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The Aerodynamics of Flapping Birdflight

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THE AERODYNAMICS OF FLAPPING BIRDFLIGHT

Clarence D. Cone, Jr.

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I. INTRODUCTION

1.1 BACKGROUND

One of the most complex, yet simultaneously beautiful and intriguing, of all natural-aerodynamic phenomena is the flapping flight of birds. Certainly no other aspect of natural aerodynamics has so captivated and puzzled the mind of man over the centuries. Early progress in elucidation of the mechanical processes involved in flapping flight was very slow, the prime technical block being the inability of the unaided human eye to follow the swift and complex motions of the flapping wing. Despite the obvious lack of fundamental knowledge of the principles underlying flapping flight, early scientists, inventors, and technicians by the score persisted in their attempts to emulate the flapping wing, hoping vainly to attain that long-sought and exalted company with the birds.1*

The barrier to human progress in flight technology imposed by the speed of the bird's wing was indeed an effective one, but in a sense this obscurity may have served a useful purpose after all. For, had man at that early date been able to view the great complexity of the avian wing in motion, as it is now so clearly revealed to us by the high-speed cameras, his despair and hopelessness of ever emulating the flight of birds might have been great enough to have precluded many of his early trials and his relentless persistence in quest of the secret of flight. The solution of human flight ultimately came, of course, not from a knowledge of flapping wings, but from a glimpse of understanding of the fixed-wing aerodynamics utilized by soaring birds. Indeed, long after human flight had been achieved and the principles of aerodynamics well established as a modern science, the actual mechanics of the flapping bird-wing remained a mystery.

With the advent of the high-speed motion picture camera came the ability to see the actual wing motions involved in flapping flight and, with this ability, some appreciation of the mechanics governing the production of lift and thrust. The camera revealed, however, a much more complex and involved aerodynamic system than had ever been imagined. Instead of the simple up-and-down motion of a hinged planar surface, the

*Superscript numbers refer to particular references listed at the end of the report.
flapping wing was seen to be executing an intricate sequence of deformations, twists, flexings, featherings, bendings, and changes of attitude and direction during the course of the flapping cycle. The flapping wing was seen to embrace the entire realm of aerodynamics, especially the more difficult areas of viscous, unsteady, aeroelastic phenomena, and to involve the complex flow about a nonrigid body with continuously changing shape. While elementary analyses of flapping flight using high-speed films allowed, in the past, some glimpse of the gross mechanics of the flapping wing and made the phenomenon understandable in a broad sense, these studies were highly qualitative and technically incomplete at best. Unfortunately, no really useful application of modern aerodynamic theory to the analysis of the flapping wing was ever developed.

Despite the inherent complexity of the subject, a somewhat more refined technical consideration by the present author of the flapping wing of birds, utilizing a large number of high-speed film segments of many species in flight, suggested that the basic aerodynamic relationships of the flapping bird-wing could be reasonably well established in quantitative form within the framework of linearized lifting-line theory. Consequently, a detailed aerodynamic analysis of the flapping flight of birds was initiated and carried out. The purpose of the present paper is to report the principal results of this study.

1.2 SCOPE

The present report has two primary intents. The first is to present in explicit terms a detailed discussion of the basic aerodynamic principles underlying the functional mechanisms of the flapping wing, proceeding stepwise through each phase of the flapping cycle in turn. The second is to develop in quantitative form the theoretical aerodynamic relationships necessary to describe the force and energy regimes of the flapping wing. With the latter relationships, it becomes possible by use of adequately detailed film data to establish much information about the force distributions, vortex wake characteristics, and energy requirements of the bird wing. As will be evident, a full quantitative analysis of the wing from photographic data using the theory developed in the subsequent chapters of
this report would be a rather involved undertaking.* Yet, the task is not impossible and it is the author's hope that such analyses will be carried out in the future. Such an analysis will permit the quantitative estimation of the power required for flapping flight for any bird on which sufficient experimental and photographic data are available, and hence allow an answer to the long-standing question of the relative efficiency between the conventional fixed-wing flight of aircraft and the flapping flight of birds. It would also provide valuable information on the physiological and ecological bases underlying the great endurance of birds, such as the arctic tern, which practice very long-distance migrations.

Although the present paper is written with the intent of making as much of its content as possible understandable and useful to the biologist, particularly the ornithologist, the report must, by the nature of the subject, be addressed to the biophysicist. The subject is highly technical in nature and can only be treated in proper and meaningful form by use of advanced aerodynamic concepts. On the other hand, the general aircraft aerodynamicist will in considering natural flapping flight require considerable familiarization study to get a proper perspective of the structure and motion of the avian wing, which possesses a large number of characteristics quite foreign to the usual aircraft wings with which he is familiar. It is not possible in the present report to cover the primary subject adequately, and simultaneously provide a full discussion of the necessary background concepts in aerodynamics, in the space allotted. Consequently, it has been necessary to exclude all detailed discussion of background concepts except in those sections where it contributes significantly to the basic analysis. For a review of pertinent aerodynamic concepts one is referred to some of the author's prior publications on soaring flight2-6,14 and to suitably advanced aerodynamics texts7.

The old adage that one picture is worth a thousand words was never more true than in reference to the flapping flight

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*The involvement would be somewhat of the same order of magnitude as the elucidation by x-ray crystallography of the structure of a complex biological macromolecule. In fact, these two apparently diverse studies actually have much in common, due to the similarities involved in the photographic solutions of spatial arrangements of structural elements in each case.
of birds. The complex, three-dimensional nature of the wing motions plus the hierarchy of relative velocities involved make the subject extremely difficult, if not impossible, to describe adequately in words alone. It is the author's conviction that a careful study of high-speed films of many different bird types under many different conditions is a prerequisite for a full understanding of the wing action in flapping flight. As detailed a verbal description of the significant wing movements as possible is given in the present paper, along with numerous sketches and photographs, but a thorough study of some of the basic flight films available with various flapping cycle sequences\textsuperscript{8-11} is highly recommended as a supplement to the present discussion.

1.3 BIONICAL APPLICATION TO ORNITHOPTER DESIGN

While this report is concerned primarily with establishing the theoretical aerodynamic basis of the flapping flight of birds, it should be noted that the analyses herein have direct application to the investigation and design of man-made flapping wing craft, the so-called ornithopters. In this sense, the results constitute a valuable bionics application of natural flapping flight. There has long been, and still remains, considerable interest in ornithopter development, primarily from the standpoint of man-powered flight. The prime interest in the flapping-wing craft lies in the (intuitive) feeling of many that the ornithopter should require less flight energy than conventional fixed-wing craft. In the past, however, the basic theory of the ornithopter has been so poorly developed that meaningful and functional designs (incorporating the most efficient features of the bird wing) have been impossible. The present theory will hopefully alleviate this situation since the results herein can be applied directly to the analysis of any proposed ornithopter design and the theoretical flight efficiency of the design evaluated. The present report also contains a number of important results regarding drag reduction mechanisms utilized by the avian wing which can be usefully employed in ornithopter design.

1.4 OUTLINE OF REPORT

The paper begins with a discussion of the classification scheme of the three principal flapping flight modes. Following that, a generalized treatment of the basic mechanical principles of flapping flight is given, and analogies between flapping
flight and forms of terrestrial locomotion and their evolutionary significances are pointed out. A detailed analysis of the aerodynamics of the three principal flapping flight modes is then presented, and this is followed by discussions of the energy requirements for flapping flight, and stability and control. The author's views on the importance of and the need for future research in flapping flight are summarized in the concluding remarks. Several appendices are included which give more detail on subjects which cannot be appropriately covered in the text.

II. CLASSIFICATION OF FLAPPING FLIGHT

The flapping flight of birds is characterized by the vigorous and clearly observable motion of the wing relative to the body. This is opposed to soaring flight of birds wherein the wing is essentially stationary relative to the body. A detailed classification of natural soaring flight is given in Reference 3. In static soaring, declivity or thermal upcurrents furnish, together with the forward (i.e., horizontal) motion of the bird, the proper relative aerodynamic velocity for production of adequate vertical lift (to counterbalance the weight) and horizontal thrust (to counterbalance the horizontal drag) to permit steady, constant-altitude (or climbing) flight. In essence, the bird is merely gliding down through a rising column of air, and hence relative to earth can maintain a constant altitude or even climb. (See Reference 2.) In dynamic soaring, such as practiced by the albatross and some other sea birds, the flight energy is also extracted from

*See Appendix I.

**In the literature, one often sees the term "motionless wings" in reference to soaring flight. The wings themselves are usually far from motionless, however, in an absolute sense, for only by (absolute) motion through the air can the necessary sustaining aerodynamic forces be obtained. What is meant by the term, of course, is "motionless relative to the bird's body." A more appropriate description from the technical standpoint is the term "fixed-wing."
moving air with a fixed wing, but the energy comes from the horizontal wind motion and the aerodynamic mechanism of the extraction process is much more involved. (See Reference 6 for a detailed treatment of dynamic soaring mechanics.) In flapping flight, the necessary flight energy comes from the bird itself, and the wing, by virtue of its motion relative to the air, produces directly the lift and thrust which sustain and propel the bird. In flapping flight there is a relative motion between the wing and the body, and consequently their separate absolute motions (relative to earth) are different. The nature and degree of this relative motion between wing and body serves as a convenient criterion for the classification of the various modes of flapping flight.

Despite the obvious variety and complexity of the wing motions associated with flapping flight, a detailed study of many high-speed films of numerous different bird species has revealed a common aerodynamic mechanism underlying all types of flapping flight, and the main difference of the wing motions in different modes has been found to be one of degree rather than of nature. In order to produce the forces necessary for flight, the wing must move relative to the air. The entire hierarchy of flapping-flight wing motions is thus merely a manifestation of the need for obtaining the proper relative velocities of the wing to produce a force system of proper magnitude and direction to allow the particular body trajectory (or directional travel) desired or required by the bird. Thus, the particular velocity pattern of the wing relative to the body serves to define a given flight mode and hence allows a convenient means for classifying flapping flight.

On the basis of the relative wing motions, flapping flight can be divided into three principal categories: fast flight, slow flight, and hovering. It must be realized, of course, that in reality the entire flapping-flight regime is continuous in nature. Starting with the stationary condition of hovering, the bird proceeds by subtle changes in wing motion progressively through the slow-flight mode and on into fast flight. Consequently, there are no sharp boundaries between each of the three flapping-flight modes; the three regions blend smoothly into one another at their extremities. Nevertheless, the overall flight regime can be meaningfully divided into the hovering; slow; and fast-speed regions, and the differences in the associated wing motions of each phase can be clearly distinguished.
Fast flight is the most commonly observed mode of flapping flight; it is the mode primarily used for normal translatory flight. Fast flight is characterized by the fact that the motion of the wing relative to the body is actually slow, while the body moves rapidly through the air; the term "fast" thus relates to the relative speed of the body (or of the translating bird as a whole). The wings in fast flight, although moving slowly relative to the body (as compared to the condition of slow flight), move rapidly relative to the air, since they move with the combined (vector) velocities of the wing relative to the body and the body relative to earth.* Consequently, the forces on the wings may be of appreciable magnitude. Fast flight must necessarily generate an appreciable horizontal thrust component with the wings to counteract the large aerodynamic drag of the body at high-flight speeds, and the wing must move with a relatively large vertical velocity component for level flight.

Slow flight is characterized by a (relatively) slow velocity of the body (relative to earth); the wing, however, moves quite rapidly relative to the body, much faster than in fast flight. The reason for this is obvious: since the body's absolute velocity is low, the aerodynamic velocity of the wing is essentially equivalent to the motion of the wing relative to the body, and hence the wing must be moved rapidly to generate the necessary forces. In slow flight, the horizontal aerodynamic drag of the body is small, and there is need for only a small thrust force. Consequently, the primary requirement in sustained slow flight is the production of vertical lift. This is accomplished by moving the wing in such a manner that the resultant wing force is essentially vertical, except when forward acceleration is desired, as in take-offs. Slow flight is used primarily during the early phases of transition from take-offs to fast flight and during the late phases of transition from fast flight to landings.

Hovering flight is characterized by the fact that the bird's body remains stationary relative to earth. In this case the wing

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*In most of the discussions of this paper, it is assumed that the air through which a bird is flying is still, relative to earth. Thus, the motion of the wing or the body relative to earth is also the velocity relative to air, and vice versa. All definitions of flapping-flight modes are, of course, based on the assumption of still air.
velocity relative to the body must be extremely rapid, and the wing must move in an essentially horizontal direction so that nearly all of the aerodynamic force acts vertically to counterbalance the bird's weight. Hovering is a very energy-consuming mode of flight and this, combined with the need for extremely rapid wing motions, makes true hovering impossible for many species of birds. True hovering, as described here, is not to be confused with pseudohovering, the latter term referring to flight into a headwind, wherein a bird, while actually performing slow or even fast flight relative to the air, is maintaining a stationary position relative to earth. The wing motions of a bird in either true or pseudohovering, as seen by a ground observer, will be quite different and can easily be distinguished. The wing motions can, in fact, be used by the experienced observer to estimate the relative wind speed at the altitude of the bird. Hovering is used as a usual flight mode only by a few species, such as the humming-bird; many species use hovering or pseudohovering for brief periods under special conditions, such as the preliminary to a vertical landing, or in aerial examination of ground food.

These three basic modes of flapping flight are subject to many alterations and variations among the avian populations. In the most general or most common type of flight, the bird performing steady flight in any one of the three primary modes executes a regular, truly periodic flapping of the wing and maintains its flight by a series of continuous, equally-spaced wing beats. In one variation of this basic theme, however, the flight remains periodic but consists of a burst of vigorous wing beats followed by a closing of the wing tightly against the body and a subsequent freefall along a ballistic trajectory. During the flapping period, the bird gains considerable forward momentum and altitude. Then during the ballistic fall, the altitude and forward momentum are reduced. By repeating this "flap-fall" pattern, the bird is able to maintain an effectively constant-altitude flight path. This type of flight is commonly observed in many of the smaller species, such as the sparrows. Just why this flight mode is used and what its advantages are, if any, over conventional flight methods is not truly known, although there are many possible explanations. For example, the flapping pattern may be keyed in with the general metabolic characteristics of the bird, wherein a rest period following vigorous flapping may be required. Alternately, it may prove to require less overall flight energy for small birds to fly regularly by a series of flaps and falls or coasts. In this connection, it should be noted that large, soaring birds with weak flapping
ability, such as the vultures and buteos, practice the flap-glide method of flight when thermal conditions are inadequate for soaring. The aerodynamic developments of the following chapters provide the theory necessary to evaluate the energy requirements of the various flight alternatives and hence offer a means for answering some of the intriguing questions just cited. Despite the various alternate flight schemes, the heart of flapping flight lies in the aerodynamics of the flapping cycle, so that the subsequent analyses are equally valid for all types of flapping flight.

For purposes of analysis, the flapping cycle can be broken up into two major parts, the downstroke or power stroke and the upstroke or recovery stroke, with short transition segments connecting the two. In the following chapters of this paper, aerodynamic analyses are carried out for "typical" flapping cycles such as might be observed for any of the larger birds, like the geese, herons, cranes, egrets, ibises, pelicans, and such. The basic analysis is developed in general terms, and hence may be applied to the wing motions of the cycle of any one of the three principal flight modes. In general, continuous periodic motion is assumed, but this is not a prerequisite for future analytical application of the aerodynamic methods developed. For the most part, the development is restricted to considerations of quasisteady, constant altitude flight. While a general discussion of the basic aspects of maneuvering flight is presented, no attempt is made to cover the quantitative details of that subject. The great scope of maneuvering flight with flapping wings is worthy of a large research effort in its own right.

Before entering into the details of the aerodynamic mechanisms of the flapping-flight cycle, however, it will be worthwhile and instructive to examine in some depth the overall force systems involved in aerial locomotion with flapping wings and to compare the basic mechanics with other types of locomotion.

III. BASIC PRINCIPLES OF FLIGHT WITH FLAPPING WINGS

The basic plan of flapping birdflight is, in its essential features, quite simple. The practical implementation of this
plan, however, requires a host of exceptionally complex aerodynamic and structural accommodations. This complexity is a result of the use by the flapping bird of unsteady aerodynamic force systems for its locomotion. On the other hand, the ability to utilize such unsteady forces allows the bird an amazing degree of flight versatility, one paralleled only by the flapping-winged insects. In the following parts of this chapter, the primary mechanical principles on which flapping birdflight is based are set forth in quantitative form in order to make the general scheme by which the bird sustains and propels itself explicitly clear and to serve as the essential basis for subsequent aerodynamic analyses.

3.1 ANALOGY BETWEEN FLAPPING FLIGHT AND TERRESTRIAL LOCOMOTION

The mechanical principles involved in flapping flight are precisely analogous to those of hopping or walking locomotion on land. So exact is the analogy that one might say quite accurately that the bird in its evolution merely walked (or more precisely, hopped) into the air. Thus, the transition from land-based locomotion to aerial locomotion is not really so great a jump at all, at least from the standpoint of the essential mechanical principles involved. From a practical standpoint, however, the "jump" is truly enormous in its significance. The details of this analogy are described in the following two sections.

3.1.1 The Mechanics of Hopping and Walking Locomotion on Solid Surfaces

We shall first consider the case of hopping locomotion because of its simplicity and symmetrical motions, i.e., the corresponding members of each pair of limbs involved in the locomotion move symmetrically with regard to the morphological plane of symmetry of the animal. The two halves of the animal are essentially mirror images during motion. Examples of biped hopping are given by the kangaroo and many species of birds; rabbits are an example of quadruped hopping. In walking locomotion, where corresponding limbs are not moved symmetrically, the identical principles apply but the asymmetry of motion makes the situation somewhat more complex.

In hopping, an animal's primary purpose is to translate its body relative to the surface of the earth. In the usual
case, on flat land, the direction of this motion is tangential (i.e., parallel) to the land surface. To accomplish this movement, the animal contracts its "locomotor" muscles which, being attached to the lever arrangement constituting the limb system, cause a deformation (usually an extension or straightening) of the limbs (e.g., the legs), as illustrated in highly simplified form in Figure 1 for the case of a biped. Here the body is represented as a concentrated mass for simplicity. The directions of rotation about the two pivot points, as caused by the muscular contractions, are as indicated. The muscular contraction results in the exertion on the body by the limb system of a resultant contact force having upward and forward components as shown. Neglecting for the moment the (relatively small) forces associated with the acceleration of the mass components of the limb system itself, the "foot" of the limb system will exert a force on the surface which is exactly equal and opposite to the resultant force the system exerts on the body. The limb system is, therefore, simply a device for creating a force to impart momentum to the body by pushing against the earth's surface.

Under the action of the applied force, the body will accelerate vertically and horizontally. The magnitude of this vertical acceleration will depend upon the difference between the vertical applied force component and the weight of the body. It also will depend upon the mass of the body, of course, but the mass is clearly specified when the weight is given, through the relation: mass = weight/gravitational acceleration constant. The horizontal acceleration will depend upon the magnitude of the applied horizontal force component and upon the body mass. The vector contact-force components in the vertical and horizontal directions vary, in general, with time, and hence the corresponding accelerations of the body will vary. The time integrals of the acceleration components, taken over the period of time during which the applied force acts, give the changes in the corresponding velocity components of the body which occur.
during this period. In turn, the time integrals of the velocity components give the displacement coordinates of the body during the period of application of the force. In alternate terminology, the vector impulse of the force exerted by the limb system on the body gives the net change in momentum experienced by the body. The vector impulse of the force on the body is also given by the negative of the vector impulse of the ground contact force by Newton's third law, still neglecting the small acceleration forces due to motion of the components of the limb system.

The upward and forward momentum imparted to the body by the limb impulse carries the body upward and forward relative to the earth. With continuing extension of the limb system, the limb force ultimately decreases to zero, but the acquired vertical momentum of the body continues to "carry" it upward for a time against the downward acting weight force, and the limb system is lifted free of the ground. During this time interval, the limb system is free to be flexed back into a "closed" position relative to the body, the body weight (and the additional downward-acting forces caused by the upward acceleration of the limb system components during recovery) being "supported" by the vertical momentum of the body. Likewise, the horizontal momentum acquired from the impulse accompanying the limb extension carries the body forward against the air resistance and against the horizontal decelerating forces introduced by the forward acceleration of the mass components of the limb system during the recovery movement of the limbs. In general, the accelerational forces associated with movement of the limb elements during both the power impulse and the recovery motions are of necessity relatively small compared to the basic weight and the primary impulsive force exerted by the limb system on the body. The nature of these acceleration forces associated with movements of the limb components will be discussed in detail later in regard to their action in the case of the flapping wing.

Ultimately, the downward-acting impulse of the weight and limb recovery forces reduces the vertical momentum of the body to zero, and the body is then accelerated downward and acquires a falling velocity. The air resistance and recovery forces being small in general compared to the imparted horizontal momentum of the body, the body continues its forward horizontal motion with relatively slight deceleration. The falling velocity ultimately brings the feet into contact with the ground, whence the relatively large vertical impact force brings the falling
velocity to zero. There may or may not be a horizontally-decelerating component of this impact force, depending on the manner in which the feet contact the ground. In general, for active forward locomotion it is desirable that there be no decelerating horizontal impact force so that the forward horizontal velocity of the body and its associated kinetic energy may be preserved, thus resulting in more rapid and efficient forward locomotion.

Upon contacting the solid surface, the recovered limb system is once again extended by muscular contraction and the impulsive force system is again brought to act on the body. Properly synchronized and continuous repetition of this periodic impulsive action results in the seemingly smooth but highly impulsive propulsion of the body over the surface. Although the motion may appear to an observer to be relatively smooth and gentle, the forces acting and their corresponding accelerations (which are felt by the animal) may be quite large. What is seen by the observer is the velocity change, which is small due to the short times over which the accelerations act. In reality, however, rapid fluctuations of very large forces may be involved.

There are, of course, many variations of the mechanical sequences described above, depending upon the particular structure and locomotion needs of the particular type of animal. These variations all involve, however, the same fundamental principles of the impulsive force regime described.

In starting from rest, the horizontal accelerating force may be much larger than that which is subsequently needed to keep the body moving horizontally once forward motion has been initiated. Likewise, in bringing the body to rest, the manner in which the feet contact the ground must be such as to produce a sufficiently large horizontal decelerating force upon contact with the ground. The desired contact force is attained by proper positioning of the various limb components relative to the body prior to contact.

Change of direction of motion is effected by increasing the horizontal contact impulse of one limb relative to that of the other, thus producing a rotation of the body about its vertical axis during the ensuing translation. As a result of this rotation, the animal upon subsequent surface contact lands facing in a slightly different direction, and the following
"hop" will carry the body in the new direction. A series of such incremental rotations may be required to effect a complete turn, as illustrated in Figure 2.

The required vertical and horizontal momentum changes of the body needed for hopping locomotion are obtained by imparting an equal and opposite momentum change to the earth, through the agent of the surface contact force, as required by the consideration of momentum. The surface contact force consists of the normal pressure on the surface and the tangential friction force (see Figure 1). In theory, hopping locomotion is relatively efficient from an energy viewpoint. For a given momentum change of the body, the velocities imparted to the body and to the earth are inversely proportional to the respective masses. Since the kinetic energy is proportional to the product of the mass and square of the velocity, the kinetic energy imparted to the earth is exceedingly small because the associated velocity change is extremely small. Thus, nearly all of the work done by the locomotor muscles during a hop goes into increasing the kinetic energy of the body. In practice, the process is usually not quite so efficient. The natural ground surface on which most animals operate is not truly solid but rather loose and springy. Consequently, the surface "gives" locally under the action of the contact force and a finite velocity is imparted to a finite mass of earth, making the locomotive process considerably less efficient than the theoretical solid-surface value. The clumps of sod thrown rearward by a trotting horse are an example of this situation. The same action can be routinely observed in the take-off of water birds, such as ducks, where the feet are used to give added forward impulse before the bird rises from the surface. The webbed feet throw large masses of water rearward as the bird "runs" along the surface. In order to minimize such losses, it is desirable that the normal contact force component of the foot be large before the rearward tangential component increases to a large value in order to increase the reactive
friction force and thus prevent wasteful rearward sliding of the foot and displacement of local earth masses.

In addition to these factors, it must be realized, of course, that the mere process of hopping (or walking) involves an inherent energy output. When the body is accelerated upward, kinetic energy is imparted. This energy comes from muscular work. Upon subsequently contacting the surface, this energy (associated with the vertical velocity component) is usually dissipated as heat of impact (through surface deformation). Only if the surface-foot impact were truly elastic could the originally imparted kinetic energy be conserved and thus eliminate the need for this inherent energy input on each jump.

The above discussion illustrates the case of biped hopping over level ground. Many species of animals also practice quadruped hopping (e.g., the rabbit). The basic principles governing the latter are precisely the same as for the former type of motion, only the impulsive forces of two synchronized limb systems must now be considered. On inclined surfaces, the weight vector has both normal and tangential components relative to the surface and these components, of course, alter the required magnitudes and directions of the surface contact forces of the feet for moving uphill or downhill. In particular the tangential friction force at the ground is altered in proportion to the steepness of the incline.

The above principles of hopping may also be directly applied to the case of both biped and quadruped walking. In walking, the recovery movements of the limbs on one side of the symmetry plane take place while the impulsive contact force of the other limbs is still acting. In this way the action of the impulsive forces which power the locomotion is made more continuous and a still smoother motion is obtained; in essence, the frequency at which the impulsive forces are applied is doubled and hence the applied force systems become more nearly continuous in their action.

3.1.2 Analogous Principles in Flapping Flight Locomotion

A bird in level flapping flight is free from direct contact with the earth. Consequently, the required momentum change necessary to balance the impulse of the bird's weight during a given time interval must be obtained by imparting a vertical
velocity to a finite mass of the air through which the bird is moving. Likewise, the impulse of the aerodynamic resistance to the forward motion of the bird through the air requires the impartation of an equivalent rearward momentum to the air in order to maintain a desired forward velocity. Thus, in the bird, the wings become the agents for creating the required systems of impulsive contact forces by pressing upon the air, which now takes the place of the earth surface in the analogous terrestrial locomotion. The wings are the analog of the terrestrial limbs, while the air becomes the analog of the earth surface. Since the required momentum change is now obtained by imparting a relatively large velocity to a relatively small mass of air, compared to the corresponding velocities and masses involved in the case of direct earth contact, it may be expected that, on a purely theoretical basis, aerial locomotion will be less efficient than terrestrial locomotion. However, in practical situations the effective difference may not be great due to the appreciable energy losses associated with the nonrigid nature of most land surfaces.

In reality, the vertical component of the contact force of the wing with the air is transmitted to the earth's surface in the form of an increased pressure at the surface, and hence the bird's weight is actually supported (ultimately) by the earth. The form of this pressure distribution is readily calculated by simple vortex theory for conventional nonflapping aircraft wings (see Reference 13, p. 186, for example); for flapping wings the same principles apply, but the calculations become very difficult because of geometrical complexities.

The flapping-wing motions of birds, to be described more fully in following sections, are directly analogous to hopping terrestrial locomotion. The downstroke of the wing by reaction with the air produces impulsive vertical and horizontal force components which act to raise the bird upward and translate it forward against the weight and aerodynamic resistance forces, respectively. It is equivalent to the extension of the limb system in hopping. In addition, the same forces as are associated with acceleration of the limb system are active in the wing mass components. Thus, during the downstroke of the wings, the body experiences a lifting force just due to the downstroke acceleration of the wing mass itself, in addition to the aerodynamic force. Although the wing mass is usually very small, the wing is highly accelerated during the downstroke and the resulting reaction force on the body may not be entirely negligible.
The resulting net vertical and horizontal forces acting on the bird's body during the downstroke accelerate it upward and forward and lead to net vertical and horizontal velocity increases or, correspondingly, to net momentum increases whose amount depends upon the magnitudes of the acceleration components and their durations.

At the completion of the downstroke, the impulsive forces exerted by the wing system on the body decrease, particularly the forward horizontal component. The body will then be accelerated downward by the weight force, or more properly the net vertical force resulting from subtraction of the upward vertical forces (aerodynamic and wing accelerational forces) from the weight, and accelerated rearward by the air resistance. During this time the wings are moved through a recovery trajectory relative to the body which brings them into position for the subsequent downstroke. The momentum components acquired during the previous downstroke are utilized during the recovery to satisfy the impulse of the net vertical and horizontal forces, so as to prevent any appreciable buildup of downward or rearward velocity increments during recovery. In this way the bird is able to maintain an effectively "level, straight-line" course at effectively "constant" speed.

From the foregoing, the importance of the proper balancing and regulation of all the various force components and their deviations (i.e., the impulse system), along with the maintenance of precise synchronization, for effective locomotion by flapping flight is evident. In the case of the bird, the arrest of its downward fall during recovery of the wings does not occur automatically by contact with the ground as in the case of terrestrial locomotion; it must be arrested by the conscious effort of the subsequent downstroke. This requires much more exacting control for flight as compared to ground movement.

3.2 MECHANICAL BASIS OF FLAPPING LOCOMOTION

3.2.1 Steady versus Unsteady Force Regimes

The straight-line movement of a body through the atmosphere at constant altitude can be accomplished by application to the body of a system of steady forces, as in the conventional fixed-wing airplane. Such motion can be accomplished just as effectively, however, by application of a suitable system of unsteady, impulsive forces, i.e., a force system whose various
vector components continuously vary with time. Although an effectively linear, constant-altitude trajectory can be achieved by an almost limitless variety of unsteady force patterns, each of these patterns must satisfy several quite specific kinematic and dynamic requirements; when the time-variation functions of certain of the force components are specified, these basic requirements serve to determine what the remaining force components must be.

In the steady straight-line flight through still air at constant altitude of a conventional airplane, the net force acting on the craft is zero. As indicated in Figure 3, where the (rigid) aircraft is represented as a point mass translating with constant velocity (V), the aerodynamic lift (L) balances the craft's weight (W), while the thrust (T) balances the aerodynamic drag (D). In the typical case of the propeller-driven craft, the thrust is also of aerodynamic origin, of course.

Such flight is wholly steady relative to the aircraft frame of reference and the center-of-gravity (c.g.) of the craft experiences no accelerations as it translates along a straight, level flight trajectory at constant speed.*

The attainment of a steady force system by the conventional airplane is made possible by the action of the revolving propeller which permits the maintenance of a steady thrust force. With this mechanical arrangement the wings are responsible only for the production of the static lift force, and hence can be

*In reality, the flight is not strictly "level," of course, but follows a curved path conforming to the curvature of the earth's surface. Also, in practice, the truly steady force balance is seldom achieved due to the normally present air turbulence, and the force balance is actually an average of the varying force conditions. We are, of course, considering the idealized case here. (See Appendix II.)
rigidly mounted to the aircraft body. In birds, however, the capability of producing a steady thrust of the conventional propeller type is precluded by the nonexistence of any mechanical means for attaining free rotation of a muscle-driven propulsion member of the propeller type.* Consequently, the bird must attain its flight goals by using its wing surfaces to create both the lift and the thrust needed to sustain and propel the body. The morphology of the wing in turn requires that the force system be unsteady, since the wing must be recovered after each power stroke. In practice, the unsteady force system of the flapping wing is essentially periodic.

In the locomotive limb systems evolved by nature to date, the structural members are living units and the need to supply these units with blood for nourishment has made necessary the direct connection of blood vessels to the limb members, thus precluding the possibility of free rotation at joints. This is not to say that evolvement of a freely rotating joint in the living system is impossible, but merely that such a joint has not been evolved to date. Indeed, there are several ways in which such a functional joint could conceivably be constructed, although the morphology and morphogenesis of such rotating joints would be rather involved. In any event, the conventional limb system of simple levers, which presently constitutes the wings of birds, has apparently been found quite adequate for avian needs. The amazing versatility of aerodynamic force variation obtainable through the controlled surface deformation and vectored motion of the wing, afforded in turn by the present lever system, is an extremely valuable asset from an overall ecological view.

3.2.2 Dynamic Relationships and Requirements

Consider the case in which the bird's body is translating along an effectively straight and level path at "constant" altitude under the action of an unsteady periodic force system. For the present we shall consider the body as a simple

*The discussion here applies to propulsion systems of the revolving propeller type. Evolvement of propulsion systems of the jet-reaction type are also a possibility in birdflight, of course, but efficiency considerations seem to preclude the successful evolvement of such systems by birds. This subject is discussed in Appendix III.
point mass and shall ignore the physical existence of the wings, considering them only as agents for applying an impulsive contact force to the body at the wing connections. As noted in Figure 4, the actual trajectory of the body will no longer be a truly linear path, but will undulate in response to the action of the impulsive force system applied to the body. The average of the vertical displacement $(\bar{z} - z_0)$ from the straight line $z = z_0$ over the duration of each undulation is zero, however, so that the effective flight path is linear, and on the average the altitude is effectively constant, $\bar{z} = z_0$.

The downstroke of the bird's wing results in the impulsive application of a contact force to the bird's body through the wing joint connection; this force has vector components acting vertically upward and horizontally forward relative to the weight force vector, which lies along the negative $z$ axis. The body also experiences the constant body weight force $W_b$ and a rearward force due to its aerodynamic resistance. This latter force component, being a contact force, can be combined with the wing-joint contact force to produce a single contact-force vector having vertical and horizontal components $F_y$ and $F_x$. This force system is illustrated in Figure 5, where the bird's body is represented as a point mass with applied forces $F_y$, $F_x$, $W_b$. The forces $F_y$ and $F_x$ are, in general, time-dependent. $V$ is the effective average velocity of linear translation of the body along the $x$ axis.
During the downstroke, $F_y > W_y$ and $F_x > 0$ (i.e., $F_x$ is positive, acting in the direction of the positive $x$ axis). Consequently, the bird is accelerating upward and forward. The instantaneous magnitudes of these accelerations are given by

$$\ddot{y} = \frac{F_y - W_y}{W_y/q} > 0$$ \hspace{1cm} (1)

$$\ddot{x} = \frac{F_x}{W_y/q} > 0$$ \hspace{1cm} (2)

and in each case are positive.

Since the applied forces $F_y$ and $F_x$ are time-variable, the accelerations will also be functions of time, and the body will experience a continuously varying acceleration. At the end of the downstroke the wings must be recovered in preparation for another downstroke. During the recovery (i.e., upstroke), the conditions

$$F_y < W_y \quad \text{and} \quad F_x < 0$$

prevail and the corresponding "decelerations" (negative accelerations) are given by

$$\ddot{y} = \frac{F_y - W_y}{W_y/q} < 0$$ \hspace{1cm} (3)

$$\ddot{x} = \frac{F_x}{W_y/q} < 0$$ \hspace{1cm} (4)

These accelerations are also, in general, functions of time. Schematic representations of the force variations over the period of one full wing cycle (downstroke-upstroke) are shown in Figure 6. The acceleration vectors $\ddot{y}$ and $\ddot{x}$ are, of course, directly proportional in magnitude to the corresponding force vectors $\vec{F}_y$ and $\vec{F}_x$, and act in the same directions. The total net force vector acting to accele-
rate the body is \( \left( \hat{F}_3 + \hat{W}_b + \hat{F}_x \right) \) and the corresponding vector accelerations are related by

\[
\left( \hat{F}_3 + \hat{W}_b + \hat{F}_x \right) = \left( \hat{x} + \hat{j} \right) \frac{g}{w_b}
\]  

The net force system may be conveniently represented in graphical form by a two-dimensional curve in the \( F_3, F_x \) plane with proper marking of curve segments corresponding to equal time increments, as shown in Figure 7. A vector drawn to any time-point on the curve gives the magnitude and direction of the net force vector acting on the body at that time. The time scale is, of course, not linear in this mode of representation. Increasing the length of the vector by the factor \( \frac{g}{w_b} \) yields the corresponding acceleration vector as a three-dimensional space curve in \( F_3 - W_b, F_x \), and \( \tau \), as illustrated in Figure 8. The curve will be periodic, the shape repeating itself every \( T \) time units along the \( \tau \) axis (a length corresponding to the flapping cycle period \( T \)). In this representation the time scale is linear. The vector variation could be equally well expressed in the cylindrical coordinates \( F_3, \Theta, \tau \); this coordinate system often proves more convenient for certain considerations where actual numerical computation is necessary. In cylindrical coordinates
\[ F = | \hat{F}_3 + \hat{W}_b + \hat{F}_x | \]  
(6)

\[ \Theta = t_{aw}^{-1} \left| \frac{\hat{F}_3 + \hat{W}_b}{|\hat{F}_x|} \right| \]  
(7)

Such graphical representations give a clear, concise picture of the impulsive nature of the vectored force system and its time variation.

While the curves of Figures 7 and 8 may take on a wide variety of shapes and still represent the condition of effectively constant altitude flight, definite requirements are imposed on the overall characteristics of the curves. Since the force variation is periodic, the conditions (i.e., altitude, velocity, acceleration, etc.) at any two instants \( t_i \) and \( t_2 \), an interval \( T \) time units apart \( (t_2 - t_i = T) \), where \( T \) is the cycle period) must be identical. This in turn requires that the vector impulse of the total force system acting on the body be zero, when the integration is carried out over any time interval \( T \) units in length (i.e., over a time interval equal to the period of the cycle)

\[ \int_{t_i}^{t_2} (\hat{F}_3 + \hat{W}_b + \hat{F}_x) \, dt = 0 \]  
(8)

This is equivalent, of course, to saying that the vector momentum of the body at any two instants \( t_i \) and \( t_i + nT \) (where \( n \) is any positive integer) must be identical.

This condition can be conveniently interpreted in terms of Figure 8, where it is seen that Equation (8) requires that the area of the surface formed by connecting the \( t \) axis and the curve with a set of elements parallel to the \( F_3 - W_b \), \( F_x \) plane, be zero. Here portions of the surface lying outside the positive first quadrant of the \( F_3 - W_b \), \( F_x \), \( t \) axis system are taken as having a negative value. Expanding Equation (8) we have the impulse conditions

\[ \int_{t_1}^{t_1 + T} (F_3 - W_b) \, dt = 0 \]

\[ \int_{t_1}^{t_1 + T} F_3(t) \, dt = -W_b T \]
In the case of the physical wing, both $F_j$ and $F_x$ are produced simultaneously by the same wing surface and hence are strongly interrelated; for a given wing under given conditions of motion $F_x$ is implied when $F_j$ is given, and vice versa. Some useful degree of altering the relation between $F_j$ and $F_x$ is provided in the bird's wing, however, by the wing's flexibility and the bird's positive control over the surface shape and contour through voluntary muscular action. The terms $F_j$ and $F_x$, particularly the latter, also involve aerodynamic forces acting on the body, as well as on the wing. The form of the functions $F_j(t)$ and $F_x(t)$ will be characteristic for each particular bird type and manner of flapping.

3.2.3 Kinematic Relationships and Requirements

The body accelerations discussed in the preceding section lead to generation of horizontal and vertical velocity components, $\dot{x}$ and $\dot{y}$ respectively, according to the relations

$$\dot{x} = \dot{x}_o + \int_0^t \ddot{x} \, dt$$

(9)

$$\dot{y} = \dot{y}_o + \int_0^t \ddot{y} \, dt$$

(10)

where $t$ denotes integration over the time interval of interest and $\dot{x}_o$ the corresponding velocity at the start of the interval. The velocities $\dot{x}$ and $\dot{y}$ are, in general, functions of time: $\dot{x} = \dot{x}(t)$, $\dot{y} = \dot{y}(t)$. Also, since the motion is periodic in flapping flight, the total time period of interest extends over one flapping cycle only.

The velocity components in turn lead to changes in position, or displacements, of the body according to the relations

$$x = x_o + \int_0^t \dot{x} \, dt$$

(11)

$$y = y_o + \int_0^t \dot{y} \, dt$$

(12)

The position coordinates of the body, like the velocity components, are functions of time and are also periodic.
These simple relations are of basic importance. The prime requirement in flapping flight is to generate an effective flight path which will meet the locomotion needs of the bird. For example, if the bird desires to fly over a forest, the maximum tree-height of which is some value $\varphi_0$, say, then to clear the tree tops the bird's minimum altitude $\varphi_m$ at any point of its actual trajectory must at least slightly exceed $\varphi_0$. Thus the boundary condition $\varphi_m > \varphi_0$ is imposed for all points of the trajectory and this in turn sets the characteristics which the physical wing system must possess in order to satisfy the flight requirements. This condition is illustrated in Figure 9.

The undulations of trajectory a (i.e., the variations of $\varphi$ about some mean value $\varphi_1$, caused by the magnitude and timing of the applied forces) during a cycle are small enough that the bird just maintains the required $\varphi_1$ altitude. In trajectory b, the undulations are much too large to permit clearance of the tree tops. Since many birds fly only at the minimum altitudes necessary to clear such obstacles and since the obstacle height may vary continuously, accuracy and safety of flight require that the amplitude of the undulations be small in such cases. As will be discussed later, the magnitude of the flight trajectory undulations is also closely associated with the flight energy requirements. Analogous conditions apply to climbing or descending flight, of course, and also to the case of maneuvering flight, with proper accounting for the altered orientation of the weight and other applied force vectors relative to the desired flight path.

From the previous discussion, it is clear that the magnitude and frequency of the trajectory undulations in flapping flight are governed directly by the properties of the impulsive force system applied to the bird's body (i.e., the wing forces, the body air resistance, and the body weight). If an essentially linear, level trajectory is desired (as is actually observed in the case of most birds for normal travel), the fluctuation of the vertical velocity component $\dot{\varphi}$ about zero
must be small or else of high frequency. Otherwise, large changes in $f$ would occur, according to Equation (12). This in turn requires that the fluctuations about zero of the vertical acceleration $\dot{y}(t)$ be small and/or of high frequency to prevent large velocity buildups in either the positive or negative directions. Since the body weight force acts continuously, the average value of the vertical force component $F_y$ must be very large relative to $W_b$ if the frequency of application of $F_y$ is small (i.e., if the bird has a long cycle period). This condition is not compatible with aerodynamic theory, however, in that a long cycle implies slow wing motions and hence low force production. For a more linear trajectory, it is necessary that the average value of $F_y$ over the period of a cycle be equal to $W_b$ and that any variations which occur in $F_y$ be small and of high frequency.

The same principle applies to the horizontal acceleration and velocity regimes. However, while for constant altitude flight, the relation

$$\int_{t_i}^{t_i+\pi T} \dot{y} \, dt = 0$$

must hold so that the net or average vertical velocity is zero, the average horizontal velocity is not zero but equals the average displacement or flight velocity $\bar{V}$,

$$\bar{V} = \frac{1}{\pi T} \int_{t_i}^{t_i+\pi T} \dot{x} \, dt$$

It is again desirable from the standpoint of accurate trajectories and efficiency of flight to maintain the instantaneous horizontal velocity $\dot{x}$ as nearly constant as possible. Consequently, for a nearly uniform horizontal velocity, the fluctuations of the force component $F_x$ must also be small and/or of high frequency; however, the average value of $F_x$ over the cycle must also be zero.*

*It must be remembered that here $F_x$ is the sum of the body resistance (which is always negative in value) and the applied wing horizontal force component (which is positive in value over much of the cycle). A detailed description of these forces follows in a later section (4.2.2).
3.2.4 Source of the Accelerational Forces

The actual physical sources of the generalized force system discussed in the previous sections are, of course, the aerodynamic contact forces associated with the surface stress distribution existing over the wings and body of the bird, and the bird's weight. The motion of the wings and body relative to the air results in the impartation of momentum to the air, the wing and body simultaneously experiencing an aerodynamic reaction force \( \hat{A} \), which can be resolved into the vertical and horizontal \( \hat{F}_v \) and \( \hat{F}_h \). This force is a resultant of all the normal (pressure, \( \rho \)) and tangential (friction, \( \tau \)) stresses integrated over the entire surface of the bird.

\[
\hat{A} = \iiint \rho \, dS + \iiint \tau \, dS \tag{15}
\]

The aerodynamic force \( \hat{A} \) is a continuous function of time, both in magnitude and in direction. In essentially level flight, the aerodynamic force component contributed by the bird's body is primarily horizontal and acts continuously to decelerate the bird. The component contributed by the wings, however, varies greatly over the course of a flapping cycle, its magnitude and direction depending upon the instantaneous shape, area, and velocity relative to the air (which, of course, may be quite different from that of the body). In reality, the relative aerodynamic velocity seen by each section of the wing is usually quite different. Additionally, the local shape and aerodynamic orientation of each section of the wing may vary continuously throughout the cycle. Hence, it can be readily appreciated that the treatment of the flapping-wing system of birds is an infinitely more involved endeavor than for the simple case of a conventional aircraft in level flight. In the case of the bird, it is no longer possible to consider simply the total "lift" and "drag" force components with respect to a constant aerodynamic velocity for the entire wing; it is necessary to consider the local force intensity on every individual element of surface.

The weight force \( \hat{W} \) is constant in magnitude and direction for all practical purposes during a given flight and acts vertically downward. This vector is given by the expression

\[
\hat{W} = \hat{q} \iiint \rho \, dV \tag{16}
\]
where the integral is taken over all volume elements of the bird. More will be said about the weight vector in the following section.

3.2.5 Rigid versus Elastic Flight Systems

A rigid flight system is by definition one in which all mass elements of the system maintain an essentially constant position relative to one another, or to some set of reference axes fixed in the system. The conventional fixed-wing aircraft is an example of a rigid flight system.* The designation of a flight system as rigid inherently implies that the system possesses a permanently fixed center of gravity (c.g.), and this fact greatly simplifies the aerodynamic analysis of such systems. The constant c.g. is usually taken as the origin of the coordinate system in such analyses, since it gives the point through which the weight vector acts, regardless of the spatial orientation of the aircraft relative to the ground surface. This condition of rigidity enormously simplifies the aerodynamic analyses of such systems.

An elastic, or more properly a nonrigid, flight system is one in which the mass elements do not maintain a constant fixed position relative to one another, but rather the mass distribution may vary considerably in relation to some fixed point in the system. The flapping bird is an excellent example of a nonrigid flight system. The position of the wings relative to some fixed point in the body, say a fixed point in the plane of symmetry of the sternum bone, for example, may vary greatly over the course of a flapping cycle. In landings and take-offs, the positions of mass elements of the neck, head, and even feet may also alter considerably. Consequently, while there exists a definite point in the bird which may be designated the "c-of-g" at any particular instant, the position of this point changes continuously as the geometry of the bird (and hence the distribution of mass) changes, and the conventional concept of a "c-of-g" has little meaning then. This situation of nonrigidity

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*No flight system is perfectly rigid, of course, there being some slight bending or deflection of any structure under load. However, these deformations are negligible and have no aerodynamic importance in the usual case.
in flapping birds introduces considerable complexity into the
analysis of the flight mechanism and requires the consideration
of each individual element of mass.

Nonrigidity of a flight system may arise from either or
both of two factors. First, it may arise from the relative
movement of two components of the system which, in themselves,
can be considered as rigid entities. An example is the motion
of a rigid (constant shape) wing relative to a rigid (constant
shape) body, through a hinging articulation at the shoulder
joint of the bird. Secondly, it may arise from a truly elastic
deformation, in which a component of the flight system alters
its shape (and hence mass distribution) as a result of the aero-
dynamic, weight, or inertial loads imposed upon it during flight.
In the flapping bird, both conditions prevail simultaneously and
must be accounted for.

The second factor above is particularly complex in nature,
since the shape change under aerodynamic load "feeds back" to
alter the aerodynamic loading (i.e., load intensity distribu-
tion) on the elastic component, which in turn causes an altera-
tion of the elastic deformation, and so on, until some stable
balance is reached. This balance may be either static wherein
the component assumes a steady shape, or it may be vibratory
wherein the shape rapidly oscillates (or flutters) over a range
of shapes between two limiting shapes. In birds, the primary
aeroelastic phenomena involve the pinion or primary wing feath-
ers, as will be discussed subsequently.

One other factor should be mentioned briefly here. The
motion of a bird's wing in flapping flight involves almost con-
tinuous acceleration (positive and negative). Hence, the force
which the body of the bird exerts on the wing during a down-
stroke (through the various muscle and tendon connections at
the wing joint) is greater than that needed to move the wing
against the existing aerodynamic forces alone; it must also
provide the impetus for the instantaneous accelerations (so-
called inertial "forces") of each element of the wing mass.*

*The nature of the accelerational force can be easily
visualized from the following elementary experiment. If a person
stands on a "weight" scale with arms extended horizontally or
overhead, his static weight will be registered. If now the arms
are forced rapidly downward by violent muscular contraction, the
weight registered by the scale (continued bottom of next page)
Since it is the time-integrated accelerations which produce the necessary local aerodynamic velocities, which in turn lead to the wing aerodynamic force, it can be readily seen that the mass distribution of the wing is intimately connected with the aerodynamic, structural, and muscular factors involved in flapping flight. The force exerted on the body by the wing is, of course, equal and opposite to that exerted on the wing by the body and hence is the primary force sustaining and propelling the bird. This nature and involvement of the wing accelerational forces will be covered in considerable detail subsequently.

3.3 MULTIFUNCTIONAL WING REQUIREMENTS IN FLAPPING FLIGHT

In the conventional airplane, the sources of lift and thrust, or more appropriately for level flight \( \mathbf{F}_y \) and \( \mathbf{F}_x \), are essentially independent. While this independence leads to great mechanical simplification for level, steady flight, it imposes some severe limitations on other aspects of the flight regime, e.g., the take-off, landing, slow flight, and hovering capabilities. In the case of the flapping bird, where such operational aspects as almost vertical pinpoint take-offs and landings, maneuverability, and hovering ability are absolute requirements for its survival, the wing must possess a host of intricate functional capabilities.

In such tasks as take-offs and landings, high aerodynamic efficiency is scarcely of concern to the bird, the main requirement being the ability to accomplish such operations accurately and safely. On the other hand, the properties of the wing must be such that long-distance level flight is possible will be much reduced. The vertical force of the muscles in accelerating the mass of the arms downward has, by reaction, "lifted" the body with an equal force. This force acts as long as the arms are being accelerated. In effect, the arms can be pushed down by the muscles only because the muscles are able to push upward against the body which is held down by its weight (conservation of momentum). If the muscles were strong enough, the entire body weight (i.e., body weight minus the weight of the arms) could be lifted from the surface by imparting a sufficiently large acceleration to the arms. The opposite effect occurs, of course, when the arms are raised.
with relatively high efficiency. For many species, however, the significance of this requirement is greatly reduced by the ability to land and takeoff from practically any terrain type. Many species possessing relatively inefficient aerodynamic wing forms make long migratory journeys by means of short hops with feeding stops in between, the situation here being that the bird is able to secure ample food to match its flight energy requirements.\textsuperscript{14}

For the bird wing, the sources of lift and propulsion for level flight are combined in the same aerodynamic surface and hence are intimately related. In addition, the host of necessary auxiliary operational requirements is accomplished by the same surface. It should not be surprising then that the flapping-bird wing constitutes one of the most complex and versatile locomotor appendages yet evolved in nature. It will be the task of the following chapter to formulate and explain in quantitative aerodynamical terms just how the flapping wing accomplishes these multifunctional requirements, and to indicate the precise manner in which the wing integrates and controls the lifting and thrust forces to achieve its varied flight goals.

IV. AERODYNAMICS OF FLAPPING FLIGHT

In this chapter we shall be concerned with the aerodynamic formulation and analysis of the three principal modes of flapping flight as defined in Chapter II, and with a consideration of the transient force systems by which the transition from one flight mode to another is accomplished. For quantitative analysis of the flapping-flight cycle, it is necessary to define the various phases of the cycle in a more precise manner than previously, and so we commence this chapter by deriving a more technical description of the cycle. Following this, the generalized aerodynamic formulation of the flapping-wing system is developed, expressing the force-velocity regime in explicit mathematical form. We then proceed with the detailed analysis in turn of each of the three flight modes by appropriate "specialization" of the generalized equations for the particular conditions of each mode. The chapter is concluded with a discussion of the nature of the dynamics of transition flight between modes.
4.1 DEFINITION OF THE FLAPPING CYCLE

4.1.1 Phases of the Typical Cycle

The complexity and diversity of the wing motions in flapping birdflight are so great that it is very difficult to formulate a single definition of a "typical" cycle which can be applied with equal precision to all flight modes. For example, while the flapping cycle can be conveniently described as consisting of the downstroke and upstroke, in very slow or hovering flight the cycle is more accurately described as consisting of the forward and rearward strokes. Additionally, there are transitional segments of the cycle occurring between the principal wing strokes in many forms of fast flight which cannot be properly ascribed as being a part of either upstroke or downstroke. In hovering flight these transitory segments may be insignificant, the wing simply "flipping" directly from one phase to the other. They may also be absent in certain types of fast flight, such as "sculling" flight, where the demarcation between the upstroke and downstroke is quite sharp. The nature of the cycle also varies with type of bird for each flight mode.

The following generalized definition of the flight cycle is based essentially on the fast-flight cycle of the larger birds, such as the swans, geese, herons, and egrets, primarily because the wing motions of these birds are sufficiently slow that they can be followed easily. The definition, however, applies to most bird species. For practical purposes, this cycle description can be applied to the other flight modes (i.e., slow and hovering flight) by considering that certain of the transitory phases are of zero duration. In general, the fast-flight cycle used for defining the "typical flapping" cycle phases is kinematically somewhat more complex than most other modes (because of the presence of the prominent transitory phases) and hence can be made to include the other flight modes by specialization of the general definition. In the analyses of this chapter, any significant deviations of the flapping cycle for particular modes from the typical cycle will be noted.

From examination of the fast-flight cycle of many of the larger birds in steady, level flight using high-speed motion picture sequences, it appears that the typical cycle can be divided into four major phases:
(1) The Power Stroke ($\Delta t_p$)*

In this phase of the cycle, the wings are producing thrust and "lift," i.e., $\hat{F}_x$ and $\hat{F}_y$ are positive; this corresponds to the greater part of the "downstroke." Formally, the power stroke commences at the instant that the wings, including the pinions, have become fully loaded and are being accelerated downward and forward relative to the body. The start of the power stroke coincides closely with the instant at which $\hat{F}_x$ first becomes positive as the wing commences the downstroke. The power stroke ends at the time the wing pinions become unloaded and the manus starts to align with the direction of flight.

(2) The Feathering Transition ($\Delta t_f$)

Toward the end of the power stroke, the inner wing begins to rise and the manus surface begins to align with the flight direction, as described in Chapter III. The transition commences at the time the manus begins to unload and align with the airstream. This transitory phase of the cycle is called the "feathering transition" and is identical to the "feathering" process used with aircraft propeller blades in order to prevent autorotation and to decrease their resistance in the engine-off conditions. The feathering phase ends when the manus has become fully aligned with the airstream and begins to retire rearward as the wing is lifted upward and rearward (relative to the body).

(3) The Recovery Stroke ($\Delta t_r$)

The recovery stroke commences at the termination of the feathering phase just described. The purpose of the recovery phase is to get the wing as rapidly as possible into position for the subsequent power stroke, since during recovery $\hat{F}_x$ is generally negative and $\hat{F}_y$ is decreased. During recovery, the inner wing is lifted upward and rearward and the manus is rapidly collapsed rearward relative to the inner wing. At the end of the recovery stroke the inner wing is raised to its maximum elevation, and the manus is on the verge of being "snapped" into alignment with the inner surface.

*The $\Delta t$ with subscript symbol is used subsequently to designate the time duration of the particular phase.
(4) The Manus-Alignment Transition ( $\Delta t_m$ )

This phase covers the period between the end of the recovery stroke and the beginning of the power stroke. During this transition the manus, which has been folded back relative to the inner wing during the recovery is very suddenly and violently snapped into alignment with the upraised inner wing, simultaneously assuming its full aerodynamic loading for the commencement of the power stroke. The manus-alignment transition is by far the most swiftly accomplished of all the cycle phases, so swiftly in fact that camera framing rates which clearly and easily resolve all other phases of the flapping cycle show the manus during alignment as merely a grossly-blurred image or shadow.

4.1.2 Typical Phase Durations for Some Larger Birds

To give some idea of the actual time durations of the various phases and the period $T$ of the flapping cycle for fast flight, Table I presents data for several large birds as determined from high-speed motion picture analysis.

Table I*

<table>
<thead>
<tr>
<th>Bird</th>
<th>Weight**</th>
<th>$T$</th>
<th>$\Delta t_r$</th>
<th>$\Delta t_s$</th>
<th>$\Delta t_r$</th>
<th>$\Delta t_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada Goose</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Fast Flight)</td>
<td></td>
<td>0.203</td>
<td>0.109</td>
<td>0.039</td>
<td>0.023</td>
<td>0.031</td>
</tr>
<tr>
<td>(Take-off)</td>
<td></td>
<td>0.179</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Fast Flight)</td>
<td></td>
<td>0.234</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Take-off)</td>
<td></td>
<td>0.179</td>
<td>0.101</td>
<td></td>
<td>0.078</td>
<td></td>
</tr>
<tr>
<td>(consecutive cycles)</td>
<td></td>
<td>0.187</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.187</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.179</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.195</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.211</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*All time durations are given in seconds.

**These figures represent the average weight of the bird; they are not the actual weights of the birds whose periods were measured.
<table>
<thead>
<tr>
<th>Bird</th>
<th>Weight</th>
<th>$T$</th>
<th>$\Delta t_r$</th>
<th>$\Delta t_e$</th>
<th>$\Delta t_v$</th>
<th>$\Delta t_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada Goose (cont'd)</td>
<td></td>
<td>0.203</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(consecutive cycles)</td>
<td></td>
<td>0.218</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(average)</td>
<td>0.197</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Snowy Egret</td>
<td>0.75</td>
<td>0.304</td>
<td>0.148</td>
<td>0.039</td>
<td>0.078</td>
<td>0.031</td>
</tr>
<tr>
<td>(Take-off)</td>
<td></td>
<td>0.320</td>
<td>0.156</td>
<td>0.039</td>
<td>0.078</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td>(average)</td>
<td>0.305</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Landing)</td>
<td>0.328</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glossy Ibis</td>
<td>4</td>
<td>0.265</td>
<td>0.172</td>
<td></td>
<td>0.093</td>
<td></td>
</tr>
<tr>
<td>(Level Flight)</td>
<td></td>
<td>0.257</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Take-off)</td>
<td></td>
<td>0.312</td>
<td></td>
<td></td>
<td>0.156</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Level Flight)</td>
<td>0.250</td>
<td>0.117</td>
<td></td>
<td>0.133</td>
<td></td>
</tr>
<tr>
<td>Roseate Spoonbill</td>
<td>3.5</td>
<td>0.203</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Tern</td>
<td>----</td>
<td>0.242</td>
<td>0.125</td>
<td></td>
<td>0.117</td>
<td></td>
</tr>
<tr>
<td>Gannet</td>
<td>5</td>
<td>0.289</td>
<td>0.179</td>
<td></td>
<td>0.110</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.226</td>
<td>0.140</td>
<td></td>
<td>0.086</td>
<td></td>
</tr>
<tr>
<td>Brown Pelican</td>
<td>8.25</td>
<td>0.296</td>
<td>0.179</td>
<td></td>
<td>0.117</td>
<td></td>
</tr>
<tr>
<td>(Landing)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Osprey</td>
<td>3.5</td>
<td>0.195</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Take-off)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Everglades Kite</td>
<td>----</td>
<td>0.335</td>
<td>0.133</td>
<td></td>
<td>0.202</td>
<td></td>
</tr>
<tr>
<td>Ducks</td>
<td>1.75</td>
<td>0.094</td>
<td>0.055</td>
<td></td>
<td>0.039</td>
<td></td>
</tr>
</tbody>
</table>
These figures give quantitative validation of several facts of casual observation. In take-off flight the wing beats are faster than in normal level flight. The birds with the lighter wing loadings (egret, kite, ibis) have slower wing beat frequencies. Heavy wing loadings (e.g., the duck) require very fast wing beats, especially for take-off. It is interesting to note the degree of variation which occurs in $T$ as the Canada Goose transitions from take-off into level flight (0.179 to 0.218 sec.). There is also appreciable variation in the period of successive cycles (Snowy Egret). The period of most of the birds cited ranges from about 0.25 to 0.30 sec. for normal cruising flight. In general, the power stroke ($\Delta t_p$) occupies the largest part of the cycle time, while the recovery ($\Delta t_r$) is accomplished in roughly half the same amount of time.

4.2 GENERALIZED AERODYNAMIC REPRESENTATION OF THE WING SYSTEM

4.2.1 The Fundamental Force System

The formulation of the force system of a bird in flapping flight is considerably more complex than that for a gliding bird. In the former case the entire flapping system must be considered as a nonrigid or deformable body, undergoing changes of shape which result in continuous changes in the spatial distribution of the mass of the system. Thus, there is no constant center-of-mass such as exists in rigid systems, and it becomes necessary to consider the various forces and motions of each individual mass element of the system.

In analysis of the flapping bird's force system, it is convenient to divide the bird's body into two parts, the body proper (consisting of all parts of the bird except the wings) and the wings, and to consider the interactions between these parts. For definiteness of concept, the wing may be pictured as consisting of all the parts "outboard" of the shoulder joint, i.e., beginning at the humerus joint with the body skeleton and extending to the pinion tips. The skin, tendons, muscles, and such, which are continuous with both wing and body at the wing-body juncture may be pictured as being cut by a plane tangent to the body at the juncture. Thus, we have two separate "free-bodies" (in mechanical terms) and can conveniently consider the force systems of each separately.
4.2.1.1 The Body Forces

Considering first the body proper, which we shall refer to hereafter simply as the "body," and considering the body to possess a constant c.g., we see that it is acted upon by two contact forces, the aerodynamic force due to the air reaction, and the force exerted by the wing on the body through the shoulder joint, tendons, and associated connecting tissue, and the gravitational force acting on the body, i.e., the body weight. These forces are illustrated schematically in Figure 10. Here \( \vec{W}_b \) denotes the body-weight (vector), \( \vec{A}_b \) the aerodynamic force acting on the body, \( \vec{F}_{wb} \) the force exerted by the wing on the body, and \( \vec{M} \) is the resultant moment produced by all the forces acting on the body. The three forces are shown in the figure as acting at a common center in the bird, taken as the center of-gravity. In general, of course, the body aerodynamic and wing forces may not intersect at the c.g., and then each produces a moment about the c.g. The resultant moment vector of these two forces is indicated as \( \vec{M} \) in the figure. It should be noted that the (relatively large) force \( \vec{F}_{wb} \) acting at any instant may not pass through the center-of-gravity and the instantaneous moment resulting from this force will tend to rotate the body about the c.g. However, the design of the bird's body geometry, especially the tail surface, is usually such that the magnitude and direction of the body aerodynamic force \( \vec{A}_b \) (which includes the contribution of the tail surfaces) produce a counter moment which cancels or at least reduces the effect of the instantaneous moment of \( \vec{F}_{wb} \). Thus, any significant rotation of the body is prevented. It is not necessary, of course, that the two moments exactly cancel at each instant; however, their averages over a complete flapping cycle must exactly cancel if a net angular rotation of the bird is to be precluded. The instantaneous resultant moment \( \vec{M} \) must be periodic and its average over any period must be equal to zero

\[
\frac{1}{T} \int_{0}^{T} \vec{M}(t) = 0
\]
Further consideration of the moment system accompanying the body force system will be deferred until the discussion of stability and control aspects in Chapter IV.

In our discussion of body forces and moments, we have assumed that the body is rigid, i.e., that the spatial distribution of its mass elements relative to one another is constant, and hence that the body has a constant center-of-gravity. In reality, the body is highly articulated, with the head, neck, legs, feet and tail being capable of a rather wide range of movement relative to some fixed point in the body, say an arbitrary point in the plane of symmetry of the sternum bone. Thus, extension or retraction of the neck or of the legs (particularly in the case of such birds as herons and egrets) can produce changes in the body c.g. Usually, however, by far the greater mass of the body elements is relatively fixed,* and since, in addition, the body appendages (i.e., neck, legs, and such) are usually held quite rigid in the normal periodic flight modes** (as is clearly evident from analysis of many motion picture films), it is quite valid to assume the body proper to be rigid during the major flight modes.

Under the action of the three forces \( \hat{A}_b \), \( \hat{F}_{wb} \), and \( \hat{W}_b \), the body will accelerate according to Newton's second law of motion:

\[
\hat{A}_b + \hat{F}_{wb} + \hat{W}_b = \frac{\hat{W}_b}{\hat{q}} \hat{a}_b
\]

Since, for convenience, it is customary to deal with the components of the acceleration \( \hat{x} \) and \( \hat{y} \), the two contact forces can be resolved into their \( \hat{x} \) and \( \hat{y} \) components to obtain the equivalent \( \hat{x} \) and \( \hat{y} \) contact forces \( F_{xb} \) and \( F_{yb} \) on the body. Thus, we have

\[
(\hat{A}_b \cdot \hat{x}) + (\hat{F}_{wb} \cdot \hat{x}) = F_{xb} \tag{17}
\]

*When the large flight muscles of the body contract during motion of the wing, there may be a shift in their own center of mass relative to the sternum; however, it is highly improbable that this factor has any significant effect on the position of the basic body c.g.

**This is not true for transition phases of flight, as will be discussed subsequently, but even then the rate at which the body c.g. changes position (relative to the sternum, say) is usually small and the condition of a quasi-steady c.g. position can be applied.
\[ (\hat{A}_b \cdot \hat{k}) + (\hat{F}_{wb} \cdot \hat{k}) = F_{\dot{y},b} \]  

where \( \hat{\cdot} \) and \( \hat{k} \) are the conventional unit direction vectors. Equations (17) and (18) then become, in scalar component form
\[ F_{x,b} = \frac{W_b}{q} \dot{x} = \hat{A}_b \cdot \hat{x} + \hat{F}_{wb} \cdot \hat{x} \]
\[ F_{\dot{y},b} - W_b = \frac{W_b}{q} \ddot{y} = \hat{A}_b \cdot \hat{k} + \hat{F}_{wb} \cdot \hat{k} - W_b \]

These are the equations of motion of the body. In general, for the flapping bird, each term in the equations is a function of time except for the constant weight factor \( W_b \). Thus, specification of the periodic vector time functions \( A_b(t) \) and \( F_{wb}(t) \) will allow determination of the velocity components \( x(t) \) and \( \dot{y}(t) \), and trajectory coordinates \( x(t) \) and \( \dot{y}(t) \) of the body c.g.

The aerodynamic body force \( \hat{A}_b \) arises from two sources: the differences in local pressure (normal to the body surfaces), and the local viscous shearing stresses (acting tangentially on the body surfaces), both stresses being due to the movement of the body through the air. \( \hat{A}_b \) is not necessarily entirely a "drag" force (i.e., contributes only to \( F_{x,b} \)), but may furnish an appreciable component of "lift" (i.e., \( F_{\dot{y},b} \)) due to low-pressure carryover between the wing surfaces. As is well known in aerodynamics, the lift distribution on a conventional airplane wing does not go to zero, or even appreciably decrease, in the region of the fuselage, corresponding to the bird's body in our present discussion. By the same token, however, \( \hat{A}_b \) may depend greatly on the condition of the wings and may vary considerably with the changes in lift on the inner segments of the wing surfaces during the flapping cycle. The flow conditions at the tail surface are also markedly affected by the (varying) downwash from the flapping wing, and this can contribute to the variation in \( \hat{A}_b \). The nature of \( \hat{A}_b \) will be discussed in more detail in section 4.2.2.1.

The contact force \( \hat{F}_{wb} \) arises in part from the force exerted by the inner end joint of the humerus on the body skeleton through the engaging socket, but a large part of it is also transmitted by the tension in the connecting tendons which move the wing relative to the body. \( \hat{F}_{wb} \) is a reflection not only
of the aerodynamic and gravitational forces acting on the wing itself, but also of the acceleration of the wing, which in turn is being caused by the contraction of the various flight muscles located within the body itself.

The body weight \( \hat{W}_b \) is, of course, regarded as constant during a given segment of flight, but it is obvious that the absolute weight may vary considerably over long periods of time, depending upon feeding conditions and such. In very long flights such as occur in long distance, nonstop migrations, the weight may decrease appreciably over the course of the flight. Under such conditions, however, the rate of change in weight, i.e., \( \frac{d\hat{W}_b}{dt} \), is so low that \( \hat{W}_b \) may be treated as essentially constant over very long periods compared to the period of a single flapping cycle. These conditions of weight variation are special cases; they can be quite accurately accounted for in any general aerodynamic treatment by considering the weight to be quasi-constant.

4.2.1.2 The Wing Forces

We turn now to a detailed consideration of the force exerted by the wing on the body through physical contact at the wing-body juncture. In order to determine \( \hat{F}_{wb} \), it is necessary to establish first the entire force picture for the wing. We are actually dealing with two wing-semispans in the case of the flapping bird, each semispan producing one-half of the total wing force in the symmetrical flight conditions under consideration here. For convenience in the following formulations, however, the forces referred to are the total wing forces, i.e., the sum of the separate (but equal) force contributions of the individual semispans.

Considering the wing as a free body acted upon by a force system as sketched in Figure 11, dynamic equilibrium requires that the following relation be satisfied:

\[
\hat{F}_{bw} + \hat{W}_w + \hat{A}_w = m_w \hat{a}_w
\]

(19)
This is shown more clearly in Figure 12, where the various vectors for the total wing system (both semispans) are shown as co-planar (the resultant force vectors all lie in the plane of symmetry of the bird, for the total wing system). The left side of Equation (19) is, of course, a vector addition. Here $F_{bw}$ is the force exerted by the body on the wing at the intersection of the wing and body (i.e., where the "free body cut" was made). $W_w$ is the total weight of the wing, and always acts vertically downward, of course. $A_w$ is the aerodynamic force acting on the wing by virtue of the relative motion between the wing and the air. The expression $m_w \ddot{A}_w$ is the net accelerational force acting on the wing.

To see the relationships of these forces more clearly, let us consider Figure 12, showing the wing during the early part of the power stroke. The wing is being forced downward by the contraction of the depressor muscles; this force appears, therefore, as a part of $F_{bw}$. As the wing moves downward initially in response to the muscle action, the wing surface assumes its full aerodynamic loading $A_w$. $A_w$ is directed forward and upward (i.e., $\dot{A}_w\hat{i}$ and $\ddot{A}_w\hat{j}$ are positive during the power stroke), as will be discussed in more detail in section 4.3. In response to the forward force component of $A_w$, the wing attempts to move forward (i.e., in the positive $x$ direction), thus applying a forward-directed force to the terminal humerus joint. The joint in turn restrains the wing from free forward motion and maintains it in an essentially lateral position by preventing free rotation of the wing about the vertical (i.e., $\hat{y}$) axis. Thus, due to the existence of the forward component of the force $A_w$, there is a rearward force component generated in $F_{bw}$ at the wing-body joint. The gravitational or weight force $W_w$, being the overall weight of the wing, is constant in both magnitude and direction. It is always oriented vertically downward, along the negative $\hat{y}$ axis, although the actual line of action of $W_w$
changes continuously with the changes occurring in the shape (i.e., the mass distribution) of the wing during the stroke (see Figure 13).

Since the various elements of the wing undergo positive \( \dot{x} \) and negative \( \dot{y} \) accelerations during the initial part of the downstroke, it is clear that the vector sum \( \mathbf{F}_{bw} + \mathbf{W}_w + \mathbf{A}_w \) is not zero. By decomposing each of these forces into its \( \dot{x} \) and \( \dot{y} \) components as shown in Figure 12, it is clear that the sum of the (negative) \( \dot{y} \) components of \( \mathbf{F}_{bw} \) and \( \mathbf{W}_w \) is greater in absolute value than the (positive) \( \dot{y} \) component of \( \mathbf{A}_w \). Consequently, there is a resultant force component acting in the negative \( \dot{y} \) direction, and this component causes the negative vertical acceleration of the wing represented by \( m_w \dot{A}_w \). Likewise, the positive \( \dot{x} \) component of \( \mathbf{A}_w \) is greater than the rearward \( \dot{x} \) component of \( \mathbf{F}_{bw} \), and this net positive \( \dot{x} \) force produces the positive forward acceleration of the wing.

While it is to be expected that the \( \dot{y} \) component of \( \mathbf{F}_{bw} \) will be negative, due to the downward component of the pull of the depressor muscle tendons on the humerus, it may at first sight appear surprising that the \( \dot{x} \) component of \( \mathbf{F}_{bw} \) is negative, i.e., that the body is in reality pulling back on the wing. That the \( \dot{x} \) component of \( \mathbf{F}_{bw} \) must act in the negative direction is quite clear, however, since if it did not, there would be no means whereby the wing could apply a forward thrust to the body proper, as will be described explicitly in the next section (4.2.1.3). The source of this rearward component of \( \mathbf{F}_{bw} \), as has already been mentioned, lies in the restraining action of the wing-body joint in reaction to the thrust component of the wing aerodynamic force \( \mathbf{A}_w \); here the term "joint" includes not only the bony structures but also the tendons and ligaments.

Equation (19) can be put into a more explicit form for purposes of analysis by expansion of the various force terms.
into their essential physical components. First defining $\hat{F}_{bw}$ for the freebody wing, we have

$$\hat{F}_{bw} = \text{force exerted at wing joint by the body on the wing}$$

Next, the weight force $\hat{W}_w$ is found from the relation

$$\hat{W}_w = g \int \int \int_{V_w} \rho_w \, dV$$  \hspace{1cm} (20)

Here $g$ = the gravitational acceleration constant (i.e., gravitational force per unit mass)

$\rho_w$ = local point mass density of the wing volume

$dV$ = differential of volume of the wing

The integral subscript $V_w$ denotes that the integration is to be carried out over the entire volume of the wing (i.e., both semispans). It should be noted that the local mass density distribution $\rho_w$ for the flapping wing is a function of four variables $\rho_w(x, y, z, t)$. The primary mass of the wing is associated with the bone and muscle components and hence is concentrated along the leading edge of the wing.

The wing aerodynamic force consists of two parts,

$$\hat{A}_w = \int \int S_w p \, d\hat{S} + \int \int S_w \hat{T} \, dS$$  \hspace{1cm} (21)

Here $p$ = local pressure at a point on the wing surface,

$d\hat{S}$ = vector differential of wing surface area

$\hat{T}$ = vector shearing stress at a point on the wing surface (tangential force/unit area of wing),

The integral subscript $S_w$ denotes that the integration is to be carried out over the entire wing surfaces exposed to the air. The shearing stress vector $\hat{T}$ is the conventional aerodynamic skin friction associated with the air flow momentum losses in the surface boundary layer.
The wing acceleration term \( m_w \ddot{\mathbf{a}}_w \) is given by the relation

\[
 m_w \ddot{\mathbf{a}}_w = \iiint_{V_w} \rho_w \ddot{\mathbf{a}}_w \, dV \tag{22}
\]

where \( \rho_w \), \( V_w \), and \( dV \) have the same definitions as in Equation (19), and \( \ddot{\mathbf{a}}_w \) is the local acceleration of each element of the wing mass. Since the wing is not rigid, it is necessary to consider the acceleration of each mass element in arriving at the net accelerational force \( m_w \ddot{\mathbf{a}}_w \).

In general, the various parameters in Equations (20), (21), and (22) are functions of space and time

\[
\begin{align*}
\rho_w &= \rho_w (x, y, z, t) \\
\mu &= \mu (x, y, z, t) \\
\hat{\tau} &= \hat{\tau} (x, y, z, t) \\
\ddot{\mathbf{a}}_w &= \ddot{\mathbf{a}}_w (x, y, z, t)
\end{align*}
\]

The domain \( S_w \) over which the surface integrations are carried out is a function of time, the wing shape and associated area \( V_w \) varying continuously throughout the period of the flapping cycle. The volume domain \( V_w \), which is associated only with the mass characteristics of the wing is constant in total value, but the coordinates of the closed surface defining the volume \( V_w \) vary continuously with time.

The expanded form of the dynamic equilibrium equation for the wing forces, using Equations (20), (21), and (22) and stated in terms of \( \mathbf{F}_{bw} \), is given by

\[
\mathbf{F}_{bw} = \iiint_{V_w} \rho_w \ddot{\mathbf{a}}_w \, dV - q \iiint_{V_w} \rho_w \, dV - \int_{S_w} \mu \hat{\tau} \, dS - \int_{S_w} \hat{\tau} \, dS \tag{23}
\]

Thus, the force exerted by the body on the wing becomes known in terms of the forces acting on the wing and the accelerations of the wing mass elements.
4.2.1.3 The Total Force System

It is now desirable to relate the body force system with that of the wing to obtain a single expression for the total force balance on the bird. To accomplish this, use is made of Newton's third law of motion as applied to the wing and body "free bodies" just considered. This law states that

\[ \hat{F}_{bw} = - \hat{F}_{wb} \]  (24)

Thus, the force \( \hat{F}_{wb} \) exerted by the wing on the body, appearing in Equation (24), is established by taking the negative of the right hand side of Equation (23), and the equation of motion for the body is then given by

\[
\frac{w_b}{q} \hat{a}_b + \iint_V \rho_w \hat{a}_w \, dV = \iiint_S \rho_b \hat{a}_b \, dS + \iiint_S \rho_w \hat{a}_w \, dS + \iint_S \rho \hat{a}_w \, dS + \iint_S \rho \hat{a}_w \, dS
  + \iint_S \hat{a}_w \, dS + \hat{q} \iint_S \rho_b \, dV + \hat{q} \iint_S \rho_w \, dV \]  (25)

or

\[ \text{acceleration force } = \hat{A} + \hat{W} \]  (26)

In terms of the \( \hat{X} \) and \( \hat{Z} \) components, Equation (25) becomes

\[
\frac{w_b}{q} \hat{x} = \iiint_S \rho_b \hat{a}_b \cdot \hat{z} \, dS + \iiint_S \rho_w \hat{a}_w \cdot \hat{z} \, dS + \iint_S \rho \hat{a}_w \cdot \hat{z} \, dS \]
\[ - \iiint_S \rho w \hat{a}_w \cdot \hat{z} \, dV \]  (27)

\[
\frac{w_b}{q} \hat{j} = \iiint_S \rho_b \hat{a}_b \cdot \hat{k} \, dS + \iiint_S \rho_w \hat{a}_w \cdot \hat{k} \, dS + \iint_S \rho \hat{a}_w \cdot \hat{k} \, dS \]
\[ - \hat{q} \iiint_S \rho_b \, dV - \hat{q} \iiint_S \rho_w \, dV - \iiint_S \rho w \hat{a}_w \cdot \hat{k} \, dV \]  (28)

where \( \hat{z} \) and \( \hat{k} \) are conventional unit orthogonal vectors. Integration of these equations yields the velocity components of the body \( \hat{x} \) and \( \hat{j} \) and a second integration yields the body trajectory coordinates \( \hat{X} \) and \( \hat{J} \).

Equations (27) and (28) constitute the generalized equations of motion for a bird in flapping flight. The force integrals involving the normal and tangential stresses \( \rho \) and \( \tau \) are of aerodynamic origin and will be discussed in considerably greater detail in the following section (4.2.2). From
the equations it can be seen that the body acceleration in the
\( X \)-direction involves only the aerodynamic forces and the
accelerations of the wing elements, while the \( j \)-acceleration
component also involves the total weight of the bird:

\[
q \iiint_V (\rho_b + \rho_w) \, dV = W = W_b + W_w
\]  

(29)

The general scheme of the force system of flapping flight
is thus clear. The contraction of the flight muscles produces
forces which are transmitted by tendons to the wing. The tendon
forces move the wing relative to the body. In the power stroke,
the motion of the wing relative to the air generates an aero-
dynamic force which tends to pull the wing forward and to resist
its downward motion. The body acts to restrain the forward
motion of the wing and hence experiences the forward pull of
the wing force. The body tendons also act to pull the wing down
against the retarding aerodynamic force and in the reaction
process exert a lifting force on the body proper. This basic
force interaction, combined with the associated total weight and
wing-mass accelerational forces, serves to sustain and propel
the bird.

4.2.2 The Aerodynamic Forces in Flapping Flight

The dynamic equilibrium relationship, expressed by Equation
(26), shows that it is the total aerodynamic force \( \hat{\mathbf{A}} \)
which is important in determining the resultant motion of the
bird. This force is obtained by integration over the entire
surface of the bird, the pressure and viscous stress being con-
tinuous over the entire surface. For practical purposes, how-
ever, it is possible to divide \( \hat{\mathbf{A}} \) into its body and wing
components \( \hat{\mathbf{A}}_b \) and \( \hat{\mathbf{A}}_w \), as in Equation (25), and to analyze
these forces separately in terms of conventional aerodynamic
precepts. By application of certain fundamental aerodynamic
relationships, it is possible to determine the local force in-
tensities contributed by various elements of the body and wings
without resorting to an actual integration of the pressure and
shearing stress distributions. The fundamental precepts in-
volved in the aerodynamic formulation of the flapping wing will
be briefly introduced here to elucidate the overall principles
involved, but the detailed application of such precepts to the
case of flapping flight will be deferred to section 4.3.
Before considering the actual natures of the forces $\hat{A}_b$ and $\hat{A}_w$ in birds, it will be useful to review briefly the concept of relative aerodynamic velocity and its relation to aerodynamic force.

4.2.2.1 Relative Velocities and Relative Forces

The aerodynamic force which acts on a body moving with constant velocity through a given fluid, such as air, depends primarily on five factors:

1. the shape of the body,
2. the size of the body,
3. the inclination of the body relative to the direction of relative motion,
4. the relative velocity between the body and the fluid, and
5. the physical properties of the fluid.

The exact relationship between the factors is

$$F = C_F \frac{1}{2} \rho V^2 S \tag{30}$$

where $F$ is the total force experienced by the body, $C_F$ is the so-called force coefficient (to be discussed in detail below), $\rho$ is the fluid mass density, $V$ is the relative velocity, and $S$ is a characteristic area of the body (i.e., the square of some characteristic length of the body).

It can be shown by the techniques of dimensional analysis that the coefficient $C_F$ depends upon the geometric shape and orientation of the body, and upon the body size and physical properties of the fluid, more specifically upon the Reynolds number $Re$ of the system, where

$$Re = \frac{\rho V \ell}{\mu}$$

Here $\rho$ and $V$ have their previous definitions while $\ell$ is a characteristic length of the body* and $\mu$ is the viscosity of the fluid. Thus, for a family of bodies having identical geometrical shapes and orientations relative to the direction

---

*The characteristic area $S$ is proportional to $\ell^2$. 
of motion but different absolute sizes, $C_F$ is a constant, provided the $Re$ of the body fluid system is maintained the same for each member of the family. This latter condition can be attained by decreasing the relative velocity $V$ as the body size increases, and/or by altering the ratio $\rho/\mu$. In general, fortunately, the variation of $C_F$ with $Re$ is small over a wide range of values of $Re$, especially for values obtaining for most flapping birds performing fast flight in air where $\rho$ and $\mu$ are constant. Hence, $C_F$ is essentially independent of body size $l$ and speed $V$ for the practical size and speed ranges characteristic of flapping birds.

In the case of a rigid, nonrotating body, $V$ has the same value for all parts of the body. For the nonrigid body, as exemplified by the flapping wing bird, the value of $V$ is grossly different for the various parts, the relative flow simultaneously being unsteady. It is valid in most cases, however, to assume local quasi-steady flow to exist, i.e., that the local forces existing due to an instantaneous local aerodynamic velocity $V$ are the same as would exist in the case of a steady relative velocity $V'$. Here $V'$ is the relative velocity between the body, or a part thereof, and the air; this local aerodynamic velocity is assumed to be the resultant of all the component air flow velocities produced locally as a result of the unsteady conditions of the total flow.

The concept of relative aerodynamic velocity is of great importance, especially in the case of the flapping wing, since the magnitude and direction of all aerodynamic forces are governed by the aerodynamic velocity. So far as the production of aerodynamic forces is concerned, it is only the relative velocity between a body and the air which is effective; the absolute velocity of either (i.e., relative to earth) is immaterial. In the case of the flapping wing, the relative velocity field about the wing must be such as to allow the generation of forces in the proper directions at the proper points in the cycle, and the basic motion patterns of the wing are entirely governed by these requirements. As will become evident from the subsequent analyses, the creation of the proper relative aerodynamic velocity by the cyclic motion of the wing (relative to the body and relative to the earth) lies at the heart of successful flight by use of flapping wings.
It may be well to mention briefly here the importance of distinguishing between the relative velocity patterns of the bird's body and of the wing in flapping flight. When the flapping-wing motion of a bird is viewed, either in the field or in motion picture films, the eye most generally discerns the motion of the wing relative to the bird's body. The motion of the wing relative to earth (and hence relative to the air for the condition of a still atmosphere), however, is quite different; it is the latter velocity which is, of course, of primary importance in the force determinations. In general, and especially in the fundamentally important forward flight modes, the bird's body is moving with appreciable forward velocity (relative to earth). This velocity must be added (vectorially) to the velocity of the wing (or parts thereof) to establish the absolute motion of the wings relative to the air.

4.2.2.2 The Body Force

The pressure distribution occurring in Equation (25) gives rise to a single resultant aerodynamic force which may be defined by the symbol $A_p$, denoting the component of total aerodynamic force due to the (normally directed) elemental pressure force. In general, if the body of the bird is translating with quasi-uniform velocity along the positive $X$ axis with quasi-uniform aerodynamic velocity $\mathbf{V}$, the resultant pressure force will be inclined upward and rearward, as shown in Figure 14. This force can be resolved into two components along the coordinate axes. The vertical component, acting along the positive $Z$ axis, for the hypothetical case shown, can be termed a "lift" force, while that acting horizontally along the negative $X$ axis is a "drag" force. These definitions are in keeping with the conventional aerodynamic definitions of lift and drag for steady flight.
The aerodynamic lift on the body arises simply from the lower average pressure on the top part of the body as compared to that on the bottom or underside. This lift is associated with the fact that the wings themselves are producing lift. The wing during the power stroke first throws air laterally outward and so reduces the air pressure over the upper surface of the body, creating an added lift which is reflected in \( \hat{A}_b \). During the latter part of the stroke, the wing throws air laterally inward under the body, creating a high pressure region which again produces lift on the body. The importance of this contribution of lift for flapping birds is discussed subsequently.

The pressure drag, i.e., the \( \hat{x} \)-component of \( \hat{A}_b \), is associated with the fact that the streamline flow around the body breaks up over the rear portions due to boundary layer separation in the region of adverse pressure gradient which normally exists on bodies of appreciable volume for a given length. This region of lowered pressure over the rear of the body results in a net force in the negative-\( \hat{x} \) direction. In most birds, this drag component is probably very small, due to the exceptionally clean streamlining of the body itself. The pressure drag of the body can be significantly altered by the pressure field of the wing, and hence the positioning of the wing relative to the body and the nature of the wing-body juncture become very important. Properly located, the wing can significantly reduce the pressure drag.

In addition to the pressure forces on the body, there exists the skin friction force [Equation (25)] arising from the tangentially-directed viscous shearing stressing \( \hat{\tau} \) at the body (i.e., the boundary layer shearing stress).* In general, the average of all \( \hat{z} \)-components of \( \hat{\tau} \) essentially cancel; however, all \( \hat{x} \)-components are negative and add to give a skin friction drag. This drag component is usually small compared to that of \( \hat{A}_b \), but is still significant because, like all drags, it dissipates energy, and this energy must be continuously supplied by the bird for sustained flight. The magnitude of the skin friction drag is usually not significantly affected by the wing-induced flows.

*Here \( \hat{\tau} = \mu (du/d\hat{z})\hat{b} \) where \( u \) is the local aerodynamic flow in the boundary layer and \( \hat{z} \) is distance above the surface. The subscript \( \hat{b} \) denotes that the derivative is evaluated at the body surface.
4.2.2.3 The Wing Force

In the same manner as with the body force, the total wing aerodynamic force \( \hat{A}_w \) may be resolved into its pressure and skin friction components. These components can in turn be resolved into \( \chi \) and \( \gamma \) components and respectively summed to yield the lift and drag forces of the wing. While the wing lift force is, like the body lift, nearly always positive in flapping flight, the wing "drag" force is both positive and negative over the period of a cycle, the positive \( \chi \) -force component constituting a negative drag or thrust. It is, of course, this propelling thrust force which lies at the heart of the flapping flight system. The total wing lift \( \hat{L}_w \) and drag \( \hat{D}_w \), as referenced to the quasi-uniform flight velocity \( \hat{\dot{V}} \) (see Figure 15), are the respective vector sums of the aerodynamic forces on each of the elemental sections of the wing. However, the actual effective aerodynamic velocity seen by such sections will vary grossly in both magnitude and direction from \( \hat{\dot{V}} \) over the course of a flapping cycle. The nature of the local aerodynamic force distribution is, therefore, intimately connected with the relative velocity of the individual wing sections.

Considering first the pressure force of the wings, we replace the wing proper by a conventional lifting line vortex, as shown in Figure 16. At any instant of time, this lifting line has a definite shape and spatial orientation in terms of the \( \chi \), \( \gamma \), and \( \hat{\gamma} \) coordinate system shown. The
motion of the bound lifting line (i.e., the wing surface) during a flapping cycle leads to a definite absolute aerodynamic velocity \( \hat{V}(s) \) where \( s \) denotes the arc length coordinate with origin at \( S_0(\neq 0) \), the wing-body juncture point. The coordinate \( s \) is given explicitly by the coordinates of the space curve constituting the lifting line

\[
S = \int_{s_0}^{s} \left[ (dx)^2 + (dy)^2 + (dz)^2 \right]^{1/2}
\]

(31)

Applying the conventional assumptions of linearized, quasi-steady wing theory to the bound lifting line, we have the situation pictured in Figure 17. At the instant \( t_i \), the local segment of the lifting line at \( S_i \) (i.e., wing section at \( S_i \)) is moving relative to the air with velocity \( \hat{V}(S_i) \). At this instant the segment experiences a resultant aerodynamic force \( \hat{A}'(S_i) \). This local force can be resolved into the conventional components \( \hat{\ell}'(S_i) \) (section lift) and \( \hat{d}'(S_i) \) (section drag), perpendicular and parallel, respectively, to the local aerodynamic velocity \( \hat{V}(S_i) \). In accordance with linearized lifting line theory, \( \hat{\ell}' \gg \hat{d}' \) and the local lift \( \hat{\ell}'(S_i) \) is assumed to be essentially equal to the magnitude of \( \hat{A}'(S_i) \). Depending upon the geometry of the wing lifting line and the relative velocity pattern \( \hat{V}(S_i) \), it is possible that a local side-force component (i.e., a force acting parallel to the \( y \)-axis) can exist; however, due to the symmetry of the total wing, any side forces
will automatically cancel out, and hence need not be considered in the overall force picture presently under analysis. Considering Figure 18, it is seen that the local (section) "lift" component is producing both positive \( \gamma \) and \( \chi \) forces, i.e., both total lift and thrust relative to the principal \( \chi \), \( \gamma \) axes. Likewise, the local (or section) "drag" component \( \delta \) is producing a component of total lift as well as total drag. Thus, the wing "drag" is actually contributing to the lift of the bird, during the downstroke, while the "lift" is actually furnishing a forward thrust component.

Invoking the Kutta-Joukowski law,

\[
\ell' = \rho V \Gamma
\]

it is seen that the local circulation distribution \( \Gamma(s) \) (which depends upon the section characteristics and angle of attack) determines the magnitude of the local lift \( \ell'(s) \). Hence, the lifting line is a vortex of strength distribution \( \Gamma(s) \) along its length. Since \( \Gamma \) must vanish at the wing tip \( s = S_0 \), i.e., \( \Gamma(S_0) = 0 \), it is required by Helmholtz's law of vortex continuity that a trailing vortex system emanate from the lifting line (Figure 18). In general, the shed vorticity takes the form of a continuous vortex sheet of intensity \( \frac{d\Gamma}{ds} \), obtained by differentiation of the lifting line circulation distribution \( \Gamma(s) \) with respect to the lifting line arc length coordinate \( s \). The initial shape of the overall vortex wake (i.e., just behind the wing) is extremely complicated for the flapping wing; this subject will be considered in detail in section 4.3.1.3.

The existence of the vortex wake of the wing results in the production of an induced downwash velocity at the wing, and it is this downwash which results in the production of the section pressure drag, \( \delta \). The nature of the downwash variation \( \delta'(s) \) is much more complex for the flapping wing than for the straight and rigid conventional aircraft wing, due to the extremely involved geometry and intensity distribution of the vortex sheet of the former.

Still further complexities are introduced by the fact that the flow conditions are all unsteady, and \( \Gamma(s) \) is changing continuously with time throughout the flapping cycle; these time-dependent changes in \( \Gamma(s) \) must be reflected in the distribution of vorticity shed into the wing wake. Consequently,
with changes in the circulation distribution $\Gamma(s)$, transverse vortices are also shed into the wing wake, along with the longitudinal vortices (i.e., vortex lines) arising from the change of $\sigma$ with $\mathbf{s}$, as sketched in Figure 19. Thus, the vortex sheet trailing from the wing-lifting line is a complex superposition of both longitudinal and transverse vortex lines with a continuous variation in intensity. This vortex sheet will be periodic in shape and intensity distribution due to the periodic flapping cycle of the wing. As will be shown subsequently in section 4.3.1.3, it is possible to construct the general shape and intensity distribution of the complex vortex sheet behind a flapping wing from a knowledge of the wing force distribution and its time variations.

In addition to the pressure-associated aerodynamic forces of the wing, there exists the skin friction force which, compared to the former, is relatively small. The skin friction force acts predominantly as a section drag, being essentially parallel to the local aero-

Figure 19a
dynamic velocity $\mathbf{V}(s)$. This force can, therefore, be included as a component of the total section drag force. The section skin friction drag is denoted by $\mathbf{d}'_s$.  

To summarize the aerodynamic force system of the wing, Figure 20 presents a representation of the forces acting at a given wing station $s$, at an instant $t$, during the cycle. Similar conditions
exist at all other wing stations, and integration of the local force intensities between $s_i$ and $s_o$ yields the total lift ($\hat{\mathbf{f}}_L$-force) and drag ($\hat{\mathbf{f}}_D$-force) of the entire wing system. The basic equations are

$$\hat{\mathbf{f}}_L(t) = 2 \int_{s_i}^{s_o} (\hat{\mathbf{x}} \cdot \mathbf{h}) \mathbf{h} \, ds + 2 \int_{s_i}^{s_o} (\hat{\mathbf{y}} \cdot \mathbf{h}) \mathbf{h} \, ds + 2 \int_{s_i}^{s_o} (\hat{\mathbf{z}} \cdot \mathbf{h}) \mathbf{h} \, ds$$

$$\hat{\mathbf{f}}_D(t) = 2 \int_{s_i}^{s_o} (\hat{\mathbf{x}} \cdot \mathbf{h}) \mathbf{h} \, ds + 2 \int_{s_i}^{s_o} (\hat{\mathbf{y}} \cdot \mathbf{h}) \mathbf{h} \, ds + 2 \int_{s_i}^{s_o} (\hat{\mathbf{z}} \cdot \mathbf{h}) \mathbf{h} \, ds$$

Here $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$, and $\hat{\mathbf{c}}$ are all functions of time. The force system pictured here is for the case where $\hat{\mathbf{a}}$ applies to the early part of the downstroke. With time, the vector picture for station $s_i$ and all other stations changes continuously as the wing passes on through the cycle. At any instant, however, Equations (33) and (34) give the total wing aerodynamic forces; $\hat{\mathbf{f}}_L(t)$ and $\hat{\mathbf{f}}_D(t)$ then determine $\hat{\mathbf{A}}_w(t)$:

$$\hat{\mathbf{A}}_w(t) = \hat{\mathbf{f}}_L + \hat{\mathbf{f}}_D$$

This general discussion of the wing aerodynamic force has been based for convenience on the early part of the downstroke, where the wing is fully extended and the nature of the force system is clearly discernable. For other phases of the cycle, the same principles apply but the situation is much more complex due to the bending of the wing. The relative velocity of the wing sections may even be such on the upstroke that the local section skin friction and pressure drags of the wing actually appear as a thrust force ($\hat{\mathbf{f}}_D$ positive) which acts to propel the bird forward (see section 4.3.1.5).

4.3 AERODYNAMIC ANALYSIS OF THE THREE FLIGHT MODES

Having now discussed the generalized form of the force system which acts on a bird in flapping flight and the nature of the aerodynamic forces involved, we proceed in this section with the consideration of the details of the specific flight modes: fast flight, slow flight, and hovering flight.

4.3.1 Fast Flight

Of the three principal modes of flapping flight, fast
flight is by far the most important, for it is by means of fast flight that the actual locomotion of most birds is accomplished, slow flight and hovering flight being limited to use under special conditions. Consequently, the analyses herein begin with a consideration of fast flapping flight. The wing motions in this mode of flight are relatively simple (compared to slow and hovering flight) and the aerodynamic forces and their variations can be followed without undue difficulty in terms of conventional aerodynamic precepts; hence, fast flight can be used as an effective starting point for developing a characteristic picture of the fundamental mechanics of propulsive flight with flapping wings.

4.3.1.1 Kinematics of the Wing Motions in Fast Flight

Fast flight is characterized by the rapid motion of the bird's body through the air; the motion of the wing (relative to the body), on the other hand, may be rather slow, especially when compared to the slow flight case. The velocity of the wing relative to the air, however, is quite large. In terms of relative velocity relationships

\[
\hat{V}_{we} = \hat{V}_{wb} + \hat{V}_{be}
\]  

(36)

where \( \hat{V}_{wb} \) = velocity of the wing relative to the body, and \( \hat{V}_{be} \) = velocity of the body relative to earth.

The velocities relative to earth are termed absolute velocities. In the case where the air through which the bird is flapping is still, relative to the earth's surface (i.e., there is no wind or thermal convection), then the velocities relative to earth [Equation (36)] are also the velocities relative to the air and we have

\[
\hat{V}_{wa} = \hat{V}_{wb} + \hat{V}_{ba}
\]  

(37)

where the subscript "a" denotes "relative to the air." These relationships of the relative velocities are extremely important since it is \( \hat{V}_{wa} \) which determines the wing aerodynamic forces and hence the speed of motion of the bird as a whole. We are, of course, using the terms "velocity of the wing relative to the air" and "velocity of the wing relative to the body" in very general terms here since, in reality, the velocities of each element of the wing are different. Formally, Equations (36)
and (37) refer to an elemental mass of the wing proper or, in a more analytically useful form, to an elemental section of the wing (i.e., to a section of the wing with elemental width $\text{d}s$ and chord length $c$ as shown in Figure 21).

The flapping motion of a bird's wing is one of an elastic, deforming body moving in three dimensions, and is so acutely complex as to make any really meaningful verbal description extremely difficult, if not impossible. From a mathematical standpoint, we can usefully picture the bird as a discrete, elastic volume in three-dimensional space $x, y, z$, the coordinates of the closed surface which defines the volume precisely specifying the external surface of the bird.* Hence, an expression of the form $f(x, y, z) = 0$ can be visualized as defining the position of the surface in space and thus specifying the shape and position of the bird at any given instant. By further making $f$ a function of time, $f(x, y, z, t) = 0$ (where $x, y, z$, and $t$ are parametric functions of $t$) then specifies the coordinates defining the bird's surface shape and position in space for all times, and the motion of the bird can be pictured as the continuous (periodic) deformation and translation of the closed surface $S$. From this function, the various relative and absolute velocities of each element of surface are defined for the complete flapping cycle. Likewise, a similar expression giving the mass (or density) distribution within the closed surface $S$ can be visualized as the periodic function $\rho = f(x, y, z, t)$. In the mathematical sense, these expressions precisely describe the entire kinematics of the bird's motion in flight; from them can then be derived all the applied-aerodynamic and mass-accelerational forces needed to define the dynamics regime of flapping flight. The determination of the explicit form of such mathematical functions as the surface

*This surface may be defined as the air-body interface, where body here refers to the entire bird, including wing, tail, and all other extremities.
equation \( \gamma(x,y,z,t) \) and the mass distribution \( \rho(x,y,z,t) \) for such a complex form as the flapping bird is obviously an impossible endeavor at present; nevertheless, the mathematical conception of the bird in flapping flight as a closed surface containing a known distribution of mass and undergoing a periodic deformation and translation in response to internal and external forces is quite a useful one for visualization of the fundamental mechanics involved.

In the use of lifting line theory, it is graphically meaningful to visualize the lifting line representing the wing as the limit of the wing surface area as the internal wing volume is shrunk to zero about the lift axis of the wing, while simultaneously maintaining the force intensity distribution along the arc span constant.

From experience, it is clear that a full appreciation of the complex three-dimensional kinematics of the flapping bird can be obtained only after a great amount of intensive study of actual slow motion films taken from many different angles and viewpoints for the same type or similar types of birds. Ultimately, it becomes possible to interpret the full three-dimensional motions simply from side-view motion pictures of the flapping cycle. As a result of such film studies, it has been found that an amazingly constant pattern of wing motion pertains for nearly all birds in a given flight mode, although minor variations of the central theme are found among species having notably different wing forms. Hence, a description of the flight kinematics of one particular species serves as a general description for flapping birdflight in general.

With the reservations expressed in the preceding paragraph in mind, we shall proceed with a summary description of the salient kinematical features of the wing motion for a complete cycle in fast flapping flight. To illustrate the description, Figure 22 has been prepared utilizing selected frames from a high-speed moving picture of a Canada

Figure 22 appears on pages 129-133 at end of report.

Snowy Egret, p. 129-130

Glossy Ibis, p. 130-131

Canada Geese, p. 132-133

All frames are 0.0077 second apart.
goose in fast-flapping flight. We join the bird at the commencement of the power stroke, when the wings are fully raised, but moving downward very rapidly. The wing moves down and forward, relative to the body with a velocity as indicated in Figure 23. Since the body is moving forward with an absolute (and aerodynamic) velocity $v_{ba}$, the wing sees an aerodynamic velocity of $v_{wa}$ which is strongly inclined downward due to the large vertical component of $v_{wb}$. In many cases, especially in very rapid forward flight, $v_{wb}$ is almost entirely vertical, with no forward component. Due to the strong inclination of the wing aerodynamic velocity, the wing lift has an appreciable positive x-component or thrust as shown; this component constitutes the propulsive force in flapping flight.

The wing is swept on downward in the power stroke, moving in an essentially circular arc as shown in Figure 24, from the near vertical position to a downward inclination of 30° or so. All during this power stroke, the wing proper is essentially planar.* The pinions are strongly bent or flexed under the heavy airload carried by the

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*In some large birds, such as the herons and egrets, the secondaries of the rearward half of the wing may be strongly bent or twisted upward due to their elasticity. This is equivalent to providing a geometrical twist to the wing, much as is done in aircraft propellers.
wing due to its high aerodynamic velocity. Since the wing is moving essentially in a circular arc with constant angular velocity about the pivot, the vertical speed component of the various wing sections relative to the air is proportional to the distance from the body, so that the outer wing sections move downward much more rapidly than the inner sections (Figure 25a). The result of this difference in vertical velocity is that the resultant section aerodynamic velocity is much larger and more heavily inclined for the outer wing sections than for the inner ones, precisely the same as in the revolving aircraft propeller. Consequently, it is characteristic that the thrust force produced by the wing during the power stroke is contributed mainly by the manus of the wing. This fact is often clearly evident in motion pictures where the pinions are seen to have a strong forward component of their flexure, as well as vertical.

The geometrical wing twist mentioned above, afforded by the bending upward of the outer wing secondaries, serves an important function in relation to the thrust force. Due to the strong inclination of $\hat{V}_{wa}$ at the outer wing sections, the sections would be above the critical angle of attack and hence would be stalled, and would create no thrust force if they were to lie in the same plane as the inner wing. However, by the automatic twisting of the outer wing under load, the proper angle of attack is maintained despite the high aerodynamic velocity inclination. Thus, high aerodynamic velocity, and hence large thrust, can be created despite the high aerodynamic velocity inclination at the tip. More will be said about the importance of such aeroelastic effects in a later section.

After the wing has reached its 30° downward inclination, the humerus begins to rise and move rearward. This is accomplished by a flexing of the wing at the elbow and simultaneously the manus is pulled into an essentially vertical position as the inner wing rises with the humerus. The downward momentum of the manus causes it to swing strongly inward as the inner wing
Figure 25b

Figure 25c

Figure 25d
continues to rise so that by the time the plane of the inner wing is essentially horizontal, the manus is situated vertically, normal to this plane. The manus tip swings forward slightly as it moved into this vertical plane, but upon becoming vertical, it is "feathered" or aligned with the airflow (i.e., relative to the general body motion) so that it becomes fully unloaded, as is evidenced by the fact that the pinions are seen to assume the normal curvature they possess when not in flight. This period, from the end of the power stroke (at 30° wing inclination) until the manus is hanging vertically and "feathered" in the airstream, constitutes the "feathering transition" as defined in section 4.1.1. During the feathering transition, the rising inner wing has a very high angle of attack relative to the average flight velocity \( \tilde{V} \), thus maintaining a full aerodynamic load. The vertical velocity with which the wing is raised is relatively small, so that the aerodynamic velocity of the inner wing is essentially \( \sqrt{V} \).

The inner wing now continues its rise above the horizontal position, thus commencing the recovery stroke. As it rises, the manus, which is essentially vertical at the end of the feathering transition, begins to rotate, tips rearward, in a plane that remains approximately normal to the plane of the rising inner wing. Simultaneously, the inner wing, which was appreciably flexed at the elbow during the feathering transition, is now straightened in its plane, being brought into line with the humerus. This straightening action gives a large segment of the inner wing (that is, the ulna-radius section) an appreciable rearward velocity component relative to the body. This combined inner wing straightening-manus retirement acts to give the majority of the wing surface, from the elbow outward, a high rearward velocity component relative to the body. Since the body is moving forward with the constant flight velocity \( \tilde{V} \), (for all practical purposes the instantaneous body velocity can be considered equal to \( \tilde{V} \) in fast flight), the outer wing during the recovery stroke has a greatly reduced, or even zero or negative aerodynamic velocity, i.e., the wing is essentially resting in still air. This disappearance of the aerodynamic velocity for the outer wing during the recovery stroke is a factor of central importance in flapping flight and, as will be shown in the next section (4.3.1.2), lies at the heart of successful flapping flight. The wing continues its rise and straightening action, while the completely unloaded manus is lifted vertically essentially in its own plane, thus experiencing an essentially zero aerodynamic force. Finally,
the humerus reaches its maximum raised position; this constitutes the end point of the recovery stroke.

At the end of the recovery stroke, the slight remaining bend in the flexed elbow is quickly straightened, the ulna-radius becoming colinear with the humerus, and the wing begins to move down once again. Simultaneously with the final instants of the straightening action and commencement of the downward movement of the wing proper, the manus is snapped with an extremely fast motion into a colinear alignment with the rest of the wing. The absolute motion of the manus at this point is the fastest of any part of the wing during any phase of the flapping cycle, the manus appearing as a blur in movie films whose speed is sufficient to catch all other wing movements very clearly. The manus motion at this time is similar to that of the tip of a cracking whip, and the mechanical aspects involved are indeed not wholly unakin. The already-initiated downward movement of the wing proper, combined with rapid straightening of the wrist, almost instantaneously snaps the manus from fully unloaded condition into a fully air-loaded condition, whence the pinions become strongly curved. In essence, the inner end of the manus (i.e., the wrist) is pulled down by the downward-forward moving wing and the simultaneous wrist straightening moves the manus in such a way that it experiences only a vertical-forward aerodynamic force during its whole alignment phase. This period, from the point of maximum humerus recovery until complete manus alignment with the wing proper, is termed the manus alignment transition. Subsequent to the manus alignment, the wing commences the power stroke for another cycle.

The foregoing verbal description, combined with the photographs of Figure 22, summarizes the essential kinematics of the flapping cycle in fast flight. For them, the main qualitative features of the relative velocity regime which pertains over the cycle for the various components of the bird are evident. In the following section (4.3.1.2), the quantitative aspects of the relative and absolute velocity relationships will be examined in more detail and related to the force system and its variations over the course of the fast flapping flight cycle.

4.3.1.2 The Wing Force System and Its Time Variation

The lifting line theory of conventional aerodynamics, although developed by Prandtl for the case of wings of very high
aspect ratio, has proven remarkably accurate for wings with aspect ratios as low as four. Consequently, it appears quite valid for our purposes to employ the precepts of lifting line theory (with appropriate modifications as necessary) for the analysis of the flapping wings of birds, since most birds with which we are concerned (i.e., the larger, efficient fliers) all have respectable aspect ratios. We shall also assume that the assumptions of conventional linearized theory are also valid for the case of the flapping wing.

In using linearized lifting line theory, we make two simplifying assumptions: (1) If the local velocity of the air (including all induced velocity components) at a section located at arc span coordinate \( S \) has a specified value, then the forces experienced by the section will be the same as in the case of true two-dimensional flow with the same velocity over the section; and (2) The instantaneous force existing on a section, corresponding to the instantaneous aerodynamic velocity to which the section is exposed, is essentially the same as would be obtained under steady-flow conditions with the same (instantaneous) section attitude and local aerodynamic velocity. Assumption (1) has been well validated and is the basis of wing design in aeronautical practice. Assumption (2), however, is not on such firm ground. If, for example, either the magnitude or direction (or both) of the local aerodynamic velocity seen by a section is changing in a continuous manner (unsteady flow), the force seen by the section may not be precisely the same at any instant as would be obtained with the same velocity conditions under steady conditions. The degree of difference, if any, depends, of course, on the rate of change in velocity, since in the limit of exceedingly slow change, the flow in both cases would be essentially steady.

As an illustration of this condition which may accompany unsteady flow with regard to an airfoil section, we can consider the case shown in Figure 25b. Here an airfoil section in two-dimensional flow is mounted on a balance in a wind tunnel and the lift (or any other desired force component) measured continuously (instantaneously) as the airfoil increases its angle of attack through steady rotation about its quarter-chord axis with constant average velocity \( \omega \). The plots of \( \alpha \) vs. \( t \) are shown as straight lines in (a) for various values of \( \omega \); the steeper slopes denote higher rotational speeds. The corresponding force variations with \( \omega \) are illustrated schematically in (b) where the forces are shown to be less (which may or may not
be the case, depending upon the airfoil shape, surface conditions, and such) for the more rapidly rotating sections. In this particular example, the section circulation \( \Gamma \) requires a small but finite time to establish the flow pattern corresponding to a given steady lift, and very rapid rates of change of \( \alpha \) would not permit the steady conditions to be established at a given \( \alpha \).

In general, the rates of change in the section angles of attack and relative aerodynamic velocities would not appear to be sufficiently rapid, especially for the relatively small wing dimensions involved in bird flight, to lead to significant variations from steady-flow section forces. Further detailed quantitative studies of this condition are desirable, however.

The phenomenon being discussed here must not be confused with the overall differences in steady and unsteady flows. We are here merely asserting that if a section sees a resultant aerodynamic velocity at a given instant, although the relative velocity may be changing with time, the instantaneous forces on the section are those which would exist in steady flow, i.e., there is no significant lag between time of the change of the relative velocity and the manifestation of the new, corresponding force regime at the section. The instantaneous relative aerodynamic velocity seen by a section in unsteady flow is itself the resultant of all the various differences (such as induced velocity patterns of altered vortex wakes) between the steady and unsteady flow regimes.

Let us consider a wing section located at coordinate on the wing at an instant \( \xi \), during the early part of the power stroke. The velocity and aerodynamic force regimes for such a section are pictured in Figure 25. As is conventional, the resultant aerodynamic force intensities (force per unit span, i.e., \( dF/ds \)) are resolved into components perpendicular and parallel to the component \( \hat{V}_{aw,n} \) of the resultant aerodynamic velocity \( \hat{V}_{aw} \), which is normal to the wing lifting line (or to the arc span line in the physical wing). \( \hat{V}_{aw} \) is also the absolute velocity of the elemental wing section. This effective aerodynamic velocity is shown in Figure 25. The normal component \( V_{aw,n} \) of \( \hat{V}_{aw} \) must be used for generality to account for the fact that \( \hat{V}_{aw} \) may not always be exactly normal to the lifting line, i.e., that there may be a component of the relative air flow \( \hat{V}_{aw} \) along the arc span line. It should be noted in particular that the general flight velocity
\( \vec{V} \) will always introduce such spanwise components in \( \hat{V}_{aw} \) whenever the wing arc span moves in other than a vertical plane relative to the body on the downstroke. However, in most cases it would be expected that \( \hat{V}_{aw,n} \) is essentially equal to \( \hat{V}_{aw} \). The normal component is the section lift \( L' \) while the parallel component is the section drag \( D' \). The lift \( L' \) arises from the circulation \( \Gamma' \) existing on the section at time \( t \), by the Kutta-Joukowski law

\[
L' = \rho \, V_{aw,n} \, \Gamma'
\]

In coefficient form

\[
L' = C_L' \frac{1}{2} \rho \, V_{aw,n}^2 \, c
\]

where \( c \) is the section chord. Combining Equations (38) and (39), the relationship between the section lift coefficient \( C_L' \) and the section circulation \( \Gamma' \) is seen to be

\[
C_L' = \frac{2\Gamma'}{V_{aw,n} \, c}
\]

In conventional airfoil practice, \( C_L' \) is obtained either theoretically (by use of such techniques as conformal mapping) or, more usually, experimentally by placing a two-dimensional model of the section in a wind tunnel and measuring the forces generated over a wide range of angles of attack \( \alpha' \). A so-called section lift curve is thus obtained, of the type shown in Figure 26. \( C_L'_{\text{max}} \) denotes the maximum attainable lift coefficient, as limited by section stall.

In the usual case in aerodynamics, a wing section is essentially rigid in form and hence \( C_L' \) is a function only of \( \alpha' \), being independent of the absolute load imposed on the section. In the case of the flapping bird wing, the wing sections are rather elastic, due to their particular materials and mode of construction, and the aerodynamically
effective shape changes under the airload imposed. The section shape change in turn causes a change in the section airload. Thus, the airfoil section shape and absolute loading interact until some stable balance, commensurate with the particular elastic properties of the section, is reached. Under such conditions, $C_L'$ for a given section becomes not only a function of $\alpha'$, but also depends on the absolute force load imposed. Hence, the $C_L'$ for the flapping bird's wing is much more involved than in the case for the conventional aircraft wing. On the other hand, with proper design of the elasticity of the section, it is possible to achieve very beneficial aeroelastic characteristics with extremely light wing structures--a fact of which the bird wing makes great use. Various types of automatic aeroelasticity action in birds' wings will be given further attention subsequently (section 4.3.1.4). Appendix IV discusses further the concepts of aeroelastic wings and wing sections; for our present purposes we shall assume that the relationship $C_L'(\alpha')$ is known for the wing or can be calculated with reasonable accuracy.

The section drag $D'$ arises from the sources previously discusses, viz., the skin friction and induced drag components. The overall section drag is given by

$$D' = C_D' \frac{1}{2} \rho V_{aw,n}^2 c$$  \hspace{1cm} (41)

The skin friction drag coefficient $C_D'$ (which also includes any pressure drag which may exist on the section) is usually obtained experimentally in the wind tunnel for conventional aircraft wings. In the case of the bird wing, $C_D'$ is also subject to the aeroelastic effect already discussed for the case of $C_L'$. The nature of the induced drag component

$$D_L' = C_{D_L'} \frac{1}{2} \rho V_{aw,n}^2 c$$  \hspace{1cm} (42)

of the flapping wing is extremely involved in the case of the bird wing, and will be treated in detail subsequently in section 4.3.1.4 after consideration of the vortex wake of the flapping wing. It will suffice here to state that a drag component $D_L'$ also exists at each section. The section induced drag $D_L'$ is related to the section lift $L'$ as per linear theory, but not at all in the simple manner as pertains for conventional straight wings of aircraft.
The major section parameters governing the wing aerodynamic force distribution and hence the total wing force can now be summarized:

\[
\begin{align*}
C_L' &= C_L' (\alpha', \hat{V}_{aw,n}, s) \\
C_D' &= C_{Df} + C_{Dz} = C_D' (\alpha', \hat{V}_{aw,n}, s) \\
\hat{V}_{aw,n} &= \hat{V}_{aw,n} (s, t) \\
C &= C(s, t) \\
\alpha' &= \alpha'(s, t)
\end{align*}
\] (43)

The section force coefficients are functions of the section angle of attack \( \alpha' \) (which is defined more precisely below), the absolute loading of the section (denoted by \( \hat{V}_{aw,n} \), i.e., \( L' = \frac{1}{2} (\hat{V}_{aw,n}) \), and the arc span coordinate \( s \), which specifies the particular section shape and associated elastic properties. The section aerodynamic velocity \( \hat{V}_{aw,n} \) is a function of the arc span coordinate and time (of the cycle). The chord length \( c \) and section angle of attack \( \alpha' \) are functions of both \( s \) and \( t \). The section angle of attack may be adequately defined here as the angle between some characteristic axis fixed in the (rigid) leading edge of the section and \( \hat{V}_{aw,n} \). This is justifiable since the wing leading edge is relatively solid and rigid in shape; it is rather the large secondaries emanating from the lifting line which are involved in determination of the elastic properties of the section (see Figure 27). The \( \alpha' \) of a section depends not only on the direction of \( \hat{V}_{aw,n} \) but also on the twist of the wing section relative to the arc-axis line through the leading edge. This twist is a combination of the natural wing twist (i.e., in the unloaded, extended wing) due to the basic structure and articulation, plus that arising as a result of the airload on the wing (due to aeroelastic effects).
Thus, if for a wing chord distribution $c(s)$, the section properties are known (i.e., if $C_{L}'$ and $C_{D}'$ are known) as a function of the relative aerodynamic velocity $\hat{V}_{aw,n}$, and additionally the aeroelastic properties of the main structure of the wing as a whole (particularly any overall wing twist, which governs $\alpha'$), the wing force distribution $L'(s)$ and $D'(s)$ at any instant can be determined for a specified pattern of $\hat{V}_{aw,n}(s)$. To obtain the instantaneous ($t = t_1$) section lift and drag coefficients, we use the relations

$$L'(s, t_1) = C_{L}'(s, V_{aw,n}) \rho V_{aw,n}^2 c(s)$$

(44)

$$D'(s, t_1) = C_{D}'(s, V_{aw,n}) \rho V_{aw,n}^2 c(s)$$

(45)

In vector form these forces are given by

$$\hat{L}'(s, t_1) = C_{L}' \frac{1}{\lambda} \rho c (\hat{V}_{aw} \cdot \hat{n}) \hat{n} \times (\hat{V}_{aw} \cdot \hat{n}) \hat{\tau}$$

(46)

$$\hat{D}'(s, t_1) = C_{D}' \frac{1}{\lambda} \rho c (\hat{V}_{aw} \cdot \hat{n}) \hat{n}$$

(47)

Strictly speaking, there may also be very small force components acting parallel to the local arc span (i.e., parallel to $\hat{n}$). These arise from the component of skin friction drag associated with the spanwise flow velocity ($\hat{V}_{aw} \cdot \hat{n}$), when such spanwise velocities exist. These forces would generally be exceedingly small in all practical cases and certainly negligible in relation to all other wing forces. Appendix V presents a brief discussion of the nature and effects of such spanwise flows in wing boundary layers. For actual flight analyses, we are not directly interested in the section lifts and drags as such, but in the resultant forces in the $\chi$ and $\gamma$ directions, for it is the resultant $\gamma$ force which will balance the bird's weight and the resultant $\chi$ force which will overcome the aerodynamic drag of the body so that the forward flight velocity $\hat{V}$ can be maintained. The total wing aerodynamic force components are thus obtained by integrating the vector section-force intensities over the span of the entire wing. Thus, at time $t_1$, we have

$$\bar{L}_{aw}(t_1) = \int_{s_i}^{s*} \rho c \left[ C_{L}' (\hat{V}_{aw} \cdot \hat{n}) \chi (\hat{V}_{aw} \cdot \hat{n}) \hat{\tau} \cdot \hat{k} \right] \hat{k} \ ds$$

(48)
where \( \tilde{L}_w \) is defined as the total wing "lift" (in relation to the general flight velocity \( \tilde{V} \)) and \( \tilde{D}_w \) is the total wing "drag" (with regard to \( \tilde{V} \)). \( \tilde{L}_w \) and \( \tilde{D}_w \) are in reality the \( \gamma \) and \( \chi \) components, respectively, of the resultant wing aerodynamic force. A third integral can be written for the side force component \( \tilde{\gamma} \) of the wing aerodynamic force, but this is always zero for symmetrical flight:

\[
\tilde{\gamma}(t_i) = \int_{s_0}^{s_i} \rho c \left[ \tilde{L}'(s_i) + \tilde{D}'(s_i,t) \right] ds = 0
\]

Equations (48) and (49) give the total wing "lift" and "drag" at the instant \( t = t_i \) during the power stroke. These equations apply, however, in precisely the same form for any time of the cycle. Thus, we have, in general,

\[
\tilde{L}_w(t) = \int_{s_c}^{s_b} \rho c \left( \tilde{V}_{aw} \cdot \hat{n} \right) \hat{n} \times (\tilde{V}_{aw} \cdot \hat{n}) \hat{\tau} \cdot \hat{k} ds
\]

\[
\tilde{D}_w(t) = \int_{s_c}^{s_b} \rho c \left[ (\tilde{V}_{aw} \cdot \hat{n})^2 \hat{n} \cdot \hat{\lambda} \right] ds
\]

By using the arc span coordinate \( s \), the integration can be theoretically carried out for the wing under any flight condition including the manus feathering, recovery, and manus alignment phase, the picture being essentially that of a continuously curving lifting line with unique, continuously varying force coefficients for each value of \( s \) between the points \( s_c \) and \( s_b \). In the actual case the low aspect ratio of some birds and the overlapping of secondaries during certain phases, such as the recovery stroke, may make it difficult to determine or specify clearly a unique value of the coefficients for each value of \( s \). Under such conditions, it is necessary to resort to a summation technique using finite areas or segments of the wing rather than a continuous integration. For most birds, however, Equations (51) and (52) are good representations of the actual wings. In Equations (51) and (52), the factors \( \tilde{C}_L' \), \( \tilde{C}_D' \), \( \tilde{V}_{aw} \), \( \tilde{n} \) and \( \hat{\tau} \) vary with both \( s \) and \( t \).

However, should we desire to perform an actual estimation of the aerodynamic forces and force distributions of the flapping
wing of a specific bird over the course of the entire cycle, we would have to obtain accurate data on the aerodynamic coefficients of the specific wing sections, as well as the time variation of the aerodynamic velocity, $\mathbf{V}_{aw}(s,t)$. A knowledge of $\mathbf{A}'(s,t)$ would also be required, particularly if the wing undergoes any voluntary $\mathbf{A}'$-influencing rotations at any of its joints during the stroke, in addition to any involuntary aeroelastic twist of the wing. Equations (51) and (52) will in theory allow the calculation of the instantaneous internal muscular forces and the power needed to drive the wing for a given system of articulation. The central role of the relative aerodynamic velocity in the flapping wing is thus clearly evident.

It is quite easy to see that, while such an estimation of the wing forces is not at all impossible, it is a most difficult and involved problem. To the complexity introduced in the calculation of the basic section lift distribution $L'(s)$ by unsteady aeroelastic flow phenomena must be added the even greater complexities of the section induced drag distribution $D_c(s)$ resulting from the intricate geometry of the vortex wake from the wing and pinions.* Still, Equations (51) and (52) state the essential aerodynamic relationships in quite explicit form and the only bar to their evaluation for a specific bird lies in determination of the coefficient and velocity variations by experimental means. Despite the complexities involved in actual computations, these equations give us a clear and lucid insight into the basic mechanics of the flapping wing.

The importance of the relative aerodynamic velocity field of the wing (that is, the observed velocity of the wing in "still" air) in determining the wing forces is obvious from the foregoing analysis. In principle, $\mathbf{V}_{aw}(s,t)$ can be experimentally determined rather precisely by high-speed photographic analysis. In practice, this proves to be a somewhat difficult task, requiring an involved three-dimensional camera system—a system most difficult to set up for birds because of their relatively great speed and mobility, especially in fast-flapping flight (as opposed to the slow flight and hovering conditions of take-offs and landings which are more easily studied). Such camera systems have been successfully utilized, however, for qualitative motion studies of birds and man and various other

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*The special aerodynamic phenomena associated with the flow about the wing tip primaries, or pinions, are a subject of great scope in themselves, and will be discussed in section 4.3.1.4.
Considering the great advances in photographic technology which have occurred since Muybridge's pioneering studies, it should be possible to obtain very accurate measurements of the wing absolute velocity field using his basic technique with modern equipment. With such data, for specific birds of measured weight and size, it should be possible to determine much about the wing force distributions and hence about the total wing forces and flight power requirements, as will be discussed subsequently. For the analytical purposes of the present paper, however, in view of the lack of precise data for quantitative treatment of actual birds, discussion will center on an analysis of the general aspects of the wing velocity field \( \hat{V}_{\text{aw}} (s,t) \) and on the associated wing force distributions. The central features of the general quantitative flight mechanics of flapping birds will be made explicitly clear, and some fundamental mechanisms which underlie the efficiency of flapping flight will be exposed.

As stated by Equation (36), the absolute velocity \( \hat{V}_{\text{we}} \) of the wing (i.e., points thereof), and hence the relative aero-dynamic velocity, can be expressed as the vector sum of the velocity of the wing (point) relative to the body \( \hat{V}_{\text{wb}} \), and the velocity of the body relative to the earth \( \hat{V}_{\text{be}} \). Advantage can be taken of this fact to determine the relative aerodynamic velocity of the wing in a rather simple way. If a high-speed film is available of the side view or projection of a bird in flapping flight, with known filming speed (i.e., the number of frames exposed per second), then by choosing some base reference point on the body of the bird, the \( x \) and \( y \) distances from the base point of various points on the wings can be measured and the changes in distribution determined for successive frames. The known body length of the bird can be used to convert the distances to actual scale. From the time-distance relations thus obtained, the \( x \) and \( y \) components of the velocity of the wing relative to the body can be determined as a function of time. If a suitable fixed reference point (i.e., fixed relative to earth) also appears in the film, the absolute velocity of the bird's body relative to earth as a function of time can be obtained in a similar manner for the entire flapping cycle. By combining the two velocities, the \( \hat{V}_{\text{we}} \) can be established for the wing points considered (neglecting any lateral velocity component which may exist). In cases where \( \hat{V}_{\text{be}} \) is essentially constant over the cycle (i.e., equal to \( \bar{V} \)), then measurement of \( \hat{V}_{\text{wb}} \) suffices, with one measurement of \( \bar{V} \) to give \( \hat{V}_{\text{we}} (\bar{V} \text{ in still air}) \). It should be emphasized that only when the air was still at the time the picture was made.
does \( \hat{v}_{we} = \hat{v}_{wa} \). If a headwind or tailwind component of the wind exists at the time of filming, the strength of this component at the position of the bird must be known in order to determine \( \hat{v}_{we} \); hence, a knowledge of the wind conditions at filming must be available for any quantitative analyses. Unfortunately, this condition is rarely known for the generally available films, and this precludes the quantitative determination of \( \hat{v}_{wa} \) or \( \hat{v}_{we} \), without the assumption of zero wind speed, \( \hat{v}_{ae} = 0 \). Since \( \hat{v}_{wa} \) in most birds in fast flight is not great (60 mph being an average maximum for fast birds, such as geese and ducks, and much less for most other birds), the existence of even a moderate headwind or tailwind at time of filming (\( \hat{v}_{ae} = 15-20 \) mph) can lead to great errors in values of \( \hat{v}_{wa} \) (and hence in the wing aerodynamic forces) estimated on the assumption \( \hat{v}_{ae} = 0 \).

There exist a number of excellent high-speed motion-picture studies of birds in flapping flight having numerous sequences or segments suitable for determining various aspects of the wing velocity relative to the body during the flapping cycle for the various modes of flight. Using sequential frames of a given flapping cycle, taken essentially at right angles to a bird's flight path and on the same level as the bird, quite accurate determinations of the longitudinal and vertical velocity components (i.e., \( \hat{v}_x \) and \( \hat{v}_y \) components) of the wing motion can be made, provided the bird is not so close to the camera that it covers too wide an arc of travel during the time period \( T \) of one cycle. Under such conditions, the successive frames are essentially perpendicular to the flight path, and hence accurate projections (or silhouettes) of the bird in the vertical plane are obtained.

4.3.1.3 The Structure of the Vortex Wake in Fast Flapping Flight

The induced drag of a conventional wing is a force of the greatest importance in flight, since it determines in large measure the amount of power that will be required for propulsion. This is especially true in the case of relatively slowly moving wings which require high angles of attack and high lift coefficients to attain their desired lift. In birds, the induced drag is probably by far the largest drag source and hence is critically important in the flight economy of the flapping wing.
In order to investigate the induced drag of the flapping wing, it is necessary to know in precise detail the spatial distribution of vorticity in the wake. From a knowledge of the vorticity, the induced downwash distribution at the wing can in principle be determined, and hence the $D_L$ of the wing calculated. This section will discuss the basic nature of the vortex sheet wake behind a flapping wing and present the quantitative relationships needed to establish its precise form and distribution of vorticity.

If, at the instant $t = t_1$, the arc span distribution of lift intensity $L'(s,t_1)$ and aerodynamic velocity $\hat{V}_{aw}(s,t_1)$ are known, then the distribution of circulation along the span is determined as

$$\Gamma'(s,t_1) = L'(\rho \hat{V}_{aw,n})^{-1}$$

from Equation (38). As required by Helmholtz's law of vortex continuity, there trails downstream of the wing a vortex sheet whose intensity distribution immediately behind the wing is as illustrated in Figure 28. We shall designate this sheet or surface of vorticity arising from the existence of the derivative $d\Gamma/ds$ at the wing as the sheet of longitudinal vortices, or simply longitudinal vortices. As we shall see, this sheet does not actually lie in the longitudinal $x-y$ plane, but the terminology is convenient since it is compatible with that of conventional fixed-wing theory.* The vortex lines constituting the longitudinal sheet follow the streamlines of flow at the wing trailing edge and hence the vortex surface just behind the wing is essentially that containing the flow vectors $\hat{V}_{aw}(s,t_1)$ at the lifting line. At each instant $t$, the longitudinal vortex sheet has a different orientation and intensity, depending upon the instantaneous pattern of $\hat{V}_{aw}(s,t)$.

*For a detailed discussion of the nature, terminology, and quantitative aerodynamic theory of steady and unsteady vortex wakes, see Cone's papers, references 6 and 16.
Unlike conventional wings, the flapping bird wing has an extremely complex motion (i.e., $\dot{V}_{aw}(s,t)$) and the total longitudinal vortex sheet, which may be pictured as an extrusion of a sheet of line vortices from the wing trailing edge with local initial extrusion velocity (relative to the wing) of $\dot{V}_{aw}$, is quite complex in both geometric form and distribution of vortex intensity over its surface. As will be shown subsequently, however, the quantitative characteristics of this sheet can be established from a knowledge of the functions $\dot{L}'(s,t)$ and $\dot{V}_{aw}(s,t)$.

In general, the magnitude and distribution along the arc span of the local lift intensity $\dot{L}'(s,t)$ are changing continuously during the course of the flapping cycle, being, as we have seen in the previous section, very small (essentially zero over the manus) during the recovery stroke and very large (over the whole wing) during the power stroke. Thus, the corresponding circulation distribution function $\Gamma(s,t)$ varies greatly over the period of the flapping cycle. When the circulation at a station $s$ on the arc span increases with time (say by an amount $d\Gamma = (d\Gamma/dt)dt$), then once again Helmholtz's law of vortex continuity (or more generally by Kelvin's law of constancy of circulation) requires that vorticity be shed from the wing, this time in the form of a transverse vortex line of strength $d\Gamma$, as illustrated in Figure 29. Since the change in $\Gamma(s)$ with $t$ is continuous, a continuous sheet of transverse vorticity will thus be generated by the wing, in addition to the longitudinal vortex sheet. For analytical purposes, the resultant vortex sheet behind the wing may be conveniently pictured as composed of a superposition of the individual sheets of longitudinal and transverse vortex lines, but in reality, of course, the sheet is generated as a single continuous entity at the wing. Like the longitudinal vortex sheet, the transverse vortex sheet (which is geometrically coincident with it) has a very complex shape and distribution of vorticity for the flapping wing.

Figure 29
For purposes of analysis, it is sufficient to construct the vortex wake for one flapping cycle, since the total wake consists of a continuous sequence of identical segments, each of length $\tilde{\mathbf{v}} \cdot \mathbf{T}$ where $\mathbf{T}$ is the cycle period. To construct the vortex sheet for a cycle, we proceed as follows. In accordance with the usual assumption of linear aerodynamic theory, we neglect any effect of the induced velocity field of the wake upon the vortex wake itself (since the induced velocities are generally relatively small), and hence assume that the vortex sheet generated by the wing during the cycle does not deform appreciably until the bird is a considerable distance away. Thus, it is possible to construct the longitudinal and transverse vortex sheets by tracing the vortex lines emanating from the wing as a result of circulation changes with the variables $s$ and $t$. Since a longitudinal vortex line will leave the wing at arc span coordinate $s$ in the same direction as $\tilde{\mathbf{v}}_{\text{w}}(s)$ at a given time $t$, the sheet of longitudinal vortices is obtained by constructing the space path traced out by each point $s$ on the arc span as the wing proceeds through the cycle. As has been discussed previously, $\tilde{\mathbf{v}}_{\text{w}}$ is essentially equal to $\mathbf{V}_{\text{w}}$ or $-\mathbf{V}_{\text{w}}$ for still air, neglecting the induced downwash velocity $\mathbf{f}_{\text{i}}$ at the wing station. At the instant $t$, a small length of longitudinal vortex filament $ds'$ is shed from a point $S$ of the wing; its strength is given by $(\mathbf{d}r/\mathbf{d}s) ds'$, corresponding to the $\mathbf{r}(s)$ existing on the wing at that instant (Figure 30). At some later time $t'$, this elemental length will lie a distance $s'$ from the wing along the space curve of the point $S$ from which the filament originated; here

$$s' = \int_{x_0}^{t} \mathbf{V}_{\text{w}} dt'$$

is obtained by integration over the elapsed time period $t - t_0$. Due to the continuous changes which occur in the magnitude and distributions of $\mathbf{r}(s)$ with time, it is clear that the vortex strips will vary continuously in intensity along their lengths.

The transverse vortex sheet is composed of transverse vortices which leave the wing in the surface of the longitudinal
sheet and parallel to the local arc span vector \( ds \). The local strength of the transverse vortex filament leaving an elemental length \( ds \) of the airfoil at time \( t \) is given by \( \frac{ds}{dt}(s) dt \). Since a strip of transverse vortices of width \( V_{we} \) is generated in time \( dt \), the strength of this elemental strip is \( \frac{ds}{dt}(s) \sqrt{s'} ds' \) where \( s' \) denotes arc length along the space path of the point \( S \). (See Figure 31.) As in the case of the longitudinal vortices, at some later time \( t' \), the transverse vortex filament which originated at the wing at time \( t \) will be \( s' \), distant from the wing along the space curve for the point \( S \). Likewise, the intensity of the longitudinal strip shed at time \( t' \) will vary along its length. Thus, the spatial distribution of the trailing vortex field can be explicitly determined for the flapping wing.

An instructive way of visualizing the vortex wake generated during one flapping cycle is as a surface consisting of a superposition of closed rings or loops of elemental vortex filaments of unit strength. The circulation distribution on the wing is thus obtained by superposition of the upstream lags of an adequate number of vortex filaments of proper length, as illustrated in Figure 32 for the simple case of a translating wing shortly after starting. Each filament is of
constant strength along its entirety, and hence the distribution of $\Gamma'$ along the starting vortex is the same as that on the wing. Since the filaments cannot end in the air, they must form a closed loop and pass on downstream to establish in totality the longitudianl and transverse vortex sheets constituting the wake. Whenever the circulation magnitude or distribution on the wing changes, new transverse filaments are shed into the wake from the wing of sufficient total strength to balance the circulation change on the wing. If the lift of the wing is increased, a new series of rings will be formed, the upstream legs of the rings remaining fixed on the wing with the downstream legs being shed as longitudinal vortices into the wake at a rate proportional to the rate of increase of wing lift. If the lift is decreased, longitudinal vortices formerly bound on the wing are released and thus form fully-closed, free vortex rings in the wake. In this case, the direction of rotation of the shed transverse vortices is the same as for those bound vortices of the wing and constitutes elemental "stopping" vortices. Thus, the wake is a superposition of closed vortex filaments, the local transverse filament density of the wake mirroring the changing conditions at the wing at the time they were shed and the longitudinal filament density reflecting the spanwise (i.e., $s$) variation of circulation on the wing at the time of shedding.

Although the figures of the preceding paragraph, for simplicity, picture the conditions for the flat linear vortex wake of a conventional wing, the principles discussed pertain without alteration directly to the vortex wake of the flapping wing. In the latter case, however, the vortex sheet is rather complex in both its geometric aspects and in its distribution of intensity. A good qualitative idea of the nature of these factors for the vortex wake of the wing in flapping flight can be obtained by consideration of the velocity and force variations discussed in the previous section (4.3.1.2). The wing is assumed to be in the final stages of the manus alignment transition, at time $t = 0$. At this instant the manus and indeed a large part of the wing is carrying essentially no aerodynamic load, and hence no circulation exists over the outer part of the wing. While there may be some aerodynamic load carried by the inner wing portion (i.e., near the body), this too is rather small. Now as the manus sweeps into alignment and the wing commences the power stroke, the circulation $\Gamma'(s)$ increases very rapidly due to the sudden increase in $\dot{X}$ and the rapid increase of the aerodynamic velocity $\dot{V}_{aw}$, as the wing is accelerated.
forward and downward by the flight depressor muscles. During the power stroke, the wing is essentially straight (i.e., fully extended) and rotates about the humerus joint in a slightly canted manner (because of the slight forward movement of the wing relative to the body).

The wing movement on the downstroke can be considered as the sum of two rotations: a rotation about a horizontal (X) axis with an angular velocity \( \dot{\omega}_X \), and a rotation about a vertical (Y) axis with angular velocity \( \dot{\omega}_Y \), where \( \dot{\omega}_X \gg \dot{\omega}_Y \). The velocity of any point on the wing (i.e., wing station) is then given by

\[
\dot{\omega}_X \times \hat{S} + \dot{\omega}_Y \times \hat{S}
\]

(54)

where \( \hat{S} \) is the (vector) connecting the humerus joint with point on the wing. These vectors are illustrated in Figure 33. The wing tip traces out a semihelical path, therefore, which has a slightly decreasing radius (measured as distance normal to the X-axis).

The body simultaneously is moving forward and slightly accelerating; for all practical purposes, however, the velocity of the body can be considered constant and equal to \( \dot{V} \). Thus, elements of the wing generate absolute space paths (i.e., paths relative to earth) which are geometrically quite similar.
to but not true helical segments originating at the beginning of the downstroke. \( S' = 0 \), see Figure 34). Consequently, the longitudinal vortex filaments of the wake will also be semi-helical segments. Superposed on this system of longitudinal filaments will be the system of transverse filaments associated with \( d\gamma/dt(s,t) \) on the wing.

The rapid generation of circulation on the wing at the commencement of the downstroke will result in the formation of a highly localized band of transverse vorticity—the starting vortex. This band will be most intense over the outer portion of the wing because the circulation there was essentially zero prior to the manus alignment. As the wing sweeps on down, however, the transverse vortices which are subsequently shed are very weak since the magnitude of the circulation loading of the wing changes only slightly once the initial starting vortex has been created. Longitudinal vortex filaments stream from the wing with essentially constant intensity of \( d\gamma/ds(s,t) \) during this period following commencement of the downstroke. As the wing velocity begins to decrease near the end of the power stroke, transverse vortices of rotation opposite from those of the starting vortex are shed into the wake at a rate proportional to \( d\gamma/dt(s) \).

In order to quantitatively relate the various spatial points of the vortex sheet with the circulation changes with \( \gamma \) and \( \tau \) on the wing, and hence to establish the distribution of vortex intensity over the sheet, we proceed as follows. At the time \( \tau = 0 \), the wing has a specific spatial position, defined precisely by the spatial equation of the arc span line at time zero, \( f(x,y,z,\tau = 0) \). The arc span at this time is essentially a straight line inclined at an angle of some 50° to 60° relative to the horizontal \( x,y \) plane and slightly rearward (10° to 20°) of the vertical \( x',y' \) plane. At time \( \tau = 0 \), the wing has a definite aerodynamic velocity distribution \( \dot{V}_{aw}(s,\tau = 0) \). This distribution can be easily determined using the \( \dot{\omega}_{x} \) and \( \dot{\omega}_{y} \) relationships discussed previously. Also, at time \( \tau = 0 \), the wing has a circulation change taking place at the rate \( d\gamma/d\tau(s,0) \).

Now, as the wing sweeps down, each point \( s' \) on the arc span generates a space curve whose arc length, measured from the location of the point at time \( \tau = 0 \), is \( s' \), as illustrated in Figure 34. Thus, the function \( s'(s,\tau) \) gives the position of any wing point \( s' \), at any time during the cycle. The wake arc length equations \( s'(\tau) = f(x,y,z,\tau) \) for any wing point \( s' \),
can be simply established using the \( \hat{\omega}_x \) and \( \hat{\omega}_y \) approximation as before. Thus, each point of the trailing vortex sheet can be specified by giving its \( s, s' \) coordinates. Since \( \nu_{aw} = (ds'/dt) \)

\[
s'(s,t) = \int_{t_0}^{t} \nu_{aw}(s,t') dt'
\]

where \( t' \) denotes a dummy variable of integration.

The longitudinal and transverse vorticity components of any point \( s, s' \) on the vortex sheet can now be determined by the fact that at the time \( t_i \), when the circulation was changing at the rates \( \frac{\pi}{\partial t} \) and \( \frac{\pi}{\partial s} \) at \( s \), the wing was shedding the corresponding vortices at point \( s, s' \). Thus, we have the relationships

\[
s'(s,t_i) \longleftrightarrow \frac{\partial \pi}{\partial t} (s,t_i)
\]
\[
s'(s,t_i) \longleftrightarrow \frac{\partial \pi}{\partial s} (s,t_i)
\]

at corresponding times. Hence, when \( s' \) is specified for a given \( s \), the corresponding vortex intensities \( \frac{\pi}{\partial t} \) (transverse) and \( \frac{\pi}{\partial s} \) (longitudinal) are specified and can be written down for all points of the vortex sheet. The total vector vorticities at point \( s, s' \) of the sheet needed for the calculation of \( D \), are determined by the fact that the longitudinal and transverse vortices lie essentially parallel to \( \nu_{aw}(s,t) \) and to \( d\zeta(s,t) \) the local vector arc length of the wing at point \( s \), at the time of shedding \( t \).

It is clear that, by use of the above relationships, the vortex wake of a flapping wing during the power stroke can be determined solely from a knowledge of \( \nu_{aw}(s,t) \) and \( \nu_{aw}(s,t) \). For the remaining phases of the cycle, the vortex wake is much less important, since the outer portion of the wing becomes essentially unloaded at the end of the power stroke and hence produces no vortex wake until commencement of the following power stroke. The inner wing near the body may continue to carry some lift loading and hence generates relatively weak longitudinal and possibly transverse vortices throughout the remainder of the cycle. The longitudinal vortex filaments from the inner wing are therefore probably continuous (although varying in intensity) through the flapping cycle. The vortex wake of the outer wing, however, is highly periodic, very
intense, and of closed form in that its wake has a very definite beginning (starting vortex) and termination (stopping vortex) bracketing the power stroke in each cycle. An overall schematic picture of the vortex wake of the flapping wing as deduced from the examination of flight films is presented in Figure 34. The relative density of the vortex lines represents roughly the intensity of the wake. The regions of strong transverse vorticity associated with the starting and stopping vortices are evident.

The vortex wake is thus a three-dimensional sheet upon which is recorded in terms of intensity and of distribution of vorticity the entire cycle history of the spatial and time-based lift variations of the wing, as well as its three-dimensional movement through space.

4.3.1.4 The Induced Drag of a Flapping Wing

(1) Approach to the Problem. The induced drag, that is, the drag associated with the velocity field induced at the wing by the vortex wake, is a factor of great importance in flapping flight since it has a critical bearing on the power requirements of the wing and hence on the efficiency of flight. Unfortunately, the vortex wake and the associated $D_\text{I}$ are the most complex features of the entire aerodynamic regime of the flapping wing, and very little, if any, previous technical consideration of these factors appears to have been made. This section is intended to make an initial start on the alleviation of this condition by providing in explicit form the fundamental mathematical setup needed to compute the $D_\text{I}$ of a flapping wing. As will become evident, calculation of the $D_\text{I}$ for a particular bird requires a considerable amount of detailed experimental data, none of which is presently available. Consequently, no actual quantitative evaluations of $D_\text{I}$ for specific birds can be carried out. Application of the techniques developed in this section must therefore await the results of carefully planned experimental programs to obtain the necessary data for particular birds. On the other hand, the techniques developed can be readily applied in principle to the analysis of hypothetical wings having specified aerodynamic and kinematic characteristics, and hence are directly applicable to the basic design of ornithopters. The actual evaluation of a real or hypothetical wing will in all probability require extensive use of electronic computers, as the complexity of the wake, as well as the need to use empirical data in tabulated form, may preclude closed-form
mathematical integrations and hence require lengthy and involved numerical analyses. It is hoped, however, that such calculations for both specific bird wings and ornithopter designs will be carried out in the future, for only then can actual quantitative figures for flapping flight efficiency be determined.

In lieu of quantitative calculations, a qualitative analysis of the apparent $D_\infty$ efficiency of the flapping wing, as compared to a fixed wing furnishing equal (average) lift, is presented, using the induced velocity relationships and vortex wake properties developed in previous sections. Based on this analysis, some tentative conclusions are drawn as to the relative efficiencies of the two wing forms.

(2) Quantitative Expressions for Calculating $D_\infty$. The basis for determining $D_\infty$ of the flapping wing is simply the law of induced velocity

$$\dot{V}_\infty (s,t) = \frac{1}{4\pi} \iint_\omega \hat{\omega}_p \times \frac{r}{r^3} \, dS_o$$  \hspace{1cm} (57)$$

where $\dot{V}_\infty$ = the velocity induced by the total wake at point $s$ of the arc span at time $t$

$\hat{\omega}_p$ = the vorticity vector at point $P$ of the vortex sheet

$\hat{r}$ = the distance vector from a point on the sheet to the wing point

$\delta S_o$ = the differential of sheet surface area.

The double integral denotes integration over the entire vortex wake (not just the segment of one cycle). Performing the integration indicated in Equation (57), the distribution of (section) induced drag becomes

$$\hat{D}_\infty (s,t) = \Gamma V_{aw}^{-2} \left[ \dot{V}_\infty \times \hat{r} \cdot \hat{V}_{aw} \right] \hat{V}_{aw}$$  \hspace{1cm} (58)$$

The total wing-induced drag is then given by

$$\overline{\hat{D}_\infty} (t) = 2 \left[ \int_0^s \Gamma V_{aw}^{-2} \left[ \dot{V}_\infty \times \hat{r} \cdot \hat{V}_{aw} \right] (\hat{V}_{aw} \cdot \hat{r}) \, d\hat{s} \right]$$  \hspace{1cm} (59)$$
The determination of \( \hat{\mathbf{q}}(s,t) \) is, for the flapping wing, no simple matter due to the complex geometry involved, both in regard to the spatial disposition of the vortex sheet and the distribution of longitudinal and transverse vorticity over the surface of the sheet. It is at once evident, however, that the direction of \( \hat{\mathbf{q}} \) will not, in general, be normal to the arc span nor perpendicular to the direction of \( \hat{\mathbf{v}}_{w} \). This is a fact of considerable importance, as will be discussed subsequently. The complexity of the wake would necessitate the use of computers to establish the vector-induced velocity field \( \hat{\mathbf{v}}(s,t) \), but could be carried out assuming the functions \( \mathbf{r}(s,t) \), \( s(x,y,z,t) \), and \( \hat{\mathbf{v}}_{w}(s,t) \) to be empirically known (in tabulated or graphical form).

Thus, it is clear from Equation (59) that the induced drag hierarchy of the flapping wing involves many features quite different from those of a conventional fixed wing. The really important question is: Do these differences in vortex wakes result in a significant difference in \( D_{\gamma} \), and more specifically, do they result in a significantly lower \( D_{\gamma} \) for the flapping wing? Before any conclusions can be drawn in regard to these questions, it is necessary to examine the differences in the wake forms in more detail.

(3) Comparison of the Vortex Wakes of Fixed and Flapping Wings. The vortex wake behind a flapping wing is, as we have seen, grossly different in geometry and distribution of vorticity from that of a conventional wing gliding or uniformly translating through the air. In the former case it is essentially a series of periodic, equally-spaced segments of sheet-like form composed of quasi-helical filaments on which is superposed an array of transverse filaments to form a complex three-dimensional network of vortex lines. In the latter case the wake is quite simple, being merely a flat sheet composed only of longitudinal vortices joining, far downstream of the wing, a single transverse starting vortex. It is this gross difference in wakes which is of interest in regard to the relative \( D_{\gamma} \) efficiency of the two wings.

One question of prime importance in regard to the vortex wake and the relative \( D_{\gamma} \) efficiency of the fixed and flapping wings involves the so-called "starting" and "stopping" (transverse) vortices. When a fixed wing starts from rest in still air, the wing develops a circulation if at a proper angle of attack, and hence a lift. A large starting vortex is shed from
the wing, of circulation equal to that on the wing but rotating in the opposite direction. The existence of a circulation implies that the air about the airfoil has been set into motion, and hence has received kinetic energy from the airfoil. This energy can be shown\textsuperscript{13} to have come from the work done by the wing in moving against the $\Omega_\alpha$ generated at the wing by the starting vortex. Hence, the kinetic energy of the air motion around and behind a three-dimensional wing comes entirely from the work done by the wing in moving against the $\Omega_\alpha$. In the fixed wing there is only one starting vortex and this is soon left far behind the wing, whence its induced effect on the wing becomes negligible. In the flapping wing, there must apparently be a starting vortex for every flapping cycle so that the wing is always in the close vicinity of a starting vortex, and hence subject to its induced velocity field. However, the following analysis of the flapping wing reveals the rather amazing fact that, in theory, a flapping wing need produce no starting vortex in the wake, a condition clearly impossible with a fixed wing. The starting induced drag of the flapping wing, therefore, can in the theoretical limit be as small as one-half that for the fixed wing.

To see how this is possible, let us consider the two semispans of a flapping wing raised to a vertical position and closely adjacent, as illustrated in plan view in Figure 35. Let the wings be rapidly impulsed into motion with velocity $V$ as shown. Each wing will theoretically generate an equal circulation $\Gamma$ as shown; the directions of circulation will be counter to one another however. By vortex continuity, a starting vortex of circulation $\Gamma$ (opposite to that on the semispan) must be generated by each semispan. But from Figure 35, it can be seen that this condition is clearly satisfied without the shedding of actual vortices into the wake since the required counter-circulation of the left semispan is precisely the circulation about the right semispan, and vice versa. Hence, vortex continuity is
satisfied without the creation of physical transverse vortices; the two semispans are simply connected by longitudinal vortex filaments corresponding to the \( \mathbf{p}(s) \) distribution on the wing. The two semispans each induce a downwash on the other and hence there will be a starting \( \check{D}_x \) on each semispan. This \( \check{D}_x \) must, of course, exist in order to provide the kinetic energy required for the absolute air flow generated about each semispan. However, there is no physical (transverse) vortex flow in the wake, and the starting \( \check{D}_x \) of the flapping wing is only one-half that of the fixed wing, where there is a free-starting vortex in the wake.

In practice, of course, the ideal situation described above is not fully realized, for the wings do not follow exactly the conditions for theoretical minimum starting \( \check{D}_x \). The wings at the time of the manus alignment and start of the subsequent power stroke are not fully vertical, although they are raised quite high. In this position, the semispans are still quite close and hence lie in each other's induced velocity fields, with a corresponding reduction in the starting drag. Such longitudinal starting vortices as are shed into the wake are probably very weak for high-starting inclinations of the semispans. By starting the wings at a lower inclination angle rather than from the fully vertical position, a component of the aerodynamic force is immediately available as wing lift to counterbalance the bird's weight, thus lessening the time during which the body may be undergoing a downward vertical acceleration. Also, the inner wing, which carries some aerodynamic lift loading throughout the cycle, would lose this lift component during the time the wings were near the vertical position, thus allowing the body to undergo a free-fall acceleration. The "overproduction" of lift that would be required on the subsequent power stroke to arrest the consequent falling velocity and then to raise the bird back to a proper altitude to maintain its quasi-level flight path might produce greater cycle-induced drags than occur with the semivertical wing start actually used. It is interesting to note in this regard that some birds which fly very fast (e.g., the gannet) do not raise their wings quite so high overhead in the fast flight mode as do the slower fliers (e.g., the herons) in the fast-flight mode; this action is quite understandable in light of the above discussion. The induced drag of a wing is purely a function of the magnitude of the wing circulation. With high speeds, the wing circulation (i.e., \( \mathbf{p}^* \)) need not be large to produce the desired forces and hence the starting \( \check{D}_x \) can be relatively small, compared to that of the more slowly-moving wing which requires a large circulation and
hence would produce a large starting vortex and starting $D_\omega$. The high inclination of the semispans at starting, however, helps reduce the starting $D_\omega$. Also, nearly all birds use very high wing inclinations on take-off, where the forward component of the wing speed is still necessarily low; the reason for this is, of course, the starting drag reductions afforded. Even though the vertical lift force may be small during the period the wings are highly inclined, the thrust force can be quite large and the desired longitudinal acceleration attained with considerably reduced power on take-off. This mechanism is of critical importance to birds with heavy wing loadings $\text{W}/\text{S}$, such as the ducks.

Thus, aerodynamic theory predicts that the bird is able by high semispan starting inclinations in fast flight to minimize the starting $D_\omega$, the lower limit being one-half the $D_\omega$ for a flat wing. The work done against this drag goes to provide the kinetic energy of the induced air flow pattern about the wing. This energy, however, is not lost to the bird but is subject to reclamation upon the subsequent stopping of the wings. If the wing near the termination of the power stroke is decelerated relatively slowly in the sense that it does not come suddenly to an absolute stop (and this is indeed the case in flapping bird flight), transverse stopping vortices will be shed from the wing. These vortices, being of counterrotation to those of the wing semispans proper, will each induce an upwash at the wing and hence will produce a component of negative $D_\omega$, i.e., a section thrust. Since the wing is moving in the direction of this thrust component, work is being done on it, and the wing is therefore reclaiming energy from the supply that it initially put into the flow through the mechanism of the starting induced drag. The stopping process may be viewed in the same manner as the starting process except that the wing semispans are not brought into very close proximity, the maximum downward inclination of the semispans being only about $30^\circ$ before the inner wing begins to rise and the manus to feather. The semispans have shed stopping vortices which induce an upwash velocity field on the entire wing and hence provide a wing thrust component so long as any of the original circulation exists, and hence helps power the forward motion of the wing and prolong its deceleration. It is clear from these effects that the flapping wing need suffer no great energy penalties because of the fact that its periodic motion involves periodic creation and extermination of the circulation loading on the wing.
This leaves the longitudinal vortex wake for primary consideration in determination of the $D_\omega$ efficiency of the flapping wing. As in the case of the transverse vortices, the longitudinal vortex wake of the flapping wing possesses some very interesting possibilities for potential induced drag reduction. Since we are primarily interested in the relative $D_\omega$ of fixed and flapping wings, it is necessary to compare the induced drag characteristics of such wings on the basis of equal wing spans, areas, and aspect ratios, carrying the same total weight $W$, and translating with the same displacement velocity $V$. A rough schematic of the relative longitudinal wake conditions for such wings is shown in Figure 36. The properties of the flapping wing will be considered first and then compared with those of the fixed wing.

In order to apply the requirements stated by Equation (57) to the analysis of the longitudinal vortex wake of the flapping wing, it is convenient to divide the wake into two parts, the near wake (consisting of the wake produced during the immediate cycle under consideration, i.e., the wake segment nearest the airfoil) and the far wake (consisting of all the remainder of the wake downstream of the wake segment of most recent origin). This division is suggested by the factor $\gamma^{-3}$ in Equation (57), which indicates that it is the near wake which is really most significant in determining the $D_\omega$.

There are three factors of primary importance in determining the element of $D_\omega$ at a point on the arc span generated by an element of the near wake: (1) the spatial orientation of the local vorticity vector at a point of the wake, relative to the arc span point on the wing, (2) the distance from the vortex sheet element to the wing point, and (3) the strength of the sheet vorticity. Two other factors are also important: (a) the length of vortex wake over which the integration [Equation (57)] must be made, and (b) the circulation $\Gamma$ existing on the wing at the time the induced drag is evaluated.
Considering the first three factors outlined above, it can be easily seen that the differential induced velocity vector \( \mathbf{\ddot{f}} \) at the wing will generally be inclined to the bound vorticity vector of the wing, and hence only a component of \( \mathbf{\ddot{f}} \) will be effective in producing \( \mathbf{D} \). By tracing the curvature of the semi-helical vortex lines of the wake, it can be seen that for many areas of the sheet the effective component of \( \mathbf{\ddot{f}} \) will be very much reduced for the entire arc span. In contrast, the induced velocity of the fixed wing wake is normal to the planar vortex wake and to the arc span and hence is fully effective in producing induced drag.

If we consider distances between wing and wake elements of the flapping wing, it can be seen that the effect of the curvature of the wake is to lengthen the distribution of \( \mathbf{v} \) as compared to a flat wake generated in the same longitudinal distance \( \mathbf{\sigma} \) travelled in one cycle, where \( \mathbf{\sigma} = \mathbf{\dot{\nu}} \mathbf{T} \) (Figure 37). Since the absolute strength of \( \mathbf{\ddot{f}} \) decreases as \( \mathbf{v}^{-1} \), even slight increases in wake distances can result in appreciable reductions in induced velocity at the wing.

This relative increase in distance for the flapping wing must be viewed, however, in light of the increased length of vortex sheet generated by the flapping wing during the power stroke, as discussed below [factor (a) of the preceding paragraph].

The intensity of the sheet vorticity also appears in Equation (57). Comparing the relative intensities of the flapping and fixed-wing wakes, it is not at all assertable that the flapping wing will have the more intense wake, despite the fact that the flapping wing must produce a large aerodynamic force during the power stroke. The situation is illustrated in Figure 38, where it is seen that the magnitude of the \( \mathbf{F}_x \) (vertical) component of the wing force must be large enough during the power stroke to compensate for the decreased magnitude of this component during the other phases of the flapping cycle so as to produce a total cycle impulse equal to that of the fixed wing.
However, the attainment of this increased vertical force component does not necessarily involve an especially high circulation on the wing. Since the relative aerodynamic velocity of the flapping wing during the power stroke is very large, much larger than that of the corresponding fixed wing, the large forces needed can be obtained with a large increase in wing circulation, and hence without a large increase in the vortex intensity of the wake.

Finally, it is seen that the $D_\omega$ acting on the wing at any instant depends upon the circulation on the wing at that instant as well as upon the wake conditions (as represented by $\hat{h}_\omega$). If $\Gamma$ is sufficiently low, the $D_\omega$ can be correspondingly low despite high wake intensities. As the flapping wing starts the power stroke, its aerodynamic velocity is relatively low, hence its circulation must be high (high $\chi$) in order to get a large wing aerodynamic force. However, as the aerodynamic velocity of the wing increases, $\Gamma$ may be correspondingly reduced while still maintaining a large aerodynamic force. If the wing circulation is thus decreased during the latter part of the power stroke, the induced drag acting during this period could be low. Near the termination of the power stroke, the wing circulation is indeed reduced essentially to zero as the wing passes into the feathering transition phase, but the nature of $\Gamma(s,t)$ during the entire last half of the power stroke is presently uncertain without quantitative measurements.

The importance of the high wing velocity in flapping flight alluded to in the above discussion, in relation to the magnitude of $D_\omega$, can also be clearly seen from the following basic aerodynamic considerations. In the case of an elliptically-loaded fixed wing translating at uniform velocity, the induced drag is determined solely by the circulation in the center plane of the wing and is given by

$$D_\omega = \frac{\pi}{8} \rho \Gamma_o^2 \tag{60}$$
The lift of the same wing is given by

\[ L = \frac{\pi}{4} \rho V b \Gamma_0 \]  

(61)

These two equations tell us that in regard to \( D_* \), \( \Gamma_0 \) is the only important parameter; the lift that will accompany this fixed drag, for a given value of \( \Gamma_0 \), is dependent upon both the wing velocity \( V \) and the wing span \( b \). Thus, for a given value of \( \Gamma_0 \) it is really the lift which varies with wing speed and dimensions, not \( D_* \), although the situation is conventionally viewed from the standpoint of fixed lift. For a given span and lift on such a wing, the total induced drag of the wing will be inversely proportional to \( V^2 \).

\[ D_* = \frac{2L^2}{w p b^4} \frac{1}{V^2} \]  

(62)

Thus, an increase in wing speed is most effective in reducing the \( D_* \) of the wing.

Now, in the case of the flapping wing, quite similar conditions will apply in regard to the role of \( \Gamma_0 \), or more correctly \( \Gamma(s,t) \), with due allowance, of course, for the alterations which will be produced by the highly nonplanar geometry and nonuniform intensity of the wake. Thus, the high velocity of the flapping wing in the power stroke can produce a given lift (i.e., basic aerodynamic force) with a relatively low value of the wing circulation and hence of the induced drag. The essential question in comparing the relative induced drag efficiencies of the flapping and fixed wings is thus obvious: Does the high speed of the flapping wing allow the production of a sufficiently large aerodynamic force such that the lift component \( L \) over a cycle produces an impulse equal to that of the steady lift of the fixed wing, and with an equivalent or lesser expenditure of total energy per cycle?

Finally, considering the far wake, it is seen that as far as induced velocity \( \vec{j} \) at the flapping wing is concerned, the vertical aspects of the wake segments become relatively unimportant from a distance standpoint, compared to the fixed-wing case, but the downwash angle considerations remain. As a result of the rapid decrease in induced velocity effects with distance [Equation (57)], the truly far wake contributes relatively little to the induced velocity field at the wing, so
that the differences in form between the flapping- and fixed-wing wakes become of negligible effect. Of perhaps more significance, however, is the segmentation of the flapping wing wake. Due to the absence of circulation on the outer wing during the feathering transition and the recovery stroke, there will be essentially no vortex wake shed during this period. Since the body is accelerated during the power stroke, the body velocity will be greatest during the transition and recovery phases and the path covered during this period will be essentially free of strong vortex wake; the fixed wing wake is, of course, continuous.

(4) Induced Drag Reduction by the Pinion System of the Flapping Wing. The wing-tips of most flapping birds, particularly those species having wings of only short or moderate span, are not simply a planar termination of the wing surface but rather are divided into several individual aerodynamic surfaces, the pinions or primary wing feathers. These feathers are usually emarginated, highly elastic, and very long and slender, relative to the other flight feathers of the wing. In the moderately loaded condition, the pinions lie relatively close to one another, giving the appearance of a continuous tip surface. In flight, however, they each deflect an appreciable but different amount under the imposed air load and the vanes become a complex system of highly curved "winglets," all anchored at their inner ends to the wing proper. The deflection of the pinions under load is such that they are spread out over an appreciable distance normal to the general plane of the wing, with the first pinion (i.e., the leading primary) being the most highly curved and the others having slightly decreasing curvature down to the last, which lies essentially in the plane of the main wing. The proper sequential curvature of the pinions in flight is attained through variation of the relative span, rachis elasticity, and surface area of each pinion.

The pinion system is most obviously exhibited in the case of land soaring birds, such as the vultures and hawks, this being one of their most characteristic features. The question of the functional purpose of the prominent pinion system in soaring birds was for many years the subject of much speculation and controversy, until Cone\cite{16} proved quantitatively that the pinions were an efficient aerodynamic device for reduction of the induced drag through spreading of the wake vorticity. The action of the pinions is actually even more pronounced in
the case of flapping birds but cannot be directly observed because of the great speed of the wing motion. By means of high-speed photography, however, the great involvement of the pinion system in both the power and recovery strokes is clearly revealed (see Figure 39). The pinion system of soaring and flapping birds is one feature of the natural wing which has no counterpart in the rigid, fixed wing of conventional aircraft.

The primary purpose of the pinions of flapping birds, as in the case of soaring birds, is the reduction of the induced drag of the wing. This it does quite efficiently. The aerodynamic theory underlying the induced drag reduction of multiplane systems in general and the pinioned or branched-tip wing in particular has previously been worked out in precise, quantitative form; hence the details of this mechanism will not be recounted here, except to mention briefly its significant role in flapping flight. The purpose of the pinions is to create a vortex wake which allows the wing's trailing vorticity to be spread out over a considerable area, thus reducing the local intensity of the wake and providing greater distances and angles between the trailing vortex sheet elements and the wing proper, as compared to the single vortex sheet from a flat-tipped wing. In this respect, the action of the pinions is akin to the effects produced by the curvature and spatial distribution of the vortex sheet of the flapping wing we have discussed. By spreading the wake, the effective induced velocity field at the wing is appreciably reduced by the pinions; this drag reduction is quite separate from all the previously considered drag-reducing features of flapping flight. It even appears that there may be additional favorable secondary interactions between the basic curved wake of the flapping wing and the curved pinions, resulting in still greater reductions in induced drag.

In comparing the $D_\omega$ of a pinioned flapping wing with that of a flat-tipped fixed wing, the drag-reducing properties
of the pinions are an important factor since the conventional fixed wing possesses no such beneficial device. The aeroelastic properties of the pinions are intimately integrated into the overall flapping wing system, for the pinions assume their proper attitudes and curvature quite automatically in response to the changing conditions of the imposed airload. When one views the gross and complex flexing of the pinions which occurs in every wing stroke and considers the countless hundreds of thousands of such flexures which occur in the course of the annual flight times of most species, the need for seasonal feather moults is quite understandable. Indeed, it is truly remarkable that under such conditions, the pinions are able to maintain their structural integrity over such long periods.

(5) General Conclusions on the Induced Drag of the Flapping Wing. Having looked at the various aspects of the vortex wake in some detail, it is now desirable to draw some conclusions, if possible, as to the induced-drag efficiency of the flapping wing. The main question is: Does the flapping wing produce its aerodynamic forces with a greater or lesser expenditure of energy than does a fixed wing furnishing an equal vertical impulse over the period of one cycle? As previously stated, this question cannot be answered in quantitative fashion until sufficient data become available for performing the necessary analyses; the mathematical basis for performing the induced-drag calculations of such analyses has been fully developed in this section, however, and it is hoped that an answer to the basic question formulated above will be forthcoming in the not too distant future. Meanwhile, we must content ourselves with some tentative conclusions about the $D_i$ based on qualitative inferences drawn from the foregoing analyses.

There appear to be five factors associated with the flapping wing which are favorable to the reduction of the induced drag:

(a) The low starting and stopping drags,

(b) The curved vortex sheet wake,

(c) The relatively high aerodynamic velocity of the wing during the power stroke,
(d) The drag reduction properties of the aeroelastic pinions,
(e) The unloading of the outer wing during the recovery stroke.

This is an impressive list of sources of induced-drag reduction; the known reduction associated with the pinions alone is highly significant. Hence, it appears most probable that the flapping wing has a relatively high aerodynamic efficiency and is at least as efficient as, and in all probability considerably more efficient than, a fixed wing of equal span traveling at the same body speed and producing an equal vertical impulse.

Additional consideration of the drag and energy requirements of the flapping wing is presented in section 4.3.1.5 and in Chapter V.

4.3.1.5 The Relationship between Lift and Thrust

In the conventional fixed-wing airplane, the lift and thrust forces are produced by independent sources. The lift is produced by the wing, the thrust by the revolving propeller. In the case of flapping-wing flight, both the sustaining and propelling forces are produced by the wing. The relative and absolute motions of the wing and body make the interaction between the lift and thrust of the flapping wing somewhat complex, and appreciably alter the conventional relationships between the wing section forces and the total lifting and propulsive forces. To clarify these relationships, the following brief analysis is presented.

The really important force components, so far as the overall translation of flapping flight is concerned, are the vertical component of the total wing aerodynamic force (which will support the bird's weight) and the horizontal component of this force (which will overcome the body drag and maintain the forward motion of the bird). Let us define, as previously, the total wing lift \( L \) and total wing drag \( D \) as the components of the total wing aerodynamic force acting respectively along the \( y \) and \( x \) axes. Forces are considered positive in sign when acting in the direction of the positive axes. Thus, specifically, a positive value of \( D \) denotes a thrust, while a negative value denotes a true drag. The \textit{section} aerodynamic forces are,
however, resolved into section lift $L'$ and section drag $D'$, normal and parallel, respectively, to the relative aerodynamic velocity $\vec{V}_{aw}$, and also mutually normal (locally) to the wing arc span. This latter specification is very important in the case of the flapping wing since the vector $\vec{L'}$ is not generally vertical, but changes its direction continuously during the cycle; the same is true of the section drag $\vec{D'}$. This is contrary to the usual fixed-wing case where $\vec{L'}$ is always vertically directed.

These relationships are presented in graphical form in Figure 40 which shows an arbitrary wing section (a), the plane of the view being taken normal to the local arc span (b). The subscripts on the vectors denote that they refer to forces in the (normal) plane of the local section. This section has a local inclination $\theta$ relative to the z-axis, as illustrated in (b), the arc span in this case being assumed for simplicity as a straight line rotating downward in a vertical plane relative to the body, i.e., about the longitudinal (x) axis. In the case where there is also a rotation of the wing about the vertical (z) axis, the situation is more geometrically complex but can be easily handled by the $\vec{\omega}_y$ concept introduced in section 4.3.1.2. Thus, the true vertical (z) and longitudinal (x) components $\vec{L'}$ and $\vec{D'}$ of the section aerodynamic force components $L'_n$ and $D'_n$ are given by

$$\vec{L'} = (L'_n \sin \varphi + D'_n \cos \varphi) \cos \theta$$  \hspace{1cm} (63)$$

$$\vec{D'} = (L'_n \cos \varphi + D'_n \sin \varphi) \cos \theta$$  \hspace{1cm} (64)$$

*In Equation (64) the sign of $D'_n$ is negative, i.e., is in the direction of the negative x-axis.
The total vertical wing lift \( \bar{L} \) and horizontal drag \( \bar{D} \) are obtained by integration over the length of the arc span of the wing

\[
\bar{L} = 2 \int_{s_i}^{s_f} \bar{L}'(s) \, ds \quad (65)
\]

\[
\bar{D} = 2 \int_{s_i}^{s_f} \bar{D}'(s) \, ds \quad (66)
\]

It is interesting to note in Equations (63) and (64) and Figure 40 that the vertical lift involves a component of the wing drag, and conversely that the overall horizontal drag (i.e., thrust) involves a component of the section lift. Hence, the basic section drag force of the wing is actually providing a favorable vertical lift. The horizontal component of the section lift \( L_n \cos \phi \cdot \cos \theta \) is actually overcoming the retarding horizontal component of the section drag and providing, in addition, a net thrust (for overcoming body drag).

Of particular interest here is the fact that the wing arc semispan has an appreciable component of its total aerodynamic force acting in a lateral direction over the course of the power stroke, due to the angle \( \phi \), and hence is not always producing its maximum vertical lift force while still expending energy for the negative drag associated with the aerodynamic force. The situation is not as adverse as may at first appear, however, for the inclination of the wings relative to the horizontal results in an increase in the relative lifting efficiency of the wing. That is, for the case of the inclined wings, at least for moderate inclinations, a given vertical lift is produced with less induced drag than in the case of a flat wing of equal projected span, provided the inclined wing has the proper circulation loading \( \Gamma(s) \). In reality, inclining the wings from the flat (\( \theta = 0 \)) position results in a decrease in overall wing efficiency if the same vertical lift has to be maintained, since the absolute \( \Gamma \) loading on the inclined wing must be increased to provide the required vertical force component and this results in an increased \( D_\infty \). However, the \( D_\infty \) does not increase proportionately with the inclination (for moderate inclinations) but more slowly because of the inherent efficiency of the inclined wing form.

To see how an optimally loaded inclined wing can produce less \( D_\infty \) for a given vertical lift than an optimally loaded plane wing of equal span producing the same vertical lift, let us consider what happens to the air flow in the case of such
wings. First of all, induced drag is simply a measure of the energy being put into the (still) air through which a wing is passing in order to create a downward momentum of the air, the reaction to which is the vertical wing lift. The lift can be produced most efficiently when for a given momentum requirement the largest possible mass of air is moved with the smallest vertical velocity.

Now when each semispan of an inclined wing is optimally loaded, it will be "attempting" to throw air outward laterally as well as downward because of the inclination of the arc span (Figure 41). However, since the opposite semispan is also attempting to throw the air in the opposite lateral direction, the air immediately above (i.e., in the "v" of the wing) the wing will become highly rarefied (very low pressure) and all outward or lateral flow will be prevented by this central low pressure region. The entire upper surfaces of the airfoil will experience, therefore, a very low pressure* (relative to that on a planar wing) with no physical outflow of air, and hence with no energy input by the wing and consequently no associated $D_L$. In terms of momentum, the very low pressure central region generated by the inclined semispans is felt over a greater region of air surrounding the airfoil and thus impulses a much larger volume (mass) of air into downward motion as it passes. Thus, relative to the planar wing, the inclined wing sets a relatively larger mass of air into vertical motion at lower velocity for the same total lift.

The mechanics of this rather complex aerodynamic action of nonplanar lifting systems has been worked out in great detail, and the basic theory along with methods for quantitatively

*The low pressure exists above the wing for the case of $\theta > 0$ (raised wing semispans). The same conditions apply, however, for the case of $\theta < 0$ (lowered semispans), only here the pressure in the central region of the wing "v" is considerably increased.
analyzing any wing form are presented in Reference 17. This reference should be consulted for a more technically detailed discussion of the properties of nonplanar lifting systems.

The drag properties of the inclined wing discussed above pertain to the case of quasi-steady flow and illustrate the fact that the lateral inclination of the section force vector in flapping flight need not have an unduly adverse effect on wing efficiency. There is an additional factor involved in the flapping wing in this respect, however, which may be of considerable importance, and this concerns the fact that the inclination of the semispans is decreasing (or increasing) with time. This condition introduces an acceleration effect into the force picture and could possibly even increase the magnitude of the lift. It should perhaps be pointed out that the effects discussed here are all automatically included in the time-space integrations of Equation (57) and hence need not be treated separately in actual calculations of wing forces and efficiencies.

From Equations (65) and (66) the overall wing force components $\mathbf{L}$ and $\mathbf{D}$ can be determined from the basic wing section characteristics $[i.e., \text{ Equations (63) and (64)}]$, whence the total forces acting on the bird's body can be established. A further consideration of these wing forces and their relation to the flapping synchrony of the bird is given subsequently in section 4.3.1.6.

4.3.1.6 Synchronization of the Wing Motions with Flight Speed

The efficient operation of the flapping wing system obviously depends upon an exacting degree of synchronization of the wing motions with the forward flight speed $\mathbf{\bar{V}}$. Since the forces generated by the wing during the flapping cycle are intimately related to the aerodynamic velocities and since the aerodynamic velocities encountered by the wing are heavily dependent upon the flight speed $\mathbf{\bar{V}}$ of the bird as determined by the wing forces, a unique balance of these mutually dependent factors must be reached for "steady" translational flight. In this section we shall explicitly examine the major parameters involved in reaching this balance and their principal relationships.

The basic features involved in synchronization of the wing motions are illustrated in Figure 42. This sketch shows the
force and velocity conditions at an arbitrary section of the outer wing during the power stroke. The symbols used have the same definitions as in the previous section. \( \vec{V} \) is the general flight velocity of (constant altitude) translation and is assumed constant over the period \( T \) of the flapping cycle. The velocity \( v \) is the downward (i.e., tangential) velocity of the section as shown, assuming the wing to be performing a simple rotation about a longitudinal axis through the shoulder joint:

\[
\dot{v} = \hat{\omega}_x \times \vec{r},
\]
where \( \vec{r} \) is the distance (vector) from the axis to the section. The value of \( \dot{v} \) is assumed to be under the control of the bird through its muscular action. The angle \( \gamma \) is defined by the relationship

\[
\gamma = \tan^{-1} \frac{v}{\vec{V}}
\]

(67)

This angle sets the inclination of the wing aerodynamic velocity vector (identical in still air to the velocity relative to earth). The section has an angle of attack \( \alpha' \) relative to the aerodynamic vector. In general, each wing section has a specific angle of attack for which \( \zeta'/\zeta' \) is a maximum, and this \( \alpha' \) constitutes the most efficient operational condition for that section. It is assumed that the bird wing for the sake of optimizing the flight efficiency operates such that the angle of attack for each section is essentially that for \( \zeta'/\zeta' \) maximum. This condition is attained by a proper twist distribution along the wing arc span, both by natural aeroelastic and structural twist and by rotation of the whole wing at the shoulder joint. Thus, the local angle of attack of the wing is under general control of the bird, somewhat independently of \( \gamma \), and is assumed to be essentially optimum (i.e., \( \alpha' \) for \( \zeta'/\zeta' \) maximum) at all normal operational flight speeds.

Let us now consider the bird in equilibrium translation flight at constant altitude with speed \( \vec{V} \), and examine the basic requirements necessary for the maintenance of this equilibrium. The fundamental requirements for continued equilibrium
of the periodic motion are that the longitudinal (x) and vertical (z) components of the impulse taken over the cycle be zero. To obtain the impulse relationships in terms of the various aerodynamic parameters, we make use of Equation (25) for the dynamic force balance of the flapping bird. The total vector impulse expressions are obtained by time integration over the period of one cycle of this equation as follows

\[
\int_0^T \left[ \frac{\mathbf{w}_b}{g} \mathbf{a}_b + \int_\mathbf{S} \rho_c \mathbf{a}_w \, dV \right] dt = \int_0^T \left[ \int_\mathbf{S}_b \mathbf{a}_b \, d\mathbf{S} + \int_\mathbf{S}_w \mathbf{a}_w \, d\mathbf{S} \right] dt + \int_0^T \mathbf{F}_d \, dV + \int_0^T \mathbf{F}_L \, dV + \int_0^T \mathbf{F}_w \, dV \right] dt = 0 \tag{68}
\]

By definition of periodic motion, the same integrals of the overall accelerations must be zero, as shown in Equation (68). The first time integral on the right-hand side is simply the total impulse of the aerodynamic force on the bird and can be expressed in component form as \( \int_0^T \mathbf{a}_b \, dt + \int_0^T \mathbf{a}_w \, dt + \int_0^T \mathbf{a}_c \, dt \). The second time integral is simply the impulse of the total weight of the bird \( \int_0^T \mathbf{w} \, dt \). Since

\[
\mathbf{\hat{D}}_b = -\mathbf{D}_b \mathbf{\hat{a}} \]
\[
\mathbf{\hat{D}} = \mathbf{D} \mathbf{\hat{a}} \]
\[
\mathbf{\hat{L}} = \mathbf{L} \mathbf{\hat{k}} \]
\[
\mathbf{\hat{W}} = -\mathbf{W} \mathbf{\hat{u}}
\]

we have, in view of the requirement that all components of impulse be zero, the following for equilibrium conditions:

\[
\int_0^T \mathbf{W} \, dt = \int_0^T \mathbf{W} \, dt \quad \text{and} \quad \int_0^T \mathbf{D} \, dt = \int_0^T \mathbf{D}_b \, dt \tag{69}
\]

The aerodynamic forces \( \mathbf{\hat{L}} \), \( \mathbf{\hat{D}} \), and \( \mathbf{\hat{D}}_b \) can be expressed in terms of the wing and body aerodynamic coefficients and relative velocities, the latter involving both the flapping velocity \( \mathbf{\hat{V}} \) and the flight velocity \( \mathbf{\hat{V}} \). The weight \( \mathbf{W} \) is a given quantity; \( \int_0^T \mathbf{W} \, dt \) is simply \( \mathbf{W} \mathbf{\hat{T}} \). Hence, if the aerodynamic characteristics of the wing and body are specified, the value of \( \mathbf{\hat{V}} \) (or for the whole wing, \( \mathbf{\hat{V}} \)) necessary to maintain a given value of \( \mathbf{\hat{V}} \) can be determined. In general,

\[
\mathbf{\hat{V}} = \mathbf{\hat{V}}(t) \quad \text{in a cycle although the variation over the greater part of the power stroke may be small.}
\]

It should be noted that the period \( \mathbf{T} \) of a flapping cycle is a function of \( \mathbf{\hat{V}} \) in that the speed of the wing in
the power stroke acts to determine the time duration of the stroke. Likewise, \( \bar{V} \) acts to determine the duration of the recovery stroke. Since the wing must be moved rearward (relative to the body) with a speed equal to \( \bar{V} \), to prevent adverse (negative) aerodynamic drag on the outer wing on recovery, the maximum duration of the recovery stroke is thus set. Hence, both \( \bar{V} \) and \( \nu \) are involved in setting the period \( T \) of the wing cycle.

Let us now consider a bird in flapping flight, moving at constant speed \( \bar{V}_1 \), and then transitioning to a new and faster speed \( \nu_2, \quad \bar{V}_2 > \bar{V}_1 \). Referring to Figure 42, the first change will be an increase in \( \omega \) and hence \( \nu \), by the bird exerting a faster contraction of the depressor muscles. This will momentarily increase \( \gamma \), resulting in a forward rotation of the resultant aerodynamic force vector on the wing, and an increase in \( \bar{D} \). The value of \( \bar{L} \) will not necessarily decrease due to this rotation, since the higher aerodynamic velocity (due to increased \( \nu \)) and (most probably) higher section angle of attack will increase the overall magnitude of the resultant aerodynamic force, thus making up for any loss in the magnitude of \( \bar{L} \) due to forward rotation of the wing force vector. The increase in \( \bar{D} \) (due both to the forward vector rotation and to the increased aerodynamic velocity) will act to accelerate the bird forward, thus increasing \( \bar{V} \). The acceleration will continue until a new condition is attained where Equation (69) is satisfied.

At the new flight speed \( \bar{V}_2 \), \( \gamma \) may have approximately the same value it had at speed \( \bar{V}_1 \), and hence the wing will be operating at essentially the same conditions of optimum angle of attack (\( \alpha'(s) \) for maximum \( \bar{L}'/D' \)). This follows because the increase in \( \nu \) is balanced by an increase in \( \bar{V} \), so that only the magnitude of the resultant aerodynamic velocity is increased. The higher aerodynamic loading on the wing at these higher speed conditions however, may, through aeroelastic deflection (or through voluntary twist of the wing), unload the
inner parts of the wing (lower the local angle of attack) and hence maintain \( \bar{\omega} \) essentially constant while increasing \( \vec{W} \).
At the higher flight speeds, the outer wing, primarily the manus, is involved in determining \( \vec{W} \), while the magnitude of \( \bar{\omega} \) is essentially unchanged.

It is evident from the foregoing analysis that an increase in forward flight speed involves an increase in section \( \psi \) and hence in \( \omega_x \) or power stroke velocity, and thus a decrease in the length of time required for the power stroke. Likewise, the wing recovery must be made more rapidly for a faster forward speed. Hence, it is quite clear why, as is commonly observed, the wing beat frequency in flapping birds increases significantly as the flight speed increases, and vice versa; the two factors are obviously inseparable.

In summary, the prime variable in the power stroke of flapping flight is \( \psi \), the wing (tangential) velocity relative to the body. This velocity ultimately determines \( \bar{\omega} \), \( \bar{\omega} \), and the angle \( \bar{\phi} \), and through these the flight velocity \( \vec{W} \). The value of \( \bar{\omega} \) and \( \bar{\omega} \) are, in addition to the aerodynamic velocity, set by the physical size and shape of the wing. In this sense both \( \bar{\omega} \) and \( \bar{\omega} \) are structurally coupled with the weight.

4.3.2 Slow Flight

The term "slow flight" refers to the flight condition wherein the bird's body moves relatively slowly through the air, i.e., \( \vec{W} \) is relatively small compared to its value in fast flight, the normal translation speed used in travelling. The wing velocity in slow flight is, however, proportionately higher. This arises from the fact that since \( \vec{W} \) is small, the wing must compensate by an increased velocity of its motion relative to the body, i.e., an increased magnitude of \( \psi \). Since \( \vec{V} \) is small, if \( \vec{V} \) were strictly an up-down motion relative to the body, the resultant aerodynamic velocity would produce wing forces in which \( \bar{\omega} \) was small and \( \bar{\phi} \) large, on a relative basis, as illustrated in Figure 43. The large value of \( \bar{\phi} \) would lead to an acceleration of the bird which would be contrary to the slow-flight condition. Simultaneously, \( \bar{\omega} \) would be small in relation to \( \vec{W} \) and hence level flight could not be maintained. To attain the desired flight conditions, it is necessary that the direction of the velocity
vector of the wing relative to the body be inclined appreciably from the vertical \( z \) axis as defined by the weight vector \( \mathbf{W} \), Figure 43(b).

This requirement is accomplished by giving the wing an appreciable rotation speed component \( \omega_\theta \) during the downstroke, in addition to the \( \omega_x \) rotation, and also by tilting the body relative to the line of action of the weight vector \( \mathbf{W} \) such that the relative wing velocity \( \mathbf{v} \) assumes an appreciable inclination as illustrated in Figure 43(b). This body rotation is secured in part by use of the tail surface for attaining trim and in part by the wing surfaces themselves, the forward inclination of the vector causing a nose-up rotation of the bird on the power stroke. The resultant wing aerodynamic force then produces a large \( \mathbf{L} \) component, one sufficient to balance the bird's weight, while simultaneously reducing the drag \( \mathbf{D} \). Alternatively, the bird may be pictured as attempting to fly forward and upward by rotation of the body, with a component of the thrust force being used to balance a part of the weight. The body inclination automatically increases the angle of attack of the inner wing and helps to maintain its lift loading.

The wing velocity \( \mathbf{v} \) must be large and the wing beat frequency in slow flight must be very high in order to produce the required vertical impulse. This is especially important since if \( \mathbf{v} \) is very low, the inner wing will furnish considerably less lift than in fast flight even though the inner wing is at a high angle of attack, and a large portion of the
aerodynamic load will be borne by the outer wing sections. In order to achieve the required vertical impulse, the speed of the recovery stroke must also be very rapid. However, since the body is moving forward at a lower velocity, the required rapid rearward motion of the wings on recovery will no longer allow the outer wing sections to maintain essentially zero aerodynamic velocity; the wing will move rearward relative to the body faster than the body is moving forward relative to the air (i.e., relative to earth). Consequently, the wing will experience an aerodynamic drag $\vec{D}$ on the recovery stroke which is positive, i.e., directed forward. Thus, the wing is able to produce a thrust force during recovery by creating an aerodynamic drag, a condition which has no counterpart in conventional fixed-wing aerodynamics. This positive reverse "drag" points up the suitability of using the symbol $\vec{D}$ to represent both "thrust" and "drag" in the flapping wing.

Due to the rapid average motions required for slow flight, and the high loadings of the wing sections (i.e., high lift coefficients), it is to be expected that the induced drag and hence the required flight power will be larger for slow-speed flight than for high-speed flight. In essence, the wing in slow-speed flight must give a smaller amount of air a larger vertical velocity in order to attain the same vertical impulse as in fast flight.

The term "slow flight" may appear to be somewhat ambiguous in that what is slow flight for one species of bird is really fast flight for another. It should be realized, however, that the terms "slow flight" and "fast flight" must be used in the sense of describing the relative speed regimes of the given species under discussion, and not for comparing the absolute flight characteristics of different species. In this manner, any possible ambiguity can be avoided and the basic definitions are then quite descriptive.

Slow flight occurs primarily in the take-off and landing of birds and is seldom observed except under these conditions. The high energy requirements for slow flight preclude its use except under such essential conditions. For instance, a bird flying about in a relatively small, closed room must perform slow flight to prevent collision with the walls and soon becomes totally exhausted. In a larger room or enclosure, where the fast-flight mode can be used while still maintaining adequate maneuverability, much longer flight durations can be achieved by the same bird. In take-offs, the bird does not usually
remain in the slow-flight condition as such but is in a continuous state of transition as it accelerates up to fast-flight speed. Hence, the wing motions of each successive flapping cycle are slightly different from the preceding one, continuously changing from those characteristic of slow flight into those of fast flight as the bird accelerates. In landing, however, the slow-flight mode is often observed for much longer periods, as the bird maneuvers about in searching for a suitable landing spot. Slow flight is also frequently observed in birds scanning terrain for food.

4.3.3 Hovering Flight

While there is no clearly defined boundary between fast and slow flapping flight, the definition of hovering is somewhat more precise. Hovering consists of flapping flight in which the bird maintains a constant position in space, relative to earth, in still air. The latter qualification is very important, since a bird may also maintain a constant position relative to earth while performing slow or even fast flight if a sufficient headwind is blowing. With this qualification, hovering flight is clearly defined by the condition $V = 0$.

Hovering is not a commonly observed flight mode of most birds; indeed many birds, especially the larger species and those with high wing loadings, are incapable of hovering flight. For certain particular species, however, hovering is an essential and constantly utilized flight mode (e.g., the hummingbirds), but this capability requires a specialized wing geometry and a high energy output, as will be discussed below. True hovering flight, as in the case of slow flight, is most often seen in association with the landing process. The bird transitions from slow flight into a hovering condition and then settles vertically to its landing position. More rarely, hovering is observed in birds, such as gulls or terns, searching the water surface for food.

When the surface wind is blowing, a bird may fly into the wind and hence reduce its ground velocity (i.e., its velocity relative to earth). The relationship involved is

$$\hat{V}_{be} = \hat{V}_{ba} + \hat{V}_{ae}$$

where $\hat{V}_{ae}$ is the wind vector. If the bird is flying into the wind, $\hat{V}_{ba}$ and $\hat{V}_{ae}$ have opposite directions, and when
these velocities become equal in magnitude, the bird will be motionless relative to earth (Figure 44). However, in this case the bird will not be in true hovering flight but in either slow or fast flight, depending upon the magnitude of $V_{ae}$, and the flight energy requirements and wing motions will be those characteristic of slow or fast flapping flight, not of hovering. The sparrow hawk and certain other species may often be seen practicing this "pseudo-hovering" type of flight in brisk winds as they search for food.

True hovering flight is the most energy-consuming mode of all types of flapping flight. This is clearly evident once again from observations of the very rapid exhaustion of most birds forced to perform hovering to remain airborne in a closed space. Since the bird is not moving forward in hovering flight, the mass of air which can be intercepted and thrown downward per unit of time to obtain the necessary vertical impulse to balance the weight impulse $WT$ is greatly reduced and hence the wing must impart a higher downward velocity to the reduced mass flow of air in the stationary vicinity of the bird. This increased velocity and decreased mass flow of the air, as discussed previously, considerably reduce the efficiency of hovering flight. The situation is precisely analogous to the case of the hovering helicopter, only much more complex in aerodynamic detail since the flow field relative to the flapping wing is grossly unsteady.

The wing motion in hovering flight is extremely complex, and the geometrical flexions of the wing surfaces are extreme. In essence, hovering is the lower limiting condition of slow flight; the body is highly canted relative to the horizontal so that the usual up-down wing motions (relative to the body) of fast flight become confined to an essentially horizontal plane. As the wing sweeps forward horizontally in the power stroke, the wing lift $\vec{L}$ acts to counterbalance the weight.
the wing drag \( \overline{D} \) acts to impart a rearward force on the body and hence to produce a negative longitudinal acceleration. Upon recovery, since the bird has no forward aerodynamic velocity, the inner wing can produce no lift to counter the action of the weight force, and it is necessary that the wing be brought into position for the subsequent power stroke with great rapidity. This requirement is especially acute since the inner wing during the power stroke will have relatively little aerodynamic velocity and hence the outer wing alone must produce a large enough lift \( \overline{C} \) such that the condition of zero vertical impulse is attained

\[
\int_r^t \overline{L} \, dt = \int_r^t W \, dt
\]

Hence, the shorter the period of the recovery stroke, the less will be the required magnitude of \( \overline{L} \) on the required power stroke to satisfy Equation (70), and the less the required power for hovering.

So critical is the difficulty of satisfying Equation (70) in hovering, involving as it does extremely rapid accelerations of the wing mass, that the bird finds it advantageous (if not, indeed, absolutely necessary) to produce a positive lift force on the recovery stroke as well as on the power stroke. This is accomplished by rapidly twisting the entire manus at the wrist by almost 180° at the end of the forward wing sweep, so that the top surface of the manus now becomes the underside of the wing surface on recovery (the manus leading edge thus pointing rearward) and strikes the air in such a manner during recovery as to produce a positive lift \( \overline{L} \) (as well as a positive drag \( \overline{D} \)). This condition is illustrated in Figure 45. This lift during the recovery stroke, in view of the rapid rearward motion of the wing, may be appreciable although the wing is not highly efficient in the production of lift when operating in this reversed condition. Apparently, the lift impulse achieved during the recovery stroke is adequate to justify

\[\overline{D} \]

\[\overline{L}\]
the extreme twisting of the manus required to attain it. The (positive) impulse of the drag \( \bar{D} \) experienced during the recovery balances the negative impulse of \( \bar{D} \) resulting from the power stroke. Hence, the basic impulse requirements

\[
\int_o^T \bar{L} \, dt = \int_o^T \bar{W} \, dt = \bar{W}T 
\]

(71)

\[
\int_o^{T'} \bar{D} \, dt = -\int_{T'}^{T} \bar{D} \, dt 
\]

(72)

are satisfied; the bird undergoes positive and negative accelerations in both the vertical (z) and horizontal (x) directions, but the reversals of the net forces occur so rapidly that no perceptible velocities can be built up in any direction and the bird hovers stationary in space. The symbol \( T \) in Equation (71) denotes the end of the power stroke. In this discussion of hovering flight, reference to the two transitional phases, the feathering transition and the manus alignment, have been omitted since they are rather ill-defined in hovering, where the transition periods between the power and recovery strokes are very short indeed.

4.3.4 Transition Flight

To complete this section on the basic aspects of flapping flight aerodynamics, the nature of the flight transitions between hovering and slow flight, and slow flight and fast flight is considered briefly.

In order to reach the fast-flight condition starting from rest, a bird must accelerate up to the cruising-flight speed. Starting out with \( \bar{V} = 0 \), the bird moves its wings relative to its body in a manner appropriate for take-off from the particular terrain involved. If the bird must rise directly upward before beginning horizontal translation, the wing is moved after the fashion of hovering, only even more vigorously, since it is then necessary that \( \int_o^T \bar{L} \, dt > \int_o^T \bar{W} \, dt \) to attain a vertical velocity. If the terrain is clear of surface obstructions such that horizontal take-off is possible, the wing is moved in a manner that is halfway between hovering (to provide the vertical acceleration) and slow flight (to provide the horizontal acceleration). Once the bird is airborne and well clear of the ground, the wing is moved in essentially the slow-flight mode
for forward acceleration. However, the wing strokes are different from constant speed flight since the bird is endeavoring to accelerate up to full flight speed. Consequently, the wing beats are more vigorous and rapid in order to provide the additional $\mathbf{D}$ needed for acceleration of the bird's mass. Under the action of the increased thrust ($\mathbf{D}$ positive), $\mathbf{V}$ is continuously increasing, so that each wing beat in sequence must be adjusted to maintain the accelerational force $\mathbf{D}$ to continue the increase in $\mathbf{V}$ with time.

The same conditions of transition maintain in the landing process. In this case, however, the bird usually commences a glide toward its landing spot upon cessation of flapping, and then enters slow flight and subsequently semihovering flight just before landing. In some cases, landing is accomplished by using an initial wing motion which creates a large negative value of $\mathbf{D}$ momentarily to arrest the forward motion, following which the bird enters the hovering or semihovering condition.

Thus, by proper motions of the wing relative to the body and by use of the tail to maintain proper balance, the bird is able to cover an extremely wide range of flight conditions and to transition smoothly and efficiently from one basic flight mode to another. The great versatility of the flapping wing offers a most valuable operational advantage to the bird in this regard, as compared to the simple fixed wing.

V. ENERGY REQUIREMENTS

Calculation of the flight energy requirements of a rigid, fixed-wing aircraft is a relatively simple, straightforward task. Calculation of the flight energy expenditures of the flapping wing of the bird, however, is a much more complex endeavor. This complexity arises from the involved geometrical aspects of the flapping wing forces and from the unsteady nature of the force-velocity regime.

If we exclude the small amount of energy dissipated through heat due to friction of the internal joints and tendon contacts within a bird (i.e., the inefficiencies of the internal
components of the flapping system), all energy required for flight must initially appear in the wake of a bird as kinetic energy of the air motion induced by the bird in its passage. This is true, indeed, of all flight, regardless of the type of propulsion system. Considering a bird flying through still air, the skin friction stresses result in the impartation of a forward velocity to the air passing between the (laminar) streamlines of the body boundary layer, and in the direction of the wing motion for the wing boundary layer. Likewise, the $D_b$ results in the production of the wake vortices with their associated induced flow velocities. Ultimately, however, through the action of viscosity, all kinetic energy of such ordered flow is reduced to the kinetic energy of random motion of the air molecules, and hence appears as an increased temperature of the air after the motion has subsided. Thus, all the energy required for flight initially appears in the wake as kinetic energy, and can be determined, in principle, by measurement of the kinetic energy content per unit length of wake.\textsuperscript{17}

For the case of the conventional fixed-wing airplane, the power required for flight is simply $DV$ where $D$ is the total drag of the entire craft (wing and body) and $V$ is the (constant) flight speed (through still air). The aircraft propeller produces a thrust force $T$ while the craft is moving forward with speed $V$ (relative to earth), and hence the energy available for flight is $TV$. At constant flight speed $V$, the power required equals the power available from the propeller

\[ DV = TV \] (73)

and $D = T$. Here the drag $D$ is that of the overall craft.

In the case of the flapping wing, things are not quite so simple. The drag forces of the wing (i.e., $D_w$) and body (i.e., $D_b$) have quite different orientations, the former varying considerably with time during the flapping cycle. To establish the energy relationships for the flapping wing, let us use the basic definitions of the preceding chapter, resolving the wing aerodynamic force into its $x$ and $y$ components $\bar{D}$ and $\bar{L}$. These forces are, of course, functions of cycle time. The bird (i.e., the body) is moving forward (relative to earth) with the flight velocity $\bar{V}$ which we shall assume, for simplicity, is constant. This is not a necessary assumption; the actual velocity of the bird relative to earth $\hat{V}_{be}$ could be used in
place of \( \vec{V} \). The force component \( \vec{D} \) is the net horizontal \((x)\) component of the total aerodynamic force of the wing and hence is parallel (or antiparallel) to \( \vec{V} \). The instantaneous net power available from the wing motion, \( P_\alpha \), is thus

\[
P_\alpha = \vec{D} \cdot \vec{V}
\]  

(74)

The total available energy per flapping cycle \( \mathcal{E}_{r,\alpha} \) is

\[
\mathcal{E}_{r,\alpha} = \int_0^T \vec{D} \cdot \vec{V}_{be} \, dt
\]

(75)

In vector form

\[
P_\alpha = \vec{D} \cdot \vec{V}_{be}
\]

(76)

It should be noted that here \( \vec{D} \) is the net \( x \)-component of the wing total aerodynamic force \([\text{see Equation (66)}]\) for definition of \( \vec{D} \), and hence involves both the section lifts and section drags of the wing. It is in essence the integrated difference between the \( x \)-components of the section lifts and the section drags and is the equivalent of the thrust force \( T \) generated by the propeller of the conventional fixed-wing aircraft. While the propeller of the conventional airplane produces all the thrust \( T \), and the aircraft body and wing produce all the drag \( D \), so that the sources of \( T \) and \( D \) are quite independent, the wing in flapping flight produces both thrust and drag in the conventional sense, and hence the two forces are not dissociable. Their resultant constitutes the force \( \vec{D} \). Hence, for those parts of the cycle where \( \vec{D} \) is positive for the flapping wing, we have a thrust (in the conventional sense) of magnitude \( \vec{D} \). When \( \vec{D} \) is negative, we have zero thrust and only (conventional) drag of magnitude \( \vec{D} \). In this sense we can compare the thrust production of the flapping wing with that of the conventional airplane propeller. As will be considered in more detail later, this action of the propeller can be profitably interpreted in terms of the dynamics of the flapping wing.

The power required by the bird for flight (\( P_\tau \)), exclusive of that for the wing drag (\( D' \)), is given by
\[ \hat{D}_b \cdot \hat{V}_{bc} = P_r \]

where \( \hat{D}_b \) is the drag of the body and \( \hat{V}_{bc} \) is the velocity of the body relative to earth. The required flight energy per cycle \( \mathcal{E}_{r,v} \) is

\[
\mathcal{E}_{r,v} = \int_0^T \hat{D}_b \cdot \hat{V}_{bc} \, dt \tag{77}
\]

For continued flight at constant average velocity \( \bar{V} \), the cycle energy available must be exactly equal to the energy required and hence

\[
\int_0^T \bar{D} \bar{V} \, dt = \int_0^T \hat{D}_b \hat{V}_{bc} \, dt \tag{78}
\]

Here the energy made available by the wing supplies the required energy for the body motion.

To establish the total energy output by the wing, i.e., the sum of the wing dissipative energy (drag) and the energy made available for accommodating the body drag \( \hat{D}_b \), the forces acting on each element of the wing must be determined. Let \( \hat{F}' \) denote the net force (vector) per unit span acting on the wing, where

\[
\hat{F}'(s) = \hat{W}' + \hat{A}' + m\hat{a}' \tag{79}
\]

In this equation the symbols have the same meaning as previously defined. The instantaneous power associated with the section motion is

\[
P_T = \hat{F}' \cdot \hat{V}_{we} \tag{80}
\]

The corresponding cycle energy of the entire wing is

\[
\mathcal{E}_T = 2 \int_{s_c}^{s_i} \int_0^T D' V_{we} \, dt \, ds \tag{81}
\]

This gives the total energy associated with the wing motion, of which \( \int_0^T \bar{D} \bar{V} \, dt \) is useful energy available to overcome the body drag. The integrals of the terms \( \hat{W}' \) and \( m\hat{a}' \) in
Equation (79) are zero* over the period of a cycle, and since
\[ \hat{A'} \cdot \hat{V}_{we} = D' \cdot V_{we} \quad (\hat{D'} \text{ is antiparallel to } \hat{V}_{we}) \]
we have

\[ \mathcal{E}_T = 2 \int_{s_0}^{s_f} \int_0^T D' V_{we} \, dt \, ds \]

The difference between \( \mathcal{E}_T \) and \( \int_0^T \vec{D} \vec{V} dt \) is similar in the case of the aircraft propeller to the power loss of the propeller, namely \( P_a(\eta - 1) \) where \( \eta \) is the conventional propeller efficiency index. However, the energy loss of the entire wing is included in the efficiency figure defined on this basis.

A convenient way to picture these overall energy relationships for the wing is to assume the wing motion relative to earth to be composed of two orthogonal vector velocities, \( \vec{v} \) and \( \vec{V} \) where \( \vec{v} \) represents the average vertical velocity of the wing and \( \vec{V} \) the forward velocity. Then the power associated with the wing consists of the two terms \( \vec{v} \vec{L} \) and \( \vec{D} \vec{V} \). The term \( \vec{v} \vec{L} \) is the power dissipated by the wing, while \( \vec{D} \vec{V} \) is the useful power made available by the wing stroke.

By inserting the proper aerodynamic velocity coefficient terms for the aerodynamic forces in Equations (76) and (77), the cycle energy requirements for the flapping wing can be determined.

The aircraft propeller can be compared quite closely with the flapping wing of birds. Considering a two-bladed revolving propeller, the downward motion of one blade can be considered as analogous to the power stroke of the bird wing. The subsequent upward arc of the propeller blade can be considered as analogous to the recovery stroke of the bird wing, only in this case the recovery stroke also produces thrust. While one blade is moving downward (power stroke), the other blade is moving upward (recovery stroke); consequently, each blade may be pictured as performing alternate power and recovery strokes, with at least one blade always in the power phase and one in the recovery phase.

* \( \vec{W} \) is a conservative force: the kinetic energy of the wing implied by \( m\vec{a'} \) is ultimately used in overcoming the section drag and hence appears as part of the \( \vec{D'} \) term.
VI. STABILITY AND CONTROL CONSIDERATIONS

In the foregoing discussion, the primary concern has been with the wing forces and their dynamics. Associated with these forces, however, are moments tending to rotate the body. As with the forces, these moments are also time-dependent, both in magnitude and direction. For level, quasi-steady flight the moments produced by the various forces must satisfy certain specific requirements; these requirements in turn put definite restrictions on the possible lines of action of the aerodynamic forces and hence act to dictate the positional relationships that can exist between the wings, body, and tail of the bird, as well as other geometric aspects of the overall structure and its articulation.

Due to the symmetry of the linear flight conditions previously discussed, moments can exist only about the lateral (spanwise) axis of the bird and hence rotations can occur only in the plane of symmetry. Consequently, for purposes of stability and control analyses in the standard flight modes, the entire force system and associated moments can be represented as acting in the symmetry plane of the bird (i.e., the x-z plane). Maneuvering flight, of course, is much more complex and requires consideration of all aspects of the bird's geometry and mass distribution. In this chapter we shall be concerned primarily with discussion of the rotational balance requirements of linear flight but shall briefly consider in qualitative fashion a few of the more important features of maneuvering.

In the case of flapping bird flight it is especially important to distinguish between stability and control. Stability (i.e., static stability) is the initial tendency of an aircraft to return to an equilibrium (trimmed) flight position following a slight perturbation. Control refers to the ability of the aircraft to maintain or follow a prescribed flight path (or maneuver) by creation of suitable forces and moments. In the case of the flapping bird, stability has little meaning, in the conventional sense, since the bird is never in a state of true static equilibrium. Control, however, is of the utmost importance, and the following analysis is primarily one of control and not stability. With adequate control, there is no absolute need for positive static stability of an aircraft (or bird). Should the craft be slightly perturbed from its intended flight conditions, it can be quickly brought back to the equilibrium
condition by imposition of a proper control action. In the case of the flapping bird, the entire flapping flight cycle can be viewed as one continuous sequence of programmed control actions, the motions of the wings and tail being such as to produce the necessary moments and counter moments needed to hold the bird's body within the desired altitude range.

Birds in general possess an enormously well-developed control capability, and hence are little concerned with the finer aspects of aerodynamic stability as such. Very slight deviations from the intended flight conditions (say due to an asymmetrical gust of wind or thermal updraft) generate asymmetrical, or otherwise discernable, forces and moments which are immediately sensed by the bird; and almost immediate, instinctive action is taken to alter the tail position or the wing shape, or speed or direction of motion, in sufficient magnitude to cancel out the undesired inputs. The extreme efficiency of the bird's control capabilities lies in the fact that the sensor-control-feedback system is intimately integrated through the nervous-muscle system and is continuous and instantaneous in its action. Remedial action to counter a perturbation is immediate, so that no significant deviational forces capable of perturbing the bird from the desired flight course ever get a chance to build up. In view of the fact that every phase of the normal flapping cycle is in reality a continuous application of control by the bird, it is not surprising that the bird possesses such an acutely sensitive control and maneuvering capability in general.

In flapping flight, it is essential that the bird's body maintain an effectively constant attitude, or at least controlled attitude variation during the cycle. This is important because the wings are attached to the body and, through their particular articulation at the shoulder joint, are in great measure controlled in their motion by the attitude of the body; rotations of the body change the possible trajectories and hence force orientations available to the wings. The three forces acting on the body are illustrated in Figure 46, where \( \hat{A}_b \) denotes the aerodynamic force acting on the body, \( \hat{F}_{wb} \) the (contact) force exerted by the wing on the body, and \( \hat{w}_b \) the body weight. These forces are all coplanar, lying in the plane of symmetry. The force \( \hat{A}_b \) here includes, in addition to the body drag, the aerodynamic force associated with the tail of the bird. The body weight acts as the center of gravity of the body. The force vectors \( \hat{A}_b \) and \( \hat{F}_{wb} \) vary with time.
in both magnitude and direction during the flapping cycle. However, they are both under the control of the bird and hence can be altered to attain the necessary control balance required. Both $A_b$ and $F_{wb}$, at least for the condition shown in Figure 46, are displaced from the body center of gravity and hence produce a moment equal in magnitude to the product of the force and its arm (normal distance) from the center of gravity. In the hypothetical case shown, $F_{wb}$ is producing a counterclockwise moment, while $A_b$ is producing a clockwise moment. The larger arm of $A_b$ compensates for its smaller magnitude so that the moment of $A_b$ essentially cancels the moment of $F_{wb}$.

In general, both the magnitude and moment arm of $A_b$ and $F_{wb}$ change during the flapping cycle. The basic requirement is that the resultant moments $\hat{M}_A$ and $\hat{M}_F$ essentially cancel at all points of the cycle. If $\hat{M}_A$ and $\hat{M}_F$ do not cancel, there will be a net moment $\hat{M}_y$ acting on the body; this moment will induce an angular acceleration $\hat{\omega}_y$ to the body about the center of gravity. If this acceleration is large (due to a large moment), the body will begin to rotate in the direction of the moment and with the angular velocity $\hat{\omega}_y$ given by

$$\hat{\omega}_y = \int_0^t \dot{\hat{\omega}}_y \, dt = I_y \int_0^t \hat{\omega}_y \, dt$$

where $I_y$ is the body's moment of inertia about the lateral axis. If $\hat{M}_y$ is large, the body reaches a high rotation velocity in a short time, relatively, and integration of the angular velocity over the same time yields a large angular rotation of the bird, a condition which is not desirable. Thus, the prime requirement is that $\hat{M}_A$ and $\hat{M}_F$ essentially cancel at all points of the cycle.

This requirement may be accomplished by making the lines of action of $A_b$ and $F_{wb}$ pass through the center of gravity.
gravity at all times. This, of course, may be overly restrictive considering the need for adequate flexibility of the wing motions for general control purposes, and the usual condition is probably that the forces pass near the center of gravity. The fact that the tail surface of most birds in fast flapping flight is not used appreciably and is, in fact, usually furled or closed to minimize area, indicates that $\mathbf{f}_{wb}$ does not produce large instantaneous moments about the center of gravity or else that the change or fluctuation in the sign of the moment produced by the wing force is so rapid during the flapping cycle that there is no chance for any appreciable net angular rotation of the bird to occur. This can be readily observed in many birds, especially the Canada goose, whose long, extended neck acts as an excellent indicator of any rotation which may occur, in high-speed films of the fast-flight mode of the bird. Such films reveal an amazing constancy of attitude for this bird during fast flight. In slow flight, conditions are quite different and many birds may be seen to execute a periodic, oscillatory rotation about the center of gravity during slow flight, especially the larger, heavier birds whose wing size prevents very rapid movement of the wings. Such birds are also generally incapable of hovering. An excellent illustration of this oscillatory rotation in slow flapping flight (and of the landing transition from a glide to slow flapping flight) is to be found in the high-speed film by Storer for the case of a brown pelican. In such cases, it is quite obvious that the bird is not designed for sustained slow flight, and hence must call upon every usable part of its geometry for attaining the necessary balance: head and neck extension, use of extended and swinging legs and feet, and maximum spreading and flexing of the tail surface. Birds which are highly adept at hovering generally have very fast wing motions while in that mode and essentially no discernable change in attitude. Apparently the rapid changes in wing force ($\mathbf{f}_{wb}$) direction are able to balance the moments successfully such that the cycle angular impulse is zero

$$\int_0^T \dot{\mathbf{M}}_y \, dt = 0$$  \hspace{1cm} (82)

and hence no net attitude change results. Further, it must also be true that

$$\int_0^{(\Delta t)} \dot{\mathbf{M}}_y \, dt = 0$$  \hspace{1cm} (83)

where $\Delta t$ denotes a small fraction of the cycle period $T$ ($\Delta t \ll T$).
The horizontal tail surface is an extremely important appendage to most birds for the slow and hovering flight modes and for maneuvering. The tail is most prominently displayed in the low-speed conditions of flight for two reasons. First, the wing motions have changed radically from the fast-flight condition so that the bird is flying in an "off-design" condition so far as moment production is concerned, and because of the low (or zero) body speed, the dynamic pressure of the airflow is very low and the tail must spread to its fullest extent in order to produce any effective aerodynamic force. Even in hovering the tail may be maximally spread, although the dynamic pressure of the airflow is zero with respect to the motion of the bird through the air. In hovering, however, the bird is essentially maintaining its lift by acceleration of the air downward at relatively high velocity; this downwash from the wings acts effectively upon the tail to produce secondary forces useful in balancing the basic moments of the wing during the flapping cycle. In addition, the horizontally-spread tail acts as an effective aerodynamic damper and automatically sets up appreciable counter moments which act to resist rotation of the body in either direction. The unfurled tail is especially effective in resisting rotation due to large impulsive moments such as might occur in hovering or slow flapping flight. Some birds, notably many of the albatrosses, have very small tails. In these cases the function of the tail is taken over by the appreciable surface area of the large webbed feet which actually extend considerably beyond the tail in fast flight and which can be lowered into the airstream to produce useful aerodynamic moments in the slow flight phase of landings and take-offs. In soaring birds, the tail serves to provide aerodynamic longitudinal stability; the degree of stability can be altered as necessary simply by opening or closing the tail so as to obtain the necessary surface area. This capability allows the bird to change its stability along with its control manipulations, and hence to achieve an efficient use of the available surface area.

In maneuvering, the bird becomes a truly remarkable machine, for the possibilities of creating the requisite aerodynamic forces and moments are immense. For example, in order to produce a rolling moment (i.e., a moment about the longitudinal x-axis), the bird can either flex one wing semispan, thus reducing its effective moment arm and its area relative to the other semispan, or it can merely twist the manus such as to give an increase or decrease in lift. The latter motion is very effective since the lift is altered at the extremity of
the wing where it has maximum moment effect. This action is similar to that of the conventional aileron control but is generally much more efficient in the bird because of the relatively smooth distribution of the force increase or decrease over the entire manus as compared to the step-like change in the case of the aileron. By flexing the wings (i.e., bending the wing at its joints, especially at the wrist and the elbow), the wing shape geometry and area can be grossly altered to attain the desired flight trajectories. With flapping wings, even greater flexure is possible through asymmetric variation of the wing flapping speed or direction in addition to voluntary twist of the wing. Many birds can go almost instantaneously from fast flight to hovering or semi-hovering flight.

One question constantly arises concerning the directional flight stability of birds in regard to the obvious lack of a vertical tail, an appendage which is almost mandatory for stability in the conventional airplane. Once again, however, the bird shows the great advantages which it has acquired through evolution. Three factors make the vertical tail unnecessary for the bird. The first is that the wing itself, with its pinioned tip and twist, is such that in gliding flight where the wings are fixed, restoring moments about the z-axis (counter yawing moments) are set up automatically whenever the wings are directionally displaced from the balanced forward position. This moment is caused by the considerably increased drag of the outer section of the forward yawing semispan, due to the disturbed flow over the pinions. The second involves the fact that in flapping flight, directional stability is of little consequence since the wings are under constant and subtle control, and any tendency of yawing rotation is immediately counteracted by the wing. The third factor concerns the horizontal tail which by twisting of its plane about the longitudinal x-axis can produce yawing moments as well as pitching moments and hence can serve as an effective control. The fact that the bird does not need a vertical tail for stability, even in low-speed gliding flight, due to the inherent directional stability built into the wing, makes possible considerable drag and weight reduction by elimination of the vertical tail surface area. Thus, we see that with proper wing design, a considerable advantage in overall flight efficiency is gained by elimination of the vertical tail in the bird. This fact has been used to good advantage in the so-called tailless sailplanes.
VII. CONCLUDING REMARKS

The generalized treatment of flapping flight aerodynamics presented in this treatise is intended to provide a basic insight into the mechanisms of flapping locomotion by birds, and to establish the general quantitative relationships by which any particular wing form can be analyzed for technical purposes. By such analyses, much can be learned about the ecological relationships of any particular species. The wing forms and flight patterns of birds cover an enormous range of variations, and these variations in turn reflect the specific adaptation of the bird to a particular system of environmental factors. As particular analyses will clearly show, the wing is not always designed for the highest aerodynamic efficiency in distance travel; accommodation of other factors may be much more influential in deciding wing form. For example, the wing of the quail, with its very small aspect ratio and short span, is designed not for efficient distance travel but for providing a means of quick escape from and travel between the thick bushes and thickets that form its habitat. Sustained flight for the quail is impossible. Additionally, many wing forms may be designed almost entirely to best accommodate one ecologically critical mode of flight, with a minimum of compromise for accommodating other modes. A prime example of this is the albatross, whose enormous span precludes the use of the wing in flapping flight during the initial stages of take-off. The bird is entirely dependent upon the existence of an appreciable surface wind to get it airborne; however, its continued survival in its present form clearly indicates that the required wind does occur in its particular environment with quite adequate frequency.

Thus, the results of the present paper provide the basis for a whole hierarchy of aeroecological studies of particular flapping bird species, studies which can in many cases establish almost the entire ecosystem relationships of the species. An example of the high degree to which the ecosystem interrelationships of a species can be quantitatively developed, along with the many lucid insights into the rationale governing its characteristic behavior, is given in the author's treatise on "Aeroecology of the North Pacific Albatrosses", for the case of dynamically soaring birds. It is hoped that the present paper will provide the basis for similar studies by avian ecologists on flapping species.
Sir D'Arcy Thompson, in his early biophysical work *On Growth and Form*, pointed out the tremendous complexities involved in the various functional mechanisms of living organisms. "And yet all the while," he commented, "with no loss of wonderment or lack of reverence, do we find ourselves constrained to believe that somehow or other, in dynamical principles and natural law, there lie hidden the steps and stages of physical causation by which the material structure was so shapen to its ends." Since these lines were written, the science of biophysics has made great strides in describing the physical bases underlying many biological phenomena. The present paper hopefully will help to lift any veil of obscurity which may have surrounded the flapping flight of birds and stimulate progress in avian aeroecology on a broad front.

Marlbank
October 1968
Some ambiguity presently exists in regard to the definition of the "wing" in flapping flight. Many authors refer to the "wings" of a bird, implying that each semispan is a separate wing. In the present paper, however, the attitude is taken that the two semispans constitute parts of the complete wing, and thus the term wing refers to the entire flight surface as in conventional aerodynamic terminology. In all except maneuvering flight, the wing semispans are really mirror images of one another, and hence at any given instant the wing is a single surface possessing a constant plane of symmetry. In maneuvering, the semispans are no longer mirror images, but the wing may still be thought of as a complex, elastic, and twisted, but still a continuous single aerodynamic surface.
APPENDIX II

Although a bird is said to be flying "level" and at constant altitude, it is actually traversing an arc in space (relative to the center of the earth). Since the bird remains a constant distance from the earth's surface (assuming "flat land"), the lift force of its wings is slightly less than the bird's weight. The remaining component of the weight acts to accelerate the bird toward the center of the earth. The general relationship is given by

\[ W = L + \frac{W \cdot V^2}{\gamma (L + R_o)} \]

Here \( L \) is the altitude above the earth's surface at which the bird is flying and \( R_o \) is the radius of the earth. In the case of birdflight, the flight speed \( V \) is so small relative to \( R_o \) that \( V^2 / (L + R_o) \) is essentially equal to zero; thus \( W = L \). Only when this ratio assumes large values (i.e., at high velocities) does the centripetal acceleration become significant in reducing the required lift for equilibrium. In the limiting case of the orbiting space satellite in a circular orbit, \( \frac{W \cdot V^2}{\gamma (L + R_o)} = W \) and no lift is required to maintain \( L \) constant.
APPENDIX III

It is conceivable that airborne creatures could utilize "jet propulsion" in a manner similar to that used by the squid in water. However, such propulsion is very inefficient except when the propelling substance is very dense, or when the object being propelled is moving at high speeds. Neither of these conditions applies in normal birdflight. For a given thrust, the bird must impart a definite time rate of increase in momentum to the air in a rearward direction. The required momentum change in the air, per second, can be attained by giving a small mass of air a large rearward velocity (as in conventional "jet" propulsion) or by giving a large mass of air a small rearward velocity. Since the kinetic energy which must be imparted to the air for given momentum change varies directly with the square of the imparted velocity, but only linearly with the mass, it is obviously more efficient, energy-wise, to propel as large an air mass as possible.

In reality, the bird can be said to use a form of "jet propulsion" so far as actual mechanical principles are concerned. The wings produce a rearward velocity in a large mass of air. Thus, flapping wings actually generate a "jet" of rearward-moving air, but the very large diameter of this air flow (Reference 18) precludes its being termed a "jet" in the conventional sense.
APPENDIX IV

Bird wings, in reality, are highly elastic structures, and hence are enormously complex aerodynamically. For example, if a wing is driven by its propulsive musculature in a given direction and with a given initial acceleration, the air loads generated on the wing surface will act to deform the surface, and these deformations will act to change the magnitude and direction of the air load on the wing, and so on. The determination of such interactions is of course extremely difficult and requires complex iterative analysis. The deformation relationships change with each change in the initial shape of the wing, with each change in wing direction of motion, and with each change in magnitude of the initial impetus applied by the wing muscles on the downstroke. Thus, by taking positive advantage of aeroelastic factors, the bird is able to accomplish an almost endless variety of flight patterns with a single aerodynamic surface. In learning to fly, the bird experiences the integrated reactions of its infinitely complex wing to the imposed motions and come to combine and utilize this knowledge in a purely instinctive and reflex manner. (See References 3 and 12.)

APPENDIX V

In lifting wings which have span-lines which are not normal to the relative aerodynamic velocity vector, there is a spanwise component of airflow. This flow contributes nothing to the general pressure distribution over the chord of the wing, but at the surface an appreciable alteration of the boundary layer structure is introduced by these lateral flows. The usual result is, especially in the case of wings with swept-back tips, that the boundary layer thickens and separates near the wing tip and can thus induce tip-stall.
APPENDIX VI

Motion Histories of Flapping Cycles

The following figures present quantitative data on the motion characteristics of the flapping wing for several larger birds. The data were obtained by carefully measuring the incremental displacements of a given point on the wing tip (usually the tip of the third primary) relative to a fixed point on the body, and the corresponding time interval, as determined from projections of high speed film frames. The following symbols are used:

\[ \Delta x, \Delta z \] = incremental longitudinal and vertical displacements, respectively, of wing tip relative to the body center of gravity

\[ \dot{x}, \dot{z} \] = instantaneous velocities of wing tip in the longitudinal and vertical directions, respectively

The birds analyzed are as follows:

Canada Goose #1
Canada Goose #2
Canada Goose #3
Snowy Egret #1
Snowy Egret #2
Canada Goose #1 - C
Canada Goose #1 - D
Canada Goose #2 - B

\[ \Delta z, \text{ft.} \]

\[ \Delta x, \text{ft.} \]
Graph showing the acceleration and velocity over time for a Canada Goose.

- X-axis: Time, seconds
- Y-axis: Acceleration, ft/sec²
- Velocity, ft/sec

Data points indicate a significant increase in acceleration followed by a period of deceleration, which is characteristic of a dive or rapid descent in flight.
1.0

\[ \Delta x, \text{ft.} \]

\[ \Delta z, \text{ft.} \]

\[ \text{Time, seconds} \]
Canada Goose #3 - D
Snowy Egret #2 - A

Time, seconds

$\Delta x, \text{ft}$

$\Delta y, \text{ft}$
126-P

Snowy Egret #2 - B

\[ \Delta x, \text{ ft.} \]

\[ \Delta z, \text{ ft.} \]
REFERENCES


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