

Logo LinX

Mathematical Magic on the Turtle's BK

by Judi Harris

Rumor has it that a large turtle was responsible for the very first square. As was inscribed in *yih king*, a Chinese book written more than three thousand years ago, a large tortoise crawled out of the Yellow River with some strange markings on her shell. When translated into numbers, the markings look like this:

4	9	2
3	5	7
8	1	6

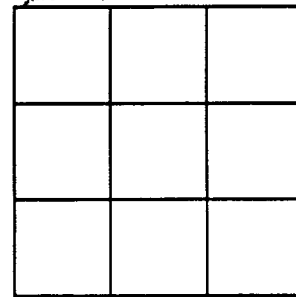
This *MAGIC SQUARE* is assumed to be the world's oldest number mystery. It was used as a divining tool in many Eastern cultures, and during the Middle Ages in Europe, it was believed to drive away disease and attract good fortune. Its apparent magic is reflected in the placement of the digits 1 - 9 so that every row, column, and diagonal sum to the same total.

Magic squares are now used as exercises in mental calculation and motivated practice in mathematical problem-solving. As such, they can be especially effective teaching tools in the beginning of the school year. When used for addition review, paper-and-pencil media are perhaps most appropriate for magic square work. But if you would rather emphasize the problem-solving aspects of magic square work (play?), some simple Logo tools can assist your students' explorations of these ancient puzzles.

Squaring Off

Using several Logo tool procedures, we could program the computer to keep track of students' digit placement trials as they experimented with magic square puzzles. Each time they placed a number in a magic square cell, the computer could show them current row, column, and diagonal totals. This may facilitate puzzle work, since it is difficult for many

students to keep nine number placements in mind at once, and having to draw a magic square grid for every attempt at puzzle solution is inconvenient. The grid itself is, in fact, an interesting Logo challenge.



Here's one solution to that challenge. All procedures are shown in Apple LogoWriter 2.0, but minor syntactical adjustments can be made to adapt the code to other versions of the language.

```
TO GRID
CG
CT
PU
SETPOS [-75 -70]
PD
REPEAT 4 [FORWARD 150 RIGHT 90]
REPEAT 2 [FORWARD 50 RIGHT 90 FORWARD
150 BACK 150 LEFT 90]
REPEAT 2 [FORWARD 50 RIGHT 90]
REPEAT 2 [FORWARD 150 BACK 150 LEFT
90 FORWARD 50 RIGHT 90]
REPEAT 2 [FORWARD 150 RIGHT 90]
END
```

For ease of use, cells, rows, and columns can be labelled as follows.

A	B	C	—
D	E	F	—
G	H	I	—
—	—	—	—

Values for individual cells and totals for rows, columns, and diagonals can be stored in global variables that are initialized with the following two procedures:

```
TO INITIALIZE.VARIABLES :LIST
IF EMPTY? :LIST [STOP]
MAKE FIRST :LIST 0
INITIALIZE.VARIABLES BUTFIRST :LIST
END

TO VARIABLE.NAMES
OUTPUT [VALUE.A VALUE.B VALUE.C
VALUE.D VALUE.E VALUE.F VALUE.G
VALUE.H VALUE.I VALUE.ROW.1
VALUE.ROW.2 VALUE.ROW.3
VALUE.COLUMN.1 VALUE.COLUMN.2
VALUE.COLUMN.3 VALUE.DIAGONAL.1
VALUE.DIAGONAL.2]
END
```

These two procedures are combined and executed with the command

```
INITIALIZE.VARIABLES VARIABLE.NAMES
```

which assigns a preliminary value of 0 to each of the 17 global variables named in VARIABLE.NAMES.

The Turtle's Accountant

To use the magic square procedures as a puzzle-solving aid, students need only type the letter of the square in which they would like to place a digit, then type the number itself. A separate procedure for each magic square cell can be written, structured like this:

```
TO A
SETPOS POS.A
LABEL :VALUE.A
TYPE [NUMBER?]
MAKE "VALUE.A FIRST READLISTCC
SETPOS POS.A
LABEL :VALUE.A
TOTAL.ROW.1
TOTAL.COLUMN.1
TOTAL.DIAGONAL.2
CC
END
```

POS.A is a subprocedure that outputs the screen position of the printed value inside cell A.

```
TO POS.A
OUTPUT [-45 55]
END
```

Notice that the turtle is directed to set its position to POS.A and print the value of the global variable "VALUE.A twice; once to erase the value that was printed there previously, and then again to insert the new value. Notice also that every time a value is changed in a cell, the corresponding row, column, and diagonal sums are adjusted with procedures structured like TOTAL.ROW.1, TOTAL.COLUMN.1 and TOTAL.DIAGONAL.2.

```
TO TOTAL.ROW.1
SETPOS POS.TOTAL.ROW.1
LABEL :VALUE.ROW.1
MAKE "VALUE.ROW.1 :VALUE.A + :VALUE.B
+ :VALUE.C
SETPOS POS.TOTAL.ROW.1
LABEL :VALUE.ROW.1
END
```

POS.TOTAL.ROW.1 is a procedure that outputs the screen position for printing the total of the cells in row 1, written similarly to the procedure POS.A.

You may be thinking that there must be a more economical way to write this program than to have nine almost-identical subprocedures for assigning values to the nine cells, and eight nearly-duplicate subprocedures for tallying row, column, and diagonal totals. You're right. Please let that be a Logo challenge to you.

Magic Turtle Tallies

Let's say that you would like to arrange the digits 1, 2, 2, 5, 6, 6, 7, 8, and 8 in the magic square below so that all rows and columns sum to the same total.

A	B	C	—
D	E	F	—
G	H	I	—
—	—	—	—

You decide to place the 7 in the center cell, so you type:

E

and press the Return key. The computer responds:

NUMBER?

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and you type:

7

and press the Return key again.

Now the screen looks like this:

A 0	B 0	C 0	—
D 0	E 7	F 0	—
G 0	H 0	I 0	—
—	—	—	—

You then try to balance the values of the rows and columns by placing the 8's and 2's in mirror-image placements, like this,

A 8	B 0	C 2	10
D 0	E 7	F 0	7
G 2	H 0	I 8	10
10	7	10	23

and the computer automatically adjusts the sums to incorporate your digit placements. Now only 1, 5, 6, and 6 remain to be positioned. Can they be arranged in the four remaining boxes so that all row and column totals are the same? If not, cell values for blocks A, C, E, G, and I can be reassigned.

Similar explorations can be conducted with the following digit lists.

- List A: 1 1 1 2 2 2 3 3 3
- List B: 0 0 1 1 2 3 4 4 6
- List C: 2 4 5 5 6 7 8 8 9
- List D: 1 1 2 2 3 3 4 5 6
- List E: 0 0 1 1 2 2 5 6 7
- List F: 2 4 4 4 6 6 6 8 8

In true Logo style, most have several correct positional puzzle solutions.

Magic Code By Mail

Hopefully, you now have enough information to help you to construct a set of Logo magic square tools so that your students can use them for problem solving. The digit lists are by no means exhaustive; I would encourage you to challenge your students to devise other lists (perhaps with two- and three-digit numbers) that will fit the magic square pattern. These can be saved in the computer's memory along with the tools so that other students can use them.

```
TO JASON'S.PUZZLE
OUTPUT [1 2 3 4 5 6 7 8 9]
END

PRINT JASON'S.PUZZLE
```

If you don't have the time or desire to code the rest of the tools, I would be glad to send you a set in either Apple LogoWriter 2.0 or IBM Logo format. Just send a 5.25" disk in a self-addressed, stamped disk mailer to me at the address below. Be sure to include a note specifying which version you would like me to copy onto the disk and return to you.

Judging by the antiquity of this mystical number puzzle, it seems that the turtle has had a long history of motivating active inquiry by revealing the magical in the mathematical. Logo use can certainly preserve and extend this delightful tradition.

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