Supplementary Materials to:

Comparisons of length-based mortality estimators and age-structured models agree for six southeastern United States stocks

Quang C. Huynh[1]*, Nancie J. Cummings[2], and John M. Hoenig[1]

[1] Virginia Institute of Marine Science, The College of William & Mary, P.O. Box 1346, Gloucester Point, VA 23062, USA

[2] Southeast Fisheries Science Center, 75 Virginia Beach Drive, Miami, FL 33149, USA

* Corresponding author's email: q.huynh@oceans.ubc.ca

Supplementary Materials A: Technical description of the mean length mortality estimators

The ML and MLCR models estimate mortality rates and change points in mortality based on the transitional behavior of the mean length and index following a change in mortality (Gedamke and Hoenig, 2006; Huynh *et al*., 2017). Assume there are k+1 time stanzas (k changes in mortality in the time series). The predicted mean length $\overline{L}_y$, abundance-based index $NPUE_y$, and weight-based index $WPUE_y$ in year y are calculated as

$$\overline{L}_y = L_\infty \frac{\sum_{i=1}^{k+1} \frac{a_{i,y} v_{i,y}}{Z_{k+2-i}} - \left(1 - \frac{L_c}{L_\infty}\right) \sum_{i=1}^{k+1} \frac{r_{i,y} s_{i,y}}{Z_{k+2-i} + K}}{\sum_{i=1}^{k+1} \frac{a_{i,y} v_{i,y}}{Z_{k+2-i}}}, \quad \text{(A.1)}$$

$$NPUE_y = \tilde{q} \sum_{i=1}^{k+1} \frac{a_{i,y} v_{i,y}}{Z_{k+2-i}}, \quad \text{(A.2)}$$

and

$$WPUE_y = \tilde{q} \sum_{i=1}^{k+1} \left\{ a_{i,y} w_{i,y} \left(1 - \frac{L_c}{L_\infty}\right)^{-Z_{k+2-i}/K} \left[Beta\left(\gamma_{k+2-i,y}; b+1, \frac{Z_{k+2-i}}{K}\right) - Beta\left(\lambda_{k+2-i,y}; b+1, \frac{Z_{k+2-i}}{K}\right) \right] \right\}$$

(A.3)

where

$$a_{i,y} = \begin{cases} 1 & i = 1 \\ \exp\left(-\sum_{j=1}^{i-1} Z_{k+2-j} d_{k+1-j,y}\right) & i = 2, \ldots, k+1 \end{cases} \quad \text{(A.4)}$$

$$v_{i,y} = \begin{cases} 1 - \exp\left(Z_{k+2-i} d_{k+1-i,y}\right) & i = 1, \ldots, k \\ 1 & i = k+1 \end{cases} \quad \text{(A.5)}$$

$$r_{i,y} = \begin{cases} 1 & i = 1 \\ \exp\left(-\sum_{j=1}^{i-1} \left[Z_{k+2-j} + K \right] d_{k+1-j,y} \right) & i = 2,\dots,k+1 \end{cases} \tag{A.6}$$

$$s_{i,y} = \begin{cases} 1 - \exp\left(-\left[Z_{k+2-i} + K \right] d_{k+1-i,y} \right) & i = 1,\dots,k \\ 1 & i = k+1 \end{cases} \tag{A.7}$$

$$w_{i,y} = \begin{cases} 1 & i = 1 \\ \exp\left(Z_{k+2-i} \sum_{j=1}^{i-1} d_{k+1-j,y} \right) & i = 2,\dots,k+1 \end{cases} \tag{A.8}$$

$$Beta(x;\alpha,\beta) = \int_0^x u^{\alpha-1}(1-u)^{\beta-1}\,du \tag{A.9}$$

$$\gamma_{k+2-i,y} = \begin{cases} 0 & i = 1,\dots,k \;\&\; y \le D_{k+1-i} \\ 1 - \left(1 - \frac{L_c}{L_\infty} \right) \exp\left(-K \sum_{j=1}^{i} d_{k+1-j,y} \right) & i = 1,\dots,k \;\&\; y > D_{k+1-i} \\ 1 & i = k+1 \end{cases} \tag{A.10}$$

$$\lambda_{k+2-i,y} = \begin{cases} 0 & i = 1,\dots,k \;\&\; y \le D_{k+1-i} \\ L_c / L_\infty & i = 1 \;\&\; y > D_{k+1-i} \\ L_c / L_\infty & i = 2,\dots,k \;\&\; D_{k+1-i} < y \le D_{k+2-i} \\ L_c / L_\infty & i = k+1 \;\&\; y \le D_{k+2-i} \\ 1 - \left(1 - \frac{L_c}{L_\infty} \right) \exp\left(-K \sum_{j=1}^{i-1} d_{k+1-j,y} \right) & i = 2,\dots,k+1 \;\&\; y > D_{k+2-i} \end{cases} \tag{A.11}$$

and

$$d_{i,y} = \begin{cases} 0 & y \le D_i \\ y - D_i & D_i < y \le D_{i+1} \\ D_{i+1} - D_i & y > D_{i+1} \end{cases} . \tag{A.12}$$

All additional variables are defined in Table A.1.

For the MLeffort model, an age-structured model approach is used (Then *et al.*, 2018).

In the first year $y = Y_1$, equilibrium effort is assumed where the abundance $N_{y,t}$ at age $t = t_c, t_c + 1, ..., t_{\max}$ is defined as

$$N_{Y_1,t} = \begin{cases} 1 & t = t_c \\ \exp\left(-\sum_{i=t_c}^{t-1}(qf_{eq} + M)\right) & t = t_c + 1, ..., t_{\max} \end{cases}, \quad \text{(A.13)}$$

where f_{eq} is the equilibrium effort, q is the catchability coefficient that scales effort to fishing mortality, $t_c = t_0 - \frac{1}{K}\log\left(1 - \frac{L_c}{L_\infty}\right)$ is the age that corresponds to length L_c from the von Bertalanffy equation, and $t_{\max}$ is the maximum age in the model. In subsequent years $y = Y_2, Y_3, ...$, the abundance $N_{y,t}$ at ages $t = t_c, t_c + 1, ..., t_{\max}$ is calculated as

$$N_{y,t} = \begin{cases} 1 & t = t_c \\ N_{y-1,t-1}\exp\left(-[qf_{y-1} + M]\right) & t = t_c + 1, ..., t_{\max} \end{cases}, \quad \text{(A.14)}$$

where f_y is the effort in year y. The mean length is

$$\overline{L}_y = \frac{\sum_a L_a N_{y,a}}{\sum_a N_{y,a}}, \quad \text{(A.15)}$$

where $L_a = L_\infty(1 - \exp[-K\{t - t_0\}])$.

Table A.1. Definitions of variables for the ML and MLCR models.

Variable	Definition
i	Index for time stanza (*i* = 1, …, *k* + 1)
j	Index for time stanzas experienced prior to time stanza *i* (*j* = 1, …, *i* - 1)
y	Calendar year
Z	Instantaneous total mortality rate (year^{-1})
D	Change point for mortality (calendar year)
L_∞	Von Bertalanffy asymptotic length
K	Von Bertalanffy growth parameter
$\tilde{q}$	Scaling parameter for index
b	Length-weight exponent

Supplementary Materials A: References

Gedamke, T., and Hoenig, J. M. 2006. Estimating mortality from mean length data in nonequilibrium situations, with application to the assessment of Goosefish. Transactions of the American Fisheries Society, 135: 476–487.

Huynh, Q. C., Gedamke, T., Porch, C. E., Hoenig, J. M., Walter, J. F., Bryan, M., and Brodziak, J. 2017. Estimating total mortality rates of mutton snapper from mean lengths and aggregate catch rates in a non-equilibrium situation. Transactions of the American Fisheries Society, 146: 803-815.

Then, A. Y., Hoenig, J. M., and Huynh, Q. C. 2018. Estimating fishing and natural mortality rates, and catchability coefficient, from a series of observations on mean length and fishing effort. ICES Journal of Marine Science, 75: 610:620.

Supplementary Materials B: Spawning potential ratio for the mean length estimators

The spawning potential ratio (*SPR*) is calculated as,

$$SPR = \frac{SSBPR(F = F_{SPR\%})}{SSBPR(F = 0)}, \quad \text{(B.1)}$$

which is the ratio of the spawning stock biomass per recruit (*SSBPR*) at $F = F_{SPR\%}$ compared to that at $F = 0$. The spawning stock biomass per recruit is

$$SSBPR(F) = \sum_{t=1}^{t_{\max}} N_t w_t m_t \quad \text{(B.2)}$$

where the abundance at age *t* (N_t) is

$$N_t = \begin{cases} 1 & t = 1 \\ N_{t-1}\exp(-Z_{t-1}) & t = 1,\ldots,t_{\max}-1 \\ \dfrac{N_{t-1}\exp(-Z_{t-1})}{1-\exp(-Z_t)} & t = t_{\max} \end{cases}, \quad \text{(B.3)}$$

the weight at age (w_t) is

$$w_t = \left(\alpha L_\infty{}^b\right)\{1-\exp[-K(t-t_0)]\}^b, \quad \text{(B.4)}$$

the maturity at age (m_t) is

$$m_t = \begin{cases} 0 & t = 0,\ldots,t_{mat}-1 \\ 1 & t = t_{mat},\ldots,t_{\max} \end{cases}, \quad \text{(B.5)}$$

and total mortality at age (Z_t) is

$$Z_t = \begin{cases} M & t = 0,\ldots,t_c-1 \\ F+M & t = t_c,\ldots,t_{\max} \end{cases}. \quad \text{(B.6)}$$

From L_c, the fully selected length, the corresponding age t_c is obtained from the inverse of the von Bertalanffy function,

$$t_c = t_0 - \frac{1}{K}\log\left(1 - \frac{L_c}{L_\infty}\right). \qquad \text{(B.7)}$$

From L_{mat}, the length of knife-edge maturity, the corresponding age t_{mat} is also obtained from the inverse of the von Bertalanffy function,

$$t_{mat} = t_0 - \frac{1}{K}\log\left(1 - \frac{L_{mat}}{L_\infty}\right). \qquad \text{(B.8)}$$

To obtain the $F_{30\%}$ reference point, Equation B.1 is solved for $F_{SPR\%}$ such that SPR = 0.3. All variables are defined in Table B.1.

Table B.1. Definition of variables for spawning potential ratio calculation.

Variable	Definition
F	Instantaneous fishing mortality rate (year^{-1})
M	Instantaneous natural mortality rate (year^{-1})
Z	Instantaneous total mortality rate (year^{-1})
α	Length-weight allometric constant
β	Length-weight allometric exponent
L_∞	Von Bertalanffy asymptotic length
K	Von Bertalanffy growth parameter
t_0	Von Bertalanffy theoretical age at length zero
$t_{\max}$	Maximum age (plus-group)
L_{mat}	Length at maturity
t_{mat}	Age at maturity
L_c	Fully selected length (knife-edge selectivity)
t_c	Fully selected age

Supplementary Materials C: Residuals in the application of the mean length-based mortality estimators

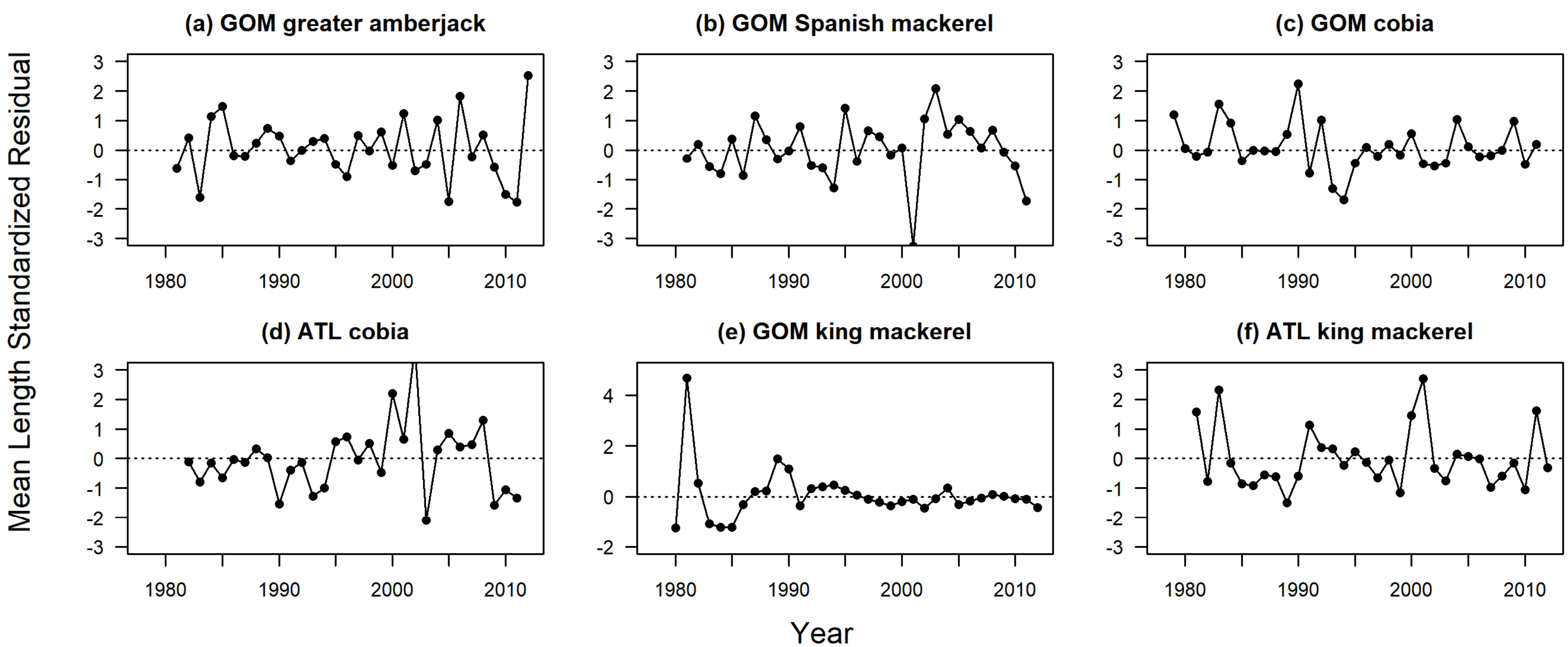

Figure C.1. Standardized residuals of mean length from the ML model. Residuals were calculated by subtracting the predicted value from the observed value and then dividing the difference by the estimated standard deviation.

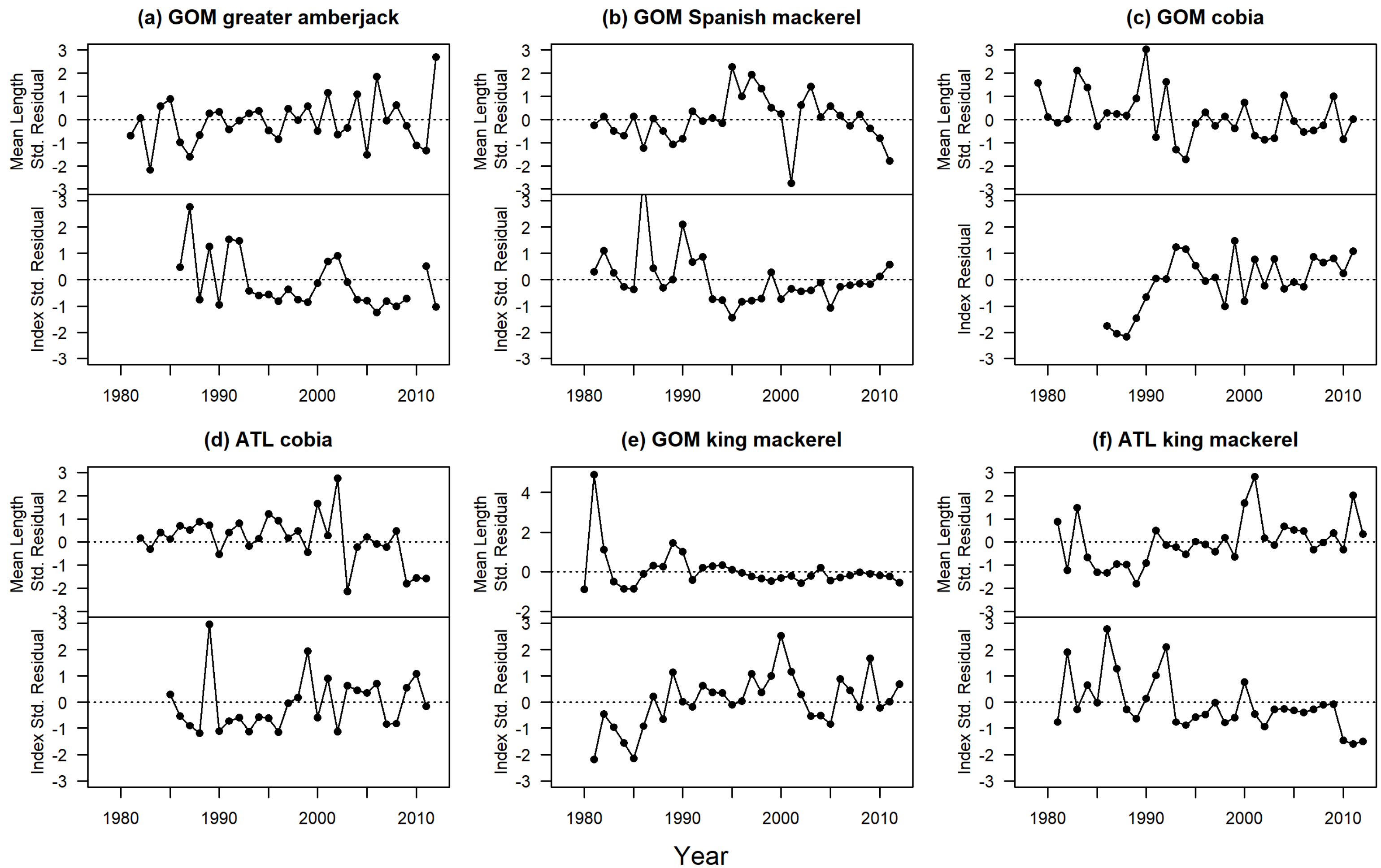

Figure C.2. Standardized residuals of mean length and index from the MLCR model. Residuals were calculated by subtracting the predicted value from the observed value and then dividing the difference by the estimated standard deviation.

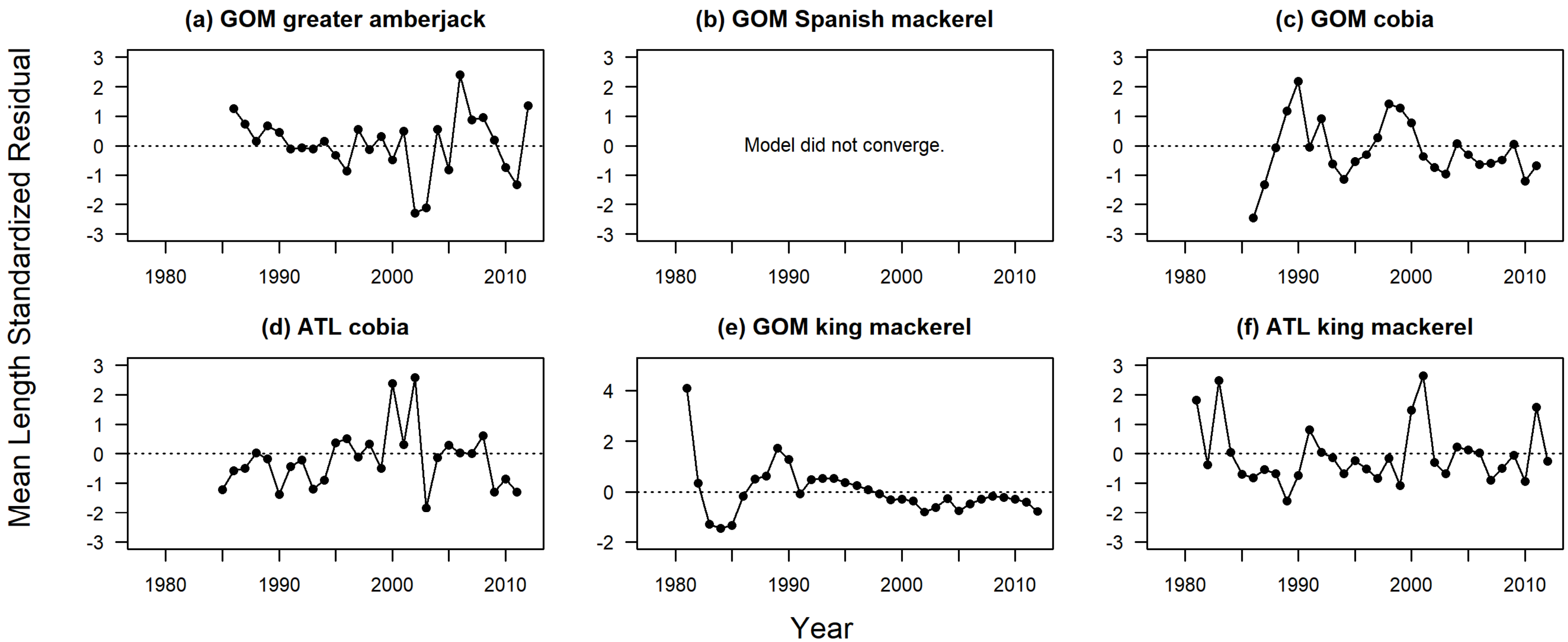

Figure C.3. Standardized residuals of mean length from the MLeffort model. Residuals were calculated by subtracting the predicted value from the observed value and then dividing the difference by the estimated standard deviation.