also impose a vertex constraint that requires all charged particles to originate from a common spatial point.

The quality of the kinematic fit is determined from the $\chi^2/NDF$ of the fit combined with the number of degrees of freedom. We cut away everything with a kinematic fit $\chi^2/NDF >$
2.25, because the $\chi^2/NDF$ distribution shown in Fig. 6.6 peaks at 1, but this cut is possibly too stringent, and the effects of this cut on our results will have to be the subject of future systematic studies

$\pi^0 \rightarrow \gamma\gamma$ Identification

Unlike for charged tracks, we can’t require that our photon showers came from the same vertex, since we don’t measure their full trajectory through the detector. After the kinematic fit, we assume that the photon showers originated from the same spatial point as the charged tracks, improving our photon momentum resolution. We perform a loose cut around the neutral pion mass, $0.12 < M_{\gamma\gamma} < 0.15$ GeV, shown in Fig. 6.7, where we see a peak in the two photon mass distribution at the $\pi^0$ mass of 135 MeV. This cut removes any events where the photons clearly did not originate from a decaying $\pi^0$.

![Graphs showing the invariant mass of the two final state photons before and after the kinematic fit, in GlueX data (left) and MC (right). The shaded regions mark events kept for further analysis.](image)

FIG. 6.7: The invariant mass of the two final state photons before and after the kinematic fit, in GlueX data (left) and MC (right). The shaded regions mark events kept for further analysis.
6.2 Selection of a Recoiling $\Delta^{++}$

To ensure that the $\pi^+\pi^-\pi^-\pi^0$ system is recoiling off of a $\Delta^{++}$ baryon, we place cuts on the invariant mass of the $p\pi^+$ system and the four-momentum transfer to that system, $-t_{\Delta^{++}} = (p_p - p_{\Delta^{++}})^2$, where $p_p$ is the four-momentum of the target proton. For the remainder of this dissertation, $-t_{\Delta^{++}}$ is referred to as $-t$.

We remove all candidate combinations with a $p\pi^+$ mass above 1.35 GeV, and those with $-t > 1 \text{ GeV}^2$. We perform the mass cut to reject events where the $\omega\pi^-$ system is not recoiling off a true $\Delta^{++}$. The $p\pi^+$ invariant mass distribution is shown in Fig. 6.8 and has a narrow peak at the $\Delta^{++}$ mass of 1.23 GeV, and the phasespace Monte Carlo is able to model the data well up to a recoil mass of approximately 1.35 GeV, as can be seen in the diagnostic plots from our amplitude fits described in Section 7.3.3 and shown in Figs. 8.20 and 8.22.

The cut on $-t$ is done to isolate the production mechanism. We expect the production mechanism of the reaction to change as a function of $-t$, similar to other reactions such as
FIG. 6.9: Four-momentum transfer squared distribution in GlueX data (left) and MC (right).

\( \gamma p \rightarrow \pi^- \Delta^{++} \), discussed in Section 1.3. Our final analysis is performed in two separate \(-t\) ranges, 0.15 – 0.30 GeV\(^2\) and 0.30 – 0.50 GeV\(^2\), to study the effect of \(-t\) on our results.

### 6.3 Two-Dimensional \( \omega \) Sideband Subtraction

In order to select \( \omega \pi^- \) events from the \( \pi^+ \pi^- \pi^- \pi^0 \) recoiling opposite the \( \Delta^{++} \), we perform a two-dimensional sideband subtraction on the two possible \( \pi^+ \pi^- \pi^0 \) mass combinations. We index the two negative pions as \( \pi_{1}^- \) and \( \pi_{2}^- \), then the \( M_1(3\pi) \) and \( M_2(3\pi) \) illustrated in Fig. 6.10 (Fig. 1 of Ref. [20]) and plotted in Fig. 6.11 are the invariant masses of \( \pi^+ \pi_{1}^- \pi^0 \) and \( \pi^+ \pi_{2}^- \pi^0 \), respectively. This sideband subtraction technique is outlined in Ref. [20].

The \( \omega \) sideband and peak regions used are illustrated in Fig. 6.12 and listed in Table 6.4.

The \( \pi^+ \pi^- \pi^- \pi^0 \) events are plotted in Fig. 6.13 and are given weights in the fitting software (AmpTools) such that at most one \( \omega \pi^- \) combination is selected from each event. When creating the input root tree for AmpTools, the selector runs through all the events,
FIG. 6.10: Illustration of two-dimensional sideband subtraction as used by Chung et al on SLAC data. Source: Fig. 1 of Ref. [20].

looking at both $\pi^+\pi^-\pi^0$ combinations made possible by the two negative pions. If both combinations have a $\pi^+\pi^-\pi^0$ invariant mass in the $\omega$ mass range, then the combination with the invariant mass closest to the nominal $\omega$ mass value of 783 MeV is chosen. These “double-$\omega$” events are each only used once, and are given a weight of +1. Next, we look at the $\pi^+\pi^-\pi^0$ combination. If it has an invariant mass in the $\omega$ mass range, and the $\pi^+\pi^-\pi^0$ combination falls anywhere besides the $\omega$ mass range, the $\pi^+\pi^-\pi^0$ event is given a weight of +1. The same weight of +1 is applied if the $\pi^+\pi^-\pi^0$ combination has an invariant mass in the $\omega$ peak region, but the $\pi^+\pi^-\pi^0$ does not. Then, if the $\pi^+\pi^-\pi^0$ combination falls in either of the sideband regions, but the $\pi^+\pi^-\pi^0$ combination does not (diagonal shaded regions in Fig. 6.10), the event is given a weight of -1/2. The same weight of -1/2 is applied if the $\pi^+\pi^-\pi^0$ combination has an invariant mass in a sideband region and the $\pi^+\pi^-\pi^0$ does not. If both $\pi^+\pi^-\pi^0$ and $\pi^+\pi^-\pi^0$ have invariant masses in a sideband region
(cross-hatched regions in Fig. 6.10), the event is given a weight of \(-5/8\). This weighting system allows us to estimate the number of true \(\omega \pi^-\) events, while removing non-\(\omega\) background and avoiding double-counting.

![Diagram showing correlation between invariant masses of two possible \(\pi^+ \pi^- \pi^0\) combinations in GlueX data (left) and MC (right).]

**FIG. 6.11:** Correlation between invariant masses of the two possible \(\pi^+ \pi^- \pi^0\) combinations in GlueX data (left) and MC (right).

<table>
<thead>
<tr>
<th>Region</th>
<th>Min (M_{\pi^+ \pi^- \pi^0}) (MeV)</th>
<th>Max (M_{\pi^+ \pi^- \pi^0}) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L)</td>
<td>690</td>
<td>735</td>
</tr>
<tr>
<td>(\omega)</td>
<td>760</td>
<td>805</td>
</tr>
<tr>
<td>(H)</td>
<td>830</td>
<td>875</td>
</tr>
</tbody>
</table>

**TABLE 6.4:** \(\omega\) peak and sideband regions used in sideband subtraction (illustrated in Figs 6.10 and 6.12).
FIG. 6.12: Illustration of \( \omega \) sideband (gray) and peak (green) regions in the \( \pi^+ \pi^- \pi^0 \) spectrum in GlueX data (left) and MC (right). Note the \( \eta \to \pi^+ \pi^- \pi^0 \) events observed in GlueX data but not MC. These \( \eta \) events are eliminated by our cuts.

FIG. 6.13: Invariant mass spectrum of \( \pi^+ \pi^- \pi^0 \) events in GlueX data (left) and MC (right), before (unshaded) and after (shaded) two-dimensional \( \omega \) sideband subtraction illustrated in Figs 6.10 and 6.12. Note that the pre-subtraction plot contains two entries per event, and is therefore double-counted.
CHAPTER 7

Partial Wave Analysis

As mentioned at the beginning of Chapter 2, Partial Wave Analysis (PWA) is the technique we use to determine the set of amplitudes that can describe the production and decay of the intermediate state $X^-$ in the reaction

$$\gamma p \rightarrow X^-\Delta^{++} \rightarrow \omega\pi^-\Delta^{++}.$$  \hspace{1cm} (7.1)

In the $\omega\pi^-$ final state, the $\pi^-$ is often referred to as the “bachelor” particle.

The software package used for this maximum likelihood fitting is called AmpTools, developed by the group at Indiana University with amplitudes encoded in the GlueX software halld_sim repository [26].

7.1 Maximum Likelihood

Maximum likelihood fitting involves calculating a set of parameters in a probability distribution function that maximizes the probability that the function fits the observed data distributions. For our reaction, $\gamma p \rightarrow \omega\pi^-\Delta^{++}$, we use the observables
\( \mathbf{x} = \{\Phi, \theta, \phi, \theta_H, \phi_H\} \), encoded in a five-dimensional vector, to describe its kinematics. As a reminder, the angle \( \Phi \) describes the angle between the beam polarization and the production plane of the intermediate meson \( X^- \), and the angles \((\theta, \phi) = \Omega\), and \((\theta_H, \phi_H) = \Omega_H\) describe the decay of \( X^- \rightarrow \omega \pi^- \) and \( \omega \rightarrow \pi^+ \pi^- \pi^0 \), respectively. These angles are more thoroughly described in Sections 2.1 and 2.2. The physics model that we fit to the data is the intensity, \( \mathcal{I}(\mathbf{x}; \boldsymbol{\theta}) = I(\Phi, \Omega, \Omega_H) \), described in Eqn. 2.30 and reproduced here

\[
I(\Phi, \Omega, \Omega_H) = \left\{ (1 - P_\gamma) \left[ \left| \sum_{i,m} [J_i]_{m}^{(-)} Im(Z^i_{m}) \right|^2 + \left| \sum_{i,m} [J_i]_{m}^{(+)} Re(Z^i_{m}) \right|^2 \right] + \\
(1 + P_\gamma) \left[ \left| \sum_{i,m} [J_i]_{m}^{(+)} Im(Z^i_{m}) \right|^2 + \left| \sum_{i,m} [J_i]_{m}^{(-)} Re(Z^i_{m}) \right|^2 \right] \right\}. \tag{7.2}
\]

The intensity is a function of \( \mathbf{x} \), encoded in the \( Z^i_{m}(\mathbf{x}) \), with \( n \) parameters \( \boldsymbol{\theta} \), corresponding to the production parameters \([J_i]_{m}^{(e)}\) in Eqn. 7.2, and we attempt to maximize the likelihood by finding the best fit in the \( n \)-dimensional phasespace. In the case where we only consider \( J^P = 1^+ \) and \( 1^- \) in the fit, \( n = 18 \) (three waves with three spin projections and two reflectivities each).

With a set of \( N \) independent observations \( \mathbf{x}_i \), which are the observed events in data, the likelihood is written as a function of the parameters \( \boldsymbol{\theta} \),

\[
\mathcal{L}(\boldsymbol{\theta}) = \frac{e^{-\mu \mu^N}}{N!} \prod_{i=1}^{N} \mathcal{P}(\mathbf{x}_i; \boldsymbol{\theta}), \tag{7.3}
\]

where \( \mathcal{P}(\mathbf{x}_i; \boldsymbol{\theta}) \) is the five-dimensional probability density, and can be written in terms of the intensity as

\[
\mathcal{P}(\mathbf{x}; \boldsymbol{\theta}) = \frac{1}{\mu} \mathcal{I}(\mathbf{x}; \boldsymbol{\theta}) \eta(\mathbf{x}), \tag{7.4}
\]

where \( \eta(\mathbf{x}) \) is the detector and selection efficiency, or the probability that an event with kinematics \( \mathbf{x} \) will both be detected and make it through all event selection criteria to be
included into the sample of \( N \) data events used as input to the fit. The model-predicted number of observed events, \( \mu \), for a certain set of parameters \( \theta \) is given by

\[
\mu = \int \mathcal{I}(\mathbf{x}; \theta)\eta(\mathbf{x})d\mathbf{x}.
\] (7.5)

Instead of maximizing the likelihood, what we do in practice is minimize \(-2\ln \mathcal{L}(\theta)\)

\[
-2\ln \mathcal{L}(\theta) = -2 \left( \sum_{i=1}^{N} \ln \mathcal{I}(\mathbf{x}_i; \theta) - \int \mathcal{I}(\mathbf{x}; \theta)\eta(\mathbf{x})d\mathbf{x} \right) + c_1,
\] (7.6)

where \( c_1 \) is a constant containing every term that does not depend on \( \theta \), and can be ignored in the \(-2\ln \mathcal{L}(\theta) \) minimization process.

The intensity is computed in AmpTools using Eqn. 7.2, rewritten here for only the \( b_1 \) amplitude, which can have the ratio of \( D \)-wave to \( S \)-wave in the decay free

\[
I_{b_1, \text{free}}(\mathbf{x}) = \left\{ (1 - P_\gamma) \left[ \left| \sum_m [1^+ S]_m^{(-)} Im(Z^S_m) + [1^+ D]_m^{(-)} Im(Z^D_m) \right|^2 
\right.ight.
\]
\[
+ \left[ \sum_m [1^+ S]_m^{(+)} Re(Z^S_m) + [1^+ D]_m^{(+)} Re(Z^D_m) \right|^2 \right]
\]
\[
+ \left(1 + P_\gamma \right) \left[ \left| \sum_m [1^+ S]_m^{(+)} Im(Z^S_m) + [1^+ D]_m^{(+)} Im(Z^D_m) \right|^2 
\right.
\]
\[
+ \left[ \sum_m [1^+ S]_m^{(-)} Re(Z^S_m) + [1^+ D]_m^{(-)} Re(Z^D_m) \right|^2 \right] \right\} \] (7.7)

or constrained (Eqn. 7.8), where we use four coherent sums, one over each combination of reflectivity and real or imaginary components of the complex-valued functions \( Z^L_m(\mathbf{x}) \). The \( Z^L_m(\mathbf{x}) \) each describe the \( S \)- and \( D \)-wave amplitudes, where \( m \) indexes the spin projection of the \( b_1 \), and \( L \) indicates an \( S \)- or \( D \)-wave. The “production factors,” \( [1^+ (L)]^{(\epsilon)}_m \), are complex numbers that represent the magnitude and phase of the corresponding amplitude

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$Z^L_m(x)$, and are free parameters in the fit. While there are four coherent sums, there are two pairs which share the same reflectivity amplitudes. There is one unconstrained phase in the fit for each pair of coherent sums, which we fix by setting one production factor to be real.

To constrain the $D/S$ ratio, we introduce a “scale factor” $s^{D/S}$, which is real-valued, and lets us constrain production factors up to a fixed or floating scale. The phase $\phi_D$ gives the difference in phase between the $D$- and $S$-waves.

$$I_{b_1}(x) = \left\{ (1 - P_\gamma) \left[ \sum_m [1^+][(-) \left( \text{Im}(Z^S_m) + s^{D/S}e^{-i\phi_D}\text{Im}(Z^D_m)) \right] \right]^2 \\
+ \sum_m [1^+](+) \left( \text{Re}(Z^S_m) + s^{D/S}e^{-i\phi_D}\text{Re}(Z^D_m)) \right] \right\}$$

(7.8)

In principle, the $D/S$ ratio should be consistent across all reflectivities and spin projections that contribute to the $b_1$. The positive reflectivity wave with spin projection $m = 0$ should have the same $D/S$ ratio as the negative reflectivity wave with spin projection $m = 1$, and so on. We can use these scale factors to force the $D/S$ ratio to be the same for each pair of $D$- and $S$-waves, such as in Eqn. 7.8, or we can allow the $D/S$ ratio to be “free” in the fit, shown in Eqn. 7.7, which will give six different $D/S$ ratios that ideally should agree with each other, at least for partial waves with non-negligible intensities. Ideally, these two approaches will give consistent results.

To normalize the probability density, we need to compute the integral $\int \mathcal{I}(x; \theta)\eta(x)dx$, which is impossible analytically. However, we can exploit the fact that the integral of a
function \( f(x) \) can be written in terms of its average value as

\[
\int_R f(x) dx = R \langle f(x) \rangle.
\] (7.9)

The average value \( \langle f(x) \rangle \) is acquired by random sampling of the function \( f(x) \) over the region \( R \), so we can integrate the intensity by generating a Monte Carlo sample with a uniform distribution in the multi-dimensional phaspace that spans the domain of the intensity function. For a sample size of \( M_g \) generated phasespace Monte Carlo events, the average value can be computed as

\[
\langle I(x; \theta) \eta(x) \rangle = \frac{1}{M_g} \sum_{i=1}^{M_g} I(x_i; \theta),
\] (7.10)

where \( M_a \) is the size of the accepted phasespace Monte Carlo sample \( M_g \) after being sent through the reconstruction and event selection process.

### 7.2 Phasespace Monte Carlo

To compute the numerical integrals described above, we generated samples of phasespace MC and ran them through the GlueX reconstruction software, with and without acceptance effects. The relative sizes of the phasespace samples were chosen to approximate the proportions of the three GlueX-I run periods. The sizes are given in Table 7.1.

<table>
<thead>
<tr>
<th>Run Period</th>
<th>2017_01</th>
<th>2018_01</th>
<th>2018_08</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Events Generated</td>
<td>50M</td>
<td>150M</td>
<td>100M</td>
<td>300M</td>
</tr>
</tbody>
</table>

TABLE 7.1: Summary of phasespace MC generated for analysis of the \( \omega \pi^- \) channel produced during GlueX Phase I.

These samples were generated with a uniform \( \omega \pi^- \) mass distribution (in the range 1 - 2 GeV) and uniform distributions in the angles \( \cos \theta, \phi, \cos \theta_H, \phi_H, \) and \( \Phi \). These
angles are defined and discussed in Sections 2.2 and 2.1. Plots of the decay angles are shown for the accepted phasespace in Fig. 7.1. The plots show a decrease in efficiency at very forward polar angles, $\theta$, which is to be expected since there's a hole in the center of the detectors where the beamline passes through. The phasespace samples were generated with a $t$-slope of 3 GeV$^{-2}$, to roughly match the data.

AmpTools takes as input both the generated phasespace MC without acceptance effects, or $M_g$ in Eqn. 7.10, and the accepted phasespace MC that has been sent through reconstruction, or $M_a$ in Eqn. 7.10, the intensity of which is summed over.

### 7.3 Modeling Mass Dependence

We have three options to model the mass dependence of the contributing waves to the $\omega \pi^-$ final state. We can perform a mass dependent fit, so called because the amplitude model used in the fit has an encoded mass dependence. We can also perform a mass independent fit, where there is no mass dependence in the amplitude model, or we can
combine the two approaches and perform a piecewise fit. Each type of mass dependence is useful for a different purpose. The three approaches are described below.

7.3.1 Mass Dependent Fit

In a mass dependent fit, the full $\omega\pi^-$ mass range is fit at once, and each partial wave amplitude used in the fit function is multiplied by a relativistic Breit-Wigner amplitude. We used results from a mass dependent fit to roughly approximate the GlueX data, in order to generate signal Monte Carlo. Since we know the exact model and waveset that was used to generate this signal Monte Carlo, we can use it to test the accuracy of our fits. The diagnostic plots for the mass dependent fit, which was performed in the $\omega\pi^-$ mass range of $1.0 - 1.4$ GeV, are shown in Fig. 7.2. These plots show how well the fits describe the intensity as functions of $\omega\pi^-$ mass and the observables $x$. The $\omega\pi^-$ mass is described well by this model (up to roughly 1.35 GeV), as are the angles $\phi$, $\cos\theta_H$, $\phi_H$, and $\Phi$. The $\cos\theta$ distribution is not described well by this fit, probably due to baryonic background that is not included in our model (see Section 8.1.1), but the goal of this fit is to obtain a roughly realistic description of the GlueX data that we can use to test our mass independent and piecewise fits.

In this mass dependent fit, the $J^P = 1^+$ partial wave is modeled using the PDG values for the $b_1(1235)$ mass and width, and the $J^P = 1^-$ partial wave is modeled with the PDG values for an excited $\rho^*(1450)$, described in Table 7.2. Isotropic background is included in the mass dependent fit, but was removed when generating the signal MC. Initial fits of this type produced an unrealistic value for the ratio of $D$-wave to $S$-wave in $b_1$ decay of $D/S = 0.047$, so the value in the signal MC was fixed to the expected value of $D/S = 0.27$. Further background and discussion on the $D/S$ ratio can be found in Sections 1.2.2, 2.1, and 3.2.1.
FIG. 7.2: Diagnostic plots from mass dependent fit to GlueX data. The plots are, left to right: Top row: $\omega\pi^-$ invariant mass, $\cos \theta$, $\phi$. Middle row: $\cos \theta_H$, $\phi_H$, $\Phi$. Bottom row: $p\pi^+$ invariant mass, $p\pi^-$ invariant mass, $p\pi^+\pi^-$ invariant mass. The black points represent the total intensity of the GlueX data, the green is the weighted phasespace Monte Carlo, the blue is the contribution from $J^P = 1^+$, and the red is the contribution from $J^P = 1^+$. 
TABLE 7.2: Model parameters \[4\] for mass dependent fit to GlueX data utilized to generate signal Monte Carlo.

7.3.2 Mass Independent Fit

We would prefer to perform our fits in as model independent a manner as possible. We attempt this by splitting the data into as many bins of $\omega\pi^-$ mass as statistics will allow and fitting each bin individually, without including a mass dependence in the fit model. In principle, this will allow us to extract the contributing partial waves without making any prior assumptions about properties such as the resonance masses and decay widths for possible $X^-$ states decaying to $\omega\pi^-$. In practice, mass independent fits to signal MC are stable when including waves up to $J = 1$, but wavesets including higher spin states are unable to correctly describe signal Monte Carlo (see discussion accompanying Fig. 8.15), so these states are not included in this analysis. This analysis is focused on the vector and axial vector contributions to the $\omega\pi^-$ final state, with an emphasis on the properties of the axial vector $b_1(1235)$.

7.3.3 Piecewise “Hybrid” Fit

A novel approach combining the stability of the mass dependent fit with the model independence of the mass independent fit is under development at GlueX. In this approach, we exploit the fact that the axial vector $b_1$ decaying to $\omega\pi$ is well-studied, and model it with a relativistic Breit-Wigner amplitude. The vector contribution is simultaneously fit mass independently along the same $\omega\pi^-$ mass range, with unique complex parameters for each mass bin. This approach produces stable results (see Section 8.2), but is computationally expensive and is most practical in the $b_1$ peak mass range, which is approximately $1.10 -$
1.35 GeV.
CHAPTER 8

Results and Discussion

8.1 Results of Mass Independent Fits

We performed mass independent fits to the GlueX data taken in the $0^\circ$ orientation (with the beam polarization parallel to the horizontal), and the signal Monte Carlo discussed in Section 7.3.1. The fit is performed in 14 mass bins with a width of 50 MeV, over the $\omega\pi^-$ mass range $1.0-1.7$ GeV. Fits are performed with the intensity defined by Eqn. 7.2. These fits are performed for wavesets including various combinations of $J^P$ waves, as well as isotropic background, described in Table 8.1. An isotropic background is an amplitude which is uniform in the observable production and decay angles and is added incoherently to the intensity function. The number of degrees of freedom in the fit is given in Eqn. 8.1 by $2J_i + 1$ $m$-projections for each wave $i$, times two reflectivities, times two to account for the real and imaginary components of each amplitude, times the number of $L$-waves, minus the two unconstrained phases in the intensity coherent sums that are fixed to zero.

$$NDF = \sum_i (2J_i + 1) \times 2 \times 2 \times (#L_i) - 2 \quad (8.1)$$
### 8.1.1 Example Fit to GlueX Data and Signal Monte Carlo

An example fit with waveset $1p1m$ is shown in Fig. 8.1, where the fit is able to match the signal Monte Carlo very well, but for the GlueX data, the coherent sum of $1^-$ waves shown in the red points don’t form a continuous distribution, which we would expect from a resonant or non-resonant $1^-$. However, the enhancement that E852 saw in the mass range above the $b_1$ was measured to be a $\rho_3$, with $J^{PC} = 3^{--}$, so it’s unsurprising that our $1p1m$ model is unable to fit the data perfectly over the full mass range. The fit fractions for this fit are shown in Table 8.2, for the $\omega\pi^-$ mass range 1.20 – 1.25 GeV as an example.

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$P$</th>
<th>$(%)$</th>
<th>$S$</th>
<th>$(%)$</th>
<th>$P$</th>
<th>$(%)$</th>
<th>$S$</th>
<th>$(%)$</th>
<th>$P$</th>
<th>$(%)$</th>
<th>$S$</th>
<th>$(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1p1m</td>
<td>+1</td>
<td>0.5 ± 0.5</td>
<td>13.4 ± 1.0</td>
<td>3.2 ± 2.0</td>
<td>0.4 ± 0.5</td>
<td>11.8 ± 0.9</td>
<td>2.8 ± 1.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>28.7 ± 1.7</td>
<td>3.0 ± 0.9</td>
<td>0.2 ± 0.3</td>
<td>25.3 ± 1.5</td>
<td>2.6 ± 0.8</td>
<td>0.1 ± 0.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1p1m</td>
<td>+1</td>
<td>1.4 ± 0.6</td>
<td>28.3 ± 0.7</td>
<td>3.4 ± 0.8</td>
<td>0.13 ± 0.04</td>
<td>2.5 ± 0.7</td>
<td>0.3 ± 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>49.6 ± 1.7</td>
<td>1.2 ± 0.4</td>
<td>1.1 ± 1.3</td>
<td>4.3 ± 1.1</td>
<td>0.11 ± 0.04</td>
<td>0.1 ± 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 8.2:** Fit fractions for partial waves from mass independent fits to GlueX data and MC, performed in the $\omega\pi^-$ mass bin 1.20 – 1.25 GeV.

The same fit is performed 25 times in each bin, using randomized starting values for the production parameters. The best fit value is the one with the smallest $-2\ln\mathcal{L}$, and the multiple fits are shown in Fig. 8.2, where each of the 25 fits is represented by a point within the relevant mass bin. The results are fairly consistent in both GlueX data and
signal Monte Carlo, however some bins show evidence for multiple minima with clusters of solutions at different intensities on the $y$-axis.

The diagnostic plots from one of the mass bins of the $1p1m$ fit are shown in Fig. 8.3 for GlueX data, performed in the $\omega\pi^-$ mass bin $1.20 - 1.25$ GeV. The diagnostic plots show the $\omega\pi^-$ invariant mass distribution, the five observable production and decay angles $\cos \theta$, $\phi$, $\cos \theta_H$, $\phi_H$, and $\Phi$, and invariant mass plots of the recoil $p\pi^+$, the proton plus the bachelor $\pi^-$, and the recoil plus the bachelor $\pi^-$. The black points in these plots represent the GlueX data or the signal Monte Carlo, and should be compared to the shaded green area, which is the phasespace Monte Carlo weighted by the results of the fit. The blue points represent the contribution to the fit from the coherent sum of $1^+$ partial waves, and the red points represent the contribution to the fit from the coherent sum of $1^-$ partial waves.

The $\omega\pi^-$ invariant mass distribution is assumed to be flat in each given mass bin, since the bins are relatively narrow, but it is not exactly flat. This bin size was chosen to
FIG. 8.2: Results of 25 fits with randomized starting parameters to GlueX data (left) and MC (right) with waveset 1p1m. The black points indicate total intensity in each mass bin, the blue points are the contribution to the intensity from all $J^P = 1^+$ partial waves, and the red points are the contribution from all $J^P = 1^-$ partial waves. The colored points are hollow, and offset on the $x$-axis for clarity.

reduce statistical error for low intensity mass bins, but it would be reasonable to try 25 MeV bins and see if that improves the stability.

The distribution of $\cos \theta$ is described well by this fit in the $b_1$ peak, but this is not the case for higher $\omega \pi^-$ mass ranges. For example, as we’ll see in Section 8.2, Fig. 8.20 is the diagnostic plot for one of the piecewise fits to the GlueX data, and the $\cos \theta$ distribution doesn’t match between weighted phasespace (green) and GlueX data (black) at very forward and backward angles. There is a peak in the forward angle which may indicate some contribution from baryonic background, such as $\gamma p \to \omega \Delta^+(1600)$, where $\Delta^+(1600) \to \Delta(1232)\pi \to p\pi^+\pi^-$, at higher values of $\omega \pi^-$ mass, and this is further indicated by the peak in the $p\pi^+\pi^-$ invariant mass plot near 1.6 GeV (bottom right panel in Fig. 8.20). This behavior is more prominent in the $1.35 - 1.40$ GeV mass bin of the mass independent fit to GlueX data, shown in Fig. 8.5. This behavior is not modeled in the
FIG. 8.3: Diagnostic plots from mass independent fit to GlueX data for the $\omega \pi^-$ mass range 1.20 – 1.25 GeV. The plots are, left to right: Top row: $\omega \pi^-$ invariant mass, $\cos \theta$, $\phi$. Middle row: $\cos \theta_H$, $\phi_H$, $\Phi$. Bottom row: $p\pi^+$ invariant mass, $p\pi^-$ invariant mass, $p\pi^+\pi^-\pi^0$ invariant mass. The black points represent the GlueX data, the green is the weighted phasespace Monte Carlo, the blue is the contribution from $J^P = 1^+$, and the red is the contribution from $J^P = 1^+$. The signal Monte Carlo, shown in Fig. 8.6. Thus, the $\omega \pi^-$ mass range chosen for our piecewise fits is 1.10 – 1.35 GeV, or the $b_1$ peak region.

The $\phi$ distribution is fairly flat, which is consistent with the expectation that the $\omega \pi^-$ production has no azimuthal dependence. There is a slight dip around $\phi = 0$, which can be attributed to detector acceptance, as shown in Fig. 7.1.

The $\cos \theta_H$ and $\phi_H$ distributions describe the decay of the $\omega$ meson to $\pi^+\pi^-\pi^0$. The angle $\cos \theta_H$ is the angle that is most sensitive to the $D/S$ ratio of the $b_1$. The intensity appears to have a $\cos 2\phi_H$ dependence, part of which can be attributed to detector acceptance, again as shown in Fig. 7.1.

The intensity has a dependence on the production angle of $\cos 2\Phi$, which is what we
FIG. 8.4: Diagnostic plots from mass independent fit to signal Monte Carlo for the \( \omega \pi^- \) mass range 1.20–1.25 GeV. The plots are, left to right: Top row: \( \omega \pi^- \) invariant mass, \( \cos \theta \), \( \phi \). Middle row: \( \cos \theta_H \), \( \phi_H \), \( \Phi \). Bottom row: \( p\pi^+ \) invariant mass, \( p\pi^- \) invariant mass, \( p\pi^+\pi^- \) invariant mass. The black points represent the signal Monte Carlo, the green is the weighted phasespace Monte Carlo, the blue is the contribution from \( J^P = 1^+ \), and the red is the contribution from \( J^P = 1^+ \).

would expect for linearly polarized photoproduction. If we were to analyze a dataset at another polarization angle, we would expect to see a similar distribution, with a phase shift dependent on the change in polarization angle.

8.1.2 Waveset Selection Through Likelihood Differences

When optimizing which waveset to use for the final fit results, it is useful to have a mathematical quantity to make meaningful comparisons between wavesets. Since our fitting program minimizes \(-2\ln(L)\), written out in Eqn. 7.6, comparing negative log likelihoods seems like an obvious choice. It’s not as easy as looking at the difference in \(-2\ln(L)\)
between wavesets, since the number of degrees of freedom in the fit also play a role. For example, a fit with more degrees of freedom may produce a smaller $-2\ln(L)$, but may be “overfit” such that this change in likelihood is not significant.

What we compare in practice is $-2\Delta \ln(L)$, which corresponds to a $\chi^2$ as described in Section 40.4.2.2 and Table 40.2 of Ref. [4], in terms of the number of degrees of freedom. If we observe a significant change in $-2 \ln(L)/NDF$, then adding those degrees of freedom to the fit is an improvement in the model. For example, in Fig. 8.7, the change in $-2 \ln(L)$ is plotted for the change in waveset from $1p$ to $1p1m$, an increase of 12 degrees of freedom. In the $b_1$ mass range from roughly $1.10 - 1.35$ GeV, the change in $2 \ln(L)$ is around 180 in the GlueX data and 300 in the Monte Carlo, corresponding to a change in $\chi^2/NDF$ of
FIG. 8.6: Diagnostic plots from mass independent fit to signal Monte Carlo for the \(\omega\pi^-\) mass range 1.35--1.40 GeV. The plots are, left to right: Top row: \(\omega\pi^-\) invariant mass, \(\cos \theta\), \(\phi\). Middle row: \(\cos \theta_H\), \(\phi_H\), \(\Phi\). Bottom row: \(p\pi^+\) invariant mass, \(p\pi^-\) invariant mass, \(p\pi^+\pi^-\) invariant mass. The black points represent the signal Monte Carlo, the green is the weighted phase space Monte Carlo, the blue is the contribution from \(J^P = 1^+\), and the red is the contribution from \(J^P = 1^+\).

15 and 25, respectively. This is consistent with the waveset used to generate the Monte Carlo, and with previous observations that the \(1^+\) is not the only amplitude that decays to \(\omega\pi^-\).

The change in \(-2\ln(\mathcal{L})\) when increasing the waveset from \(1m\) to \(1p1m\), adding 14 degrees of freedom, is plotted in Fig. 8.8, where we see that the change in \(-2\ln(\mathcal{L})\) in the \(b_1\) mass range is roughly 900 in the GlueX data and 2500 in the Monte Carlo, corresponding to a change in \(\chi^2/NDF\) of 64 and 180, respectively. Since the Monte Carlo sample was generated using predominantly \(b_1\) contribution, and previous measurements have shown strong \(b_1\) contribution to \(\omega\pi\), this is consistent with our expectations, especially if we look
FIG. 8.7: Difference in $2\ln(L)$ between the fit wavesets $1p$ and $1p1m$, in GlueX data (left), and MC (right). The increase in $NDF$ from $1p$ to $1p1m$ is 12.

at the Monte Carlo in the $b_1$ peak range. We choose to adopt the $1p1m$ waveset over $1p$ or $1m$ alone.

FIG. 8.8: Difference in $2\ln(L)$ between the fit wavesets $1m$ and $1p1m$, in GlueX data (left), and MC (right). The increase in $NDF$ from $1m$ to $1p1m$ is 14.

The mass independent fits using the $1p$ and $1m$ wavesets are not shown here, as the entire intensity is taken up by the respective $J^P$ wave, so the plots are trivial. The fits with waveset $1p1m$ were shown and discussed in Section 8.1.1.

Figure 8.9 shows a similar fit to Fig. 8.1, but in this case, the $D/S$ ratio is not required to be consistent across reflectivities and spin projections. This adds 8 degrees of
freedom to the fit, and corresponds to a change in $\chi^2/NDF \approx 160/8 \approx 20$, as illustrated in Fig. 8.10 in the data around the $b_1$ mass range, and $\chi^2/NDF \approx 20/8 \approx 2.5$ in the Monte Carlo. However, the fit fractions obtained from this fit, shown in Table 8.3, give a $D$-wave that is sometimes twice as strong as the $S$-wave, which is unexpected, based on previous measurements, and inconsistent with the constrained case shown in Table 8.2. We opt not to adopt this waveset, as the freedom introduced by releasing $D/S$ appears to lead to overfitting and inconsistent $D/S$ ratios between waves with different spin projections and reflectivities.

<table>
<thead>
<tr>
<th>$J_m^P L$</th>
<th>Refl</th>
<th>$1^+_1 S$ (%)</th>
<th>$1^+_0 S$ (%)</th>
<th>$1^+_1 S$ (%)</th>
<th>$1^+_1 D$ (%)</th>
<th>$1^+_0 D$ (%)</th>
<th>$1^+_1 D$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1p1mdfree, Data</td>
<td>+1</td>
<td>0.7 ± 0.5</td>
<td>6.9 ± 2.3</td>
<td>1.7 ± 1.1</td>
<td>5.4 ± 1.8</td>
<td>10.3 ± 2.3</td>
<td>5.3 ± 2.5</td>
</tr>
<tr>
<td>-1</td>
<td>11.1 ± 2.2</td>
<td>2.7 ± 0.9</td>
<td>4.1 ± 2.1</td>
<td>26.6 ± 2.8</td>
<td>7.9 ± 2.3</td>
<td>0.3 ± 0.7</td>
<td></td>
</tr>
<tr>
<td>1p1mdfree, MC</td>
<td>+1</td>
<td>1.5 ± 0.5</td>
<td>27.3 ± 0.9</td>
<td>2.6 ± 0.8</td>
<td>0.8 ± 0.4</td>
<td>4.6 ± 0.7</td>
<td>0.4 ± 0.3</td>
</tr>
<tr>
<td>-1</td>
<td>48.2 ± 1.5</td>
<td>1.2 ± 0.4</td>
<td>1.4 ± 1.5</td>
<td>3.3 ± 0.6</td>
<td>0.1 ± 0.1</td>
<td>0.2 ± 0.2</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 8.3: Fit fractions for partial waves from mass independent fits to GlueX data and MC, for the $1^+[D/S]$ ratio unconstrained, performed in the $\omega\pi^-$ mass bin 1.20 – 1.25 GeV.

Figure 8.11 shows the fit using the waveset with a vector, axial vector, and isotropic background (1p1miso), and the difference in likelihood between that and 1p1m is shown in Fig. 8.12. One degree of freedom is added by including the isotropic background, corresponding to a change in $\chi^2/NDF \approx 90$ in the data and 0 in the Monte Carlo. We do not adopt this waveset either, as the inclusion of the isotropic background takes over the intensity in the $b_1$ peak region, which is physically unrealistic.

Figure 8.13 shows the fit using the waveset 0m1p1m, and the difference in likelihood between that and 1p1m is shown in Fig. 8.14. Four degrees of freedom are added when adding $0^-$ to the waveset, which corresponds to a change of $\chi^2/NDF \approx 50/4 \approx 13$ in GlueX data, and $\chi^2/NDF \approx 4/4 \approx 1$ in the Monte Carlo. The $0^-$ wave could conceivably be a reasonable addition to our waveset, but further study, such as generating new Monte Carlo with a $0^-$ resonance in its waveset to see if we can reliably extract it, is necessary.
FIG. 8.9: Results of fit to GlueX data (left) and MC (right) with waveset 1p1mdfree. The black curve indicates total intensity in each mass bin, the blue points are the contribution to the intensity from all $J^P = 1^+$ partial waves, and the red points are the contribution from all $J^P = 1^-$ partial waves.

Figure 8.15 shows the fit using the waveset 1p1m2m, and the difference in likelihood between that and 1p1m is shown in Fig. 8.16. The addition of the $2^-$ wave adds 40 degrees of freedom to the fit, corresponding to a change of $\chi^2/NDF \approx 350/40 \approx 9$ in the data, and $\chi^2/NDF \approx 100/40 \approx 2.5$ in the Monte Carlo. The inclusion of the $2^-$ wave prevents the fit from returning the correct amplitudes, as is evident from the fit to Monte Carlo in Fig. 8.15. Note that the $2^-$ wave was not included in the Monte Carlo generation, but the dominant contribution in and below the $b_1$ peak is described in the fit by the $2^-$ wave, completely inconsistent with what we generated.

The fit waveset we opt to use for our piecewise fits is 1p1m, since in our studies, none of the improvements in likelihood achieved by increasing the waveset were worth the additional degrees of freedom, which often resulted in overfitting. The axial vector contribution to the $\omega\pi$ channel is well-documented, and is the focus of our study. The vector contribution has been seen in $e^+e^- \rightarrow \omega\pi$ decay, and is therefore a reasonable
FIG. 8.10: Difference in $2 \ln(L)$ between the fit wavesets $1p1m$ and $1p1mdfree$, in GlueX data (left), and MC (right). The increase in $NDF$ from $1p1m$ to $1p1mdfree$ is 8.

candidate for the background contribution.
FIG. 8.11: Results of fit to GlueX data (left) and MC (right) with waveset 1p1miso. The black curve indicates total intensity in each mass bin, the blue points are the contribution to the intensity from all $J^P = 1^+$ partial waves, the red points are the contribution from all $J^P = 1^-$ partial waves, and the gray points are the contribution from isotropic background.

FIG. 8.12: Difference in $2\ln(\mathcal{L})$ between the fit wavesets 1p1m and 1p1miso, in GlueX data (left), and MC (right). The increase in $NDF$ from 1p1m to 1p1miso is 1.
FIG. 8.13: Results of fit to GlueX data (left) and MC (right) with waveset $0m1p1m$. The black curve indicates total intensity in each mass bin, the blue points are the contribution to the intensity from all $J^P = 1^+$ partial waves, the red points are the contribution from all $J^P = 1^-$ partial waves, and the magenta points are the contribution from all $J^P = 0^-$ partial waves.

FIG. 8.14: Difference in $2\ln(L)$ between the fit wavesets $1p1m$ and $0m1p1m$, in GlueX data (left), and MC (right). The increase in $NDF$ from $1p1m$ to $0m1p1m$ is 4.
FIG. 8.15: Results of fit to GlueX data (left) and MC (right) with waveset 1p1m2m. The black curve indicates total intensity in each mass bin, the blue points are the contribution to the intensity from all $J^P = 1^+$ partial waves, the red points are the contribution from all $J^P = 1^-$ partial waves, and the green points are the contribution from all $J^P = 2^-$ partial waves.

FIG. 8.16: Difference in $2\ln(L)$ between the fit wavesets 1p1m and 1p1m2m, in GlueX data (left), and MC (right). The increase in $NDF$ from 1p1m to 1p1m2m is 40.
8.2 Results of Piecewise Fits

We performed piecewise fits, as introduced in Section 7.3.3, to the GlueX data in two beam polarization orientations, $0^\circ$ and $90^\circ$ (the other two were left for future work due to time constraints), and signal Monte Carlo, using a fit waveset modeling the $J^P = 1^+$ as a Breit-Wigner amplitude, whose mass and width are fixed to the $b_1$ PDG values of 1.235 GeV and 0.142 GeV, respectively. The $J^P = 1^-$ wave is modeled using five independent piecewise bins of width 50 MeV each. The fit is performed in the $\omega\pi^-$ mass range where the $b_1$ has been previously measured to be prominent, $1.10 - 1.35$ GeV.

<table>
<thead>
<tr>
<th>$J^P_{mL}$</th>
<th>Refl</th>
<th>$1^+_0 S$ (%)</th>
<th>$1^+_1 S$ (%)</th>
<th>$1^+_0 D$ (%)</th>
<th>$1^+_1 D$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>+1</td>
<td>0.8 ± 0.3</td>
<td>0.9 ± 0.3</td>
<td>1.7 ± 0.3</td>
<td>0.4 ± 0.1</td>
</tr>
<tr>
<td>Data</td>
<td>-1</td>
<td>37.8 ± 1.2</td>
<td>4.7 ± 0.8</td>
<td>3.0 ± 0.5</td>
<td>0.12 ± 0.04</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>+1</td>
<td>1.2 ± 0.5</td>
<td>2.2 ± 0.8</td>
<td>0.6 ± 0.1</td>
<td>0.03 ± 0.01</td>
</tr>
<tr>
<td>Data</td>
<td>-1</td>
<td>37.4 ± 1.7</td>
<td>4.4 ± 1.2</td>
<td>0.3 ± 0.1</td>
<td>0.06 ± 0.02</td>
</tr>
<tr>
<td>Signal</td>
<td>+1</td>
<td>1.4 ± 0.2</td>
<td>2.6 ± 0.4</td>
<td>1.7 ± 0.1</td>
<td>0.16 ± 0.02</td>
</tr>
<tr>
<td>MC</td>
<td>-1</td>
<td>50.4 ± 0.7</td>
<td>1.5 ± 0.2</td>
<td>3.0 ± 0.1</td>
<td>0.09 ± 0.01</td>
</tr>
</tbody>
</table>

TABLE 8.4: Fit fractions for partial waves from piecewise fits to GlueX data and MC, in the $-t$ range $0.15 - 0.30$ GeV$^2$.

<table>
<thead>
<tr>
<th>$J^P_{mL}$</th>
<th>Refl</th>
<th>$1^+_{-1} S$ (%)</th>
<th>$1^+_{0} S$ (%)</th>
<th>$1^+_{-1} D$ (%)</th>
<th>$1^+_{0} D$ (%)</th>
<th>$1^+_{-1} D$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>+1</td>
<td>0.30 ± 0.01</td>
<td>0.30 ± 0.01</td>
<td>1.7 ± 0.2</td>
<td>0.31 ± 0.04</td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>-1</td>
<td>10.4 ± 0.1</td>
<td>2.77 ± 0.04</td>
<td>0.7 ± 0.1</td>
<td>0.19 ± 0.02</td>
<td></td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>+1</td>
<td>1.1 ± 0.4</td>
<td>6.2 ± 0.8</td>
<td>0.9 ± 0.2</td>
<td>0.2 ± 0.1</td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>-1</td>
<td>20.0 ± 1.6</td>
<td>6.2 ± 0.8</td>
<td>0.7 ± 0.1</td>
<td>0.08 ± 0.04</td>
<td></td>
</tr>
<tr>
<td>Signal</td>
<td>+1</td>
<td>1.3 ± 0.2</td>
<td>2.2 ± 0.3</td>
<td>1.9 ± 0.1</td>
<td>0.14 ± 0.02</td>
<td></td>
</tr>
<tr>
<td>MC</td>
<td>-1</td>
<td>51.2 ± 0.6</td>
<td>0.07 ± 0.02</td>
<td>0.14 ± 0.02</td>
<td>0.005 ± 0.001</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 8.5: Fit fractions for partial waves from piecewise fits to GlueX data and MC, in the $-t$ range $0.30 - 0.50$ GeV$^2$.

The fit fractions extracted from these piecewise fits are shown in Tables 8.4 and 8.5. It’s clear that the prominent contributors to the $b^-_1$ are the waves with $m = +1$ spin projection in negative reflectivity, and $m = 0$ spin projection in positive reflectivity, in the $-t$ range $0.15 - 0.30$ GeV$^2$. In the higher $-t$ range of $0.30 - 0.50$ GeV$^2$, the negative...
reflectivity contribution to the $b_1$ is diminished in both sets of GlueX data, while the positive reflectivity contribution is slightly greater.

Partial wave intensities for the lower $-t$ range are plotted in Figs. 8.17 and 8.18 for GlueX data and Fig. 8.19 for signal Monte Carlo. The fit results (green) in each plot are the intensities for each individual partial wave, therefore the sum of the partial waves in all plots should match the total intensity (black) in each. The corresponding partial wave intensity and diagnostic plots for the higher $-t$ range are included in Appendix B.

Plots of the diagnostic histograms for the low $-t$ fits are shown in Figs. 8.20, 8.21, and 8.22. The piecewise fits describe the signal Monte Carlo well, but have trouble with the $\cos\theta$ distribution for the GlueX data. This is likely due to some smaller waves that could not be included in the fit, or possibly baryonic background, such as $\gamma p \to \omega\pi^+\Delta^0$ or $\gamma p \to \omega\Delta^+(1600)$.

The nominal $D/S$ ratios extracted from these piecewise fits are given in Table 8.6. Note that the errors come from the covariance matrix in the fitter used in AmpTools (MINUIT) and are likely underestimated. A better description of the errors should come from methods like bootstrapping [27] or, as we will use in Section 8.3, parameter scans. The 90 PERP fit in the lowest $-t$ bin is returning results from a false minimum, which is one reason why parameter scans are so important.

<table>
<thead>
<tr>
<th>Low $-t$</th>
<th>$D/S$ Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data, 0 PARA</td>
<td>0.300 ± 0.026</td>
</tr>
<tr>
<td>Data, 90 PERP</td>
<td>0.133 ± 0.011</td>
</tr>
<tr>
<td>Signal MC</td>
<td>0.261 ± 0.004</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High $-t$</th>
<th>$D/S$ Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data, 0 PARA</td>
<td>0.279 ± 0.019</td>
</tr>
<tr>
<td>Data, 90 PERP</td>
<td>0.193 ± 0.019</td>
</tr>
<tr>
<td>Signal MC</td>
<td>0.277 ± 0.008</td>
</tr>
</tbody>
</table>

**TABLE 8.6:** Nominal ratio of $D/S$ wave in $b_1^- \to \omega\pi^-$ decay extracted from piecewise fits. Upper rows are the results from the $-t$ range $0.15 - 0.30$ GeV$^2$, and lower rows are the results from the $-t$ range $0.30 - 0.50$ GeV$^2$. 
8.2.1 Reflectivity Contributions

In the lower $-t$ range, $0.15 - 0.30$ GeV$^2$, the partial waves that contributed most strongly in the $b^-_1$ decay are the positive reflectivity $m = 0$, and the negative reflectivity $m = +1$, with the negative reflectivity wave roughly twice as strong as the positive. This indicates that the $b^-_1$ is produced predominantly by unnatural reflectivity particle exchange, such as the $\pi^-$, with a smaller contribution from a natural exchange such as a $\rho^-$. The beam asymmetry measurement of the charged exchange reaction $\gamma p \rightarrow \pi^- \Delta^{++}$ discussed in Section 1.3 also saw a preference for unnatural exchange at low values of $-t$. As we increase the $-t$ range to $0.30 - 0.50$ GeV$^2$, the contribution from negative reflectivity significantly decreases, indicating that as four-momentum transfer increases, the $b^-_1$ is produced more and more dominantly through natural particle exchange, at least up to $0.5$ GeV$^2$. For further study, finer binning in $-t$ could provide a more detailed description of the dependence of the exchange naturality on $-t$. Our main limitation in a study like this one would be statistical.
FIG. 8.17: Partial waves extracted from piecewise fit to 0 PARA GlueX data. The top row is the $1^+ S$ wave, middle row is $1^+ D$ wave, and bottom row is the $1^- P$ wave. The columns are, from left to right, spin projection $m = +1, 0, -1$. The black points represent the total intensity of the GlueX data, the green is the weighted phasespace MC for each partial wave, the blue is the contribution from negative reflectivity, and the red is the contribution from positive reflectivity.
FIG. 8.18: Partial waves extracted from piecewise fit to 90 PERP GlueX data. The top row is the $1^+ S$ wave, middle row is $1^+ D$ wave, and bottom row is the $1^- P$ wave. The columns are, from left to right, spin projection $m = +1, 0, -1$. The black points represent the total intensity of the GlueX data, the green is the weighted phasespace MC for each partial wave, the blue is the contribution from negative reflectivity, and the red is the contribution from positive reflectivity.
FIG. 8.19: Partial waves extracted from piecewise fit to signal Monte Carlo. The top row is the $1^+S$ wave, middle row is $1^+D$ wave, and bottom row is the $1^-P$ wave. The columns are, from left to right, spin projection $m = +1, 0, -1$. The black points represent the total intensity of the signal Monte Carlo, the green is the weighted phasespace Monte Carlo for each partial wave, the blue is the contribution from negative reflectivity, and the red is the contribution from positive reflectivity.
FIG. 8.20: Diagnostic plots from piecewise fit to 0 PARA GlueX data. The plots are, left to right: Top row: $\omega\pi^-$ invariant mass, $\cos \theta$, $\phi$. Middle row: $\cos \theta_H$, $\phi_H$, $\Phi$. Bottom row: $p\pi^+$ invariant mass, $p\pi^-$ invariant mass, $p\pi^+\pi^-$ invariant mass. The black points represent the total intensity of the GlueX data, the green is the weighted phasespace Monte Carlo for each partial wave, the blue is the contribution from $J^P = 1^+$, and the red is the integrated contribution from $J^P = 1^+$. 
FIG. 8.21: Diagnostic plots from piecewise fit to 90 PERP GlueX data. The plots are, left to right: Top row: $\omega\pi^-$ invariant mass, $\cos \theta$, $\phi$. Middle row: $\cos \theta_H$, $\phi_H$, $\Phi$. Bottom row: $p\pi^+$ invariant mass, $p\pi^-$ invariant mass, $p\pi^+\pi^-$ invariant mass. The black points represent the total intensity of the GlueX data, the green is the weighted phasespace Monte Carlo for each partial wave, the blue is the contribution from $J^P = 1^+$, and the red is the integrated contribution from $J^P = 1^+$. 
FIG. 8.22: Diagnostic plots from piecewise fit to signal Monte Carlo. The plots are, left to right: Top row: $\omega\pi^-$ invariant mass, $\cos \theta$, $\phi$. Middle row: $\cos \theta_H$, $\phi_H$, $\Phi$. Bottom row: $p\pi^+$ invariant mass, $p\pi^-$ invariant mass, $p\pi^+\pi^-$ invariant mass. The black points represent the total intensity of the signal Monte Carlo, the green is the weighted phasespace Monte Carlo for each partial wave, the blue is the contribution from $J^P = 1^+$, and the red is the integrated contribution from $J^P = 1^+$. 
8.3 $D/S$ Ratio of the $b_1^-(1235)$ Meson

We use the resulting amplitude parameters from the piecewise fits described in Section 8.2 to perform a likelihood scan of the $D/S$ ratio. In this procedure, we hold the $D/S$ ratio fixed at values across the range from $0.1 - 0.9$, with step sizes of 0.02, using the results from the piecewise fit as starting parameters for each fit in the scan, where all the other parameters in the model are allowed to float. The difference in $-2\ln(\mathcal{L})$ from the interpolated minimum of these scans are plotted against the fixed $D/S$ values in Fig. 8.23, where spline interpolation is utilized to precisely determine the minimum and error, shown in Table 8.7. The error on the value is given by a change of one unit in $-2\Delta\ln(\mathcal{L})$, which corresponds to an uncertainty of $1\sigma$ (see Section 40.4.2.2 of Ref. [4]).

<table>
<thead>
<tr>
<th>Low $-t$</th>
<th>$D/S$ Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data, 0 PARA</td>
<td>$0.318^{+0.033}_{-0.035}$</td>
</tr>
<tr>
<td>Data, 90 PERP</td>
<td>$0.277^{+0.008}_{-0.007}$</td>
</tr>
<tr>
<td>Signal MC</td>
<td>$0.284 \pm 0.007$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High $-t$</th>
<th>$D/S$ Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data, 0 PARA</td>
<td>$0.275^{+0.026}_{-0.025}$</td>
</tr>
<tr>
<td>Data, 90 PERP</td>
<td>Not Reliable</td>
</tr>
<tr>
<td>Signal MC</td>
<td>$0.286^{+0.009}_{-0.011}$</td>
</tr>
</tbody>
</table>

TABLE 8.7: Ratio of $D/S$ wave in $b_1^- \to \omega\pi^-$ decay extracted using parameter scan. Upper rows show results for the $-t$ bin from $0.15 - 0.30$ GeV$^2$, lower rows are results for the $-t$ bin from $0.30 - 0.50$ GeV$^2$.

The values extracted from the Monte Carlo in both $-t$ ranges are consistent within $2\sigma$ with the generated value of 0.27. A likelihood scan with finer granularity may determine these better. In the $0^\circ$ orientation, the values extracted from the GlueX data are consistent with previous measurements [7]. In the $90^\circ$ orientation, the fit to the lower $-t$ dataset, shown in the center left panel of Fig. 8.23, shows a local minimum around $D/S = 0.13$, which is where the nominal piecewise fit was trapped in Section 8.2. Even though this value is consistent with previous measurements, the proximity of the local minimum to the
true minimum means that the error bars on that measurement are artificially small, so we choose not to include it in our final $D/S$ ratio calculation. In the upper $-t$ range for the $90^\circ$ orientation, all but three of fits used in the parameter scan failed to converge, and the value obtained from interpolation is not meaningful, so it is not used in our calculation either.

For our final $D/S$ ratio measurement, we take the statistical average of the results from the two reliable parameter scans, and add their statistical errors in quadrature, giving a value of

$$D/S_{\text{GlueX}} = 0.296 \pm 0.021,$$  \hspace{1cm} (8.2)

which is consistent with E852’s result of $D/S = 0.269 \pm 0.009 \,[7]$. 

FIG. 8.23: Results of likelihood scan to GlueX data in $0^\circ$ (upper row) and $90^\circ$ (middle row) orientations and MC (bottom row), for the $-t$ bin from $0.15 - 0.30$ GeV$^2$ (left column) and from $0.30 - 0.50$ GeV$^2$ (right column). The black points indicate the change in $-2\ln(L)$ from the minimum, extracted from the parameter scan, and the red dashed lines are the results of spline interpolation. Note that the $90^\circ$ high $-t$ plot (middle right) does not have enough data points to interpolate a meaningful value and is included only for completeness.
8.4 Future Systematic Studies

Systematic studies will allow us to account for any uncertainty in our measurement that is not statistical. For example, we can see how much our results depend on the cuts we make on the data, we can apply our procedure to statistically independent datasets to test for robustness, and we can calculate how much our results depend on quantities that we assume are well-known in our initial analysis. All of these studies will provide information on how sensitive we are to choices or assumptions made in the analysis, since these are all sources of non-statistical error.

The simplest systematic study would involve applying our fit procedure to another pair of statistically independent datasets, which would tell us how azimuthally uniform our detector is. GlueX collects data with the photon beam polarization in four different orientations, which are in two perpendicular pairs. We’ve already fit one of the perpendicular pairs, with the beam polarized parallel and perpendicular to the horizontal (0/90). The other pair has the beam polarized at a 45° and at a 135° angle to the horizontal (45/135). The difference between our perpendicular datasets, if statistically significant, will provide one form of systematic uncertainty.

Another systematic study would involve varying our event selection cuts. For example, we removed all events with a $p\pi^+$ mass greater than 1.35 GeV, in an attempt to balance removal of non-$\Delta^{++}$ events with keeping signal $\Delta^{++}$ events. We could have easily made that cut at 1.325 or 1.375 GeV, or elsewhere, and it is not obvious where best to place the cut and how much influence the placement of the cut will have on our final results. To test this, we would need to take several datasets with various cuts on the $p\pi^+$ mass and apply the fit procedure to each of them. We’d need to decide what a reasonable cut range is on the $p\pi^+$ mass, and then the systematic error associated with that cut would be the nominal value of the parameter of interest, such as the $D/S$ ratio, plus or
minus the difference between that value and the nominal values associated with the most extreme cuts. The same principle applies to other cuts we’ve made, notably the $\chi^2/NDF$ associated with the kinematic fit, which indicates our dependence on detector resolution. Varying the $\omega$ sideband subtraction will also be an important systematic study, and the variation of the sideband widths in our study will provide knowledge of the systematic error associated with removal of non-$\omega$ background.

Further, we should calculate what would happen if our detector acceptance was not correctly modeled by Monte Carlo. For example, the charged pion tracking efficiencies agree between data and Monte Carlo within roughly 5%, but we can take the ratio of these efficiencies, as shown in Figs. C.3, C.4, C.5, and C.6, and use those ratios to create weights for our accepted phasespace Monte Carlo sample used in the fit. This adjustment of the acceptance weight will provide a better accounting of the differences between data and Monte Carlo, and comparison of the results with the nominal MC will tell us how much systematic error is associated with those differences.

A final systematic study would involve testing the dependence of our results on quantities that we assume are well known. For example, when we perform our fits, we assume that the photon beam polarization fraction $P_\gamma$ is simply 0.35, when it is actually a measured quantity with a statistical error of 3% and a systematic uncertainty of 1.5%. To measure how much that uncertainty propagates into our final results, we would want to perform our fits with $P_\gamma$ at each of its upper and lower bounds, and see what effect this change has on our results. We would expect for it to have the most effect on our measured reflectivities, since in Eqn. 2.30, the term $(1 - P_\gamma)$ multiplies the negative reflectivity coherent sums, and $(1 + P_\gamma)$ multiplies the positive reflectivity coherent sums. A similar principle applies to the Dalitz parameters ($\alpha$, $\beta$, and $\gamma$) used in calculating the $\omega$ decay, discussed in Section 2.3.
CHAPTER 9

Summary and Outlook

9.1 Summary

We’ve presented the first analysis of the charged $b_1^- \to \omega \pi^-$ decay in photoproduction. Previous measurements of the $\omega \pi$ system, as discussed in Chapter 3, have either studied the neutral channel in photoproduction [16] or $e^+e^-$ annihilation [21, 22], or the charged channel in pion production [7, 20], none of which involve charged $t$-channel exchange in the production mechanism. However, the previous pion and photoproduction experiments have indicated that the axial vector $b_1$ is a significant contributor to the $\omega \pi$ system, and the $e^+e^-$ annihilation experiments have observed vector contributions to that final state, indicating that a vector state is a reasonable background for this analysis. Previous measurements of the charged $b_1^- \to \omega \pi^-$ [7] have measured the ratio of $D$-wave to $S$-wave in its decay to be $D/S = 0.269 \pm 0.009$, and theoretical calculations [23] are in agreement with that value, giving a ratio of $D/S = 0.27 \pm 0.20$.

We used a sample of exclusive $\gamma p \to \pi^+\pi^+\pi^-\pi^-\pi^0p$ events collected at the GlueX experiment, outlined in Chapter 4, and applied strategic cuts, described in Chapter 6, to
extract a set of $\omega \pi^-$ events recoiling off a $\Delta^{++}$. In Chapter 5, we described our study of the GlueX drift chamber system’s ability to track charged pions through analysis of exclusive production of an $\omega$ meson decaying to three pions.

We performed a partial wave analysis on the extracted $\omega \pi^-$ events, using a resonance amplitude model for vector-pseudoscalar production and decay in the reflectivity basis, as described in Chapter 7. We used mass independent fitting to test various fit wavesets to use in our final piecewise fitting procedures. These piecewise fits show that at low four-momentum transfer squared $-t$, the $b_1^-$ meson is produced primarily through unnatural exchange, with a smaller amount of natural exchange. The amount of unnatural exchange decreases at higher values of $-t$, which has been observed in previous measurements of charged exchange mechanisms. We used a parameter scan to determine the ratio of $D$-wave to $S$-wave in $b_1^-$ decay, which is measured to be $D/S_{\text{GlueX}} = 0.296 \pm 0.021$, slightly higher than, but still consistent with, previous measurements.

### 9.2 Outlook

This analysis was performed on datasets taken with the photon beam polarized at 0° and 90° to the horizontal, and should be repeated for the datasets taken at 45° and 135°. These will provide consistency checks and allow us to account for any azimuthally dependent detector inefficiencies. As a further check, all four polarization samples of the GlueX-I dataset should be fit at once. This method has added stability to fits performed to other GlueX final states, for example, the $\eta'/\pi$.

The $t$-slope in the phasespace Monte Carlo is not a perfect match to the GlueX data, and needs to be optimized to better model the production mechanism. The systematic studies outlined in Section 8.4 also need to be performed.

Extending the fit code to reliably extract spin $J = 2$ waves and above is necessary,
especially in the search for the exotic $J^{PC} = 2^{+-}$ meson claimed by E852 in Ref. [7]. The excited $\rho^*$ meson we use in our signal Monte Carlo is not consistent with the $\rho_3$ measured by E852 in this channel, but our fits are not yet able to reliably extract amplitudes with spins above $J = 1$, so we had to model the background with $JP = 1^-$ waves instead.

The neutral $\omega \pi^0$ channel is under investigation by our collaborators at University of Regina, and our analyses will in principle provide consistency checks for properties like the $D/S$ ratio of the $b_1$, and will also allow us to compare charged and neutral production mechanisms, as discussed in Section 8.2.1.

Understanding the decay of the $b_1$ meson will allow us to continue our PWA journey by studying more complicated channels. One of these channels shares a final state with this one, $\gamma p \rightarrow \omega \pi^+ \pi^- p$. The data cut away by our $\Delta^{++}$ mass cut consists of an $\omega \pi^+ \pi^-$ system recoiling off a proton. Since the $b_1$ decays prominently through $\omega \pi^-$, it is reasonable to assume that a portion of the events in that $\omega \pi^+ \pi^-$ system decayed from $b_1^+ \pi^-$, which our theory colleagues have predicted to be the dominant decay mode of the exotic $\pi_1(1600)$ [10].
APPENDIX A

Formalism for Photoproduction of Two-Pseudoscalar Systems

The formalism used in our work on vector-pseudoscalar systems is an extension of the formalism for two-pseudoscalar systems used by colleagues in both GlueX and JPAC.

A.1 The $\eta\pi$ System

We start by considering the $\eta\pi$ system at GlueX. This is a similar system to the $\omega\pi$, made simpler by the fact that both the $\eta$ and the pion are spinless particles. This two-pseudoscalar formalism is discussed in detail in Section II and Appendix D of Ref. [15].

The reaction producing the $\eta\pi^0$ system at GlueX is formally written out as

$$\gamma(\lambda, p_{\gamma}) p(\lambda_1, p_N) \rightarrow \pi^0(p_\pi) \eta(p_\eta) p(\lambda_2, p'_N), \quad (A.1)$$

where the helicities ($\lambda, \lambda_1, \lambda_2$) of the beam, target, and recoil particles are defined in the helicity reference frame, which is the rest frame of the $\eta\pi^0$ system with the $z$-axis in the
direction opposite the recoil system (in this case the proton). The amplitude for this reaction is described by \( A_{\lambda,\lambda_1,\lambda_2}(\Omega) \), where \( \Omega \) is the spherical angle describing the direction of the \( \eta \) meson in the \( \eta\pi^0 \) helicity frame. The quantity that we fit, the measured intensity, can be described in terms of the differential cross-section

\[
I(\Omega, \Phi) \equiv \frac{d\sigma}{dt d\Omega_{\eta\pi^0} d\Phi} \quad (A.2)
\]

\[
= \kappa \sum_{\lambda,\lambda',\lambda_1,\lambda_2} A_{\lambda,\lambda_1,\lambda_2}(\Omega) \rho^\gamma_{\lambda,\lambda'}(\Phi) A^*_{\lambda',\lambda_1,\lambda_2}(\Omega) \quad (A.3)
\]

\[
= I^0(\Omega) - P_\gamma I^1(\Omega) \cos 2\Phi - P_\gamma I^2(\Omega) \sin 2\Phi, \quad (A.4)
\]

where \( \rho^\gamma_{\lambda,\lambda'}(\Phi) \), the photon spin density matrix, describes the dependence on \( \Phi \), the angle of linear polarization of the incoming photon beam measured with respect to the \( \eta\pi^0 \) production plane. All numerical factors are contained in the phase space factor

\[
\kappa = \frac{1}{(2\pi)^3} \frac{1}{4\pi} \frac{1}{2\pi} \frac{\lambda^{1/2} \left( m_{\eta\pi^0}^2, m_{\pi^0}^2, m_\eta^2 \right)}{16 m_{\eta\pi^0} (s - m_N^2)^2} \frac{1}{2}, \quad (A.5)
\]

where the Mandelstam variable \( s = (p_\gamma + p_N)^2 \) is the total energy squared and \( \lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ca) \) is the triangle function.

These components of the intensity function are defined in Eqn. 4 of Ref. [15] and are given here, suppressing the helicity indices of the target and recoil baryons (these will be reintroduced later)

\[
I^0(\Omega) = \frac{\kappa}{2} \sum_\lambda A_\lambda(\Omega) A^*_\lambda(\Omega), \quad (A.6)
\]

\[
I^1(\Omega) = \frac{\kappa}{2} \sum_\lambda A_{-\lambda}(\Omega) A^*_\lambda(\Omega), \quad (A.7)
\]

\[
I^2(\Omega) = i \frac{\kappa}{2} \sum_\lambda \lambda A_{-\lambda}(\Omega) A^*_\lambda(\Omega). \quad (A.8)
\]
If we plug these definitions into Eqn. A.4, we can rewrite the intensity in terms of the amplitudes $A_{\lambda}(\Omega)$

$$I(\Omega, \Phi) = \frac{\kappa}{2} \sum_{\lambda} [A_{\lambda}(\Omega) A_{\lambda}^*(\Omega) - P_{\gamma} \cos 2\Phi A_{-\lambda}(\Omega) A_{\lambda}^*(\Omega) - i\lambda P_{\gamma} \sin 2\Phi A_{-\lambda}(\Omega) A_{\lambda}(\Omega)].$$

(A.9)

The value of beam photon helicity, $\lambda$, can take on the values of $\pm 1$. Making the sum over $\lambda$ explicit, we obtain

$$I(\Omega, \Phi) = \frac{\kappa}{2} \left[ A_+ (\Omega) A_+^*(\Omega) - P_{\gamma} \cos 2\Phi A_- (\Omega) A_+^*(\Omega) - iP_{\gamma} \sin 2\Phi A_- (\Omega) A_+^*(\Omega) 
+ A_- (\Omega) A_-^*(\Omega) - P_{\gamma} \cos 2\Phi A_+ (\Omega) A_-^*(\Omega) + iP_{\gamma} \sin 2\Phi A_+ (\Omega) A_-^*(\Omega) \right]$$

(A.10)

$$= \frac{\kappa}{2} \left[ A_+ (\Omega) A_+^*(\Omega) + A_- (\Omega) A_-^*(\Omega) - P_{\gamma} e^{2i\Phi} A_- (\Omega) A_+^*(\Omega) - P_{\gamma} e^{-2i\Phi} A_+ (\Omega) A_-^*(\Omega) \right].$$

(A.11)

If we fold the phase-dependence into the helicity-basis amplitudes

$$\tilde{A}_\pm (\Omega, \Phi) \equiv e^{\mp i\Phi} A_\pm (\Omega),$$

(A.12)

we can write the intensity in terms of them

$$I(\Omega, \Phi) = \frac{\kappa}{2} \left[ \tilde{A}_+ (\Omega, \Phi) \tilde{A}_+^*(\Omega, \Phi) + \tilde{A}_- (\Omega, \Phi) \tilde{A}_-^*(\Omega, \Phi) 
- P_{\gamma} \tilde{A}_- (\Omega, \Phi) \tilde{A}_+^*(\Omega, \Phi) - P_{\gamma} \tilde{A}_+ (\Omega, \Phi) \tilde{A}_-^*(\Omega, \Phi) \right]$$

(A.13)

$$= \frac{\kappa}{4} \left[ (1 - P_{\gamma}) \left| \tilde{A}_+ (\Omega, \Phi) + \tilde{A}_- (\Omega, \Phi) \right|^2 + (1 + P_{\gamma}) \left| \tilde{A}_+ (\Omega, \Phi) - \tilde{A}_- (\Omega, \Phi) \right|^2 \right].$$

(A.14)
The helicity-basis amplitudes $A_\lambda(\Omega)$ are expanded into partial waves, $T_{\lambda m}^l$

$$A_\lambda(\Omega) = \sum_{lm} T_{\lambda m}^l Y_l^m(\Omega). \quad (A.15)$$

where the angular dependence is contained in the spherical harmonics $Y_l^m(\Omega)$.

### A.1.1 Two-Pseudoscalar Reflectivity

The partial wave amplitudes for a two-pseudoscalar system in the reflectivity basis, $(^{(e)}V_{m_1;\lambda_1}\lambda_2)$, are defined using a linear combination of the partial wave amplitudes in the helicity basis

$$(^{(e)}V_{m_1;\lambda_1}\lambda_2) = \frac{1}{2} \left[ T_{+1m_1;\lambda_1}\lambda_2 - \epsilon (-1)^m T_{-1-m_1;\lambda_1}\lambda_2 \right], \quad (A.16)$$

note that this equation does not involve the naturality of the resonance, since only natural parity states are accessible to $\eta\pi$.

From parity invariance, we infer

$$(^{(e)}V_{m_1;\lambda_1}\lambda_2) = \epsilon (-1)^{\lambda_1-\lambda_2} (^{(e)}V_{m_1;\lambda_1}\lambda_2), \quad (A.17)$$

which allows us to define two sets of partial waves

$$[l]^{(e)}_{m;0} = (^{(e)}V_{m;+\lambda_1\lambda_2}, \quad [l]^{(e)}_{m;1} = (^{(e)}V_{m;+\lambda_1\lambda_2}, \quad (A.18)$$

which allows us to define two sets of partial waves

$$[l]^{(e)}_{m;0} = (^{(e)}V_{m;+\lambda_1\lambda_2}, \quad [l]^{(e)}_{m;1} = (^{(e)}V_{m;+\lambda_1\lambda_2}, \quad (A.18)$$

corresponding to nucleon helicity non-flip and flip, respectively. Here $[l] = S, P, D, \cdots$ for $l = 0, 1, 2, \cdots$. For each $[l]$, there are $2 \times 2 \times (2l + 1)$ complex partial waves $[l]^{(e)}_{m;k}$, since $\epsilon = \pm 1$, $k = 0, 1$, and $m = -l, \cdots, l$. 

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APPENDIX B

Additional Plots from Fits to GlueX Data and Signal Monte Carlo

B.1 Piecewise Fit Results in the $-t$ Range $0.30-0.50$ GeV$^2$

B.1.1 GlueX Data, 0 PARA Orientation
FIG. B.1: Partial waves extracted from piecewise fit to 0 PARA GlueX data. The top row is the $1^+ S$ wave, middle row is $1^+ D$ wave, and bottom row is the $1^− P$ wave. The columns are, from left to right, spin projection $m = +1, 0, −1$. The black points represent the total intensity of the GlueX data, the green is the weighted phasespace MC for each partial wave, the blue is the contribution from negative reflectivity, and the red is the contribution from positive reflectivity.
FIG. B.2: Diagnostic plots from piecewise fit to 90 PERP GlueX data. The plots are, left to right: Top row: $\omega \pi^-$ invariant mass, $\cos \theta$, $\phi$. Middle row: $\cos \theta_H$, $\phi_H$, $\Phi$. Bottom row: $p\pi^+$ invariant mass, $p\pi^-$ invariant mass, $p\pi^+\pi^-$ invariant mass. The black points represent the total intensity of the GlueX data, the green is the weighted phasespace Monte Carlo for each partial wave, the blue is the contribution from $J^P = 1^+$, and the red is the integrated contribution from $J^P = 1^+$. 
FIG. B.3: Partial waves extracted from piecewise fit to 90 PERP GlueX data. The top row is the $1^+S$ wave, middle row is $1^+D$ wave, and bottom row is the $1^-P$ wave. The columns are, from left to right, spin projection $m = +1, 0, -1$. The black points represent the total intensity of the GlueX data, the green is the weighted phase space MC for each partial wave, the blue is the contribution from negative reflectivity, and the red is the contribution from positive reflectivity.
FIG. B.4: Diagnostic plots from piecewise fit to 90 PERP GlueX data. The plots are, left to right: Top row: $\omega \pi^-$ invariant mass, $\cos \theta$, $\phi$. Middle row: $\cos \theta_H$, $\phi_H$, $\Phi$. Bottom row: $p\pi^+$ invariant mass, $p\pi^-$ invariant mass, $p\pi^+\pi^-$ invariant mass. The black points represent the total intensity of the GlueX data, the green is the weighted phasespace Monte Carlo for each partial wave, the blue is the contribution from $J^P = 1^+$, and the red is the integrated contribution from $J^P = 1^+$. 
FIG. B.5: Partial waves extracted from piecewise fit to signal Monte Carlo. The top row is the $1^+S$ wave, middle row is $1^+D$ wave, and bottom row is the $1^-P$ wave. The columns are, from left to right, spin projection $m = +1, 0, -1$. The black points represent the total intensity of the signal Monte Carlo, the green is the weighted phasespace Monte Carlo for each partial wave, the blue is the contribution from negative reflectivity, and the red is the contribution from positive reflectivity.

B.1.3 Signal Monte Carlo
FIG. B.6: Diagnostic plots from piecewise fit to signal Monte Carlo. The plots are, left to right:
Top row: $\omega \pi^-$ invariant mass, $\cos \theta, \phi$. Middle row: $\cos \theta_H, \phi_H, \Phi$. Bottom row: $p\pi^+$ invariant mass, $p\pi^-$ invariant mass, $p\pi^+\pi^-$ invariant mass. The black points represent the total intensity of the signal Monte Carlo, the green is the weighted phasespace Monte Carlo for each partial wave, the blue is the contribution from $J^P = 1^+$, and the red is the integrated contribution from $J^P = 1^+$. 
APPENDIX C

Two-Dimensional Plots of Charged Pion Efficiency

The efficiencies in 3-momentum vs $\theta$ can be viewed in a 2D plot, as shown in Figs C.1 and C.2, for $\pi^+$ and $\pi^-$, respectively.
FIG. C.1: $\pi^+$ efficiency as a function of 3-momentum vs $\theta$
FIG. C.2: $\pi^-$ efficiency as a function of 3-momentum vs $\theta$
C.1 Two-dimensional efficiency data/MC ratios in $p$ vs $\theta$

We can also consider the ratios of efficiencies for data/MC, as seen in Figs [C.3], [C.4], [C.5] and [C.6] for Method 1 and 2 and both $\pi^+$ and $\pi^-$.  

FIG. C.3: Method 1 $\pi^+$ efficiency ratio (data/MC) as a function of 3-momentum vs $\theta$. 
FIG. C.4: Method 2 $\pi^+$ efficiency ratio (data/MC) as a function of 3-momentum vs $\theta$.

FIG. C.5: Method 1 $\pi^-$ efficiency ratio (data/MC) as a function of 3-momentum vs $\theta$. 

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FIG. C.6: Method 2 π⁻ efficiency ratio (data/MC) as a function of 3-momentum vs $\theta$. 
C.2 Analysis-Level Cuts

Cuts on $P(\chi^2)$

We placed cuts on $P(\chi^2)$ of the charged track hypothesis to see how much the efficiencies depend on track quality.

![Graph showing $P(\chi^2)$ of charged track hypothesis in the case of a missing $\pi^+$](image)

FIG. C.7: $P(\chi^2)$ of charged track hypothesis in the case of a missing $\pi^+$

As shown in Figs C.8, C.9, C.10, and C.11, placing cuts on $P(\chi^2)$ of the charged track hypothesis does not effect the efficiency very much.
FIG. C.8: $\pi^+$ efficiencies as function of missing $\theta$, with $P(\chi^2)$ cuts applied
FIG. C.9: $\pi^+$ efficiencies as function of missing 3-momentum, with $P(\chi^2)$ cuts applied
FIG. C.10: $\pi^-$ efficiencies as function of missing $\theta$, with $P(\chi^2)$ cuts applied
FIG. C.11: $\pi^-$ efficiencies as function of missing 3-momentum, with $P(\chi^2)$ cuts applied
BIBLIOGRAPHY


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