INVESTIGATION OF A TWO-DIMENSIONAL NEGATIVE RESISTANCE OSCILLATOR

A Thesis

Presented to

The Faculty of the Department of Physics

The College of William and Mary in Virginia

In Partial Fulfillment

Of the Requirements for the Degree of

Master of Arts

By

Leo Rogon

1966

APPROVAL SHEET

This thesis is submitted in partial fulfillment of

the requirements for the degree of

Master of Arts

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Approved, August 1966

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ACKNOWLEDGMENTS

The author would like to express his gratitude to Professor Frederic R. Crownfield, Jr., under whose guidance this thesis was done, for his patient guidance, help and encouragement throughout this investigation. The author also wishes to thank the faculty and staff of the Physics Department for their suggestions and help, with special thanks going to Professor James D. Lawrence and Professor Howard R. Gordon. An expression of appreciation also goes to the author's wife, Shirley Rogon, for her help in preparing the manuscript and her efforts in his behalf.

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ABSTRACT

The theory of "negative resistance" and steady state oscillation for a singly resonant LRC circuit is reviewed. The average negative resistance R_n is defined and conditions for steady state oscillation are established in terms of average effective parallel impedance, using a plot of average negative resistance versus amplitude for the negative resistance device. The extension of the theory to a doubly resonant LRC circuit is discussed and an attempt is made to compare the theory with data, using the tunnel diode as the negative resistance device. Qualitative agreement is satisfactory, but quantitative agreement is poor due to a lack of accurate determination of the tunnel diode characteristics and circuit reactance changes which are not accounted for in the theory.

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INTRODUCTION

While studies of coupled non-linear oscillators go back to the early twenties, it is believed that a reexamination of some general properties of such non-linear oscillatory systems would be fruitful. Most oscillatory phenomena in nature involve the non-linear behavior of one or more elements of the system involved. These elements may be distributed through the volume of the system as in a plasma or they may be lumped as in an electrical circuit, where each element is a localized component of the circuit. Deviations from linearity may be classified as large or small amplitude effects. Relaxation oscillations are typical of large amplitude non-linear behavior of a system. Small amplitude sinusoidal oscillations on the other hand still require non-linear effects in their description, although they can be treated by almost-linear ("quasi-linear") methods, such as those discussed in this thesis.

Since all real devices have losses, self-sustained oscillation can occur only if the system contains a source of energy such as a battery. The manner in which a battery can supply losses to an oscillating circuit frequently involves the concept of "negative resistance."

The non-linear behavior of a one dimensional oscillator has been extensively analyzed[1,2] and is well understood. The behavior of the non-linear oscillator with several degrees of freedom has also been discussed[2,3] but is less well understood than the one-dimensional case.

Further investigation seems desirable in order to develope some "intuition" or "feeling" as to the physical behavior of multi-mode oscillation, since this is not always clear in terms of the mathematical description. A considerable interest has arisen recently with regard to multi-mode oscillations, particularly in the areas of lasers and of plasma physics[4]. This thesis will, however, be devoted to the simple case of two coupled circuits containing a single non-linear element in the form of a tunnel diode, in the hope of clarifying some of the general properties of multi-mode oscillation.

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[1] E. V. Appleton and Balth. Van Der Pol, "Oscillation Hysteresis in Simple Triode Generators," Phil. Mag. 43, 177 (1922)

[2] Nicholas Minorsky, Nonlinear Oscillations (D. Van Nostrand Co., Inc., New York, 1962)

[3] Balth. Van Der Pol, "Oscillation Hysteresis in a Triode Generator with Two Degrees of Freedom," Phil. Mag. 43, 700 (1922)

[4] Conference on Phenomena in Ionized Gases, 7th Belgrade 1965. Proceedings (to be published), Gradevinska Rnjiga, Belgrade 1966.

SMALL AMPLITUDE NEGATIVE RESISTANCE OSCILLATORS

The equation most often used to describe the motion of an oscillating system has the form

$$m\ddot{x} + D\dot{x} + kx = 0 \quad . \tag{1}$$

The term in the equation involving D is the "damping" term and describes the dissipative effects on the system. If D = 0, the system conserves energy and oscillates with arbitrary constant amplitude. If $D > 0$, there is damping, the system dissipates energy and oscillates with an amplitude which decreases with time. If $D < 0$ the energy increases and the system oscillates with increasing amplitude.

Since no real physical systems are lossless, such a system cannot be described by an equation of the type D = 0, since this implies no damping. On the other hand it cannot be described by an equation of the type $D > 0$ or $D < 0$, since these imply decay to zero or infinite growth of oscillation respectively. A system of self-sustained oscillation at a definite amplitude requires that the damping of the system be positive when the oscillations become too large and negative when the oscillations become too small. That is, the damping in a system of self-sustained oscillation must be a function of its amplitude of oscillation; the energy required to overcome losses must be supplied, for example, by a battery.

In an electrical circuit this type of non-linear behavior may be obtained by including in the circuit a suitable "negative resistance device," such as a tunnel diode. Figure (1) is a plot of the current - voltage characteristic of a typical tunnel diode. Over a portion of its I V characteristic the tunnel diode exhibits a "negative resistance:" i. e. a decrease in voltage causes an increase in current. Also of importance is the fact that this "negative resistance" is confined to only a limited portion of the characteristic, preventing the current from increasing without limit for large decreases in voltage. If such a device is put into a circuit it can feed power into the circuit, the power originating from the battery used to bias the diode to the negative resistance region of its characterisitc.

We will proceed next to discuss such a negative resistance os cillator, following Brunetti[5], and describing the extension of his method to the case of two degrees of freedom. Suppose that a negative resistance device such as a tunnel diode is connected across the simple LRC circuit shown in figure (2). (Connections for biasing the negative resistance device are omitted in this figure.) If a sinusoidal voltage is present across A B the circuit is able to make up for losses because the negative resistance device R_n feeds power to

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[5]Cledo Brunetti, "The Clarification of Average Negative Resistance with Extensions of Its Use," Proc. I. R. E., 25, 1595

FIGURE 1. CURRENT-VOLTAGE CHARACTERISTIC OF IN2939 TUNNEL DIODE.

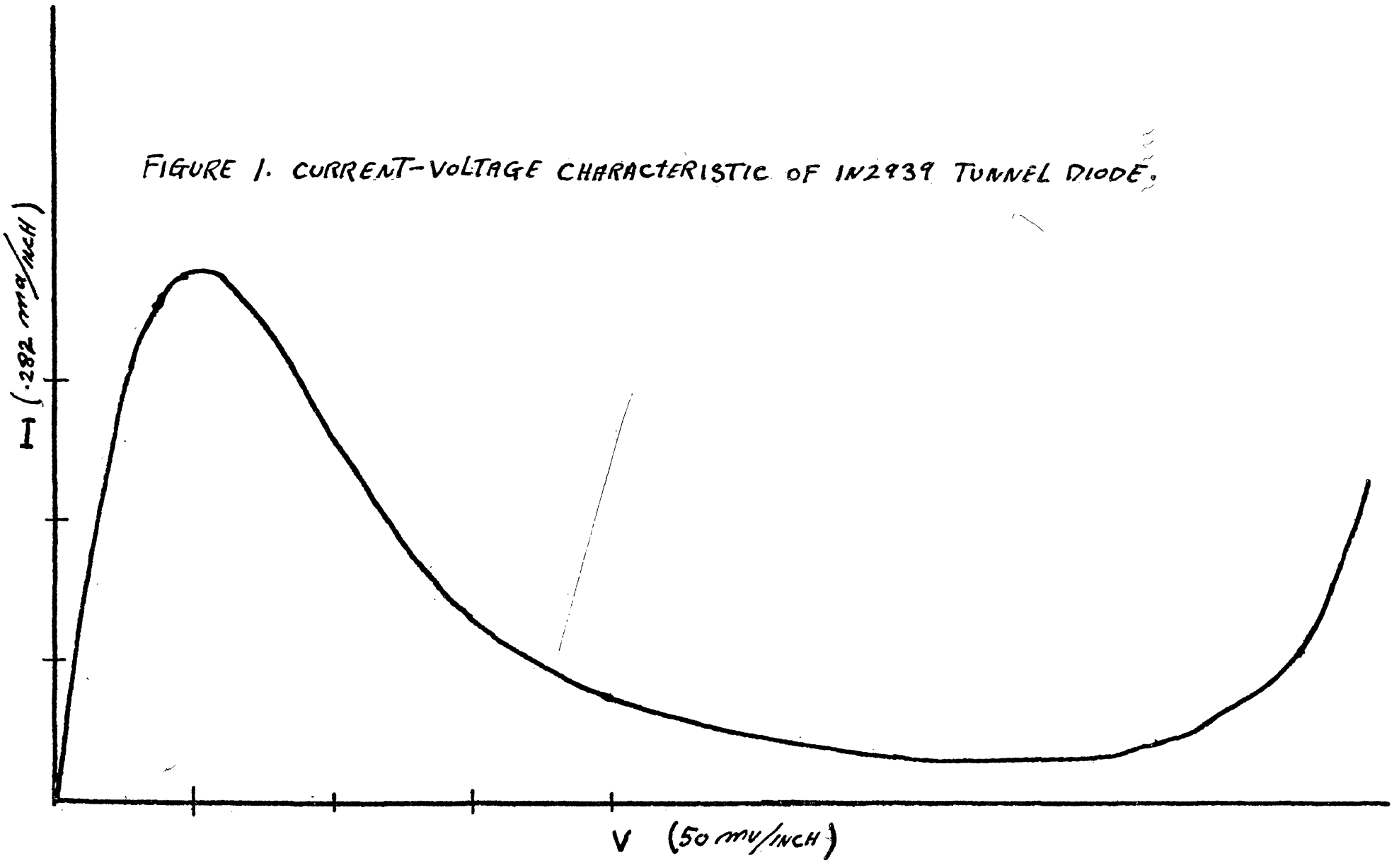
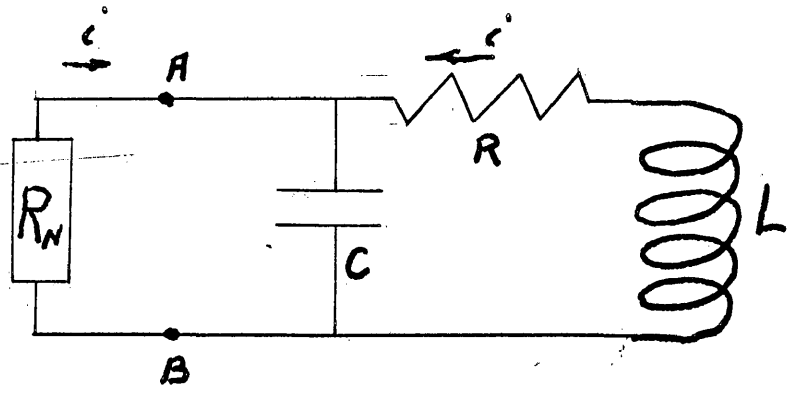


FIGURE 2. LRC CIRCUIT CONTAINING NEGATIVE RESISTANCE R_N.



it; the instantaneous value of i depends on the instantaneous value of the sinusoidally changing voltage. We are thus dealing with a quantity whose value changes periodically along with the voltage across the circuit. The I - V relation is non-linear. Since energy conservation must hold we may expect that the average power supplied by the negative resistance device must just equal the circuit losses. In the steady state, the average may be taken over one cycle of the voltage. The circuit is then able to sustain oscillation. The average effective negative resistance R_n may be defined by requiring that the power fed into the circuit averaged over one cycle of oscillation be given by:

$$P = -\frac{V^2}{2R_N} \quad , \tag{2}$$

where V is the maximum value of the oscillation voltage, assumed sinusoidal.* The average effective resistance of the device is thus:

$$-R_N = \frac{V^2}{2P} \quad .$$

If the voltage is given by

$$v = V \sin \omega t \tag{3}$$

the power delivered to the circuit averaged over a single cycle of

---

*The assumption of "small amplitude" is somewhat subtl/ hidden in this assumption which is well verified experimentally in many cases, (cf. Brunetti[5]), but is obviously not true for relaxation oscillations.

oscillation is:

$$P = \frac{1}{T}\int_0^T i v \, dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} i v \, dt \,, \tag{4}$$

where T is the period of oscillation and i the current resulting from v. The characteristic relationship between the current and voltage of the negative resistance device can be represented by the series:

$$i = \alpha v + \beta v^2 + \gamma v^3 + \delta v^4 + \epsilon v^5 + \xi v^6 + \zeta v^7 + \cdots, \tag{5}$$

where $\alpha, \beta, \gamma, \ldots$ are constants. Using equation (3) this becomes:

$$i = \alpha V \sin \omega t + \beta V^2 \sin^2 \omega t + \gamma V^3 \sin^3 \omega t + \ldots$$

With the use of trigonometric identities the form of this series can be changed to yield:

$$i = I_0 + I_I \sin \omega t + I_{II} \cos 2\omega t + I_{III} \sin 3\omega t + \ldots, \tag{6}$$

where:

$$I_0 = \beta \frac{V^2}{2} + \frac{3}{8}\delta V^4 + \cdots$$

$$I_I = \alpha V + \frac{3}{4}\gamma v^3 + \frac{5}{8}\epsilon V^5 + \frac{35}{64}\zeta V^7 + \cdots \tag{7}$$

$$I_{II} = -\beta \frac{V^2}{2} - \delta \frac{V^4}{2} - \cdots$$

If this expression for i is substituted in the integral for the average power delivered to the circuit, the only term which remains after integration is the $I_I \sin \omega t$ term; i e.:

$$\frac{\omega}{2\pi}\int_0^{2\pi/\omega} i v \, dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} (I_I \sin \omega t)(V \sin \omega t) \, dt$$

The other terms vanish since in each case the integration is over one complete cycle of the oscillation. Then:

$$P = \frac{\omega}{2\pi} I_1 V \int_0^{2\pi/\omega} \sin^2 \omega t \, dt = \frac{I_1 V}{2} \tag{8}$$

and since $-R_N = \frac{V^2}{2P}$ we have:

$$-R_N = \frac{V}{I_1} = \frac{1}{\alpha + \frac{3}{4}\gamma V^2 + \frac{5}{8}\epsilon V^4 + \frac{35}{64}\eta V^6 + \cdots} \tag{9}$$

It can be seen that R_n depends only on the maximum value of the amplitude of oscillation V and on the constants: $\alpha, \gamma, \epsilon, \eta, \ldots$

Considering figure (2), we know that at junction A:

$i = -\frac{V}{R_n}$ and also: $i = V \cdot$ (admittance of circuit).

Equating we get:

$$-\frac{1}{R_N} = j\omega C + \frac{1}{R + j\omega L} = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2} \tag{10}$$

Since the circuit is operating under steady state conditions, the real and imaginary parts of either side of equation (10) must be separately equal, giving the two independent equations:

$$\omega C = \frac{\omega L}{R^2 + \omega^2 L^2} \; ; \; -\frac{1}{R_N} = \frac{R}{R^2 + \omega^2 L^2} \tag{11}$$

$$R^2 + \omega^2 L^2 = \frac{L}{C} \; ; \; R^2 + \omega^2 L^2 = -RR_N$$

$$R_N = -\frac{L}{RC}. \tag{12}$$

Thus under steady state conditions the frequency of oscillation of the circuit is such as to make its effective impedance a pure resis-

tance of magnitude $\frac{L}{RC}$. Using equations (9) and (12) we have:

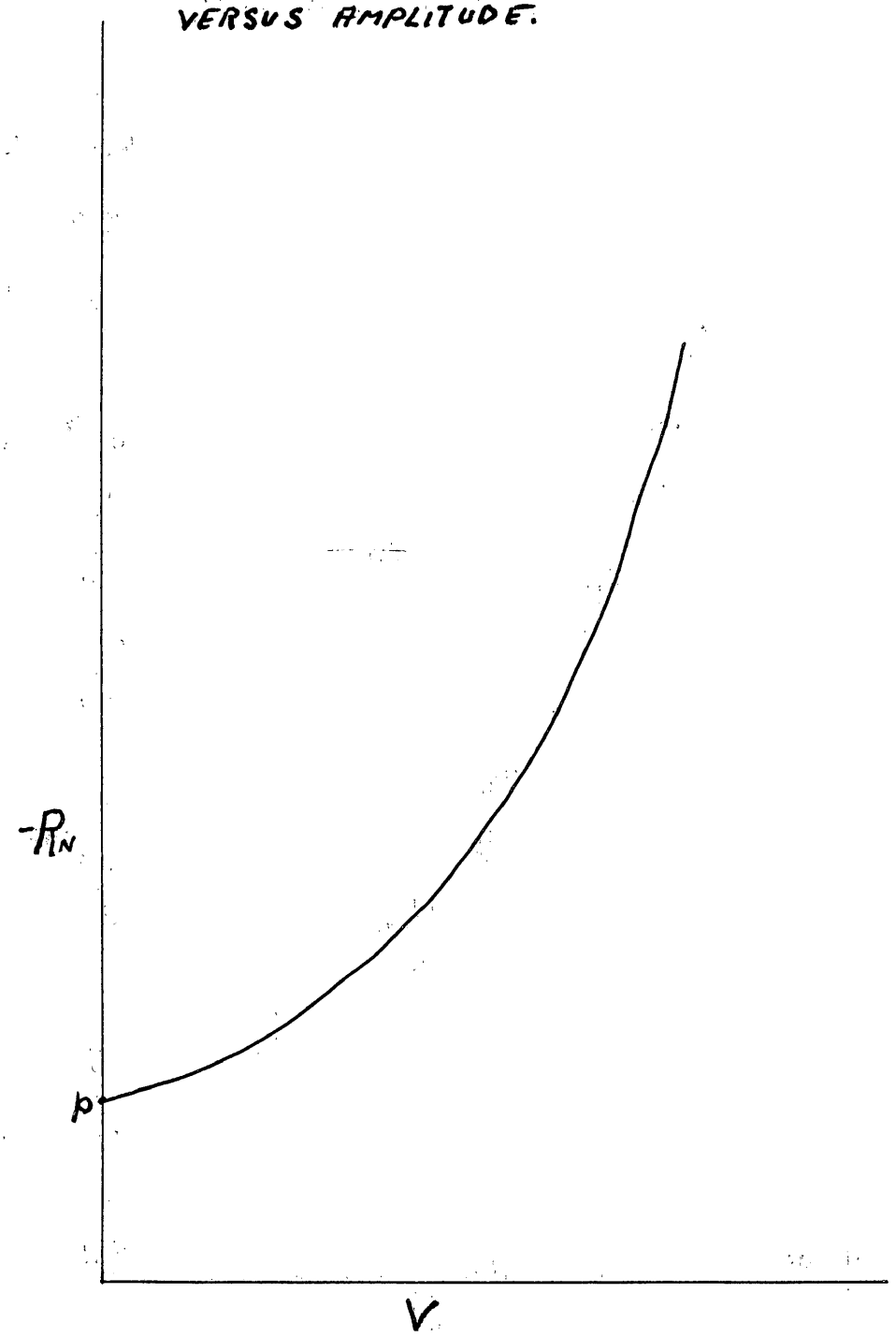
$$-\frac{L}{RC} = \frac{1}{\alpha + \frac{3}{4}\gamma V^2 + \frac{5}{8}\epsilon V^4 + \frac{35}{64}\zeta V^6 + \cdots} \tag{13}$$

The left side of equation (13) is the effective impedance of the circuit and the right side is the effective average negative resistance.

If one plots a graph of $-R_n$ as a function of amplitude V, it is possible to predict conditions for oscillation. A typical plot of this kind is sketched in figure (3).

If R(eq), the equivalent parallel resistance of an LRC circuit, is such that the line R(eq) = constant intersects the $-R_n$ versus V curve, steady state oscillation is possible. An increase in R(eq) will result in oscillations of larger amplitude while a decrease will decrease the amplitude. As R(eq) approaches ρ the amplitude tends smoothly to zero. A plot of this kind for a particular negative resistance device clearly shows the conditions on R(eq) for self-sustained oscillation of a particular LRC circuit and predicts the amplitude of oscillation for any particular value of R(eq). However, from equation (13) it can be seen that the function need not be the monotomically increasing type just considered; the shape of the plot of $-R_n$ versus V is determined by the characteristic I - V curve of the negative resistance device and the bias point, corresponding to V = 0. We are dealing here with a small amplitude device which swings over only a portion of the I - V characteristic. The exact portion over which the oscillation swings is determined by the bias on the negative resistance device and by R(eq).

FIGURE 3. TYPICAL PLOT OF $-R_N$ VERSUS AMPLITUDE.



Many positions on its I - V characteristic are possible.

Another negative resistance versus amplitude curve is shown in figure (4). Some of our data show this effect, and other examples are shown in Brunetti[5].

The question now arises as to whether values of R(eq) situated below the $-R_n$ intercept yield two amplitudes for stable oscillation, since these values of R(eq) yield two values of V. To investigate this we note that the average power delivered to the circuit by the negative resistance device is

$$-\frac{V^2}{2R_n}$$

and the average power dissipated by the circuit is

$$\frac{V^2}{2R(eq)}.$$

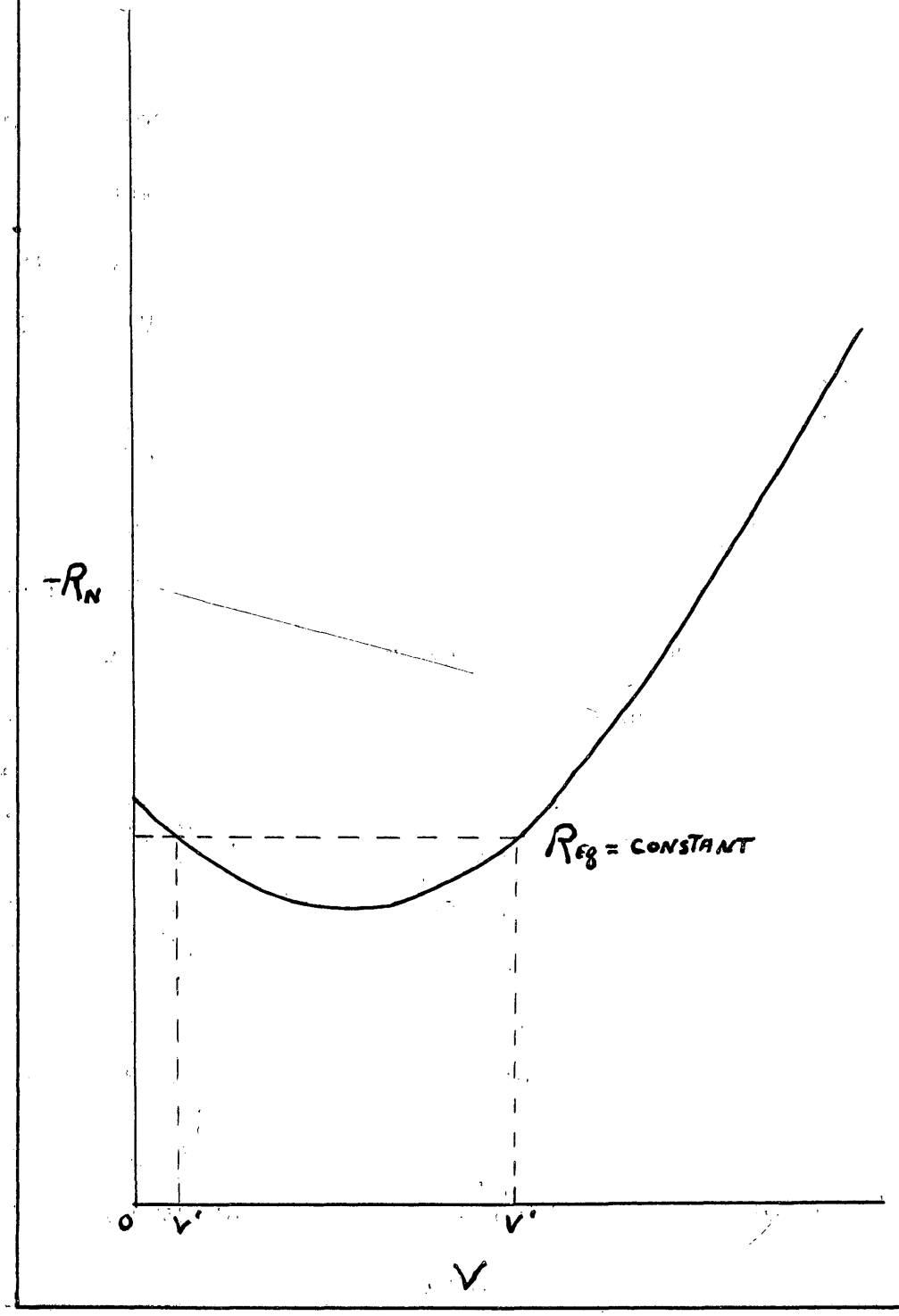
The difference between the average power absorbed per cycle by the circuit and that supplied by the negative resistance device is:

$$\Delta P = \frac{V^2}{2R_{(eq)}} - \frac{(-V^2)}{2R_n} \qquad (14)$$

$$= \frac{V^2}{2}\left(\frac{1}{R_n} + \frac{1}{R_{(eq)}}\right).$$

When ΔP is negative, the negative resistance device is supplying more power per cycle than the circuit is dissipating. This results in an increase of the energy in the LRC circuit, and hence in the amplitude of oscillation. When ΔP is positive, the negative resistance device supplies less power per cycle than the circuit dissipates.

FIGURE 4. SKETCH OF $-R_N$ VERSUS AMPLITUDE PLOT.



This results in a decrease in energy and a corresponding decrease in amplitude. In order that an oscillation of amplitude V be stable it is necessary that ΔP be positive if V increases in amplitude i. e. damp the amplitude down to V again. It is also necessary that ΔP be negative if V decreases in amplitude i. e. push the amplitude up to V again. For stable oscillation then, we must have the condition:

$$\frac{d}{dV}(\Delta P) > 0 \qquad (15)$$

Taking the derivatives this becomes:

$$\frac{d}{dV}(\Delta P) = \frac{\partial(\Delta P)}{\partial R_n}\frac{dR_n}{dV} + \frac{\partial(\Delta P)}{\partial V}$$

$$\frac{\partial(\Delta P)}{\partial R_n} = -\frac{V^2}{2R_n^2}\,; \quad \frac{\partial(\Delta P)}{\partial V} = V\left(\frac{1}{R_n} + \frac{1}{R_{neg}}\right).$$

Thus for stable oscillation:

$$-\frac{V^2}{2R_n^2}\frac{dR_n}{dV} + V\left(\frac{1}{R_n} + \frac{1}{R_{neg}}\right) > 0$$

We already know that for amplitude V to be possible: $R_n = -R(eq)$ thus the condition for stable oscillation with amplitude V becomes:

$$\frac{d(-R_n)}{dV} > 0 \qquad (16)$$

We thus see that values of R(eq) below the $-R_n$ intercept represent conditions for stable oscillation, as long as the curve there has a positive slope.

Consideration of figure (4) shows, then, that if the value of R(eq) is decreased the oscillations decrease in amplitude but persist, even for values of R(eq) below the $-R_n$ intercept. The oscillations continue to persist until a value of R(eq) is reached which corresponds to the

ordinate of the curve at the minimum, where its slope goes negative. At this time the oscillation vanishes. It is interesting to note that if now the value of R(eq) is increased slightly, bringing it back above the minimum of the curve again, oscillation will not be initiated. In order to initiate oscillation the value of R(eq) must be increased to its value at the $-R_n$ intercept of the curve. The positive slope region below the $-R_n$ intercept corresponds to a type of metastable condition where oscillation will persist if R(eq) is brought into this region from above the intercept, although oscillation will not be initiated here unless they are started by some external source, such as a starting pulse, noise, etc. The negative slope region of the curve corresponds to conditions which are unstable for self-sustained oscillation. It is interesting to note that if R(eq) is increased from zero the oscillation amplitude discontinuously jumps to an amplitude corresponding to a position on the positive slope portion of the curve and from this point increases continuously as the value of R(eq) is increased. More complicated dependance of $-R_n$ on V is discussed by Brunetti[5].

We next consider the behavior of a simple negative resistance oscillator with two degrees of freedom, such as the one shown in figure (5). Since the two main subsystems here are the negative resistance device and the circuit to which it is connected, it is convenient to consider their interaction with each other in terms of the separate systems. That is, again consider a negative resistance device connected across an "effective" parallel circuit. Such a system

FIGURE 5. LRC CIRCUIT WITH TWO DEGREES OF FREEDOM.

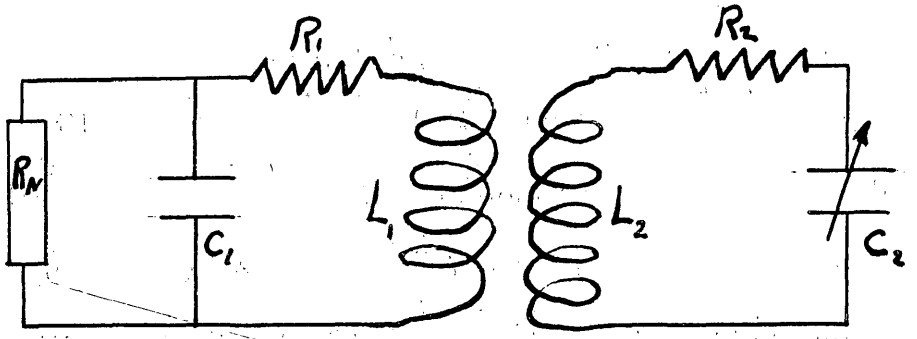
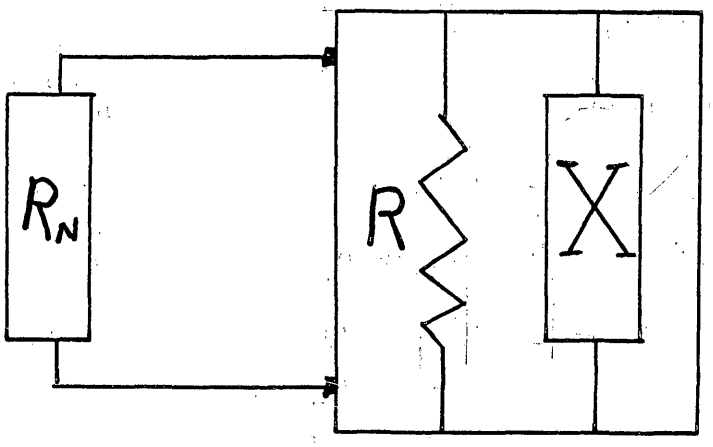


FIGURE 6. EQUIVALENT CIRCUIT FOR FIGURE 5.



is represented in figure (6). If one obtains $-R_n$ versus amplitude curves for the negative resistance device, proper consideration of the behavior of our two dimensional oscillator to which it is connected should predict conditions for oscillation, as well as stability and amplitude. As was the case for the one dimensional oscillator, in order for the circuit to oscillate the total effective equivalent parallel impedance of the coupled circuits must be resistive and greater than the minimum value of $-R_n$ (and to be self-starting must exceed the intercept value of $-R_n$); the frequency must be one for which the susceptance is zero. The problem differs slightly now, since we are dealing here with two circuits coupled together whose total effective impedance is a function of the frequency of oscillation, and the susceptance may vanish at more than one frequency. (Actually two parallel resonant and one series resonant, for the circuit shown and for the case when both circuits are tuned to approximately the same frequency.) The question arises quite naturally then as to the possibility of multi-mode oscillation of a system such as this in the event that the equivalent parallel resistance is high enough at both parallel resonant frequencies simultaneously.

It should be possible to arrive at stability conditions for multi mode oscillation by considerations analogous to those above, except now for multi-mode oscillation:

$$v = a \sin \omega_1 t + b \sin (\omega_2 t + \lambda),$$

where a and b are the amplitudes and ω_1 and ω_2 the frequencies of the modes. λ is a phase angle.

It is this expression for v which is now substituted into equation (5):

$$i = f(v) = f\left(a \sin \omega_1 t + b \sin (\omega_2 t + \lambda) \right) .$$

But i is also given by:

$$i = \frac{"V"}{Z(\omega)} = \frac{a \sin \omega_1 t}{Z(\omega_1)} + \frac{b \sin (\omega_2 t + \lambda)}{Z(\omega_2)} ,$$

where $Z(\omega_1)$ is the impedance at frequency ω_1 and $Z(\omega_2)$ that at ω_2. Equating these two expressions for i:

$$f(a \sin \omega_1 t + b \sin (\omega_2 t + \lambda)) = \frac{a \sin \omega_1 t}{Z(\omega_1)} + \frac{b \sin(\omega_2 t + \lambda)}{Z(\omega_2)} .$$

If ω_1 and ω_2 are not commensurate, we may expand f(v) in a "double Fourier Series" in $\omega_1 t$, $\omega_2 t + \lambda$, and as we did in the one dimensional case, we may require that the coefficients of $\sin \omega_1 t$, $\cos \omega_1 t$, $\sin (\omega_2 t + \lambda)$ and $\cos(\omega_2 t + \lambda)$ be identical on both sides of the equation, giving four independent equations in a and b. For cubic f(v) these equations are the same ones arrived at by Van Der Pol.[3] These lead to four possibilities between a and b:

$a > 0 , \quad b = 0$

$a = 0 , \quad b > 0$

$a = 0 , \quad b = 0$

$a > 0 , \quad b > 0$

The last $a > 0$ and $b > 0$, leads to a contradiction in the equations and is thus impossible. We conclude therefore, that multi mode oscillation is not possible for a cubic f (v)

Stability considerations are also possible, except that one should consider effects of the changing of each amplitude alone, and find the changes in power supplied to or absorbed from the circuit by R_n. These should lead to equations in a and b and changes in power which will again be the same as Van Der Pol's and stability conclusions will be the same.

EQUIPMENT AND PROCEDURE

The negative resistance device used in these experiments was a type 1N2939 tunnel diode. The I - V characteristic of the device was obtained by inserting it in the resistance bridge shown in figure (7). It can be shown that if the bridge is initially balanced (i. e. balanced with no tunnel diode in the circuit), inserting the diode in the circuit unbalances the bridge in such a way that V' is proportional to the current through the diode. V represents the voltage across the diode[6].

The I – V characteristic of the tunnel diode was plotted directly, using a Model 135 Moseley Autograf X Y recorder. To do this the bridge was first balanced without the diode in place. After balancing, the diode was placed across the 10-Ω resistor and the voltage across the bridge was slowly increased. The 10-Ω resistor shunting the diode prevented it from switching, thus giving values of I and V in its negative resistance region. I and V were fed directly to the X and Y axes of the recorder respectively. The recorder traced out their values as the power supply voltage was raised. The diode was then removed and the characteristic curve of a 1 KΩ resistor was obtained to callibrate

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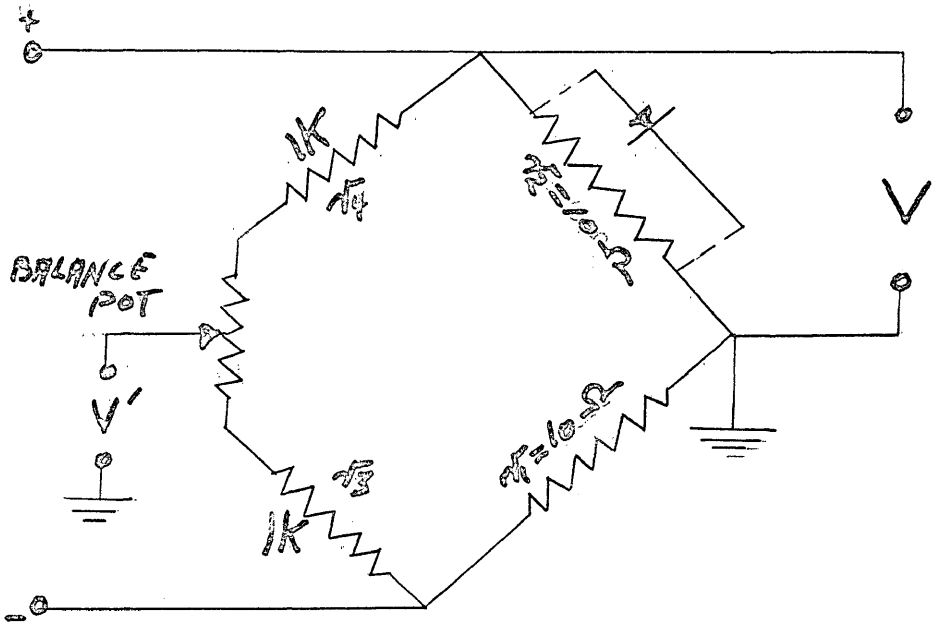
[6]Alvin M. Goodman, "Tunnel Diode Test Set," The Review of Scientific Instruments, 31, 288 (1960)

FIGURE 7. CIRCUIT USED TO OBTAIN CURRENT VOLTAGE CHARACTERISTIC OF TUNNEL DIODE.



the current scale on the recorder. Figure (1) is the I - V characteristic obtained for the 1N2939 tunnel diode.

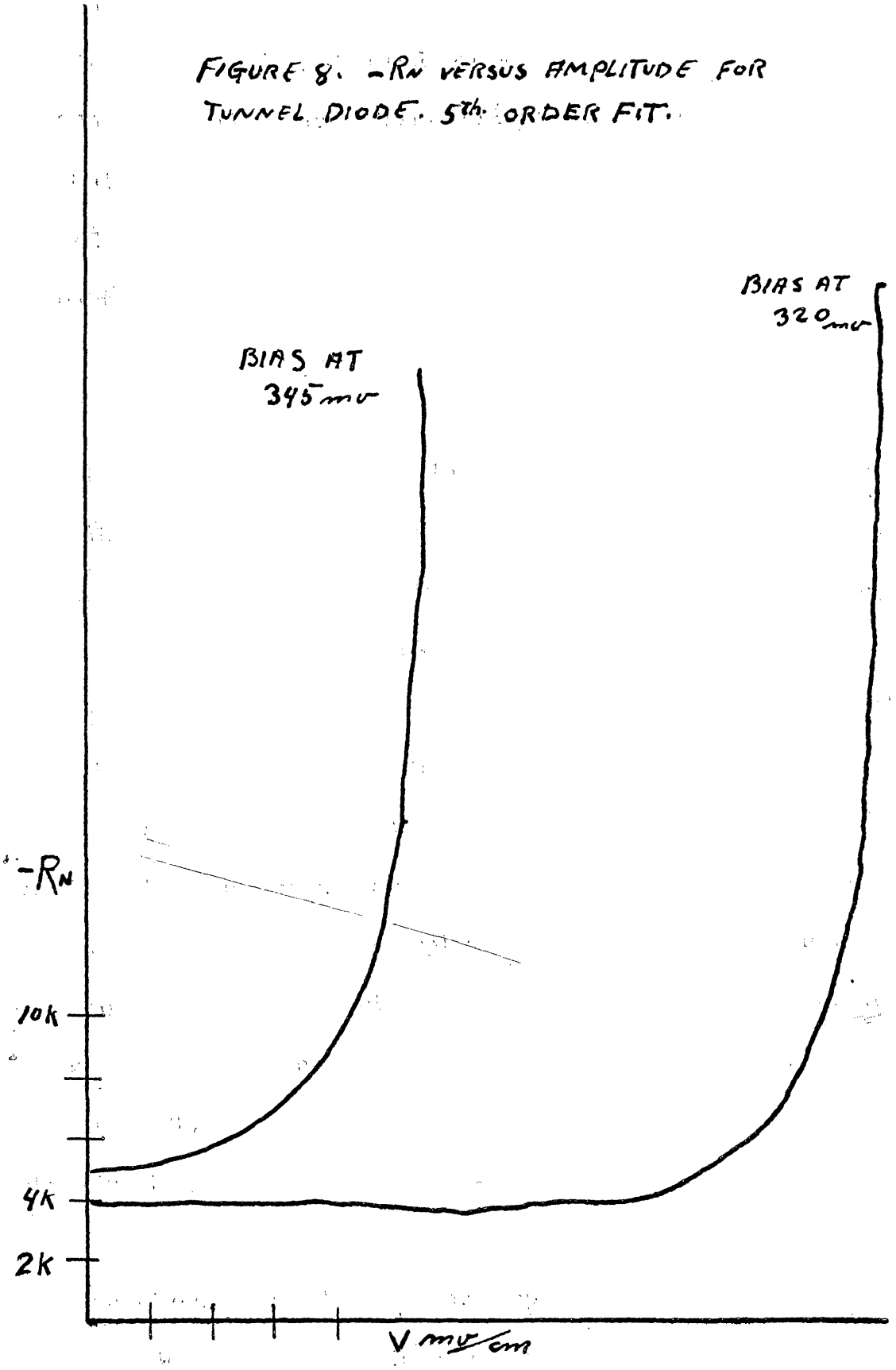
Values of I and V were taken directly off of the curve and a 1620 IBM digital computer was used to obtain a least-squares fit to a fifth degree polynomial. A seventh degree polynomial fit was also obtained for comparison. Then since:

$$-R_n = \frac{1}{\alpha + \frac{3}{4}\gamma V^2 + \frac{5}{8}\epsilon V^4 + \cdots} \quad ,$$

the machine was programmed to calculate $-R_n$ for a range of values of V from this equation, first using the fifth degree polynomial fit and then again for the seventh. Figure (8) shows two of these curves each representing a different bias point on the I -V characteristic. On the basis of the calculated variance of fit no preference was distinguishable between fifth and seventh order fits for the characteristic. It is suspected that the fit of a polynomial to the characteristic of the diode is poor. It is plain, however, that changing the bias on the tunnel diode quite drastically influences the amplitude of oscillation. The plots plainly show a decrease in oscillation amplitude as the bias on the diode is increased. This agrees with what was seen experimentally.

Figure (9) is a schematic drawing of the negative resistance coupled circuit oscillator used for these experiments. The resonant frequency of the secondary circuit was varied by tuning the variable 100 pf. capacitor. A regulated power supply was needed, since the proper operation of the tunnel diode is very sensitive to slight voltage

FIGURE 8. $-R_N$ VERSUS AMPLITUDE FOR TUNNEL DIODE. 5th ORDER FIT.



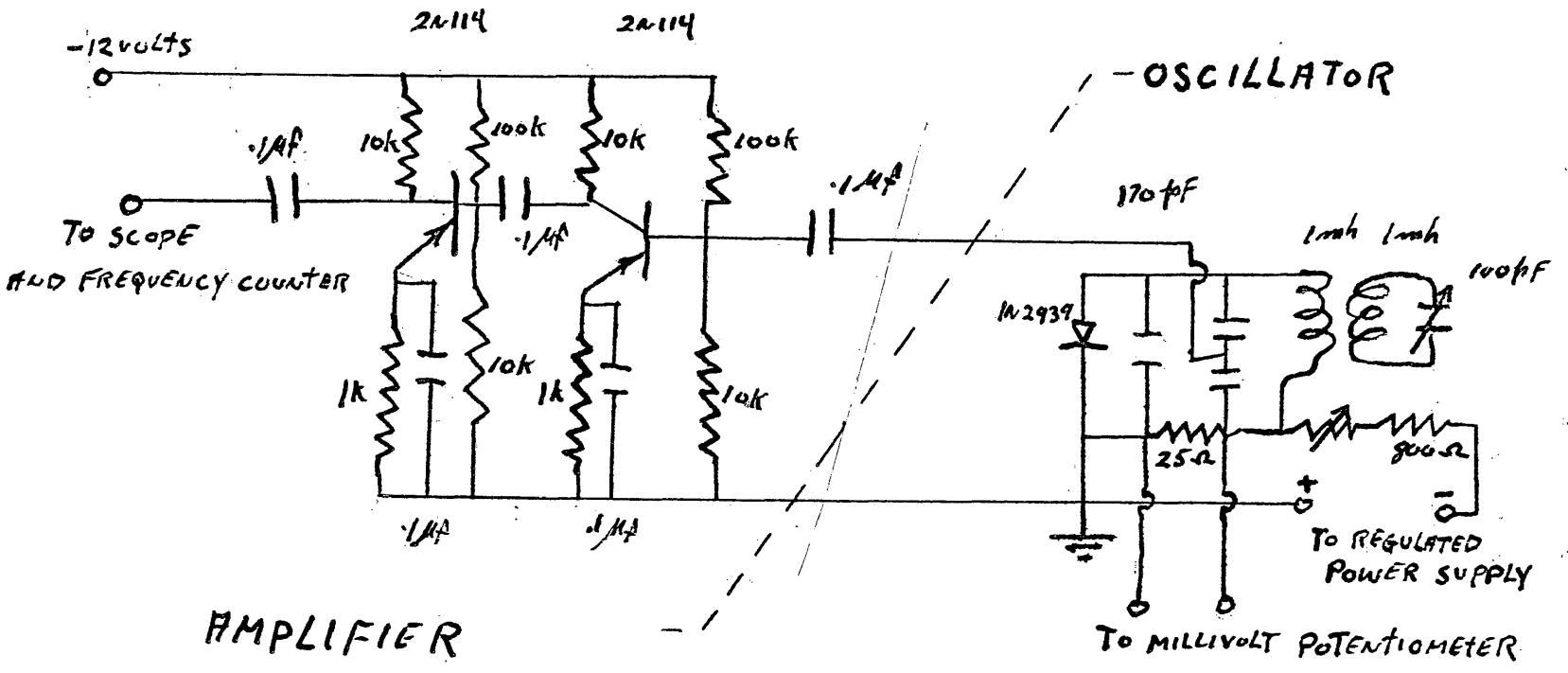


FIGURE 9. CIRCUIT DRAWING OF TWO-DIMENSIONAL NEGATIVE RESISTANCE OSCILLATOR.

fluctuations. Voltage fluctuations of less than .5 mv were needed for consistant data. Measurements of voltages across the diode could be made with a Leeds and Northrup Millivolt Potentiometer.

The tunnel diode was disconnected from the circuit and effective total impedance measurements of the coupled circuits were obtained, under conditions of resonance. At resonance the effective impedance was a pure resistance and measurements could be made without the complication of phase angles. Figure (10) shows a block diagram of the set-up used for the procedure.

To avoid difficulties due to an unbalanced signal generator, a differential amplifier plug-in was used and (A - B) sent to the vertical plates of the scope. V_R and V_X were displayed on the vertical and horizontal plates of an oscilloscope respectively and the circuit swept with a signal generator, until the resulting ellipse degenerated into a straight line. The values of V_R and V_X were then read directly off of the scope screen and R_X could then be computed, since at resonance:

$$\frac{V_X}{V_R} = \frac{R_X}{R_S} \quad \text{and} \quad R_X = \frac{V_X}{V_R} R_S .$$

In this way the total effective parallel impedance of the coupled circuits was obtained for a range of values of secondary circuit capacitance. The resulting measurements are plotted in figure (14a).

Whereas the total effective impedance of the one dimensional oscillator is just $\frac{L}{RC}$, it can be seen that the total effective impedance of the two dimensional oscillator is a function of its frequency of oscilla

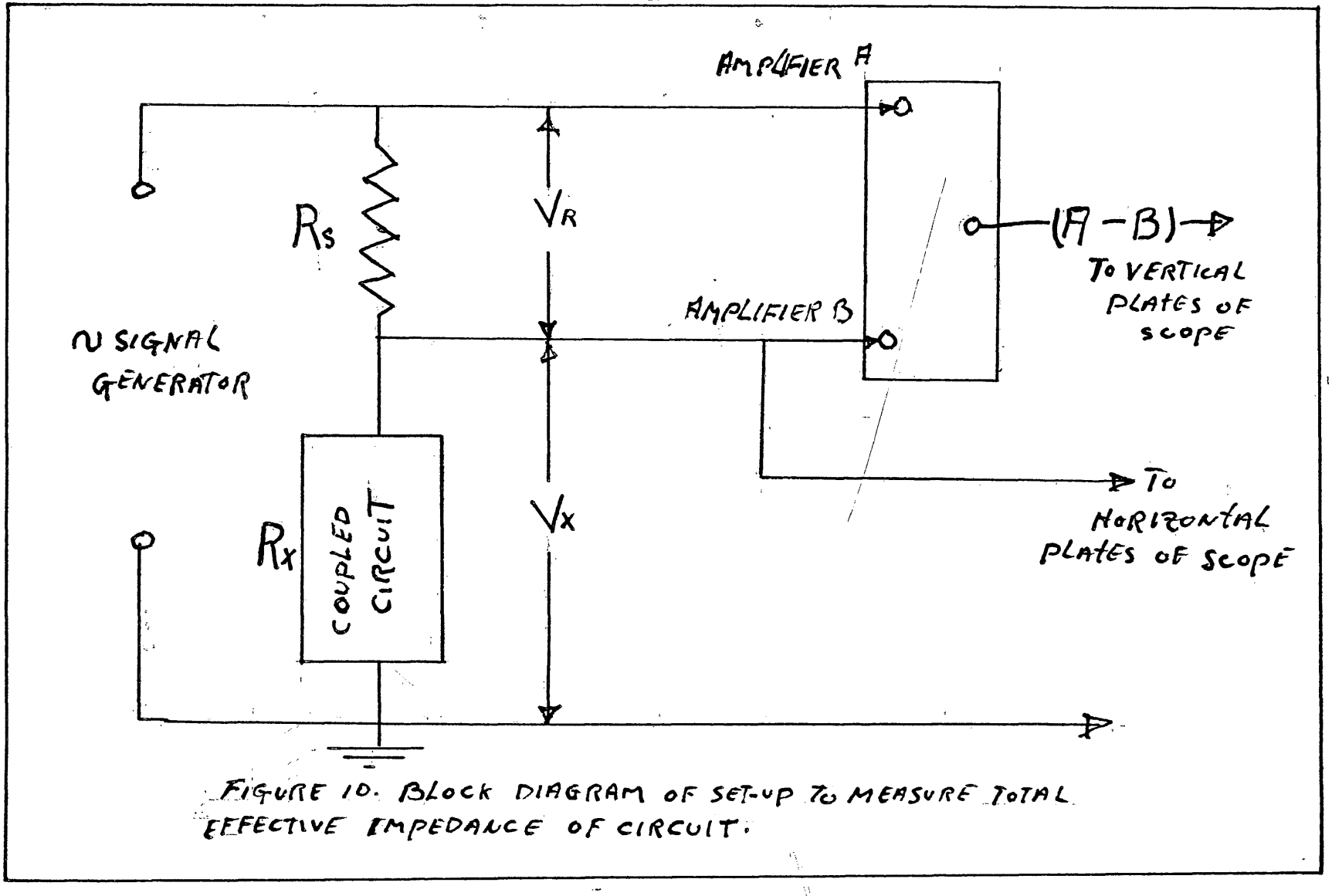


FIGURE 10. BLOCK DIAGRAM OF SET-UP TO MEASURE TOTAL EFFECTIVE IMPEDANCE OF CIRCUIT.

FIGURE 14b. FREQUENCY VERSUS DIAL SET FOR COUPLED CIRCUITS.

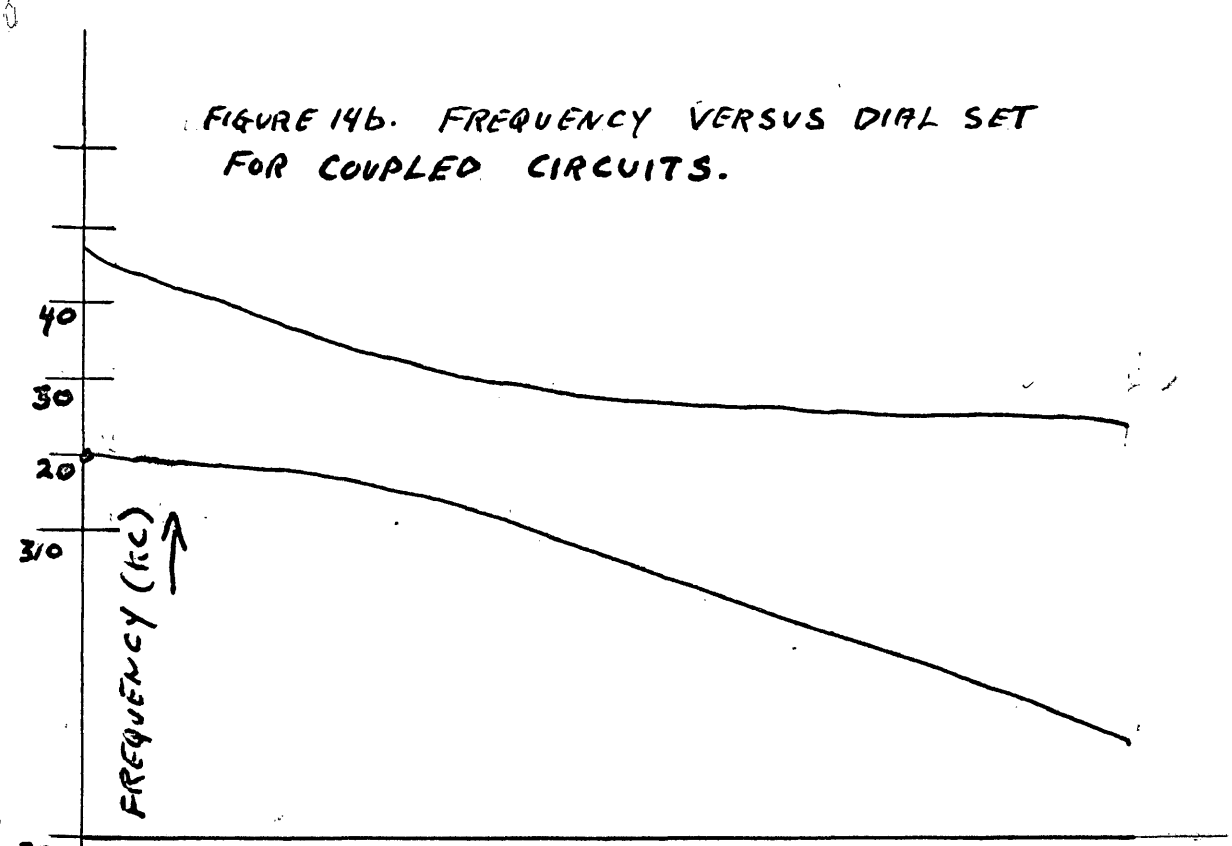
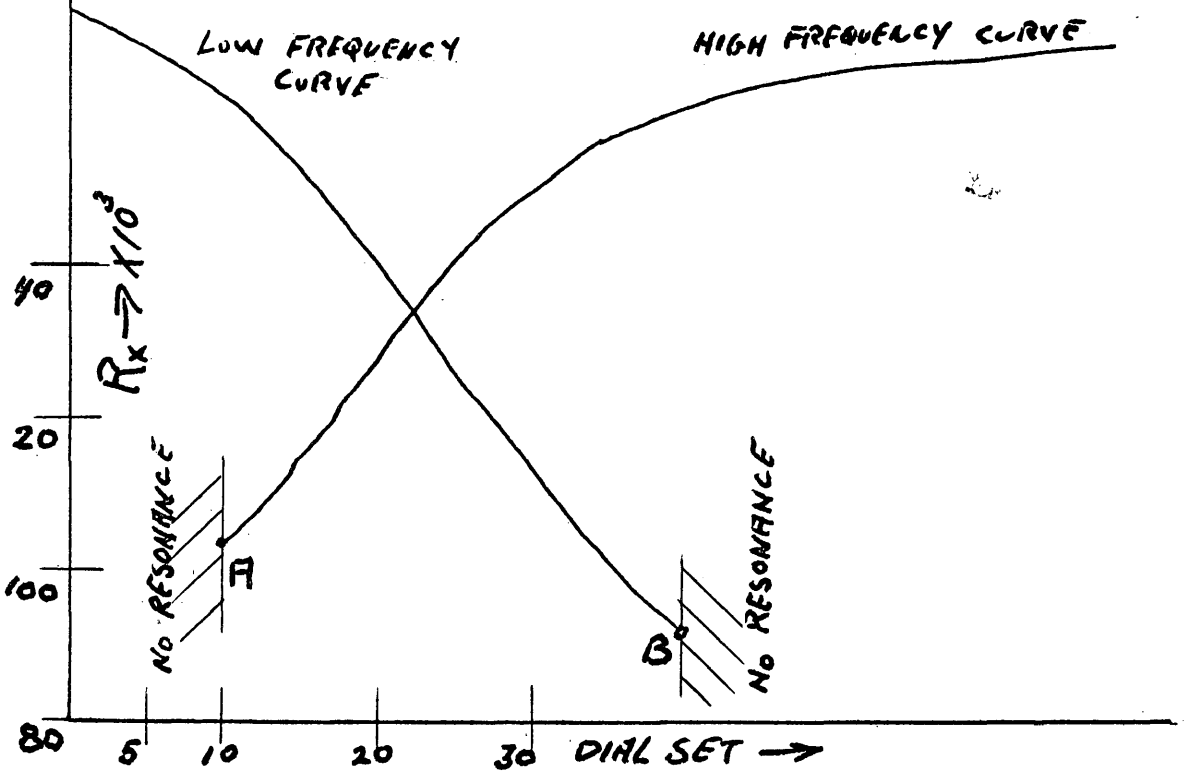


FIGURE 14a. EFFECTIVE PARALLEL IMPEDANCE VERSUS DIAL SET.



tion. As might be expected, when the two circuits are tuned to frequencies near to each other, the coupled circuit possesses two resonant frequencies. Since the circuit can be resonant at two frequencies at once, a question of interest is the one of possible multi mode oscillation. Of interest also are the conditions under which the circuit will oscillate in one mode or the other.

To investigate the behavior of the frequency of oscillation of the system, frequency and amplitude measurements were made as the capacitance (and thus the resonant frequency) of the secondary circuit was varied. The data is in qualitative agreement with the behavior which would be expected on the basis of the curves in figures (8) and (14). Comparison of these curves show that a decrease in the effective parallel resistance of the circuit is accompanied by a corresponding decrease in amplitude of oscillation. Figure (14a) shows that at a capacitor setting (in the primary) of five the frequency of oscillation of the circuit is such as to make its effective parallel impedance (R_X) lie on the low frequency curve. Figure (14b) is a plot of frequency versus dial setting for the circuit. It can be seen by comparing figures (8) and (14) that as the dial setting is increased, R_X is decreased and both the frequency and amplitude of oscillation should correspondingly decrease. Figures (11), (12) and (13) show that this was verified experimentally. If the dial setting is increased further, R_X is decreased until it becomes less than $-R_n$ or until the circuit ceases to be resonant in this

FIGURE 11. FREQUENCY AND AMPLITUDE VERSUS DIAL SET.

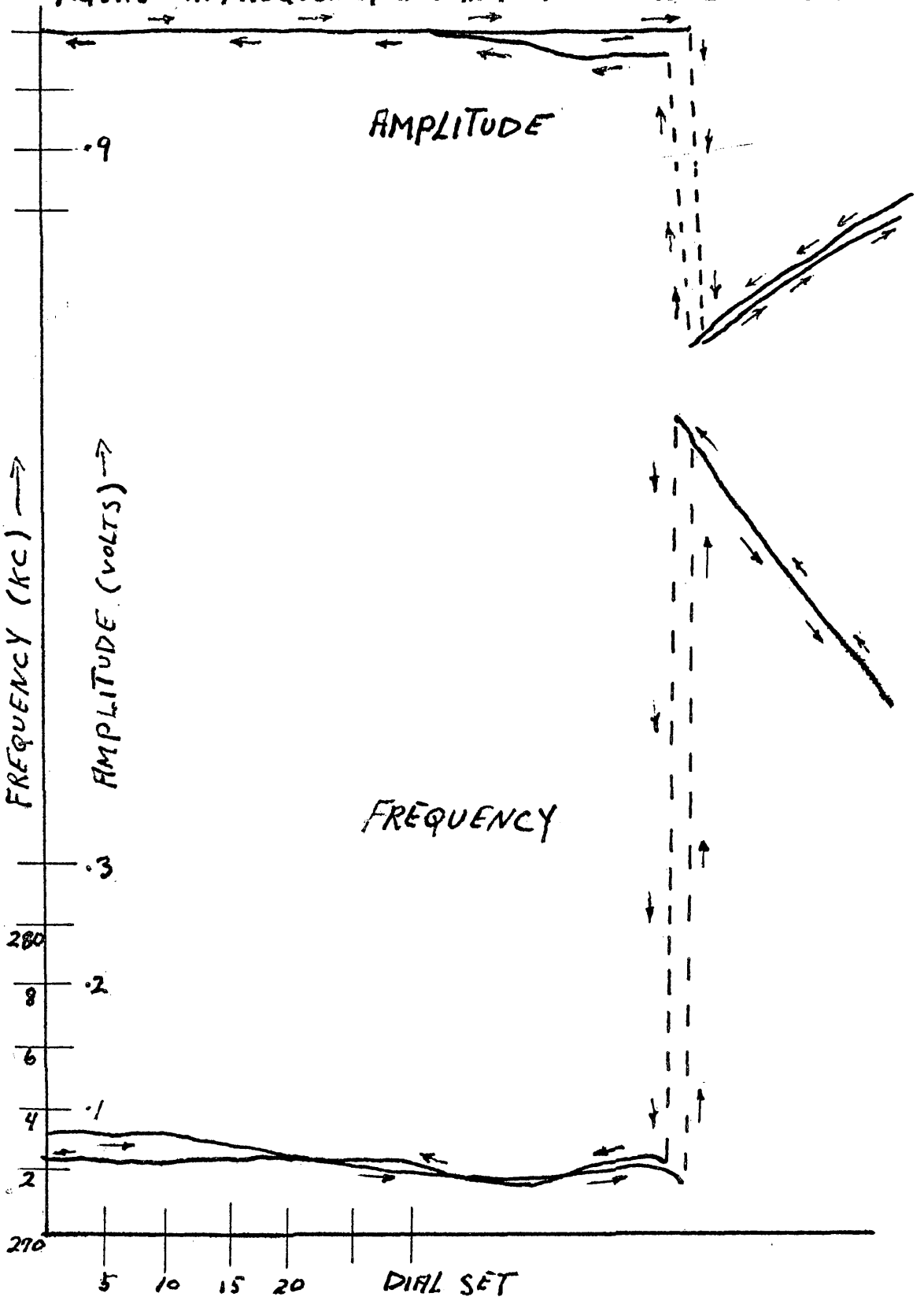


FIGURE 12. FREQUENCY AND AMPLITUDE VERSUS DIAL SET.

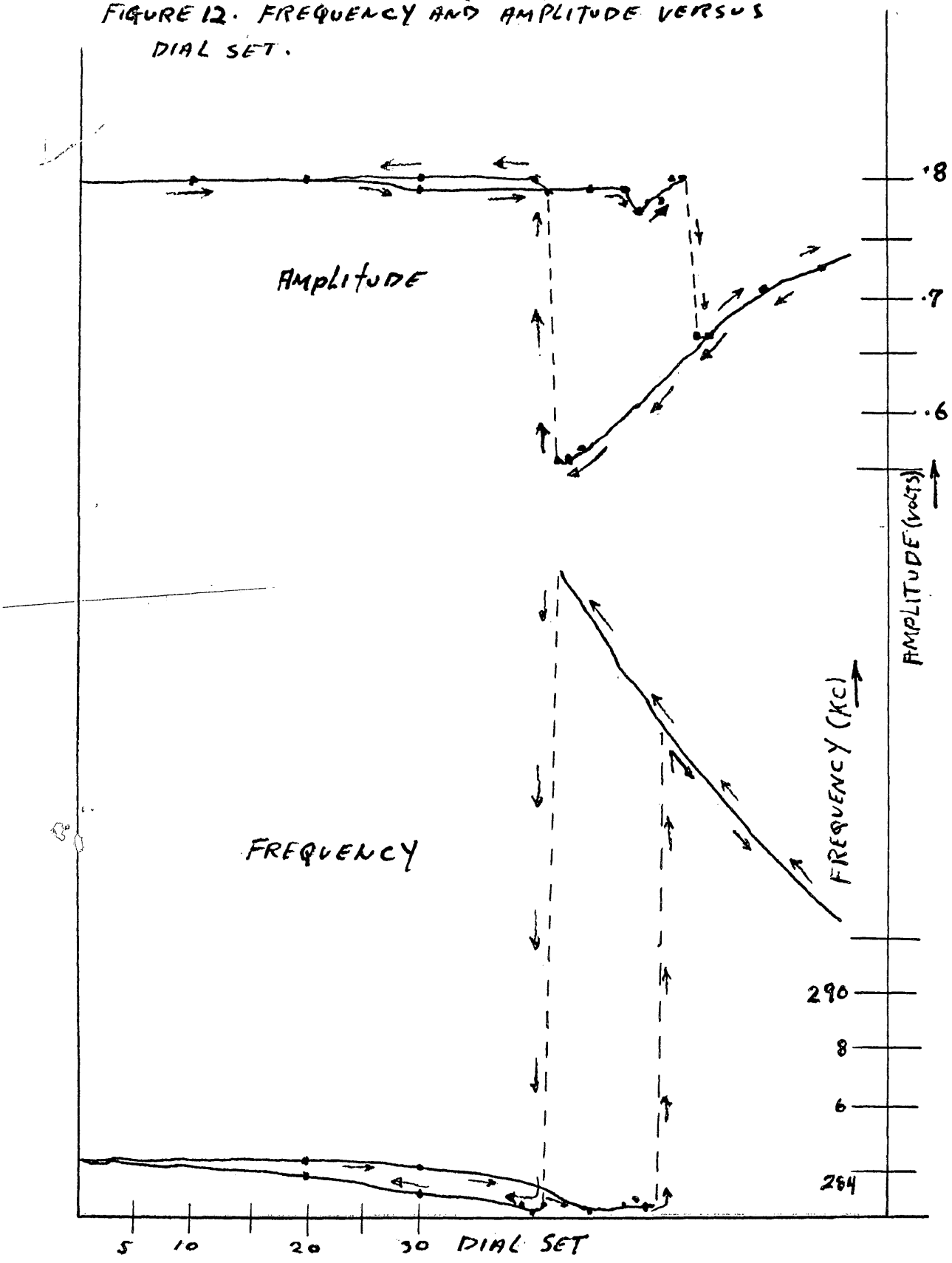
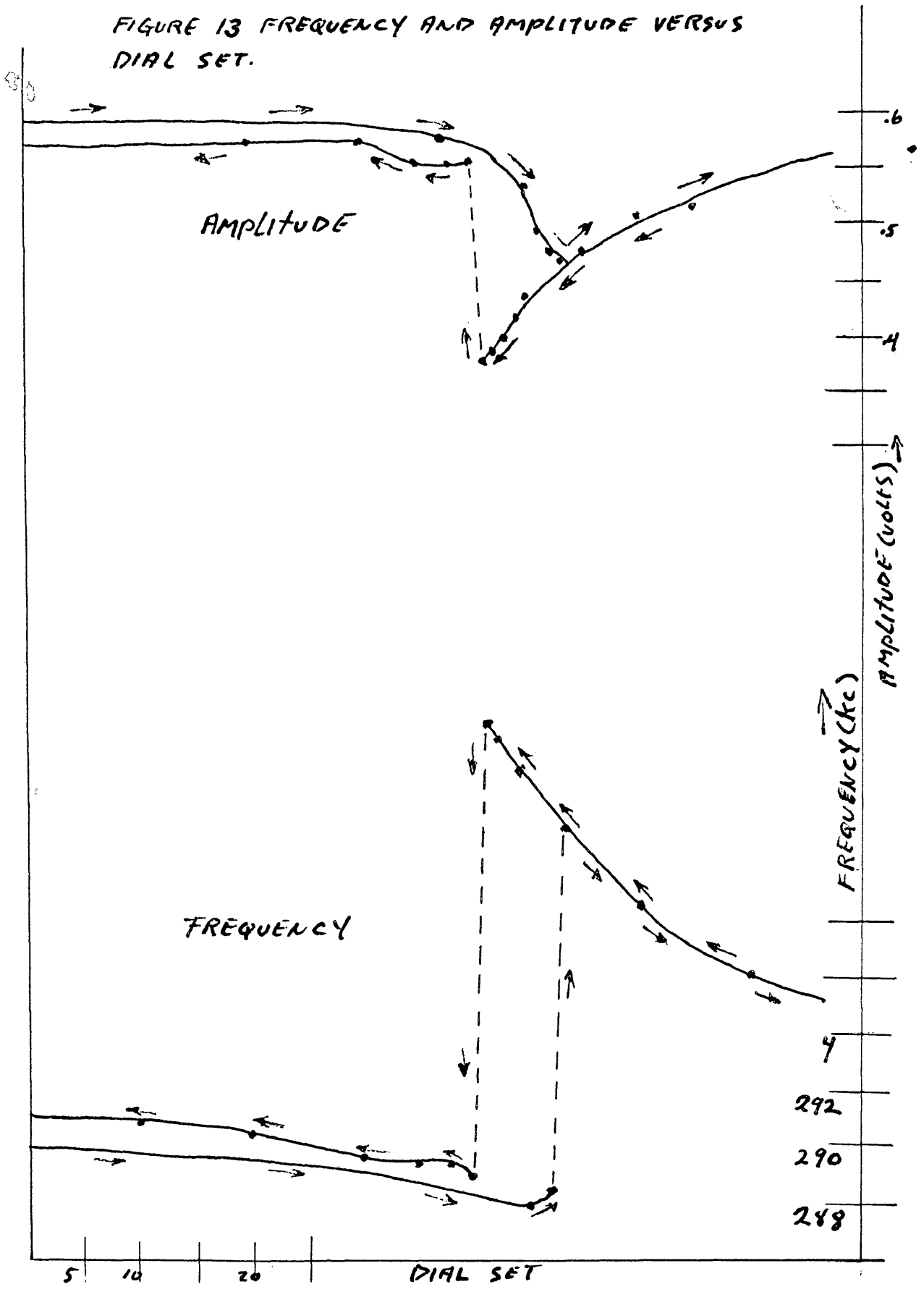


FIGURE 13 FREQUENCY AND AMPLITUDE VERSUS DIAL SET.



range (point B on figure (14a)), and the frequency jumps to the higher frequency range with a corresponding increase in amplitude. If the dial setting is now decreased again (the frequency of oscillation now being on the high range) the frequency now increases until it reaches A (or where it is such that $R_X < -R_n$) and jumps to the low frequency band again with its corresponding increase in amplitude. This characteristic behavior can be followed in the plots of figures (11), (12) and (13) quite easily. It is interesting to note that the jumps do not occur in the same place with increasing frequency as they do with decreasing frequency causing a (frequency and amplitude) "hysterisis" effect. Figure (14a) shows that the circuit is doubly resonant between points A and B, and singly resonant elsewhere. No data was obtained showing multi-mode oscillation. These results are similar to those of Van Der Pol.[3]

The curves in figure (14a) are shifted to the left relative to those of figures (11), (12) and (13) due to changes in circuit capacitance, when the circuit was disconnected from the tunnel diode for the measurement of figure (14a). The resonant frequency of the operating circuit was near 45 (on the dial set scale) and if the point of intersection of these curves is moved to this position, the frequency jumps predicted are relatively near where they were observed. It is quite clear from figures (11), (12) and (13) that these frequency jumps are also strongly effected by the bias on the diode. The width of the hysteresis loop increases with bi bias, as would be expected from the decrease in R_n, but a quantitative

comparison is not possible. It is believed that the lack of accuracy of the low-current portion of the tunnel diode characteristic is largely responsible for the lack of detailed comparison between theory and experiment.

CONCLUSIONS

Qualitative agreement of experimental results with theory is satisfactory. It was found experimentally that changes in circuit reactance occured when connections were altered in order to measure the equivalent parallel resistance. Since this effect is not included in the theory and because of lack of accurate measurements of diode characteristics quantitative agreement is not good. The changes in circuit reactance are probably partly due to the tunnel diode and possibly operating at a lower frequency would minimize this effect. Another alternative would be to use a non-reactive negative resistance device. It is not obvious how well the polynomial fits the tunnel diode characteristic. The least squares estimates of "goodness of fit" provided by the computer program did not allow us to distinguish clearly between polynomials of different orders. Amplitude measurements off of the $-R_n$ versus amplitude plot are doubtful.

In conclusion, we have found good qualitative agreement with theory and increased our understanding of negative resistance oscillators. We believe, however, that the incorporation of some of these suggestions and/or the direct measurement of R_n as suggested by Brunetti[5] would lead to better detailed agreement.

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