

INFORMATION TO USERS

This reproduction was made from a copy of a document sent to us for microfilming. While the most advanced technology has been used to photograph and reproduce this document, the quality of the reproduction is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help clarify markings or notations which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure complete continuity.
2. When an image on the film is obliterated with a round black mark, it is an indication of either blurred copy because of movement during exposure, duplicate copy, or copyrighted materials that should not have been filmed. For blurred pages, a good image of the page can be found in the adjacent frame. If copyrighted materials were deleted, a target note will appear listing the pages in the adjacent frame.
3. When a map, drawing or chart, etc., is part of the material being photographed, a definite method of "sectioning" the material has been followed. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again--beginning below the first row and continuing on until complete.
4. For illustrations that cannot be satisfactorily reproduced by xerographic means, photographic prints can be purchased at additional cost and inserted into your xerographic copy. These prints are available upon request from the Dissertations Customer Services Department.
5. Some pages in any document may have indistinct print. In all cases the best available copy has been filmed.

**University
Microfilms
International**

300 N. Zeeb Road
Ann Arbor, MI 48106

8506713

Hunter, John William

DEFORMATION IN A CHIRAL BAG MODEL

The College of William and Mary in Virginia

PH.D. 1984

University
Microfilms
International 300 N. Zeeb Road, Ann Arbor, MI 48106



PLEASE NOTE:

In all cases this material has been filmed in the best possible way from the available copy. Problems encountered with this document have been identified here with a check mark ✓.

1. Glossy photographs or pages _____
2. Colored illustrations, paper or print _____
3. Photographs with dark background _____
4. Illustrations are poor copy _____
5. Pages with black marks, not original copy _____
6. Print shows through as there is text on both sides of page _____
7. Indistinct, broken or small print on several pages ✓ _____
8. Print exceeds margin requirements _____
9. Tightly bound copy with print lost in spine _____
10. Computer printout pages with indistinct print _____
11. Page(s) _____ lacking when material received, and not available from school or author.
12. Page(s) _____ seem to be missing in numbering only as text follows.
13. Two pages numbered _____. Text follows.
14. Curling and wrinkled pages _____
15. Dissertation contains pages with print at a slant, filmed as received _____
16. Other _____

University
Microfilms
International

DEFORMATION IN A CHIRAL BAG MODEL

A Dissertation

Presented to

The Faculty of the Department of Physics
The College of William and Mary in Virginia

In Partial Fulfillment

Of the Requirements for the Degree of
Doctor of Philosophy

by


John W. Hunter

1984

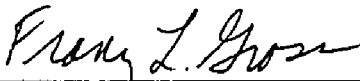
APPROVAL SHEET

This dissertation is submitted in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy


John W. Hunter


Approved, July 1984


Franz L. Gross


Carl E. Carlson


Morton Eckhause


Edward A. Remler


Hans C. von Baeyer



Carl M. Andersen

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	v
LIST OF TABLES	vi
LIST OF FIGURES	vii
ABSTRACT	viii
I. INTRODUCTION	2
II. LAGRANGIAN AND PERTURBATION HIERARCHY	16
III. SOLUTIONS FOR THE DEFORMED BAG	31
IV. NUMERICAL FITS	40
V. NLC, STABILITY, HIGHER ORDER DEFORMATION OF THE NUCLEON	43
VI. CALCULATION OF ϕ_3	47
VII. CALCULATION OF THE COUPLING CONSTANTS $g_{\pi NN}$, $g_{\pi \Delta \Delta}$	61
VIII. THE MAGNETIC MOMENTS	74
IX. APPROXIMATIONS AND SOME LOOSE ENDS IN THE BAG MODEL	89
APPENDIX A.	
TECHNIQUES FOR DEFORMED BAGS	97
APPENDIX B.	
PION AND GLUON EFFECTS	121
APPENDIX C.	
IDENTITIES AND THE SECOND-ORDER QUARK SOLUTIONS	136
APPENDIX D.	
MATRIX ELEMENTS OF SPIN AND ISOSPIN	142
APPENDIX E.	
CHIRAL INVARIANCE	146

APPENDIX F.

MATRIX ELEMENTS USED IN THE DERIVATION OF	$g_{\pi NN}, g_{\pi \Delta \Delta},$	
$\mu_{NN}, \mu_{\Delta \Delta}$	158

APPENDIX G.

THE CLASSICAL PION FIELDS	$\phi_{NN}, \phi_{ND}, \phi_{\Delta \Delta}$	173
---------------------------	--	-------	-----

APPENDIX H.

NUMERICAL AND ANALYTICAL INTEGRALS	177
------------------------------------	-------	-----

APPENDIX I.

VIOLATION OF ANGULAR MOMENTUM CONSERVATION	186
--	-------	-----

REFERENCES	187
------------	-------	-----

ACKNOWLEDGMENTS

I would like to thank my wife Heather and daughter Lindsey for the moral support they both gave me. They faithfully endured my preoccupation and long hours. My wife's family as well as my own have also been extremely steadfast.

I particularly want to thank my father R. O. Hunter and mother Lola Hunter without whose help I could not have completed my doctorate. Other benefactors include Jim Hunter, Robert Hunter, Jr., Duncan Hunter, Dan and Sue Clayton, Steve Blanks and Art Griesser.

People who collaborated with, or helped me regarding technical aspects of my dissertation include Franz Gross, Carl Carlson, Ed Remler, Fred Myhrer and especially Morty Eckhause.

LIST OF TABLES

	Page
I. Gluon and Pion Effects on Bag Shape	9
II. The Bag Constant and α_c	9
III. All Energy Contributions in the Bag	10
IV. Contributions to the π -nucleon Coupling Constants	65
V. Contributions to the $\pi - \Delta$ Coupling Constant	73
VI. The ratio $g_{\pi\Delta\Delta} / g_{\pi NN}$	73
VII. The Nucleon Magnetic Moments	86
VIII. The Δ Magnetic Moments	87

LIST OF FIGURES

	Page
1. The Δ Shape	7
2. Nucleon Energy as a Function of Radius ($f_{\pi} = 93.3 \text{ MeV}$) . . .	11
3. Nucleon Energy as a Function of Radius ($f_{\pi} = 114 \text{ MeV}$) . . .	11
4. Δ Energy as a Function of Radius ($f_{\pi} = 93.3 \text{ MeV}$)	12
5. Δ Energy as a Function of Radius ($f_{\pi} = 114 \text{ MeV}$)	12
6. "Feynman" diagrams for ϕ_3	60
7. "Feynman" diagrams for Magnetic Moments	88
8. Cloudy Bag Form Factor	90
9. Classical Color-electric and Magnetic Self Energies	94
10. MIT Bag Violation of Chiral Symmetry	150

ABSTRACT

A chiral bag model including the effects of quarks, gluons and pions is calculated perturbatively to second order. The bag shape is determined from the requirement that the pressure exerted by the fields be zero at the bag surface. To second order the nucleon is found to be spherical, while the Δ is oblate or prolate for spin projections $|m_z|=3/2$, $|m_z|=1/2$ respectively. The coupling constants $g_{\pi\Delta\Delta}$ and $g_{\pi NN}$ are calculated and the ratio $g_{\pi\Delta\Delta}/g_{\pi NN}$ is an improvement over previous work. The nucleon and Δ magnetic moments are also calculated.

DEFORMATION IN A CHIRAL BAG MODEL

I. INTRODUCTION AND SUMMARY

A. Deformation of the Bag

This dissertation presents a systematic treatment to second order of a classical chiral bag. A major feature is our inclusion of deformation effects generated by gluon and pion fields. A key to the calculation is the nonlinear boundary condition (henceforth NLC) which arises when one varies the Lagrangian with respect to the bag surface. As is shown in Chapter V, the NLC is merely the statement that the pressure at any point on the bag surface be zero. Previous bag papers have settled for an angle-averaged NLC which automatically implies a spherical surface. This compromises the bag physics and is unnecessary.

The original MIT bag phenomenology includes a volume energy in order to stabilize the bag. This is accomplished by introducing a constant B into that part of the Lagrangian density which describes the interior of the bag. The inward pressure generated by the constant B cancels the outward radial pressure due to the S -state quarks. Since B is a constant, a spherical bag can result only if the pressure is independent of angle, which fortunately is true for S -state quarks. As soon as one considers excited quark states or gluon or pion effects the pressure becomes angle dependent. This shows up in the NLC, which acquires angle-dependent terms, and one needs a mechanism to counter them locally. The pressure terms which can have an angular dependence

are of the following form for gluons and pions respectively.

$$g^2 \langle S.F. | \sum_{ij} \left[\frac{3}{2} b_i \cdot \hat{\lambda} b_j \cdot \hat{\lambda} - \frac{1}{2} b_i \cdot b_j \right] \tau_i \cdot \tau_j | S.F. \rangle \quad (1.1)$$

where S.F. refers to the SU(6) spin-flavor wave functions of either the nucleon or Δ . The indices i and j refer to the quarks in the bag. One may observe that the matrix elements in (1.1) have a spin 2, $\ell=2$, $J=0$ behavior. They are essentially the familiar S_{12} of nuclear physics. Eq. (1.1) doesn't involve any integration so we look to the spin behavior of the nucleon and Δ SU(6) wave functions for guidance. One sees that (1.1) must be zero for any spin 1/2 system. This includes the nucleon. The Δ , however, will not be zero since it is spin 3/2. In fact (1.1) then behaves like the Legendre polynomial $P_2(z)$. The bag surface must somehow conform so that the pressure is still locally zero. We anticipate a deformation of the same form as the pressure. (See Chapter III for intuitive discussion)

The remedy is to solve the bag equations for an oblate or prolate bag since it happens that the pressure is independent of the azimuthal angle. The deformed-bag solutions are worked out as perturbations around a spherical geometry in a simple power series in the eccentricity ϵ . The eccentricity is then found to be a direct

function of the strengths of the pion and gluon fields, as well as the mean bag radius. (We did not know until the end of the calculation that ϵ was actually small enough to justify a perturbation expansion.) Fortunately the spherical solutions for the pions and gluons are unchanged by the deformation in this second-order calculation since they are already of second order in the small quantities g and g_c , where $g=1/f_\pi$ and g_c = the strong coupling constant. Deformation effects on them would be of order $\epsilon^2 g^2$ and $\epsilon^2 g_c^2$, and are ignored. The only fields affected are the three original S-state quarks. These acquire, in addition to the usual $J=1/2$ structure, additional small $J=3/2$ and $J=5/2$ components which are calculated in Chapter III. (Appendix A develops the necessary machinery for a deformed bag.)

Once the fields have been determined, both the average bag radius and the eccentricity are fixed by the NLC. The average radius is fixed by the angle-independent part of the NLC (equivalent to minimizing the energy with respect to mean radius), while ϵ is fixed by requiring the angle-dependent part of the NLC to vanish.

In Chapter IIIB the shapes of the nucleon and Δ are obtained. Naturally to second order in the calculation the nucleon is spherical; ϵ_N^2 is identically zero. Aside from the reasoning given earlier one might have guessed this on the basis that quadrupole moments are prohibited for $J=1/2$ objects. However, it is not clear how the bag shape relates to its quadrupole moment. It may be possible that a higher-order calculation will show a deformed nucleon.

We would like to briefly discuss an alternative scheme which makes for a deformed nucleon. A possible mechanism for nucleon deformation at order g^2 is shown for the nucleon in the paper by Ma and Wambach.¹ This would involve configuration mixing, which entails assuming one of the original S-state quarks is somehow replaced by a D-state quark. The total wave function is then antisymmetrized appropriately. They minimize the total energy with respect to the deformation parameter and find that the pion pressure has induced a finite deformation of the nucleon. The simple reason for this is that the quark spins in the nucleon no longer add up to $1/2$. This is a non-perturbative approach and one must note that for a given radius the D-state energies will be higher than the S-state energies. We regard using excited states for the basis of a perturbation scheme as unnatural, and probably unrealistic for the nucleon. However, the configuration mixed states could be reasonable for describing the Roper resonance or the N^* particles. Indeed the use of them will automatically generate deformation even in $J=1/2$ systems, so perhaps the Roper is deformed. Ma and Wambach¹ also look for nucleon deformation without configuration mixing. There they find no deformation so we are in agreement. In the absence of configuration mixing they found the Δ to be strictly oblate. We believe this is only half the story since our Δ was both oblate and prolate depending on spin projections. In fact they overlooked calculating the shape of the Δ for $m_z = 1/2$. When they calculate this we believe they will reproduce our results qualitatively.

Our calculation includes gluons whose angular pressure tends to partly cancel the pions' angular pressure so that the bag is not radically deformed.

The Δ shape is shown in Fig. 1. A simple mnemonic is obtained if one thinks of a water balloon. If all the quarks spins line up $m_z = \pm 3/2$ and the water balloon is oblate due to rotation. If the spins are mostly in the x and y directions, then, $m_z = \pm 1/2$ and our water balloon is prolate. Of course this mnemonic should not be taken literally.

The reader can track backward from Eq. (3.17) to find all the deformation contributions. Table I presents the eccentricity of an $m_z = \pm 3/2$ Δ for several values of m_π and f_π (note that we use an oblate geometry for our calculations; however, prolate geometry corresponds to ϵ^2 negative; see Chapter IIIA and Appendix A). One finds that pion effects outweigh the gluon effects in determining the bag shape. Equation (3.17) shows that this strongly depends on the bag radius along with α_c . For example, if either α_c or R increased much, then the previously oblate (prolate) bag would be found to be prolate (oblate). Finally, note from Table I that the use of massive pions in the bag equations leads to significantly smaller pion pressure and consequently a smaller amount of deformation.

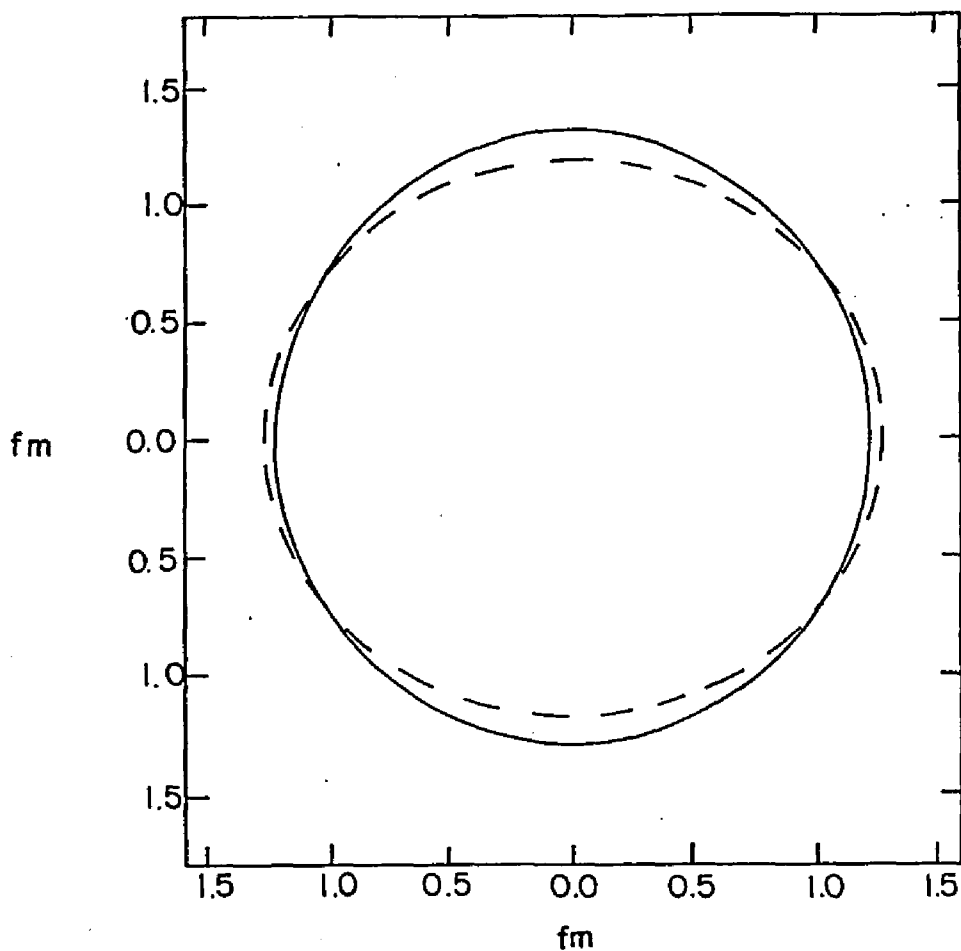


FIG. 1. The shape of the Δ bags. The solid curve represents the prolate $|m_z| = \frac{1}{2}$ case, while the dashed curve is oblate and $|m_z| = \frac{3}{2}$. A nominal value of $f_\pi = 93.3$ MeV and $m_\pi = 138$ MeV was used in obtaining these figures. The mean radius of both Δ 's is $R = 1.249$ fm.

B. Bag Energy and Stability

We have fitted the nucleon and Δ masses for the case of both massive and massless pion fields. This fixes the two free parameters B and α_c for four choices of f_π and m_π (see Table II). The separate contributions to the bag energy are summarized in Table III.

We note that the pion mass softens the pion pressure significantly and allows for a larger bag. This effectively increases the energy splitting due to gluon effects and is desirable. However, one can see from Figs. 2-4 that the pion mass does not change the dependence of the bag energy on R very much.

A problem basic to chiral bags is a $-R^{-3}$ [Eq. (4.2d)] energy dependence due to the pions. As the energy stands now, this is unreasonable because both nucleon and Δ bags would be highly metastable, as shown in Figs. 2-5. We had hoped that the increased energy due to deformation alone might overwhelm the negative R^{-3} pion-energy term. The argument was based on the observation that the deformation energy (Appendix A) is of order $+\epsilon^4/R$ where ϵ^2 is a linear function of g^2 , Eq. (3.17). Now g^2 has dimensions of $(\text{mass})^{-2}$ so the pion contribution to ϵ^4 must be proportional to $g^4 R^{-4}$ in order to be dimensionless. This means that the pion contribution to the deformation energy goes as R^{-5} , and is positive. However, even though the deformation energy would help the Δ in a fourth-order calculation, it does not aid the nucleon since $\epsilon_N=0$ identically.

TABLE I. Gluon and pion contributions to the bag eccentricity for two values of f_π , and both massive and massless pions. (Note that ϵ_Δ^2 simply changes sign for $|m_z| = \frac{1}{2}$ and the bag is prolate.)

$f_\pi = 1/g$ (MeV)	m_π (MeV)	$\epsilon_\Delta^2(m = 3/2)$		Total
		Gluon	Pion	
93.3	0	-0.054	0.242	0.188
	138	-0.062	0.197	0.135
114	0	-0.071	0.150	0.079
	138	-0.075	0.123	0.048

TABLE II. The bag constant and α_c obtained for two values of f_π , and both massive and massless pions.

f_π (MeV)	m_π (MeV)	B (MeV/fm ³)	α_c
93.3	0	38.60	1.593
	138	36.72	1.808
114	0	34.65	2.080
	138	33.75	2.192

TABLE III. All the energy contributions to the bag, along with the bag radius. We use both massive and massless pions as well as two values of f_π . Other parameters are given in Table II.

	f_π (MeV)	m_π (MeV)	R (fm)	E_Q (MeV)	E_V (MeV)	E_g (MeV)	E_π (MeV)	$E_{c.m.}$ (MeV)	E_B (MeV)
N	93.3	0	0.926	1304.0	128.4	-118.8	-212.6	-162.0	939.0
		138	0.999	1209.3	153.1	-125.0	-148.2	-150.2	939.0
	114	0	1.098	1099.9	192.0	-130.8	-85.5	-136.6	939.0
		138	1.115	1082.9	196.0	-135.8	-68.7	-135.4	939.0
Δ	93.3	0	1.227	983.9	298.9	89.7	-18.3	-122.2	1232.0
		138	1.249	966.6	299.8	100.0	-14.4	-120.0	1232.0
	114	0	1.278	945.2	302.6	112.4	-10.8	-117.4	1232.0
		138	1.289	936.4	303.0	117.4	-7.7	-117.1	1232.0

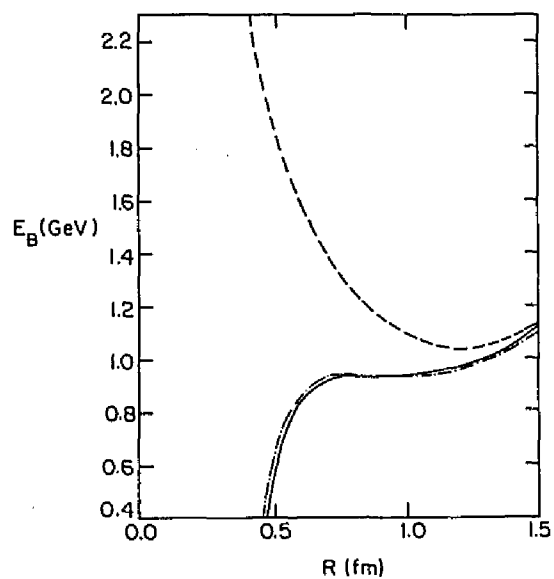


FIG. 2. Nucleon bag energy as a function of radius. Here $f_\pi=93.3$ MeV. The solid curve is for massless pions while the dash-dot curve is for massive pions. Also shown is a dashed curve which is the total energy minus the pion contribution.

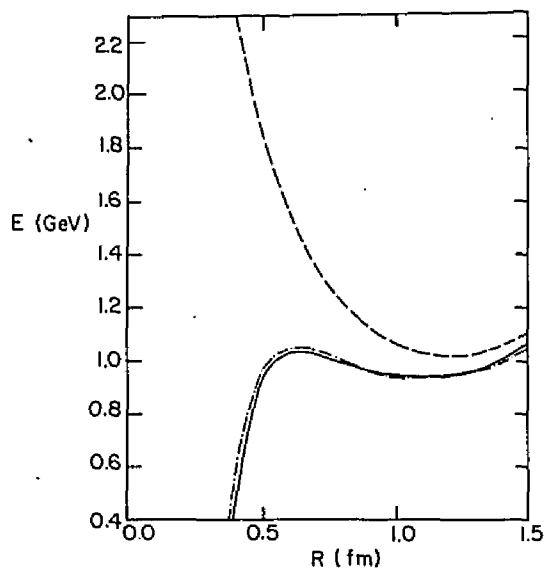


FIG. 3. Nucleon bag energy as a function of radius. Here $f_\pi=114$ MeV. For further explanation, see Fig. 2 caption.

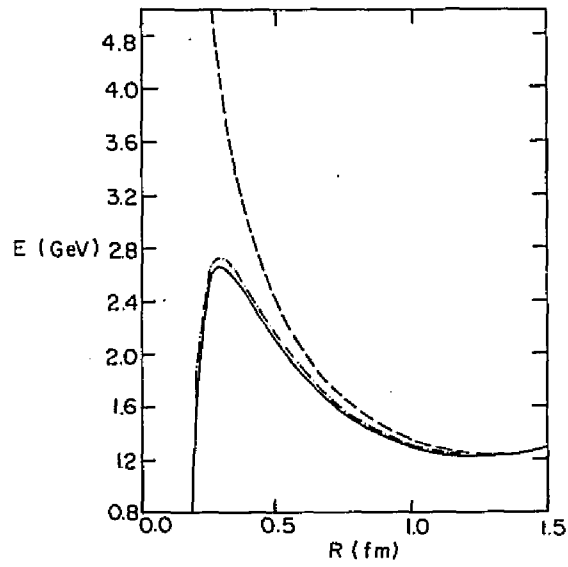


FIG. 4. Δ bag energy as a function of radius. Here $f_\pi = 93.3$ MeV. For further explanation, see Fig. 2 caption.

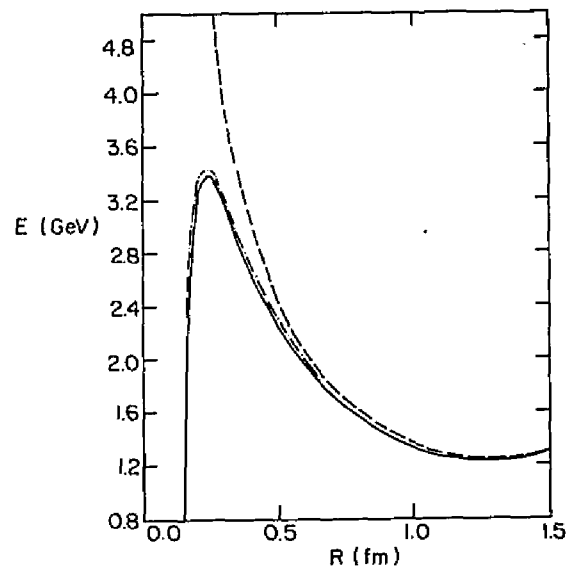


FIG. 5. Δ bag energy as a function of radius. Here $f_\pi = 114$ MeV. For further explanation, see Fig. 2 caption.

C. Coupling Constants and Magnetic Moments

A physical consequence of Δ deformation is reflected in the pion- Δ coupling constant $g_{\pi\Delta\Delta}$. The coupling constant is calculated in Chapter VII and shown to have two different values depending on the Δ spin-projection. This is a controversial result and is discussed in Appendix I. The ratio of pion-nucleon coupling to pion- Δ coupling is also calculated. It is an improvement over the results of other bag calculations⁸ we have seen which are off the experimental number by around 100%. The ratio we obtain is

$$\left. \frac{g_{\pi\Delta\Delta}}{g_{\pi NN}} \right|_{|m_z| = 3/2} = .741 \quad (1.2)$$

$$\left. \frac{g_{\pi\Delta\Delta}}{g_{\pi NN}} \right|_{|m_z| = 1/2} = .773 \quad (1.3)$$

The experimental ratio² is

$$\frac{g_{\pi\Delta\Delta}}{g_{\pi NN}} = .598 \pm .147 \quad (1.4)$$

Another physical quantity which however is not affected by deformation is the ratio of the proton magnetic moment to the neutron magnetic moment. This is calculated in Chapter VIII. We get

$$\frac{\mu_p}{\mu_n} = -1.439 \quad (1.5)$$

The experimental ratio is

$$\frac{\mu_p}{\mu_n} = -1.461 \quad (1.6)$$

The Δ magnetic moments are also calculated but we have no experimental comparison. The deformation effects on the Δ are less than 1%.

D. Organization of Dissertation

Chapter II is devoted to the derivation and details of a perturbation hierarchy for a chiral bag. Simple formulas for the bag energy are obtained there.

Chapter III summarizes the necessary technique for a non-spherical bag and calculates the deformation via the NLC.

Chapter IV describes the simple numerical fits which determine the mean radii along with α_c and B.

Chapter V relates the NLC to pressure and discusses nucleon deformation in higher order.

Chapters VI and VII calculate the pion field of $O(g^3)$, and picks off the various coupling constants.

Chapter VIII is a calculation of magnetic moments.

Chapter IX discusses classical versus quantum field theory in the bag model and explains the energies at which bag models might be appropriate.

The Appendices are mainly technical. However, Appendix E discusses chiral symmetry.

II. ENERGY AND EQUATIONS OF MOTION

A. Exact Equations from the Lagrangian

Inside the bag, the equations of motion are dictated by the original MIT Lagrangian. Outside the bag we use a Lagrangian whose chiral symmetry is broken by massive pions. The surface terms which couple the inner and outer Lagrangians are discussed in Ref. 3. Stability of the action under variation of the three fields results in three field equations and three so-called linear boundary equations. When a variation is taken with respect to the bag surface there emerges the well known but frequently ignored nonlinear boundary condition (NLC).

The Lagrangian is

$$\begin{aligned}
 \mathcal{L} = & \left[\frac{i}{2} \bar{\Psi} \not{\partial} \Psi - \frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} - g_c \bar{\Psi} \not{A}^a \frac{\lambda^a}{2} \Psi \right] \theta_V - B \theta_V \\
 & + \frac{1}{2} \left[\bar{\Psi} (1 + i g \underline{\tau} \cdot \underline{\phi} \gamma^5) \Psi (1 + g^2 \phi^2)^{-\frac{1}{2}} \right] \kappa^\mu \kappa_\mu \Delta_S \\
 & + \frac{1}{2} \left[(1 + g^2 \phi^2) D_\mu \underline{\phi} \cdot D^\mu \underline{\phi} - \frac{1}{4} g^2 (D_\mu \phi^2)^2 - m_\pi^2 \phi^2 \right] \theta_V
 \end{aligned} \tag{2.1}$$

where Ψ is the quark field, A_μ^a is the gluon vector potential with color a , $\underline{\phi}$ is the isovector pion field, and $\phi = |\underline{\phi}|$. Sums over quark isospin and colors are suppressed in (2.1), but are implied, and $\not{A} = A^\mu \gamma_\mu$. The generators of the SU(3) color group are λ^a , and of the SU(2) isospin

group \mathcal{G} . The symbol Θ_V is unity inside the bag and zero outside, Θ_S is unity outside and zero inside, and Δ_S is a δ function on the surface of the bag which has outward normal \hat{n} . For a spherical bag, $\hat{n}=(0,\hat{r})$. The two coupling constants are

$$g = \frac{1}{f_\pi} \tag{2.2}$$

$$g_c = \sqrt{4\pi \alpha_c}$$

where f_π is the pion decay constant with experimental value of 93.3 MeV, and α_c is the color fine-structure constant. The color field tensor is

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g_c f_{abc} A_b^\mu A_c^\nu \tag{2.3}$$

where f_{abc} are the SU(3) structure constants. Two "covariant" derivatives which will be employed are

$$D_{ab}^\mu = \delta_{ab} \partial^\mu - g_c f_{abc} A_c^\mu$$

$$D^\mu = (1 + g^2 \phi^2)^{-1} \partial^\mu \tag{2.4}$$

The action, which is stationary, is

$$S = \int d^4x \mathcal{L}$$

Varying this with respect to $\bar{\Psi}$, A_μ^a , and ϕ gives three equations

$$i \not{\partial} \Psi = g_c A^a \frac{\lambda^a}{2} \Psi \quad (2.5a)$$

$$D_{ab}^{\mu\nu} F_{\nu\mu}^b = -g_c \bar{\Psi} \gamma_\nu \frac{\lambda^a}{2} \Psi \quad (2.5b)$$

$$D_\mu D^\mu \phi + m_\pi^2 \phi = 0 \quad (2.5c)$$

(where the first two hold inside the bag and the last outside) and three boundary conditions on the surface of the bag

$$i \not{n} \Psi = (1 + g^2 \phi^2)^{-1/2} (1 + i g \underline{\alpha} \cdot \underline{\phi} \gamma^5) \Psi \quad (2.6a)$$

$$n_\nu F_a^{\nu\mu} = 0 \quad (2.6b)$$

$$n^\mu \partial_\mu \phi = - (1 + g^2 \phi^2) \bar{\Psi} \gamma^5 g \frac{\underline{\alpha}}{2} \Psi \quad (2.6c)$$

Note that the Dirac equation for Ψ has a source term of order g_c^2 since the color field A will be proportional to g_c . The source of the pion

field, boundary condition (2.6c), shows that it is of order g . If the bag is static, $i\cancel{\not{D}} = -i\vec{\gamma} \cdot \hat{n}$.

Finally, varying the action with respect to the surface of the bag gives the NLC. This is a local condition which must be satisfied at every point on the bag surface S :

$$\begin{aligned}
 \mathcal{B} = - \left\{ \frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} - \frac{i}{2} \bar{\Psi} \cancel{\not{D}} \Psi + g_c \bar{\Psi} A^a \frac{\lambda^a}{2} \right. \\
 \left. + \kappa^\mu \partial_\mu \frac{\bar{\Psi}}{2} (1 + i g \cancel{\not{\tau}} \cdot \cancel{\not{\phi}} \gamma^5) (1 + g^2 \phi^2)^{-1/2} \right. \\
 \left. + \frac{1}{2} \left[(1 + g^2 \phi^2) D_\mu \cancel{\not{\phi}} \cdot D^\mu \cancel{\not{\phi}} - \frac{g^2}{4} (D_\mu \phi^2)^2 - \mu_\pi^2 \phi^2 \right] \right\}
 \end{aligned}$$

This can be simplified with the help of Eq. (2.5a):

$$\begin{aligned}
 \mathcal{B} = - \left\{ \frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} \right. \\
 \left. + \kappa^\mu \partial_\mu \frac{\bar{\Psi}}{2} (1 + i g \cancel{\not{\tau}} \cdot \cancel{\not{\phi}} \gamma^5) \Psi (1 + g^2 \phi^2)^{-1/2} \right. \\
 \left. + \frac{1}{2} \left[(1 + g^2 \phi^2) D_\mu \cancel{\not{\phi}} \cdot D^\mu \cancel{\not{\phi}} - \frac{g^2}{4} (D_\mu \phi^2)^2 - \mu_\pi^2 \phi^2 \right] \right\} \quad (2.7)
 \end{aligned}$$

B. Perturbative Equations to Second Order

The next step is to establish a perturbation scheme to solve the three field equations with their respective linear boundary conditions and the NLC. Following Jaffe,⁴ Chodos and Thorn,³ and

others, we expand the fields and energy perturbatively assuming both g and g_c are small quantities of similar order:

$$\begin{aligned}
 \underline{\phi} &= g \underline{\phi}_1 + g^3 \underline{\phi}_3 + \dots \\
 \psi &= \psi_0 + g^2 \psi_{2\pi} + g_c^2 \psi_{2g} + \dots \\
 w &= w_0 + g^2 w_{2\pi} + g_c^2 w_{2g} + \dots \\
 A_a^\mu &= g_c A_{a1}^\mu + g_c^3 A_{a3}^\mu + \dots
 \end{aligned} \tag{2.8}$$

The equations to zeroth order in the coupling constants involve the quark fields only:

$$-i \vec{\alpha} \cdot \vec{\nabla} \psi_0 = w_0 \psi_0 \tag{2.9a}$$

$$i \not{V} \psi_0 = \psi_0 |_s \tag{2.9b}$$

$$B = - \frac{1}{2} h^\mu \partial_\mu \sum_{i=1}^3 \overline{\psi_{0i}} \psi_{0i} |_s \tag{2.9c}$$

$$1 = \int_V d^3x \psi_0^\dagger \psi_0 \tag{2.9d}$$

The last term is the normalization condition, and in (2.9c) the sum over the three quarks has been explicitly included.

The pion and gluon fields first arise in $O(g, g_c)$, but as all physical observables depend on the square of these fields (or on products of currents and fields), only terms of $O(g^2, g_c^2)$ are physical,

and the equations are not complete until all terms of $O(g^2, g_c^2)$ are included.

Before developing the specific equations which determine the pion and gluon fields, we make a few general remarks. In what follows we will use the pion field as an example, although similar comments also apply to the gluon fields. It is convenient to denote that part of the pion field which arises from the i th quark by ϕ_i . Bilinear products of the pion field then take the form $\sum_{i,j} \phi_i \cdot \phi_j$ where the double sum is over both i and j . There are two options; the sum can be unrestricted, or it can be restricted so that the $i=j$ terms are excluded. In a truly field theoretic calculation the $i=j$ terms are crucial and must be included in some fashion. This is discussed at length in Chapter IX. For practical reasons we exclude them in our energy fits. If we included them we wouldn't be able to fit the nucleon. (See Figure 2). Perhaps this omission is reasonable since we are doing a classical calculation where the self energies are meaningless anyway. Since we dropped the $i=j$ terms in the pion sector it would be consistent to drop them in the gluon sector as well. Presumably one should absorb the $i=j$ terms into the mass renormalization and compute the energy from sums over $i \neq j$. However, it is well known⁵ that this procedure cannot be carried out for the color-electric field. In order to satisfy the boundary condition (2.6b), which for the electric components becomes

$$\hat{r} \cdot \vec{E}^a \Big|_s = 0 \quad (2.10)$$

it is necessary to include the $i=j$ terms in the sum and to require that the state be a color singlet. If the quarks all have the same mass and are all in the same state, which is the case for the nucleons and Δ 's treated here, this condition means that the color-electric field is zero everywhere inside the bag, and can be ignored. This will be assumed in the remainder of this dissertation. For consistency we will also neglect the time component of the color current, which would be the source for the color electric field. For further discussion see Chapter IX.

We now return to the system of equations which completely specify the solution to $O(g^2, g_c^2)$. It is sufficient to write these for a spherical bag (see Chapter IA). There are four kinds of equations. First, the gluon fields are determined by

$$\vec{\nabla} \times \vec{A}_i^a = \vec{B}_i^a \quad (2.11a)$$

$$\vec{\nabla} \cdot \vec{A}_i^a = 0 \quad (2.11b)$$

$$\nabla^2 \vec{A}_{ii}^a = - \vec{j}_i^a = - \psi_{oi}^\dagger \vec{\alpha} \frac{\lambda^a}{2} \psi_{oi} \quad (2.11c)$$

$$\hat{r} \times \vec{B}_i^a \Big|_s = 0 \quad (2.11d)$$

where the index i ranges from 1 to 3 and labels the three quarks in the bag. It will be suppressed whenever possible. In this model the source

of the gluon field is the inhomogeneous term in (2.11c) which may be interpreted as the color current density of the i th quark. That part of the total gluon field generated by the i th quark is written \vec{A}_i^a . Note that the static color-magnetic fields⁵ satisfy boundary conditions on the surface identical to the boundary conditions for the usual electric field at the surface of a conductor.

The second set of equations give the pion field to $O(g)$

$$\begin{aligned} \partial_\mu \partial^\mu \underline{\phi}_{1,i} + m_\pi^2 \underline{\phi}_{1,i} &= 0 \\ \eta^\mu \partial_\mu \underline{\phi}_{1,i} &= - \bar{\psi}_{0,i} \gamma^5 \frac{\tau_z}{2} \psi_{0,i} \Big|_s \\ &= i \bar{\psi}_{0,i} \gamma^5 \frac{\tau_z}{2} \psi_{0,i} \Big|_s \end{aligned} \quad (2.12)$$

The source of the pion field is the boundary term involving $\psi_{0,i}$ and again that part of the field generated by the i th quark is denoted by

$$\underline{\phi}_i.$$

The third set of equations gives us the quark fields ψ_2 and also involves the second-order energies $w_{2\pi}$ and w_{2g} . Before we write these equations, we call attention to the fact that $w_{1\pi}$ and w_{1g} are actually operators in the quark spin-isospin space; they have a value which depends on how the three quarks in the baryon are coupled together. Furthermore, the operator form of $w_{1\pi}$ and w_{1g} does not commute with the $\hat{b} \cdot \hat{r}$ factor in ψ_0 ; hence the order of terms is important. Since the partial time derivative commutes with $\hat{b} \cdot \hat{r}$ and the space derivatives $\vec{b} \cdot \vec{\nabla}$ do not commute with $\hat{b} \cdot \hat{r}$, it is clear that the

energy terms should be written to the right of any Dirac wave function (and to the left of all conjugate wave functions). The equations for $\psi_{2\pi}$ and ψ_{2g} are, therefore,

$$\begin{aligned}
 -i \vec{\alpha} \cdot \vec{\nabla} (g^2 \psi_{2\pi i} + g_c^2 \psi_{2g i}) &= W_0 (g^2 \psi_{2\pi i} + g_c^2 \psi_{2g i}) \\
 + g^2 \psi_{0i} W_{2\pi i} + g_c^2 \psi_{0i} W_{2g i} &+ g_c^2 \left[\vec{\alpha} \frac{\lambda^a}{2} \psi_{0i} \right] \sum_{j \neq i} \vec{A}_{ij} \quad (2.13a)
 \end{aligned}$$

$$\begin{aligned}
 i \not{Y} (g^2 \psi_{2\pi i} + g_c^2 \psi_{2g i}) &= g^2 \psi_{2\pi i} + g_c^2 \psi_{2g i} \\
 + i g^2 (\underline{\tau} \gamma^5 \psi_{0i}) \cdot \sum_{j \neq i} \underline{\phi}_{ij} \quad (2.13b)
 \end{aligned}$$

Note that the sums on the right-hand sides (RHS's) of these equations exclude the $i=j$ term. This means that the color fields \vec{A} and pion field $\underline{\phi}$ which interact with the i th quark will arise only from the other quarks in the bag, and contributions to the quark self-energy are neglected (as discussed above). Also, (2.13a) includes only the magnetic interaction; the electric interaction is neglected. Since g and g_c are independent, we can separate Eqs. (2.13) as follows:

$$-i \vec{\alpha} \cdot \vec{\nabla} \psi_{2\pi i} = W_0 \psi_{2\pi i} + \psi_0 W_{2\pi i} \quad (2.14a)$$

$$i \not{Y} \psi_{2\pi i} = \psi_{2\pi i} + i (\underline{\tau} \gamma^5 \psi_{0i}) \cdot \sum_{j \neq i} \underline{\phi}_{ij} \quad (2.14b)$$

and

$$-i \vec{\alpha} \cdot \vec{\nabla} \psi_{2gi} = \omega_0 \psi_{2gi} + \psi_0 \omega_{2gi} + \left[\vec{\alpha} \frac{\lambda^a}{2} \psi_{0i} \right] \cdot \sum_{j \neq i} A_{ij}^a \quad (2.15a)$$

$$i \psi \psi_{2gi} = \psi_{2gi} \quad (2.15b)$$

These equations are solved in Appendix B.

Finally, the last equation is the NLC. Before writing this down, observe that the normalization of ψ is affected by ψ_2 . If the original definition of ψ_0 is retained (including its normalization), then the correct normalization condition to $O(g^2, g_c^2)$ is (no summation over repeated indices is implied)

$$1 = \int_V \left[(1 + \delta N_i) \psi_{0i}^+ + g^2 \psi_{2\pi i}^+ + g_c^2 \psi_{2g i}^+ \right] \cdot \left[(1 + \delta N_i) \psi_{0i} + g^2 \psi_{2\pi i} + g_c^2 \psi_{2g i} \right] d^3x \quad (2.16)$$

Hence

$$\delta N_i = -\frac{1}{2} \int_V \left[g^2 (\psi_{2\pi i}^+ \psi_{0i} + \psi_{0i}^+ \psi_{2\pi i}) + g_c^2 (\psi_{2g i}^+ \psi_{0i} + \psi_{0i}^+ \psi_{2g i}) \right] \quad (2.17)$$

and the NLC, to $O(g^2, g_c^2)$, becomes

$$\mathcal{B} = -\frac{1}{2} \sum_{i \neq j} \vec{B}_i \cdot \vec{B}_j - \frac{i}{2} u^\mu \partial_\mu \sum_{i \neq j} (\vec{\nabla} \psi_{0i} g^2 \sum \gamma^5 \psi_{0i}) \cdot \psi_{0j} \\ - \frac{1}{2} u^\mu \partial_\mu \sum_i \vec{\nabla} \psi_{0i} \psi_{0i} - \sum_i \delta N_i u^\mu \partial_\mu \psi_{0i} \psi_{0i}$$

$$\begin{aligned}
& - \frac{1}{2} g^2 n^\mu \partial_\mu \sum_i \left(\bar{\psi}_{2\pi i} \psi_{0i} + \bar{\psi}_{0i} \psi_{2\pi i} \right) \\
& - \frac{1}{2} g_c^2 n^\mu \partial_\mu \sum_i \left(\bar{\psi}_{2g_i} \psi_{0i} + \bar{\psi}_{0i} \psi_{2g_i} \right) \\
& - \frac{1}{2} g^2 \sum_{i \neq j} \left(\partial_\mu \phi_{1,i} \cdot \partial^\mu \phi_{1,j} - m_\pi^2 \phi_{1,i} \cdot \phi_{1,j} \right) \quad (2.18)
\end{aligned}$$

The color vector potential has been replaced with the color-magnetic field and the electric field has been set equal to zero. To evaluate the RHS of Eq. (2.17) it is necessary to perform a double sum over all three quarks in the bag, and evaluate the spin, isospin operations for each hadron of interest. In performing the double sum over quarks i and j , we will elect not to retain the $i=j$ term, as discussed above.

A complete solution to $O(g^2, g_c^2)$ is obtained by first solving Eqs. (2.9)–(2.18) for a deformed bag with a small eccentric ϵ of $O(g, g_c)$. The eccentricity will be uniquely determined from the NLC as discussed in Chapter III.

C. The Bag Energy

The Hamiltonian density for the bag can be obtained from the Lagrangian in the usual manner. To simplify the discussion we will use the fact that $\vec{E}^a=0$, and \vec{A}^a and ϕ are independent of time (these terms will be absent in the final result in any case). Then

$$\mathcal{H} = \left[\frac{\partial \bar{\Psi}}{\partial x} \frac{\partial \mathcal{L}}{\partial (\partial \bar{\Psi} / \partial x)} + \frac{\partial \mathcal{L}}{\partial (\partial \Psi / \partial x)} \frac{\partial \Psi}{\partial x} \right] - \mathcal{L} \quad (2.19)$$

and the bag energy is

$$E = \int d^3x \mathcal{H} = E'_Q + E'_\pi + E_g + E_V \quad (2.20)$$

where

$$E'_Q = \int_V -\frac{i}{2} (\Psi^\dagger \vec{\alpha} \cdot \vec{\nabla} \Psi) \quad (2.21a)$$

$$E'_\pi = \int_S \frac{1}{2} \bar{\Psi} (1 + i g \vec{\tau} \cdot \vec{\phi} \gamma^5) \Psi (1 + g^2 \phi^2)^{-1/2} \\ + \int_V \frac{1}{2} \left[(1 + g^2 \phi^2) \vec{D} \phi \cdot \vec{D} \phi - \frac{1}{4} g^2 \vec{D} \phi^2 \cdot D \phi^2 + \mu_\pi^2 \phi^2 \right] \quad (2.21b)$$

$$E_g = \int_V \left[\frac{1}{2} \vec{B}^a \cdot \vec{B}^a - g_c \Psi^\dagger \vec{\alpha} \cdot \vec{A}^a \frac{\lambda^a}{2} \Psi \right] \quad (2.21c)$$

$$E_V = \frac{4\pi}{3} R^3 B \quad , \quad (2.21d)$$

where we have suppressed all sums over different quarks in the bag.

We will now use the equations of motion to simplify these expressions. The boundary condition (2.6a) shows that

$$i\bar{\psi}\psi = \bar{\psi} (1 + i g \underline{\alpha} \cdot \underline{\phi} \gamma^5) \psi (1 + g^2 \phi^2)^{-1/2} \quad (2.22)$$

is exactly zero on the surface of the bag (because the RHS is real while the left-hand side is imaginary). Integrating the $(\vec{D}\phi)^2$ term by parts, and using the equation of motion (2.5c) and the boundary condition (2.6c) (recall that the outward normal to the surface is positive) gives

$$E_{\pi}^{\prime} = \frac{1}{2} \int_S g (\bar{\psi} \gamma^5 \underline{\alpha} \cdot \underline{\phi} \psi) \cdot \underline{\phi} - \frac{g^2}{2} \int_V \left[m_{\pi}^2 (\phi^2)^2 + \frac{1}{4} \vec{D}\phi^2 \cdot \vec{D}\phi^2 \right] \quad (2.23)$$

To order g^2 the volume term can be ignored, and (2.9b) used to reduce the surface term to

$$E_{\pi}^{\prime} = -\frac{i}{2} \int_S g^2 \left[\bar{\psi}_0 \gamma^5 \underline{\alpha} \cdot \underline{\phi}_0 \psi_0 \right] \cdot \underline{\phi}_1 \quad (2.24)$$

As is well known, to order g_c^2 the color-magnetic interaction term is twice as large as the B^2 term, and of opposite sign, so the magnetic energy to order g_c^2 is

$$E_g = -\frac{1}{2} \int_V \vec{B}^a \cdot \vec{B}^a \quad (2.25)$$

The quark "kinetic"-energy term can be reduced if we first use the Dirac equation and boundary conditions (2.13) to determine the

second-order energy terms w_{2g} and $w_{2\pi}$. If we multiply (2.13a) by ψ_0^\dagger and integrate over V we obtain, to $O(g^2, g_c^2)$,

$$\begin{aligned} & \int_V \psi_0^\dagger (-i \vec{\alpha} \cdot \vec{\nabla}) (g^2 \psi_{2\pi} + g_c^2 \psi_{2g}) \\ &= w_0 \int_V \psi_0^\dagger (g^2 \psi_{2\pi} + g_c^2 \psi_{2g}) + g^2 w_{2\pi} + g_c^2 w_{2g} \\ &+ g_c^2 \int_V (\psi_0^\dagger \vec{\alpha} \frac{\lambda^a}{2} \psi_0) \cdot \vec{A}_{1,a} \end{aligned} \quad (2.26)$$

The LHS of this equation can be integrated by parts, and the $\vec{\nabla}$ operator eliminated by using (2.9a). The boundary term which comes from the integration by parts can be written using either (2.13b) or (2.9b). Averaging these two gives

$$i \int_S \bar{\psi}_0 \not{x} (g^2 \psi_{2\pi} + g_c^2 \psi_{2g}) = \frac{i}{2} \int_S g^2 (\bar{\psi}_0 \gamma^5 \not{x} \psi_0) \cdot \underline{\phi}_1 \quad (2.27)$$

Collecting terms gives

$$\begin{aligned} & \frac{i}{2} g^2 \int_S (\bar{\psi}_0 \gamma^5 \not{x} \psi_0) \cdot \underline{\phi}_1 \\ &= g^2 w_{2\pi} + g_c^2 w_{2g} + g_c^2 \int_V (\psi_0^\dagger \vec{\alpha} \frac{\lambda^a}{2} \psi_0) \cdot \vec{A}_{1,a} \end{aligned} \quad (2.28)$$

Using the fact that g and g_c are independent gives relations for $w_{2\pi}$ and w_{2g} :

$$w_{2\pi} = i \int_S (\bar{\psi}_0 \gamma^5 \not{x} \psi_0) \cdot \underline{\phi}_1 \quad (2.29a)$$

$$W_{2g} = - \int_V (\psi_0^\dagger \vec{\alpha} \frac{\lambda^a}{2} \psi_0) \cdot \vec{A}_{1,a} \quad (2.29b)$$

These results agree with those obtained in Appendix B from the solution for ψ_2 ; we have merely reproduced the well known Chodos and Thorn³ result that the second-order energies can be obtained without solving for ψ_2 . Note that the g_c^2 terms can cancel from the RHS of Eq. (2.26), and that $W_{2\pi}$ is twice as large and of opposite sign from E'_{π} .

The quark energy can now be obtained directly from (2.26).

Using the fact that the state is normalized, we obtain

$$E'_q = W_0 + g_c^2 W_{2\pi} \quad (2.30)$$

Collecting together all the terms and inserting sums over quarks gives our final expression for the bag energy to $O(g^2, g_c^2)$:

$$E_B = 3W_0 + \frac{4}{3} \pi R^3 B - \frac{1}{2} \int_V \sum_{\substack{i,j \\ i \neq j}} \vec{B}_i^a \cdot \vec{B}_j^a \\ + g^2 \frac{i}{2} \int_V \sum_{\substack{i,j \\ i \neq j}} (\bar{\psi}_{0i} \gamma^5 \frac{\lambda^a}{2} \psi_{0i}) \cdot \vec{\phi}_{ij} \quad (2.31)$$

This will be evaluated in Chapter V.

III. SOLUTIONS FOR THE DEFORMED BAG

A. The Dirac Equation for a Deformed Bag

Equations (2.9) are the traditional Dirac equation, boundary condition and normalization for the lowest-order quark wave function and energy. If the bag is spherical, the solution is well known:

$$\psi_0 = N \begin{pmatrix} j_0(w_0 r) \\ i \vec{\sigma} \cdot \hat{r} j_1(w_0 r) \end{pmatrix} \chi \quad (3.1)$$

where χ is the two-component spinor of the quark, and

$$\rho = w_0 R = 2.04 \quad (3.2)$$

$$N^2 = \frac{\rho}{8\pi R^3 j_0^2(\rho)(\rho-1)} \quad (3.3)$$

In the rest of this chapter we will suppress all two-component spinors; all expressions will therefore be matrices or operators on the two-component spin-isospin space of the quarks.

The remainder of this section will be devoted to a brief summary of how to treat the bag if it is not spherical. For a detailed discussion, see Appendix A.

We anticipate a slight deviation from sphericity. The form of deviation is based on knowledge of the form of the pressure of $O(g^2, g_c^2)$. A simplified example follows. Assume the pressure is mainly a function of radius but has an additional small portion with a $P_2(z)$ dependence.

$$\text{Pressure} = -B_0 + p_0(r) + g^2 \Delta p(r) P_2(z)$$

where B_0 is the external vacuum pressure. $p_0(r)$ is the pressure from the S-state quarks and $g^2 \Delta p(r) P_2(z)$ might be the pressure from gluons or pions. If g^2 were zero we would simply say $B_0 = p_0(R)$ and be finished. This would determine B_0 . Since $g^2 \neq 0$ we must find a way to eliminate the g^2 term. The solution is to deform the surface. We guess

$$r|_s = R + \epsilon^2 R P_2(z)$$

Next evaluate the pressure by Taylor series at the new bag surface. We get

$$\begin{aligned} \text{Pressure} \Big|_s &= -B_0 + p_0(R) + \left. \frac{\partial p_0(r)}{\partial r} \right|_R \cdot \epsilon^2 R P_2(z) \\ &+ g^2 \Delta p(R) P_2(z) + O(g^2 \epsilon^2) \end{aligned} \quad (3.4)$$

Now making the pressure zero is simple. We have $B_0 = p_0(R)$ and

$$\left. \frac{\partial p_0(r)}{\partial r} \right|_R \epsilon^2 R P_2(z) = -g^2 \Delta p(R) P_2(z) \quad (3.5)$$

In practice one wouldn't use a naive Taylor series as we have done for the left hand side of (3.5). (The Taylor series is a simple

way of estimating the change in pressure exerted by the original quarks in a perturbed domain. In general one must calculate the pressure formally via the NLC after solving for the quark fields within a perturbed surface.) Nevertheless, one now knows what shape of bag to anticipate. The following is a technical description of the solution to the Dirac fields in the deformed cavity.

One first guesses the form of the bag radius as discussed above.

$$r \Big|_s = R - \frac{1}{3} \epsilon^2 R P_2(z) \quad (3.6)$$

The principal effect of the deformation on the lowest order equations (2.9) is to modify the boundary conditions. The outward normal to the bag surface is no longer purely in the direction of the radius vector. As shown in Appendix A,

$$\begin{aligned} -i \vec{\gamma} \cdot \hat{n} &\simeq -i \vec{\gamma} \cdot \hat{r} + i \epsilon^2 (\vec{\gamma} \cdot \hat{r} z^2 - \gamma_z^z) \\ &\equiv i \not{y}_0 + i \epsilon^2 \Delta \not{y} \end{aligned} \quad (A51)$$

$$\hat{n} \cdot \vec{\nabla} \simeq \frac{\partial}{\partial r} + \frac{\epsilon^2}{r} z (1-z^2) \frac{\partial}{\partial z} \quad (A53)$$

The extra term in (A51) will force us to add new terms to the elementary solution (3.1). To see what the form of these new terms must be, apply (A51) to (3.1). This gives

$$\begin{aligned}
 -i \vec{\gamma} \cdot \hat{n} \psi_0(w_0 r) &= \psi_0(w_0 r) \\
 + \epsilon^2 N \begin{pmatrix} j_1(w_0 r) \\ -i \vec{\sigma} \cdot \hat{r} j_0(w_0 r) \end{pmatrix} (\vec{z}^2 - \vec{\sigma} \cdot \hat{r} \sigma^z z) & \quad (3.7)
 \end{aligned}$$

to be evaluated at the surface where

$$r = R \left(1 - \frac{\epsilon^2}{3} P_2(z) \right) = R + \epsilon^2 \Delta R$$

The quark energy must be expanded as well: $w = w_0 + \epsilon^2 w_{2d} + \dots$. Hence if the new solution is ψ , and

$$\psi = \psi_0(w_0 r) + \epsilon^2 \psi_{2d}(w_0 r) \quad (3.8)$$

then ψ_{2d} must be a solution of the free Dirac equation inside the bag and satisfy the boundary condition

$$\begin{aligned}
 (i \not{y}_0 - 1) \psi_{2d}(\rho) &= -i \Delta \not{y} \psi_0(\rho) - (w_{2d} R + w_0 \Delta R) i \not{y}_0 \psi_0'(\rho) \\
 &+ (w_{2d} R + w_0 \Delta R) \psi_0'(\rho) \quad (3.9)
 \end{aligned}$$

where we dropped the term $O(\epsilon^4)$. A solution of the free Dirac equation which has the correct angular dependence to satisfy (3.9) can be constructed from selected $J=3/2$ and $J=5/2$ solutions with positive parity:

$$\psi_{2d} = a_1 \psi^{3/2} + a_2 \psi^{5/2} \quad (3.10)$$

These are

$$\psi^{3/2} = N \begin{pmatrix} j_2(\omega_0 r) \\ -i \vec{b} \cdot \hat{r} j_1(\omega_0 r) \end{pmatrix} \begin{pmatrix} \frac{3}{2} \vec{b} \cdot \hat{r} b_z^2 & -\frac{1}{2} \end{pmatrix} \quad (3.11a)$$

and

$$\psi^{5/2} = N \begin{pmatrix} j_2(\omega_0 r) \\ i \vec{b} \cdot \hat{r} j_3(\omega_0 r) \end{pmatrix} \begin{pmatrix} \frac{5}{2} z^2 - \vec{b} \cdot \hat{r} b_z^2 & -\frac{1}{2} \end{pmatrix} \quad (3.11b)$$

where the two-component spinors of the quarks have again been suppressed. The constants a_1, a_2 , and $w_{2\lambda}$, as determined from (3.9) are

$$\begin{aligned} a_1 &= -\frac{2\rho(2\rho+1)}{45} \\ a_2 &= \frac{2\rho^2(\rho-2)}{5(15-8\rho)} \\ w_{2\lambda} &= 0 \end{aligned} \quad (3.12)$$

The relations between the $j_\lambda(\rho)$ given in Appendix C were used to simplify a_1 and a_2 .

The solutions (3.8) and (3.10) satisfy all of the Eqs. (2.9) except the NLC. (There is no change in the normalization of ψ to order ϵ^2 because $\psi_{2\lambda}$ is orthogonal to ψ_0). To test the NLC we use (A53), which gives, to order ϵ^2 :

$$\begin{aligned}
B &= -\frac{3}{2} N^2 \left\{ \frac{d}{dr} (j_0^2 - j_1^2) + \epsilon^2 \frac{d}{dr} (j_1' j_1 - j_0' j_0) - \frac{2}{3} \rho P_2(z) \right. \\
&\quad \left. + \epsilon^2 \left[a_1 \frac{d}{dr} (j_0 j_1 + j_1'^2) + a_2 \frac{d}{dr} (j_0 j_2 - j_1 j_3) \right] \cdot 2 P_2(z) \right\} \\
&= \frac{3\rho}{4\pi R^4} + \epsilon^2 \left(\frac{\rho P_2(z)}{6\pi R^4 (\rho-1)} \right) \left(\frac{10\rho^4 - 26\rho^3 - 5\rho^2 + 54\rho - 36}{8\rho - 15} \right) \quad (3.13)
\end{aligned}$$

where we have summed over all three quarks and assumed normalized two-component spinors. Note that the first term is precisely the relation between B and R which is usually obtained by minimizing the bag energy for the case when gluons and pions are ignored. The second term, which is proportional to ϵ^2 , depends on $\cos\theta$, and hence if $\epsilon \neq 0$ there is no way to satisfy (3.13) at every point on the surface of the bag with a single bag constant B . As we shall see below, when pions and gluons are added to the bag, the NLC will pick up new angle-dependent terms which go like $P_2(z)$. These can be canceled by the angle-dependent term in (3.13) if ϵ^2 has the correct value; in this way ϵ^2 becomes a function of g^2 and g_c^2 .

Before turning to the calculation of the g^2 and g_c^2 terms, we emphasize that ϵ^2 is $O(g^2, g_c^2)$ when the deformation is treated perturbatively. Hence a calculation to second order need not include effects due to deformation of the bag in any terms already of order g^2 or g_c^2 ; such terms are of fourth order in small quantities. To second

order, the only contributions which arise from deformation are those given above. Therefore, when the pion and gluon field effects are calculated, one may assume a spherical bag.

Note that we have deferred the question of classical stability to Chapter IX. In any event, satisfying the NLC is a necessary condition for stability.

B. Deformation Via the NLC

The calculation of the g^2 and particularly the g_c^2 terms is fairly involved, and so these have been relegated to Appendix B. The g^2 contribution to the NLC is in Eq. (B12), while the g_c^2 contribution is in Eq. (B39). Adding Eqs. (B12), (B39), and 3.13) gives the net result which is Eq. (2.18) [i.e., the complete NLC for an oblate (prolate) bag which includes pions, gluons, and quarks]. We have

$$\beta = B_2 P_2(z) + B_0 \quad (3.14a)$$

where

$$B_0 = \frac{3\rho}{4\pi R^4} + B_{0\pi} \delta_{0,B}^{\pi} + B_{0g} \delta_{0,B}$$

$$B_2 = \frac{\rho \epsilon^2}{6\pi R^4 (1-\epsilon)} \left(\frac{10\rho^4 - 26\rho^3 - 5\rho^2 + 54\rho - 36}{8\rho - 15} \right) \quad (3.14b)$$

$$+ \delta_{2,B}(\mu) B_{2g} + \delta_{2,B}^{\pi}(\mu) B_{2\pi}$$

and the Δ 's are defined via

$$\langle B m | \bar{P}_2^I | B m \rangle = \delta_{2,B}^I(m) P_2(z) \quad (3.15)$$

$$\langle B m | \bar{P}_2 | B m \rangle = \delta_{2,B}(m) P_2(z)$$

[Equation (B13) defines these spin, isospin operators.] The B (baryon) subscript on the δ 's tells whether it is a nucleon or Δ bag. The m denotes spin projection. In Appendix D one finds

$$\delta_{2,N}^I = \delta_{2,N} = 0$$

$$\delta_{2,\Delta}^I(m) = \delta_{2,\Delta}(m) = \begin{cases} +6, & |m| = 3/2 \\ -6, & |m| = 1/2 \end{cases} \quad (3.16)$$

The only way to satisfy (3.14) at all points on the bag surface is to set $B_2=0$. This gives the desired relation for the eccentricity. Using (3.16) we find

$$\epsilon_N^2 = 0$$

$$\epsilon_\Delta^2 = \pm \left(\frac{3(8\rho - 15)}{\rho(\rho - 1)(10\rho^4 - 26\rho^3 - 5\rho^2 + 54\rho - 36)} \right) \times \left(\frac{g^2}{4\pi} \frac{1}{R^2} F_\pi(\gamma) - \alpha_c F_g \right) \quad (3.17)$$

where the positive sign is for $|m|=3/2$ and the negative for $|m|=1/2$, and

$$\begin{aligned}
 F_{\pi}(\eta) &= \frac{1}{4} \rho^2 \left\{ 1 - 2 \frac{h_1}{\eta h_1'} (4\rho^2 - 4\rho - 3) + \left(\frac{h_1}{\eta h_1'} \right)^2 (1 - \eta) \right\} \\
 &= 3.203 \quad (\text{if } \eta = 0) \\
 F_g &= 6 \left(\frac{4\rho - 3}{12\rho} \right)^2 + \frac{1}{6} \left(\frac{1}{\rho} - \frac{4}{3} (\rho - 1)^2 \right) \\
 &= 0.108 \tag{3.18}
 \end{aligned}$$

Hence the nucleon is not deformed to $O(g^2, g_c^2)$, while the $|m|=3/2$ and $|m|=1/2$ Δ have different shapes.

Specific values of ϵ_{Δ}^2 for the four cases treated in the next chapter are summarized in Table 1 in the Introduction.

IV. NUMERICAL FITS

As shown in Appendix A, the deformation does not affect the energy to $O(\epsilon^2)$. Therefore, to $O(\epsilon^2, g_c^2, g^2)$ the energy is

$$E = E_Q + E_V + E'_{c.m.} + E_g + E_\pi \quad (4.1)$$

where $[\mu(\rho)]$ is defined in Eq. (C10)]

$$E_Q = \frac{3\rho}{R} \quad (4.2a)$$

$$E'_{c.m.} = \frac{Z_0}{R} + E_{c.m.} \quad (4.2b)$$

$$\begin{aligned} E_g &= 0.058 \frac{\alpha_c}{R} \bar{p}_0 \\ &= \frac{\alpha_c}{R} \left(\mu^2(\rho) + 2\rho \int_0^\rho \frac{\mu^2(\rho')}{\rho'^2} d\rho' \right) \bar{p}_0 \end{aligned} \quad (4.2c)$$

$$E_\pi = -\frac{g^2}{96\pi} \left(\frac{\rho}{\rho-1} \right)^2 \left(\frac{\eta+1}{\eta^2+2\eta+2} \right) \frac{1}{R^3} \bar{p}_0^I \quad (4.2d)$$

The volume energy E_V was given in Chapter II. While the definition of E_g is identical to that used in Chapter II, E_Q and E_π differ; $W_{2\pi}$ formerly included with E'_Q is now incorporated into E_π . We have lumped

a zero point energy Z_0/R and a c.m. correction into E'_{cm} , but herein we will take $Z_0=0$ for simplicity, and take the center-of-mass energy to be merely

$$E_{c.m.} = -\frac{3}{8} \frac{\rho}{R} = -\frac{0.76}{R} \quad (4.3)$$

which is an approximation based on the assumption that the main contribution to the bag energy is due to the quark kinetic energy.⁴ The pion and gluon terms both involve matrix elements of spin and isospin operators, which are evaluated in Appendix D.

In doing our fits, we will vary α_c and B, for two different choices of $f_\pi=g^{-1}$ and m_π . We will use the values $f_\pi=93.3$ MeV (a value in good agreement with the experimentally determined pion decay constant), and $f_\pi=114$ MeV. The latter value has been arrived at by calculating the axial charge and then forcing the Goldberger-Treiman relation to be exact.⁶ We shall solve for the radii of the nucleon and Δ using both values, and compare results. For m_π we will use both the experimental value of 138 MeV and $m_\pi=0$.

We have four variables and four equations. The variables are R_N , R , B, and α_c while the four equations are

$$\frac{\partial E}{\partial R_N} = 0 \quad (4.4a)$$

$$E(R_N) = M_N = 939 \text{ MeV} \quad (4.4b)$$

$$\frac{\partial E}{\partial R_A} = 0 \quad (4.4c)$$

$$E(R_A) = M_A = 1232 \text{ MeV} \quad (4.4d)$$

These may be solved numerically. The results are shown in Tables II and III, already discussed in Chapter I.

V. THE NLC, STABILITY, HIGHER ORDER DEFORMATION OF THE NUCLEON

A. The Non-Linear Boundary Condition

The NLC is obtained by assuming the action is stationary with respect to variations in the confining volume. For a static cavity and classical fields the NLC is merely the requirement that the pressures exerted by the fields be zero at all points on the surface. The stress energy tensor is known to contain the pressure in its diagonal cartesian components, so we would like to compare the two. The stress energy tensor is defined in terms of general fields ϕ as:

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\nu \phi)} \partial^\mu \phi - g^{\mu\nu} \mathcal{L} \quad (5.1)$$

The conditions of energy and momentum conservation are

$$\partial_\nu T^{\mu\nu} = 0 \quad (5.2)$$

We know that $T^{\mu\nu}$ is a contr. variant tensor so this tells us the type of transformation we need to go from Minkowski coordinates to those more suitable for the bag. The transformation is:

$$T^{\mu\nu} = \Lambda^\mu_\alpha \Lambda^\nu_\beta T^{\alpha\beta} \quad (5.3)$$

where the transformation operator is the outward normal.

$$\Lambda_{\alpha}^{\mu} \equiv \frac{\partial u}{\partial x^{\alpha}} = \hat{n}_{\alpha}^{\mu} \quad (5.4)$$

If we consider the MIT bag without gluons we find

$$p = T^{\mu\nu} = \partial_{\nu} \left(-\frac{1}{2} u_{\mu} \partial^{\mu} \bar{\psi} \psi - \beta \right) \quad (5.5)$$

The condition of equilibrium is that the pressure evaluated inside plus the pressure outside be zero. One has

$$-\frac{1}{2} u_{\mu} \partial^{\mu} \bar{\psi} \psi - \beta = 0 \Big|_s \quad (5.6)$$

This is exactly the NLC.

B. Stability of the Bag

It is a well know pathology of chiral bag models that the pion field becomes stronger for smaller radii and causes metastability with respect to radius. Figures 2 and 3 show this. At least in the classical sense both figures show stable points as a function of radius. I am more concerned with the stability of the bag with respect to arbitrary deformation. The NLC is known to be a necessary but not a sufficient condition for stability.⁷ The stability condition is:

$$(\epsilon^2 - \epsilon_0^2)^2 \frac{\partial^2 E}{\partial \epsilon^2} + 2(\epsilon^2 - \epsilon_0^2)(r-R) \frac{\partial^2 E}{\partial \epsilon^2 \partial r} + (r-R)^2 \frac{\partial^2 E}{\partial r^2} > 0 \quad (5.7)$$

The NLC has merely satisfied

$$\left. \frac{\partial E}{\partial R} \right|_{\epsilon^2} = 0, \quad \left. \frac{\partial E}{\partial \epsilon^2} \right|_R = 0 \quad (5.8)$$

Eq. (5.8) could easily be satisfied at some kind of saddle point so I believe the stability of the $\hat{\Lambda}$ and even the nucleon should be checked. This will be done in the future.

C. Higher Order Deformation of the Nucleon

The mechanism for deformation arose from operators of the form.

$$\frac{3}{2} b_i \cdot \hat{\lambda} b_j \cdot \hat{\lambda} - \frac{1}{2} b_i \cdot b_j \quad (5.9)$$

This is a scalar operator which has a spin two and orbital angular momentum of two also. It therefore has zero expectation value with respect to the nucleon, which is spin 1/2.

A consequence of the spin behavior of (5.9) is that no spin 1/2 bag models will have deformation to $O(g^2, g_c^2)$. (Note the discussion of configuration mixing in the Chapter 1.) This is true for the Roper

resonance as well as spin 1/2 systems involving strange quarks. It will require additional work to see whether any of those spin 1/2 systems, (especially the nucleon) have deformation in higher order. We have checked some likely contributions to deformation at $O(g^4)$ and $O(g^6)$ and found none. There are lots of contributions unfortunately and most of them haven't been checked. Our feeling is that there may be a general proof which shows the nucleon is not deformed to all orders.

VI. CALCULATION OF

We intend to ultimately calculate $g_{\pi\Delta\Delta}$ and $g_{\pi NN}$ to $O(g^3, g_c^2 g, \epsilon^2 g)$. In order to do this one needs to know the pion field at that order. Equations (2.5c) and (2.6c) are required.

$$D_\mu D^\mu \underset{\sim}{\phi} + m_\pi^2 \underset{\sim}{\phi} = 0 \quad (6.1)$$

$$i \gamma^\mu \partial_\mu \underset{\sim}{\psi} = -(1 + g^2 \phi^2) \bar{\Psi} \gamma^5 g \frac{\tau_z}{2} \Psi \Big|_s \quad (6.2)$$

where

$$\underset{\sim}{\phi} = g \underset{\sim}{\phi}_1 + g^3 \underset{\sim}{\phi}_3 + \dots \quad (6.3)$$

$$\psi = \psi_0 + g^2 \psi_{2\pi} + g_c^2 \psi_{2g} + \epsilon^2 \psi_{2d} + \dots \quad (6.4)$$

The solution $\underset{\sim}{\phi}_{1,i}$ was obtained in Appendix B. Index i refers to the i th quark.

$$\underset{\sim}{\phi}_{1,i} = b h_r (m_{\pi r}) \tau_z^i \vec{b}_i \cdot \hat{r} \quad (6.5)$$

Equations (6.1) and (6.2) may now be solved for $\underset{\sim}{\phi}_{3,i}$ since we have already gotten solutions to ψ_{2d} , $\psi_{2\pi}$, ψ_{2g} , as well as $\psi \Big|_s, \hat{h} \Big|_s$

The Dirac wave functions are

$$\psi_{2\pi i} = N \hat{w}_{2\pi} \begin{pmatrix} r j_0' \\ i \vec{b}_i \cdot \hat{r} j_1' \end{pmatrix} p_{0i}^I \quad (6.6a)$$

$$+ N c_1 \begin{pmatrix} j_2 \\ -i \vec{b}_i \cdot \hat{r} j_1 \end{pmatrix} p_{2i}^I \quad (6.6b)$$

$$+ \frac{N(2p-3)}{2(p-1)} \frac{\hat{w}_{2\pi}}{w_0} \begin{pmatrix} j_0 \\ i \vec{b}_i \cdot \hat{r} j_1 \end{pmatrix} p_{0i}^I \quad (6.6c)$$

$$\psi_{2g i} = N \hat{w}_{2g} \begin{pmatrix} r j_0' \\ i \vec{b}_i \cdot \hat{r} j_1' \end{pmatrix} p_{0i} \quad (6.7a)$$

$$+ \begin{pmatrix} K_1 \\ i \vec{b}_i \cdot \hat{r} K_3 \end{pmatrix} p_{0i} \quad (6.7b)$$

$$+ \begin{pmatrix} K_2 \\ i \vec{b}_i \cdot \hat{r} K_4 \end{pmatrix} p_{2i} \quad (6.7c)$$

$$\psi_{2d i} = N a_1 \begin{pmatrix} j_2 \\ -i \vec{b}_i \cdot \hat{r} j_1 \end{pmatrix} p_{2i}^{a_1} + \quad (6.8a)$$

$$+ N a_2 \begin{pmatrix} j_2 \\ i \vec{b} \cdot \hat{r} j_3 \end{pmatrix} p_{2i}^{a_2} \quad (6.8b)$$

The constants and quark operators are summarized below.

$$N^2 = \frac{1}{8\pi R^3} \frac{P}{j_0^2 (\rho-1)} \quad (6.9)$$

$$\hat{W}_{2\pi} = - \frac{w_0 b h_1(mR)}{6(\rho-1)} \quad (6.10)$$

$$b = \frac{N^2 j_0^2}{m h_2(mR)} \quad (6.11)$$

$$C_1 = \frac{2}{9} \rho b h_1(mR) \quad (6.12)$$

$$\hat{W}_{2g} = \frac{8}{9} N^2 \int_0^R r^3 dr j_0 j_1 H(r) \quad (6.13)$$

$$\hat{W}_{2g} R = 9.451 \times 10^{-3} \quad (6.14)$$

$$a_1 = - \frac{2\rho}{45} (2\rho+1) \quad (6.15)$$

$$a_2 = \frac{z}{5} \frac{\rho^2(\rho-2)}{(15-8\rho)} \quad (6.16)$$

$$\mathcal{P}_{0i}^{\mathbb{I}} = \sum_{j \neq i} \vec{b}_i \cdot \vec{b}_j \tau_i \cdot \tau_j \quad (6.17)$$

$$\mathcal{P}_{2i}^{\mathbb{I}} = \sum_{j \neq i} \left(\frac{3}{2} \vec{b}_i \cdot \hat{r} \vec{b}_j \cdot \hat{r} - \frac{1}{2} \vec{b}_i \cdot \vec{b}_j \right) \tau_i \cdot \tau_j \quad (6.18)$$

$$\mathcal{V}_{0i} = \sum_{j \neq i} \vec{b}_i \cdot \vec{b}_j \quad (6.19)$$

$$\mathcal{V}_{2i} = \sum_{j \neq i} \left(\frac{3}{2} \vec{b}_i \cdot \hat{r} \vec{b}_j \cdot \hat{r} - \frac{1}{2} \vec{b}_i \cdot \vec{b}_j \right) \quad (6.20)$$

$$\mathcal{P}_{2i}^{a_1} = \frac{3}{2} \vec{b}_i \cdot \hat{r} b_i^z z - \frac{1}{2} \quad (6.21)$$

$$\mathcal{V}_{2i}^{a_2} = \frac{5}{2} z^2 - \vec{b}_i \cdot \hat{r} b_i^z z - \frac{1}{2} \quad (6.22)$$

The radius, surface normal and normal gradient are to $O(\epsilon^2)$:

$$\begin{aligned} \Lambda &= R - \frac{\epsilon^2}{3} R P_2(z) \\ &= R + \epsilon^2 \Delta R \end{aligned} \quad (6.23)$$

$$\begin{aligned}
 \hat{h} &= \hat{r} - \epsilon^2 (\hat{r} z^2 - \hat{z} z) \\
 &= \hat{h}_0 + \epsilon^2 \Delta \hat{h}
 \end{aligned}
 \tag{6.24}$$

$$\hat{h} \cdot \vec{\nabla} = \frac{\partial}{\partial r} + \epsilon^2 z (1-z^2) \frac{1}{r} \frac{\partial}{\partial z}
 \tag{6.25}$$

Equations (6.1) and (6.2) may now be solved for ϕ_{3i} . Equation (6.1) is at $O(g^3)$:

$$\partial_\mu \partial^\mu \phi_{3i} + m^2 \phi_{3i} = 0
 \tag{6.26}$$

Equation (6.2) involves the expansion parameters g, ϵ, g_c which we shall assume are of the same order. In the following, ψ_{2i} is the sum of three fields $\psi_{2\pi i}, \psi_{2g i}, \psi_{2\lambda i}$. The expansion parameters won't be shown but one can pick off the appropriate parameter by seeing the type of wave function and assuming all terms have been divided by g^3 . Rewritten (with the aid of (2.6)) and with different terms labeled, we have for (6.2) ;

$$\begin{aligned}
\kappa_0^\mu \partial_\mu \phi_{3i} \Big|_R &= -4\kappa^\mu \partial_\mu \phi_{1i} \quad - \kappa_0^\mu \partial_\mu (\kappa \Delta R \phi_{1i}') \quad (1) \quad (2) \\
&+ i \bar{\psi}_{2i} \gamma^5 \frac{\kappa}{2} \psi_{0i} \quad + i \bar{\psi}_{0i} \gamma^5 \frac{\kappa}{2} \psi_{2i} \quad (3) \quad (4) \\
&+ i \kappa_0 \Delta R \bar{\psi}_{0i}' \gamma^5 \frac{\kappa}{2} \psi_{0i} \quad + i \kappa_0 \Delta R \bar{\psi}_{0i} \gamma^5 \frac{\kappa}{2} \psi_{0i}' \quad (5) \quad (6)
\end{aligned} \tag{6.27}$$

The radial solutions to (6.26) are well known. In the following radial solutions the subscript denotes the angular momentum of the corresponding angular solution. $H_\ell(i\kappa r) \equiv j_\ell(i\kappa r) + i\eta_\ell(i\kappa r)$

$$\begin{aligned}
H_0(i\kappa) &= \frac{e^{-\kappa}}{\kappa} \\
H_1(i\kappa) &= i \frac{e^{-\kappa}}{\kappa} \left(1 + \frac{1}{\kappa} \right) = i h_1(\kappa) \\
H_2(i\kappa) &= \frac{e^{-\kappa}}{\kappa} \left(1 + \frac{2}{\kappa} + \frac{3}{\kappa^2} \right) \\
H_3(i\kappa) &= -i \frac{e^{-\kappa}}{\kappa} \left(1 + \frac{6}{\kappa} + \frac{15}{\kappa^2} + \frac{15}{\kappa^3} \right)
\end{aligned} \tag{6.28}$$

We shall solve for ϕ_{3i} in (6.27) by working out the angular dependences of terms ① through ⑥. Once the angular dependence is known, the radial solution is easily chosen from (6.28).

Term ① gives a contribution to $\frac{\partial \phi_{3i}}{\partial r}$ of the form.

$$\textcircled{1} = - \left(\frac{\epsilon^2}{g^2} \right) z (1-z^2) \left(\frac{1}{R} \frac{\partial}{\partial z} b_{h_1(\mu R)} \vec{b}_i \cdot \hat{r} z_i \right) \quad (6.29)$$

The angular behavior of ① lies in the terms

$$z (1-z^2) \frac{\partial}{\partial z} \vec{b}_i \cdot \hat{r} \quad (6.30)$$

This may be rewritten after explicitly working the derivative as

$$-\frac{z}{3} P_2(z) \vec{b}_i \cdot \hat{r} - \frac{1}{3} \vec{b}_i \cdot \hat{r} + z \vec{b}_i^z \quad (6.31)$$

It is necessary to decompose (6.31) into separate terms involving different values of orbital angular momentum. The following helpful identity may be derived.

$$P_2(z) \vec{b}_i \cdot \hat{r} = \left(-\frac{1}{5} \vec{b}_i \cdot \hat{r} + \frac{3}{5} z \vec{b}_i^z \right)_{\ell=1} + \left(P_2(z) \vec{b}_i \cdot \hat{r} + \frac{1}{5} \vec{b}_i \cdot \hat{r} - \frac{3}{5} z \vec{b}_i^z \right)_{\ell=3} \quad (6.32)$$

where the first term transforms like a combination of Y_{1m} and the second term transforms like a combination of Y_{3m} . Eq. (6.31) is

$$\left(-\frac{1}{5} \vec{b}_i \cdot \hat{r} + \frac{3}{5} z \vec{b}_i^z \right)_{\ell=1} - \frac{z}{3} \left(P_2(z) \vec{b}_i \cdot \hat{r} + \frac{1}{5} \vec{b}_i \cdot \hat{r} - \frac{3}{5} z \vec{b}_i^z \right)_{\ell=3} \quad (6.33)$$

The contribution of ① to $\underset{\sim}{\phi}_{3i}$ is now readily found. We know $\underset{\sim}{\phi}_{3i}$ must have $l=1$ and $l=3$ behavior.

$$\begin{aligned} \underset{\sim}{\phi}_{3i} &= A_1 H_1(imr) \underset{\sim}{\chi}_i \left(-\frac{1}{5} \vec{b}_i \cdot \hat{r} + \frac{3}{5} z b_i^z \right) \\ &+ A_3 H_3(imr) \underset{\sim}{\chi}_i \left(-\frac{2}{3} \right) \left(P_2(z) \vec{b}_i \cdot \hat{r} + \frac{1}{5} \vec{b}_i \cdot \hat{r} - \frac{3}{5} z b_i^z \right) \end{aligned} \quad (6.34)$$

Coefficients A_1 and A_3 are found by matching the respective l terms in

$\frac{\partial}{\partial \lambda} \underset{\sim}{\phi}_{3i}$ to those in ①

$$A_1 = -\frac{\omega_0 b h_1}{\rho i m H_1'} \left(\frac{\epsilon^2}{g^2} \right)$$

$$A_3 = -\frac{\omega_0 b h_1}{\rho i m H_3'} \left(\frac{\epsilon^2}{g^2} \right) \quad (6.35)$$

Term ② has an angular momentum dependence similar to ①.

Term ② is :

$$\textcircled{2} = \frac{\partial}{\partial r} \left(\frac{m R}{3} h_1'(mr) b P_2(z) \underset{\sim}{\chi}_i \vec{b}_i \cdot \hat{r} \right) \left(\frac{\epsilon^2}{g^2} \right) \quad (6.36)$$

Eq. (6.32) enables a solution. For ② one has

$$\begin{aligned} \phi_{3i} = & B_1 H_1(imr) \underline{\alpha}_i \left(-\frac{1}{5} \vec{b}_i \cdot \hat{r} + \frac{3}{5} z b_i^z \right) \\ & + B_3 H_3(imr) \underline{\alpha}_i \left(P_2(z) b_i \cdot \hat{r} + \frac{1}{5} b_i \cdot \hat{r} - \frac{3}{5} z b_i^z \right) \end{aligned} \quad (6.37)$$

where

$$\begin{aligned} B_1 = & \frac{\mu R b h_1''}{3 i H_1'} \left(\frac{\epsilon^2}{g^2} \right) \\ B_3 = & \frac{\mu R b h_3''}{3 i H_3'} \left(\frac{\epsilon^2}{g^2} \right) \end{aligned} \quad (6.38)$$

Now turn to the contributions ③ and ④. First solve for the pion field resulting from wave functions (6.6a) and Ψ_0

$$\phi_{3i} = A H_1(imr) \left(\underline{\alpha}_i \vec{b}_i \cdot \hat{r} P_{0i}^I + P_{0i}^{I+} \vec{b}_i \cdot \hat{r} \underline{\alpha}_i \right) \quad (6.39)$$

$$A = \frac{N^2 \hat{W}_{3\pi}}{im H_1'} \left(\frac{2j_0^2 R}{\rho} \right) \quad (6.40)$$

The wave functions (6.6b) and Ψ_0 give

$$\phi_{3i} = B H_1(imr) \left(\underline{\alpha}_i \vec{b}_i \cdot \hat{r} P_{2i}^I + H.C. \right) \quad (6.41)$$

$$B = \frac{N^2 C_1}{i \omega H_1'} \left(\frac{2\rho-3}{\rho} \right) j_0^2 \quad (6.42)$$

Wave functions (6.6c) and ψ_0 give

$$\tilde{\phi}_{3i} = C H_1(i\omega r) \left(\frac{\gamma_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{0i}^\pm + H.C. \right) \quad (6.43)$$

$$C = \frac{N^2 (3-2\rho) \hat{\omega}_{2\pi} j_0^2}{i \omega H_1' (\rho-1) \omega_0} \quad (6.44)$$

Similarly the contribution to ③ and ④ from wave functions (6.7a), (6.7b), (6.7c) and ψ_0 are.

$$\tilde{\phi}_{3i} = D H_1(i\omega r) \left(\frac{\gamma_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{0i} + H.C. \right) \quad (6.45)$$

$$D = \frac{2 N^2 \hat{\omega}_{2g} j_0^2 R}{i \omega H_1' \rho} \quad (6.46)$$

$$\tilde{\phi}_{3i} = E H_1(i\omega r) \left(\frac{\gamma_i}{2} b_i \cdot \hat{r} \mathcal{P}_{0i} + H.C. \right) \quad (6.47)$$

$$E = - \frac{N j_0}{i m H_1'} (K_1 + K_3) \left(\frac{g_c^2}{g^2} \right) \quad (6.48)$$

$$\tilde{\phi}_{3i} = F H_1 (i m r) \left(\frac{\gamma_i}{2} \vec{b}_i \cdot \hat{r} P_{2i} + H.c. \right) \quad (6.49)$$

$$F = - \frac{N j_0}{i m H_1'} (K_2 + K_4) \left(\frac{g_c^2}{g^2} \right) \quad (6.50)$$

The contribution from wave functions (6.8a), (6.8b) and ψ_0 are.

$$\tilde{\phi}_{3i} = G H_1 (i m r) \left(\frac{\gamma_i}{2} \vec{b}_i \cdot \hat{r} P_{2i}^{a_1} + H.c. \right) \quad (6.51)$$

$$G = \frac{N^2 a_1 j_0^2}{i m H_1'} \rho (2\rho - 3) \left(\frac{e^2}{g^2} \right) \quad (6.52)$$

$$\tilde{\phi}_{3i} = \beta_3 H_3 (i m r) \left(\frac{10}{3} \right) \frac{\gamma_i}{2} \left(P_{2(2)} \vec{b}_i \cdot \hat{r} + \frac{1}{5} \vec{b}_i \cdot \hat{r} - \frac{7}{5} z b_i^z \right) \quad (6.53)$$

$$\beta_3 = - \frac{N^2 a_2 \rho^2}{i m H_3'} (j_1 j_2 + j_3 j_0) \left(\frac{e^2}{g^2} \right) \quad (6.54)$$

Next the contribution of ⑤ and ⑥ to $\tilde{\phi}_{3i}$ is:

$$\begin{aligned} \phi_{3i} &= \alpha_1 H_1(i\mu r) \frac{\gamma_i}{2} \left(-\frac{1}{5} \vec{b}_i \cdot \hat{r} + \frac{3}{5} z b_i^z \right) \\ &+ \alpha_3 H_3(i\mu r) \frac{\gamma_i}{2} \left(P_2(z) b_i \cdot \hat{r} + \frac{1}{5} b_i \cdot \hat{r} - \frac{3}{5} z b_i^z \right) \end{aligned} \quad (6.55)$$

$$\alpha_1 = \frac{2}{3} \frac{N^2 \rho}{i\mu H_1'} \left(j_1' j_0 + j_0' j_1 \right) \left(\frac{\epsilon^2}{g^2} \right) \quad (6.56)$$

$$\alpha_3 = \frac{2}{3} \frac{N^2 \rho}{i\mu H_3'} \left(j_1' j_0 + j_0' j_1 \right) \left(\frac{\epsilon^2}{g^2} \right) \quad (6.57)$$

Below is a summary of ϕ to $O(g^3, \epsilon^2 g, g_c^2 g)$. Terms involving similar matrix elements have been grouped, although the matrix elements have not been evaluated for a particular hadron:

Summary:

$$\phi_i = g \phi_{1i} + g^3 \phi_{3i}$$

$$\phi_{1i} = b h_1(\mu r) b_i \cdot \hat{r} \gamma_i$$

$$\begin{aligned} \phi_{3i} &= (A + C) H_3(i\mu r) \left(\frac{\gamma_i}{2} \vec{b}_i \cdot \hat{r} P_{0i}^T + H.C. \right) \\ &+ B H_1(i\mu r) \left(\frac{\gamma_i}{2} \vec{b}_i \cdot \hat{r} P_{2i}^T + H.C. \right) + \end{aligned}$$

$$\begin{aligned}
& + (D+E) \mathcal{H}_1(i\nu r) \left(\frac{\alpha_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{0i} + H.C. \right) \\
& + F \mathcal{H}_1(i\nu r) \left(\frac{\alpha_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{2i} + H.C. \right) \\
& + \left(A_1 + B_1 + \frac{\alpha_1}{2} + \frac{5}{2} G \right) \mathcal{H}_1(i\nu r) \alpha_i \left(-\frac{1}{5} \vec{b}_i \cdot \hat{r} + \frac{2}{5} z b_i^z \right) \\
& + \left(-\frac{2}{3} A_3 + B_3 + \frac{\alpha_3}{2} + \frac{5}{3} \beta_3 \right) \mathcal{H}_3(i\nu r) \alpha_i \left(\mathcal{P}_2(z) \vec{b}_i \cdot \hat{r} + \frac{1}{5} \vec{b}_i \cdot \hat{r} - \frac{3}{5} z b_i^z \right) .
\end{aligned} \tag{6.58}$$

The different contributions to the pion field are shown in Figure 6.

One may note that there are no diagrams where the two pions don't couple to the same quark. This is peculiar to our classical perturbation scheme and is unavoidable. Presumably a true quantum field theoretic treatment would have to include the omitted diagrams since they are of $O(g^3)$. There is a classical treatment which includes the missing diagrams⁶. Said treatment sums the quark energies $w_{\pi i}$ over all the quarks in the chosen baryon, obtaining a number which is used instead of the operator $\hat{w}_{\pi i} \mathcal{P}_{0i}^z$ in Eqs. (6.6a) and (6.6c). We believe this is premature since the wave function so obtained ($\Psi_{\pi i}$) won't satisfy its linear boundary condition (2.14b).

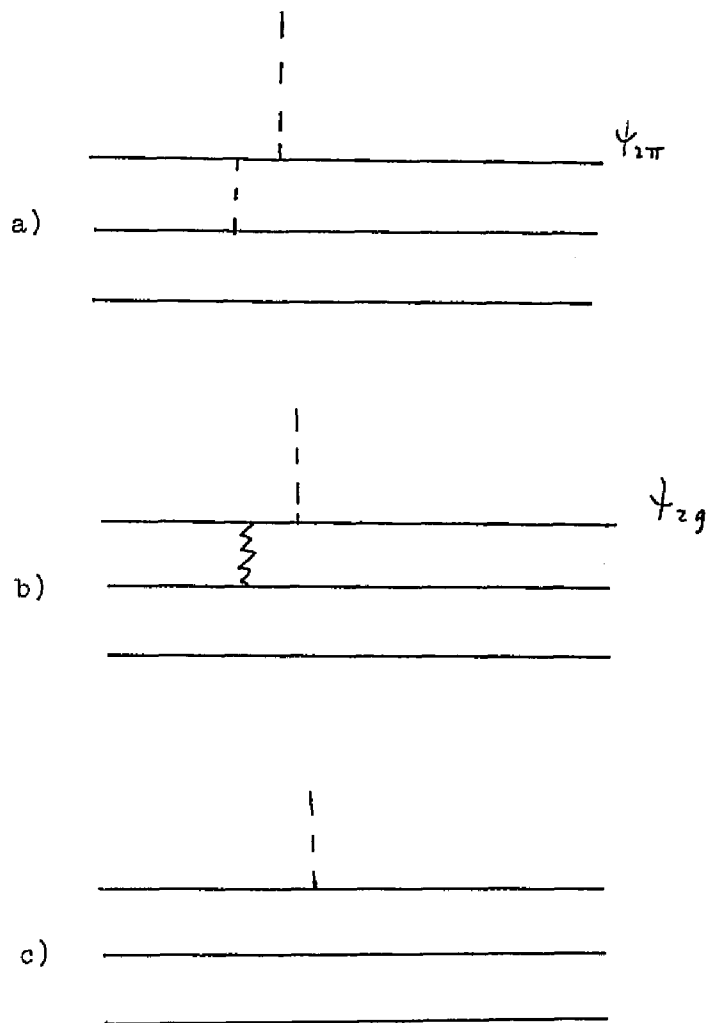


FIG. 6. Three types of contributions to ϕ_3 due to a) pions, b) gluons, c) deformation.

VII. DERIVATION OF THE COUPLING CONSTANTS

A. Derivation of $g_{\pi NN}$, $g_{\pi AA}$

The asymptotic form of the pion field resulting from a pointlike nucleon is shown in Appendix G. It is:

$$\underset{\sim}{\phi}_{NN} = \frac{g_{\pi NN}}{8\pi} \frac{\mu_\pi}{M_N} \frac{e^{-\mu_\pi r}}{r} \left(1 + \frac{1}{\mu_\pi r} \right) \vec{b}_N \cdot \hat{r} \underline{\chi}_N \quad (7.1)$$

We can determine $g_{\pi NN}$ in our bag model by putting $\underset{\sim}{\phi} = g \underset{\sim}{\phi}_1$ and $g^3 \underset{\sim}{\phi}_3$ in a similar form to Eq. (7.1) and then comparing. Recall the nucleon is not deformed to $O(g^2, g_c^2)$ so one ignores the ϵ^2 part of Eq. (6.58).

$$\underset{\sim}{\phi}_{NN} = g \sum_i \underset{\sim}{\phi}_{1i} + g^3 \underset{\sim}{\phi}_{3i} + \dots \quad (7.2)$$

$$\sum_i \underset{\sim}{\phi}_{1i} = \sum_i b h_i(\mu_\pi r) \vec{b}_i \cdot \hat{r} \underline{\chi}_i \quad (7.3)$$

$$\begin{aligned} \sum_i \underset{\sim}{\phi}_{3i} &= (A+C) \mathcal{H}_1(\mu_\pi r) \sum_i \left(\frac{\underline{\chi}_i}{\frac{1}{2}} \vec{b}_i \cdot \hat{r} \mathcal{P}_{0i}^{\underline{I}} + H.C. \right) \\ &+ B \mathcal{H}_1(\mu_\pi r) \sum_i \left(\frac{\underline{\chi}_i}{\frac{1}{2}} \vec{b}_i \cdot \hat{r} \mathcal{P}_{2i}^{\underline{I}} + H.C. \right) \\ &+ (D+E) \mathcal{H}_1(\mu_\pi r) \sum_i \left(\frac{\underline{\chi}_i}{\frac{1}{2}} \vec{b}_i \cdot \hat{r} \mathcal{P}_{0i} + H.C. \right) \\ &+ F \mathcal{H}_1(\mu_\pi r) \sum_i \left(\frac{\underline{\chi}_i}{\frac{1}{2}} \vec{b}_i \cdot \hat{r} \mathcal{P}_{2i} + H.C. \right) \end{aligned} \quad (7.4)$$

The various sums and coefficients must be evaluated for a nucleon.

Appendix F works out the sums. They are

$$\sum_i \vec{b}_i \cdot \hat{r} \tau_i = \frac{5}{3} \vec{b}_N \cdot \hat{r} \tau_N \quad (7.5)$$

$$\sum_i \left(\frac{\tau_i}{2} \vec{b}_i \cdot \hat{r} \rho_{0i}^{\pm} + H.C. \right) = \frac{34}{3} \vec{b}_N \cdot \hat{r} \tau_N \quad (7.6)$$

$$\sum_i \left(\frac{\tau_i}{2} \vec{b}_i \cdot \hat{r} \rho_{2i}^{\pm} + H.C. \right) = -\frac{2}{3} \vec{b}_N \cdot \hat{r} \tau_N \quad (7.7)$$

$$\sum_i \left(\frac{\tau_i}{2} \vec{b}_i \cdot \hat{r} \rho_{0i} + H.C. \right) = -\frac{2}{3} \vec{b}_N \cdot \hat{r} \tau_N \quad (7.8)$$

$$\sum_i \left(\frac{\tau_i}{2} \vec{b}_i \cdot \hat{r} \rho_{2i} + H.C. \right) = -\frac{2}{3} \vec{b}_N \cdot \hat{r} \tau_N \quad (7.9)$$

The coefficients are evaluated at $R_N=1$ Fermi and $m_\pi=138$ MeV.

$$b = 1.977 \times 10^{-2} \left(\frac{1}{R^2} \right) \quad (7.10)$$

$$A = \frac{2.153 \times 10^{-4}}{i} \left(\frac{1}{R^4} \right) \quad (7.11)$$

$$B = -\frac{1.624 \times 10^{-4}}{i} \left(\frac{1}{R^4} \right) \quad (7.12)$$

$$C = -\frac{1.121 \times 10^{-4}}{i} \left(\frac{1}{R^4} \right) \quad (7.13)$$

$$D = -\frac{1.929}{i} \times 10^{-4} \left(\frac{1}{R^2} \right) \left(\frac{g_c^2}{g^2} \right) \quad (7.14)$$

$$E = \frac{2.215}{i} \times 10^{-4} \left(\frac{1}{R^2} \right) \left(\frac{g_c^2}{g^2} \right) \quad (7.15)$$

$$F = -\frac{9.894}{i} \times 10^{-5} \left(\frac{1}{R^2} \right) \left(\frac{g_c^2}{g^2} \right) \quad (7.16)$$

We will break $g_{\pi NN}$ into two parts.

$$g_{\pi NN} = g_{\pi NN}^{(0)} + \Delta g_{\pi NN} \quad (7.17)$$

The first part comes from $g_{\pi NN}^{(0)}$, where we use a spin identity and compare the result to Eq. (7.1)

$$\begin{aligned} \frac{5}{3} g b \frac{e^{-m_r r}}{m_r} \left(1 + \frac{1}{m_r r} \right) \vec{b}_N \cdot \hat{r} \hat{\zeta}_N \\ = \frac{g_{\pi NN}}{8\pi} \frac{m_\pi}{M_N} \frac{e^{-m_r r}}{r} \left(1 + \frac{1}{m_r r} \right) \vec{b}_N \cdot \hat{r} \hat{\zeta}_N \end{aligned} \quad (7.18)$$

This gives us

$$g_{\pi NN}^{(0)} = 8\pi \frac{M_N}{(m_\pi)^2} \frac{5}{3} g b = 17.023 \quad (7.19)$$

This is a well known result. More interesting is $\Delta g_{\pi NN}$ for which one needs $g^3 \phi_3$

$$g^3 \phi_3 = \left\{ \frac{g^3}{R^4} (1.278 \times 10^{-3}) + 4\pi \alpha_c \frac{g}{R^2} (4.023 \times 10^{-5}) \right\} \times \frac{e^{-kr}}{kr} \left(1 + \frac{1}{kr} \right) \vec{b}_N \cdot \hat{r} \vec{\zeta}_N \quad (7.20)$$

Comparing this with Eq. (7.2) we can pick off $\Delta g_{\pi NN}$.

$$\Delta g_{\pi NN} = \frac{8\pi M_N}{R (k_N)^2} \left\{ \frac{g^3}{R^3} (1.278 \times 10^{-3}) + \frac{4\pi \alpha_c g}{R} (4.023 \times 10^{-5}) \right\} \quad (7.21)$$

We evaluate this for the nominal value $\alpha_c = 1.808$. The result is

$$\Delta g_{\pi NN} = 3.425 \quad (7.22)$$

The pion contribution to $\Delta g_{\pi NN}$ is 2.953 and the gluon contribution is 0.472. Including effects to $O(g^3, g_c^2 g)$ we have obtained

$$g_{\pi NN} = g_{\pi NN}^{(0)} + \Delta g_{\pi NN} = 20.448 \quad (7.23)$$

A breakdown of the contributions is shown in Table IV.

TABLE IV. Contributions to the pion-nucleon coupling constant $g_{\pi NN}$.

$g_{\pi NN}^{(0)}$	17.02
$\Delta g_{\pi NN}$ (pion)	2.95
$\Delta g_{\pi NN}$ (gluon)	0.47
$g_{\pi NN}$ (total)	20.45

B. Derivation of $g_{\pi\Delta\Delta}$

The pion field resulting from a pointlike Δ is shown in Appendix G. It is

$$\underset{\sim}{\phi}_{\Delta\Delta} = \frac{g_{\pi\Delta\Delta}}{8\pi} \left(\frac{m_\pi}{M_\Delta} \right) \frac{e^{-m_\pi r}}{r} \left(1 + \frac{1}{m_\pi r} \right) \vec{S} \cdot \hat{r} T \quad (7.24)$$

where S and T are the usual spin 3/2, isospin 3/2 operators. $\underset{\sim}{\phi}_{3i}$ is shown in Eq. (6.58) and we note that the term with $H_3(imr)$ which transforms like an $\ell=3$ object cannot be put in the form of Eq. (7.24). In addition there is an $H_1(imr)z\delta_i^z$ term which even though it has $\ell=1$, still cannot match Eq. (7.24).

Clearly the $\ell=3$ term as well as the $z\delta_i^z$ term represent differences with the usual pion field which results from a pointlike source. Originally form factors were a way of putting in the finite size of nuclear matter. It appears that if we are to take quark bags seriously, not only will the form factors be functions of q^2 , but they might also have spin dependence. Certainly the unusual aspects of $\underset{\sim}{\phi}_{\Delta\Delta}$ must result in an O.P.E. which differs from those in the past.

For our purpose of calculating $g_{\pi\Delta\Delta}$ we are forced to ignore those parts of the $\underset{\sim}{\phi}_{\Delta\Delta}$ which don't correspond to the classical fields. We just write out the parts of Eq. (6.58) which can be put in the same form as Eq. (7.24).

$$\phi_{\sim 1} = \sum_i b_{h_1(lmr)} \vec{b}_i \cdot \hat{r} \alpha_i \quad (7.25)$$

$$\begin{aligned} \phi_{\sim 3} &= \left(-\frac{A_1}{5} \quad -\frac{B_1}{5} \quad -\frac{\alpha_1}{10} \quad -\frac{G}{2} \right) \mathcal{H}_1(lmr) \sum_i \alpha_i \vec{b}_i \cdot \hat{r} \\ &+ (A+C) \mathcal{H}_1(lmr) \sum_i \left(\frac{\alpha_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{0i}^{\mp} + \text{H.C.} \right) \\ &+ B \mathcal{H}_1(lmr) \sum_i \left(\frac{\alpha_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{2i}^{\mp} + \text{H.C.} \right) \\ &+ (D+E) \mathcal{H}_1(lmr) \sum_i \left(\frac{\alpha_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{0i} + \text{H.C.} \right) \\ &+ F \mathcal{H}_1(lmr) \sum_i \left(\frac{\alpha_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{2i} + \text{H.C.} \right) \end{aligned} \quad (7.26)$$

The sums are worked out in Appendix F. They are:

$$\sum_i \vec{b}_i \cdot \hat{r} \alpha_i = \frac{4}{3} T_{\sim} \vec{S} \cdot \hat{r} \quad (7.27)$$

$$\sum_i \left(\frac{\alpha_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{0i}^{\mp} + \text{H.C.} \right) = \frac{8}{3} T_{\sim} \vec{S} \cdot \hat{r} \quad (7.28)$$

$$\sum_i \left(\frac{\alpha_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{2i}^{\mp} + \text{H.C.} \right) = \frac{8}{3} T_{\sim} \vec{S} \cdot \hat{r} \quad (7.29)$$

$$\sum_i \left(\frac{\alpha_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{0i} + \text{H.C.} \right) = \frac{8}{3} T_{\sim} \vec{S} \cdot \hat{r} \quad (7.30)$$

$$\sum_i \left(\frac{\alpha_i}{2} \vec{b}_i \cdot \hat{r} \mathcal{P}_{2i} + \text{H.C.} \right) = \frac{8}{3} T_{\sim} \vec{S} \cdot \hat{r}$$

The necessary coefficients which have been evaluated at $R = 1.249$ Fermi and $m_{\pi} = 138$ MeV are:

$$b = \frac{3.162 \times 10^{-2}}{R^2} \quad (7.31)$$

$$A_1 = \frac{1.314 \times 10^{-2}}{i} \left(\frac{1}{R^2} \right) \left(\frac{E^2}{g^2} \right) \quad (7.32)$$

$$B_1 = \frac{-3.319 \times 10^{-2}}{i} \left(\frac{1}{R^2} \right) \left(\frac{E^2}{g^2} \right) \quad (7.33)$$

$$\alpha_1 = \frac{4.215 \times 10^{-2}}{i} \left(\frac{1}{R^2} \right) \left(\frac{E^2}{g^2} \right) \quad (7.34)$$

$$G = \frac{7.762 \times 10^{-3}}{i} \left(\frac{1}{R^2} \right) \left(\frac{E^2}{g^2} \right) \quad (7.35)$$

$$A = \frac{3.272 \times 10^{-4}}{i} \left(\frac{1}{R^4} \right) \quad (7.36)$$

$$B = \frac{-2.470 \times 10^{-4}}{i} \left(\frac{1}{R^4} \right) \quad (7.37)$$

$$C = \frac{-1.703 \times 10^{-4}}{i} \left(\frac{1}{R^4} \right) \quad (7.38)$$

$$D = \frac{-2.926 \times 10^{-4}}{i} \left(\frac{1}{R^4} \right) \left(\frac{g_c^2}{g^2} \right) \quad (7.39)$$

$$E = \frac{3.542 \times 10^{-4}}{i} \left(\frac{1}{R^2} \right) \left(\frac{g_c^2}{g^2} \right) \quad (7.40)$$

$$F = -\frac{1.582 \times 10^{-4}}{i} \left(\frac{1}{R^2} \right) \left(\frac{g_c^2}{g^2} \right) \quad (7.41)$$

Now break $g_{\pi\Delta\Delta}$ into two parts, where the first part comes from $g_{\sim 1}^{\phi_1}$ and the second part from $g_{\sim 3}^{\phi_3}$.

$$g_{\pi\Delta\Delta} = g_{\pi\Delta\Delta}^{(0)} + \Delta g_{\pi\Delta\Delta} \quad (7.42)$$

Using (7.25), (7.27) and (7.31) we calculate $g_{\pi\Delta\Delta}^{(0)}$ first

$$g_{\sim 1}^{\phi_1} = g \cdot \frac{3.162 \times 10^{-2}}{R^2} \frac{e^{-m_r r}}{m_r} \left(1 + \frac{1}{m_r} \right) \frac{4}{3} \frac{T \vec{s} \cdot \hat{r}}{\sim} \quad (7.43)$$

Comparing this with Eq. (7.24) gives

$$\begin{aligned} g_{\pi\Delta\Delta}^{(0)} &= 8\pi \left(\frac{M_\Delta}{m_\pi} \right) \frac{1}{m_\pi R} \left(\frac{g}{R} \right) (3.162 \times 10^{-2}) \left(\frac{4}{3} \right) \\ &= 18.324 \end{aligned} \quad (7.44)$$

Next calculate $\Delta g_{\pi\Delta\Delta}$ from ϕ_3 .

We will show the $|m_z|=3/2$ case.

$$g^3 \underset{\sim}{\phi}_3 = \left\{ \epsilon^2 (-1.055 \times 10^{-2}) - 1.334 \times 10^{-3} + \alpha_c (-6.270 \times 10^{-3}) \right\} \\ \times \frac{e^{-mr}}{r} \left(1 + \frac{1}{mr} \right) \vec{S} \cdot \hat{r} \underset{\sim}{T} \quad (7.45)$$

Comparing this with (7.24) we get

$$\Delta g_{\pi\Delta\Delta} = 8\pi \frac{M_\Delta}{m_\pi} \\ \times \left\{ \epsilon^2 (-1.055 \times 10^{-2}) - 1.334 \times 10^{-3} + \alpha_c (-6.271 \times 10^{-3}) \right\} \quad (7.46)$$

This is evaluated and is

$$\Delta g_{\pi\Delta\Delta} = -3.163 \quad (7.47)$$

A breakdown of both $|m_z|=3/2$ and $|m_z|=1/2$ is shown in Table V.

C. The Ratio $g_{\pi\Delta\Delta}/g_{\pi NN}$

One obtains the SU(6) ratios $f_{\pi NN} : f_{\pi\Delta\Delta} : f_{\pi N\Delta}$ by assuming non relativistic quarks with a pseudovector coupling to the pion field of the form

$$\mathcal{L} = \frac{f_{\pi qq}}{m_\pi} \psi^\dagger \sum_i \vec{t}_i \cdot \vec{\nabla} \underline{z}_i \cdot \phi \psi \quad (7.48)$$

where Ψ is the nucleon or Δ operator. This gives the SU(6) result $\frac{f_{\pi\Delta\Delta}}{f_{\pi NN}} = 4/5$. The pseudoscalar coupling constants are simply related at $q^2=0$ by

$$\frac{g_{\pi NN}}{2 M_N} = \frac{f_{\pi NN}}{m_\pi} \quad (7.49)$$

$$\frac{g_{\pi\Delta\Delta}}{2 M_\Delta} = \frac{f_{\pi\Delta\Delta}}{m_\pi} \quad (7.50)$$

Combining the SU(6) result with Eqns. (7.49) and (7.50) gives

$$\frac{g_{\pi\Delta\Delta}}{g_{\pi NN}} = \frac{M_\Delta}{M_N} \left(\frac{4}{5} \right) = 1.05 \quad (7.51)$$

Experimental results give a much smaller ratio. Arndt et al² give a value

$$\left. \frac{g_{\pi\Delta\Delta}}{g_{\pi NN}} \right|_{\text{Experiment}} = (.57 \pm .14) \times \text{SU}(6) \text{ Ratio} \\ = .598 \pm .147 \quad (7.52)$$

The only calculation we have seen, was done by Duck and Umland⁸ using a cloudy bag model. They get a result which is even larger than the SU(6) ratio:

$$\frac{g_{\pi\Delta\Delta}}{g_{\pi NN}} = 1.26 \quad (7.53)$$

Our ratio is barely within the experimental error of Arndt et al.² We have

$$\left. \frac{g_{\pi\Delta\Delta}}{g_{\pi NN}} \right|_{|m_z|=3/2} = \frac{15.16}{20.45} = .741$$

$$\left. \frac{g_{\pi\Delta\Delta}}{g_{\pi NN}} \right|_{|m_z|=1/2} = \frac{15.80}{20.45} = .773 \quad (7.54)$$

These ratios include the effects of pions, gluons, and deformation of the Δ . Table VI gives a breakdown of various contributions to said ratios. Note that both $|m_z|=3/2$ and $|m_z|=1/2$ cases were close to the experimental ratio but the former was within experimental error. In fact the ratio was much more dependent on pion and gluon effects than deformation.

TABLE V. Contribution to the pion- Δ coupling constant for the two cases of Δ spin projection $|m_{\pm}| = \frac{1}{2}$, $|m_{\pm}| = \frac{3}{2}$.

	$ m_{\pm} = \frac{1}{2}$	$ m_{\pm} = \frac{3}{2}$
$g_{\pi\Delta\Delta}^{(0)}$	18.32	18.32
$\Delta g_{\pi\Delta\Delta}$ (pion)	-0.30	-0.30
$\Delta g_{\pi\Delta\Delta}$ (gluon)	-2.54	-2.54
$\Delta g_{\pi\Delta\Delta}$ (ϵ^2)	+0.32	-0.32
$g_{\pi\Delta\Delta}$ (total)	15.80	15.16

TABLE VI. The ratio of the pion- Δ coupling constant to the pion-nucleon coupling constant.

	$ m_{\pm} = \frac{1}{2}$	$ m_{\pm} = \frac{3}{2}$
$\frac{g_{\pi\Delta\Delta}}{g_{\pi NN}}$ (without deformation)	0.757	0.757
$\frac{g_{\pi\Delta\Delta}}{g_{\pi NN}}$ (with deformation)	0.773	0.741

VIII. THE MAGNETIC MOMENTS

A Quark Contributions to Magnetic Moments

The electromagnetic current operator for Dirac fields is

$$J_i^\mu = e \bar{\Psi}_i \gamma^\mu Q_i \Psi$$

Q_i is the quark charge operator and e is the charge of a positron. If the magnetic field is constant then the electromagnetic vector potential \vec{A} may be chosen to be

$$\vec{A} = \frac{1}{2} \vec{B} \times \vec{r} \quad (8.1)$$

The electromagnetic interaction Hamiltonian is

$$-\int \vec{J} \cdot \vec{A} d^3r \equiv -\vec{\mu} \cdot \vec{B} \quad (8.2)$$

So the magnetic moment operator is

$$\mu_i = \frac{1}{2} e \int \bar{\Psi}_i Q_i \vec{r} \times \vec{\gamma} \Psi_i d^3r \quad (8.3)$$

The index i refers to the i th quark and the moment is taken in the \hat{z} direction. Naturally the total moment involves a sum over all three quarks. The charge operator for up and down quarks only is

$$Q_i = \frac{1}{6} + \frac{\hat{z}_i}{2} \quad (8.4)$$

We wish to obtain a result to $O(g^2, e^2, g_c^2)$ and so will use the wave functions shown in Chapter VI. Below is a recital of the well known $O(1)$ moment before we turn to the new contributions.

$$\mu_i = \frac{1}{2} e \int \bar{\psi}_0 \vec{r} \times \vec{\gamma} Q_i \psi_0 r^3 dr d\Omega \quad (8.5)$$

$$= \frac{1}{2} e N^2 \int \left[i j_0 j_1 (\hat{r} \times \vec{b}_i) \vec{b}_i \cdot \hat{r} - i j_0 j_1 (\vec{b}_i \cdot \hat{r}) (\hat{r} \times \vec{b}_i) \right] \cdot Q_i r^3 dr d\Omega \quad (8.6)$$

After working the integral and summing over the quarks one has

$$\vec{\mu} = \frac{eR(4\rho-3)}{12\rho(\rho-1)} \sum_i Q_i \vec{b}_i \quad (8.7)$$

Note that in comparison with the non-relativistic quark model, instead

of e/m , Eq. (8.7) has eR . The neutron and proton moments are reproduced by (8.7) for $R_n=1.5$ Fermi. Since we are using $R_n=1.0$ Fermi, both neutron and proton moments will be $2/3$ too small unless the $O(g^2, g_c^2)$ contributions help us.

The quark wave functions $\Psi_{2\pi i}$ will give a moment of the form

$$\vec{\mu}_i = \frac{1}{2} e g^2 \int \left[\bar{\Psi}_{0i} \hat{r} \times \vec{\gamma} Q_i \Psi_{2\pi i} + \bar{\Psi}_{2\pi i} \hat{r} \times \vec{\gamma} Q_i \Psi_{0i} \right] r^3 dr d\Omega \quad (8.8)$$

where the second term is just the Hermitian conjugate of the first term.

We will calculate the first term using $\Psi_{2\pi i} = (6.6a)$.

$$\begin{aligned} \vec{\mu}_i = \frac{1}{2} e g^2 N^2 \hat{W}_{2\pi} \int & (j_{0,i} \vec{b}_i \cdot \hat{r} j_{1,i}) \begin{pmatrix} 0 & \hat{r} \times \vec{b}_i \\ -\hat{\lambda} \times \vec{b}_i & 0 \end{pmatrix} Q_i \begin{pmatrix} r j_{0,i}' \\ i \vec{b}_i \cdot \hat{r} r j_{1,i}' \end{pmatrix} \\ & \cdot \mathcal{P}_{0i}^{\pm} r^3 dr d\Omega \end{aligned} \quad (8.9)$$

The angular integral is facilitated using

$$\begin{aligned} (\hat{r} \times \vec{b}_i) (\vec{b}_i \cdot \hat{r}) &= -(\vec{b}_i \cdot \hat{r}) (\hat{r} \times \vec{b}_i) \\ \int d\Omega \hat{r}^i \hat{r}^j &= \int d\Omega \frac{\delta^{ij}}{3} \\ (\vec{b}_i \times \vec{b}_i) &= 2i \vec{b}_i \end{aligned} \quad (8.10)$$

We obtain the general form of some constants times a matrix element times a radial integral. Upon working (8.9), adding its Hermitian conjugate and summing over the three quarks one has

$$\vec{M} = \frac{4\pi}{3} e g^2 N^2 \frac{\hat{W}_{2\pi}}{W_{45}} \left[\sum_i Q_i \vec{b}_i \cdot \vec{p}_{0i} + p_{0i}^+ \vec{b}_i \cdot Q_i \right] \mathbb{I}_\pi^{(1)} \quad (8.11)$$

where

$$\mathbb{I}_\pi^{(1)} = \int_0^{\rho} \chi^4 d\chi \left[j_0 j_1' + j_1 j_0' \right] \quad (8.12)$$

The value of $\mathbb{I}_\pi^{(1)}$ is given in Appendix H and the matrix elements are worked in Appendix F. We will put all the moments in the form of Eq. (8.11) and ultimately summarize and evaluate them. The next two contributions from $\Psi_{\pi i}$, namely $g^2 \downarrow_{\pi i}^{(2)} = (6.6b)$ and $g^2 \downarrow_{\pi i}^{(3)} = (6.6c)$ give

$$\begin{aligned} \vec{M} = & \pi e g^2 N^2 C_1 \left\{ \sum_{\substack{i,j \\ i \neq j}} \left[i \underline{\tau}_i \cdot \underline{\tau}_j \vec{b}_j \times \vec{b}_i + \frac{2}{3} \underline{\tau}_i \cdot \underline{\tau}_j (\vec{b}_j \cdot \vec{b}_i) \vec{b}_i \right] Q_i \right. \\ & \left. + Q_i \left[i \vec{b}_i \times \vec{b}_j \underline{\tau}_i \cdot \underline{\tau}_j + \frac{2}{3} \vec{b}_i (\vec{b}_i \cdot \vec{b}_j) \underline{\tau}_i \cdot \underline{\tau}_j \right] \right\} \mathbb{I}_\pi^{(2)} \quad (8.13) \end{aligned}$$

$$\mathbb{I}_\pi^{(2)} = \int_0^{\rho} \chi^3 d\chi \left[-j_2 j_1 + j_1 j_2 \right] \quad (8.14)$$

and

$$\vec{\mu} = \frac{8\pi}{3} \frac{e g^2 N^2}{\omega_0^4} \left(\frac{(2p-3) \hat{\omega}_{2p}}{2(p-1) \omega_0} \right) \left\{ \sum_i Q_i \vec{b}_i p_{oi} + H.C. \right\} \mathcal{I}_\pi^{(3)} \quad (8.15)$$

$$\mathcal{I}_\pi^{(3)} = \int_0^p x^3 dx j_{0j_1} \quad (8.16)$$

There are three contributions from ψ_{ig} ; namely $\frac{2}{g} \psi_{2g}^{(1)}$ = (6.7a), $\frac{2}{g} \psi_{2g}^{(2)}$ = (6.7b) and $\frac{2}{g} \psi_{2g}^{(3)}$ = (6.7c). Respectively these give

$$\vec{\mu} = \frac{4\pi}{3} \frac{e g_c^2 N^2}{\omega_0^5} \hat{\omega}_{2g} \left\{ \sum_i \left[Q_i \vec{b}_i p_{oi} + H.C. \right] \right\} \mathcal{I}_g^{(1)} \quad (8.17)$$

$$\vec{\mu} = \frac{4\pi}{3} \frac{e g_c^2 N}{\omega_0^4} \left\{ \sum_i \left[Q_i \vec{b}_i p_{oi} + H.C. \right] \right\} \mathcal{I}_g^{(2)} \quad (8.18)$$

$$\begin{aligned} \vec{\mu} = & -\pi e g_c^2 \frac{N}{\omega_0^4} \left\{ \sum_{\substack{i,j \\ i \neq j}} \left[i (\vec{b}_j \times \vec{b}_i) + \frac{2}{3} (\vec{b}_j \cdot \vec{b}_i) \vec{b}_i \right] Q_i \right. \\ & \left. + Q_i \left[i (\vec{b}_i \times \vec{b}_j) + \frac{2}{3} \vec{b}_i (\vec{b}_j \cdot \vec{b}_i) \right] \right\} \mathcal{I}_g^{(3)} \end{aligned} \quad (8.19)$$

where the integrals are

$$I_g^{(1)} = \int_0^p [j_1' j_0 + j_0' j_1] x^4 dx \quad (8.20)$$

$$I_g^{(2)} = \int_0^p [j_1' k_1 + j_0' k_3] x^3 dx \quad (8.21)$$

$$I_g^{(3)} = \int_0^p [j_0' k_4 + j_1' k_2] x^3 dx \quad (8.22)$$

Next are the two contributions from $\psi_{2\lambda i}$. $\epsilon^2 \psi_{2\lambda i}^{(1)} = (6.8a)$ and $\epsilon^2 \psi_{2\lambda i}^{(2)} = (6.8b)$

$$\vec{\mu}^{(1)} = 2\pi e \epsilon^2 \frac{N^2 a_1}{\omega_0^4} \left\{ \sum_i Q_i \left[(\delta^{k3} b_i^3 - b_i^k) + \frac{2}{3} \vec{b}_i \right] \right\} I_{\lambda}^{(1)} \quad (8.23)$$

$$\mu^{(2)} = 0 \quad (8.24)$$

$$I_{\lambda}^{(1)} = \int_0^p [j_0 j_1 - j_1 j_2] x^3 dx$$

The $\psi_{2\lambda}^{(2)}$ integral is zero because of the Wigner-Eckart theorem. Also note the $\psi_{2\lambda}^{(1)}$ matrix element has a $\delta^{k3} b_i^3 - b_i^k$ part which is zero when $\vec{\mu}$ is in the \hat{z} direction. The remaining matrix element in (8.23) is

$$\sum_i Q_i \frac{z}{r} \vec{b}_i \quad (8.25)$$

The last quark contribution to $\vec{\mu}$ arises from deformed geometry acting on the lowest order quark wave function. In fact Eq. (8.6) has a portion in addition to (8.7) which we shall now work.

Consider the integral

$$\vec{\mu}_i = \frac{1}{2} e \int_V \bar{\psi}_{0i} \hat{r} \times \vec{\gamma} Q_i \psi_{0i} r^3 dr d\Omega \quad (8.26)$$

Break this into two integrals

$$\textcircled{1} = \frac{1}{2} e \int_0^R \bar{\psi}_{0i} \hat{r} \times \vec{\gamma} Q_i \psi_{0i} r^3 dr d\Omega \quad (8.27)$$

$$\textcircled{2} = \frac{1}{2} e \int_R^{R+\Delta R} \bar{\psi}_{0i} \hat{r} \times \vec{\gamma} Q_i \psi_{0i} r^3 dr d\Omega \quad (8.28)$$

But ① has already been done giving result (8.7). We only need the $O(\epsilon^2)$ part of ② so write $I(\Delta R)$ as a Taylor series :

$$I(\Delta R) = I(0) + \Delta R I'(0) + \dots \quad (8.29)$$

One gets

$$I(0) = 0 \quad (8.30)$$

and

$$I'(0) = \frac{1}{2} \epsilon R^3 \bar{\psi}_{0i} \hat{r} \times \vec{\gamma} Q_i \psi_{0i} \quad (8.31)$$

$$\Delta R = -\frac{\epsilon^2}{3} R P_2(z) \quad (8.32)$$

giving

$$\textcircled{2} = -\frac{\epsilon^2}{6} \epsilon R^4 \int d\Omega P_2(z) \bar{\psi}_{0i} \hat{r} \times \vec{\gamma} Q_i \psi_{0i} \quad (8.33)$$

This is easily integrated using the identity

$$\int d\Omega \hat{r}^k \hat{r}^j P_2(z) = 4\pi \left[\frac{-\delta^{ij}}{15} + \frac{\delta^{i3} \delta^{j3}}{5} \right] \quad (8.34)$$

giving

$$\textcircled{2} = \frac{4\pi}{15} e \epsilon^2 N^2 R^4 j_0^2 \left\{ \sum_i \left[\frac{2}{3} \vec{b}_i^k - i (\vec{b}_i \times \vec{b}_i^3)^k \right] Q_i \right\} \quad (8.35)$$

where the subscript i indicates which quark and the superscript k refers to a cartesian index. One sees the second term is zero since the \hat{z} direction is taken for the magnetic moment.

B. Pion Part of the Magnetic Moment

The pion charge current is

$$\vec{J} = ie \left[\phi_+ \vec{\nabla} \phi_- - \phi_- \vec{\nabla} \phi_+ \right] \quad (8.36)$$

to $O(g^2)$ this is

$$\vec{J} = ig^2 e \left[\phi_{1+} \vec{\nabla} \phi_{1-} - \phi_{1-} \vec{\nabla} \phi_{1+} \right], \quad (8.37)$$

where $\phi_{1\pm}$ are related to ϕ_1 by

$$\phi_{1\pm} = \sum_i b_{h_1(\mu r)} \vec{b}_i \cdot \hat{r} \zeta_i^{\pm} \quad (8.38)$$

The magnetic moment is

$$\vec{\mu} = \frac{1}{2} e \int \vec{r} \times \vec{J} d^3 r \quad (8.39)$$

which becomes

$$\begin{aligned} \frac{1}{2} e g^2 b^2 \sum_{i,j} \int h_i^2(\mu r) (\vec{b}_i \cdot \hat{r}) (\hat{r} \times \vec{b}_j) \cdot \\ \cdot \left[\tilde{\chi}_i^+ \tilde{\chi}_i^- - \tilde{\chi}_i^- \tilde{\chi}_i^+ \right] r^2 dr d\Omega \end{aligned} \quad (8.40)$$

$$= \frac{e}{2} g^2 b^2 \sum_{i,j} \int h_i^2 \frac{(\vec{b}_i \times \vec{b}_j)}{3} (\tilde{\chi}_i \times \tilde{\chi}_j)^z r^2 dr d\Omega \quad (8.41)$$

or

$$\vec{\mu} = \frac{2\pi}{3} e g^2 \frac{b^2}{\mu^3} \sum_{i,j} \left[\vec{b}_i \times \vec{b}_j (\tilde{\chi}_i \times \tilde{\chi}_j)^z \right] I_{\phi\phi} \quad (8.42)$$

$$I_{\phi\phi} = \int_{\mu\sigma R}^{\infty} h_i^2(x) x^2 dx \quad (8.43)$$

C. Summary of Nucleon Magnetic Moments

The necessary matrix elements are worked out in Appendix F.

They are:

$$\sum_i Q_i b_i^3 = \frac{S^3}{3} + \frac{10}{3} \frac{T^3}{\sim} S^3 \quad (8.44)$$

$$\sum_i \left[Q_i b_i^3 P_{0i}^{\perp} + \text{H.C.} \right] = \frac{4}{3} S^3 + \frac{136}{3} \frac{T^3}{\sim} S^3 \quad (8.45)$$

$$\begin{aligned} & \sum_{\substack{i,j \\ i \neq j}} \left\{ \left[i \tau_j \cdot \tau_i (\vec{b}_j \times \vec{b}_i)^3 + \frac{2}{3} \tau_j \cdot \tau_i (\vec{b}_j \cdot \vec{b}_i) b_i^3 \right] Q_i \right. \\ & \left. + Q_i \left[i (\vec{b}_i \times \vec{b}_j)^3 \tau_i \cdot \tau_j + \frac{2}{3} b_i^3 (\vec{b}_i \cdot \vec{b}_j) \tau_i \cdot \tau_j \right] \right\} \quad (8.46) \\ & = \frac{8}{9} S^3 - \frac{16}{9} \frac{T^3}{\sim} S^3 \end{aligned}$$

$$\sum_i \left[Q_i b_i^3 P_{0i} + \text{H.C.} \right] = \frac{4}{3} S^3 - \frac{8}{3} \frac{T^3}{\sim} S^3 \quad (8.47)$$

$$\sum_{\substack{i,j \\ i \neq j}} \left\{ \left[i(\vec{b}_j \times \vec{b}_i)^3 + \frac{2}{3} (\vec{b}_j \cdot \vec{b}_i) b_i^3 \right] Q_i \right. \\ \left. + Q_i \left[i(\vec{b}_i \times \vec{b}_j)^3 + \frac{2}{3} b_i^3 (\vec{b}_i \cdot \vec{b}_j) \right] \right\} = \frac{8}{9} S^3 - \frac{16}{9} \tilde{T}^3 S^3 \quad (8.48)$$

$$\sum_{i,j} \left[(\vec{b}_i \times \vec{b}_j)^3 (\tilde{\alpha}_i \times \tilde{\alpha}_j)^3 \right] = -\frac{126}{3} \tilde{T}^3 S^3 \quad (8.49)$$

After evaluating the integrals in Appendix H and calculating the various coefficients and matrix elements one has eight contributions to the nucleon moment. These are broken down in Table VII. The Δ magnetic moments are also calculated and displayed in Table VIII. "Feynman" diagrams for the moment contributions are shown in Figure 7.

TABLE VII. Contributions to proton and neutron magnetic moments. The nominal values used were $\alpha_c = 1.808$, $R_N = 1 \cdot \text{Fermi}$, $g = \frac{1}{97.3 \text{ MeV}}$

Source of contribution	Proton	Neutron
$\psi_0^+ \psi_0$.8265	-.5510
$\psi_{2\pi}^+ \psi_0 + \text{H.C.}$	-.0325	.0430
$\psi_{2g}^+ \psi_0 + \text{H.C.}$	0	.0489
$\phi_1 \cdot \phi_1$.3042	-.3042
Total (units of $\frac{e}{2m_0}$)	1.0982	-.7633
Total (units of $\frac{e}{2M_N}$)	2.5586	-1.7783

TABLE VIII. Contributions to Δ^+ and Δ^0 magnetic moments. Parameters were $\alpha_c = 1.808$, $R_D = 1.249$ Fermi, $g = \frac{1}{92.7 \text{ MeV}}$, $\epsilon^2 = .135$

Source of contribution	Δ^+	Δ^0
$t_0^+ t_0$.5510	-.2755
$t_{1\pi} t_0 + \text{H.C.}$.0178	0
$t_{2g} t_0 + \text{H.C.}$.1465	0
$t_{2d} t_0 + \text{H.C.} \frac{R+DR}{R}$.0033	-.0016
$\phi_1 \cdot \phi_1$.0421	-.0421
Total (units of $\frac{e}{2M\nu}$)	.7607	-.3192
Total (units of $\frac{e}{2M\nu}$)	1.7723	-.7437

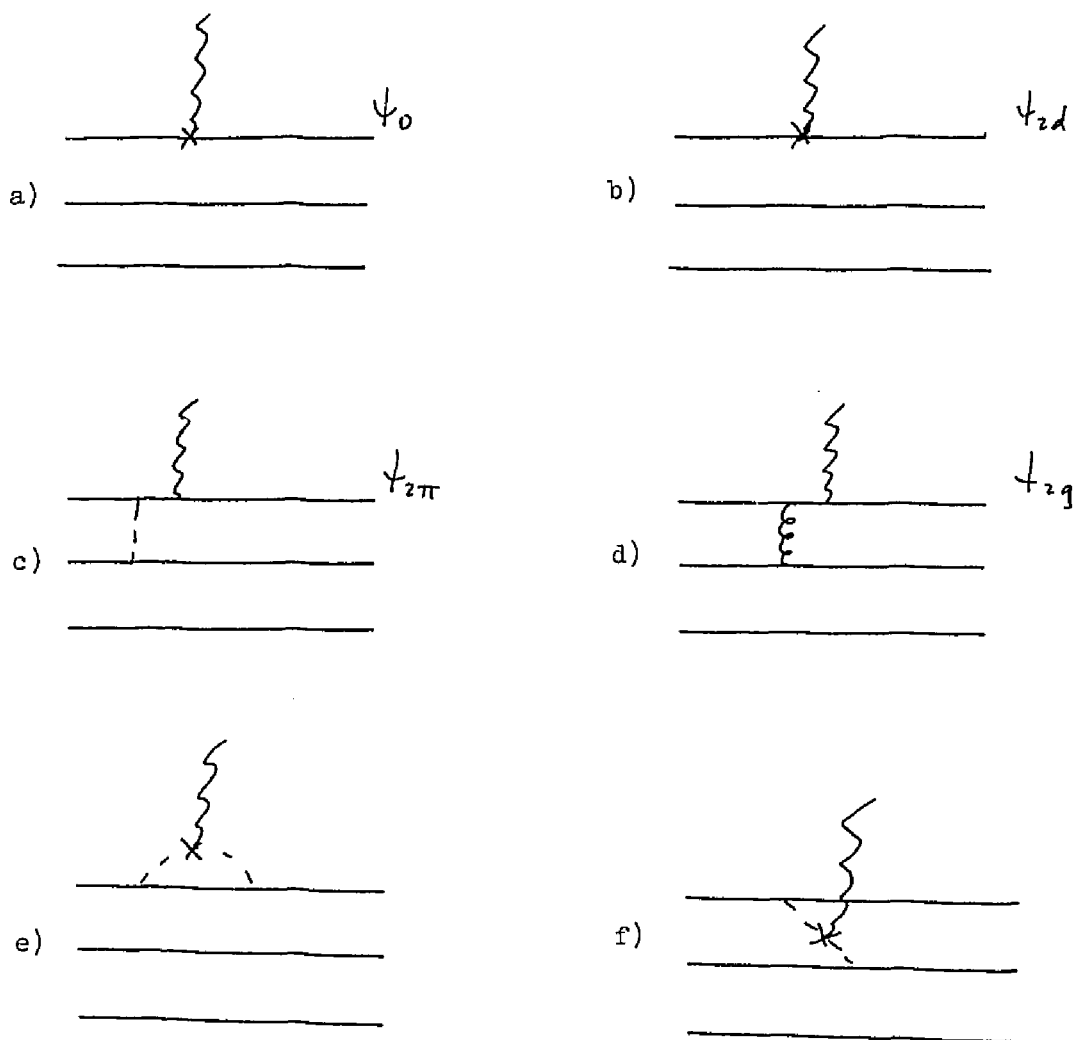


FIG. 7. Contributions to magnetic moments where a) is due to the original S-state quarks. b) is due to ψ_{2d} . c) is due to $\psi_{2\pi}$. d) is due to ψ_{2g} . e) and f) are the pion contributions.

IX. APPROXIMATIONS AND SOME LOOSE ENDS IN THE BAG MODEL

A Pointlike Pion in Quantum Field Theory

The Lagrangian Eq. (2.1) we use is unusual and ambiguous in the sense that it is a "hybrid" of the old nuclear physics and the more recent quantum chromodynamics. Our modern impression is that the pion should no longer be treated field theoretically as a pointlike object but instead as a composite of quarks and gluons. To be consistent with the MIT interpretation of hadrons as bag like objects the pion must have a radius and quark substructure like the baryons. This means that the surface delta function interaction at the confinement radius must be an approximation and at the very least the pion bags should interact with the baryon bags in some finite volume which might involve the pion radius as well.

The use of phenomenological pointlike pions seems reasonable in a low energy regime where the pion wave length would be much greater than the pion bag radius. This reasonableness at low energies is borne out in the cloudy bag calculation of the pion form factor which is compared with a best fit Gaussian form factor in Thomas' review article.⁹ We reproduce Thomas' figure in Figure 8. One notices good agreement between the form factors at KR up to 3 which is precisely the long wavelength regime if we assume the pion radius about half the

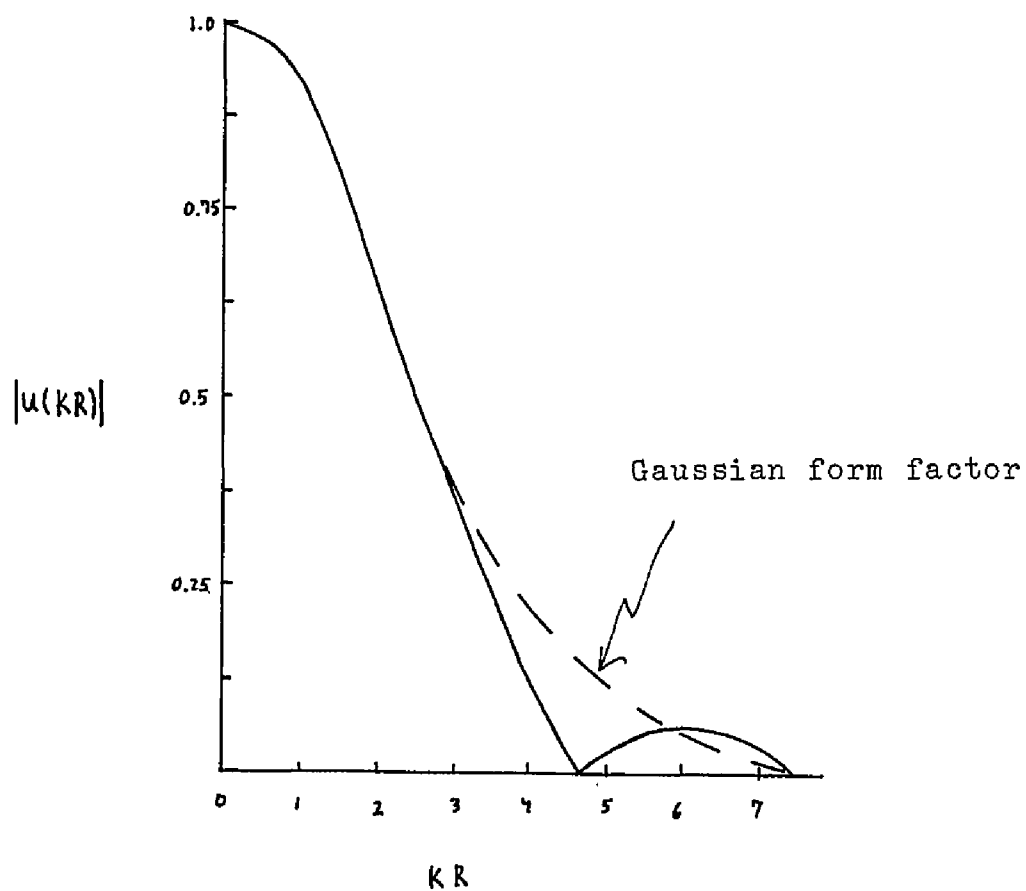


FIG. 8. Cloudy bag form factor compared to best fit Gaussian form factor.

nucleon radius. That is if

$$k R_N \leq 3$$

$$\frac{2\pi}{\lambda} (2 R_\pi) \leq 3 \tag{9.1}$$

Then

$$\lambda \geq 4 R_\pi \tag{9.2}$$

The discrepancies crop up in the oscillatory nature of the cloudy bag form factor at shorter wavelengths. The cloudy bag form factor is merely

$$U(KR) = \frac{3}{KR} \left(\frac{\sin(KR)}{(KR)^2} - \frac{\cos(KR)}{KR} \right) \tag{9.3}$$

The wiggleness of $U(KR)$ at high momenta is an artifact of the surface coupling of the pointlike pions. Any smoothing of the interaction, say via extended pions would likely make for a more satisfactory form factor at high momenta. To date chiral bags have not included pion structure so one is hesitant to trust results which depend on high pion momenta.

This dissertation doesn't comprise a true quantum field theoretic treatment of a chiral bag, but such a treatment has been developed by Chin.¹⁰ He discovers that a proper relativistic, many body

calculation of the $O(g^2)$ quark self energies is linearly divergent. His treatment is perturbative in the same sense as ours, but the main improvement is a correct calculation of the second order energy shift which we have merely treated classically. The classical treatment discards highly excited intermediate states and only allows the nucleon and Δ . Chin finds that if one includes pions coupling to radially excited and higher angular momentum quarks, then the self energy becomes negative infinity. We quote him "Such a divergence is traceable to the unconventional surface pion coupling which possesses no radial integrations to suppress high $-J$ intermediate states. Consequently the $1S_{1/2}$ state couples to all higher angular momentum states with undiminished strength and the self energy diverges." Whether one can extract the pion induced Lamb shift and renormalize the pion sector of the chiral bag model is not known. Even if one could renormalize the bag model one wouldn't trust the high momentum details of the renormalization which are certainly the most important parts. The point of view taken herein is that chiral bag models which ignore intermediate excited states are essentially the same as the classical models. The quantization of the quark and pion fields as in the cloudy bag is merely a form of bookkeeping. Likewise one isn't too surprised to find the renormalizations of the bare coupling constants as in ¹¹ to be small, since for example there are no anti-nucleons to renormalize pion propagators. For these reasons we trust the classical chiral bag versions like this one as much as the semi-quantized versions.

The reason for excluding the classical quark self energies in the energy fits is purely practical. We were unable to obtain local minima if we included them. Frankly we don't have much intuition regarding the real self energies consistent with truly extended pions. It is probable that the singularity in the energy fits at $R=0$ is due to the use of pointlike pions and their surface coupling to the quarks. Presumably if our Lagrangian were modified to display the pion structure at higher momenta, then not only would the bags become more stable, but one might actually obtain renormalization.

B. Self Energies in Cavity QCD

The quark self energies in the past have been treated in somewhat unusual ways.⁵ Prior to any knowledge of the true self energies at the one loop level, the self energies were by turns absorbed into the renormalized but presupposed state independent quark masses in the color-magnetic case, or included classically in the color-electric case. See Figure 9. We now know both treatments are inadequate. Regarding the former case, Hansson and Jaffe¹² have recently shown that the self energy of a confined quark may be separated into two parts. The first part is state independent and in fact zero for massless quarks. The second part is state dependent and is

$$\Delta E = \frac{.9 \alpha_c}{R} \quad (9.4)$$

Since this is radius-dependent it would give a different shift to quarks in different hadrons.

In both color-electric and magnetic cases the intermediate states should properly include all possible excited states. This inclusion of all states is the hallmark of a quantized field theory as properly regarded by Chin¹⁰ in his work on the pion effects on self energy. Unlike Chin's result with pions Hansson and Jaffe¹² show that the self energies due to gluon emission and re-absorption are finite. They have not yet finished the renormalization procedure since there remain more calculations of, for example, gluon self energies. Their quark self energy in (9.4) is large and likely harmful to fits of hadron spectroscopy. It is also very sensitive to the surface sharpness and they show it reduces by a factor of two if the loop momenta is arbitrarily cutoff at 1 GeV. A cutoff of this sort might easily arise if the surface were fuzzier as a true quantum surface should be. In fact it is reasonable to expect that the gluons and quarks might have different confinement ranges.¹³ One knows the gluons may roam without quarks though the quarks need to be around gluons for reasons of gauge invariance.

C. Conclusion

The net result of this discussion is that we think it plausible to stick with the classical bag regarding self energies. One

can see that inclusion of true quantum field theoretic self energies only leads to results which diverge from our physical notions about

1. Finite sized pions
2. Hadron spectroscopy
3. Quark and gluon confinement

The inclusion of classical self energies has been reduced to a practical selection process out of necessity. I believe it would be illusory to pretend to calculate renormalizations a la Q.E.D. for either the confined gluon coupling constant which probably varies near the surface anyway or the bare pion-quark coupling constant whose renormalization has to depend on excited intermediate states. For these reasons g_c is fitted phenomenologically and g is taken to be $1/f_\pi$ which is a low energy result from PCAC.

APPENDIX A
TECHNIQUES FOR DEFORMED BAGS

1. Bag Geometry

This section develops the machinery for imposing the linear boundary condition in a deformed bag. Since our original paper¹⁴ was published we found that oblate/prolate geometry is not general enough to accommodate a situation involving pressures of $O(g_c^4, g^4, g_c^2 g^2)$; which may be desirable to calculate in the future. Fortunately, the remedy is even simpler and more natural than the approach in the paper, and also verifies that our original $O(g^2, g_c^2)$ solution is correct.

We will briefly rederive the necessary identities to $O(\epsilon^2)$ using a general geometry and then compare these results to those in our paper which used oblate/prolate geometry. Then we will present identities to $O(\epsilon^4)$ using a general geometry.

Using the intuitive arguments in Chapter III one sees that in order to cancel gluon and pion pressures which are angle-dependent one merely distorts the surface, giving it the same angle-dependence. We have

$$r|_s = R - \frac{\epsilon^2}{3} R P_2(z) + O(\epsilon^4) \quad (A1)$$

where the $-1/3$ factor was chosen with hindsight. The Dirac wave function and energy are written similarly

$$W = W_0 + \epsilon^2 W_{2d} + \dots \quad (\text{A2})$$

$$\psi = \psi_0(w_0 r) + \epsilon^2 \psi_{2d}(w_0 r) + \dots \quad (\text{A3})$$

To implement the linear boundary condition one needs to know the surface normal. We have from (A1)

$$r - R + \frac{\epsilon^2 R P_2(z)}{3} \Big|_S = 0 \equiv S \quad (\text{A4})$$

The surface normal is defined via

$$\hat{n} = \frac{\vec{\nabla} S}{\sqrt{\vec{\nabla} S \cdot \vec{\nabla} S}} \quad (\text{A5})$$

Use the following

$$\vec{\nabla} = \hat{r} \frac{\partial}{\partial r} + \frac{\hat{\theta}}{r} \frac{\partial}{\partial \theta} + \frac{\hat{\phi}}{r \sin \theta} \frac{\partial}{\partial \phi} \quad (\text{A6})$$

$$\vec{\nabla} S = \hat{r} - \hat{\theta} \epsilon^2 \sin \theta \cos \theta \quad (\text{A7})$$

$$\hat{n} \Big|_S = \hat{r} - \hat{\theta} \epsilon^2 \sin \theta \cos \theta \quad (\text{A8})$$

Now we need to express $\hat{\theta}$ in terms of \hat{r} and \hat{z} . Recall that for spherical geometry

$$\sqrt{\frac{x^2+y^2}{z^2}} = \tan \theta \quad (\text{A9})$$

From this we obtain

$$\hat{\theta} = \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi - \hat{z} \sin \theta \quad (\text{A10})$$

$$\hat{\theta} = \hat{r} \frac{\cos \theta}{\sin \theta} - \frac{\hat{z}}{\sin \theta} \quad (\text{A11})$$

So we find for Eq. (A8)

$$\hat{n} = \hat{r} - \epsilon^2 (\hat{r} \cos^2 \theta - \hat{z} \cos \theta) \quad (\text{A12})$$

Finally, the NLC requires knowledge of the normal derivative. This may be found with the help of the following identity

$$\frac{\partial}{\partial(r \cos \theta)} = (1-z^2) \frac{1}{r} \frac{\partial}{\partial z} + z \hat{r} \cdot \vec{\nabla} \quad (\text{A13})$$

In Eq. (A13) and from here on $z = \cos \theta$. We obtain

$$\hat{n} \cdot \vec{\nabla} = \hat{r} \cdot \vec{\nabla} - \epsilon^2 z(z^2-1) \frac{\partial}{\partial z} \quad (\text{A14})$$

Note the essential Eqs. (A4), (A12) and (A14) are the same as the corresponding equations in our previous paper which

used the oblate/prolate geometry. In that paper ϵ^2 was the eccentricity squared.

$$\epsilon^2 \equiv \epsilon^2 \Big|_{\text{paper}} \quad (\text{A15})$$

In a calculation of $O(\epsilon^4)$ one would generally assume the following (oblate/prolate geometry being unnecessarily restrictive.)

$$r \Big|_s = R - \frac{\epsilon^2}{3} R P_2 + \epsilon^4 (a P_0 + b P_2 + c P_4) \quad (\text{A16})$$

This would give the following identities

$$\begin{aligned} \hat{h} \Big|_s &= \hat{r} - \epsilon^2 (\hat{r} z^2 - \hat{z} z) \\ &+ \epsilon^4 \left[3b + \frac{35}{2} c z^2 - \frac{15}{2} c \right] (\hat{r} z^2 - \hat{z} z) \\ &+ \frac{\epsilon^4}{3} \left[-\hat{r} z^2 + \hat{z} z \left(\frac{3}{2} z^2 - \frac{1}{2} \right) \right] \end{aligned} \quad (\text{A17})$$

$$\begin{aligned} \hat{h} \cdot \vec{\nabla} \Big|_s &= \frac{\partial}{\partial r} - \epsilon^2 \left(\frac{z^3 - z}{R} \right) \frac{\partial}{\partial z} \\ &+ \epsilon^4 \left[3b + \frac{35}{2} c z^2 - \frac{15}{2} c \right] \left(\frac{z^3 - z}{R} \right) \frac{\partial}{\partial z} \\ &+ \epsilon^4 \left[\frac{z^4}{2} - \frac{z^2}{2} \right] \frac{\partial}{\partial r} \\ &+ \frac{\epsilon^4}{3} \left[\frac{3}{2} z^2 - \frac{1}{2} \right] \left(\frac{z - z^3}{R} \right) \frac{\partial}{\partial z} \end{aligned} \quad (\text{A18})$$

2. Dirac Wave Functions

We need the solutions to the homogeneous Dirac equation. These solutions will be used as candidate wave functions later when we perturb around the lowest energy Dirac wave function. The free Dirac Hamiltonian is

$$H = \vec{\alpha} \cdot \vec{p} + \beta m \quad (\text{A19})$$

There are three conserved quantities associated with this Hamiltonian. They are

$$\begin{aligned} \vec{J} &= \vec{l} + \frac{\vec{S}}{2} \\ J^z &= l^z + \frac{S^z}{2} \end{aligned} \quad (\text{A20})$$

Where \vec{S} operates on upper and lower components, i.e.

$$\vec{S} = \begin{pmatrix} \vec{S} & 0 \\ 0 & \vec{S} \end{pmatrix} \quad (\text{A21})$$

The other conserved quantity is

$$K = \beta (\vec{S} \cdot \vec{l} + 1) \quad (\text{A22})$$

This has eigenvalues

$$K^2 = (J + 1/2)^2 = K^2$$

so that

$$k = \pm (J + 1/2) \quad (\text{A23})$$

We may write the wave function like so

$$\psi = \begin{pmatrix} g(r) Y_a(j, l_a, J^z) \\ i f(r) Y_b(j, l_b, J^z) \end{pmatrix} \quad (\text{A24})$$

Then, using the property $\vec{\sigma} \cdot \hat{r} Y_b = -Y_a$, one obtains for solutions (assuming zero mass)

$$\psi = \begin{pmatrix} j_k \\ -i \vec{\sigma} \cdot \hat{r} j_{k-1} \end{pmatrix} Y_a \quad \begin{array}{l} l_a = J + 1/2, \quad k = +(J + 1/2) \\ l_b = J - 1/2 \end{array}$$

$$\psi = (-1)^{k+1} \begin{pmatrix} j_{-k-1} \\ i \vec{\sigma} \cdot \hat{r} j_{-k} \end{pmatrix} Y_a \quad \begin{array}{l} l_a = J - 1/2, \quad k = -(J + 1/2) \\ l_b = J + 1/2 \end{array} \quad (\text{A25})$$

The parity of Y_a will be the parity of ψ . The lowest energy, odd-parity wave function has $J=1/2$, $k = +(J+1/2)$, $l_a=1$, $l_b=0$

$$\psi = \begin{pmatrix} j_1 \\ -i \vec{\sigma} \cdot \hat{r} j_0 \end{pmatrix} Y_a = \begin{pmatrix} -\vec{\sigma} \cdot \hat{r} j_1 \\ i j_0 \end{pmatrix} Y_b \quad (\text{A26})$$

where

$$Y_b = \begin{pmatrix} \sqrt{J^z + 1/2} & Y_{0, J^z - 1/2} \\ \sqrt{-J^z + 1/2} & Y_{0, J^z + 1/2} \end{pmatrix} \quad (\text{A27})$$

For $J^z = +1/2, -1/2$ one has respectively

$$Y_b = \begin{pmatrix} Y_{0,0} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ Y_{0,0} \end{pmatrix} \quad (\text{A28})$$

Similarly, the lowest energy even parity wave function has $J=1/2$, $k = -(J+1/2)$, $l_a = 0$, $l_b = 1$

$$\psi = \begin{pmatrix} j_0 \\ i b \cdot \hat{r} j_1 \end{pmatrix} Y_a \quad (\text{A29})$$

Again for $J^z = +1/2, -1/2$

$$Y_a = \begin{pmatrix} Y_{0,0} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ Y_{0,0} \end{pmatrix} \quad (\text{A30})$$

In the case of $J=1/2$ we may write out the even and odd parity solutions like this

$$\psi_{1/2}^{+P} = \begin{pmatrix} j_0 \\ i b \cdot \hat{r} j_1 \end{pmatrix} \chi, \quad \psi_{1/2}^{-P} = \begin{pmatrix} -j_1 b \cdot \hat{r} \\ i j_0 \end{pmatrix} \chi \quad (\text{A31})$$

Since the positive parity $J=1/2$ state is the lowest energy state, we identify $\sum_a \bar{Y}_a$ and hence χ with the usual $SU(6)$ wave functions. For example, we might have

$$\Delta_{3/2}^{++} \Big|_{\text{S.F.}} = u \uparrow_1 u \uparrow_2 u \uparrow_3 \equiv \chi_1 \chi_2 \chi_3 \quad (\text{A32})$$

The subscript S.F. stands for spin-flavor. The total wave function, assuming all quarks are in the S-state, would be

$$\Psi = \begin{pmatrix} j_0(w_{0r1}) \\ i b_1 \cdot \hat{r} j_1(w_{0r1}) \end{pmatrix} \chi_1 \begin{pmatrix} j_0(w_{0r2}) \\ i b_2 \cdot \hat{r} j_1(w_{0r2}) \end{pmatrix} \chi_2 \begin{pmatrix} j_0(w_{0r3}) \\ i b_3 \cdot \hat{r} j_1(w_{0r3}) \end{pmatrix} \chi_3 \\ \times \psi_{123}(\text{color}), \quad (\text{A33})$$

where $\psi_{123}(\text{color})$ is antisymmetric in color.

The linear boundary condition relates higher order quark solutions to the lower order solutions and ultimately to the $J=1/2$ even parity solution. Since the relation is linear one ends up with the higher order solutions having ordinary $SU(6)$ spin-flavor wave functions being left multiplied by the Dirac structure. Likewise, the $\psi(\text{color})$ is left multiplied.

In order to choose the right higher angular momentum wave functions we must examine the perturbation scheme ordained by the linear boundary condition. The important questions are:

1. What type of parity will the candidate wave functions have?
2. Since the interaction Hamiltonian should couple the ground state Hamiltonian to the negative energy states of the spherical bag, how will the negative energy states manifest themselves?
3. How can we write the wave functions compactly without lots of Clebsch-Gordan coefficients?

The answer to question 1. lies in the linear boundary condition.

$$i\gamma_5 \psi(wr) \Big|_s = \psi(wr) \quad (\text{A34})$$

This is a way of writing the interaction Hamiltonian involving the bag surface and the Dirac wave functions. The strong interaction conserves parity so we can only use Y_{jm} 's of even l values to describe the surface. One can write the perturbed wave functions formally as follows:

$$\psi_k \cong \psi_k^{(0)} + \Delta \psi_k \quad (\text{A35})$$

$$\Delta \psi_k = \sum_n |\psi_n\rangle \frac{\langle \psi_n | H_{int} | \psi_k \rangle}{(E_n - E_k)} \quad (\text{A36})$$

where H_{int} comprises the non-spherical part of the surface interaction.

Eq. (A36) also exhibits the excitation of negative energy modes of the spherical bag. This is because the index n refers to the complete basis set of Dirac wave functions in the spherical bag. We shall re-address question 2. after obtaining explicit solutions to the linear boundary condition.

The simplest way to write the complicated momentum dependence of the Dirac wave functions for a spherical cavity is to use combinations of the $SU(6)$ spinors and various components of \hat{p} and \vec{b} . One should write out the Clebsches by hand for comparison. The following are all positive energy and positive parity wave functions

$$\psi_0 = \begin{pmatrix} j_0 |wr| \\ i b \cdot \hat{p} j_1 |wr| \end{pmatrix} \chi, \quad J = 1/2, J^z = \pm 1/2$$

$$\psi = \begin{pmatrix} j_2 \\ -i b \cdot \hat{p} j_1 \end{pmatrix} \left[\frac{3}{2} b \cdot \hat{p} b^z - 1/2 \right], \quad J = 3/2, J^z = \pm 1/2$$

$$\psi = \begin{pmatrix} j_2 \\ i \vec{b} \cdot \hat{r} j_3 \end{pmatrix} \left[\frac{5}{2} z^2 - \vec{b} \cdot \hat{r} b^2 z - \frac{1}{2} \right], \quad J = 5/2, \quad J^z = \pm 1/2$$

$$\psi = \begin{pmatrix} j_4 \\ -i \vec{b} \cdot \hat{r} j_3 \end{pmatrix} \left[\frac{15}{8} \vec{b} \cdot \hat{r} b^2 \left(\frac{2}{3} z^3 - z \right) - \frac{15}{8} z^2 + \frac{3}{8} \right], \quad J = 7/2, \quad J^z = \pm 1/2$$

$$\psi = \begin{pmatrix} j_4 \\ i \vec{b} \cdot \hat{r} j_5 \end{pmatrix} \left[-\frac{3}{2} \vec{b} \cdot \hat{r} b^2 \left(\frac{2}{3} z^3 - z \right) + \frac{63}{8} z^4 - \frac{21z^2}{4} + \frac{3}{8} \right],$$

$$J = 9/2, \quad J^z = \pm 1/2 \quad (A37)$$

The Pauli spinors have been suppressed on all but the first wave function. These wave functions aren't normalized. Notice the $J=1/2$ wave function should be $2J+1=2$ different angular wave functions and it is, since χ may be spin up or spin down. However, we have only shown two of each of the $J=3/2$, $J=5/2$ and $J=7/2$ wave functions. This is because it is known that to $O(q^2, q_c^2)$ the Δ has a pressure which has purely $Y_{2,0}$ dependence. The following calculation will show the effect of evaluating the pressure using the $J=3/2$, $J^z=\pm 3/2$ wave function beating against the $J=1/2$, $J^z=\pm 1/2$ wave function. The $J=3/2$, $J^z=\pm 3/2$ wave function is

$$\psi^1 = \begin{pmatrix} j_2 \\ -i \vec{b} \cdot \hat{r} j_1 \end{pmatrix} \left[\vec{b} \cdot \hat{r} \vec{b} \cdot (\hat{r} \times -\hat{r} \gamma) \right] \chi \quad (A38)$$

The outward pressure exerted on a spherical bag by the quarks is

$$p = -\frac{1}{2} \frac{\partial}{\partial r} \bar{\psi} \psi \quad (\text{A39})$$

If ψ is mostly ground state along with a small amount of ψ' the pressure will be

$$p = -\frac{1}{2} \frac{\partial}{\partial r} \left\{ \bar{\psi}_0 \psi_0 + a(\bar{\psi}_0 \psi' + \bar{\psi}' \psi_0) \right\} \\ \equiv p_0 + \Delta p \quad (\text{A40})$$

The second term in Eq. (A40) has the following angle and spin dependence

$$\chi^\dagger \hat{b} \cdot \hat{r} \hat{b} \cdot (\hat{r}^\alpha - \hat{r}^\beta) \chi \quad (\text{A41})$$

$$= \sin^2 \theta e^{\mp 2i\phi} \quad (\text{A42})$$

where the minus is for spin up and the plus is for spin down. The third term has a similar behavior and when added to the second we obtain a Δp which is spin independent. Aside from normalization constants it is

$$\Delta p = -a \sin^2 \theta \cos 2\phi \frac{\partial}{\partial r} (j_0 j_2 + j_1^2) \quad (\text{A43})$$

This is a linear combination of $Y_{1,\pm 2}$, not $Y_{2,0}$, and so we don't use ψ' as a candidate wave function.

In a strict sense only ψ_0 shall be an eigenfunction of the spherical bag. We are taking it as the ground state and solving for the quark energy as a perturbation around the lowest eigenenergy of ψ_0 . The higher J states will be evaluated at $w_0 = \frac{2.04}{R}$ and not at their natural eigenenergies. This is where the negative energy states manifest themselves. Any state $\psi(wr)$ is composed of some parts of the original spherical negative energy states if it has an energy argument which is not an eigenenergy of the spherical Hamiltonian. To verify this merely note that a negative energy state is

$$\psi^N = i \gamma^2 \psi^*(wr) \quad (\text{A44})$$

One may ascertain, for example, that the $J=1/2$ positive energy wave function is not orthogonal to the $J=1/2$ negative energy wave function unless both wave functions are evaluated at their correct eigenenergies. In this example those energies happen to be $\pm w_0$ respectively.

As an aside, note that if the pressure is evaluated for a purely negative energy wave function, it is negative.

3. Effects of Deformation on Energy and Wave Functions

The following is a second order calculation of the deformation energy and wave functions.

Following the scheme outlined in Chapter III, the wave function ψ and energy w can be expanded perturbatively.

$$\begin{aligned}\psi &= \psi_0 + \epsilon^2 \psi_{2d} + \epsilon^4 \psi_{4d} \\ w &= w_0 + \epsilon^2 w_{2d} + \epsilon^4 w_{4d}\end{aligned}\tag{A45}$$

The Dirac equation for the first two orders is

$$\begin{aligned}-i \vec{\alpha} \cdot \vec{\nabla} \psi_0 &= w_0 \psi_0 \\ -i \vec{\alpha} \cdot \vec{\nabla} \psi_{2d} &= w_0 \psi_{2d} + w_{2d} \psi_0\end{aligned}\tag{A46}$$

The structure of (A46) shows that ψ_{2d} will have a homogeneous and an inhomogeneous part, so that the full solution to second order becomes

$$\psi = \psi_0 + \epsilon^2 \left(\psi_{2d}^H + \psi_{2d}^I \right)\tag{A47}$$

The form of the inhomogeneous solution, which can be obtained by differentiating the lower order equation with respect to w_0 , is

$$\psi_{2d}^I = \left(\frac{d}{dw_0} \psi_0 \right) w_{2d}\tag{A48}$$

The form of the homogeneous solution and the energy will be determined by the boundary condition

$$i\psi|_s = \psi \quad (\text{A49})$$

where \hat{n} is the outward normal to the bag surface (A14).

We need

$$\hat{n}|_s = \hat{r} - \epsilon^2 (\hat{r} z^2 - \hat{z} z) \quad (\text{A50})$$

$$\begin{aligned} i\psi|_s &= -i\vec{\gamma} \cdot \hat{n} = -i\vec{\gamma} \cdot \hat{r} + i\epsilon^2 (\vec{\gamma} \cdot \hat{r} z^2 - \gamma^3 z) \\ &= i\psi_0 + \epsilon^2 i \Delta\psi \end{aligned} \quad (\text{A51})$$

$$\begin{aligned} r|_s &= R - \frac{\epsilon^2}{3} R P_2(z) \\ &= R + \epsilon^2 \Delta R \end{aligned} \quad (\text{A52})$$

$$\hat{n} \cdot \vec{\nabla}|_s = \frac{\partial}{\partial r} + \epsilon^2 z(1-z^2) \frac{1}{R} \frac{\partial}{\partial z} \quad (\text{A53})$$

The candidate wave functions are chosen from among those in (A37). Also, include a term of the form (A48), as well as a renormalization constant which will ultimately be zero. The argument of all the Bessel functions is $w_0 r$. Written symbolically the wave function to order ϵ^2 is

$$\psi = N \left\{ \psi_0 + \epsilon^2 a_1 \psi_2^{3/2} + \epsilon^2 a_2 \psi_2^{5/2} + \epsilon^2 (w_2 \Delta r) \psi_0' + \epsilon^2 \frac{\delta N}{N} \psi_0 \right\} \quad (\text{A54})$$

Explicitly this is

$$\begin{aligned} \psi = N \left\{ \begin{pmatrix} j_0(w_0 r) \\ i b \cdot \hat{r} j_1(w_0 r) \end{pmatrix} \right. \\ + \epsilon^2 a_1 \begin{pmatrix} j_2(w_0 r) \\ -i b \cdot \hat{r} j_1(w_0 r) \end{pmatrix} \left[\frac{3}{2} \vec{b} \cdot \hat{r} b^2 z - \frac{1}{2} \right] \\ + \epsilon^2 a_2 \begin{pmatrix} j_2(w_0 r) \\ i b \cdot \hat{r} j_3(w_0 r) \end{pmatrix} \left[\frac{5}{2} z^2 - \vec{b} \cdot \hat{r} b^2 z - \frac{1}{2} \right] \\ \left. + \epsilon^2 w_2 \Delta r \begin{pmatrix} j_0'(w_0 r) \\ i b \cdot \hat{r} j_1'(w_0 r) \end{pmatrix} + \epsilon^2 \frac{\delta N}{N} \begin{pmatrix} j_0(w_0 r) \\ i b \cdot \hat{r} j_1(w_0 r) \end{pmatrix} \right\} \quad (\text{A55}) \end{aligned}$$

When ψ is evaluated at the surface in (A49), one sees from (A52) that the arguments of the Bessel functions will have an $O(\epsilon)$ part. This will only express itself to $O(\epsilon^2)$ through the Taylor expansion of ψ_0 . The other portions of ψ are already of $O(\epsilon^2)$.

Applying the linear boundary condition one gets the usual condition at $O(1)$. From here on the arguments of the parts of ψ are implicitly $w_0 r = \rho$.

$$i\psi_0 \psi_0 = \psi_0 \quad (\text{A56})$$

This gives the condition $j_0(\rho) = j_1(\rho)$. The solution having lowest energy is

$$w_{0R} = \rho = 2.0428 \quad (\text{A57})$$

The $O(\epsilon^2)$ equation is

$$\begin{aligned} & i\Delta\psi\psi_0 + i\psi_0 w_{0R}\Delta R\psi_0 + i\psi_0 a_1 \psi^{3/2} + i\psi_0 a_2 \psi^{5/2} \\ & + i\psi_0 (w_{2LR}) \psi_0' + i\psi_0 \frac{\delta N}{N} \psi_0 \\ & = w_{0R}\Delta R\psi_0' + a_1 \psi^{3/2} + a_2 \psi^{5/2} \end{aligned} \quad (\text{A58})$$

This is really two equations involving the upper and lower components of the linear boundary condition. Both equations yield the same results so we present only the upper equation.

$$\begin{aligned} & (-j_1 z^2 + \vec{b}^z z \vec{b} \cdot \hat{r} j_1) + w_{2LR} j_1' - \frac{\rho}{3} P_2(z) j_1' \\ & - a_1 j_1 \left(\frac{3}{2} \vec{b} \cdot \hat{r} b^z z - \frac{1}{2} \right) + a_2 j_3 \left(\frac{5}{2} z^2 - \vec{b} \cdot \hat{r} b^z z - \frac{1}{2} \right) + \frac{\delta N}{N} j_1 \\ & = w_{2LR} - \frac{\rho}{3} P_2(z) j_0' + a_1 j_2 \left(\frac{3}{2} \vec{b} \cdot \hat{r} b^z z - \frac{1}{2} \right) \\ & + a_2 j_3 \left(\frac{5}{2} z^2 - \vec{b} \cdot \hat{r} b^z z - \frac{1}{2} \right) + \frac{\delta N}{N} j_0 \end{aligned} \quad (\text{A59})$$

For this equation to be correct the coefficients of 1 , $z \hat{b} \cdot \hat{r} b^z$, z^2 must all be zero separately. This yields three equations. (Note that $z \hat{b} \cdot \hat{r} b^z$ may be written as $z z^2 - z \hat{b} \cdot \hat{r} b^z$.) The terms involving $\frac{\delta U}{\nu}$ cancel since $j_0 = j_1$. Below are the results for a_1 , a_2 , w_{2d} obtained by solving the three equations.

$$a_1 = -\frac{2\rho(2\rho+1)}{45}$$

$$a_2 = \frac{2\rho^2(\rho-2)}{5(15-8\rho)}$$

$$w_{2d} = 0 \tag{A60}$$

Some of the Bessel function identities from Appendix C were used in simplifying the j_ℓ and j'_ℓ terms.

4. Calculation of Deformation Energy to $O(\epsilon^4)$

It is desirable to know the energy shift to $O(\epsilon^4)$. We also want to verify the perturbative scheme to $O(\epsilon^4)$ using the candidate wave functions in (A37). The following calculation was done in the oblate/prolate geometry of reference 15. A calculation involving gluon and pion pressure terms of $\mathcal{O}(g^4, g^2 g_c^2, g_c^4)$ would necessitate the more general even parity geometry worked out in Section 1. Since we have only calculated the gluon and pion field pressures to $O(g^2, g_c^2)$, the more general geometry is unnecessary. (With hindsight it would have been easier.)

In the following calculation we use an expansion parameter c^2 which is an artifact of the Flammer¹⁵ notation. At $O(\epsilon^2)$, $\frac{c^2}{\rho^2} = \epsilon^2$ is the actual eccentricity of an elliptical body of revolution. The normal and radius as a function of $z = \cos\theta$ may be worked out perturbatively in oblate/prolate geometry. (We require incompressibility of volume in order to identify a mean bag radius in using the oblate/prolate geometry.) Recall in Flammer¹⁵ that if c^2 is positive/negative due to pion and gluon effects, the bag is oblate/prolate. At the end of this calculation we will use the nominal value of $\frac{c^2}{\rho^2} = \epsilon^2 = .135$ to evaluate the w_{4d} , and obtain an energy in MeV. The normal and radius evaluated at the bag surface are

$$\hat{n}_s = \hat{r} - \frac{c^2}{\rho^2} (\hat{r} z^2 - \frac{2}{3} z) - \frac{c^4}{\rho^4} \left[\hat{r} \left(\frac{2}{6} z^2 - \frac{2}{3} z^4 \right) + \frac{2}{3} \left(z^3 - \frac{2}{3} z \right) \right]$$

$$r_s = R - \frac{c^2}{3\rho^2} R P_2(z) + \frac{c^4}{6\rho^4} \left(\frac{9}{4} z^4 - \frac{5}{2} z^2 + \frac{1}{4} \right)$$

$$= R + c^2 \Delta R_1 + c^4 \Delta R_2$$

(A61)

Now in choosing the candidate wave functions which will allow us to solve the linear boundary condition in this geometry we must make sure that total wave function is normalized. That is

$$1 = \frac{1}{w_0^3} \int_0^{2\pi} \lambda \phi \int_{-1}^1 dz \int_0^1 w_0 [R + C^2 \Delta R_1 + C^4 \Delta R_2] \chi^2 dx \quad \psi^\dagger \psi \quad (\text{A62})$$

The following is the appropriate wave function, including normalization terms

$$\begin{aligned} \psi = N & \left\{ \begin{pmatrix} j_0 \\ i\vec{b} \cdot \hat{r} j_1 \end{pmatrix} + C^2 \alpha_1 \begin{pmatrix} j_2 \\ -i\vec{b} \cdot \hat{r} j_1 \end{pmatrix} \left[\frac{3}{2} \vec{b} \cdot \hat{r} b^2 z - \frac{1}{2} \right] \right. \\ & + C^2 \alpha_2 \begin{pmatrix} j_2 \\ i\vec{b} \cdot \hat{r} j_3 \end{pmatrix} \left[\frac{5}{2} z^2 - \vec{b} \cdot \hat{r} b^2 z - \frac{1}{2} \right] \\ & + C^4 \delta N_1 \begin{pmatrix} j_0 \\ i\vec{b} \cdot \hat{r} j_1 \end{pmatrix} \\ & + C^4 w_{41} \begin{pmatrix} r j_0' + \frac{\delta N_2}{w_0} j_0 \\ i\vec{b} \cdot \hat{r} (r j_1' + \frac{\delta N_2}{w_0} j_1) \end{pmatrix} \\ & + C^4 \alpha_1 \begin{pmatrix} j_2 \\ -i\vec{b} \cdot \hat{r} j_1 \end{pmatrix} \left[\frac{3}{2} \vec{b} \cdot \hat{r} b^2 z - \frac{1}{2} \right] \\ & + C^4 \alpha_2 \begin{pmatrix} j_2 \\ i\vec{b} \cdot \hat{r} j_3 \end{pmatrix} \left[\frac{5}{2} z^2 - \vec{b} \cdot \hat{r} b^2 z - \frac{1}{2} \right] \\ & + C^4 \beta_1 \begin{pmatrix} j_4 \\ -i\vec{b} \cdot \hat{r} j_3 \end{pmatrix} \left[\frac{15}{8} \vec{b} \cdot \hat{r} b^2 \left(\frac{2}{7} z^3 - z \right) - \frac{15}{8} z^2 + \frac{3}{8} \right] \\ & \left. + C^4 \beta_2 \begin{pmatrix} j_4 \\ i\vec{b} \cdot \hat{r} j_5 \end{pmatrix} \left[-\frac{3}{2} \vec{b} \cdot \hat{r} b^2 \left(\frac{2}{7} z^3 - z \right) + \frac{63}{8} z^4 - \frac{21}{4} z^2 + \frac{3}{8} \right] \right\} \quad (\text{A63}) \end{aligned}$$

This wave function is evaluated at the surface where the arguments $w_0 r$ have angular terms. To $O(c^4)$ these contributions to the wave function are made explicit in c^2 and c^4 via Taylor expansion of the first three terms in (A62). These additional terms are

$$\begin{aligned}
 N \left\{ c^2 w_0 \Delta R_1 \begin{pmatrix} j_0' \\ i \vec{b} \cdot \hat{r} j_1' \end{pmatrix} + c^4 w_0 \Delta R_2 \begin{pmatrix} j_0' \\ i \vec{b} \cdot \hat{r} j_1' \end{pmatrix} \right. \\
 + \frac{c^4}{2} (w_0 \Delta R_1)^2 \begin{pmatrix} j_0'' \\ i \vec{b} \cdot \hat{r} j_1'' \end{pmatrix} + c^4 a_1 w_0 \Delta R_1 \begin{pmatrix} j_2' \\ -i \vec{b} \cdot \hat{r} j_1' \end{pmatrix} \left[\frac{3}{2} \vec{b} \cdot \hat{r} b^z z^{-\frac{1}{2}} \right] \\
 \left. + c^4 a_2 w_0 \Delta R_1 \begin{pmatrix} j_2' \\ i \vec{b} \cdot \hat{r} j_3' \end{pmatrix} \left[\frac{5}{2} z^2 - \vec{b} \cdot \hat{r} b^z z - \frac{1}{2} \right] \right. \quad (A64)
 \end{aligned}$$

To $O(c^4)$ our wave function is (A63) and (A64) where now all the arguments are ρ . Next apply the linear boundary condition

$$i \chi \psi|_s = \psi \quad (A65)$$

This involves lots of terms. We shall use the top equation of (A65) in the following. The bottom equation should give the same result even though it may not appear identical. It is a good check on the algebra to do the lower equation as well. Eq. (A65) was solved to $O(c^2)$ in the last section so we proceed to $O(c^4)$ in the top equation, obtaining

$$\frac{j_1}{\rho^4} \left[\frac{3}{2} z^4 - \frac{3}{4} z^2 + (\vec{b} \cdot \hat{r} b^z - 2z) (z^3 - \frac{2}{3} z) \right] +$$

$$\begin{aligned}
& + a_1 \frac{j_1}{\rho^2} \left[\frac{3}{2} \vec{b} \cdot \hat{r} b^2 z^3 - \frac{z^2}{2} - \frac{z^2}{2} - \frac{\vec{b} \cdot \hat{r} b^2 z}{2} \right] \\
& + a_2 \frac{j_3}{\rho^2} \left[-\frac{5}{2} z^4 + \vec{b} \cdot \hat{r} b^2 z^3 + \frac{z^2}{2} + 5z^4 - 2\vec{b} \cdot \hat{r} b^2 z^3 - 2z^2 \right] \\
& + W_{4dR} \left[j_1' + \frac{\delta N_2 j_1}{\rho} \right] + \delta N_1 j_1 \\
& + \alpha_1 j_1 \left[\frac{1}{2} - \frac{3}{2} \vec{b} \cdot \hat{r} b^2 z \right] + \alpha_2 j_3 \left[\frac{5}{2} z^2 - \vec{b} \cdot \hat{r} b^2 z - \frac{1}{2} \right] \\
& + \beta_1 j_3 \left[\frac{15}{8} \vec{b} \cdot \hat{r} b^2 (z - \frac{2}{3} z^3) + \frac{15}{8} z^2 - \frac{3}{8} \right] \\
& + \beta_2 j_5 \left[\frac{3}{2} \vec{b} \cdot \hat{r} b^2 (z - \frac{2}{3} z^3) + \frac{63}{8} z^4 - \frac{21}{4} z^2 + \frac{3}{8} \right] \\
& + \frac{j_1'}{3\rho^3} \left(\frac{3}{2} z^2 - \frac{1}{2} \right) \left[-z^2 + \vec{b} \cdot \hat{r} b^2 z \right] \\
& + \frac{j_1'}{6\rho^3} \left[\frac{9}{4} z^4 - \frac{5}{2} z^2 + \frac{1}{4} \right] + \frac{j_1''}{18\rho^2} \left[\frac{9}{4} z^4 - \frac{3}{2} z^2 + \frac{1}{4} \right] \\
& + \frac{a_1 j_1'}{3\rho} \left(\frac{3}{2} z^2 - \frac{1}{2} \right) \left[\frac{3}{2} \vec{b} \cdot \hat{r} b^2 z - \frac{1}{2} \right] \\
& + \frac{a_2 j_3'}{3\rho} \left(\frac{3}{2} z^2 - \frac{1}{2} \right) \left[-\frac{5}{2} z^2 + \vec{b} \cdot \hat{r} b^2 z + \frac{1}{2} \right] \\
& = \delta N_1 j_0 + W_{4dR} \left(j_0' + \frac{\delta N_2 j_0}{\rho} \right) \\
& + \alpha_1 j_2 \left(\frac{3}{2} \vec{b} \cdot \hat{r} b^2 z - \frac{1}{2} \right) + \alpha_2 j_2 \left(\frac{5}{2} z^2 - \vec{b} \cdot \hat{r} b^2 z - \frac{1}{2} \right)
\end{aligned}$$

$$\begin{aligned}
& + \beta_1 j_4 \left[\frac{15}{8} \hat{b} \cdot \hat{r} b^z \left(\frac{1}{3} z^3 - z \right) - \frac{15}{8} z^2 + \frac{1}{8} \right] \\
& + \beta_2 j_4 \left[-\frac{3}{2} \hat{b} \cdot \hat{r} b^z \left(\frac{1}{3} z^3 - z \right) + \frac{63}{8} z^4 - \frac{21}{4} z^2 + \frac{3}{8} \right] \\
& + \frac{j_0'}{6\rho^3} \left[\frac{9}{4} z^4 - \frac{5}{2} z^2 + \frac{1}{4} \right] + \frac{j_0''}{18\rho^2} \left[\frac{9}{4} z^4 - \frac{3}{2} z^2 + \frac{1}{4} \right] \\
& + \frac{a_1 j_2'}{3\rho} \left(\frac{1}{2} - \frac{3}{2} z^2 \right) \left[\frac{3}{2} \hat{b} \cdot \hat{r} b^z z - \frac{1}{2} \right] \\
& + \frac{a_2 j_2'}{3\rho} \left(\frac{1}{2} - \frac{3}{2} z^2 \right) \left[\frac{5}{2} z^2 - \hat{b} \cdot \hat{r} b^z z - \frac{1}{2} \right]
\end{aligned}$$

(A66)

This is really five equations involving the coefficients of 1 , $\hat{b} \cdot \hat{r} b^z z$, z^2 , $\hat{b} \cdot \hat{r} b^z z^3$, z^4 . The five unknowns are $\alpha_1, \alpha_2, \beta_1, \beta_2, W_{4d}$. After solving the five equations we obtain W_{4d}

$$\begin{aligned}
W_{4d} = & \frac{1}{2(j_1' - j_0')} \left\{ \frac{(j_1' - j_0')}{45\rho^3} + \frac{(j_0'' - j_1'')}{90\rho^2} \right. \\
& + \frac{a_2(j_3' - j_2')}{15\rho} - \frac{a_1(j_1' + j_2')}{15\rho} + \frac{2a_2 j_3}{15\rho^2} \\
& \left. + \frac{a_1 j_1}{5\rho^2} + \frac{j_1}{15\rho^4} \right\}
\end{aligned}$$

(A67)

This may be reduced using the identities in Appendix C as well as the values already determined for a_1 and a_2 . Note the terms involving δN_1 and δN_2 disappeared. This is because $j_0=j_1$. The reduced form of (A67) is

$$W_{4d} = \frac{1}{R} \frac{\rho}{2(\rho-1)} \left(\frac{1}{75(15-8\rho)\rho^4} \right) \left(40 - 60\rho + \frac{50\rho^2}{9} + \frac{260\rho^3}{9} - \frac{100\rho^4}{9} \right) \quad (\text{A68})$$

substituting $\rho = 2.0428$ we get

$$W_{4d} = \frac{1}{R} \left[3.691 \times 10^{-3} \right] \quad (\text{A69})$$

So the energy shift due to one quark in this deformed bag is

$$\Delta E = c^4 W_{4d} = \left(\frac{c^2}{\rho^2} \right)^2 \cdot \rho^4 W_{4d} \quad (\text{A70})$$

Using the nominal value¹⁴ for $\frac{c^2}{\rho^2} = .135$ as well as $R=1.249$ Fermi which was the radius of the Δ and multiplying by $\frac{1}{\hbar c}$, we find

$$\Delta E = .185 \text{ MeV} \quad (\text{A71})$$

All three quarks would combine for a deformation energy of .56 MeV which is pretty small.

APPENDIX B
 PION AND GLUON EFFECTS

This appendix contains the solutions of the bag model with pion effects to order g^2 and gluon effects to order g_c^2 .

1. Treatment of the Pion to Second Order

First we solve Eqs.(2.12) and (2.14) and consider the modifications required in the NLC, (2.18).

The source of the lowest-order pion field ϕ_i is the surface term (2.12b). Using (3.1) this gives

$$\frac{\partial}{\partial r} \phi_{i\dot{i}} = - N^2 j_0^2(\rho) \tau_i^{\dot{i}} \vec{b}_i \cdot \hat{r} \quad (B1)$$

where we continue to suppress the two-component spinors. Eq.(B1) is an operator in the spin-isospin space of the quarks, and the matrix elements which must eventually be taken will be postponed until the last step.

Eq.(B1) shows that ϕ_i must be a P wave independent of time, and (2.12a) shows that ϕ_i has the radial dependence of a spherical Hankel function with imaginary argument. The solution is

$$\phi_{i\dot{i}}(r) = b h_1(\mu_{\pi} r) \tau_i^{\dot{i}} \vec{b}_i \cdot \hat{r} \quad (B2)$$

where b is determined from the boundary condition (B1)

$$b = \frac{N^2 j_0^2(\rho)}{k_\pi h_1'(\eta)} = \frac{\rho}{8\pi \eta h_1'(\eta) \rho^2(\rho-1)} \quad (\text{B3})$$

and

$$h_1(x) = \frac{e^{-x}}{x} \left(1 + \frac{1}{x} \right)$$

$$h_1'(x) = -\frac{e^{-x}}{x} \left(1 + \frac{2}{x} + \frac{1}{x^2} \right) \quad (\text{B4})$$

and $\eta = k_\pi R$.

Now we proceed with the solution for $\psi_{2\pi}$. Following Chodos and Thorn³ and Vento¹⁶ we will write

$$\psi_{2\pi} = \psi_{2\pi}^{\text{I}} + \psi_{2\pi}^{\text{H}} \quad (\text{B5})$$

where $\psi_{2\pi}^{\text{I}}$ is a particular solution of the inhomogeneous equation (2.14a) and $\psi_{2\pi}^{\text{H}}$ is a solution of the homogeneous equation which will be adjusted to satisfy the boundary condition (2.14b). The inhomogeneous solution for the i th quark is

$$\psi_{2\pi i}^{\text{I}} = N \begin{pmatrix} r j_0'(w_{0r}) \\ i \vec{\sigma} \cdot \hat{r} r j_1'(w_{0r}) \end{pmatrix} w_{2\pi i} \quad (\text{B6})$$

where the prime will always refer to differentiation with respect to the full argument (w_{or} in this case) and we are careful to write w_{ini} , an operator in the quark spin-isospin space, to the right as discussed in Chapter II. The structure of the homogeneous solution is determined by the spin structure of the last term in the boundary condition (2.14b), which is

$$i \underline{\alpha}_i \cdot \underline{\phi}_{1i} \gamma^5 \psi_{0i} \Big|_s = N b h_{1j_0} \begin{pmatrix} 1 \\ i \vec{b}_i \cdot \hat{v} \end{pmatrix} \left[\frac{2}{3} \mathcal{P}_{2i}^{\mp} + \frac{1}{3} \mathcal{P}_{0i}^{\mp} \right] \quad (\text{B7})$$

where $h_1 = h_1(\eta)$ and we introduce the operators

$$\begin{aligned} \mathcal{P}_{0i}^{\mp} &= \sum_{j \neq i} \vec{b}_i \cdot \vec{b}_j \underline{\alpha}_i \cdot \underline{\alpha}_j \\ \mathcal{P}_{2i}^{\mp} &= \sum_{j \neq i} \left(\frac{2}{3} \vec{b}_i \cdot \hat{v} \vec{b}_j \cdot \hat{v} - \frac{1}{2} \vec{b}_i \cdot \vec{b}_j \right) \underline{\alpha}_i \cdot \underline{\alpha}_j \end{aligned} \quad (\text{B8})$$

Note that the sum $\frac{2}{3} \mathcal{P}_{2i}^{\mp} + \frac{1}{3} \mathcal{P}_{0i}^{\mp}$ commutes with $\vec{b}_i \cdot \hat{v}$ but the separate terms do not. Eg. (B7) suggests a homogeneous solution with a $J=3/2$ structure. Specifically

$$\psi_{2ni}^{\#} = N C_1 \begin{pmatrix} j_2(w_{or}) \\ -i j_1(w_{or}) \vec{b}_i \cdot \hat{v} \end{pmatrix} \mathcal{P}_{2i}^{\mp} \quad (\text{B9})$$

where C_1 is a constant. Finally, combining (B6), (B7) and (B9) gives

$$C_1 = \frac{2}{3} \frac{b h_1 j_0}{(j_0 + j_2)} = \frac{2}{9} b \rho h_1$$

$$W_{2\pi i} = \frac{-b h_1 j_0}{3R(j_1 + j_2)} \rho_{0i}^T = \frac{-w_0 b h_1}{6(\rho-1)} \rho_{0i}^T \quad (\text{B10})$$

It is easily demonstrated that (B10) agrees with (2.29a). Note that our complete solution looks like the result in Ref. 6 if we let the pion mass go to zero.

The correction to the normalization, Eq.(2.17), comes entirely from the inhomogeneous term (B6), and is

$$\begin{aligned} \delta N_{\pi i} &= -\frac{1}{2} \int d^3x \left[g^2 \psi_{2\pi i}^+ \psi_{0i} + g^2 \psi_{0i}^+ \psi_{2\pi i} \right] \\ &= -\frac{g^2 (3-2\rho) W_{2\pi i}}{2(\rho-1) w_0} \end{aligned} \quad (\text{B11})$$

We will need this result for the NLC.

Finally, the contributions to the NLC from the pion field can be obtained from (2.18), (B2), (B5), and (B11). Specifically, if we call that part of (2.18) which is due to the pion field B_π , then

$$\begin{aligned} B_\pi &= -g^2 \frac{\partial}{\partial r} \sum_{i \neq j} \frac{\partial \phi_{1i}}{\partial r} \cdot \phi_{ij} - \sum_i \delta N_\pi \frac{\partial}{\partial r} (\bar{\psi}_{0i} \psi_{0i}) \\ &\quad - \frac{1}{2} g^2 \frac{\partial}{\partial r} \sum_i (\bar{\psi}_{2\pi i} \psi_{0i} + \bar{\psi}_{0i} \psi_{2\pi i}) \\ &\quad + \frac{1}{2} g^2 \sum_{i \neq j} \left(\nabla^k \phi_{1i} \cdot \nabla^k \phi_{ij} + m_\pi^2 \phi_{1i} \cdot \phi_{ij} \right) \end{aligned}$$

$$= B_{0\pi} \overline{\mathcal{P}}_0^{\pm} + B_{2\pi} \overline{\mathcal{P}}_2^{\pm} \quad (\text{B12})$$

where the new spin-isospin operators are

$$\begin{aligned} \overline{\mathcal{P}}_0^{\pm} &= \sum_i \mathcal{P}_{0i}^{\pm} = \sum_{\substack{i,j \\ i \neq j}} \vec{b}_i \cdot \vec{b}_j \tau_i \cdot \tau_j \\ \overline{\mathcal{P}}_2^{\pm} &= \sum_i \mathcal{P}_{2i}^{\pm} = \sum_{\substack{i,j \\ i \neq j}} \left(\frac{3}{2} \vec{b}_i \cdot \hat{r} \vec{b}_j \cdot \hat{r} - \frac{1}{2} (\vec{b}_i \cdot \vec{b}_j) \right) \tau_i \cdot \tau_j \end{aligned} \quad (\text{B13})$$

Calculating the quantities $B_{0\pi}$ and $B_{2\pi}$ can be facilitated using the relations in Appendix C

$$B_{0\pi} = - \frac{g^2}{4\pi} \left(\frac{\rho^2}{32\pi R^6 (\rho-1)^2} \right) \left[\frac{1}{3} - \frac{2}{3} \frac{h_1}{\gamma h_1'} - \frac{1}{3} \left(\frac{h_1}{\gamma h_1'} \right)^2 (2+\eta^2) \right] \quad (\text{B14a})$$

$$\begin{aligned} B_{2\pi} = - \frac{g^2}{4\pi} \left(\frac{\rho^2}{16\pi R^6 (\rho-1)^2} \right) \left\{ \frac{1}{3} - \frac{2}{9} \left(\frac{h_1}{\gamma h_1'} \right) (4\rho^2 - 4\rho - 3) \right. \\ \left. + \frac{1}{3} \left(\frac{h_1}{\gamma h_1'} \right)^2 (1-\eta^2) \right\} \quad (\text{B14b}) \end{aligned}$$

It can be shown that (B14a) is precisely what would be obtained if the pion energy were differentiated with respect to R . Specifically,

$$B_{0\pi} = - \frac{1}{4\pi R^2} \frac{d}{dR} \left\{ \left(\frac{g^2}{4\pi} \right) \left(\frac{\rho^2}{24(\rho-1)^2} \frac{h_1}{\gamma h_1' R^3} \right) \right\} \quad (\text{B15})$$

Comparing this with the pion-energy result obtained in (2.31) shows explicitly that the \bar{p}_0^z part of (B12) is equivalent to the requirement that the bag radius minimize the energy. When matrix elements are evaluated, the angular term p_1^z will generate an angular dependence of the form $P_2(z)$, which must be cancelled by deforming the bag. This term, and a similar term obtained from the gluon fields will be cancelled by the term proportional to ϵ^2 in (3.13). We now turn to a treatment of gluonic effects to second order.

2. Treatment of the Gluons to Second Order

The solution for the gluon fields to first order is familiar⁵. Eqs.(2.11) are Maxwell's equations involving eight colored vector fields instead of one. The color current is

$$\vec{j}^a = \psi_{0i}^+ \vec{\alpha} \frac{\lambda_i^a}{2} \psi_{0i} = -N^2 \lambda_i^a j_0(w_{0i}) j_1(w_{0i}) (\hat{r} \times \vec{t}_i) \quad (\text{B16})$$

This is the source term for the transverse colored gluon field, which satisfies (2.11b) and (2.11c). The solution to these two equations is

$$\vec{A}_{1i}^a = -\frac{1}{4\pi} \frac{\lambda_i^a}{2} (\vec{\alpha} \times \vec{t}_i) H(r) \quad (r < R) \quad (\text{B17})$$

where

$$H(r) = \frac{8\pi}{3} N^2 \left[\int_0^r \frac{r'^3}{r^3} dr' j_0 j_1 + \int_r^R dr' j_0 j_1 + C_2 \right] \quad (\text{B18})$$

and C_2 is an integration constant to be determined by the boundary condition (2.11d). The color-magnetic field generated by the i th quark is

$$B_i^a(r) = -\frac{g_c}{4\pi} \frac{\lambda_i^a}{2} \vec{F} (\vec{F} \cdot \vec{b}_i) \frac{H'(r)}{r} + \frac{g_c}{4\pi} \frac{\lambda_i^a}{2} \vec{b}_i \left[r H'(r) + 2 H(r) \right] \quad (\text{B19})$$

To satisfy the boundary condition (2.11d) at the surface of a spherical bag requires

$$R H'(R) + 2 H(R) = 0 \quad (\text{B20})$$

which gives

$$C_2 = \frac{1}{2} \int_0^R \frac{r'^3}{r^3} dr' j_0 j_1 \quad (\text{B21})$$

This completely determines the color vector potential and magnetic field to order g_c .

To obtain the solution ψ_{2g} , we must solve equations (2.15). Following a procedure similar to that used for pions, we write

$$\psi_{2g} = \psi_{2g}^1 + \psi_{2g}^2 \quad (\text{B22})$$

where in this case both ψ_{2g}^1 and ψ_{2g}^2 are solutions of inhomogeneous Dirac equations

$$(i\vec{\alpha}\cdot\vec{\nabla} + w_0) \psi_{2g,i}^1 = -\psi_{0,i} w_{2g,i} \quad (\text{B23a})$$

$$\begin{aligned} (i\vec{\alpha}\cdot\vec{\nabla} + w_0) \psi_{2g,i}^2 &= -\left(\frac{\lambda^a}{2} \vec{\alpha} \psi_{0,i}\right) \cdot \sum_{j \neq i} \vec{A}_{ij}^a \\ &= -\frac{N}{6\pi} H(r) \begin{pmatrix} r j_1 \\ i \vec{b} \cdot \hat{r} j_0 \end{pmatrix} \left[\frac{2}{3} \mathcal{P}_{2,i} - \frac{2}{3} \mathcal{P}_{0,i} \right] \end{aligned} \quad (\text{B23b})$$

where we made use of the relation

$$\sum_a \lambda_i^a \lambda_j^a = -\frac{8}{3} \quad (i \neq j) \quad (\text{B24})$$

which holds for colorless three-quark states. Note that we again write $w_{2g,i}$ to the right. Two new operators have been introduced in (B23b). They are

$$P_{oi} = \sum_{j \neq i} \vec{b}_i \cdot \vec{b}_j$$

$$P_{2i} = \sum_{j \neq i} \left(\frac{3}{2} \vec{b}_i \cdot \hat{r} \vec{b}_j \cdot \hat{r} - \frac{1}{2} \vec{b}_i \cdot \vec{b}_j \right) \quad (\text{B25})$$

These are similar to the operators of (B8) without the isospin operators. Note that while the linear combination of these operators which enters (B23b) does not commute with $\vec{b}_i \cdot \hat{r}$, the operator \vec{A}_{ij}^a does commute because \vec{A}_{ij}^a depends on the spin variables of the other two quarks only.

A solution to (B23a) is, as before

$$\psi_{2gi}^1 = N \begin{pmatrix} r j_{i0}' / |w_{0i}| \\ i \vec{b}_i \cdot \hat{r} r j_{i1}' / |w_{0i}| \end{pmatrix} w_{2gi} \quad (\text{B26})$$

The solution to (B23b) is considerably more complicated, and details are given in Appendix C. Briefly, the inhomogeneous term suggests a solution of the form

$$\psi_{2gi}^2 = \begin{pmatrix} K_1(r) P_{oi} + K_2(r) P_{2i} \\ i \vec{b}_i \cdot \hat{r} [K_3(r) P_{oi} + K_4(r) P_{2i}] \end{pmatrix} \quad (\text{B27})$$

Substituting this into (B23b) gives a set of coupled first-order differential equations for the K's

$$w_{0i} K_1 - \frac{1}{r^2} \frac{d}{dr} (r^2 K_3) = \frac{N}{4\pi} r j_{i1}' / |w_{0i}| H(r) \quad (\text{B28a})$$

$$\omega_0 K_2 - r \frac{d}{dr} \left(\frac{K_4}{r} \right) = -\frac{N}{9\pi} r j_1(\omega_0 r) H(r) \quad (\text{B28b})$$

$$\omega_0 K_3 + \frac{d}{dr} (r K_1) = \frac{N}{9\pi} r j_0(\omega_0 r) H(r) \quad (\text{B28c})$$

$$\omega_0 K_4 + \frac{1}{r^3} \frac{d}{dr} (r^3 K_2) = -\frac{N}{9\pi} r j_0(\omega_0 r) H(r) \quad (\text{B28d})$$

These equations can be separated into four independent second-order inhomogeneous equations

$$\theta_0 K_1(r) = \frac{N}{9\pi} \frac{j_0(\omega_0 r)}{r^2} \frac{d}{dr} (r^3 H(r))$$

$$\theta_2 K_2(r) = -\frac{N}{9\pi} j_0(\omega_0 r) r \frac{d}{dr} H(r)$$

$$\theta_1 K_3(r) = -\frac{N}{9\pi} j_1(\omega_0 r) r^2 \frac{d}{dr} \left(\frac{H(r)}{r} \right)$$

$$\Theta_1 K_4(r) = \frac{N}{9\pi} \frac{j_1(\omega_0 r)}{r} \frac{d}{dr} (r^2 H(r)) \quad (\text{B29})$$

where

$$\Theta_l = \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} + \omega_0^2 - \frac{l(l+1)}{r^2} \quad (\text{B30})$$

is the operator associated with the spherical Bessel functions j_l and y_l . These equations are easily solved using Green's-function techniques. Including the homogeneous solutions regular at the origin we obtain

$$K_1 = \frac{N}{9\pi} \int_0^R dr' G_0(r, r') j_0(\omega_0 r') \frac{d}{dr'} [r'^3 H(r')] + A_1 j_0(\omega_0 r)$$

$$K_2 = -\frac{N}{9\pi} \int_0^R r'^3 dr' G_2(r, r') j_0(\omega_0 r') \frac{d}{dr'} H(r') + A_2 j_2(\omega_0 r)$$

$$K_3 = -\frac{N}{9\pi} \int_0^R r'^4 dr' G_1(r, r') j_1(\omega_0 r') \frac{d}{dr'} \left[\frac{H(r')}{r'} \right] + A_3 j_1(\omega_0 r)$$

$$\begin{aligned} \hat{K}_y = & \frac{N}{9\pi} \int_0^R r' dr' G_1(r, r') j_1(\omega_0 r') \frac{d}{dr'} \left[r'^2 H(r') \right] \\ & + A_4 j_1(\omega_0 r) \end{aligned} \quad (\text{B31})$$

where the A_i are constants of integration, and the kernel of the Green's function is

$$\begin{aligned} G_2(r, r') = \omega_0 \left\{ \eta_2(\omega_0 r) j_2(\omega_0 r') \Theta(r-r') \right. \\ \left. + j_2(\omega_0 r) \eta_2(\omega_0 r') \Theta(r < r') \right\} \end{aligned} \quad (\text{B32})$$

Two relations between the four constants A_i can be obtained by requiring that (B31) satisfy the original coupled equations (B28). For this purpose it is sufficient to use only two of the four equations; Eqs. (B28a) and (B28c) are satisfied for all values of r if

$$A_1 - A_3 = -\frac{N}{9\pi} j_0(\gamma_1 + \gamma_0) \rho R^2 H(R) \quad (\text{B33})$$

where $\gamma_2 = \gamma_2(\rho)$. Similarly, (B38b) and (B38d) are satisfied for all values of r if

$$A_2 + A_4 = -\frac{N}{9\pi} j_0(\gamma_1 - \gamma_2) \rho R^2 H(R) \quad (\text{B34})$$

Two more relations are obtained by requiring that the total ψ_{2g} satisfy the boundary condition (2.15b). From (B22), (B26), and (B27) this gives

$$[\mathcal{K}_1(R) - \mathcal{K}_3(R)] \mathcal{P}_{0i} + NR(j_0' - j_1') w_{2gi} = 0$$

$$\mathcal{K}_2(R) = \mathcal{K}_4(R) \quad (\text{B35})$$

where we have exploited the fact that w_{2gi} cannot depend on \mathcal{P}_{2i} . From (B35) we obtain

$$w_{2gi} = \frac{8}{9} N^2 \int_0^R r^3 dr j_0 j_1 H(r) \mathcal{P}_{0i} \quad (\text{B36})$$

$$A_2 j_2 - A_4 j_1 = \frac{N}{9\pi} \left\{ j_0 (\gamma_2 j_2 + \gamma_1 j_0) \rho R^2 H(R) - \frac{\rho^2}{R^2} (\gamma_1 + \gamma_2) \int_0^R r^3 dr j_1 (j_0 - j_2) H(r) \right\} \quad (\text{B37})$$

This completely determines ψ_{2g} except for an arbitrary additional term $\psi_{0i} \delta N_{gi}$, which must be chosen to preserve the normalization of the full solution to order g_c^2 . Note that (B36) agrees with (2.29b).

As discussed in the second chapter, it is not necessary to know ψ_{2g} in order to determine the energy to $O(g_c^2)$. To this order the term containing (B36) cancels, and need not be determined at all. However, if we wish to determine the eccentricity, we need to calculate that part of the contribution of ψ_{2g} to the NLC which will give rise to an angle-dependent term of the form $P_2(z)$. It is clear that only the matrix elements of the \mathcal{P}_{2i} operator will give a $P_2(z)$, and hence only the contribution of this term to the NLC will be worked out now.

In preparation for this computation, the relations (B34) and (B37) can be solved for A_2 and A_4

$$A_2 = \frac{N}{9\pi} \left[\eta_2 j_0 \rho R^2 H(R) - \frac{\rho^2}{R^2} \left(\frac{\eta_2 + \eta_1}{j_1 + j_2} \right) \int_0^R r^3 dr j_1 (j_0 - j_2) H(r) \right]$$

$$A_4 = -\frac{N}{9\pi} \left[\rho R^2 \eta_1 j_0 H(R) - \frac{\rho^2}{R^2} \left(\frac{\eta_2 + \eta_1}{j_1 + j_2} \right) \int_0^R r^3 dr j_1 (j_0 - j_2) H(r) \right] \quad (B38)$$

The gluon contribution to the NLC, which will be denoted by B_g , becomes

$$B_g = -\frac{1}{2} \sum_{i \neq j} \vec{B}_i^a \cdot \vec{B}_j^a - \sum_i \delta N_{gi} \frac{\partial}{\partial r} \overline{F}_{0i} \dagger_{0i}$$

$$- \frac{1}{2} g_c^2 \frac{\partial}{\partial r} \sum_i (\overline{F}_{2gi} \dagger_{0i} + \overline{F}_{0i} \dagger_{2gi})$$

$$= B_{0g} \overline{\mathcal{P}}_0 + B_{2g} \overline{\mathcal{P}}_2 \quad (B39)$$

where

$$\overline{\mathcal{P}}_0 = \sum_i \mathcal{P}_{0i} = \sum_{\substack{i,j \\ i \neq j}} \vec{b}_i \cdot \vec{b}_j$$

$$\overline{\mathcal{P}}_2 = \sum_i \mathcal{P}_{2i} = \sum_{\substack{i,j \\ i \neq j}} \left(\frac{3}{2} \vec{b}_i \cdot \hat{r} \vec{b}_j \cdot \hat{r} - \frac{1}{2} \vec{b}_i \cdot \vec{b}_j \right) \quad (B40)$$

The quantity B_{0g} , not needed in this calculation, is given in Appendix C. The B_{2g} factor, after considerable manipulation becomes

$$B_{2g} = \frac{\alpha_c}{2\pi R^4 (\rho-1)^2} \left\{ \left(\frac{4\rho-3}{12\rho} \right)^2 + \frac{1}{36} \left(\frac{1}{\rho} - \frac{4}{3} (\rho-1)^2 \right) \right\}$$

(B41)

The first term in curly brackets is the contribution of the B^2 term. The rest of the terms come from ψ_{2g} .

APPENDIX C

IDENTITIES AND THE SECOND-ORDER QUARK SOLUTIONS

This appendix includes some useful identities and details of the second-order quark wave functions, ψ_{2j} .

Many bag formulas require knowledge of spherical Bessel functions on the bag surface where $j_1(\rho) = j_0(\rho)$. Using recursion relations, it is always possible to reduce $j_\ell(\rho) = j_\ell$ to j_0 times a polynomial in $\frac{1}{\rho}$. Useful identities are

$$j_2 = j_0 \left(\frac{3}{\rho} - 1 \right)$$

$$j_3 = j_0 \left(\frac{15}{\rho^2} - \frac{5}{\rho} - 1 \right)$$

$$y_0 = j_0 \left(1 - \frac{1}{\rho} \right)$$

$$y_1 = -j_0 \left(\frac{1}{\rho^2} - \frac{1}{\rho} + 1 \right)$$

$$y_2 = -j_0 \left(\frac{3}{\rho^3} - \frac{2}{\rho^2} + \frac{2}{\rho} + 1 \right)$$

$$j_0' = -j_0$$

$$j_1' = j_0 \left(1 - \frac{2}{\rho} \right)$$

$$j_2' = j_0 \left(-\frac{q}{\rho^2} + \frac{3}{\rho} + 1 \right)$$

$$j_3' = j_0 \left(-\frac{60}{\rho^3} + \frac{20}{\rho^2} + \frac{2}{\rho} - 1 \right)$$

$$j_0^2 (2\rho^2 - 2\rho + 1) = 1 \quad (C1)$$

The remainder of this appendix will be devoted to discussion of the solution Ψ_{2g} . The solutions (B31) can be simplified by integrating by parts, giving

$$\begin{aligned} K_1(r) = & \frac{N}{9\pi\omega_0^2} \left[j_0(s) \int_1^{\rho} (\eta_1 j_0 + \eta_0 j_1) \rho'^3 H d\rho' + \eta_0(s) \int_0^1 2j_0 j_1 \rho'^3 H d\rho' \right] \\ & + j_0(s) \left[\frac{N}{9\pi\omega_0^2} j_0 \eta_0 \rho^3 H + A_1 \right] \end{aligned}$$

$$\begin{aligned} K_2(r) = & \frac{N}{9\pi\omega_0^2} \left[j_2(s) \int_1^{\rho} (\eta_1 j_0 - \eta_2 j_1) \rho'^3 H d\rho' + \eta_2(s) \int_0^1 (j_1 j_0 - j_2 j_1) \rho'^3 H d\rho' \right] \\ & + j_2(s) \left[-\frac{N}{9\pi\omega_0^2} \eta_2 j_0 \rho^3 H + A_2 \right] \end{aligned}$$

$$\begin{aligned} K_3(r) = & \frac{N}{9\pi\omega_0^2} \left[j_1(s) \int_1^{\rho} (j_0 \eta_1 + \eta_0 j_1) \rho'^3 H d\rho' + \eta_1(s) \int_0^1 2j_1 j_0 \rho'^3 H d\rho' \right] \\ & + j_1(s) \left[-\frac{N}{9\pi\omega_0^2} \eta_1 j_1 \rho^3 H + A_3 \right] \end{aligned}$$

$$\begin{aligned} K_4(r) = & \frac{N}{9\pi\omega_0^2} \left[j_1(s) \int_1^{\rho} (j_2 \eta_1 - \eta_0 j_1) \rho'^3 H d\rho' + \eta_1(s) \int_0^1 (j_2 j_1 - j_1 j_0) \rho'^3 H d\rho' \right] \\ & + j_1(s) \left[\frac{N}{9\pi\omega_0^2} \eta_1 j_1 \rho^3 H + A_4 \right] \end{aligned} \quad (C2)$$

In these expressions $\gamma = w \text{ or } \dots$, and $j_\ell = j_\ell(\rho')$ when under the ρ' integral. To obtain these use the identities

$$\frac{1}{x^{\ell+1}} \frac{d}{dx} (x^{\ell+1} j_\ell) = j_{\ell-1}, \quad x^\ell \frac{d}{dx} \left(\frac{j_\ell}{x^\ell} \right) = -j_{\ell+1} \quad (C3)$$

Using the forms (C2) and the identities (C3) it is easy to verify the conditions (B33), (B34), and (B36), (B37).

The correction to the normalization condition, defined in Eq. (2.17), can also be obtained from (C2). The equation is

$$\begin{aligned} \delta N_{2g_i} = & -\frac{g_c^2}{2} \int d^3x \left[(\psi_{2g_i}^{1+} \psi_{0i} + \psi_{0i}^+ \psi_{2g_i}^1) \right. \\ & \left. + 2N(j_0 K_1 + j_1 K_3) \rho_{0i} \right], \quad (C4) \end{aligned}$$

where the K_2 and the K_4 functions do not contribute because ρ_{2i} integrates to zero. The first term in (C4) has structure similar to $\delta N_{\pi i}$ given in Eq. (B11). The second term requires evaluation of the expression

$$\begin{aligned} \mathbb{I}_1 = & -g_c^2 \cdot \frac{4}{9} \frac{N^2}{w^2} \int_0^{\rho} r^2 dr \left[(j_0^2 + j_1^2) \int_0^{\rho} (y_1 j_0 + y_0 j_1) \rho^3 H d\rho' \right. \\ & \left. + (j_0 y_0 + j_1 y_1) \int_0^{\rho} 2 j_0 j_1 \rho^3 H d\rho' \right] \quad (C5) \end{aligned}$$

This can be integrated by parts and reduced to

$$I_1 = -g_c^2 \frac{4}{9} \frac{N^2}{\omega_0^5} \left\{ H \rho^2 j_0^2 \left(-\rho^2 + \frac{\rho}{2} - \frac{3}{8} \right) + \left(\frac{1}{2} - 2 j_0^2(\rho-1) \right) \int_0^\rho H \rho'^3 j_0 j_1 d\rho' \right\} \quad (C6)$$

In addition to this term, there is a term coming from the constant terms involving A_1 and A_3 . Using (B33) this last term gives

$$\begin{aligned} I_2 &= -g_c^2 N^2 \int d^3x \left\{ j_0^2(\omega_0 r) \left[A_1 + \frac{N}{9\pi\omega_0^2} j_0 y_0 \rho^3 H \right] \right. \\ &\quad \left. + j_1^2(\omega_0 r) \left[A_3 - \frac{N}{9\pi\omega_0^2} j_0 y_1 \rho^3 H \right] \right\} \\ &= -\frac{g_c^2}{2N} (A_1 + A_3) - \frac{g_c^2 N^2}{\omega_0^5} \frac{4(\rho-1)}{9} \rho^3 j_0^2 H \end{aligned} \quad (C7)$$

Hence, combining these expressions gives

$$\mathcal{F} N_{2gi} = -g_c^2 \frac{(3-2\rho)}{2(\rho-1)} \frac{\omega_{2gi}}{\omega_0} + (I_1 + I_2) \mathcal{P}_{0i} \quad (C8)$$

Instead of leaving this nonzero, as in the treatment of the pion contributions, $A_1 + A_3$ will be determined from the requirement that $\delta N_{2gi} = 0$. Using (B36) this gives

$$A_1 + A_3 = \frac{N}{9\pi\omega_0^2} \frac{1}{j_0^2 \rho^2 (\rho-1)} \times \left[\frac{\left(\frac{3}{2}\rho - \frac{5}{2} + 2j_0^2(\rho-1)^2\right)\Lambda}{(\rho-1)} + H\rho^2 j_0^2 \left(\frac{\rho}{2} + \frac{3}{8}\right) \right] \quad (C9)$$

where

$$\begin{aligned} \Lambda &= \int_0^\rho \rho'^3 j_0 j_1 H d\rho' \\ &= \frac{9\omega_0^5}{16\pi N^2} \left[\frac{\mu^2(\rho)}{\rho} + 2 \int_0^\rho \frac{\mu^2(\rho')}{\rho'^2} d\rho' \right] \\ \mu(\omega_0 r) &= \frac{8\pi}{3} N^2 \frac{1}{r} \int_0^r dr' r'^3 j_1(\omega_0 r') j_0(\omega_0 r') \quad (C10) \end{aligned}$$

The solution ψ_{2gi} is now completely determined, and the contributions of ψ_{2g} to the NLC can be computed. The computations require frequent integrations by parts and use of the identities involving Bessel functions. The quantity β_{0g} becomes

$$B_{0g} = \frac{\alpha c}{4\pi} \frac{1}{R^4} \left[\mu^2(\rho) + 2\rho \int_0^\rho \frac{\mu^2(\rho')}{\rho'^2} d\rho' \right]$$

(C11)

where the first term is the $\vec{B} \cdot \vec{B}$ contribution and the second term comes from ψ_{2g} . Comparing this with the result reported in Eq. (4.2) shows that the angle independent part of the NLC is equivalent to the requirement that the derivative of the energy with respect to the mean radius be zero.

APPENDIX D

MATRIX ELEMENTS OF SPIN AND ISOSPIN

This section contains the transition operators necessary for calculation of spin-isospin matrix elements. The matrices explicitly shown here were obtained from Pandharipande and Smith.¹⁷

A typical intermediate state matrix element looks like

$$\langle \Delta | \sum_i b_i^k \tau_i^\ell | N \rangle \quad (D1)$$

where k and ℓ are the usual Cartesian indices. Such matrix elements can be rewritten in terms of transition operators \mathcal{S}^k and \mathcal{T}^ℓ , both of which describe transitions from spin-1/2 to spin-3/2 states, and therefore have the same form. The explicit form of the \mathcal{S} operator is

$$\begin{aligned} \mathcal{S}^x &= \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 0 \\ 0 & -\sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} & 0 \\ 0 & 1 \end{bmatrix} \\ \mathcal{S}^y &= \frac{i}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} & 0 \\ 0 & 1 \end{bmatrix} & \mathcal{S}^z &= \sqrt{\frac{2}{3}} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \end{aligned} \quad (D2)$$

There are three helpful identities regarding spin-isospin matrix elements. These are

$$\begin{aligned}
 \langle N | \sum_i b_i^{\ell} \tau_i^k | N \rangle &= \frac{20}{3} \langle N | S_{1/2}^{\ell} T_{1/2}^k | N \rangle \\
 \langle \Delta | \sum_i b_i^{\ell} \tau_i^k | N \rangle &= \frac{4}{\sqrt{2}} \langle \Delta | \mathcal{S}^{\ell} \mathcal{T}^k | N \rangle \\
 \langle \Delta | \sum_i b_i^{\ell} \tau_i^k | \Delta \rangle &= \frac{4}{3} \langle \Delta | S_{3/2}^{\ell} T_{3/2}^k | \Delta \rangle
 \end{aligned} \tag{D3}$$

where S_j and T_j are the usual spin and isospin operators on the space of states with spin j , and \mathcal{S} and \mathcal{T} are the transition operators. These may be obtained by explicit computation from the SU(6) quark wave functions of the N and Δ . In addition to Eq. (D3) two further identities of importance are

$$\begin{aligned}
 \mathcal{S} \cdot \mathcal{S}^{\dagger} = \mathcal{T} \cdot \mathcal{T}^{\dagger} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 \mathcal{S}^{\dagger} \cdot \mathcal{S} = \mathcal{T}^{\dagger} \cdot \mathcal{T} &= 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
 \end{aligned} \tag{D4}$$

With these identities one may readily work out the matrix elements needed in Eq. (3.14). The basic approach, shown below for $\overline{\mathcal{P}}_0^{\mathbf{I}}$, is to insert a complete set of intermediate states and use (D3)

$$\begin{aligned}
\langle N | \bar{P}_0^{\mathbb{I}} | N \rangle &= \langle N | \sum_{\substack{i,j \\ i \neq j}} \vec{b}_i \cdot \vec{b}_j \tau_i \cdot \tau_j | N \rangle \\
&= \sum_{B=N,\Delta} \langle N | \sum_i b_i^x \tau_i^x | B \rangle \langle B | \sum_j b_j^x b_j^x | N \rangle - 27 \\
&= \left(\frac{20}{3}\right)^2 \frac{9}{16} + \frac{16}{2} \mathcal{J}^\dagger \cdot \mathcal{J} \mathcal{J}^\dagger \cdot \mathcal{J} - 27 \\
&= 25 + 32 - 27 = 30
\end{aligned} \tag{D5}$$

The calculation of \bar{P}_0 and \bar{P}_2 is simplified by the conservation of isospin, which ensures that only diagonal terms will enter into the state sum in (D5). Finally, the quantities $\delta_{\mathcal{A},B}$ and $\delta_{\mathcal{A},B}^{\mathbb{I}}$ introduced in Eq. (3.15) are

$$\begin{aligned}
\delta_{0,N}^{\mathbb{I}} &= 30 \\
\delta_{0,N} &= -6 \\
\delta_{0,\Delta} &= \delta_{0,\Delta}^{\mathbb{I}} = 6
\end{aligned} \tag{D6}$$

The δ_2^i were given in Eq. (3.16).

In addition to (D3) there are other identities which come in handy. These are

$$(S_{1/2} \times S_{1/2})^i = i S_{1/2}^i$$

$$(S_{3/2} \times S_{3/2})^i = i S_{3/2}^i$$

$$(S \times S_{1/2})^i = -\frac{i}{2} S^i$$

$$(S_{3/2} \times S)^i = \frac{5}{2} i S^i$$

$$(S \times S^+)^i = \frac{2}{3} i S_{3/2}^i$$

$$(S^+ \times S)^i = -\frac{4}{3} i S_{1/2}^i \quad (D7)$$

Naturally there are equivalent relations between isospin operators.

APPENDIX E
CHIRAL INVARIANCE

1. Motivation: A Warm-up With Isospin

The lowest mass baryons are the nucleon and Δ . These are taken to be composed of up and down quarks, whose masses are small in order to satisfy hadron spectroscopy. Now isospin invariance is the most accurate hadron symmetry and is naively dependent on the mass of the up and down quarks being equal. This is easily shown by writing out the isovector current and taking its divergence. The isovector current is

$$\vec{V}^{\mu} = \sum_i \bar{q}_i \gamma^{\mu} \frac{\vec{\lambda}}{2} q_i \quad (\text{E1})$$

The sum over i refers to all the up and down quarks in the baryon. The divergence of (E1) is facilitated via the QCD equations of motion.

$$i \vec{\not{D}} q_i = m_i q_i + g_s \vec{A}^a \frac{\lambda^a}{2} q_i \quad (\text{E2})$$

$$-i \bar{q} \overleftarrow{\not{D}} = m_i \bar{q}_i + g_s \bar{q}_i \overleftarrow{A}^a \frac{\lambda^a}{2} \quad (\text{E3})$$

We know now that isospin symmetry is badly broken at the SU(2) level. "Indeed the u and d quark masses are typically of order 5 and 10 MeV respectively."⁹ The reason hadronic isospin is still good is because the constituent quark mass for a light confined quark is several hundred MeV larger than the current quark mass. This means the percentage difference in constituent masses (which is a function of current quark masses) is almost zero.

We have just seen how one starts with a theory like flavor SU(3) and grudgingly breaks it with the strange quark mass only to have it broken again at the SU(2) level by the u and d quark mass differences. Fortunately the latter are small on the scale of hadron masses.

2. The Assumption of Conserved Axial Current

The naive assertion that m_u and m_d were equal led us to the conclusion that isospin is conserved at the hadronic level, as well as ^{at} the quark level. It was later shown that isospin is not conserved at the quark level; however the mechanism of confinement saves isospin at the hadron level. It is therefore plausible that if one assumes $m_u = m_d = 0$, consistent with axial current conservation at the quark level, then perhaps axial current will be mostly conserved at the hadron level. This is strongly supported by the Goldberger-Treiman relation which shows that (among other things) $\partial_\mu \tilde{A}^\mu$ is proportional to the mass of the lightest hadron known.

3. Quark Handedness Under Parity Transformation

The three vector current operators are taken to satisfy

$$\partial_\mu \tilde{V}^\mu = 0 \quad (\text{E4})$$

This implies the space integrals of V^0 are constants of motion.

$$\dot{\tilde{Q}}_V = \frac{d}{dt} \int V^0 d^3x = 0 \quad (\text{E5})$$

Now if we assume $m_U = m_D = 0$ then we obtain a conserved axial current.

$$\tilde{A}^\mu = \sum_i \bar{q}_i \gamma^\mu \gamma^5 \frac{\tau_i}{2} q_i \quad (\text{E6})$$

where

$$\partial_\mu \tilde{A}^\mu = 0 \quad (\text{E7})$$

Again there are constants of motion

$$\dot{Q}_A = 0 \quad (\text{E8})$$

The chiral $SU(2) \times SU(2)$ group often denoted $SU(2)_L \times SU(2)_R$ is generated by the linear combinations

$$\underline{Q}_{\pm} \equiv \frac{1}{2} (\underline{Q}_V \pm \underline{Q}_A) \quad (\text{E9})$$

along with the parity operator $P(t)$. These satisfy the following equal time commutation relations

$$[\underline{Q}_+^l(t), \underline{Q}_+^m(t)] = i \epsilon^{lmn} \underline{Q}_+^n(t) \quad (\text{E10})$$

$$[\underline{Q}_-^l(t), \underline{Q}_-^m(t)] = i \epsilon^{lmn} \underline{Q}_-^n(t) \quad (\text{E11})$$

$$[\underline{Q}_+^l(t), \underline{Q}_-^m(t)] = 0$$

This means that right-handed u,d quarks may transform into right-handed u,d quarks and left-handed u,d quarks transform into left-handed u,d quarks. However, left-handed quarks can't mix with right-handed quarks. Naturally the original MIT bag version of confinement violated conservation of handedness since any quark striking the bag surface would reverse its momentum but maintain its spin, thereby changing handedness or chirality. (See Fig. 10 which was taken from Thomas⁹.) Similarly a parity inversion should transform a left-handed particle into a right-handed particle. As a consequence one finds that the parity operator $P(t)$ connects the generators $\underline{Q}_-(t)$ and $\underline{Q}_+(t)$ which form the $SU(2)_L \times SU(2)_R$ group. Following Lee¹⁸ we define the parity operator $P(t)$

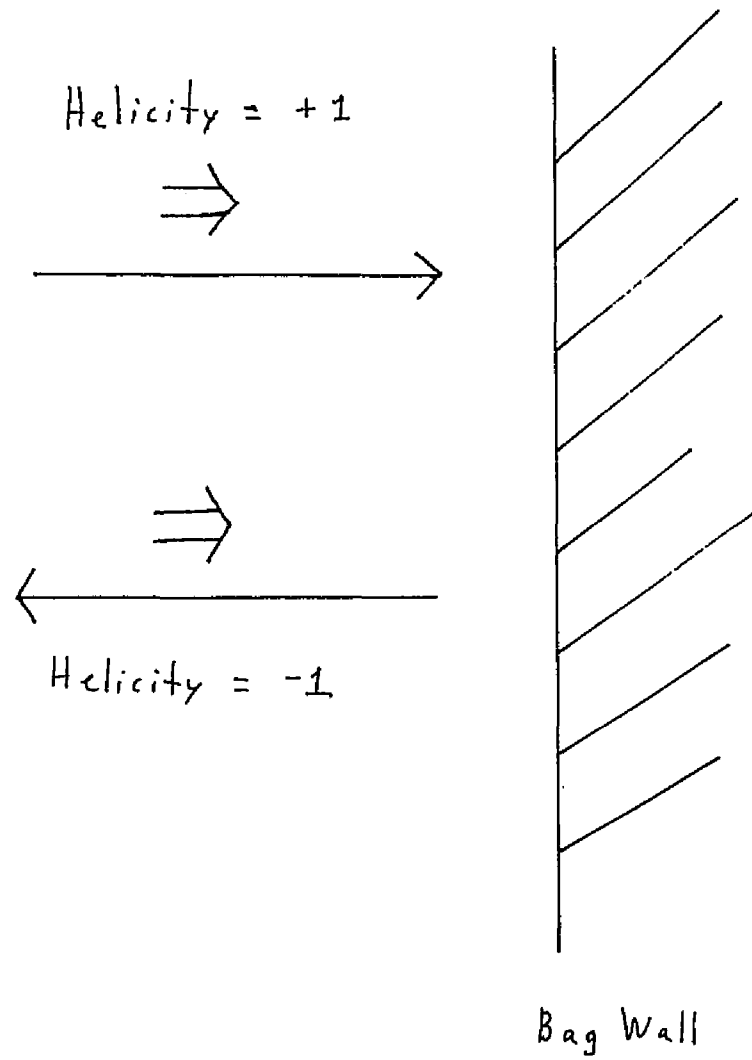


FIG. 10. Violation of chiral symmetry at the MIT bag boundary.

$$P(t) q(\vec{\lambda}, t) P^\dagger(t) = \eta \gamma^0 q(-\vec{\lambda}, t), \quad (\text{E12})$$

with q the quark field. One finds

$$P(t) \underline{Q}_V(t) P^\dagger(t) = Q_V(t) \quad (\text{E13})$$

$$P(t) \underline{Q}_A(t) P^\dagger(t) = -Q_A(t) \quad (\text{E14})$$

Therefore

$$P(t) \underline{Q}_+(t) P^\dagger(t) = \underline{Q}_-(t) \quad (\text{E15})$$

$$P(t) \underline{Q}_-(t) P^\dagger(t) = \underline{Q}_+(t) \quad (\text{E16})$$

The last two equations merely reiterate the idea that left and right-handed quarks transform into each other under the parity operation.

4. The Pion as a Goldstone Boson

(The following is nearly verbatim from reference 18.)

If one assumes axial current conservation as well as vector current conservation then

$$\dot{\underline{Q}}_A = 0 \quad (\text{E17})$$

$$\dot{\underline{Q}}_V = 0 \quad (\text{E18})$$

Consider the proton $|p\rangle$ at rest. It is an eigenstate of the strong interaction Hamiltonian H_{st}

$$H_{st} |p\rangle = m_p |p\rangle \quad (\text{E19})$$

It is an eigenstate of parity and we choose the phase η in (E12) so that

$$P |p\rangle = |p\rangle \quad (\text{E20})$$

Now operating the Q_v on $|p\rangle$ will yield a linear superposition of the neutron and proton. Next extend the reasoning to Q_a . Due to (E17) we have

$$\left[\tilde{Q}_A, H_{st} \right] = 0 \quad (\text{E21})$$

This means

$$H_{st} \tilde{Q}_A |p\rangle = \tilde{Q}_A H_{st} |p\rangle \quad (\text{E22})$$

Use (E19) for the right hand side

$$H_{st} \tilde{Q}_A |p\rangle = m \tilde{Q}_A |p\rangle \quad (\text{E23})$$

Equation (E23) implies there are states $\tilde{Q}_a |p\rangle$ which are degenerate with the proton; however the parity of these states differs from the parity of the proton (or neutron) by a minus sign. That is

$$\begin{aligned}
 P Q_A |p\rangle &= P Q_A P^{-1} P |p\rangle \\
 &= -Q_A |p\rangle
 \end{aligned}
 \tag{E24}$$

We know that none of these states exist in nature so we conclude that $Q_a |p\rangle$ must be one of the continuum states $|p\pi\rangle, |n\pi\rangle$. These have the correct parity and if the pions were massless they would ^{also} have the right mass. We see that "The assumption of axial-current conservation must be used in conjunction with the approximation of zero mass pseudoscalar mesons. Together they will be referred to as the two conditions of CAC. It is important that we distinguish these unusual features of CAC from those (more conventional ones) of CVC."¹⁸ Using arguments similar to those leading to (E24) but replacing $|p\rangle$ with $|0\rangle$ we see that the vacuum is not an eigenstate of Q_a . If it were it would have the wrong parity. Instead, we obtain the "Nambu-Goldstone realization of chiral symmetry"¹⁸ wherein there are as many massless pseudoscalar bosons as there are generators for which $Q_a |0\rangle \neq 0$. (Details of the Goldstone theorem may be found in reference 18.) In the case of SU(2) flavor we obtain

$$Q_A |0\rangle \approx A |\pi^+\rangle + B |\pi^-\rangle + C |\pi^0\rangle
 \tag{E25}$$

If we extended masslessness to the strange quark as well we would have nine generators of SU(3) flavor and they could reasonably give rise to Goldstone bosons π , K , and η . These are all light on the scale of hadron masses, and have negative parity.

5. Chiral Invariance in the Bag Model

As stated previously, the MIT bag model did not conserve axial current even under the assumption of massless u,d quarks. This is due to the unusual surface coupling in the Lagrangian which acts like a mass term. We refer to

$$\mathcal{L} = \bar{\psi} \psi \Delta_s \quad (\text{E26})$$

One observes that under the chiral quark transformation (which obtains in QCD for massless u,d quarks)

$$\begin{aligned} \psi &\rightarrow e^{i\frac{\tau \cdot \epsilon}{2} \gamma^5} \psi \\ \bar{\psi} &\rightarrow \bar{\psi} e^{i\frac{\tau \cdot \epsilon}{2} \gamma^5} \end{aligned} \quad (\text{E27})$$

$$\mathcal{L} \rightarrow \bar{\psi} e^{i\tau \cdot \epsilon \gamma^5} \psi \Delta_s \quad (\text{E28})$$

Clearly the Lagrangian (E26) is not invariant under the chiral transformation. Another way to see this is by writing out the axial current in the bag:

$$\vec{A}^a = \bar{\psi} \gamma^\mu \gamma^5 \frac{\tau^a}{2} \psi \partial_\mu \quad (\text{E29})$$

Taking the divergence of this we obtain

$$\partial_\mu \hat{A}^\mu = \bar{\Psi} \gamma^\mu \gamma^5 \frac{\gamma_\mu}{i} \Psi + h_\mu \Delta_s \quad (\text{E30})$$

There should be some other contribution to the axial current whose divergence cancels (E30) but is such that

$$\partial_\mu \hat{A}^\mu = f_\pi m_\pi^2 \phi \quad (\text{E31})$$

A Lagrangian which satisfies (E31) namely (2.1) was used by Vento et al.¹⁶ Like most chiral Lagrangians its progenitor is the σ model of Gell-Man and Levy.¹⁹ Our choice of (2.1) among the various chiral bag Lagrangians is based on its minimal surface coupling as well as ease of calculation. Eq.(2.1) does not describe a renormalizable theory (see Chapter IX); however, it has a history of successes in the low energy regime which is where we intend to use it. Mainly we are looking for deformation effects at the classical level so it makes sense to have a Lagrangian that embodies PCAC which is a low energy result. The biggest difference between our model and the cloudy bag model¹¹ for example, is that we don't allow pions inside. For an outstanding review of chiral bag models see Thomas⁹.

The axial current in our model is obtained by first noting the following variations in the fields

$$\begin{aligned}
 \psi' &= \psi - i \frac{\underline{x} \cdot \underline{\epsilon}}{2} \gamma^5 \psi \\
 &\equiv \psi + \underline{f}_2(\psi) \cdot \underline{\epsilon}
 \end{aligned}
 \tag{E32}$$

$$\begin{aligned}
 \bar{\psi}' &= \bar{\psi} - i \bar{\psi} \gamma^5 \frac{\underline{x} \cdot \underline{\epsilon}}{2} \\
 &\equiv \bar{\psi} + \underline{f}_2(\bar{\psi}) \cdot \underline{\epsilon}
 \end{aligned}
 \tag{E33}$$

$$\begin{aligned}
 \phi' &= \phi - \frac{\underline{\epsilon}}{g} - g \underline{f}_2(\phi) (\underline{\epsilon}) \\
 &\equiv \phi + \underline{f}_2(\phi) \cdot \underline{\epsilon}
 \end{aligned}
 \tag{E34}$$

In particular the transformation (E34) is not obvious but was constructed in an involved way.⁹ The current is

$$\underline{A}^\mu = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi_i)} \underline{f}_i
 \tag{E35}$$

where ψ_i refers to the fields in equations (E32) thru (E34).

We obtain for the axial current

$$\underline{A}^\mu = \bar{\psi} \gamma^\mu \gamma^5 \frac{\underline{x}}{2} \psi \theta_V + \frac{1}{g} \partial^\mu \phi \theta_V
 \tag{E36}$$

It is still not divergenceless due to the pion mass terms in (2.1). The divergence of (E36) may be worked easily by hand or more formally by noting that

$$\partial_\mu \underline{A}^\mu = \frac{\partial \mathcal{L}}{\partial \psi_i} \underline{f}_i
 \tag{E37}$$

where \mathcal{L}' is that part of the Lagrangian which isn't invariant, in our case the pion mass term which was put in for that reason:

$$\mathcal{L}' = -\frac{1}{2} m^2 \phi^2 \theta_V \quad (\text{E38})$$

One obtains from (E37)

$$\partial_\mu \tilde{A}^\mu = \frac{1}{g} m_\pi^2 \tilde{\phi} \quad (\text{E39})$$

APPENDIX F

MATRIX ELEMENTS USED IN THE DERIVATION OF $g_{\pi NN}$, $g_{\pi\Delta\Delta}$, M_{NN} , $M_{\Delta\Delta}$

1. Nucleon Matrix Elements Required For $g_{\pi NN}$ Calculation

We need to know five types of matrix elements.

They are

$$\langle N | \sum_i \vec{b}_i \cdot \hat{\sigma}_i \tau_i^z | N \rangle \quad (\text{F1})$$

$$\langle N | \sum_i \left[\frac{\tau_i^+}{2} \vec{b}_i \cdot \hat{\sigma}_i \mathcal{P}_{0i}^{\mp} + \text{H.C.} \right] | N \rangle \quad (\text{F2})$$

$$\langle N | \sum_i \left[\frac{\tau_i^-}{2} \vec{b}_i \cdot \hat{\sigma}_i \mathcal{P}_{2i}^{\mp} + \text{H.C.} \right] | N \rangle \quad (\text{F3})$$

$$\langle N | \sum_i \left[\frac{\tau_i^z}{2} \vec{b}_i \cdot \hat{\sigma}_i \mathcal{P}_{0i} + \text{H.C.} \right] | N \rangle \quad (\text{F4})$$

$$\langle N | \sum_i \left[\frac{\tau_i^z}{2} \vec{b}_i \cdot \hat{\sigma}_i \mathcal{P}_{2i} + \text{H.C.} \right] | N \rangle \quad (\text{F5})$$

Matrix element (F1) has already been worked in Appendix D.

$$\langle N | \sum_i \vec{b}_i \cdot \hat{r} \tau_i | N \rangle = \frac{5}{3} \vec{b} \cdot \hat{r} \tau \quad (\text{F6})$$

Matrix element (F2) involves some intermediate state sums with both nucleons and Δ 's in the intermediate states. We shall require use of the identities in Appendix D. Note that the second term in (F2) is just the Hermitian conjugate of the first term. The first term is

$$\otimes_{\text{Res}} = \langle N | \sum_{\substack{i,j \\ i \neq j}} \frac{\tau_i}{2} \vec{b}_i \cdot \hat{r} \vec{b}_i \cdot \vec{b}_j \frac{\tau_j}{2} | N \rangle \quad (\text{F7})$$

In (F7) we denote by the subscript Res, the restriction $i \neq j$. The procedure in evaluating \otimes_{Res} will be to work out the \otimes = unrestricted sum first and then subtract off the $i=j$ terms to get \otimes_{Res} .

$$\otimes = \langle N | \sum_{i,j} \frac{\tau_i}{2} \vec{b}_i \cdot \hat{r} \vec{b}_i \cdot \vec{b}_j \frac{\tau_j}{2} | N \rangle \quad (\text{F8})$$

This may be broken into four terms which will be labeled.

$$\begin{aligned}
 \textcircled{*} &= \frac{1}{2} \sum_{i,j} \left\{ \begin{array}{l} \textcircled{1} \\ \vec{b}_j \cdot \hat{r} \tau_j \\ -i \vec{b}_j \cdot \hat{r} (\tau_i \times \tau_j) \\ \textcircled{3} \\ -i \hat{r} \cdot (\vec{b}_i \times \vec{b}_j) \tau_j \\ \textcircled{4} \\ -\hat{r} \cdot (\vec{b}_i \times \vec{b}_j) (\tau_i \times \tau_j) \end{array} \right. \quad (F9)
 \end{aligned}$$

① is simply

$$\textcircled{1} = \frac{5}{2} \vec{b} \cdot \hat{r} \tau \quad (F10)$$

② will involve only nucleon intermediate states since the expectation value of isospin between nucleon and Δ is zero. The necessary identities are Eq. (F6) and

$$\tau \times \tau = 2i\tau \quad (F11)$$

$$\begin{aligned}
 \textcircled{2} &= -\frac{i}{2} \sum_{\substack{j, \lambda \\ \Delta', N'}} \langle N | \tau_i \times \tau_j | \Delta' \rangle \langle \Delta' | \tau_j \vec{b}_j \cdot \hat{r} | N \rangle \quad (F12) \\
 &= -\frac{i}{2} \frac{5}{3} (\tau \times \tau) \vec{b} \cdot \hat{r}
 \end{aligned}$$

$$\textcircled{2} = \frac{5}{3} \vec{b} \cdot \hat{r} \tau \quad (F13)$$

We perform ③ in a similar fashion using

$$\vec{b} \times \vec{b} = 2i\vec{b} \quad (F14)$$

(F15)

There will be contributions from nucleon and Λ intermediate states in (4). The helpful identities are Eqs. (D7)

$$\vec{S}^+ \times \vec{S} = -\frac{4}{3} i \vec{S} = -\frac{2}{3} i \vec{6} \quad (\text{F16})$$

$$\vec{T}^+ \times \vec{T} = -\frac{4}{3} i \vec{T} = -\frac{2}{3} i \vec{7} \quad (\text{F17})$$

We obtain

$$\textcircled{4} = \frac{22}{3} \vec{6} \cdot \hat{r} \vec{7} \quad (\text{F18})$$

So that

$$\textcircled{*} = \frac{29}{6} \vec{6} \cdot \hat{r} \vec{7} \quad (\text{F19})$$

Double this to get

$$\textcircled{*} + \textcircled{*}^+ = \frac{29}{3} \vec{6} \cdot \hat{r} \vec{7} \quad (\text{F20})$$

The $i=j$ terms are easily subtracted off. We present the value of (F2) for the cases of $i=j$ and $i \neq j$.

$$\begin{aligned} & \langle N | \sum_i \left[\frac{29}{3} \vec{6}_i \cdot \hat{r}_i \mathcal{P}_{0i}^{\text{I}} + \text{H.c.} \right] | N \rangle \\ &= \frac{29}{3} \vec{6} \cdot \hat{r} \vec{7} \Big|_{i=j} \quad , \quad \frac{34}{3} \vec{6} \cdot \hat{r} \vec{7} \Big|_{i \neq j} \end{aligned} \quad (\text{F21})$$

The values of Eqs. (F2), (F4) and (F5) may be derived similarly. They are

$$\begin{aligned} & \langle N | \sum_i \left[\frac{\tau_i}{2} \vec{b}_i \cdot \hat{r} p_{2i}^{\dagger} + \text{H.C.} \right] | N \rangle \\ &= -\frac{2}{3} \vec{b} \cdot \hat{r} \tau \Big|_{i=j} , \quad -\frac{2}{3} \vec{b} \cdot \hat{r} \tau \Big|_{i \neq j} \end{aligned} \quad (\text{F22})$$

$$\begin{aligned} & \langle N | \sum_i \left[\frac{\tau_i}{2} p_{0i} + \text{H.C.} \right] | N \rangle \\ &= +\frac{13}{3} \vec{b} \cdot \hat{r} \tau \Big|_{i=j} , \quad -\frac{2}{3} \vec{b} \cdot \hat{r} \tau \Big|_{i \neq j} \end{aligned} \quad (\text{F23})$$

$$\begin{aligned} & \langle N | \sum_i \left[\frac{\tau_i}{2} \vec{b}_i \cdot \hat{r} p_{2i} + \text{H.C.} \right] | N \rangle \\ &= -\frac{2}{3} \vec{b} \cdot \hat{r} \tau \Big|_{i=j} , \quad -\frac{2}{3} \vec{b} \cdot \hat{r} \tau \Big|_{i \neq j} \end{aligned} \quad (\text{F24})$$

Note, there is a large difference between the $i=j$ terms and the $i \neq j$ terms in Eqs. (F21) and (F23).

2. Δ Matrix Elements Required For $g_{\pi\Delta\Delta}$ Calculation

We need the following five matrix elements

$$\langle \Delta | \sum_i \vec{b}_i \cdot \hat{v} \tau_i | \Delta \rangle \quad (\text{F25})$$

$$\langle \Delta | \sum_i \left[\frac{\tau_i}{2} \vec{b}_i \cdot \hat{v} \mathcal{P}_{0i}^I + \text{H.C.} \right] | \Delta \rangle \quad (\text{F26})$$

$$\langle \Delta | \sum_i \left[\frac{\tau_i}{2} \vec{b}_i \cdot \hat{v} \mathcal{P}_{2i}^I + \text{H.C.} \right] | \Delta \rangle \quad (\text{F27})$$

$$\langle \Delta | \sum_i \left[\frac{\tau_i}{2} \vec{b}_i \cdot \hat{v} \mathcal{P}_{0i} + \text{H.C.} \right] | \Delta \rangle \quad (\text{F28})$$

$$\langle \Delta | \sum_i \left[\frac{\tau_i}{2} \vec{b}_i \cdot \hat{v} \mathcal{P}_{2i} + \text{H.C.} \right] | \Delta \rangle \quad (\text{F29})$$

Matrix element (F25) is found in Appendix D.

$$\langle \Delta | \sum_i \vec{b}_i \cdot \hat{v} \frac{\tau_i}{2} | \Delta \rangle = \frac{4}{3} \mathcal{T} \vec{S} \cdot \hat{v} \quad (\text{F30})$$

T and S are the usual isospin and spin operators for an isospin 3/2, spin 3/2 system. We next work (F26) noting

the second term is just the Hermitian conjugate of the first. The first term is

$$\otimes_{Res} = \langle \Delta | \sum_{\substack{i,j \\ i \neq j}} \frac{\tau_i}{2} \vec{b}_i \cdot \hat{r} \vec{b}_i \cdot \vec{b}_j \tau_i \cdot \tau_j | \Delta \rangle \quad (F31)$$

The Res notation is the same as in Section 1. We'll work the unrestricted sum first.

$$\otimes = \langle \Delta | \sum_{i,j} \frac{\tau_i}{2} \vec{b}_i \cdot \hat{r} \vec{b}_i \cdot \vec{b}_j \tau_i \cdot \tau_j | \Delta \rangle \quad (F32)$$

This is four terms.

$$\otimes = \frac{1}{2} \sum_{i,j} \left\{ \begin{array}{l} \textcircled{1} \tau_j \vec{b}_j \cdot \hat{r} \quad - \quad \textcircled{2} i \vec{b}_j \cdot \hat{r} (\tau_i \times \tau_j) \\ - \quad \textcircled{3} i \hat{r} \cdot (\vec{b}_i \times \vec{b}_j) \tau_j \quad - \quad \textcircled{4} (\tau_i \times \tau_j) (\vec{b}_i \times \vec{b}_j) \cdot \hat{r} \end{array} \right\} \quad (F33)$$

Using identity (D3) we get

$$\textcircled{1} = 2 \sum_{\underline{\lambda}} T \vec{s} \cdot \hat{r} \quad (F34)$$

There will be both nucleon and Δ intermediate states in $\textcircled{2}$; however, only the Δ states will contribute.

$$\begin{aligned} \textcircled{2} &= -\frac{i}{2} \sum_{\substack{i,j \\ N', \Delta}} \langle \Delta | \tau_i \times | N' \rangle \langle N' | \tau_j \vec{b}_j \cdot \hat{r} | \Delta \rangle \\ &= -\frac{i}{2} \cdot 2 \cdot \frac{4}{3} \left(\underline{T} \times \underline{T} \right) \vec{s} \cdot \hat{r} \end{aligned} \quad (F35)$$

$$\textcircled{2} = \frac{4}{3} \underline{T} \vec{S} \cdot \hat{r} , \quad (\text{F36})$$

where we used identity (D3) and

$$\underline{T} \times \underline{T} = i \underline{T} \quad (\text{F37})$$

Next, perform $\textcircled{3}$ similarly using

$$\vec{S} \times \vec{S} = i \vec{S} \quad (\text{F38})$$

$$\textcircled{3} = \frac{4}{3} \underline{T} \vec{S} \cdot \hat{r} \quad (\text{F39})$$

The nucleon and Δ intermediate states will both contribute to $\textcircled{4}$. Using identities (D7)

$$\underline{T} \times \underline{T}^+ = \frac{2}{3} i \underline{T} \quad (\text{F40})$$

$$\vec{S} \times \vec{S}^+ = \frac{2}{3} i \vec{S} , \quad (\text{F41})$$

we get

$$\textcircled{4} = \frac{8}{3} \underline{T} \vec{S} \cdot \hat{r} \quad (\text{F42})$$

Adding $\textcircled{1}$, $\textcircled{2}$, $\textcircled{3}$ and $\textcircled{4}$ we get

$$\textcircled{*} = \frac{22}{3} \underline{T} \vec{S} \cdot \hat{r} \quad (\text{F43})$$

Double this for

$$\otimes + \otimes^{\dagger} = \frac{44}{3} \underline{\tau} \vec{S} \cdot \hat{r} \quad (\text{F44})$$

The $i=j$ terms are easy to calculate and subtract off. Below is (F26) for the $i=j$ and $i \neq j$ cases.

$$\begin{aligned} & \langle \Delta | \sum_i \left[\frac{\tau_i}{2} b_i \cdot \hat{r} \rho_{0i}^{\dagger} + \text{H.c.} \right] | \Delta \rangle \\ &= \frac{44}{3} \underline{\tau} \vec{S} \cdot \hat{r} \Big|_{i=j}, \quad \frac{8}{3} \underline{\tau} \vec{S} \cdot \hat{r} \Big|_{i \neq j} \end{aligned} \quad (\text{F45})$$

The other identities are derived similarly.

$$\begin{aligned} & \langle \Delta | \sum_i \left[\frac{\tau_i}{2} b_i \cdot \hat{r} \rho_{2i}^{\dagger} + \text{H.c.} \right] | \Delta \rangle \\ &= \frac{8}{3} \underline{\tau} \vec{S} \cdot \hat{r} \Big|_{i=j}, \quad \frac{8}{3} \underline{\tau} \vec{S} \cdot \hat{r} \end{aligned} \quad (\text{F46})$$

$$\begin{aligned} & \langle \Delta | \sum_i \left[\frac{\tau_i}{2} b_i \cdot \hat{r} \rho_{0i} + \text{H.c.} \right] | \Delta \rangle \\ &= \frac{20}{3} \underline{\tau} \vec{S} \cdot \hat{r} \Big|_{i=j}, \quad \frac{8}{3} \underline{\tau} \vec{S} \cdot \hat{r} \Big|_{i \neq j} \end{aligned} \quad (\text{F47})$$

$$\begin{aligned} & \langle \Delta | \sum_i \left[\frac{\tau_i}{2} b_i \cdot \hat{r} \rho_{2i} + \text{H.c.} \right] | \Delta \rangle \\ &= \frac{8}{3} \underline{\tau} \vec{S} \cdot \hat{r} \Big|_{i=j}, \quad \frac{8}{3} \underline{\tau} \vec{S} \cdot \hat{r} \end{aligned} \quad (\text{F48})$$

3. Matrix Elements For μ_{NN} Calculation.

There are five different forms of matrix elements required. They are

$$\sum_i Q_i \vec{b}_i \quad (\text{F49})$$

$$\sum_i \left[Q_i \vec{b}_i \rho_{oi}^\dagger + \text{H.C.} \right] \quad (\text{F50})$$

$$\begin{aligned} & \sum_{i,j} \left\{ \left[i \underline{\tau}_j \cdot \underline{\tau}_i (\vec{b}_j \times \vec{b}_i) + \frac{2}{3} \underline{\tau}_j \cdot \underline{\tau}_i (\vec{b}_j \cdot \vec{b}_i) \vec{b}_i \right] Q_i \right. \\ & \left. + Q_i \left[i (\vec{b}_i \times \vec{b}_j) \underline{\tau}_i \cdot \underline{\tau}_j + \frac{2}{3} \vec{b}_i (\vec{b}_i \cdot \vec{b}_j) \underline{\tau}_i \cdot \underline{\tau}_j \right] \right\} \quad (\text{F51}) \end{aligned}$$

$$\sum_i \left[Q_i \vec{b}_i \rho_{oi} + \rho_{oi}^\dagger \vec{b}_i Q_i \right] \quad (\text{F52})$$

$$\begin{aligned} & \sum_{i,j} \left\{ \left[i \vec{b}_j \times \vec{b}_i + \frac{2}{3} (\vec{b}_j \cdot \vec{b}_i) \vec{b}_i \right] Q_i \right. \\ & \left. + Q_i \left[i (\vec{b}_i \times \vec{b}_j) + \frac{2}{3} \vec{b}_i (\vec{b}_i \cdot \vec{b}_j) \right] \right\}, \quad (\text{F53}) \end{aligned}$$

where

$$Q_i = \frac{1}{6} + \frac{\tau_i^3}{2} \quad (\text{F54})$$

We'll work (F49) first.

$$\sum_i Q_i \vec{b}_i = \sum_i \left[\frac{1}{6} + \frac{\tau_i^3}{2} \right] \vec{b}_i \quad (\text{F55})$$

This is easily done using identity (D3)

$$\sum_i Q_i \vec{b}_i = \frac{\vec{1}}{3} + \frac{10}{3} \tau^3 \vec{1} \quad (\text{F56})$$

Next work (F50). We know the second term in (F50) is the Hermitian conjugate of the first so we just double the first term, giving us for (F50)

$$\sum_{i,j} \left[\vec{b}_j - i \vec{b}_i \times \vec{b}_j \right] \left[\frac{\tau_i \cdot \tau_j}{3} + \tau_j^3 - i (\tau_i \times \tau_j)^3 \right] \quad (\text{F57})$$

This is six terms which shall be labeled

$$\sum_{i,j} \left\{ \begin{array}{l} \textcircled{a} \quad \vec{b}_j \tau_i \cdot \tau_j \quad \textcircled{b} \quad + \vec{b}_j \tau_j^3 \quad \textcircled{c} \quad - i \vec{b}_j (\tau_i \times \tau_j)^3 \\ \textcircled{d} \quad - i (\vec{b}_i \times \vec{b}_j) \frac{\tau_i \cdot \tau_j}{3} \quad \textcircled{e} \quad - i (\vec{b}_i \times \vec{b}_j) \tau_j^3 \quad \textcircled{f} \quad - (\vec{b}_i \times \vec{b}_j) (\tau_i \times \tau_j)^3 \end{array} \right\} \quad (\text{F58})$$

One sees that \textcircled{a} will involve a sum over intermediate states but only the nucleon intermediate states will contribute. Using the usual identities in Appendix C one has

$$\textcircled{a} = \frac{10}{3} \vec{S} \quad (\text{F59})$$

The other five pieces of Eq.(F58) may be performed routinely with the result

$$\textcircled{b} = 20 \vec{S} \vec{T}^3 \quad (\text{F60})$$

$$\textcircled{c} = \frac{40}{3} \vec{S} \vec{T}^3 \quad (\text{F61})$$

$$\textcircled{d} = \frac{100}{9} \vec{S} \quad (\text{F62})$$

$$\textcircled{e} = \frac{40}{3} \vec{S} \vec{T}^3 \quad (\text{F63})$$

$$\textcircled{f} = \frac{528}{9} \vec{S} \vec{T}^3 \quad (\text{F64})$$

The $i=j$ terms are easy to subtract off. We present below the results for Eqs.(F50) through (F53) for the cases $i=j$ and $i \neq j$.

$$\begin{aligned} & \sum_{i,j} \left[Q_i \vec{b}_i \rho_{0i}^{\vec{T}} + \text{H.C.} \right] \\ &= \frac{22}{3} \vec{S} + \frac{316}{3} \vec{S} \vec{T}^3 \Big|_{i=j}, \quad \frac{4}{3} \vec{S} + \frac{136}{3} \vec{S} \vec{T}^3 \Big|_{i \neq j} \quad (\text{F65}) \end{aligned}$$

$$\sum_{i,j} \left\{ \left[i \underline{x}_j \cdot \underline{x}_i (\vec{b}_j \times \vec{b}_i) + \frac{2}{3} \underline{x}_j \cdot \underline{x}_i (\vec{b}_j \cdot \vec{b}_i) \vec{b}_i \right] Q_i + H.C. \right\} \quad (F66)$$

$$= \frac{8}{9} \vec{S} - \frac{16}{9} \vec{S} \vec{T}^3 \Big|_{i=j}, \quad \frac{8}{9} \vec{S} - \frac{16}{9} \vec{S} \vec{T}^3 \Big|_{i \neq j}$$

$$\sum_i \left[Q_i b_i \psi_{0i} + H.C. \right]$$

(F67)

$$= \frac{10}{3} \vec{S} + \frac{52}{3} \vec{S} \vec{T}^3 \Big|_{i=j}, \quad \frac{4}{3} \vec{S} - \frac{8}{3} \vec{S} \vec{T}^3 \Big|_{i \neq j}$$

$$\sum_{i,j} \left\{ \left[i (\vec{b}_j \times \vec{b}_i) + \frac{2}{3} (\vec{b}_j \cdot \vec{b}_i) \vec{b}_i \right] Q_i + H.C. \right\}$$

$$= \frac{8}{9} \vec{S} - \frac{16}{9} \vec{S} \vec{T}^3 \Big|_{i=j}, \quad \frac{8}{9} \vec{S} - \frac{16}{9} \vec{S} \vec{T}^3 \Big|_{i \neq j} \quad (F68)$$

5. Matrix Elements For $\mu_{\Delta\Delta}$ Calculation

The forms of matrix elements were given in Eqs. (F49) through (F53). The algebraic process and use of the identities in Appendix D are parallel to that in the last section. We present the solutions in the same form except now S and T are spin and isospin 3/2 operators.

$$\sum_i Q_i \vec{b}_i = \frac{\vec{S}}{3} + \frac{2}{3} \vec{S} \tilde{T}^3 \quad (\text{F69})$$

$$\begin{aligned} & \sum_i \left[Q_i \vec{b}_i p_{0i}^{\text{I}} + \text{H.C.} \right] \\ &= \frac{22}{3} \vec{S} + \frac{44}{3} \vec{S} \tilde{T}^3 \Big|_{i=j}, \quad \frac{4}{3} \vec{S} + \frac{8}{3} \vec{S} \tilde{T}^3 \Big|_{i \neq j} \quad (\text{F70}) \end{aligned}$$

$$\begin{aligned} & \sum_{i,j} \left\{ \left[i \tau_j \cdot \tau_i (\vec{b}_j \times \vec{b}_i) + \frac{2}{3} \tau_j \cdot \tau_i \vec{b}_j \cdot \vec{b}_i \vec{b}_i \right] Q_i + \text{H.C.} \right\} \\ &= \frac{8}{9} \vec{S} + \frac{16}{9} \vec{S} \tilde{T}^3 \Big|_{i=j}, \quad \frac{8}{9} \vec{S} + \frac{16}{9} \vec{S} \tilde{T}^3 \Big|_{i \neq j} \quad (\text{F71}) \end{aligned}$$

$$\begin{aligned} & \sum_{i,j} \left[Q_i \vec{b}_i p_{0i} + \text{H.C.} \right] \\ &= \frac{10}{3} \vec{S} + \frac{20}{3} \vec{S} \tilde{T}^3 \Big|_{i=j}, \quad \frac{4}{3} \vec{S} + \frac{8}{3} \vec{S} \tilde{T}^3 \Big|_{i \neq j} \quad (\text{F72}) \end{aligned}$$

$$\sum_{i,j} \left\{ \left[i \vec{b}_j \times \vec{b}_i + \frac{2}{3} (\vec{b}_j \cdot \vec{b}_i) \vec{b}_i \right] Q_i + \text{H.c.} \right\}$$

$$= \frac{8}{9} \vec{S} + \frac{16}{9} \vec{S} \mathcal{I}^3 \Big|_{i=j}, \quad \frac{8}{9} \vec{S} + \frac{16}{9} \vec{S} \mathcal{I}^3 \Big|_{i \neq j} \quad (\text{E73})$$

APPENDIX G

THE CLASSICAL PION FIELDS ϕ_{NN} , $\phi_{N\Delta}$, $\phi_{\Delta\Delta}$

In order to extract the coupling constants from the various fields developed in Chapter VI one needs to know the classical solutions for ϕ . We present the fields here for completeness, although the solutions are well known. We follow Ref. 20 in the following. The pion field will be that resulting from a nucleon source which in the process of pion emission stayed a nucleon. The pion fields resulting from nucleon-becomes- Δ and Δ -becomes- Δ are simple generalizations of the ϕ_{NN} with the only difference being the coupling constants and types of isospin and spin matrix elements.

The wave equation for the classical ϕ_{NN} is

$$\left(-\nabla^2 + m_\pi^2 - w^2\right) \phi_{\sim} = \frac{f_{\pi NN}}{m_\pi} \vec{\nabla} \cdot (\psi^\dagger \vec{\sigma} \psi) \quad (G1)$$

The solution for $w=0$ is

$$\phi_{\sim} = \frac{f_{\pi NN}}{4\pi m_\pi} \int \frac{e^{-m|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} 4\pi \vec{s} \cdot \vec{\nabla}' (\psi^\dagger \psi) d^3r' \quad (G2)$$

Assume a simple nuclear charge distribution

$$\psi^+(\vec{r}') \downarrow(\vec{r}') = \frac{1}{\frac{4\pi R^3}{3}} \Theta_r(r'-R) \quad (\text{G3})$$

The gradient acting on the charge distribution is

$$\nabla' \psi^+ \downarrow = - \frac{r' \Delta s}{\frac{4\pi R^3}{3}} \quad (\text{G4})$$

Now rewrite $S \cdot \hat{r}$

$$S \cdot \hat{r} = \sqrt{\frac{4\pi}{3}} \sum_a S^a \star Y_{1a} \quad (\text{G5})$$

where $S^a \star$ are the spherical components of \vec{S} . Next use the expansion in Jackson on pg. 742:

$$\frac{e^{-m|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} = -4\pi m \sum_{l=0}^{\infty} j_l(imr) h_l^{(1)}(imr) \sum_{m=-l}^l Y_{lm}^*(\theta', \phi') Y_{lm}(\theta, \phi) \quad (\text{G6})$$

One finds

$$\phi_{\sim}(r) = \frac{f_{\pi NN}}{\pi R} j_1(imR) h_1^{(1)}(imr) \int \vec{S} \cdot \hat{r} \quad (\text{G7})$$

which in the limit of zero radius becomes

$$\phi_{\sim NN} = - \frac{f_{\pi NN}}{4\pi} 2 \int 2 \vec{S} \cdot \hat{r} \frac{e^{-mr}}{r} \left(1 + \frac{1}{mr} \right) \quad (\text{G8})$$

In order to find asymptotic fields for the case of $\phi_{N\Delta}$ and ϕ_{NN} we need the analogous source terms corresponding to Eq. (G1). These may be obtained from the non-relativistic Lagrangians for both fields

$$\mathcal{L}_{\pi N\Delta} = \frac{f_{\pi N\Delta}}{m_\pi} \langle \Delta | \vec{J} \cdot \vec{\nabla} \vec{\sigma} \cdot \phi | N \rangle \quad (\text{G9})$$

$$\mathcal{L}_{\pi\Delta\Delta} = \frac{f_{\pi\Delta\Delta}}{m_\pi} \langle \Delta | \vec{S}_{3/2} \cdot \vec{\nabla} \vec{T}_{3/2} \cdot \phi | \Delta \rangle, \quad (\text{G10})$$

where we are using the transition operators shown in Appendix D for (G9). The equations of motion are

$$(-\nabla^2 + m_\pi^2 - W^2) \phi_{\vec{\lambda}} = \frac{f_{\pi N\Delta}}{m_\pi} \vec{\nabla} \cdot (\psi_\Delta^+ \vec{J} \vec{\sigma} \psi_N) \quad (\text{G11})$$

$$(-\nabla^2 + m_\pi^2 - W^2) \phi_{\vec{\lambda}} = \frac{f_{\pi\Delta\Delta}}{m_\pi} \vec{\nabla} \cdot (\psi_\Delta^+ \vec{S}_{3/2} \vec{T}_{3/2} \psi_\Delta) \quad (\text{G12})$$

Naturally, since these are non-relativistic, we may think of the right hand side of the two equations as a uniform transition spin-isospin and spin-isospin density, having

the same spatial distribution as (G4). It is easy to compare the resultant pion fields to (G8). We will summarize all the fields using the more convenient pseudoscalar coupling constants. The coupling constants are defined as follows

$$\frac{g_{\pi NN}}{2M_N} = \frac{f_{\pi NN}}{m_\pi} \quad (\text{G13})$$

$$\frac{g_{\pi N\Delta}}{(M_N + M_\Delta)} = \frac{f_{\pi N\Delta}}{m_\pi} \quad (\text{G14})$$

$$\frac{g_{\pi\Delta\Delta}}{2M_\Delta} = \frac{f_{\pi\Delta\Delta}}{m_\pi} \quad (\text{G15})$$

The fields are

$$\tilde{\phi}_{NN} = -\frac{g_{\pi NN}}{8\pi} \left(\frac{m_\pi}{M_N} \right) \tilde{T}_{1/2} \tilde{S}_{1/2} \cdot \hat{r} \frac{e^{-m_\pi r}}{r} \left(1 + \frac{1}{m_\pi r} \right) \quad (\text{G16})$$

$$\tilde{\phi}_{N\Delta} = -\frac{g_{\pi N\Delta}}{4\pi} \left(\frac{m_\pi}{M_N + M_\Delta} \right) \tilde{T} \tilde{S} \cdot \hat{r} \frac{e^{-m_\pi r}}{r} \left(1 + \frac{1}{m_\pi r} \right) \quad (\text{G17})$$

$$\tilde{\phi}_{\Delta\Delta} = -\frac{g_{\pi\Delta\Delta}}{8\pi} \left(\frac{m_\pi}{M_\Delta} \right) \tilde{T}_{3/2} \tilde{S}_{3/2} \cdot \hat{r} \frac{e^{-m_\pi r}}{r} \left(1 + \frac{1}{m_\pi r} \right) \quad (\text{G18})$$

APPENDIX H

NUMERICAL AND ANALYTICAL INTEGRALS

1. Integrals Used in Magnetic Moments Calculation

There were needed several integrals involving just spherical Bessel functions. These were done analytically and checked on a computer. They were

$$\int_0^p j_0 j_1 x^3 dx = 1.0255 \quad (\text{H1})$$

$$\int_0^p (j_0 j_1' - j_1 j_0') x^4 dx = - .7918 \quad (\text{H2})$$

$$\int (j_1 j_0 - j_2 j_1) x^3 dx = .7594 \quad (\text{H3})$$

In addition integrals $I_g^{(2)}$ and $I_g^{(3)}$ were calculated. In the following

$$I_g^{(2)} = \int_0^P (j_1 K_1 + j_0 K_3) x^3 dx = \frac{N}{9\pi} (.1421) \quad (H4)$$

$$I_g^{(3)} = \int_0^P (j_0 K_4 + j_1 K_2) x^3 dx = \frac{N}{9\pi} (-.14924) \quad (H5)$$

We shall first calculate $I_g^{(2)}$

Recall Eq. (C2)

$$K_1 = \frac{N}{9\pi w_0^2} \left\{ j_0(\xi) \int_f^P H \rho^3 (\eta_0 j_1 + \eta_1 j_0) d\rho' \right. \\ \left. + \eta_0(\xi) \int_0^2 H \rho^3 j_0 j_1 d\rho' \right\} \\ + j_0(\xi) \left[\frac{N j_0 \eta_0 \rho^3 H}{9\pi w_0^2} + A_1 \right] \quad (H6)$$

$$K_3 = \frac{N}{9\pi w_0^2} \left\{ j_1(\xi) \int_f^P H \rho^3 (\eta_0 j_1 + \eta_1 j_0) d\rho' \right. \\ \left. + \eta_1(\xi) \int_0^2 H \rho^3 j_0 j_1 d\rho' \right\} \\ + j_1(\xi) \left[-\frac{N}{9\pi w_0^2} j_1 \eta_1 \rho^3 H + A_3 \right] \quad (H7)$$

Break $I_g^{(2)}$ into two parts. The first part will contain the pieces of K_1 and K_3 involving integrals. The last part will have $j_0(\xi)[\quad]$ and $j_1(\xi)[\quad]$ terms. The first part sans a factor of $\frac{N}{9\pi w_0^2}$, is

$$\begin{aligned}
I_1 = & \int_0^p (j_1 j_0 + j_0 j_1) z^3 dz \int_{-z}^p (\eta_0 j_1 + \eta_1 j_0) H z^3 dz \\
& + \int_0^p (j_1 \eta_0 + j_0 \eta_1) z^3 dz \int_0^z 2 H z^3 j_0 j_1 dz \quad (H8)
\end{aligned}$$

We need the indefinite integrals of the outer nested integrands in order to integrate by parts. They are

$$\int dz 2 j_0 j_1 z^3 = -z \sin^2 z + \frac{3}{2} z - \frac{3}{4} \sin 2z \quad (H9)$$

$$\int dz (j_1 \eta_0 + j_0 \eta_1) z^3 = z \cos z \sin z - \frac{3}{2} \sin^2 z \quad (H10)$$

Integrate I_1 by parts, getting

$$\begin{aligned}
I_1 = & \int_0^p \left[-z \sin^2 z + \frac{3}{2} z - \frac{3}{4} \sin 2z \right] (\eta_0 j_1 + \eta_1 j_0) H z^3 dz \\
& + \left[p \sin p \cos p - \frac{3}{2} \sin^2 p \right] \int_0^p 2 H z^3 j_0 j_1 dz \\
& - \int_0^p \left[z \cos z \sin z - \frac{3}{2} \sin^2 z \right] 2 H z^3 j_0 j_1 dz \quad (H11)
\end{aligned}$$

This is easy to integrate numerically provided we know H

$$H(z) = \frac{8\pi}{3} \frac{N^2}{W_0} \left\{ \frac{3}{4z^2} - \frac{3}{4} \frac{\sin z \cos z}{z^3} - \frac{3}{8} \frac{j_0^2(\rho)}{\rho} \right\} \quad (H12)$$

The answer is

$$I_1 = \frac{-.98421}{R^2} \quad (\text{H13})$$

Now work I_2 :

$$I_2 = \left[\frac{N}{9\pi W_0^2} j_0 \rho^3 H + A_1 \right] \int_0^{\rho} j_1 j_0 x^3 dx$$

$$+ \left[\frac{-N}{9\pi W_0^2} j_1 \rho^3 H + A_3 \right] \int_0^{\rho} j_1 j_0 x^3 dx \quad (\text{H14})$$

We need the following constants

$$A_1 = \frac{N}{9\pi} \left[1.241 \times 10^{-1} \right] \quad (\text{H15})$$

$$A_3 = \frac{N}{9\pi} \left[9.589 \times 10^{-2} \right] \quad (\text{H16})$$

$$H(\rho) = \frac{.3034}{R^2} \quad (\text{H17})$$

which together with (H14), give

$$I_2 = \frac{N}{9\pi} (-.3779) \quad (\text{H18})$$

Now multiply I_1 by $\frac{N}{9\pi W_0^2}$ and add to I_2

$$I_9^{(1)} = \frac{N}{9\pi W_0^2} I_1 + I_2$$

$$I_9^{(2)} = \frac{N}{9\pi} \left[.1421 \right] \quad (\text{H19})$$

Next perform $I_g^{(3)}$

$$I_g^{(3)} = \int_0^P (j_0 K_4 + j_1 K_2) x^3 dx \quad (\text{H20})$$

This requires K_2 and K_4 . Recall Eq. (C2)

$$\begin{aligned} K_2 = \frac{N}{9\pi\omega_0^2} & \left\{ j_2(\xi) \int_{\xi}^P (y_1 j_0 - y_2 j_1) \rho^3 H d\rho' \right. \\ & + y_2(\xi) \int_0^{\xi} (j_1 j_0 - j_2 j_1) \rho^3 H d\rho' \\ & \left. + j_2(\xi) \left[-\frac{N}{9\pi\omega_0^2} y_2 j_0 \rho^3 H + A_2 \right] \right\} \quad (\text{H21}) \end{aligned}$$

$$\begin{aligned} K_4 = \frac{N}{9\pi\omega_0^2} & \left\{ j_1(\xi) \int_{\xi}^P (j_2 y_1 - y_0 j_1) \rho^3 H d\rho' \right. \\ & + y_1(\xi) \int_0^{\xi} (j_2 j_1 - j_1 j_0) \rho^3 H d\rho' \\ & \left. + j_1(\xi) \left[\frac{N}{9\pi\omega_0^2} y_1 j_1 \rho^3 H + A_4 \right] \right\} \quad (\text{H22}) \end{aligned}$$

As before, break $I_g^{(3)}$ into two parts. The first part contains the pieces of K_2, K_4 involving integrals. The second part will contain $j_2(\zeta)[\]$ and $j_1(\zeta)[\]$ terms. The first part sans $\frac{N}{9\pi W_0}$ is

$$I_1 = \int_0^{\rho} j_0 j_1 z^3 dz \int_z^{\rho} (j_2 y_1 - y_0 j_1) \rho'^3 H d\rho' \quad (\text{H23a})$$

$$+ \int_0^{\rho} j_0 y_1 z^3 dz \int_0^z (j_2 j_1 - j_1 j_0) \rho'^3 H d\rho' \quad (\text{H23b})$$

$$+ \int_0^{\rho} j_1 j_2 z^3 dz \int_z^{\rho} (y_1 j_0 - y_2 j_1) \rho'^3 H d\rho' \quad (\text{H23c})$$

$$+ \int_0^{\rho} j_1 y_2 z^3 dz \int_0^z (j_1 j_0 - j_2 j_1) \rho'^3 H d\rho' \quad (\text{H23d})$$

Again we need some indefinite integrals :

$$\int j_0 j_1 z^3 dz = -\frac{z \sin^2 z}{2} + \frac{z}{4} - \frac{3}{4} \sin z \cos z \quad (\text{H24})$$

$$\int j_0 y_1 z^3 dz = \frac{1}{2} \sin^2 z + \frac{z^2}{4} - \frac{1}{4} z \sin z - \frac{\cos 2z}{8} \quad (\text{H25})$$

$$\int j_1 j_2 z^3 dz = \sin^2 z \left(\frac{z}{2} - \frac{3}{z} \right) + \frac{9}{4} \sin z \cos z + \frac{3}{4} z \quad (\text{H26})$$

$$\int j_1 y_2 z^3 dz = -j_0 y_2 z^3 + \int j_0 y_1 z^3 dz \quad (\text{H27})$$

Now integrate (H23a) and (H23c) by parts to get

$$\begin{aligned} & \int_0^p \left[-z \frac{\sin^2 z}{2} + \frac{z}{4} - \frac{3}{4} \sin z \cos z \right] \\ & \quad \times [j_2 y_1 - y_0 j_1] z^3 H dz \\ & + \int_0^p \left[\sin^2 z \left(\frac{z}{2} - \frac{3}{z} \right) + \frac{9}{4} \sin z \cos z + \frac{3}{4} z \right] \\ & \quad \times [y_1 j_0 - y_2 j_1] z^3 H dz \end{aligned} \quad (\text{H28})$$

The numerical evaluation of this gives

$$\frac{8\pi}{3} \frac{N^2}{w_0} \left[-.0511 \right] \quad (\text{H29})$$

Next, integrate (H23b) and (H23d) by parts to get

$$\begin{aligned} & j_0 \gamma_2 \rho^3 \int_0^\rho (j_2 j_1 - j_1 j_0) H z^3 dz \\ & - \int_0^\rho j_0 \gamma_2 z^3 (j_2 j_1 - j_1 j_0) H z^3 dz, \end{aligned} \quad (\text{H30})$$

which is

$$\frac{8\pi}{3} \frac{N^2}{w_0} \left[-.0695 \right] \quad (\text{H31})$$

Now add (H29) and (H31), remembering to multiply by $\frac{N}{9\pi w_0^2}$, to get

$$I_1 = \frac{N}{9\pi} \left[-.0486 \right] \quad (\text{H32})$$

Finally, work I_2

$$\begin{aligned} I_2 = & \left[\frac{N}{9\pi w_0^2} \gamma_1 j_1 \rho^3 H + A_4 \right] \int_0^\rho j_0 j_1 x^3 dx \\ & + \left[\frac{-N}{9\pi w_0^2} \gamma_2 j_0 \rho^3 H + A_2 \right] \int_0^\rho j_1 j_2 x^3 dx \end{aligned} \quad (\text{H33})$$

We need A_2 and A_4 . See Eq. (B38)

$$A_2 = \frac{N}{9\pi} \left\{ \gamma_2 j_0 \rho H R^2 - \frac{1}{\omega_0^2} \frac{(\gamma_2 + \gamma_1)}{(j_2 + j_1)} \int_0^{\rho} (j_1 j_0 - j_1 j_2) z^3 H dz \right.$$

$$\left. A_4 = -\frac{N}{9\pi} \left\{ \gamma_1 j_0 \rho H R^2 - \frac{1}{\omega_0^2} \frac{(\gamma_2 + \gamma_1)}{(j_2 + j_1)} \int_0^{\rho} (j_1 j_0 - j_1 j_2) z^3 H dz \right. \right.$$

(H34)

This gives

$$I_2 = \frac{N}{9\pi \omega_0^2} \frac{(\gamma_2 + \gamma_1)}{(j_2 + j_1)} \int_0^{\rho} (j_1 j_0 - j_1 j_2) z^3 H dz + \int_0^{\rho} (j_0 j_1 - j_1 j_2) z^3 dz, \quad (\text{H35})$$

which is

$$I_2 = \frac{N}{9\pi} [-.1007] \quad (\text{H36})$$

Finally add (H32) to (H36) giving

$$I_9^{(3)} = \frac{N}{9\pi} [-.1492]. \quad (\text{H37})$$

APPENDIX I

VIOLATION OF ANGULAR MOMENTUM CONSERVATION

Our deformed bag model and models like it have an important deficiency. They violate conservation of angular momentum. This is apparent from the Dirac wave function in (3.10). These wave functions conserve J_z but violate total J conservation. The violation arises from the perturbative nature of our calculation. We have minimized the energy via the NLC and then chosen the $SU(6)$ spinors germane to the nucleon or Δ . In principle one should project out of the system of quarks, gluons and pions those states which have the correct quantum numbers of the nucleon or Δ . Only after this step should the energy of the system be minimized. Unfortunately it is not clear how one could implement this technique perturbatively.

REFERENCES

1. Z. Y. Ma, J. Wambach, Stony Brook Preprint (1983).
2. R. A. Arndt, J. B. Cammarata, Y. N. Goradia, R. H. Hackman, V.L. Teplitz, D.A. Dicus, R. Aaron, and R. S. Longacre, Phys. Rev. D. 20 651 (1979).
3. A. Chodos and C. B. Thorn, Phys. Rev. D. 12 2733 (1975).
4. R. L. Jaffe, in Pointlike Structure inside and Outside Hadrons. proceedings of the Seventeenth International School of Subnuclear Physics, Erice, Italy, 1979, edited by A. Zichichi (Plenum, New York, 1982).
5. T. De Grand, R. L. Kaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12 2060 (1975).
6. I. Hulthage, F. Myhrer, and Z. Xu, Nucl. Phys. A364, 322 (1981).
7. C. De Tar, Phys. Rev. D. 17 323 (1978).
8. I. M. Duck and E. A. Umland, Phys. Latt. 116B 308 (1982).
9. A. W. Thomas, CERN report, 1982 (unpublished).
10. S. A. Chin, Nucl. Phys. A382 355 (1982).
11. S. Theberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D 22 2838 (1980).
12. T. H. Hanson and R. L. Jaffe, Phys. Rev. D 28 882 (1983).
13. P. Hasenfratz and J. Kuti, Physics Reports 40 No. 2 75 (1978).
14. F. Gross and J. Hunter, Phys. Rev. D 28 2266 (1983).
15. C. Flammer, Spheroidal Wave Functions (Stanford University Press, Stanford, California, 1957).
16. V. Vento et al., Nucl. Phys. A345 413 (1980).

17. R. A. Smith and V. R. Pandharipande, Nucl. Phys. A256 327 (1976).
18. T. D. Lee, Particle Physics and Introduction to Field Theory, Harwood Academic, Chur, London, New York (1981).
19. M. Gell-Mann and M. Levy, Nuovo Cim. 16, 53 (1960).
20. V. Vento, Ph.D. thesis, Stony Brook, May (1980) unpublished.