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BY THE METHOD OF LEAST SQUARES
A User's Manual
by
John D. Boon, III
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Special Report No. 186
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PREFACE

A common objective in the treatment of measured tides is the mathematical separation by harmonic analysis of certain periodic components which can be used to simulate the astronomical tide. Traditionally, astronomical or predicted tides have been represented by the sum of several such components, each one due to a particular aspect of the tide-producing forces. Once determined, the same set of parameters governing each periodic component may be used to predict the tide during any desired time period at a given location. These parameters are known as the tidal constants for that location.

Although a number of different methods have been developed for determining tidal constants, both before and since the advent of electronic computing techniques, one rarely finds that any of these are intended for practical application by nonspecialists. This is particularly true in the case of the scientist or technician who has a collection of tidal records and would like to utilize them to accurately predict tides by some convenient means. This manual has therefore been written with the general user in mind. It is assumed that the reader has some familiarity with general reference works on the subject of tides.
Introduction

The method of least squares was first applied in the analysis of tides by Horn (1960). An excellent description of its use has been given by Dronkers (1964) who mentions that official tide tables in Germany have since been prepared by this means. The least squares algorithm is exceptionally easy to program on a digital computer and requires very little memory space. It is most efficient when used to obtain a fixed number of tidal constituents from a fixed number of discrete tidal height (or current) measurements representing a standard series length. For the average user, an appropriate series consists of hourly measurements covering the synodic month of approximately 29 days. This series will normally yield ten of the major tidal constituents making up the astronomical tide; several others may be indirectly obtained by the inference formulas of Schureman (1958). For a discussion of the types of constituents normally used in tidal analysis, the reader is referred to Defant (1958).

This manual describes two main programs written in Fortran IV, one for the determination of tidal constants (program HAMELS) and one which utilizes the constants to generate predicted tides (program ASTRO). In addition to the basic derivation of tidal constants, we have included the computational steps necessary to convert these results to a standardized form which enables the prediction of tides for any period within the present century. This is accomplished in both programs by subroutine ORBEL.
Mathematical Development - Method of Least Squares

Let $h_t$ represent a series of tidal height measurements obtained at hourly intervals of time $t = -n, -n+1, \ldots, 0, \ldots, n-1, n$ so that $t = 0$ at the midpoint of the series. The total number of measurements is then $N = 2n+1$.

The harmonic series used to approximate the measurements using $k$ tidal constituents is

$$h(t) = H_0 + \sum_{i=1}^{k} A_i \cos a_i t + \sum_{i=1}^{k} B_i \sin a_i t$$

(1)

Here $a_i$ is the speed of the $i$th constituent in degrees per mean solar hour and $A_i$, $B_i$ are coefficients representing the constituent amplitude. $H_0$ represents the mean height of tide in the series. By reducing the measurements to zero mean with no linear trend, $H_0$ may be neglected in equation (1) and we obtain the required least squares fit by choosing coefficients $A_i$, $B_i$ such that

$$E = \sum_{t=-n}^{n} [h(t) - h_t]^2$$

is a minimum. This will occur when

$$\frac{\partial E}{\partial A_j} = \frac{\partial E}{\partial B_j} = 0; \ j = 1, 2, \ldots, k$$

By carrying out the above, $2k$ simultaneous equations are generated:

$$\sum_{t=-n}^{n} [h(t) - h_t] \cos a_j t = 0$$

(2)
\[ \sum_{t=-n}^{n} [h(t) - h_t] \sin_a t = 0 \quad (3) \]

Substituting equation (1) in the above equations and rearranging terms, the normal equations are

\[ \sum_{i} A_i \sum_{t} \cos_a t \cos_a t + \sum_{i} B_i \sum_{t} \sin_a t \cos_a t = \sum_{t} h_t \cos_a t \quad (4) \]

\[ \sum_{i} A_i \sum_{t} \sin_a t \sin_a t + \sum_{i} B_i \sum_{t} \sin_a t \sin_a t = \sum_{t} h_t \sin_a t \quad (5) \]

Due to the selection of a central time origin, we also have

\[ \sum_{t} \sin_a t \cos_a t = \sum_{t} \cos_a t \sin_a t = 0 \]

as well as the identities

\[ \sum_{t} \cos_a t \cos_a t = \frac{1}{2} C(a_i - a_j) + \frac{1}{2} C(a_i + a_j) = S_{ij} \]

\[ \sum_{t} \sin_a t \sin_a t = \frac{1}{2} C(a_i - a_j) - \frac{1}{2} C(a_i + a_j) = D_{ij} \]

where

\[ C(0) = N \]

and

\[ C(\xi) = \frac{\sin(N\xi/2)}{\sin(\xi/2)} , \quad \xi = a_i + a_j \text{ or } a_i - a_j \neq 0 \]

The final set of 2k equations are then

\[ \sum_{i} A_i S_{ij} = \sum_{t} h_t \cos_a t \quad (6) \]

\[ \sum_{i} B_i D_{ij} = \sum_{t} h_t \sin_a t \quad (7) \]

which must be solved for the 2k unknowns \( A_i \) and \( B_i \), \( i = 1, 2, \ldots, k \).
To illustrate, assume that $k = 3$ tidal constituents are being sought. Written in full, the equations in (6) and (7) are

\[ A_1 S_{11} + A_2 S_{21} + A_3 S_{31} = \Sigma h_t \cos a_1 t \]
\[ A_1 S_{12} + A_2 S_{22} + A_3 S_{32} = \Sigma h_t \cos a_2 t \]  \hspace{1cm} (6)
\[ A_1 S_{13} + A_2 S_{23} + A_3 S_{33} = \Sigma h_t \cos a_3 t \]

\[ B_1 D_{11} + B_2 D_{21} + B_3 D_{31} = \Sigma h_t \sin a_1 t \]
\[ B_1 D_{12} + B_2 D_{22} + B_3 D_{32} = \Sigma h_t \sin a_2 t \]  \hspace{1cm} (7)
\[ B_1 D_{13} + B_2 D_{23} + B_3 D_{33} = \Sigma h_t \sin a_3 t \]

or in matrix form

\[
\begin{pmatrix}
S_{11} & S_{12} & S_{13} \\
S_{12} & S_{22} & S_{23} \\
S_{13} & S_{23} & S_{33}
\end{pmatrix}
\begin{pmatrix}
A_1 \\
A_2 \\
A_3
\end{pmatrix}
= 
\begin{pmatrix}
\Sigma h_t \cos a_1 t \\
\Sigma h_t \cos a_2 t \\
\Sigma h_t \cos a_3 t
\end{pmatrix}  \hspace{1cm} (6)
\]

\[
\begin{pmatrix}
D_{11} & D_{12} & D_{13} \\
D_{12} & D_{22} & D_{23} \\
D_{13} & D_{23} & D_{33}
\end{pmatrix}
\begin{pmatrix}
B_1 \\
B_2 \\
B_3
\end{pmatrix}
= 
\begin{pmatrix}
\Sigma h_t \sin a_1 t \\
\Sigma h_t \sin a_2 t \\
\Sigma h_t \sin a_3 t
\end{pmatrix}  \hspace{1cm} (7)
\]

noting that $S_{ij} = S_{ji}$ and $D_{ij} = D_{ji}$ since these symbols represent a summation of cosine and sine products, respectively, in which the order is immaterial. Multiplying both sides of (6) and (7) by the inverse of the respective square matrices, the required coefficients are
A computational advantage of the least squares method is now apparent. Given a standard set of \( k \) tidal constituents and a standard series containing \( N \) tidal measurements, one can obtain the inverse matrices in advance so that only the data summation vectors on the right of (6) and (7) require computation prior to matrix multiplication during a run.

Tidal Constants - Amplitude, Speed and Phase

Having obtained a set of coefficients by the method just described, it is convenient to express each tidal constituent in the form of a simple sine wave of amplitude \( R_i \), speed \( a_i \), and phase \( \zeta_i \) using

\[
h_i(t) = R_i \cos(a_i t - \zeta_i)
\]  

where

\[
R_i = \sqrt{A_i^2 + B_i^2}
\]

\[
a_i = \frac{360^\circ}{T_i}, \quad T_i = \text{period of } i\text{th constituent}
\]

\[
\zeta_i = \tan^{-1}\left( \frac{B_i}{A_i} \right)
\]
The phase angle, \( \zeta_i \), fixes the position of the wave form relative to the time origin. In reference to series time, this means that a high water phase occurs at \( t = \zeta_i/a_i \) hours after (or if \( \zeta_i \) is negative, before) zero hour at the midpoint of the series. If standard time is to be used in equation (8) the angle \( a_i t_0 \) must first be added to \( \zeta_i \), \( t_0 \) being the standard time at the midpoint of the series.

**Goodness of Fit Criteria**

An important step following an analysis is to ask how well the approximation made with equation (1) represents the input data. One way of answering is through an analysis of variance or sum of squares partitioning among the individual constituents used in the analysis.

The total sum of squares for the tidal measurements is

\[
SS_{\text{total}} = \sum_{t} h_{i}^2 - \left[ \sum_{t} h_{i} \right]^2 / N
\]

and the sum of squares contributed by the ith constituent is

\[
SS_i = \sum_{t} h_{i}(t)^2 - \left[ \sum_{t} h_{i}(t) \right]^2 / N
\]

in which \( h_{i}(t) \) values are generated using equation (8) at series times \( t = -n, -n+1, \ldots, 0, \ldots, n-1, n \). One then obtains the percent sum of squares accounted for by the ith constituent as the fraction \( SS_i / SS_{\text{total}} \times 100 \).

Totaling the percentages for all of the constituents used gives an indication of the strength of that particular model of
the tide. It should be noted, however, that a combined sum of less than 100% is to be expected; in an area where the astronomical range is small but weather-related disturbances are pronounced, the combined sum may be less than 50% using any number of astronomical constituents.

**Tidal Prediction Model**

Before accurate predictions can be made, specifically in years other than that of the analytical series, certain adjustments to the tidal constants become necessary. As presented by Schureman (1958), the required harmonic model of the tide is

\[
h(t) = H_0 + \sum_i f_i H_i \cos[a_i t + (V_0 + u)_i - \kappa_i]
\]  

(9)

where

\(H_0 = \text{height of mean sea level above model datum}\)

\(f_i = \text{nodal factor for reducing mean amplitude to the required amplitude in the year of prediction}\)

\(H_i = \text{mean amplitude of } i\text{th constituent during 18.6 year-period of lunar node cycle}\)

\((V_0 + u)_i = \text{the local equilibrium phase of } i\text{th constituent in the year of prediction}\)

\(\kappa_i = \text{phase of } i\text{th constituent relative to the local equilibrium phase}\)

Each of the above arguments associated with equation (9) are essential elements in the tidal analysis and prediction programs of this manual. The following explanations apply to their use in these programs:
$H_0$ - If the predicted heights are to refer to a model datum of mean sea level, $H_0$ must then be zero. Other values depend upon the datum selected. For example, $H_0$ should equal approximately one-half the mean tidal range at the station if the model datum is mean low water. For a discussion of tidal datums, see Marmer (1951), Boon and Lynch (1972), or Swanson (1974).

$f_i, H_i$ - These arguments are related by $R_i = f_i H_i$ where $R_i$ is the expected amplitude of the $i$th constituent in the year of the prediction. A slight variation in $R_i$ for most (but not all) constituents occurs from one year to the next, the value in any one year depending upon the position of the lunar nodes within an 18.6-year cycle. Subroutine ORBEL computes nodal factors, $f_i$, to convert $R_i$ to $H_i$ in program HAMELS, $H_i$ to $R_i$ in program ASTRO.

$(V_0 + u)_i$ - The equilibrium phase of a constituent is the phase of an imaginary sine curve representing the so-called equilibrium tide in the absence of friction and other factors. In the case of the main lunar constituent, $M_2$, this would mean that high water occurs just as the moon transits the meridian of the place in question. Thus, the $M_2$ equilibrium phase simply expresses lunar position (hour angle) in relation to the time origin and meridian in use. Greenwich equilibrium phases refer to Greenwich mean time (G.M.T.) and the prime meridian at Greenwich, England. Subroutine ORBEL computes the latter for up to 37 selected constituents during any year between 1900 and the year 2000.
Comparing equations (8) and (9), one sees that \( \zeta_i = \kappa_i - (V_o + u)_i \) or \( \kappa_i = \zeta_i + (V_o + u)_i \) as illustrated in the following diagram:

\[ \text{sine curve representing } i\text{th tidal constituent} \]

The phase angle \( \kappa_i \) is used in equation (9) because it alone behaves as a true constant from one year to the next. Except for certain solar constituents, \((V_o + u)_i\) and hence \( \zeta_i \) vary in a uniform manner determined by celestial mechanics (the combined symbol for the equilibrium phase signifies that it consists of a slowly changing component and a more rapidly changing component).

A primed kappa symbol indicates that the constituent phase angle, \( \kappa_i' \), has been adjusted so that Greenwich \((V_o + u)_i\) values may be used in place of local \((V_o + u)_i\) in equation (9). This adjustment simply accounts for the difference in longitude between the tide station and the meridian of Greenwich and for the use of local standard time in the predictions. Each \( \kappa_i \) to \( \kappa_i' \) conversion is made according to the formula

\[ \kappa_i' = \kappa_i + P_i L - \frac{a_i S}{15} \]
where

\[ L = \text{west longitude in degrees of station at which predictions are to be made} \]

\[ S = \text{west longitude in degrees of time meridian used (e.g., 75^\circ W for U.S. Eastern Standard time)} \]

\[ P_i = \text{number of daily cycles of ith constituent (diurnal constituents = 1, semidiurnal = 2, etc.)} \]

Both \( k_1 \) and \( k_i' \) are included in the output of program HAMELS. Only \( k_i' \) values, however, are used as input to program ASTRO.
REFERENCES CITED


APPENDIX A

Program HAMELS
Instructions for using Program HAMELS

I. Preliminary Input

The initial input required by Program HAMELS includes a master list of constituent speeds and other associated parameters for 37 tidal constituents, a set of inference formula constants, and the inverse matrices matching the number and order of the selected constituents. The inverse matrices supplied in this publication must be used with a standard series of 697 observations. Other matrices for a series of different length may be computed using Program LESCO, Appendix C.

The complete card listing of preliminary input data for the standard series is given at the end of this appendix. The array values in each matrix have been multiplied by 1000 to eliminate extraneous leading zeroes and to conform with computational steps taken in the program.

II. Station Control Card

A four-letter station code is used to identify the tide station supplying the tidal data. The latitude and longitude of the station must be given as coded numbers in which the first three digits represent degrees and the remainder minutes and tenths (e.g., 03515.1 = 35° 15.1'). Other control information must be coded as indicated in the user comments listed at the beginning of the program.

III. Tide Data Cards

Twelve hourly heights per card for the first 58 cards (696 heights) followed by the 59th card containing the final height.
A condition of the least squares method is that the total number of heights must be odd.

III. Data Output and Plot

The main output from Program HAMELS consists of a set of harmonic constants obtained by the least squares analysis of the input data. A printer plot subroutine then graphically displays both the observed series of tidal heights and the predicted heights based on the above harmonic constants. Finally a plot of the residual between the observed and computed heights is printed.

The scale of the plot may be changed as needed by insertion of new range limits in the calling statements for subroutine PLOT which are found at the end of the main program.
C PROGRAM HAMELS - HARMONIC ANALYSIS METHOD OF LEAST SQUARES
C PROGRAM COMPUTES TIDAL CONSTANTS FOR 10 MAIN CONSTITUENTS BY LEAST SQUARES PLUS 15 SECONDARY CONSTITUENTS BY INFERENCES FORMULAS OF SCHUREMAN.
C TIDES DUE TO DISTURBING SECONDARY CONSTITUENTS t2, Pi, P3 ARE REMOVED FROM THE RECORD PRIOR TO FINAL DETERMINATION OF CONSTANTS.
C 1. MASTER LIST OF CONSTITUENT INFORMATION
C A. 37 TIDAL CONSTITUENT CARDS (I2, IA, A4, I2, A8, 4X, 9F3.1)
C 2. 15 INFERENCE FORMULA CARDS (2F6.3, 313)
C 3. STATION CONTROL CARD (1X, A4, 2F7.1, 13, 1X, 14, 3I3)
C 4. TIDAL DATA CARDS - 59 CARDS FOR STANDARD SERIES OF 29 DAYS
C SUBROUTINES REQUIRED - ORBEL, PLOT
C 00000010
C 00000020
C 00000030
C 00000040
C 00000050
C 00000060
C 00000070
C 00000080
C 00000090
C 00000100
C 00000110
C 00000120
C 00000130
C 00000140
C 00000150
C 00000160
C 00000170
C 00000180
C 00000190
C 00000200
C 00000210
C 00000220
C 00000230
C 00000240
C 00000250
C 00000260
C 00000270
C 00000280
C 00000290
C 00000300
C 00000310
C 00000320
C 00000330
C 00000340
C 00000350
C 00000360
C 00000370
C 00000380
C 00000390
C 00000400
C 00000410
C 00000420
C 00000430
C 00000440
C 00000450
C 00000460
C 00000470
C 00000480
C 00000490
C 00000500
C 00000510
C 00000520
C 00000530
C 00000540
C 00000550
C 00000560
C 00000570
C 00000580
C PROGRAM HAMELS - HARMONIC ANALYSIS METHOD OF LEAST SQUARES
C PROGRAM Computes TidAl constants FOR 10 MAIN CONSTITUENTS BY LEAST SQUARES PLUS 15 SECONDARY CONSTITUENTS BY INFERENCES FORMULAS OF SCHUREMAN. TIDES DUE TO DISTURBING SECONDARY CONSTITUENTS t2, Pi, P3 ARE REMOVED FROM THE RECORD PRIOR TO FINAL DETERMINATION OF CONSTANTS. 
C INPUT REQUIRED - 
C 1. MASTER LIST OF CONSTITUENT INFORMATION
C A. 37 TIDAL CONSTITUENT CARDS (I2, IA, A4, I2, A8, 4X, 9F3.1) 
C NOS(I) - NATIONAL OCEAN SURVEY REFERENCE NO. FOR ITH CONSTITUENT 
C SYM(I) - SYMBOL FOR ITH CONSTITUENT (A4) 
C A(I,1) - SPEED OF ITH CONSTITUENT IN OEG/MSH (F8.4) 
C A(I,J) - ORBITAL ELEMENT INDICES FOR ITH CONSTITUENT (8F3.1) 
C CT(I) - ITH CO~STITUENT TYPE, CIURNAL=1, SEMIOIURNAL=2, ETC. (F3.1) 
C B. 15 INFERENCE FORMULA CARDS (2F6.3, 313) 
C SCA(I) - AMPLITUDE PARAMETER, ITH INFERRED CONSTITUENT (F6.3) 
C SCE(I) - PHASE PARAMETER, ITH INFERRED CONSTITUENT (F6.3) 
C IC1(I) - INFERRED PARAMETER INDEX (13) 
C IC2(I) - INFERRED PARAMETER INDEX (13) 
C IC3(I) - INFERRED PARAMETER INDEX (13) 
C 2. LEAST SQUARES INVERSE MATRICES - 10 CARDS EACH MATRIX 
C SINV(l,J)X1000 (10F8.6) 
C DINVCl,J)X1000 (10F8.6) 
C 3. STATION CONTROL CARD (1X, A4, 2F7.1, 13, 1X, 14, 3I3) 
C XIDEN - STATION CODE (A4) 
C XLAT - STATION LATITUDE (F7.1) 
C XLON - STATION LONGITUDE (F7.1) 
C LTM - LONGITUDE OF TIME MERIDIAN (13) 
C IYR - YEAR OF OBSERVATIONS (14) 
C IDS - MONTH STARTING OBSERVATIONS (13) 
C IDS - DAY STARTING OBSERVATIONS (13) 
C TS - TIME OF FIRST OBSERVATION (F4.2) 
C 4. TIDAL DATA CARDS - 59 CARDS FOR STANDARD SERIES OF 29 DAYS 
C MM(I) - HOURLY HEIGHT OF TIDE (32X, 12F4.2) 
C SUBROUTINES REQUIRED - ORBEL, PLOT 
C C ------------------------------- DIMENSION HCS(10), HSN(10), AA(10), BB(10), SINW(10, 10), DINV(10, 10) 
C DIMENSION A(37, 9), J5(12), JE(12), HH(697), HI(25), R(25), ZETA(25) 
C DIMENSION VOU(37, 37), F(37, 37), NOS(37), SCA(25), SCE(25), T(697), PHH(697) 
C DIMENSION IC1(25), IC2(25), IC3(25), SS(25), SYM(37), CT(37) 
C REAL KAPA(25), KAPR(25), KPRMK(25) 
C NHH=697 
C NPH=1 
C M=10 
C N=15 
C C READ MASTER LIST OF CONSTITUENT SLEEDS, ORBITAL ELEMENT INDICES. 
C C SECONDARY CONSTITUENT INDICES. 
C C C N1=M+1 
C N2=M+N 
C READ(S,1) (NOS(I), SYM(I), A(I,J, J=1,9), CT(I), I=1,37) 
C 1 FORMAT(12, 1X, A4, 1X, F8.4, 4X, 9F3.1) 
C 46 FORMAT(2F6.3, 313) 
C C READ LEAST SQUARES INVERSE MATRICES(X1000) 
C C
C READ STATION CONTROL CARD
C READ HOURLY HEIGHTS
C
READ(5,2J XIDEN,XLAT,XL,ON,LTM,IYR,MS,IDS,TS
XLON=XL+ON/60.
2 FORMAT(1X,A4,F7.1,F3.0,F4.1,13,1X,14,213,F4.1)
43 FORMAT(32X,12F4.2)
C COMPUTE JULIAN START AND MIDPOINT DATES, ZULU START AND MIDPOINT TIMES
C COMPUTE LEAP YEAR ADJUSTMENT
C
DEGRAD=57.29577951
ZIP=0.1
LY=1
XLEAP=(2000.-IYR)/4.-LEAP
IF(XLEAP.GT.ZIP)LY=0
JS(1)=1
JE(1)=31
JS(2)=32
JE(2)=59+LY
K=1
DO 5 I=3,12
JS(I)=JE(I-1)+1
JE(I)=JS(I)+29+K
K=1-K
IF(I.EQ.7)K=1
5 CONTINUE
TZ=LTM/15
ZS=TS+TZ
ZDS=(JS(MS)+IDS-2)*24.*ZS
ZDM=ZDS+NPH/2
JD=ZDM/24+1
ZM=ZDM-(JD-1)*24
CALL ORBEL(IYR,JDS,JDM,ZS,ZM,NOS,A,VOU,F)
C REMOVE LINEAR TREND, ADJUST DATA TO ZEROC MEAN
C COMPUTE TOTAL SUMS OF SQUARES FOR SERIES
C
JUMP=0
SX=0.0
SX2=0.0
SY=0.0
SY2=0.0
DO 7 I=1,NHH
SX=SX+I
SY=SY+I
SX2=SX2+I*I
SY2=SY2+I*Y(I)
7 CONTINUE
600 XBAR=SX/NHH
YBAR=SY/NHH
$S_X = S_X - X_BAR \cdot S_Y$

$S_X^2 = S_X^2 - X_BAR \cdot S_X$

$C_Z = Y_BAR - B_Z \cdot X_BAR$

$T(1) = 1 - 1$

$PHH(1) = 0.0$

$HH(1) = HH(1) - B_Z \cdot I - C_Z$

8 CONTINUE

602 TSS = 0.0

DO 11 I = 1, NHH

TSS = TSS + HH(I)

11 CONTINUE

GO TO 15

16 SY = 0.0

DO 17 I = 1, NHH

SY = SY + HH(I)

17 CONTINUE

$Y_BR = S_Y / NHH$

DO 18 I = 1, NHH

$H_H(I) = H_H(1) - Y_BR$

18 CONTINUE

C------------------------------------
C COMPUTE COSINE AND SINE SUMMATION VECTORS
C---------------------------------------------

606 JUMP = 1

15 CONTINUE

DO 13 J = 1, M

$H_CS(J) = 0.0$

$H_SN(J) = 0.0$

II = (NHH - 1) / 2

ARG1 = A(J, 1) / DEGRAD

DO 13 K = 1, NHH

ARG = ARG1 * II

$H_CS(J) = H_CS(J) + HH(K) \cdot COS(ARG)$

$H_SN(J) = H_SN(J) + HH(K) \cdot SIN(ARG)$

II = II + 1

13 CONTINUE

C----------------------------------
C COMPUTE LEAST SQUARES COEFFICIENTS
C-----------------------------

DO 30 I = 1, M

$AA(I) = 0.0$

$BB(I) = 0.0$

DO 30 J = 1, M

AA(I) = AA(I) + SIN(V1, J) \cdot H_CS(J)

BB(I) = BB(I) + COS(V1, J) \cdot H_SN(J)

30 CONTINUE

AA(I) = AA(I) / 1000

BB(I) = BB(I) / 1000

10 CONTINUE

C----------------------------------
C COMPUTE CONSTITUENT AMPLITUDES AND PHASES
C---------------------

DO 14 I = 1, M

$ZETA(I) = ATAN2(BB(I), AA(I)) / DEGRAD$

$ZETA(I) = ZETA(I) + A(I, 1) \cdot (NHH / NPH - 1) / 2$

IDUMP = ZETA(1) / 360
ZETA(I)=ZETA(I)-1DU*360
IF(ZETA(I).LT.0)ZETA(I)=ZETA(I)+360.
ARG=AA(I)*AA(I)+BB(I)*BB(I)
R(I)=SQRT(ARG)

14 CONTINUE
IF(JUMP.GT.0)GO TO 19

C COMPUTE INFERRED SECONDARY CONSTITUENTS T2,P1,K2
C REMOVE T2,P1,K2 FROM DATA

DO 4 I=1,6
  KAPA(I)=VOU(I)-CT(I)*XLOA+AL(I,1)*TZA(I)
  H(I)=R(I)/F(I)
4 CONTINUE

H(20)=H(2)*SCA(20)
KAPA(20)=KAPA(2)+SCE(20)*(KAPA(2)-KAPA(11))
H(23)=H(4)*SCA(23)
KAPA(23)=KAPA(4)+SCE(23)*(KAPA(4)-KAPA(6))
H(25)=H(2)*SCA(25)
KAPA(25)=KAPA(2)+SCE(25)*(KAPA(2)-KAPA(11))
ZETA(20)=KAPA(20)-VOU(27)+2*XLOA-A(20,1)*TZA
ZETA(23)=KAPA(23)-VOU(30)+XLOA-A(23,1)*TZA
ZETA(25)=KAPA(25)-VOU(35)+2*XLOA-A(25,1)*TZA
H(20)=H(20)*SCA(21)
KAPA(20)=KAPA(20)-L*360.
KAPR(20)=KAPA(20)+CT(I)*XLOA-A(20,1)*TZA
KPRMK(20)=KAPR(20)-KAPA(20)
H(20)=H(20)/F(I)

6 CONTINUE
604 IF(JUMP.EQ.0)GO TO 16

C COMPUTE REMAINING SECONDARY CONSTITUENTS
C

DO 20 I=1,M
  J=NDS(I)
  K=IC2(I)
  L=IC3(I)
  KAPA(I)=KAPA(K)+SCE(I)*(KAPA(K)-KAPA(L))
  LL=L*KAPA(I)/360.
  KAPA(I)=KAPA(I)-LL*360.
  IF(KAPA(I).LT.0)KAPA(I)=KAPA(I)+360.
  KAPR(I)=KAPA(I)+CT(I)*XLOA-A(I,1)*TZA
  KPRMK(I)=KAPR(I)-KAPA(I)
  H(I)=H(I)/F(I)

20 CONTINUE
ISUBC=NOS(1)
ZETA(I)=KAPA(I)-VOU(ISUBC)+CT(I)*XLOM+AT(I)*TZ
21 CONTINUE
C COMPUTE PERCENT SUMS OF SQUARES FOR EACH CONSTITUENT
C
SST=O.0
DO 22 J=1,N2
L=NO(J)
SS(J)=O.0
II=O
ARG1=A(J,1)/OEGRA
ZETAL=ZETA(J)/DEGRAD
DO 23 K=1,NHH
ARG=ARG1*II-ZETAL
Y=F(LJ*H(J)*CCS(ARG)
SY=SY+Y
SS(J)=SS(J)+Y*Y
PHH(K)=PHH(K)+Y
II=II+1
23 CONTINUE
SS(JJ)=SS(J)-SY*SY/NHH*100./SS
SST=SST+SS(J)
SSCJ)=SS(J)+1.E-4
22 CONTINUE
C PRINT RESULTS
C
XLOM=XLM*100.+ON
WRITE(6,24) XLOM,XLAT,XLOM,LTM,IMR,MS,IDS,T5,NHH
24 FORMAT('HAMELS- HARMONIC ANALYSIS. METHOD OF LEAST SQUARES ' , / , 'STATION •,A4,2F7.1,13, 1 W',3X,'YEAR ',14,/ , '29 DAY SERIES STARTING •,t3,'-',12,3X,F5.1,' HRS 1 ,4X,I4,08SERVATIONS 1 ,//1X00002630
WRITE(6,26) (SYM(I),A(I,1),H(I),KAPA(I),KAPR(I),KPRM(I),1SS(I),1=1,N2) 00002680
3 FORMAT(4X,I2,3X,A4,2X,F8.4,F7.3 1 F8.2 1 FB.1,2X,F8.2 1 2X,F7.2) 00002700
26 FORMAT(/,1X,'SERIES MSL ',f6.2,41X,F6.2)
C PLOT RESULTS
C
WRITE(6,35) XIDEN,XLAT,XLOM,LTM,IMR,MS,IDS,T5,NHH
35 FORMAT('OBSERVED HOUVY HEIGHTS(X) AND PREDICTED H00002280
WRITE(6,36) (XLOM,TS,MS,IDS,IYR)
36 FORMAT('RESIDUAL(X)= OBSERVED MINUS PREDICTED HOU00002280
WRITE(6,37) (HH(I),HH(I)-PHH(I),1=1,NHH)
37 FORMAT(1X,'1Y HEIGHTS VERSUS TIME',/1X,'29-DAY SERIES STARTING AT •,F4+lO0002280
CALL PLOT(NHH,HH,PHH,T,MIN,MAX,MIN,MAX,MIN,TINCR)
38 FORMAT('1Y HEIGHTS VERSUS TIME',/1X,'29-DAY SERIES STARTING AT •,F4+l00002280
HH(1)=HH(I)-PHH(I)
CALL PLOT(NHH,HH,PHH,T,MIN,MAX,MIN,MAX,MIN,TINCR)
39 FORMAT(1X,'1Y HEIGHTS VERSUS TIME',/1X,'29-DAY SERIES STARTING AT •,F4+l00002280
HH(1)=HH(I)-PHH(I)
CALL PLOT(NHH,HH,PHH,T,MIN,MAX,MIN,MAX,MIN,TINCR)
STOP
END
SUBROUTINE ORREL(YR, JS, JM, ZS, ZM, NOS, A, VOU, F)
C ------------------------------
C SUBROUTINE COMPUTES GREENWICH EQUILIBRIUM PHASES AND NODAL FACTORS
C FOR ANY OR ALL OF 37 TIDAL CONSTITUENTS USED IN STANDARD HARMONIC
C ANALYSIS AND TIDAL PREDICTION. COMPUTATIONS ARE BASED ON ORBITAL ELEMENT
C FORMULAE BY SCHUREMAN(1958) FOR EITHER HARMONIC ANALYSIS OR TIDAL PREDICTIONS
C
1. INPUT VARIABLES
C
YR- YEAR OF OBSERVATIONS/PREDICTIONS
JS- JULIAN DAY BEGINNING THE MONTH IN WHICH SERIES STARTS
JM- JULIAN DAY CONTAINING MIDPOINT OF SERIES
ZS- GREENWICH MEAN (ZULU) TIME AT START OF SERIES
ZM- GREENWICH MEAN (ZULU) TIME AT MIDPOINT OF SERIES
NOS(I)- NATIONAL OCEAN SURVEY REFERENCE NUMBER FOR ITH CONSTITUENT
A(I,J)- ORBITAL ELEMENT INDICES (J=2,9) FOR ITH CONSTITUENT

2. OUTPUT VARIABLES
C
VOU(I)- GREENWICH EQUILIBRIUM PHASE FOR ITH CONSTITUENT
F(I)- NODAL FACTOR FOR ITH CONSTITUENT

DIMENSION VOU(37), F(37), A(37,9), NOS(37)

DEGAD=97.29577951
LYC=(YR-1901)*.25
S=263.849224+129.38482*(YR-1900)+13.176397*(JS+LYC)+.549016*ZS

IDUMP=S/360
S=S-IDUMP*360
P=336.72317+40.662466*(YR-1900)+.111404*(JS+LYC)+.0046418*ZS

IDUMP=P/360
P=P-IDUMP*360
H=79.203854-.236725*(YR-1900)+.985647*(JS+LYC)+.041069*ZS
P1=281.220810+.017179*(YR-1900)+.0000071*(JS+LYC)+.000002*ZS
PM=334.272321+.6662466*(YR-1900)+.111404*(JM+LYC)+.0046418*ZM

IDUMP=PM/360
PM=PM-IDUMP*360
PM=PM/DEGAD

XN=259.209010-19.328186*(YR-1900)-.052954*(JM+LYC)-.0022064*ZM

XN=XN/DEGAD
XN2=2.0*XN

ARG=913694-.0356926*COS(XN)
QT=ARCCOS(ARG)
Q12=2.0*QT

ARG=987056*SIN(XN)/SIN(Q1)

XNU=ARCTAN(ARG)

ARG=206727*SIN(XN)+(1. - .0194926*COS(XN))

ARG2=9979852+206727*COS(XN)-.0020418*COS(XN2)

XI=ATAN2(ARG, ARG2)

ARG=ATAN2(XNI, XNU2)

ARG2=COS(XNI)+.334766/SIN(Q12)

XNU=ATAN2(ARG, ARG2)*DEGAD

ARG=ATAN2(XN, XNU2)+.726184*(SIN(Q1)*SIN(Q1))

XNDP=ATAN2(ARG, ARG2)*DEGAD

PP=PM-X1

X1=X1*DEGAD

ARG=ARCTAN(PP)

ARG3=0.5*Q1
ARG2=(COS(ARG3)*COS(ARG3)/(6.*SIN(ARG3)*SIN(ARG3)))-COS(PP2)
R=ATAN2(ARG,ARG2)*DEGRAD
UL2=2.0*(XI-XNU)-R
ARG=SIN(PP)*((1.0+COS(OI)-1.0))
ARG2=COS(PP)*(7.0*COS(OI)+1.0)
Q=ATAN2(ARG,ARG2)*DEGRAD
UM1=(XI-XNU)+Q
DO 1 J=1,37
 1 CONTINUE
VOUT(1)=A(J,2)+A(J,3)*H+A(J,4)*P+A(J,5)*P1+A(J,6)*90.+A(J,7)*XI*A
1(J,8)*XNU*A(J,3)*XNU
TF=VOUT(1)/360
VOUT1=VOUT(1)-TDUMP*360
TF(VOUT(1))=1.0,1,1
DO 3 VOUT(1)=VOUT(1)+360.
3 CONTINUE
VOUT(18)=VOUT(18)+UM1
VOUT(33)=VOUT(33)+UL2
VOUT(35)=VOUT(35)-XNOP
XNU=XNU/DEGRAD
PP2=COS(PP2)
XNU2=COS(XNU2)
SQ=SIN(ARG3)*SIN(ARG3)
SOC=COS(ARG3)*COS(ARG3)
QT1=1.0*SQ*PP2/SOC
QT2=3.0*SQ/SOC
SQ1=SQ*SQ1
SQ2=SQ1*SQ1
SQ3=SIN(OI)
F(I)=1.000
2 CONTINUE
F(1)=SOC*SOC+.91544
F(3)=F(1)
F(4)=SORT(.8965*SQ*SQ3+.6001*SQ3*XNU+.1006)
F(5)=F(1)*F(1)
F(6)=SQ*SQC-.37988
F(7)=F(1)*F(1)*F(1)
F(8)=F(1)*F(1)*F(1)
F(9)=F(1)
F(10)=F(5)
F(11)=F(1)
F(12)=F(1)
F(13)=F(1)
F(14)=F(1)
F(15)=SQ1*SQ3+.016358
F(16)=F(1)
F(17)=SORT(2.31+1.435*PP2)*F(6)
F(19)=SQ3/.72137
F(20)=1.327757-1.991635*SQ2
F(23)=F(1)
F(24)=SQ2/.1578
F(25)=F(6)
F(26)=F(6)
F(27)=F(6)
F(28)=F(6)
F(29)=F(6)
F(31)=F(1)
F(32)=SQC*SQC*SQC/.8758
F(33)=SORT(1.-QT1+QT2)*F(1)
F(34)=F(1)*F(1)*F(4)
F(35)=SORT(.2044*SQ2+2.*7702*SQ2*XNU2+.0981)
F(36)=F(5)*F(5)
2 CONTINUE
VOUT(I)=VOUT(I)+360.
DO 2 I=1,37
VOUT(I)=VOUT(I)+360.
RETURN
END
SUBROUTINE PLOT(L,ORD1,ORD2,AB,FMIN1,FMAX1,FMIN2,FMAX2,AMIN,XMIN,XINC)
C SUBROUTINE PLOTS ON PRINTER ONE OR TWO ARRAYS OF Y VALUES (ACROSS PAGE)
C AGAINST ONE ARRAY OF X VALUES (DOWN PAGE)
C USAGES CALL PLOT(N,Y1,Y2,X,Y1MIN,Y1MAX,Y2MIN,Y2MAX,XMIN,XINC)
C DESCRIPTION OF PARAMETERS
C N - NUMBER OF POINTS TO BE PLOTTED
C Y1 - ARRAY CONTAINING FIRST SET OF Y VALUES (LENGTH N)
C Y2 - ARRAY CONTAINING SECOND SET OF Y VALUES (LENGTH N)
C X - ARRAY CONTAINING SET OF X VALUES (LENGTH N)
C Y1MIN - MINIMUM VALUE CHOSEN FOR Y1 SCALE
C Y1MAX - MAXIMUM VALUE CHOSEN FOR Y1 SCALE
C Y2MIN - MINIMUM VALUE CHOSEN FOR Y2 SCALE
C Y2MAX - MAXIMUM VALUE CHOSEN FOR Y2 SCALE
C XMIN - MINIMUM VALUE OF THE X ARRAY
C XINC - INCREMENT FOR EACH VALUE OF X
C PLOT SYMBOLS Y1- X Y2- + COINCIDENT POINTS- *
C NOTE: Y SCALE RESOLUTION IS 100 COLUMNS FULL SCALE
C IF ONLY ONE Y ARRAY IS TO BE PLOTTED THEN THE SAME ARRAY NAME
C MUST BE USED FOR BOTH Y1 AND Y2
C
DIMENSION ORD1(1),ORD2(1),AB(1),PLT(105),YCALE(11)
DATA PLT/105*'/ISYM/' /ISYMD'/'X'/'ISYME'/'ISYMY/**'/
DATA ' ' /ISYM/' /ISYME'/'ISYMY/**'/
LA=6
YNCR1=(FMAX1-FMIN1)/10.
YNCR2=(FMAX2-FMIN2)/10.
FMINA=AMIN
YNCR=AINC*10.
5 LATCH=1
LINE=1
K=2
1 IF(YNCR2-YNCR1)<105,104,105
104 IF(FMIN2-AMIN)<105,103,135
103 K=1
105 M=ISYMX
YNCR=YNCR1
FMINA=FMIN1
NO 110 J=1,
YCALF(J)=FMINA
DO 17 I=1,11
17 YCALF(I)=FMINA+YNCR*(I-1)+5.0E-5
WRITE(LA,14) M,YCALE
14 FORMAT('0.2X,'/'ISYMY */X',13X,'**',21('....**'))
M=ISYMY
YNCR=YNCR2
110 FMINA=FMIN2
FINC1=YNCR1/10.
FINC2=YNCR2/10.
FINL=YNCR/10.
FI=FMINA
ICN=ISYMX
IZERD1=(0-FMIN1)/FINC1+1.5
JZER02=(0-FMIN2)/FINC2+1.5
DO 7 J=1,L
IPLT(JZER02)=ISYM
IPLT(JZER02)=ISYM
IP1=ORD1(J)-FMIN1)/FINC1+1.5
IPT2=(ORD2(J)-FMIN2)/FINC2+1.5
IF(IPT1-105)=50,64,51
61 IP1=1
IPLT(IP1)=ISYM
GO TO 65
60 IF(IP1=163,63,64
63 IP1=1
IPLT(IP1)=ISYM
GO TO 65
64 IPLT(IP1)=ISYM
65 IF(IP1-15)=43,44,41
41 IPT2=1
IPLT(IP2)=ISYM
GO TO 45
40 IF(IP2=43,43,44
43 IP2=1
IPLT(IP2)=ISYM
GO TO 45
44 IPLT(IP2)=ISYM
45 IF(IP2-IP1)=50,46,50
46 IF(IPLT(IP1)=ISYM)=49,50,49
49 IPLT(IP1)=ISYM
50 IPT3=AR(J)-FMINA)/FINL+1.5
77 IF(IP3=LATCH)=70,71,72
71 GO TO (8,9),LINE
8 IF(I=FI
ITEMFI=(ITEMFI)*24.
ITEMFI=ITEMFI+1
ITEMFI=ITEMFI+.006
FI=ITEMFI+ITEMFI*.01
WRITE(LA,10) FI, IPLT
10 FORMAT('F.I',1XF10.2,'E',10,5A1)
ICT=1
.IINF=2
GO TO 12
9 WRITE(LA,11) IPLT
11 FORMAT('F.I',11X,'E',10,5A1)
ICT=ICT+1
IF(ICT=6)=12,13,12
13 LINE=1
12 IPLT(IP1)=ISYMB
IPLT(IP2)=ISYMB
FI=FMINA+LATCH*FINL+.0E-6
LATCH=LATCH+1
GO TO 7
72 GO TO (73,74),LINE
73 WRITE(LA,75) FI
75 FORMAT('F.I',1XF10.2,'**')
ICT=1
LINE=2
GO TO 79
74 WRITE(LA,76)
76 FORMAT('F.I',11X,'**')
ICT=ICT+1
IF(ICT=5)=70,78,79
78 LINE=1
79 F1=FMINA+LATCH*FINL+5.0E-6
   LATCH=LATCH+1
   GO TO 77
70 IF(J-1)81,80,81
81 ICON=ISYMP
80 WRITE(LA,30) ICON,IPLT
30 FORMAT(A1,13X,105A1)
   IPLT(IPT1)=ISYMP
   IPLT(IPT2)=ISYMP
7 CONTINUE
   WRITE(LA,94)
94 FORMAT(*,13X,*'..*'),21(*...*))
   RETURN
   END
MASTER LIST OF CONSTITUENT INFORMATION - PROGRAM HAMELS

A. TIDAL CONSTITUENT CARDS

1  M2  28.9841042  -2  2  0  0  0  2  -2  0  2
2  S2  30.0000000  0  0  0  0  0  0  0  2
3  N2  28.4397796  -3  2  1  0  2  -2  0  2
4  K1  15.0410686  0  1  0  1  0  0  -1  1
5  M4  57.9682084  -4  4  0  0  0  4  -4  0  4
6  Q1  13.9430356  -2  1  0  0  1  2  -1  0  1
7  M6  86.9523176  -6  6  0  0  0  6  -6  0  6
8  S4  60.0000000  0  0  0  0  0  0  0  0  4
9  S6  90.0000000  0  0  0  0  0  0  0  0  6
10  M8  115.9364169  -8  8  0  0  0  8  -8  0  8
11  NU2  28.5125830  -3  4  -1  0  2  -2  0  2
12  MU2  27.9682084  -4  4  0  0  0  2  -2  0  2
14  2N2  27.8935484  -4  2  2  0  0  2  -2  0  2
15  D1  16.1391017  2  1  0  1  -2  -1  0  1
16  LAM2  29.4556254  -1  0  1  0  2  -2  0  2
18  M1  14.4966939  -1  1  0  0  1  0  0  0  1
19  J1  15.5854433  1  1  -1  0  1  0  -1  0  1
25  RHO1  13.4715145  -3  3  -1  0  1  2  -1  0  1
26  Q1  13.3986609  -3  1  1  0  -1  2  -1  0  1
27  T2  29.9589333  0  -1  0  1  0  0  0  0  2
28  R2  30.0410667  0  1  0  -1  2  0  0  0  2
29  RQ1  12.8542862  -4  1  2  0  -1  2  -1  0  1
30  P1  14.9589314  0  1  0  0  -1  0  0  0  1
33  L2  29.5284789  -1  2  -1  0  2  0  0  0  2
35  K2  30.0821373  0  2  0  0  0  0  0  0  2
38  MK3  44.0251728  -2  3  0  0  1  2  -2  -1  3
40  MN4  57.4238338  -5  5  4  1  0  4  -4  0  4
47  S1  15.0000000  0  0  0  0  2  0  0  0  1
50  MM  0.0443747  1  0  -1  0  0  0  0  0  0
51  SSA  0.0821373  0  2  0  0  0  0  0  0  0
52  SA  0.0410666  0  1  0  0  0  0  0  0  0
53  MSF  1.0158958  2  -2  0  0  0  0  0  0  0
54  MF  1.0980331  2  0  0  0  0  0  0  0  0
58  2SM2  31.0158958  2  -2  0  0  0  2  -2  0  2
59  M3  43.4761563  -3  3  0  2  3  -3  0  3
62  2MK3  42.9271398  -4  3  0  0  -1  4  -4  1  3
67  MS4  58.9841042  -2  2  0  0  0  2  -2  0  4
### B. INFEERENCE FORMULA CARDS

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LEAST SQUARES INVERSE MATRICES - 29 DAY SERIES

(697 OBSERVATIONS OF HOURLY HEIGHT OF TIDE)

SINV (x1000)

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SAMPLE OUTPUT - PROGRAM HAMELS

HAMELS- HARMONIC ANALYSIS METHOD OF LEAST SQUARES

STATION HRVA 3656.8 7619.9 75W YEAR 1970
29 DAY SERIES STARTING 12-1 0.0 HRS 697 OBSERVATIONS

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SERIES MSL 5.37

73.68
ORIGINATED HOURLY HEIGHTS (X) AND PREDICTED HOURLY HEIGHTS (+) VERSUS TIME
29-DAY SERIES STARTING AT 0.00HRS 12-1 1970

(X) -5.00 -4.00 -3.00 -2.00 -1.00 0.00 1.00 2.00 3.00 4.00 5.00
HRVA
RESIDUAL(X) = OBSERVED MINUS PREDICTED HOURLY HEIGHTS VERSUS TIME
72-DAY SERIES STARTING AT 0.0 HRS 12-1 1970
APPENDIX B

Program ASTRO
Instructions for using Program ASTRO

I. Preliminary Input

As in Program HAMELS, a master list of constituent information is initially required as input to the program. The list includes 37 tidal constituents normally used by the National Ocean Survey in official U.S. tide table publications. The list is presented at the end of this appendix.

II. Station ID Card, Tidal Datum Card

The station ID card identifies the tide station, its location, and the local time meridian used. The tidal datum card names the datum used (for labeling purposes), fixes its position relative to mean sea level, and gives the number of tidal constituents to be used in the predictions.

III. Tidal Constants Cards

Each tidal constituent to be used in the predictions must be represented by a tidal constants card. These may include any combination of the 37 constituents referred to in the master list. Cards containing the sets of tidal constants (amplitude and phase) may be entered in any order as long as the correct NOS reference number appears on each card.

IV. Station Control Card

This card specifies the year and the month series in which predictions are wanted as well as their type (hourly heights or times and heights of highs and lows). Various combinations may
be achieved for a given station by entering one or more control cards, followed by a blank card to terminate the program.
**Program Author**

Astronomical Tide Prediction Program for

Computation of Hourly Heights and/or Times and Heights of High and Low Tides at Stations Having Known Tidal Constants

**Input Variable List**

- XTUEN - Station Identifier Code (A4)
- XLAT, XLON - Station Latitude and Longitude (2F7.1)
- LTM - Longitude of Time Meridian Used at Station (13)
- IYR - Year of Required Predictions (14)
- MS, ME - Starting Month, Ending Month (213)
- ITYPE - Selects Hourly Heights (01) or Highs and Lows (02) (13)
- DATNM - Name of Model Datum (14)
- HO - Height of Local Mean Sea Level Above Model Datum (F5.3)
- NTC - Number of Tidal Constituents Used (13)
- NOS - National Ocean Survey Reference No. for ith Constituent
- A(I,I) - Speed of ith Tidal Constituent in Deg/MSH (F8.4)
- A(I,J) - Orbital Element Indices for ith Constituent (8F3.1)
- H(I) - Mean Amplitude of ith Tidal Constituent (F5.3)
- XKP(I) - Phase of ith Constituent Adjusted for Station Location (F5.3)

**Called Variables**

- F(I) - Nodal Factor Reducing H to Year of Prediction
- VOL(I) - Greenwich Equilibrium Phase for ith Constituent

**Input Sequence and Format**

1. Station ID Card - XTUEN, XLAT, XLON, LTM (1X, A4, 2F7.1, 13)
2. Tidal Datum Card - DATNM, HO, NTC (1X, A4, 1X, F5.3, 13)
3. Tidal Constants Card - ITYPE, XKP(I) (1X, F5.3, 1X, F5.1)
4. Station Control Cards - IYR, ME, ITYPE (1X, I4, 2X, I3)
5. Additional Station Control Cards if Desired
6. Blank Card to Terminate Job

Subroutines Required

- DOP, CONSUM

- DIMENSION F(37), H(37), VOL(37), XKP(37), HO(366, 2), XLW(366, 2)
- DIMENSION JS(12), JE(12), ND(367), THW(366, 2), HW(366, 2), TLW(366, 2)
- DIMENSION A(37, 9), DMTL(366), DMN(366), NOS(37), KNOS(37)

- READ MASTER LIST OF CONSTITUENT SPEEDS, ORBITAL ELEMENT INDICES

- 44 FORMAT(12, 6X, F8.4, 4X, F8.3, 1)

- READ STATION ID CARD
- READ TIDAL DATUM CARD
- READ TIDAL CONSTANTS CARDS

- 1 FORMAT(1X, A4, 2F7.1, 1X, I2)
- 3 FORMAT(1X, A4, 1X, F5.3, I3)
- 4 FORMAT(12, 2X, F5.3, 2X, F5.1)

- READ STATION CONTROL CARD

- 200 READ(5, 7) IYR, MS, ME, ITYPE
- 2 FORMAT(1X, I4, 2X, I3)
IF(YR.EQ.0)GO TO 101

C COMPUTE JULIAN DATES STARTING AND ENDING EACH MONTH
C ADJUST FOR LEAP YEAR, NON-LEAP YEAR

DEGRAD=57.29577951
ZIP=0.1
LY=1
XLEAP=(2000.-YR)/4.-LEAP
IF(XLEAP.GT.ZIP)LY=0
JS(1)=1
JF(1)=31
JS(2)=32
JF(2)=31+LY
K=1
DO 5 I=3,12
JF(I)=JF(I-1)+1
JS(I)=JF(I)+29+K
K=K-1
IF(JF(I).EQ.7)K=1
5 CONTINUE
ZM=12.*(LY+1)
CALL ORBEL(YR,1,183,0.0,LM,NOS,A,VOU,F)

C COMPUTE HOURLY HEIGHTS

6 DO 8 K=MS,ME
J1=JS(K)
J2=JF(K)
ND(J1)=I
8 ND(I+1)=ND(I)+1
DO 9 J=1,24
9 HH(I,J)=HO
10 CONTINUE
11 CONTINUE
9 CONTINUE

C PRINT HOURLY HEIGHTS

WRITE(6,12) XIDEN,XLAT,XLON,LTM,YR,K,NTC
12 FORMAT('EXPECTED HOURLY HEIGHT OF TIDE IN FEET ABOVE ','A4','
1,'A4',' YEAR ','A4','MONTH','
1,'A4','NO. CONST. ','A12')
WRITE(6,13) DATNM
13 FORMAT('PREDICTED HOURLY HEIGHT OF TIDE IN FEET ABOVE ','A4','
1,DATUM')
WRITE(6,94) DATNM,HO
94 FORMAT('PREDICTED HOURLY HEIGHT OF TIDE IN FEET BELOW MSL')
COMPUTE TIMES AND HEIGHTS OF HIGHS AND LOWS

    7 DO 15 K=Ms, M+1
    15    J1=JS(K)
    16 J2=JE(K)
    17 ND(J1)=1
    18 DO 16 I=J1, J2
    19    THW(I, 2)=1000000.
    20    TLW(I, 2)=1000000.
    21    HW(I, 1)=1000000.
    22    XlW(I, 2)=1000000.
    23 N=1
    24 M=1
    25 NSkip=1
    26 ND(I+1)=ND(I)+1
    27 XJT=(I-1)*24
    28 CALL CONSUM(Knos, XJT, A, Vou, XKP, F, HD, H, NTC, SUM, DSUM)
    29 SUMO=SUM
    30 DSUM=DSUM
    31 IF(DSUM=DSUM)120, 17, 20
    32 17 XJT=XJT+1.
    33 CALL CONSUM(Knos, XJT, A, Vou, XKP, F, HD, H, NTC, SUM, DSUM)
    34 IF(DSUM=DSUM)19, 20, 18
    35 18 THW(I, N)=0.0
    36 HW(I, N)=SUMO
    37 NSUMO=-1.
    38 N=N+1
    39 M=M+1
    40 NSkip=5
    41 G0 77 20
    42 19 TLW(I, M)=0.0
    43 XlW(I, M)=SUMO
    44 DSUMO=1.
    45 M=M+1
    46 NSkip=5
    47 G0 77 21
    48 20 NQ=0
    49 DO 25 J=1, 24
    50 NSkip=NSkip-1
    51 IF(NSkip.GT.0) GO TO 25
    52 XJT=((I-1)*24)+J
    53 CALL CONSUM(Knos, XJT, A, Vou, XKP, F, HD, H, NTC, SUM, DSUM)
    54 SAVE=DSUMO
    55 IF(DSUM=DSUMO)23, 22, 28
    56 22 IF(J.EQ.24) GO TO 25
    57 IF(SAVE=DSUMO)27, 26, 26
    58 26 THW(I, N)=J
    59 HW(I, N)=SUM
    60 DSUMO=-1.
    61 N=N+1
    62 GO TO 21
    63 27 TLW(I, M)=J
    64 XlW(I, M)=SUM
    65 DSUMO=1.
M=M+1
GO TO 21
QO=Q0+1
XJ=(J-1)+0.1*Q0
XT=(J-1)+24)+XJ
CALL CONSUM(KNOS,XJT,A,VOU,KXP,F,HO,H,NTC,SUM,DSUM)
IF(DSUM*DSUM)-ABS(DSUM)
IF(DSUM*DSUM)30,30,28
30 DFF=ABS(DSUM)-ABS(DSUM)
IF(DFF)31,29,29
29 SUM=SUMO
XJ=XJ-0.1
31 IF(SAVE-DSUM)33,32,32
32 THW((N)=XJ
33 SUM=SUMO
XJ=SUM
34 DSUM0=-1.
35 M=M+1
21 NQ=O
NSKP=5
GO TO 25
25 CONTINUE
C---------------------------------------------
C PRINT TIMES AND HEIGHTS OF HIGHS AND LOWS
C---------------------------------------------
55 WRITE(6,12) XIDFN,XLAT,XLON,LTM,IYR,K,NTC
12 FORMAT(6,12) XIDFN,XLAT,XLON,LTM,IYR,K,NTC
34 FORMAT(/1X,*PREDICTED TIMES AND HEIGHTS OF HIGH AND LOW TIDE*/1X
1,*TIME IN HRS*,5X,*HEIGHT IN FEET ABOVE *,A4,* DATUM*)
147 IF(HO.EQ.O)GO TO 336
148 WRITE(6,337) DATNM,HO
337 FORMAT(/1X,A4,* DATUM IS *,F5.3,* FEET BELOW MSL*)
338 WRITE(6,338)
339 FORMAT(/1X,*DAY*,3X,*THW*,3X,*HW*,2X,*TLW*,3X,*LW (AM)*,2X,*THW*,3
1X,*HW*,2X,*TLW*,3X,*LW (PM)*)
35 WRITE(6,350)(DAY,(THW(I,J),HW(I,J),TLW(I,J),XW(1,J),J=1,2),I=1,J=1)
35 FORMAT(3I12,1X,4(1X,F4.1),5X,4(1X,F4.1)/)
15 CONTINUE
101 STOP
E"
SUBROUTINE CONSUS(KNOS, XJT, A, VOU, XKP, F, HD, H, NTC, SUM, DSUM)

DIMENSION A(37,9), VOU(37), F(37), H(37), XKP(37), KNOS(37)

DEGRAD = 57.29577951

SUM = HO

DSUM = 0.0

DO 1 J = 1, NTC

1 I = KNOS(J)

ARG = (A(I, I) * XJT + VOU(I) - XKP(J)) / DEGRAD

SUM = SUM + F(I) * H(J) * COS(ARG)

DSUM = DSUM - A(I, I) * F(I) * H(J) * SIN(ARG)

1 CONTINUE

RETURN

END
MASTERY LIST OF CONSTITUENT INFORMATION - PROGRAM ASTRO

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<th>No.</th>
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SAMPLE OUTPUT - PROGRAM ASTRO

HRVA 3656.8 7619.9 T.M. 75W
YEAR 1970 MONTH 12 NO. CONST. 25

PREDICTED HOURLY HEIGHT OF TIDE IN FEET ABOVE MSL DATUM

| D/HR | 0   | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  | 21  | 22  | 23  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|      | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | 2.7 | 2.8 | 2.9 | 3.0 | 3.1 |
APPENDIX C
Program LESCO
Instructions for using Program LESCO

I. Purpose

Program LESCO is used to compute the inverse of the least squares coefficient matrices required by Program HAMELS. The inverse matrices may be computed for an observational series of virtually any length but the program is limited to a maximum of ten tidal constituents (two 10 X 10 matrices) in present form. If more than ten constituents are needed, both the main program and subroutines INVER and MAPROD must be modified accordingly.

II. Input Required

The required information must be entered on cards in the following order:

A. Data information card (I3, I4, I3)
   M - number of tidal constituents (I3)
   N - total number of observations (I4)
   NPD - number of observations per day (I3)

B. Constituent Cards (A3, 2X, F13.10)
   C - constituent symbol (A3)
   W - constituent speed (F13.10)

Note: The order of the constituent cards determines the order in which the constituents must appear in Program HAMELS.

III. Output

A listing of the cosine summation (S) and sine summation (D) matrices are printed along with their respective inverse matrices and a set of verification (identity) matrices obtained as the products S X SINV and D X DINV.
The inverse matrices are multiplied by a factor of 1000 to eliminate extraneous leading zeroes and allow retention of significant figures in the lesser elements of the array output. The remainder of the output consists of a deck of punched cards containing

\[
\begin{align*}
SINV \times 1000 & \quad (10F8.6) \\
DINV \times 1000 & \quad (10F8.6)
\end{align*}
\]
PROGRAM LESCO - GENERATES COEFFICIENT MATRICES AND COMPUTES THE INVERSE MATRICES FOR THE LEAST SQUARES METHOD OF HARMONIC ANA"LYSIS. THE INVERSE MATRICES ARE MULTIPLIED BY 1000 DUE TO THE SMALL VALUES OF THE NON-PIVOT ELEMENTS.

INPUT REQUIRED

1. MASTER LIST OF CONSTITUENT AND INPUT DATA INFORMATION
   M = ORDER OF COEFFICIENT MATRIX (MAXIMUM = 10)
   N = NUMBER OF OBSERVATIONS IN TIDAL SERIES (N MUST BE ODD)
   C(I) = CONSTITUENT SYMBOL FOR ITH CONSTITUENT
   W(I) = ITH CONSTITUENT SPEED (OEG/MSH)

2. SUBROUTINES REQUIRED - INVERS, MAPR

-----------------------------------------------------------------------
DOUBLE PRECISION W, S, D, SINV, DINV, SSINV, PDINV, MS, WSN, SFS, WD, WDN, SFD
DIMENSION W(10), C(10), S(10,10), D(10,10), SINV(10,10)
DATA IR/5/, IW/6/, IP/7/
-----------------------------------------------------------------------
READ MATRIX ORDER(M), SERIES LENGTH
READ M, N, NPD, CONSTITUENT SYMBOLS AND CONSTITUENT SPEEDS
-----------------------------------------------------------------------
READ(IP,1) M,N,NPD
MTEST=N/2
MTEST= N-MTEST#2
FORMAT(13,14,13)
WRITE(IW,102)
102 FORMAT(*1/,//IX,'IN EVEN, PROGRAM CANCELLED')
GO TO 100
FORMAT(13,14,13)
WRITE(1W,18)
18 FORMAT(*1/,//IX,MATRIX COEFFICIENTS - PROGRAM LESCO*,//IX,'RECORD LENGTH- ',13,'DAYS',//IX,'TOTAL NO. OBS.- ',14,'/IX,'NO. CONSTITUENTS- ',13,'/IX,'CONSTITUENT SPEEDS (OEG/MSH)*
WRITE(1W,19)
19 FORMAT(*1/,//IX,A4,2X,F8.4)
-----------------------------------------------------------------------
GENERATE COEFFICIENT MATRIX
-----------------------------------------------------------------------
DO 3 I=1,M
DO 3 J=1,M
IF (I-J) 45,45,43
FORMAT(13,14,13)
WRITE(1W,19)
3 FORMAT(*1/,//IX,A4,2X,F8.4)
GO TO 3

45 \( WS = (W(I) + W(J)) / 114.591559 \)

WSN = WS\#N

SFS = DSIN(WSN) / DSIN(45)

IF(I < J)5, 4, 5

4 \( S(I,J) = (WS + SFS) / 2. \)

GO TO 3

5 \( WD = (W(I) - W(J)) / 114.591559 \)

WDN = WD\#N

SFD = DSIN(WDN) / DSIN(WD)

S(I,J) = (SFD + SFS) / 2.

GO TO 3

CONTINUE

C WRITE MATRIX, INVERSE MATRIX, MATRIX PRODUCTS

C-----------------------------------------------------------------

0039 WRITF(IW,6)

0040 6 FORMAT(*1'E1X,'S1, /1X)

0041 DO 30 I=1,M

0042 30 WRITF(IW,7) (S(I,J), J=1,M)

0043 WRITF(IW,8)

0044 7 FORMAT(*,10F12.6)

0045 8 FORMAT(*1'E1X, /1X, D1, /1X)

0046 DO 82 I=1,M

0047 82 WRITF(IW,7) (D(I,J), J=1,M)

0048 CALL INVERSTM(5, SINV)

0049 IF (M.EQ.0) GO TO 100

0050 CALL INVERSTM(10, DINV)

0051 IF (M.EQ.0) GO TO 100

0052 WRITF(IW,9)

0053 9 FORMAT(*1'E1X, /1X, SINV X 1000*, /1X)

0054 DO 84 I=1,M

0055 84 WRITF(IW,7) (SINV(I,J), J=1,M)

0056 WRITF(IW,10)

0057 10 FORMAT(*1'E1X, /1X, DINV X 1000*, /1X)

0058 DO 86 I=1,M

0059 86 WRITF(IW,7) (DINV(I,J), J=1,M)

0060 DO 84 I=1,M

0061 84 WRITF(IW,66) (SINV(I,J), J=1,M)

0062 66 FORMAT(10F8.6)

0063 DO 70 J=1,M

0064 70 WRITF(IW,66) (DINV(I,J), J=1,M)

0065 WRITF(IW,20)

0066 20 FORMAT(*1'E1X, /1X, CHECK S*SINV*, /1X)

0067 CALL MAPR0D(M, S, SINV)

0068 WRITF(IW,21)

0069 21 FORMAT(*1'E1X, /1X, CHECK D*DINV*, /1X)

0070 CALL MAPR0D(M, D, DINV)

0071 100 STOP

0072 END
SUBROUTINE INVERS(M,F,A)

C SUBROUTINE INVERS - COMPUTES THE INVERSE OF THE M X M MATRIX IN
C PLACE. THE METHOD IS GAUSS-JORDAN ELIMINATION USING MAXIMUM PIVOT
C STRATEGY. IF THE PIVOT IS LESS THAN .0000001, THE PROGRAM IS
C TERMINATED.
C
C 1. INPUT VARIABLES
C M - ORDER OF COEFFICIENT MATRIX
C F - COEFFICIENT MATRIX
C
C 2. OUTPUT VARIABLES
C A - INVERSE OF COEFFICIENT MATRIX
C
REAL*8 A,F,PIVOT
DIMENSION IROW(10),JCOL(10),JORD(10),Y(1),A(10,10),F(13,10)
DATA I5,W6/,IP/7/
EPS=.00000001
DO 2 I=1,M
DO 2 J=1,M
2 A(J,J)=F(I,J)
IF (M.LE.10) GO TO 6
M=0
RETURN
5 ON 18 K=1,M
KMI=K-1
PIVOT=0.
DO 11 I=1,M
DO 11 J=1,M
11 IF (I.EQ.I) GO TO 9
IF (J.EQ.J) GO TO 11
IF (ABS(A(I,J)).LE.EPS) GO TO 13
PIVOT=I
JORD(K)=J
CONTINUE
8 CONTINUE
IF (I .NE. J) GO TO 11
IF (ABS(A(I,J)).LE.EPS) GO TO 13
WRITE(W,205)
M=0
RETURN
660
6
13 IROW=IROW(K)
14 A(IROW,K)=A(IROW,K)/PIVOT
GO TO 14
DO 18 I=1,M
  AIJCK=A(I,JCOLK)
10  IF (I.EQ.IROWK) GO TO 16
  A(I,JCOLK)=AIJCK/PIVOT
20  DO 17 J=1,M
  17 IF (J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
  CONTINUE
  IROW=IROW(I)
  JCOL=JCOL(I)
20  JORD(IROW)=JCOL
28  DO 20 J=1,M
  30  Y(JCOL) = A(IROW,J)
  DO 28 I=1,M
  28 A(I,J)=Y(J)
  DO 30 J=1,M
  20 FORMAT(*0*,*N TOO BIG*)
  205 FORMAT(*0*,*MAGNITUDE OF PIVOT IS LESS THAN EPS*)
SUBROUTINE MAPROD(M,A,AINV)

C SUBROUTINE MAPROD - MULTIPLIES THE COEFFICIENT MATRIX BY ITS INVERSE
C MATRIX TO INSURE THE PRODUCT IS THE IDENTITY MATRIX.
C 1. INPUT VARIABLES
C M - ORDER OF COEFFICIENT MATRIX
C A - COEFFICIENT MATRIX
C AINV - INVERSE OF COEFFICIENT MATRIX
C 2. OUTPUT VARIABLES
C AAINV - PRODUCT OF A AND AINV (THIS SHOULD BE THE IDENTITY MATRIX)

DOUBLE PRECISION A,AINV,AAINV
DIMENSION A(10,10),AINV(10,10),AAINV(10,10)
DATA IR/5/,IW/6/,IP/7/
DO 10 I=1,M
   DO 10 J=1,M
      AAINV(I,J)=0.
   DO 10 K=1,M
      AAINV(I,J)=AAINV(I,J)+A(I,K)*AINV(K,J)/1000.
10 CONTINUE
DO 15 I=1,M
   WRITE(IW,11)(AAINV(I,J),J=1,M)
11 FORMAT(*0',10F12.7)
RETURN
END