Timing and Magnitude of Soil Erosion in Owens Valley, California: A Combined 137Cs and 210Pbex Approach

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Timing and Magnitude of Soil Erosion in Owens Valley, California:
A Combined $^{137}$Cs and $^{210}$Pb$_{ex}$ Approach

A thesis submitted in partial fulfillment of the requirement for the degree of Bachelors of Science in Geology from The College of William and Mary

by

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Abstract
Soil erosion poses a major threat to the sustainability of soil resources in both natural and agricultural ecosystems and is considered a key contributor to desertification in arid and semi-arid regions. While studies have tried to resolve linkages between soil erosion and other environmental influences, attempts to connect soil erosion to these processes over time is often inhibited by the lack of a detailed decadal-scale soil erosion history. The study presented here examines erosion rates in Owens Valley, California and possible linkages to a recent episode of drought, water withdrawal, and live vegetation loss in 1986-1992. The study used a modification of the standard $^{137}$Cs and $^{210}$Pb$_{ex}$ technique in which a mass balance advection-diffusion erosion model is used to connect $^{137}$Cs and $^{210}$Pb$_{ex}$ soil bulk inventories to past erosion. Cs-137 is generally more sensitive to total erosion while Pb-210 inventories are sensitive to both total erosion and erosion timing. By combining the two isotope bulk inventories in a model and comparing them with measurements collected in the field, it was possible to constrain both erosion timing and magnitudes. This is in contrast to single-isotope models which do not present results with regards to erosion timing. This study found a higher likelihood of erosion in more recent years, particularly during and following the drought period of 1986-1992. However, significant spatial variability was observed and some soil cores were more consistent with episodic erosion while others were consistent with a relatively constant erosion rate. The average erosion magnitude for the region was $1.2\pm 0.65$ cm over 54 years or $2.5$ t ha$^{-1}$ yr$^{-1}$. 
Timing and Magnitude of Soil Erosion in Owens Valley, California: A Combined $^{137}$Cs and $^{210}$Pb$_{ex}$ Approach

Guy Nathaniel Evans

Introduction

Soil erosion poses a major threat to the sustainability of soil resources in both natural and agricultural ecosystems (Eswaran, Reich and Beinroth 1999). In arid and semi-arid regions, soil erosion and removal of soil nutrients is considered a key contributor to desertification, which inhibits the reestablishment of plant communities following periodic climate changes and drought (Elmore et al. 2008, Schlesinger and Pilmanis 1998). Airborne particulates generated by soil erosion may also contribute to atmospheric pollution and potentially play a significant role in climate and ecosystem dynamics on the local, regional, and global scales (Schlesinger and Pilmanis 1998).

In order to better understand the processes of desertification and soil deflation, many authors have attempted to compare records of regional hydrology, land use, and vegetation cover to changes in soil erosion and atmospheric dust production over time (Elmore et al. 2008, Gill and Gillette 1991, Schlesinger and Pilmanis 1998). Studies of current conditions have suggested a system of complex relationships by which soil erosion and physical dust formation may be associated with both long-term drought and short-term precipitation events (Reheis 2006, Elmore et al. 2008, Kaste 2008, Cahill et al. 1996). However, attempts to connect these processes over past climates and environmental conditions are limited by the lack of a detailed decadal-scale soil erosion history, which is necessary for comprehensive comparison between soil erosion and other, time dependent environmental factors.

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One commonly used method by which to determine mid-range (~40 year) erosion histories is the measurement of $^{137}$Cs and $^{210}$Pb$_{ex}$ radioactivity and concentration within the soil profile. Cesium-137 is a weapons fallout radionuclide associated with atmospheric testing of nuclear weapons during the 1950s and 1960s. On the other hand, $^{210}$Pb$_{ex}$ is a continually produced natural product of the $^{238}$U-$^{230}$Th decay series formed through the radioactive decay and subsequent fallout of $^{222}$Rn which diffuses into the atmosphere.

Both $^{137}$Cs and $^{210}$Pb$_{ex}$ are atmospherically deposited radionuclides, the activity of which in the soil profile is partly controlled by past occurrences of sediment erosion and deposition. The radioactivity profiles and total bulk inventory concentrations of these isotopes over depth have been linked to historic erosion rates through the application of a quantitative advection-diffusion type model (Kato, Onda and Tanaka 2010, Wallbrink and Murray 1999, Walling and He 1999a, Walling and He 1997). Reviews of several of these models are given by Walling and He (1999b) and Ritchie and McHenry (1990).

Studies of this sort typically define results based on either $^{137}$Cs or $^{210}$Pb$_{ex}$, while those few which investigate both $^{137}$Cs and $^{210}$Pb$_{ex}$ generally consider the results for each isotope independent of the other and then compare across isotopes in order to constrain variability or to confirm results (Walling, Collins and Sichingabula 2003, Kato, Onda and Tanaka 2010). Wallbrink and Murray (2003) took advantage of the different depositional histories of $^{210}$Pb$_{ex}$ and $^{137}$Cs by investigating variations in the $^{210}$Pb$_{ex}$/$^{137}$Cs ratio with depth in the soil profile in order to account for spatial variation of the initial isotope deposition.

Another common feature of nearly all current models relating $^{137}$Cs or $^{210}$Pb$_{ex}$ concentration with depth or bulk inventory is the assumption of a time
independent constant erosion rate. For this reason, models only claim to yield an average mean erosion rate from the start of $^{137}$Cs deposition to the date of collection, some 40-50 years (Ritchie and McHenry 1990, Kato, Onda and Tanaka 2010, Walling and He 1997, Walling and He 1999a,b, Walling, Collins and Sichingabula 2003). The assumption of a constant erosion rate through time automatically precludes comparisons with historic records of other environmental influences. This assumption may also lead to the calculation of long term mean erosion rates for a given site which are not explained by both $^{137}$Cs and $^{210}$Pb$_{ex}$ inventory concentrations caused by the potentially episodic or time dependent nature of soil erosion. This is because $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk inventory concentrations, particularly $^{210}$Pb$_{ex}$, are sensitive to both erosion timing and

![Diagram](image)

**Figures 1a, b and c.** The concentration of $^{210}$Pb$_{ex}$ with depth approaches an equilibrium condition where inputs through deposition are balanced by loss through radioactive decay. This resembles an exponential distribution because $^{210}$Pb$_{ex}$ is transported downward through the soil through time (a). When erosion occurs, $^{210}$Pb$_{ex}$ is removed from the system (b). With continued fallout, $^{210}$Pb$_{ex}$ eventually stabilized back toward the equilibrium condition (c). For this reason, an older, large magnitude erosion event may give the same inventory value as a more recent, lower magnitude event.
erosion magnitude. Because $^{210}\text{Pb}_{\text{ex}}$ is continually deposited, older, higher-magnitude events could be confused with newer lower-magnitude erosion periods if only the bulk inventory concentration of $^{210}\text{Pb}_{\text{ex}}$ is to be considered (figure 1a-c).

Having been deposited largely in association with a specific historical event, $^{137}\text{Cs}$ concentrations are less sensitive to erosion timing than $^{210}\text{Pb}_{\text{ex}}$. However, in environments where downward transport of $^{137}\text{Cs}$ through the soil column is relatively rapid, a time dependency of results will occur as $^{137}\text{Cs}$ becomes protected from erosion events by the intervening soil layers (figures 2a,b,c).

$^{137}\text{Cs}$ Concentration

(A) Initial deposition of $^{137}\text{Cs}$ is advected downward into the soil profile, slowly decaying such that concentrations are lower with deeper depths in the soil profile (a). Erosion removes some of the $^{137}\text{Cs}$ (b). This amount is determined in part by the rate and timing of the erosion and the rate of advection into the soil. Cesium which is removed is not replaced and the total bulk inventory (c) will tend to be more sensitive to erosion magnitude than erosion timing.

In contrast to previous studies, the research presented in this article seeks to exploit differences in the deposition histories of both $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$, seeking agreement between response in the two isotope’s responses to various time dependent erosion scenarios before determining possible erosion histories and
rates. Also this study considers $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk inventory concentrations, rather than concentration depth profiles, which are easier and more precise to measure. The goal of this project is to better constrain possible erosion histories such that they may be compared with other known events of environmental importance.

**Study site: Owens Valley**

The study area for this research is Owens Valley, California. This site is advantageous because detailed records of hydrologic, land use and live vegetation cover have already been compiled and several on-going studies continue to investigate possible links between soil erosion, live vegetation cover, land use, and water management (City of Los Angeles and County of Inyo. 1990a, Elmore et al. 2000, Hollett 1991).

The Owens Valley of California is a naturally endoheric basin located 400 km (240 mi) north of Los Angeles (figures 2a-c). Supported by a depressed fault block between the uplifted Sierra Nevada and White-Inyo Mountains, Owens Valley is a 120km (72mi) long remnant of the large Pliocene-Pleistocene Waucobi Lake and is primarily underlain by Waucobi Lake sediments (Bachman 1978). Elmore et al. (2003) divided the valley into two broad geomorphologic regions: alluvial fan deposits with deep water tables and the valley floor with a generally shallow water table. The alluvial deposits source from the surrounding mountain slopes and tend to be dominated by relatively low grade and constant slopes.

Situated within the rain shadow of the Sierra Nevada, Owens Valley receives only 13cm of precipitation a year. However, the valley receives significant quantities of water as runoff from the surrounding mountains. This includes the Owens River, which historically drained into Owens Lake at the southern end of the valley (Hollett
et. al. 1991). In 1913, the City of Los Angeles began diverting water from Owens Valley through the Los Angeles Aqueduct (figure 3c). This diversion was completed in 1970, when the City completed a second aqueduct, further drawing down groundwater supplies in the valley and leading to an increased reliance on
groundwater pumping (Hollett et. al. 1991). Groundwater removal has led to significant changes in the plant and ecological communities native to Owens Valley in which there is a transition from groundwater dependent perennials to xeric and precipitation dependent species as well as an increase in exotic, precipitation dependent annuals particularly during and after periods of drought (Elmore et al. 2003). What remains of Owens Lake, dry since the 1920s, is a wet playa, characterized by mobile dunes, cemented and puffy crusts, and an increase in subsequent aeolian sediment transport processes (Cahill et. al. 1996).

Though not an historically important source of airborne dust particles (Reynolds et. al. 2006, Reheis, Budhain and Lamothe 2001), Owens Valley is currently the largest natural producer of PM$_{10}$ (diameter < 10 µm) airborne particulate pollution in the United States (Gill and Gillette 1991). Geochemical studies suggest that dust from the valley can be found as much as 400 km east of the valley itself (Reheis et. al. 2002). Recently, the practice of water spreading has begun as a means of recharging the groundwater aquifer during abnormally wet years, which alleviates some of the effect of pumping (Elmore et al. 2003). However, this practice is unlikely to be continued during periodic drought. Investigations have also been undertaken into the use of sand fences and local vegetation to stabilize otherwise mobile dunes (Cahill et. al. 1996). While effective where implemented, these are clearly reactive rather than preventative measures against soil erosion.

Precipitation data of the region reveals a relative drought from 1987-1992, which was the focus of Elmore et al. (2003) (figures 4a,b and c). Other perturbations in the recent climate record show that increases in airborne dust fluxes often coincide with strong El Niño
and La Niña events, for example in 1984-85, 1987-88, 1989-1991, and 1997-99 and an additional drought period between 1995 and 1997 (Reheis 2006). This study seeks to compare soil erosion histories as determined by analysis of $^{137}$Cs and $^{210}$Pb bulk concentration inventories to decadal scale climate changes, particularly the drought of 1906-1992.

**Figures 4a,b, and c.**
Graph of % live cover for sites delimited by change in %live cover in response to a drought between 1986-1992, marked in gray (a) and no change in %live cover groundwater in response to drought because groundwater levels remained stable during the drought or vegetation was dependent on precipitation (b). The temporary decrease in % live cover corresponds to an increase in groundwater extraction from 1986-1990 in response to a drought 1986-1992. Graphs and figures from Elmore et al. (2003)
Cesium-137 and $^{210}\text{Pb}_{\text{ex}}$ and sediment erosion models

Cesium-137 is perhaps the most common radioisotope used as a proxy for soil erosion rates. Within Earth systems, $^{137}\text{Cs}$ is convenient because it occurs almost exclusively as atmospheric fallout from nuclear weapons testing. Because of the nature of its synthesis, $^{137}\text{Cs}$ is largely found within sediments deposited concurrent to large-scale nuclear weapons tests during the period 1955-1970 with peak values occurring in 1963 (ERDA ref). Cesium-137 is often concentrated within the upper 10cm of the soil profile with the maximum concentration peak in the upper few centimeters (Walling and He 1997). Because deposition of $^{137}\text{Cs}$ is associated with a particular historical event and not continuous, concentrations of $^{137}\text{Cs}$ within a given soil profile are more sensitive to total erosion than to the timing of that erosion as $^{137}\text{Cs}$ removed through erosion is not replaced except through a redeposition of sediment.

In contrast, $^{210}\text{Pb}$ occurs naturally as part of the $^{238}\text{U}$-$^{230}\text{Th}$ decay series; forming as the granddaughter product of $^{222}\text{Rn}$, some of which is released into the atmosphere. While $^{210}\text{Pb}$ is less commonly used than $^{137}\text{Cs}$, it is continuously produced and is seen as a possible replacement isotope for atmospheric radioisotope techniques as the latent concentration of $^{137}\text{Cs}$ decreases over time (Walling, Collins and Sichingabula 2003). Lead-210 in the soil column occurs in two forms. A base concentration of $^{210}\text{Pb}$ occurs in the soil as a result of radioactive decay within soil layers, while fallout or excess $^{210}\text{Pb}$ occurs as a result of $^{222}\text{Rn}$ decay in the atmosphere. This excess $^{210}\text{Pb}$ or $^{210}\text{Pb}_{\text{ex}}$ is deposited at a more or less constant rate for a given site and occurs in greatest abundance at the soil surface, decreasing exponentially with depth (Turkebian et. al. 1977, Walling and He 1999). Because it is constantly being produced and deposited, $^{210}\text{Pb}_{\text{ex}}$ is sensitive to erosion timing, but
also to erosion rate, as smaller, more recent episodes of erosion may produce similar bulk inventory concentrations as those seen following older but higher magnitude events.

Once in the soil, both $^{137}$Cs and $^{210}$Pb adhere strongly to small particulate matter or colloids and are considered chemically stable and largely inert within the soil column (Kretzschmar and Schäfer 2005, Ritchie and McHenry, 1990). The slight alkalinity of Owens Valley soils also promotes the attachment of these $^{137}$Cs and $^{210}$Pb to colloidal particles (Kretzschmar and Schäfer 2005). As such, the primary transport mechanisms appear to be in association with these particles. Assuming that these particles move either in association with gravity and water moving downward through the soil column or through a Fickian diffusion-like mechanism caused by random, direction non-biased movements of soil particles within column (salt efflorescences, bioturbation, etc.), $^{137}$Cs and $^{210}$Pb$_{ex}$ profiles and bulk inventories is response to erosion may be modeled through the application of a calibrated mass-balance advection-diffusion model with erosion and deposition of sediment occurring at the

Figure 5a,b and c For the model, a continuous soil depth profile (a) is discretized (b) to form an initial model profile. As a whole, the bulk inventory of this profile is affected by input in the form of atmospheric fallout and erosion. Within each depth interval (c), additional inputs (black) and outputs (white) are connected to advection, diffusion and radioactive decay.
top of the modeled soil column (figure 5a, b, and c).

Because volumetric concentrations of $^{137}$Cs and $^{210}$Pb$_{ex}$ tend to be greater near the top of the soil profile, lower volumetric concentrations of these isotopes indicate greater erosion rates. Conversely, a higher than expected value for the overall $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk inventory indicates sediment deposition and accumulation. Within most models, known inputs of $^{137}$Cs or $^{210}$Pb$_{ex}$ are added to the simulated soil surface as deposition from the atmosphere. Inputs are then alternatively advected and diffused within the model system or eroded out of it. For simplicity, transport within soil columns is most often represented by constant advection and diffusion terms (He and Walling 1997, Kato, Onda and Tanaka 2010, Ritchie and McHenry 1990). However, some authors, especially Walling and He (1999b), question this assumption and argue for correction factors most of which account for differences in mean sediment size between $^{137}$Cs and $^{210}$Pb$_{ex}$ associated particles. Because $^{137}$Cs ($t_{1/2} = 30.2$ a) and $^{210}$Pb$_{ex}$ ($t_{1/2} = 23.2$ a) decay on the decadal scale, a percentage of the concentration will also be lost to radioactive decay and this portion should be taken into account within the mass balance considerations of the model (Walling and He 1999b).

Such mass balance advection-diffusion erosion models have generally been used in space for time substitution experiments to calculate the long-term erosion effects of changing land use, from agriculture to deforestation to animal grazing practices (He and Walling 1997, Walling and He 1999a, Wallbrink and Murray 1999, Walling, Collins and Sichingabula 2003, Kato, Onda and Tanaka 2010).
Methods

Measurement of Cs-137 and Pb-210

Data for this project was collected as part of an on-going study being conducted as per the collaborative research proposal presented in Kaste 2008.

Soil cores obtained from 100m radius plots in Owens Valley with centers located at previously installed City of Los Angeles groundwater and wind monitoring stations on relatively low grade (<%5) flat plains or slopes. Different plots were then categorized between stable and non-stable sites according to the findings of Elmore et al. (2003), which categorized site based on vegetation response to a drought period between 1986-1992. Observations conducted in the field also took note of exposed plant roots and other obvious signs of sediment erosion or deposition on these sites, bearing in mind that point erosion could exhibit significant spatial variability even within the confines of a particular plot. The soils tested in this study consisted of both aridisols and mollisols. However, the non-stable sites analyzed in this study are all identified as mollisols.

At each site, several 12cm soil cores were collected with a cylindrical bulb planter. Soil samples below this depth were collected with a standard bulk density core tube in order to confirm that the entire bulk inventory of $^{137}\text{Cs}$ or $^{210}\text{Pb}_{\text{ex}}$ had been accurately collected. Each 12cm core was then analyzed for bulk $^{137}\text{Cs}$ or $^{210}\text{Pb}_{\text{ex}}$ isotopic inventories using gamma spectrometry according to the method provided in Murray et al. (1987). Several cores from stable sites identified by Elmore et al. 2003 were also sectioned and analyzed for radioisotope concentrations with depth in order to create a calibration profile for the advection and diffusion terms used in the model.

$^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$ advection-diffusion and sediment erosion model
In order to quantitatively relate radionuclide depletion in soils to erosion, I developed a one-dimensional discretized mass-balance advection-diffusion type model. Once this model is calibrated with a measured $^{137}\text{Cs}$ and $^{210}\text{Pb}_{ex}$ concentration profile with depth, an erosion function is added to the model. This alteration allows us to simulate the effects of a limited period constant erosion event on the expected $^{137}\text{Cs}$ and $^{210}\text{Pb}_{ex}$ bulk inventories. Field bulk inventory values can then be compared with those from the model in order to determine likely erosion scenarios as determined by both timing and erosion magnitude.

In order to construct the model, first, a continuous soil profile from 0-30cm is discretized into 1cm segments. This is the scale of the samples collected to create the calibration profile and calibrate the advection and diffusion transport terms. The discretized time interval for the model is one month. A one month discretization was chosen because this is the scale of the atmospheric isotopic record for $^{90}\text{Sr}$ recorded by the Health and Safety Laboratory: U.S. Atomic Energy Commission in New York City, which was used as the atmospheric input fallout function for $^{137}\text{Cs}$ (figure 6). The $^{90}\text{Sr}$ data set ends in December 1976 due to a lack of further deposition and $^{137}\text{Cs}$ fallout is consequently set to zero after this period. The use of atmospheric $^{90}\text{Sr}$ concentration data as a direct proxy for atmospheric $^{137}\text{Cs}$ concentration is justified as $^{90}\text{Sr}$ occurs in equivalent ratio to $^{137}\text{Cs}$ in the fallout debris of nuclear explosions and like $^{137}\text{Cs}$, $^{90}\text{Sr}$ has no significant natural source (Ritchie and McHenry 1990).
Because results are normalized for a given site based on local reference inventories based on the $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$ bulk inventories of a locally determined stable site, it is not critical that specific $^{90}\text{Sr}$ monthly fluxes differ in magnitude from those of $^{137}\text{Cs}$. However, the distribution of atmospheric $^{90}\text{Sr}$ concentration as a proxy for $^{137}\text{Cs}$ deposition over time is important as the depth profile of the $^{137}\text{Cs}$ concentration is dependent on fallout as a time dependent function. For calculation of the $^{210}\text{Pb}_{\text{ex}}$ concentration depth profile, a constant input fallout function is used and calibrated to steady state with $^{210}\text{Pb}_{\text{ex}}$ radioactive decay for the $^{210}\text{Pb}_{\text{ex}}$ bulk inventory of stable reference cores.

To start the model, time is set to January 1954 (the start of the $^{137}\text{Cs}$ deposition record) and each of the 30 1cm segments within a depth profile is given an initial isotope concentration of 0Bq/cm$^3$. Then, the fallout value for $^{137}\text{Cs}$ corresponding to January 1954 is added to the top 1cm segment of the $^{137}\text{Cs}$ model and a constant fallout value for $^{210}\text{Pb}_{\text{ex}}$ is added to the top 1cm segment of the $^{210}\text{Pb}_{\text{ex}}$ model.

In the second time interval, representing February 1954, the $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$ concentrations from the previous time interval are adjusted for movement through

**Figure 6.** $^{90}\text{Sr}$ fallout over time. Also a weapons fallout nuclide, $^{90}\text{Sr}$ is a good proxy for $^{137}\text{Cs}$ and occurs in the same ratios over time. If $^{137}\text{Cs}$ amounts are to be used instead of ratios, a local control site is needed to calibrate this value.
advection- and diffusion-like processes. Advection and diffusion terms are calculated independently of each other and added together to achieve a net movement at each 1cm depth interval. Advection is calculated by multiplying a constant advection term $A$ by the total concentration within any depth interval. The magnitude of $A$ determines the velocity with which $^{210}$Pb and $^{137}$Cs travel down the soil profile from any 1cm interval to the one immediately below it. The model is closed below 30cm such that the isotopes are conserved in the last interval, which represents all depths greater than 30cm.

The change in concentration over time attributed to advection is calculated by the function

$$\frac{\Delta C_{A(x,t)}}{\Delta t} = A \nabla C_{(x,t-\Delta t)}$$

where $\Delta C_{A(x,t)}$ is the change in concentration attributed to advection at depth interval $x$ at time $t$, $\Delta t$ is equal to $\frac{1}{12}$ years and $\Delta x$ is equal to 1cm. $A$ is an experimentally determined model advection constant representing the speed of advection and $\nabla C_{(x,t-\Delta t)}$ is the concentration gradient at the discretized depth interval $x$ at time $t-\Delta t$. Advection is a unidirectional process and may be considered either in the downward (if $A > 0$) or upward (if $A < 0$) direction.

In contrast to advection, the apparent diffusion-like movement of particles is not direction specific. Rather, the non-biased movement of any particular particle up or down creates the effect that particles appear to shift down gradient through time. Such direction non-biased processes are likely to include expansion contraction processes such as wetting and drying, salt efflorescences, possible expansion due to temperature or bioturbation. The speed at which this apparent movement occurs is a
function of the change in concentration gradient \( \nabla^2 C_{(x,t-\Delta t)} \) evaluated for the depth interval \( x \) at time \( t-\Delta t \), and the empirically determined model diffusion constant \( D \).

\[
\frac{\Delta C_{D(x,t)}}{\Delta t} = D\nabla^2 C_{(x,t-\Delta t)}
\]

The ground-air interface found at the surface of the depth profile is considered a non-diffusion, non-advection surface.

In order to find the total bulk inventory for the date of sample collection which was February 2008, the model runs for a total of 649 months and the sum of all depths in the entire profile is taken to determine the bulk inventory. Because the time period 1954-2008 is on the same order of magnitude as the radioactive half-lives of \(^{137}\)Cs and \(^{210}\)Pb (30.23 and 20.4 years respectively) it was important to make adjustments for radioactive decay according to the formula

\[
C_t = C_{t-\Delta t} e^{-\lambda \Delta t}
\]

where \( \Delta t \) is the time step of the model in years or \( \frac{1}{12} \), \( C_t \) is the isotopic concentration at a given time interval, \( C_{t-\Delta t} \) is the isotopic concentration at the previous time interval. The relationship between the two is determined by the radioactive decay factor \( e^{-\lambda \Delta t} \) where \( \lambda = \frac{\ln(2)}{t_{1/2}} \) and \( t_{1/2} \) is the half-life of the isotope (\( \lambda_{^{137}\text{Cs}} = 0.029 \text{ yr}^{-1}, \lambda_{^{210}\text{Pb}} = 0.034 \text{ yr}^{-1} \)).

Because the only radionuclide input to the stable site, non-erosion depth profile as a whole comes through fallout deposition and the only output was through radioactive decay, this value could have been achieved without the formation of a depth profile. However, the generation of a depth profile is important because it
influences the effect that erosion has on $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$ bulk inventories once an erosion scenario is considered. The distribution of $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$ concentrations with depth are a function of the terms $A$ and $D$, which are calibrated according to a measured control profile taken in the field (discussed below). The combined effect of atmospheric fallout, radioactive decay, advection and diffusion is modeled by the formula

$$C_{(x,t)} = e^{-\lambda \Delta t} \left[ C_{(x,t-\Delta t)} - A \Delta C_{(x,t-\Delta t)} + D \Delta^2 C_{(x,t-\Delta t)} \right] + F_{(x,t)}$$

where $F_{(x,t)}$ is the fallout function for time $t$ and depth interval $x$. This is set to the atmospheric concentration of $^{90}\text{Sr}$ at the soil surface or $x=0$ and is set to 0 for all other depths, $x>0$.

**Calibration of the model advection and diffusion constants $A$ and $D$**

Diffusion and advection terms are calibrated using data from $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$ profiles measured at stable field sites for cores we believe to represent a non-erosion condition. These are identified by a high $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$ bulk inventory similar to that which was expected for a stable surface.

First, depth profiles were generated for both the model and collected field data by normalizing the concentration at each depth interval to the total radionuclide concentration of the profile (figure 7). Because of some experimental uncertainty in the field data with regards to the collection of a specific 1cm depth interval, several of these intervals are bulked together leaving three depth intervals at 1cm, 3cm, and 5cm. Below 5cm, the $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$ activities were found to be less than 5% of the total bulk inventory and thus discarded for the purposes of calibration. The calculated model depth profile was similarly bulked to create three depth intervals at 1cm, 3cm and 5cm for direct comparison while calibrating for appropriate advection and diffusion rate constants. These constants are assumed to be the same for both $^{137}\text{Cs}$
and $^{210}\text{Pb}_{\text{ex}}$ as well as for every depth and time interval. This assumption is based on the sparse nature of the calibration data, which prevented a more detailed analysis of variations in advection and diffusion rates across depths. The coarseness of $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$ concentration measurements with depth also encouraged the use of bulk inventory data for the two isotopes rather than analyses of soil profiles.

In order to calibrate the model, different advection and diffusion terms were tried in order to find the best fit for both $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$. Using the advection and diffusion rates calculated from the $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$ control profiles. This calibration ratio was 1.50 for the aridisol and 1.64 for the mollisol.

Calibration of the advection diffusion model yielded a diffusion rate of 3.6 mm$^2$/year and an advection rate of 0.06 mm/year. This means that in each simulated year of the model run, 0.06% of each isotope was transported downward 1 centimeter.

![Mollisol Calibration](image)

**Figure 7** Values for the calibration of the model were collected from a control site core. The values calculated are 3.6 mm$^2$/year for diffusion and 0.06 mm/year for advection. Thus, most movement occurs through diffusion.
while during the same period 3.6% of the gradient was diffused a distance of 1cm on average.

Running the Erosion Model and Finding Possible Scenarios

Erosion scenarios tested by this model are constrained to single erosion events of constant erosion rate, starting within a given year and continuing for a set duration. This is based on the hypothesis that recent vegetation decline in non-stable sites might have led to greater erosion rates and that evidence for this should be found in the $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk inventory values for these sites. Constant erosion rates are chosen because it is the simplest type of erosion scenario which could account for changes in the timing of erosion while also considering the long-term constant erosion rate proposed by most previous studies. More complicated solutions could not be tested without adherence to $^{137}$Cs and $^{210}$Pb$_{ex}$ concentrations with depth as well as bulk inventories. However, this was not attempted as concentration depth profiles collected in the field of the two isotopes were considered too coarse for this sort of detailed analysis. Instead, possible erosion scenarios were calculated based on the bulk inventory of the two isotopes.

In order to calculate an expected $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk inventory for particular erosion scenarios, a grid array is assembled and composed of different single-event constant-erosion scenarios varying in erosion rate, erosion durations and erosion start times. Erosion rates are tested varying from 0.0cm/year to 6.0cm/yr with intervals of 0.12cm/yr. Duration of the erosion event is allowed to vary from 0-50 years at 1 year intervals. Start time for the erosion event varies from 1954-2008 with intervals of 0.5 years.

For each point on the grid, iterations of the model run for the period January 1954 – February 2008 adding the erosion equation
\[
\frac{\Delta C}{\Delta t} = \frac{E \nabla C}{\Delta t}
\]

where \( E \) is the erosion rate in cm/month and \( \nabla C \) is the concentration gradient evaluated at depth interval \( x \), to the non-erosion advection-diffusion decay equation for iterations in which the particular erosion scenario tested called for erosion. This results in the equation

\[
C_{x,t} = e^{\frac{-\lambda}{n}} \left[ C_{x,t-\Delta t} - A \nabla C_{x,t-\Delta t} + D \nabla^2 C_{x,t-\Delta t} + E \nabla C_{x,t-\Delta t} \right] + F_{x,t}
\]

for all time intervals in which erosion is set to occur according to a given erosion scenario. For instance, the erosion scenario with erosion rate=0.3cm/month, duration=5 years and start of erosion=1973 would call for the erosion value \( E=0.3 \) and the term \( E \nabla C_{x,t-\Delta t} \) would added to the non-erosion equation for the years 1973-1978.

Unlike advection and diffusion, the top of the soil profile is not closed to erosion and this process constitutes an additional export of \(^{137}\text{Cs}\) and \(^{210}\text{Pb}_{\text{ex}}\) and associated shift in depth coordinates in order to account for the new surface level. The erosion function \( E \Delta C_{x,t-\Delta t} \) removes all material from the top \( E \) centimeters of the profile and shifts the reference frame for the rest of the profile upward by the same quantity \( E \) centimeters (figures 8a, b and c). Final expected bulk inventories for a given erosion scenario were reported as a percentage of the total concentration inventory calculated for completely non-erosion scenarios (figures 9a,b).

One normalized inventory values are recorded for all erosion scenarios tested, those erosion scenarios that explain both \(^{137}\text{Cs}\) and \(^{210}\text{Pb}_{\text{ex}}\) concentrations of non-stable sites are identified by determining and graphing the intersection between two.
Figures 8a, b, and c
An explanation of the mass balance formula used to calculate new concentration profiles following advection or erosion. Fig. 4a is the profile before advection or erosion. With advection, the entire profile is shifted downward and $\Delta C/\Delta t$ for a given depth is equal to the advection rate times $\alpha - \beta$, which is negative $\Delta C/\Delta x$. For erosion (4c), the process is similar except that the difference $\Delta C/\Delta x$ must be shifted upwards and the quantity $\Delta C/\Delta x$ times $E$, the erosion rate, is subtracted rather than added.

Figures 9a and b
Figure 9a shows isosurfaces for normalized Cs-137 inventories of 80%-80% with 20% represented by the light color and 20% represented by the dark color. Because Cs-137 is only deposited during the interval 1955-1970, the inventory is largely related to total erosion rather than erosion timing. Pb-210 on the other hand is very sensitive to both erosion timing and total erosion (9b). By combining the two inventories, both erosion timing and total erosion can be constrained for a limited type of erosion scenario.

given $^{137}\text{Cs}$ and $^{210}\text{Pb}_{ex}$ isovalue surfaces corresponding to normalized radioisotope inventories of non-stable sites. Inventory percentages of non-stable sites are
determined as a percentage of the mean average for stable sites. Tolerances for the surface intersection were compared at 25%, 50%, 75%, and 100% of the total standard deviation for the measurements of isotopic activity or 60Bq/m$^2$ for $^{137}$Cs and 180Bq/m$^2$ for $^{210}$Pb$_{ex}$.

This method generated surfaces comprised of many different erosion scenarios that constrain the range of measured $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk concentration inventories. Additional analysis of the possible solutions for erosion scenarios generated by the model are made by creating a frequency distribution curve of the erosion scenarios calculated to be consistent with the measured isotopic data in order to gauge the most likely times of erosion occurrence. For each year, erosion scenarios indicating erosion during that year are summed according to the erosion rate indicated by each particular erosion scenario and this sum was normalized such that the sum of all possible scenarios, adjusting for erosion rate, was equal to 1 or 100%. This analysis was conducted for sites 12 and 21 only, because

**Figure 10** Cs-137 and Pb-210 bulk inventories for reference, stable and non-stable sites. Stable and Non-stable sites were identified in Elmore et al. (2003) and this designation refers to the stable or non-stable reaction of live vegetation cover during the drought period 1986-1992.

“Reference sites” are soil cores collected from stable sites as identified by Elmore et al (2003), which appear to match the expected Cs-137 bulk inventory of 1240Bq/m$^2$ as determined in Simon et al. (2004). It is important to distinguish these because erosion can occur on a smaller scale even at a generally stable site cover during the drought period 1986-1992.
these sites offered the highest number of measured cores, using measurement errors of 30Bq/m² for $^{137}\text{Cs}$ and 90Bq/m² for $^{210}\text{Pb}_{ex}$. Results were then tested for sample variability and sensitivity to variations in the advection and diffusion rate constants.

**Results**

**Concentration and $^{137}\text{Cs}$ and $^{210}\text{Pb}_{ex}$ Bulk Inventory Results**

Concentrations of $^{137}\text{Cs}$ and $^{210}\text{Pb}_{ex}$ varied exhibited significant spatial variability for both stable and non-stable sites. However, radioisotope concentrations observed for stable sites were generally higher than those for non-stable sites and yielded a statistically different population. The mean bulk inventory of $^{137}\text{Cs}$ for stable sites was $626\pm330\text{Bq/m}^2$, while the mean $^{210}\text{Pb}_{ex}$ concentration was $1119\pm275\text{Bq/m}^2$. However, taking into consideration the possibility that non-stable areas may exist within generally stable sites, it appears that the local reference inventory for Owens Valley is likely closer to the higher measurements recorded for stable sites. Taking this into account, the reference inventory for Owens Valley is recalculated as $1221\pm127\text{Bq/m}^2$ for $^{137}\text{Cs}$ and $1390\pm256\text{Bq/m}^2$ for $^{210}\text{Pb}_{ex}$ (figure 9). The value for $^{137}\text{Cs}$ is similar to the value proposed for $^{137}\text{Cs}$ deposition by Simon et al. (2004) of $\sim3500\text{Bq/m}^2$ or $1240\text{ Bq/m}^2$ if this value is age corrected for radioactive decay since the peak year of $^{137}\text{Cs}$ deposition in 1963.

**Calibration Results**

Calibration of the advection diffusion model yielded a diffusion rate of 0.003 cm²/month and an advection rate of 0.0005cm/month. This means that for every year, 0.06% of each isotope was transported downward 1 centimeter while during the same period 3.6% of the gradient was diffused a distance of 1cm on average.

**Model Results**
Solutions to erosion scenarios supported by measurements of both $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk measurements showed possible erosion scenarios to be well constrained with regards to total erosion amount, but poorly constrained with regards to duration of erosion or erosion rate (figures 11 a-e). Generated solutions at many sites were also well constrained for erosion timing if less than experimental error was assumed. However, few were well constrained with time at higher or full allowances for

**Figures 11a-e** Different projections of a solutions set at a) 100%, b) 75%, c) 50%, and d) 25% of experimental error. Figure 10e shows the solution set to be well constrained with regards to total erosion magnitude.
measurement uncertainties. There were also some cores for which no solution was found. This was typically the case when little to no $^{210}\text{Pb}_{\text{ex}}$ was found but some $^{137}\text{Cs}$ was still present. It was generally assumed that these sites could be explained through a combination of deposition and erosion. This could occur if rapid deposition of sediment following the initial deposition of $^{137}\text{Cs}$ created a sufficiently large separation between the $^{137}\text{Cs}$ peak concentration and the $^{210}\text{Pb}_{\text{ex}}$ accumulating on the surface that the later isotope could then be removed through erosion with lesser effects on the bulk inventory of $^{137}\text{Cs}$ at the site. However, no further analysis of these sites was conducted in this study.

Cases in which bulk inventories of $^{137}\text{Cs}$ and $^{210}\text{Pb}_{\text{ex}}$ were very close to the mean bulk inventories for stable control sites were poorly constrained for timing yielding low total erosion quantities. Calculated erosion timing and total erosion showed high variability within sites and averages ranged from the highest at site 20, 1.7cm, to the lowest at site 25 with 0.8cm. However, the number of samples collected for these sites was relatively small.

<table>
<thead>
<tr>
<th>Site #</th>
<th>total erosion (cm)</th>
<th>Standard Deviation (1σ)</th>
<th>Sample size</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.86</td>
<td>± 0.5</td>
<td>11</td>
</tr>
<tr>
<td>17</td>
<td>1.40</td>
<td>± 0.49</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>1.67</td>
<td>± 0.85</td>
<td>3</td>
</tr>
<tr>
<td>21</td>
<td>1.44</td>
<td>± 0.68</td>
<td>14</td>
</tr>
<tr>
<td>25</td>
<td>0.80</td>
<td>± 0.53</td>
<td>5</td>
</tr>
<tr>
<td>All</td>
<td>1.20</td>
<td>± 0.65</td>
<td>38</td>
</tr>
</tbody>
</table>

Table 1. shows total erosion average and standard deviation between soil cores for several non-stable sites. Sites 12 and 21 with higher numbers of samples were singled out for further analysis. Other analyses were made assuming all non-stable sample cores were representative of the same population.
within specific sites and the average for all non-stable sites measured was $1.2\pm 0.65\text{cm}$ over 54 years or $0.2\text{mm/year}$ (table 1). At a bulk density of $1050\text{kg/m}^3$ as measured for one of the soil cores in the field, this yields an erosion rate of $2.5 \text{ t ha}^{-1}\text{yr}^{-1}$.

**Erosion Timing Results**

Further analyses of possible erosion timings conducted on sites 12 and 21 showed results for cores on each of the two sites to be broken into several different categories.

Site 12 consisted of three categories of probable erosion years. Of 11 cores, 10 yielded possible solutions (figure 1a). Two of the cores suggested erosion events from the 1950s and 1960s or during $^{137}\text{Cs}$ deposition. Two more cores suggested the most likely erosion scenarios to occur during the years between 1980 and 1990, while six cores indicated a likelihood of more recent erosion from within the past two decades.

**Figures 12a and b.** show the densities of solution sets for Sites (a) 12 and (b) 21. Erosion years are normalized for erosion rates and total number of solutions. Calculations are made at $\frac{1}{2}$ the experimental standard deviation. Letters and numbers indicate the identities of cores plotted.
with a peak occurring after the year 2005.

Probable solutions for site 21 were broken into 4 different categories out of a total of 15 cores (figure 12b). Six cores exhibited a relatively even distribution of probable erosion years for all years in the data set, 1954-2008. One core exhibited likely erosion years between 1973 and 1984. Three core exhibited likely erosion years of between 1987 and 1997 while five cores exhibited many solutions for both after 1990 and showed peaks after 2005. When all non-stable sites were treated as a single site and numbers of solutions calling for erosion during each year were summed, more

Figure 13 A double y plot. On the left, this shows the percent of all erosion scenarios found to be in agreement with the $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk inventories of a non-stable site against the years in which these scenarios state erosion will occur. These values are normalized for the erosion rate set to occur in a given year. On the right is the number of cores belonging to a particular category of erosion scenarios. This corresponds to the height of the arrows, which indicate the period for which erosion is thought to occur. For instance, the long arrow in the middle at a height of 7 indicates that there are 6 cores whose $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk inventory measurements correspond to a steady long term erosion rate. The gray box indicates the drought period 1986-1992.
solutions called for erosion during more recent years, post-1985 than prior (figure 13). Numbers of solutions appeared to increase steadily from the 1950s onward before reaching a local maximum at 1988, during the drought period identified by Elmore et al. (2003). A second peak in numbers of solutions was found after the year 2005. However, this may be because of changes in sensitivity in the model. Values for $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk inventory are not well distinguished for such recent episodes of erosion and similar tolerances will tend to give an artificially large number of solutions during this interval.

**Sensitivity to Advection and Diffusion Rates**

Calculations of total erosion magnitude and erosion timing appeared to be relatively robust in response to changes in advection and diffusion rate. Results for total erosion magnitude and erosion timing were recalculated for advection and diffusion rates ±25% and ±50% of the originally calibrated rates. Because transport within the soil column appeared to be

**Figure 14a and b** Recalculation of total erosion magnitude and start time of erosion scenarios explaining $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk inventory measurements based on variations in advection (a) and diffusion (b) rates. Numbers before the comma indicate the site number. Numbers and letters after the comma identify the soil core.
dominated by diffusion, results were more sensitive to changes in the diffusion rate. However, even with a 50% swing in the diffusion rate, total calculated erosion rates changed by less than 20% of the total erosion magnitude, amounting to less than 0.4cm for 2.0cm of total erosion and erosion timing varied by less than five years in either direction (figure 14 a and b).

**Conclusions and Discussion**

Model analyses of $^{137}$Cs and $^{210}$Pb$_{ex}$ bulk inventory soil concentrations in comparison to erosion rate timing and duration showed $^{137}$Cs to be primarily sensitive to total erosion magnitude while concentrations of $^{210}$Pb$_{ex}$ were sensitive to both total erosion magnitude and erosion timing. In theory, this should provide a mechanism by which to constrain both erosion timing and erosion magnitude. However, the significant spatial variability and experimental uncertainty involved in this technique makes definitive conclusions about erosion timing difficult to make. Erosion magnitude on the other hand was well constrained but demonstrated significant spatial variability between cores within a given plot.

When lower than experimental uncertainty was assumed, the combined isotope technique not only could be used to constrain erosion timing on the decadal scale but could also be used to distinguish between categories of cores on a given test plot. Despite spatial variability within the plots themselves, sites 12 and 21 exhibited similar categories of cores and suggested areas of relatively old or continuously eroding area as well as areas which appeared to have been eroded more recently within the past few decades.

Additionally, isotopic measurements from several cores were consistent with erosion between the years 1985 and 1995. This would correspond roughly with the
drought period reported in Elmore et al. (2003). It also fits well with the observation reported in Reheis (2006) that increased dust deposition occurs at sites downwind of alluvial and dry-playa sources during dry and very wet years. There are also several cores where the radioisotope inventories suggest that most erosion scenarios extend to the period beyond the end of the drought. It is possible this could indicate some indirect effects of the drought on erosion, for example the removal of plant live cover, remained active even following the drought period. The existence of roughly the same categories of cores at both site 12 and 21 could indicate some sort of causal mechanism in which certain categories of cores are found to be common even as plots themselves prove spatially variable.

In general, the $^{137}$Cs and $^{210}$Pb$_{ex}$ combined isotope method used in this study seems promising in its ability to constrain total erosion and erosion timing. Experimental uncertainty remains a significant issue. However, results here show that the combined $^{137}$Cs and $^{210}$Pb$_{ex}$ isotope technique could have some interesting promise should some of these problems of experimental uncertainty be resolved and it is robust with regards to advection and diffusion rates, which may be difficult to measure.

The technique presented here provides non-unique solutions with regards to erosion rate and erosion duration. This is likely due to the relatively low advection rate relative to the erosion rate such that downward movement of particles by advection does not significantly decrease the quantity of radionuclide particles removed through erosion. A wide variability of results is also indicated for very small (<0.5cm) erosion totals and the method is ineffective for erosion events older than the deposition of $^{137}$Cs.
Additionally, the model used here only tested for purely erosion and non-deposition scenarios with only a single period of erosion and only for a constant erosion rate over these periods. While this may be an improvement over previous models which assume a constant erosion rate over all time periods, analyses of $^{137}$Cs and $^{210}$Pb$_{ex}$ inventories rather than profiles will inevitably lead to non-unique solutions if both erosion and deposition are allowed, or possibly if multiple erosion events are allowed or if variable erosion rates are considered. In order to better constrain these factors, it will be necessary to conduct a detailed analysis of $^{137}$Cs and $^{210}$Pb$_{ex}$ concentrations profiles with depth within the soil column. This again speaks to the problem of constraining experimental uncertainty.

Acknowledgements

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References


City of Los Angeles and County of Inyo. 1990a. Technical Appendix F. Green book for the long-term groundwater management plan for the Owens Valley and Inyo County. In Draft EIR: water from the Owens Valley to supply the second Los Angeles Aqueduct; 1970 to 1990, and 1990 onward, pursuant to a long term groundwater management plan. State Clearing House no. 89080705, Department of Water and Power, City of Los Angeles, California, USA.


Appendix A: Matlab code for Cs-137 bulk inventory calculation

% Author: Guy Evans, Dept. of Geology, College of William and Mary
% Script describing atmospheric radionuclide transport in Aridisols in Owens Valley California. 
% Sr-90 input data, with radioactive decay, advection, diffusion, and erosion.
% Produces array

% Input parameters
half_life=30.23; isotope is Cs-137, half-life 30.23 years
K_diff=0.003; diffusion constant
K_adv=0.0005; advection constant
T_final_year=2008; date year of model end (sample collection date)
T_final_month=2; date month of model end
time_begin=1954; year of model begin (fixed)

% Output parameters
max_duration=50;
max_erosion=0.5; maximum tested erosion rate in cm/month (0<max_erosion<1)

% Adjusted parameters
K_decay=log(2)/(half_life*12); monthly decay rate
T_final_adj=T_final_year+T_final_month/12; adjusted decimal time

% Setup of initial C_prior vector and initial conditions
% C_prior is stepwise results matrix. Results records output matrices

% Set up physical grid
res_depth=1; depth resolution in depth intervals per cm, set to 1
z=0:1/res_depth:30;
E_dt=ones(31,1)*[0:0.01:max_erosion];

% Creates Sr-90 Input vector "I_full"
T=1954:1/12:T_final_adj; Time Jan 1954-end (res_month per month)
cont=repmat(0,(T_final_adj)*12-(1976*12+10)+1,1); filling zeros for Sr-90 after Oct 1976
NYC_Sr=xlsread('M_database.xls'); NYC Sr-90 input data (monthly iteration) Jan 1954 to end
I_Sr=[NYC_Sr(1:274,2);cont];
I_T_Sr=[T; I_Sr']; makes a 2 column by length(time) matrix of dates and Sr-90 fallout data
zeros=repmat(0,length(z)-1,length(I_T_Sr)); makes a grid of zeros for time and depth not=0cm
I_full=[I_T_Sr;zeros]; creates input vector with depth in rows and time in columns,
% First row is date in years % second row is Sr-90 input

% Set up graph coordinate arrays:
% graph_ii, (erosion rate)
% graph_jj, (erosion timing)
% graph_kk, (erosion duration)

e_dt=0:0.01:max_erosion;
T_e_i=(time_begin:1:T_final_year)';
erosion_duration=0:0.5:max_duration;
for n=time_begin:1:T_final_year;
    if n==time_begin;
        ...
```matlab
    graph_i=e_dt;
    elseif n>time_begin;
        graph_i=cat(1,graph_i, e_dt);
    end
end
for m=0:0.5:max_duration;
    if m==0;
        graph_ii=graph_i;
    elseif m>0;
        graph_ii=cat(3,graph_ii,graph_i);
    end
    'compiled graph_ii';
end
for p=0:0.01:0.5;
    p;
    if p==0;
        graph_j=T_e_i;
    elseif p>0;
        graph_j=cat(2,graph_j, T_e_i);
    end
end
for m=0:0.5:max_duration;
    if m==0;
        graph_jj=graph_j;
    elseif m>0;
        graph_jj=cat(3,graph_jj,graph_j);
    end
    'compiled graph_jj';
end
for m=0:0.5:max_duration;
    if m==0;
        graph_k=repmat(m,length(T_e_i'),length(e_dt));
        graph_kk=graph_k;
    elseif m>0;
        graph_k=repmat(m,length(T_e_i'),length(e_dt));
        graph_kk=cat(3,graph_kk, graph_k);
    end
    'compiled graph_kk';
end
i=graph_ii;%3-D e_dt values
j=graph_jj;%3-D T_e_i values
k=graph_kk;%3-D duration values

%I,R,A,D/dt and erosion loop

%Concentration profile with depth
%constant erosion rate 0<E_dt<6cm/yr for time period
%'E_duration' starting at T_E_initial ending at T_final

for T_E_initial=time_begin:1:T_final_year;%varies time of erosion start
    T_E_initial
    for duration=0:0.5:max_duration;%varies duration of erosion period (6 month intervals) for max_duration duration;
```
C_prior = repmat(0, length(z), length(E_dt));
time = time_begin;

while time <= T_final_adj
    time;
    % first distribute...
    column_id = round(((time - time_begin) * 12 + 1)); % identifies column in
    I_full corresponding to time 'time'
    CR_dt = C_prior * exp(-K_decay); % monthly loss to decay
    I_dt = [repmat(I_full(2, column_id), 1, length(E_dt));
            repmat(0, length(z) - 1, length(E_dt))]; % input vector dt
    A_dt = [repmat(0, 1, length(E_dt));
            diff(C_prior) * K_adv]; % movement due to advection dt
    D_dt = [repmat(0, 1, length(E_dt));
            diff(C_prior, 2) * K_diff;
            repmat(0, 1, length(E_dt))]; % movement due to diffusion dt (check edges)
    C_distributed = CR_dt + I_dt + A_dt + D_dt; % concentration profile with
    % depth after I, R, A, D/dt

    % then erode...
    % E_dt = ones(31, 1) *[0:0.01:1]; (reminder)
    % monthly erosion rate in cm, not IFESLEIF because erosion btw 0 and
    % 1.
    % Is erosion occurring?
    % If yes erosion...
    if time > T_E_initial && time <= T_E_initial + duration;
        'erode';
    % to erode
    C_mid = [diff(C_distributed);
             repmat(0, 1, length(E_dt))];
    C_final = C_distributed + (E_dt * C_mid);
    % produces a 2D 31X102 matrix. Columns represent erosion rates
    % 0:0.01:1 cm per month.
    % If no erosion...
    elseif time <= T_E_initial || time > (T_E_initial + duration);
        'not erode';
    % to not erode
    C_final = C_distributed;
    end

    C_prior = C_final; % reset loop for next time iteration
    time = time + (1/12); % advance time one month
end

if duration == 0;
    results_ddt = C_final; % sets initial value for results_edt
elseif duration == 0;
    results_ddt = cat(3, results_ddt, C_final); % third dimension is duration
end
invent = sum(C_final);
inventnormal = invent(1, 1);
end

if T_E_initial == time_begin;
    results_dyi = results_ddt;
elseif T_E_initial == time_begin;

results_dyi=cat(4,results_dyi,results_ddt);
end
inventory_results_Cs=sum(results_dyi,1)./inventnormal.*100;
end
'done'

%time_begin=1954;
%max_duration=10;
%max_erosion=0.5;
%T_final_year=2008;

for dur=(1:1:max_duration*2+1);
dur;
    for es=(1:1:T_final_year-time_begin+1);
es;
        for er=(1:1:max_erosion*100+1);
er;
            if er==1;
                rate_matrix=inventory_results_Cs(1,er,dur,es);
            elseif er>1;
                rate_matrix=cat(2,rate_matrix,inventory_results_Cs(1,er,dur,es));
            end
        end
        if es==1;
            start_matrix=rate_matrix;
        elseif es>1;
            start_matrix=cat(1,start_matrix,rate_matrix);
        end
    end
    if dur==1;
        duration_matrix=start_matrix;
    elseif dur>1;
        duration_matrix=cat(3,duration_matrix,start_matrix);
    end
end
results_matrix_Cs=duration_matrix;
ed
'done_results'
Appendix A: Matlab code for Pb-210 bulk inventory calculation

%Author: Guy Evans, Dept. of Geology, College of William and Mary
%script describing atmospheric radionuclide transport in Aridisols in
%Owens Valley California.
%Sr-90 input data, with radioactive decay, advection, diffusion,
%and erosion.
%Produces array

%Input parameters
half_life=20.4; isotope is Cs-137, half-life 30.23 years
K_diff=0.003; diffusion constant
K_adv=0.0005; advection constant
T_final_year=2008; date year of model end (sample collection date)
T_final_month=2; date month of model end
T_final_adj=T_final_year+T_final_month/12; adjusted decimal time

%Output parameters
time_begin=1954; year of model begin (fixed)
max_duration=50;
max_erosion=0.5; maximum tested erosion rate in cm/month

%adjusted parameters
K_decay=log(2)/(half_life*12); monthly decay rate

%setup of initial C_prior vector and initial conditions
C_prior is stepwise results matrix. Results records output matrices

%set up physical grid
res_depth=1; depth resolution in depth intervals per cm, set to 1cm
z=0:1/res_depth:30;
E_dt=ones(31,1)*[0:0.01:max_erosion];

%formation of Pb_210 Input vector
T=1954:1/12:T_final_adj; Time Jan 1954-end (res_month per month)
ATM_Pb=repmat(0.183,(T_final_adj)*12-(1954*12)+1,1);%ATM Pb-210 input
I_T_Pb=[T;ATM_Pb'];%makes a 2column by length(time) matrix of dates
zeros=repmat(0,length(z)-1,length(I_T_Pb));%makes a grid of zeros for
and Sr-90 fallout data
I_full=[I_T_Pb;zeros];%creates input vector with depth in rows and
time in columns,
%first row is date in years %second row is Sr-90 input
E_dt=ones(31,1)*[0:0.01:max_erosion];

%set up graph coordinate arrays:
%graph_i, (erosion rate)
%graph_jj, (erosion timing)
%graph_kk, (erosion duration)

e_dt=0:0.01:max_erosion;
T_e_i=(time_begin:1:T_final_year)';
erosion_duration=0:0.5:max_duration;
for n=time_begin:1:T_final_year;
    if n==time_begin;
        graph_i=e_dt;
    elseif n>time_begin;
        graph_i=cat(1,graph_i, e_dt);
end
end
for m=0:0.5:max_duration;
    if m==0;
        graph_ii=graph_i;
    elseif m>0;
        graph_ii=cat(3,graph_ii,graph_i);
    end
    'compiled graph_ii';
end

for p=0:0.01:0.5;
    p;
    if p==0;
        graph_j=T_e_i;
    elseif p>0;
        graph_j=cat(2,graph_j, T_e_i);
    end
end
for m=0:0.5:max_duration;
    if m==0;
        graph_jj=graph_j;
    elseif m>0;
        graph_jj=cat(3,graph_jj,graph_j);
    end
    'compiled graph_jj';
end
for m=0:0.5:max_duration;
    if m==0;
        graph_k=repmat(m,length(T_e_i'),length(e_dt));
        graph_kk=graph_k;
    elseif m>0;
        graph_k=repmat(m,length(T_e_i'),length(e_dt));
        graph_kk=cat(3,graph_kk, graph_k);
    end
    'compiled graph_kk';
end

i=graph_ii;%3-D e_dt values
j=graph_jj;%3-D T_e_i values
k=graph_kk;%3-D duration values

%I,R,A,D/dt and erosion loop

%Concentration profile with depth
%constant erosion rate 0<E_dt<6cm/yr for time period
%'E_duration' starting at T_E_initial ending at T_final

for T_E_initial=time_begin:1:T_final_year;%varies time of erosion
    start
    T_E_initial

    for duration=0:0.5:max_duration;%varies duration of erosion
        period (6 month intervals) for max_duration
duration;
        C_prior=repmat(0,length(z),length(E_dt));
        time=time_begin;

    end
while time<=T_final_adj
    time;
    %first distribute...
    column_id=round((time-time_begin)*12+1); %identifies column in
    I_full corresponding to time 'time'
    CR_dt=C_prior*exp(-K_decay); %monthly loss to decay
    I_dt=[repmat(I_full(2,column_id),1,length(E_dt));repmat(0,length(z)-
    1,length(E_dt))]; %Input vector dt
    A_dt=[repmat(0,1,length(E_dt));diff(C_prior)*K_adv]; %movement due to
    advection dt
    D_dt=[repmat(0,1,length(E_dt));diff(C_prior,2)*K_diff;repmat(0,1,leng
    th(E_dt))]; %movement due to diffusion dt (check edges)
    C_distributed=CR_dt+I_dt+A_dt+D_dt; %concentration profile with
    depth after I,R,A,D/dt

    %then erode...
    E_dt=ones(31,1)*[0:0.01:1]; %reminder
    %monthly erosion rate in cm, not IFESLEIF because erosion btw 0
    %and 1.
    %Is erosion occurring?
    %If yes erosion...
    if time>T_E_initial && time<=T_E_initial+duration;
        'erode';
        %to erode
        C_mid=[diff(C_distributed);repmat(0,1,length(E_dt))];
        C_final=C_distributed+(E_dt.*C_mid);
        %produces a 2D 31X102 matrix. Columns represent erosion
        %rates
        %0:0.01:1 cm per month.
    %If no erosion...
        elseif time<=T_E_initial||time>(T_E_initial+duration);
            'not erode';
            %to not erode
            C_final=C_distributed;
        end
    C_prior=C_final; %reset loop for next time iteration
    time=time+(1/12); %advance time one month
end

%sum over depth profile for inventory values and normalize to non-
%erosion.
    if duration==0;
        results_ddt=C_final;
    elseif duration==0;
        results_ddt=cat(3,results_ddt, C_final);
    end
    invent=sum(C_final);
    inventnormal=invent(1,1);
end

if T_E_initial==time_begin;
    results_dyi=results_ddt;
else T_E_initial==time_begin;
    results_dyi=cat(4,results_dyi,results_ddt);
end
inventory_results_Cs=sum(results_dyi,1)./inventnormal.*100;
end
'done'

%compile inventory values into array results_matrix_Cs
for dur=(1:1:max_duration*2+1);
dur;
    for es=(1:1:T_final_year-time_begin+1);
es;
        for er=(1:1:max_erosion*100+1);
er;
            if er==1;
rate_matrix=inventory_results_Cs(1,er,dur,es);
            elseif er>1;
rate_matrix=cat(2,rate_matrix,inventory_results_Cs(1,er,dur,es));
            end
    end
    if es==1;
start_matrix=rate_matrix;
    elseif es>1;
start_matrix=cat(1,start_matrix,rate_matrix);
    end
end

if dur==1;
duration_matrix=start_matrix;
else dur>1;
duration_matrix=cat(3,duration_matrix,start_matrix);
end
results_matrix_Pb=duration_matrix;
end
'done_results'