

**TEACHING SLOPE OF A LINE USING THE GRAPHING CALCULATOR
AS A TOOL FOR DISCOVERY LEARNING**

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TEACHING SLOPE OF A LINE USING THE GRAPHING CALCULATOR AS A TOOL FOR DISCOVERY LEARNING

ABSTRACT

Discovery learning is one of the instructional strategies sometimes used to teach Algebra I. However, little research is available that includes investigation of the effects of incorporating the graphing calculator technology with discovery learning. This study was initiated to investigate two instructional approaches for teaching slope of a line in Algebra I. One approach involves the graphing calculator as a tool in a discovery learning setting. The second approach involves using the graphing calculator to reinforce traditional instruction. An urban public school division located in southeastern Virginia was the site for this investigation. Two Algebra I classes from each of two middle schools and two Algebra I classes from each of three high schools were involved in this study. The experimental groups completed a discovery learning activity, while the control groups used traditional instruction. This study is an investigation of whether there was a difference in student achievement in slope of a line when one discovery learning activity was completed prior to formal instruction. It was concluded that student achievement did not increase with the inclusion of one discovery learning based activity. Further study is needed to evaluate if discovery learning is effective if utilized throughout the unit on slope of a line, if additional professional development focused on discovery learning is necessary, or if a series of discovery learning activities would increase student achievement.

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**Teaching Slope of a Line Using the Graphing Calculator as
a Tool for Discovery Learning**

Chapter 1: Introduction

Because of the development of new technologies, the instructional tools used for teaching Algebra I have changed over the years from use of the slide rule to the use of computers and graphing calculators. When the graphing calculator was introduced into the classroom approximately twenty years ago, a controversy erupted almost immediately. Teachers, parents, and much of the public feared that basic mathematics skills would be lost as students potentially became calculator dependent (Herrera & Owens, 2001). There is now research available to indicate that graphing calculator technology, when properly used, supports instruction and student learning (Heller et al., 2005; Bos, 2007).

Since the introduction of the graphing calculator into the classroom, there have been claims of advantages and disadvantages regarding handheld technology over this twenty-year span. Potential advantages to graphing calculator use may include increased conceptual understanding. For example, the graphing calculator may be used as a tool to create concrete imagery of abstract mathematical ideas (Lee, 2007; Lopez, 2001). When students discuss the resulting display after graphing a function, they engage in mathematical communication as recommended by the National Council of Teachers of Mathematics (NCTM) Principles (2000) (Waites & Demana, 1998). Graphing calculator use also permits real world problem solving where the data would be too cumbersome to calculate by hand (Lopez, 2001). The use of the graphing calculator as a tool allows for equity in the classroom as the calculator permits a common starting point for all students (NCTM, 2000). Students then have the ability to explore mathematical ideas using the graphing calculator, potentially leading to more in-depth mathematical understanding

(White & Gerson, 2006).

Graphing calculator capabilities include basic computations, visual representations of abstract mathematical equations, confirming solutions calculated by hand using paper and pencil, exploring original ideas, or answering “what if” questions. Classroom use of the graphing calculator may include verifying solutions to problems, visualizing an abstract idea, and finding a solution to what is too difficult to solve with paper and pencil. The optimal classroom use could be a blending of traditional instruction and technology integration.

While there are many advantages to graphing calculator use, potential disadvantages also exist. Lack of knowledge or experience on the part of students and/or teachers may inhibit effective use of the graphing calculator. If teachers do not understand the capabilities of graphing calculator use, students may not receive optimal instruction (Cavanagh & Mitchelmoer, 2010; Waites & Demana, 2001). Finally, a perceived deterioration of basic skills may be considered a disadvantage of graphing calculator use in the classroom as there are claims that the students are not “doing the math” since they are using the graphing calculator (Pomerantz, 1997).

Statement of Problem

The objective of high school ultimately is for students to graduate and enter college or successfully enter the workplace. Challenges arise with students who are considered at risk of failure. Because of the algebra for all students initiative, Algebra has been considered a gatekeeper course for moving into advanced mathematics courses needed for college or career readiness. An increased number of middle school students are completing Algebra I by the eighth grade. These students tend to be above average,

as they are the students who take advanced mathematics courses in high school (Henrico Public Schools, 2008). Many of the average/struggling learners take Algebra I at the high school level and the traditional methods of instruction may not be as effective with this group of students. Students enter the algebra classes at different foundational levels (Heller et al., 2005; Bos, 2007; Morgatto, 2008; Spielhagen, 2006). Some students enter at a level where they are ready to move at an accelerated pace while others lack foundational skills. A number of students struggle with Algebra I, especially at the high school level. This becomes a concern as Algebra I is a requirement for high school graduation and is necessary for many trade certifications as delineated in the Virginia Department of Education (2012) College and Career Ready Mathematics Performance Expectations. Additionally, graphing calculator use is mandated in the Virginia Department of Education (VDOE) Algebra I Standards of Learning (SOL) document that guides instruction in Virginia classrooms (VDOE, 2012). As the “Algebra for All” initiative strives to place more students in Algebra I, and not separate college bound students from other students, Morgatto (2008) suggested that placement in Algebra I should be based on student needs and readiness so algebra is delivered successfully to all students.

How can Algebra I be successfully delivered to all students? The What Works Clearinghouse (2012) offered suggestions to assist students who may potentially struggle with Algebra I. Suggestions include increasing number sense skills as well as assisting students in developing a deeper understanding of fractions. These skills can be increased through algebra “boot camps” or algebra labs (i.e., computer labs that provide prerequisite skill building). Additionally, conceptual understanding can be enhanced

through technology-based tools. Heller et al. (2005), Lapp, Cyrus, Dick, and Dunham (2000), and Kersaint (2007) found similar results that improved conceptual understanding with grade nine Algebra I students were observed when the graphing calculator was incorporated into instruction.

These high school students may experience more success using different instructional strategies and tools. The discovery learning instructional strategy involves student exploration and creation of understanding through problem solving or investigations. Students take ownership of their own learning while building new knowledge based on prior learning. This process may increase motivation for learning (Castronova, 2002). The introduction of the graphing calculator technology paired with discovery learning may provide an effective tool to increase success in Algebra I. The combination of handheld technology and discovery learning might assist the average/struggling learners grasp concepts such as slope of a line more effectively when these topics are introduced in Algebra I.

Discovery learning allows students to interact with their learning and develop their own understanding (Castronova, 2002). The graphing calculator may serve as a tool that assists students in developing their own understanding of slope by interacting with their environment while learning. The question arises regarding whether the graphing calculator used in a discovery learning environment could assist average/struggling learners to understand better the concept of slope than if they were taught in a traditional method.

This study is an investigation of whether discovering the concept of slope through discovery learning paired with graphing calculator technology increased the achievement

of the average/struggling learner in a high school classroom. Second, the study is focused on gains in achievement where the sequencing of graphing calculator use was studied. Sequencing the graphing calculator use as a tool in discovery learning followed by direct instruction was studied. Interest in this topic grew from the fact that graphing calculator use as a tool is recommended during the instruction of Algebra I, although the specifics of how to use the graphing calculator are not detailed. Ultimately, the goal of Algebra for All is to increase the number of students successfully completing Algebra I. The approach of pairing discovery learning with graphing calculator technology was studied to determine if this approach would provide a method that would assist more students in successfully completing Algebra I, thus permitting these students entry into mathematics courses needed for college and career readiness.

The Virginia Standards of Learning refer to graphing calculator use as a tool for students to check their work. For the teacher, the VDOE does provide lessons in the Enhanced Scope and Sequence (2012) where the graphing calculator can be used during instruction. While this particular resource is available, it is categorized as a resource so the lessons may or may not be used by classroom teachers. Graphing calculator use is also present in the Common Core State Standards launched by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officials (CCSSO). These organizations call for graphing calculator use when students experiment with properties of functions (Common Core State Standards Initiative, 2009). Graphing calculator use can assist in teaching the Common Core Standards' focus on conceptual understanding as well as procedures. Additionally, the National Council of Teachers of Mathematics (NCTM) Technology Principle

recommends this type of technology to enhance instruction (NCTM, 2001). The principle states, “Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction.” Again, the use of the calculator is that of a tool to enhance learning. The graphing calculator as a tool can be one of equity, exploration, and verification. Earlier research recommended that graphing calculators be used as an equity tool to allow all students to have a common starting point (Waits & Demana, 1998), which can allow students to explore algebraic concepts without being held back by having to graph functions by hand, which can be time consuming. Additionally, the graphing calculator has been referred to as a partner in mathematical investigation where students explore with the use of the graphing calculator (White & Gerson, 2006). Lastly, this tool allows students the ability to verify what they have created by hand (VDOE, 2012).

Conceptual Framework

The conceptual framework for this study is focused on the use of graphing calculator technology and discovery learning for teaching the concept of slope of a line in Algebra I courses (see Figure 1). The learning experience was situated in components of discovery learning, critical exploration, the use of mathematical tools and direct instruction. The learning experience placed the teacher in the role of a facilitator who guided students through the task and structured the learning process that provided students the opportunity to create their own ideas about slope of a line prior to receiving direct instruction. Teachers provided all students a challenging task that created active engagement and opportunities for communication. Students explored the concept of

slope by exploring the changes in the graph and developed ideas based on these explorations. This exploration task was followed by direct instruction. As a result, an experience where students interacted with the teacher during the learning process developed. Students also experienced the opportunity to interact with other students through collaboration and communication while solving mathematical problems and through discussions during the guided practice portion of the lesson. Students completed the assessment that provided the teacher with feedback. The feedback from the assessment was also provided to the students. Specifically, this study was an investigation of the use of graphing calculator technology as the learning tool and a blending of discovery learning followed by direct instruction to teach the concept of slope. The sequencing of discovery learning and direct instruction was the focus of the study. The resulting learning experience incorporated critical exploration of algebraic problems and situations as the assigned task.

Purpose of Study

This study is focused on two instructional approaches for teaching slope of a line in Algebra I. One approach involved the graphing calculator as a tool in a discovery learning setting. The second approach involved using the graphing calculator to reinforce traditional instruction. With many advanced students completing Algebra I in middle school, students taking Algebra I in high school tend to be average/struggling learners. Delivering instruction from a discovery learning approach as opposed to the sole use of direct instruction may enable these average/struggling learners to understand slope of a line more clearly. The discovery learning approach could facilitate students' creating their own meaning of the concept of slope. This study also was an investigation of

whether there is a difference with advanced learners using the same instructional approaches.

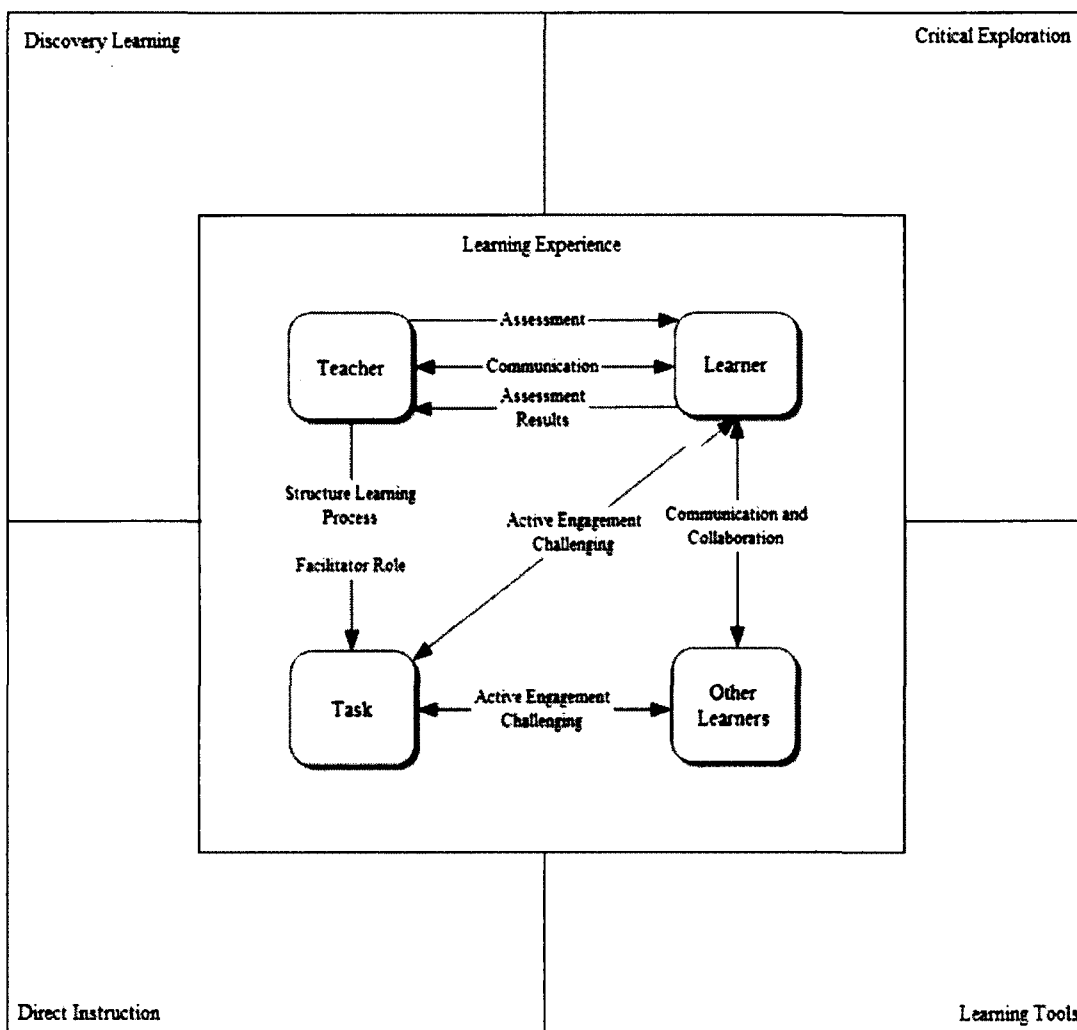


Figure 1. Conceptual Framework for the Learning Experience

Definition of Key Terms

The key terms that will be used throughout this paper are listed below.

Graphing calculator – a handheld calculator that is capable of plotting graphs, solving simultaneous equations, and performing numerous other tasks with variables.

Graphing calculators are considered a tool in the instruction of mathematics.

Handheld technology – a computing device that is held in one's hand, such as a

graphing calculator.

Paper-and-pencil – a method/tool used to find solutions to mathematical problems (VDOE, 2009)

Direct instruction – “a model for teaching that emphasizes well-developed and carefully planned lessons designed around small learning increments and clearly defined and prescribed teaching tasks. It is based on the theory that clear instruction eliminating misinterpretations can greatly improve and accelerate learning” (National Institute for Direct Instruction, 2011).

Discovery Learning – “an approach to instruction through which students interact with their environment by exploring and manipulating objects, wrestling with questions and controversies, or performing experiments” (Ormrod, 1995, p. 443).

Average /struggling learner – a student who did not meet the prerequisite requirements, defined by the focus school division, to take Algebra I in middle school. Prerequisite requirements for taking Algebra I in the focus school division are an end-of-year grade of an A or B in grade six mathematics and a score in the pass advance range on the grade six state mathematics assessment. The same prerequisite skills, based on grade 7 achievement, apply to students taking Algebra I in grade 8.

Advanced learner – a student who meets the prerequisite requirements in this school division to take Algebra I in middle school. Prerequisite requirements in the focus division are an end-of-year grade of an A or B in grade six mathematics and a score in the pass advance range on the grade six state mathematics assessment. The same prerequisite skills, based on grade 7 achievement, apply to students taking Algebra I in grade 8.

Research Questions

This study is a comparison of the effectiveness of two different instructional approaches for teaching the slope of a line in Algebra I to average/struggling learners as well as advanced learners. The research questions are:

1. Is there is a difference in achievement of average/struggling learners in high school Algebra I when taught by development of the concept of slope through graphing calculator technology and discovery learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing activities using graphing calculator technology?
2. Is there is a difference in achievement of advanced learners in middle school Algebra I when taught by development of the concept of slope through graphing calculator technology and discovery learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing activities using graphing calculator technology?
3. Is there a difference in achievement between the middle school and high school Algebra students when taught by development of the concept of slope through graphing calculator technology and discovery learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing activities using graphing calculator technology?

Significance of Study

Information and results derived from this study could be used by mathematics leaders and mathematics teachers to develop effective instructional modules to teach Algebra I to average/struggling learners. Curricular materials focusing on real-life algebraic examples could be created incorporating the discovery learning approach into the activities. Study results could also be used to develop professional development topics for teachers focusing on pairing graphing calculator technology and discovery learning. Findings from this study could also affect how graphing calculator use is defined in state and/or national standards. Additionally, advanced learners in middle school may also benefit from the inclusion of discovery learning lessons that teach or extend the objectives in the mathematics curriculum.

Chapter 2: Literature Review

The graphing calculator is an integral part of instruction in today's algebra classroom. Handheld technology began in 1947 when the Curta was designed to perform the functions of addition, subtraction, multiplication, and division. In 1967, Texas Instruments created a calculator capable of performing basic functions. Hewlett Packard followed in 1972 with the development of the first Pocket Calculator. During the 1980s, graphing calculators were developed that introduced calculators with algebraic capabilities. In 1998, graphing calculators with upgradeable operating systems and downloadable software were introduced (Department of Education, Newfoundland, and Labrador, 2004). It was during the late 1980s and early 1990s that graphing calculators also entered the classroom as a tool to enhance instruction. Currently, thirty-seven states permit or require graphing calculator use on state assessments. The SAT, ACT, and the Praxis also require or permit graphing calculator use (Texas Instruments, 2012).

Classroom use of the graphing calculator has the potential to provide a deeper level of mathematics understanding but is not intended to replace pencil-and-paper computational skills. Computational skills refer to the ability to perform the four operations of addition, subtraction, multiplication, and division. These computational skills are the foundational skills in mathematics and are often referred to as basic mathematics skills. These basic skills need to be expanded to include problem solving in conjunction with these basic operations (Schwartz, 1999). The basic operation skills may be moved to a more rigorous level using calculator technology. Specifically, the National Council of Teachers of Mathematics Technology Principle (NCTM, 2000) calls for basic skills to be fostered rather than replaced by the use of technology during instruction. For

instance, students use the technology to increase or deepen understanding of mathematics by using the technology as a tool to verify mathematics calculations or graphs. The graphing calculator permits students to check the work they have done, not replace mathematics completed by hand. This NCTM Technology Principle has been incorporated into Virginia state standards for mathematics instruction.

The Virginia Department of Education (VDOE, 2009) Standards of Learning (SOL) refer to the paper-and-pencil method as a way for students to record their thinking while performing mathematical computations. Additionally, in conjunction with basic skills, graphing calculator technology is also specifically referenced in the VDOE SOL for Algebra I. The VDOE Algebra I Curriculum Framework suggests that the graphing calculator should be used as a tool to check pencil-and-paper mathematics (VDOE, 2009). Specifically, algebraic topics are taught and then practiced without using the calculator technology and then the results are confirmed using the graphing calculator. The process of incorporating the graphing calculator technology into classroom instruction has created a move from teaching algebraic concepts with pencil-and-paper only to teaching algebraic concepts with the integration of graphing calculator technology and pencil-and-paper. While graphing calculator integration into classroom instruction can be beneficial to the learning process, caution should be taken to maximize the potential advantages to instruction and minimize potential disadvantages created by the infusion of graphing calculator technology into the algebra classroom.

In introductory algebra courses, there are advantages and potential disadvantages of using the graphing calculator technology as part of classroom instruction. These advantages and disadvantages created by graphing calculator use must be considered

when integrating this calculator technology into the curriculum and into classroom instruction. The introduction of the graphing calculator technology should be integrated into instruction so the disadvantages are minimized and the advantages are maximized. To maximize the advantages and minimize the disadvantages, attention should be given to how and when the graphing calculator technology is integrated into the curriculum and integrated into classroom instruction.

Potential Advantages

Potential advantages of using the graphing calculator as a tool for teaching certain algebraic skills are realized when graphing calculator integration is designed in a purposeful manner. The desired result of graphing calculator use in the algebra classroom is to support effective algebra instruction. Integration that focuses on graphing calculator use as a tool for instruction can create a gateway to increased student achievement in mathematics. The graphing calculator becomes the tool potentially to aid in advancing academic success for students. Potential advantages may include increased conceptual understanding, effective curriculum integration of graphing calculator technology, a common starting point, an exploration tool, concrete imagery, math talk, real-world-based problem solving, an equity tool, and a tool for discovery learning. Finally, graphing calculator use is a mathematical tool utilized to perform basic calculations so instructional time can be spent on in-depth algebraic content.

Conceptual Understanding

Conceptual understanding refers to an integrated and functional grasp of mathematical ideas (National Research Council, 2001). The NCTM Learning Principle addresses conceptual understanding and technology integration by stating:

Conceptual understanding is an essential component of the knowledge needed to deal with novel problems and settings. Moreover, as judgments change about the facts or procedures that are essential in an increasingly technological world, conceptual understanding becomes even more important. For example, most of the arithmetic and algebraic procedures long viewed as the heart of the school mathematics curriculum can now be performed with handheld calculators. Thus, more attention can be given to understanding the number concepts and the modeling procedures used in solving problems. (2000)

Conceptual understanding may be achieved through the integration of calculator technology into the curriculum, the use of discovery learning, concrete imagery of abstract algebraic ideas, and math talk as a means to communicate about mathematical ideas.

Curriculum Integration

Curriculum and technology integration is defined by Digital Learning Environments, Tools and Technologies for Effective Classrooms as “the goal of technology integration is to use technology seamlessly so that the technology itself becomes a transparent and integral tool to teach core curriculum” (2011). This integration may be accomplished by using the approach suggested by the NCTM (2000) technology principle, which is to foster intuitions but not replace basic understandings. Basic understandings are developed by using mental mathematics computation, pencil-and-paper computation, and technology-assisted computation, as all are important components of mathematics (Waits & Demana, 1999). Integrating technology into instruction may aid in teaching these basic understandings. Kersaint (2007) discussed the

concept of technological pedagogical content knowledge (TPCK) as the “interweaving of technology, pedagogy, and content,” which brings together the idea of curriculum content that is taught using effective instructional strategies with the support of technology.

While planning for teaching curriculum content, decisions teachers make about the partnership of curriculum and technology use can hinder or enhance how students learn important mathematical content. Heller, Curtis, Jaffe, and Verboncoeur, (2005) administered an end-of-course algebra test to 458 students in two suburban schools. Students used graphing calculators during instruction for algebraic calculations and graphing but not during the assessment. Heller et al. (2005) found that the manner in which the technology is used in classroom instruction ultimately influences the end of course scores in a positive manner. They further described that the greater exposure students had to calculators during instruction; the higher the end-of-course scores as compared to students who did not use graphing calculators at all. It was also reported that when calculators were used to teach linear inequalities, non-functions, and quadratic functions a “more is better” approach resulted in higher end-of-course scores.

Achieving end-of-course success may be accomplished through a written and taught curriculum that effectively blends traditionally taught mathematics with technology-assisted instruction such as graphing calculator technology. Lapp et al. (2000) called for careful integration of calculator use into the curriculum, since the calculator used as a tool only is not enough to develop understanding of important concepts. They observed this happening as students interpreted real time data using a motion detector with the graphing calculator. Students did not understand that the graph that was created with these tools was displaying information relating to distance. Lapp et

al. (2000) found the problems students experienced while using the graphing calculator fell into four areas: the inability to connect graphs with the physical concepts, the inability to connect graphs with the real world, difficulty making the transition between graphs and physical events, and difficulty building concepts through communication. These problem areas may be eliminated or minimized through curriculum integration. To aid in concept development and understanding, a focus on student needs should be considered when incorporating graphing calculator use into classroom instruction. These needs may require structured guidance provided by curricular materials to direct students throughout the experience to focus what students examine. As calculators are integrated into mathematics instruction, the need for more curricular materials and instructional strategies may be required.

Haas (2005) found in a meta-analysis of thirty-five studies that technology could change the look of the mathematics classroom. The students would not complete all assignments using only paper and pencil, but rather they would interact with the technology to study mathematics concepts. He further discussed how a lesson might look very different from a traditionally taught lesson. For example, a new skill is presented and instruction occurs within a real life problem situation. The students use graphing calculators to model the mathematics that is occurring thus eliminating time-consuming computations and graphing usually completed using a pencil-and-paper method. Students are encouraged to communicate with each other and share their thinking process as they work through the problem situation. Lastly, the teacher reviews concepts and provides feedback as closure to the lesson. This type of lesson format may require additional curricular support so algebraic concepts are blended effectively with the use of

technology.

Waites and Demana (2000) described a second view of curriculum integration where paper-and-pencil methods were to be used during concept development and calculator use was to be incorporated during extension activities and when students are generalizing concepts. They suggested that technology should be “integrated into the fabric of classroom practice” (Waites & Demana, 2000, p.) This use of technology may provide a common starting point for all students as they begin the learning process.

Common Starting Point

Graphing calculator use allows all students to begin at a common starting point when solving complex problems and constructing their own conceptual knowledge (Lee, 2007). Conceptual knowledge may develop by including reasoning, problem solving, and representations as essential components in classroom instruction (Herrera & Owens, 2001; NCTM, 2000). In algebra courses, skill and idea development may be enhanced using graphing calculator technology during instruction. For instance, instruction may be enhanced when all students have the capability to see identical graphs (Lee, 2007; Vavilis & Vavilis, 2004). As students are studying slope of a line, they may be required to graph a linear equation in their notes. This particular task may take some students very little time while other students may require more time to graph the equation. Teachers often wait until most students have completed the graph before continuing with instruction. Students who complete the graph quickly have the potential to become bored while frustration may build in those students who require more time to complete the graph. Graphing calculator technology affords all students the opportunity to enter the equation in the graphing calculator and eliminate lost instructional time since the amount of time

needed to create the graph by hand is reduced significantly. Lee (2007) found that instructional time could be saved when students set up the domain and range for a given graph based on teacher direction so all graphs appear identical. This graph set up procedure can be completed quickly using technology resulting in all students being ready to examine the identical graph so instructional time can be spent analyzing the graph. The analysis may focus on specific components of the graph or focus on simply exploring the various components of the graph.

Exploration Tool

Many students are curious by nature. They will explore and experiment with a new video game, drawing on their prior knowledge to make meaning of the new game. Could this curiosity and sense of experimentation be introduced into the algebra classroom through graphing calculator technology? Martin found “placing graphics calculators in the hands of students gives them the power and freedom to explore mathematical territory that may be unfamiliar to the teacher” (2008, p.20). The advanced students could potentially be held back by topics unfamiliar to the teacher or topics beyond the curricular requirements. This may specifically apply to gifted students who want to explore beyond the constraints of classroom instruction. The Virginia Department of Education suggested the following for gifted students:

Appropriately, differentiated curricula for gifted students refer to curricula designed in response their cognitive and effective needs. Such curricula provide emphasis on both acceleration and enrichment opportunities for (i) advanced content and pacing of instruction, (ii) original research production, (iii) problem finding and solving, (iv) higher level thinking that leads to the generation

products, and (v) a focus on issues, themes, and ideas within and across areas of study. (2012)

Incorporating graphing calculator technology into classroom lessons may result in many advantages for gifted students, while average students may also experience gains through graphing calculator experiences. Algebraic graphing calculator explorations focused on conceptual learning may include discovery experiences, creating concrete imagery, experiencing deeper problem solving and applications, communicating with other students, and communicating with teachers. Students can test their ideas or conjectures, produce multiple representations, and develop conceptual understandings. While graphing calculator use has the potential to enhance student understanding of mathematics through exploration, attention should be given to the specifics of these graphical images.

Concrete Imagery

Students can use graphing calculators to create concrete imagery and receive immediate feedback, which may enrich their learning experience. As students explore mathematics concepts with the graphing calculator they can observe multiple views of the same function and observe the related patterns in the function tables. Bos (2007) suggested that the graphing calculator has the power to allow students to see multiple representations of patterns. Observations of these patterns occur in equation format, graphical format, or table format. When eleventh grade at-risk students utilized graphing calculator technology, their achievement levels on portions of a state assessment were at higher levels than their counterparts who did not use technology during instruction (Bos, 2007). Relationships between these multiple representations can be reinforced using

graphing calculators. Additionally, student visual reasoning skills and graph interpretation skills are enhanced (Hennessy, Fung, & Scanlon, 2001). Hennessy et al. (2001) surveyed undergraduate students and found that by generating multiple graphs easily using graphing calculators more time was available for analysis and interpretation. They also extended this finding from university students to secondary students. Hennessy et al. found “portable graphing technologies present a unique opportunity to help mathematics students (at secondary and university entry level) develop concepts and skills in a traditionally difficult curriculum area” (2001, p.282). Graphing calculators allow students to test ideas and receive instantaneous feedback since graphical images are produced quickly (Bos, 2007). Lapp et al. cited a study by Brasell (1987) which found “that a delay of even twenty seconds between the conclusion of a physical event and the graphical display makes a difference in the students’ ability to link the graph and the physical concept” (2000, p. 504). This further supports the benefits of immediate feedback as well as connections. The NCTM Connections Standard indicates that seeing mathematics as a connected whole and the understanding of the connections between ideas builds the foundation for success in future mathematics courses (NCTM, 2000). The blending of graphing calculators, concrete materials, tables, and representations assist students in gaining an understanding of abstract topics (Herrera & Owens, 2001; Lapp & Cyrus, 2000; Lopez, 2001). While abstract topics may pose hurdles for some students, the graphing calculator may be an effective tool when used to overcome these hurdles. Since the graphing calculator creates images very quickly, it has the potential to begin mathematical discussions in the classroom,

Math Talk

The NCTM Communication Standard (2000) suggested that students should organize their thinking using communication skills, such as the think aloud process for students based upon their own work. Sharing of mathematical thinking with peers and teachers is recommended. This recommendation would expand the practice of working on problems in isolation and receiving feedback from the teacher in the form of a graded paper to becoming verbal feedback from classmates and from the teacher. Opportunities to discuss their work with classmates and with the teacher during the time they are working on the problems should be provided. Discussion-based activities in conjunction with technology represent the changing emphasis of mathematics instruction as presented in a review of articles in the NCTM publications over the last three decades (Hallagan, Carlson, Finnegan, Nylén & Sochia, 2006). Discussion-based activities such as looking at the work of others analytically allow students to use the language of mathematics during the learning process (NCTM, 2000; Haas, 2006). These student discussions go beyond comparing solutions to discussing why a particular solution is correct or incorrect. Students would work together, using graphing calculator technology, to determine if particular solutions are correct. White and Gerson (2006) refer to graphing calculator technology as a pseudo-collaborator in the learning process where students interact with the graphing calculator and interact with other students as they work collaboratively on mathematics problems.

The communication between students can assist in creating a deeper mathematical understanding for students. As students work with the graphing calculator, they encounter opportunities where they can discuss what they observe on the calculator

screen with other students. Graphing calculator technology integration into mathematics instruction allows opportunities for students to develop deeper understanding by using the calculator to explore ideas, discuss the findings of others, and receive instantaneous feedback that can be used to confirm or refute a solution. These classroom activities support the NCTM (2000) technology principle that calls for student engagement with other students and with the teacher as they communicate about what is observed on the graphing calculator screen during mathematical investigations. According to the NCTM technology principle, students become actively involved in instruction when they have the opportunity to manipulate data and explore outcomes using the graphing calculator (NCTM, 2000). The graphing calculator technology allows students the opportunity to explore real life data that is often cumbersome and not easily manipulated by hand.

Real World Problem Solving

The NCTM Curriculum and Evaluation Standards (2000) and the Virginia Standards of Learning (2012) call for instruction to rise out of real world problem situations. Mathematical models and simulations permit students to observe a real world problem situation through a mathematical graph or formula. These real world problems move students from solving discrete mathematical skills to connecting these skills into a model or simulation creating a more meaningful learning environment for students.

Graphing calculator technology potentially could give students control over their learning environment as it aids in concrete imagery creation and pace of learning (Lopez 2001). This imagery could provide a learning environment rich in problem solving where the use of the graphing calculator technology allows problems to reflect real world mathematics applications. Real-life examples often produce graphs rich in data but are

too difficult to compute by hand. Lopez (2001) cited one example used in a professional development session for algebra teachers where parabolas were used to create the McDonald's restaurant logo on the graphing calculator screen. This activity could create an experience in the classroom where students could use the graphing calculator to explore the properties of parabolas while working with equations of parabolas to complete the task. As more lessons involve problem solving and real-life situations, students should have the advantage of using all available tools and resources such as graphing calculator technology.

Additionally, students can become actively involved in problem solving when using the graphing calculator to solve problems that are based in their world. Students have the opportunity to delve deeper into mathematical topics and visualize real-world mathematics modeled in the classroom with the aid of the graphing calculator. When real world examples, science experiments for instance, were used in conjunction with graphing calculators, students demonstrated better understanding of the concepts (Steele, 2006). When real world problems are used in classrooms, "technology can create environments for higher cognitive domains, for problem solving and conjecturing, to assure student success" (Bos, 2007, p.366). Additionally, technology can place more emphasis on conceptual understanding than on simply emphasizing the procedural steps in an algorithm. For example, when incorporating the graphing calculator as a data collection device, into an activity involving a bouncing ball experiment, the teacher was able to observe students using higher order thinking skills. Additionally, the teacher was able to incorporate deeper exploration into the lesson (Bowman, Koirala, Edmonds & Davis, 2000). This type of lesson moves students from working with equations of

parabolas using graph paper to looking at how the graph relates to the bouncing ball problem given a real world situation. The mathematical model of this real world situation is created using the graphing calculator as a mathematical tool.

Mathematical Tool

Once students have learned to perform calculations by hand, the graphing calculator can perform the calculations so instructional time can be spent discussing more in-depth mathematics content. Active involvement with the graphing calculator technology allows students to explore mathematical ideas, connect mathematics to real world ideas, and communicate ideas to peers and teachers (NCTM, 2000). Using graphing calculator skills during the learning process enables students to communicate mathematically about what they are learning (Waits & Demana, 1998). This is an opportunity for the graphing calculator to become a precursor to additional and more in-depth mathematics topics through concrete imagery and communication. The graphing calculator can be used as a tool for guided practice or as a tool for reinforcement of previously learned skills and concepts. While graphing calculator technology is important to classroom instruction, paper-and-pencil methods of instruction in algebra classrooms should not be abandoned. During instruction, students still need to practice new concepts in the traditional sense with paper-and-pencil (Vavilis & Vavilis, 2004; Waits & Demana, 2000). Waits and Demana (1998) suggested some steps for effective graphing calculator use in the classroom. First, the practice should be in pencil-and-paper format and graphing calculators should not be used at this point in the lesson. Second, graphing calculators should be used to correct or confirm results obtained through pencil-and-paper methods. Finally, the calculator should be used to solve

problems or equations, especially when pencil-and-paper methods are too time consuming. While the steps discussed above were delineated in 1998, they still hold true today in that state standards still require paper-and-pencil methods and the use of the graphing calculator is used as a tool (VDOE, 2012).

When is the most appropriate time to use the graphing calculator? Waits and Demana (1998) refer to the calculator as a better “tool” than paper and pencil in some instances. They related the role of paper-and-pencil strategies changing resulting from graphing calculator technology; however, they still indicate that paper-and-pencil methods have an important role in the algebra curriculum. The advantage of graphing calculator use as a tool for students may result in a deeper understanding of mathematics concepts and mathematical ideas becoming stronger in students by using the graphing calculator as a tool in the classroom. White and Gerson (2006) defined the graphing calculator as a pseudo-collaborator where students used the graphing calculators as a “partner” to explore ideas and investigate representations. This is an instance where the exploration with the graphing calculator may be used in collaboration with instructional methods such as discovery learning. Technology may play an important role in changing the emphasis in the classroom to student-centered conjectures where the activities students experience are discussion-based (Hallagan, Carlson, Finnegan, Nylén, & Sochia, 2006). When teachers select open-ended and exploration-based tasks, students of varied ability levels can become the center of the learning environment and achieve discoveries that would not have occurred without the use of the graphing calculator (Chamblee, Slough & Wunsch, 2008).

Equity Tool

The NCTM Equity Principle states “excellence in mathematics education requires equity—high expectations and strong support for all students” (NCTM, 2000, p.12). “Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students” (NCTM, 2000, p.12). Equity is an integral part of all NCTM Standards and Principles for School Mathematics. This principle also states that technology can and should be used as a tool to achieve equity in the classroom and should be accessible to all students (NCTM, 2000). The graphing calculator can “provide a vehicle for all students to engage in doing real mathematics” (Waits & Demana, 1998). This vehicle may create a common starting point for students, be used as an exploration tool, and be used as a mathematical instructional tool.

Discovery Learning

Discovery learning is a method of instruction where active, hands-on learning is present in the learning environment. Castronova (2002) discussed the three main attributes of discovery learning. The first attribute involves student exploration through problem solving and creating understanding through generalizations. The second attribute focuses on students taking ownership of their own learning since the students set the pace for learning experiences. The third attribute states that students build new knowledge based on prior knowledge. These components may also lead to higher student motivation in learning. The attributes of discovery learning may lead to engaging educational experiences for students. Through technology advancements, discovery learning may become easier since students have instant access to digital worlds through

computers and through the Internet. This allows students to be members of larger learning communities that reach beyond the confines of the classroom (Castronova, 2002).

Within the classroom setting, what does discovery learning look like? Students are engaged actively and usually completing hands-on activities. Emphasis is placed on the processes students use during the lesson. Feedback is necessary for students as part of the learning process. The learning may also include failure since learning takes place from experiencing failure as well as success (Castronova, 2002). Within the classroom setting, when students create their own understanding of concepts, they tend to be more motivated to learn (White-Clark, DiCarlo, & Gilchrist, 2008). White-Clark et al. (2008) also noted that exposure to viewing equations entered into the graphing calculator before encountering the chapter on graphing allowed students to see function representations on a regular basis. As students continue to see the representation of functions, they may see patterns and begin to create understanding by drawing on previous learning experiences.

Graphing calculator use has been linked to students creating their own understanding of mathematics. Herrera and Owens cited Jerome Bruner stating that by using well-chosen problems “students can do investigation and discovery rather than being told the relevant concepts and expected to practice skills” (2001, p. 85). As discussed earlier, well-chosen problems may provide students the opportunity to create mathematical models of real life situations. In this instance, students create understanding with a combination of graphing calculator use, pencil-and-paper techniques, explorations, teacher guidance, and their own prior knowledge.

The graphing calculator provides a wealth of potential advantages that may move

students to deeper understandings through experiences focusing on higher level thinking skills. While advantages for deeper understanding are possible, it is necessary to minimize potential disadvantages and potential misconceptions.

Potential Disadvantages/Misconceptions

The advantages of graphing calculator use tend to be more visible than the potential disadvantages or misconceptions of graphing calculator use during mathematics instruction. There are potential problems or misconceptions that may result from graphing calculator use in the classroom and caution should be taken so that potential disadvantages do not minimize the effects of the potential advantages. Potential disadvantages may arise in the areas of curriculum integration, calculator mechanics, teacher ability effectively to use the graphing calculator, and the potential deterioration of basic skills.

Curriculum Integration

The instructional practices teachers select aid in the successful or unsuccessful integration of graphing calculator technology (Kersaint, 2007). Hennessy et al. (2001) noted that the use of graphing calculator technology should be firmly embedded within and inseparable from the mathematics activity. A combination of graphing calculators and curricular materials must be paired so it guides students to examine appropriate mathematical topics (Lapp & Cyrus, 2000). If a concerted effort is not made to connect the calculator output to the mathematics, effectiveness diminishes. The graphing calculator should not be used as a tool isolated from the mathematics, rather, purposeful attention should be given to connecting the graphing calculator output to curriculum topics.

Teachers and curriculum leaders need to determine where graphing calculator use best fits into the curriculum (Lee, 2007; Kersaint, 2007). If curricular fit is not purposeful, it can undermine student learning resulting in mathematical misconceptions. The following ideas may lead to potential disadvantages of calculator use. When calculators and curriculum are designed specifically to be used together, conceptual understanding is improved (Heller, 2005). To build conceptual understanding, the curriculum should stress communication skills in conjunction with graphing calculator skills. The use of student communication and graphing calculator activities can aid in developing mathematical concepts (Lapp & Cyrus, 2000). The focused selection of mathematical tasks assigned to students may avert potential misconceptions. Teachers must select appropriate tasks to assign to students to take advantage of the technology (NCTM, 2000). If teachers frame their instruction by using procedural steps to compute with an algorithm or complete all mathematics tasks with graphing calculators, students will be missing pieces of conceptual understanding. Lee (2007) found:

teachers who view mathematics as a dynamic field, who emphasize understanding concepts as opposed to mechanical procedures, and who prefer the construction and understanding of the concept over memorization of procedures will try to use technology to help their students construct their own knowledge and understanding of mathematics. (pp. 126-127)

Lopez (2001) indicated that mathematical concepts naturally emerge when patterns arise out of using graphing calculators with problem solving requiring drawings or models. These advantages are lost if the teacher is unable to integrate and teach with graphing calculator technology in an effective manner.

Teachers need training to integrate graphing calculator technology into their lessons effectively. If effective and sustained professional development is not provided, teachers may struggle to make the connections between graphing calculator use and the mathematics topics that will be taught. Effective professional development for teachers needs to move beyond “show and tell” workshops and focus more on how to teach specific skills in the curriculum using the graphing calculator while also being cognizant of the limitations of the calculator (Chamblee, Slough & Wunsch, 2008). If professional development is not in-depth, sustained and focused on curriculum integration, teachers will not achieve the level of proficiency necessary to help students make the mathematical connections during instruction. Mathematical connections resulting from the graphing calculator experiences can lead to greater conceptual understanding for students especially when the graphing calculator technology is embedded in the mathematics curriculum and teachers are knowledgeable of the mechanics of the graphing calculator.

Calculator Mechanics.

The advantages of graphing calculator use may provide enhanced student learning and understanding. The students have the capabilities of viewing data in real time and in a concrete manner. These mathematical models provide concrete images of abstract topics. While students should possess the ability to interpret the output on the calculator screen, they also need the ability to input the mathematical information correctly. The entering of data requires knowledge of the mechanics of the graphing calculator. As technology is updated and teachers learn the new capabilities of the calculator, care should be taken to discuss the limitations of the technology (Waits & Demana, 2001;

Cavanagh & Mitchelmore, 2003). One limitation comes from misconceptions that may result from simply setting up the viewing window. The viewing window is where the graphs or other calculator output will appear. A thorough explanation of this component often is omitted in professional development sessions. Cavanagh and Mitchelmore (2003) conducted a 2 day workshop with 12 teachers who had minimal experience with the graphing calculator technology. They found teachers experienced troubles with setting up the scale for the graphs as well as the interpretations of the graphs. They reported that teachers, and in turn students, may fall victim to some of the following misconceptions.

Calculator-created misconceptions may occur when using the graphing calculator in the following ways: scales and zoom functions, accuracy and approximation, interpreting decimal coordinates when tracing, understanding the viewing window, and problems understanding the effect of pixels. Cavanagh and Mitchelmore (2003) observed that students experienced difficulties working with the calculator when the scales were unequal. The scale setup of the calculator marks the intervals of the graph. Each mark on the axis may represent 1, 2, 10, or any number as determined by the user. Students experienced difficulties interpreting graphs when the graphs had different scale settings on the x-axis and y-axis. To investigate the graphs in more or less detail, the zoom function of the calculator is available. Students had the ability to use the zoom function, which changes the viewing setting, but could not explain the effect using the zoom function has on the graph (Cavanagh & Mitchelmore, 2003). In a study with 18 high school students, some students stated that if the Xscl was finer, the cursor would be more precise to the actual value of the point when tracing (Ward, 1998). In actuality, the Xscl

and Yscl will change the viewing window, but it will not affect the trace function. The screen captures below were created to show how the same function has three different appearances depending on the window setting as shown in Figure 2, Figure 3, and Figure 4.

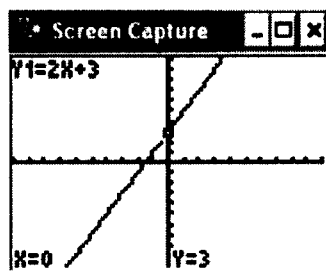


Figure 2. Standard Window

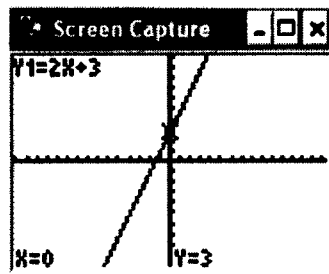


Figure 3. Square Window

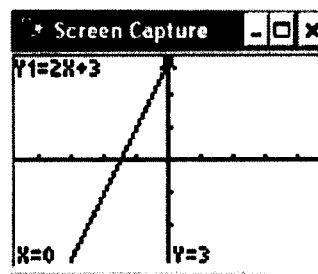


Figure 4. Decimal Window

Students may incorrectly identify the graphs as not representing the same function if they view the function in an unfamiliar window. The window setting adjusts the number of pixels visible in the graph and thus changes the portion of the graph that is seen by the user. The purpose of these window settings is to create a variety of views of the graph. The trace function of the calculator identifies some of the points of the function that is graphed. The standard window sets the x-axis and y-axis with a range of -10 to 10 with a scale of 1 on each axis. The square window has setting of -15.16... to 15.16... on the x-axis and a range of -10 to 10 on the y-axis with a scale of 1 on each axis. The third window is the decimal window with an x-axis setting of -4.7 to 4.7 with a scale of one and the y-axis setting range is -3.1 to 3.1 with a scale of one.

When a function is graphed in the standard window and the trace component is used, the x and y values are decimal values and they are not considered “friendly,” meaning numbers may be decimals with many digits. When the same function is graphed in the square window, the coordinates are still not “friendly” numbers but more of the

graph is visible in the viewing window. The decimal window will display coordinates as decimals to the tenths place. The calculator will “sample only a discrete number of function values and connect the associated points” (Waits & Demana, 2001). Students may not realize that there are values “in between” the points they trace on the screen since many students often based their answers on only the highlighted pixels. Waits and Demana (2001) also noted that errors in understanding could occur when a calculator, a discrete device that samples only certain points on the line, is used to model a continuous function with infinite points. To minimize misconceptions a thorough understanding of the calculator is required by both students and teachers.

Teacher Ability

The ability of teachers to incorporate graphing calculators into instruction may influence the learning students take away from lessons using graphing calculators. After sustained professional development teachers moved toward looking for mathematical applications with the graphing calculator (Chamblee et al., 2008; North Central Regional Educational Laboratory, 1994). If teachers have not reached this level, the students may not gain the optimal benefits of the graphing calculator capabilities during instruction.

One area where misconceptions can occur is the ability of the teacher to use and teach with the graphing calculator. In a study involving three teachers who viewed mathematics as dynamic rather than dormant, Lee (2007) called for time to be allowed for teachers to learn the graphing calculator capabilities and limitations and become comfortable with its use during classroom instruction. Kersaint (2007) found that pre-service teachers focused on using the graphing calculator rather than focusing on connecting the graphing calculator to the mathematics being taught. These findings were

determined from a course focusing on technology, pedagogy, and content knowledge (TPCK). As students use the calculator to model and investigate functions, teachers should design instruction so students do not become isolated when using the graphing calculator. In a study focusing on graphing calculator use in two pre-calculus classes, it was suggested that the calculator can become a “black box” of sorts, where the students are working with the calculator as a private device and not sharing their work or their thinking with other students or the teacher (Doerr & Zangor, 2000). Teachers may incorporate overhead projection screens or smartboards as a way for students to share their work with others in the classroom setting.

Classroom use of the calculator can be used to extend or support instruction. However, there is a fine line between support through scaffolding and dependence when using the graphing calculator (Hennessy et al., 2001). Students should exhibit confidence in their work whether they determine solutions using the graphing calculator or using paper-and-pencil methods. Again, caution should be exercised so the calculator technology does not replace student understandings but rather provides scaffolding to higher levels of understanding. Increased student understanding may result if students have the opportunity to explore beyond the confines of the classroom. If teachers do not encourage exploration, the advantage of graphing calculator use may limit potential extensions and explorations in the learning process. The graphing calculator may assist students to use mathematical modeling as well as exploration to make sense of quantitative information if teachers are effective users of technology during instruction (Shaffer & Kaput, 1999). In addition, they suggest that the graphing calculator is a computational media that manifests as a fifth stage of cognitive development where there

is potential for long-term effects. As students experience the technology infused learning environment, concerns over the perceived deterioration of basic skills has arisen.

Perceived Deterioration of Basic Skills

There is a perception that calculator use has contributed to a decline in student abilities to perform basic skills. Pomerantz (1997) discussed myths regarding the use of calculators and the decline of numeracy. Numeracy is said to be lost since students are using calculators to perform numerical computations. While tedious computational skills may be performed with the calculator, it is important for students to possess estimation skills to verify calculator output. The calculator may replace rote drill that requires a use of minimal higher-level thinking skills. Specifically, the calculator can assist with basic computations, which permits additional time for higher-level problem solving. The level of problem solving required of students has become more rigorous and more prominent in state learning objectives (VDOE, 2012). Many opponents to calculator use claim that learning took place without calculators when they learned mathematics. The teaching of mathematics was primarily drill and practice using many sheets of paper to practice using mathematical algorithms and basic computations (Pomerantz, 1997). Calculators can expand instructional time and permit mathematics to move beyond basic computing and move toward rich problem solving. Waits and Demana (2000) cited a British study conducted between 1986 and 1992 by Hembre and Dessart that found students who used graphing calculators and were never taught paper-and-pencil skills developed their own skills over the years. This suggests that the use of the graphing calculator does not degrade students' basic skills but may aid in strengthening these skills. However, the question arises as to the optimal blending and sequencing of calculator use as an integral

part of instruction.

Conclusions

The introduction of the graphing calculator has allowed more student access to technology (Waits & Demana, 1998). Purposeful steps should be considered so the misconceptions created by graphing calculator use do not inhibit the learning outcomes. Teacher comfort level with graphing calculator use and understanding of graphing calculator limitations would appear to be a key factor leading to effective use in the classroom. Attention to the purposeful integration of graphing calculator technology into the curriculum and classroom instruction may increase conceptual understanding. Students benefit most when graphing calculators are incorporated seamlessly into the curriculum and into classroom instruction. Benefits such as student ability to produce identical graphs quickly through calculator use may result in additional class time that can be spent on discussion and concept development (Lee, 2007).

The calculator may also be used as a “partner in exploration” where students use calculators as an integral part of instruction (White & Gerson, 2006). White and Gerson (2006) further claimed that students could engage in more discovery learning with the use of graphing calculator technology. Effective integration of graphing calculator technology permits students to become actively involved with their learning experiences while using the graphing calculator. Calculator use permits more student-centered conjectures and discussion-based activities to be included during instruction (Hallagan et al., 2006).

Studies have also shown that instruction may be hampered by ineffective use of the graphing calculator (Cavanagh & Mitchelmore, 2003; Ward 1998). Misconceptions

may arise when classroom teachers have gaps in their knowledge of the specifics of the graphing calculators. Focused and sustained professional development opportunities for in-service teachers may assist in filling the gaps in understanding involving graphing calculator use and provide richer learning experiences for students.

When graphing calculator technology is used in the classroom, it has been shown to be a tool for closing the achievement gap and bringing equity to the classroom (Nzuki & Masingila, 2006). Before graphing calculators became an affordable tool for teaching algebra, schools had to rely on computers for graphing capabilities, leaving its use mainly to affluent school divisions. As graphing calculators became available, more students gained access to technology-integrated tools during instruction.

When graphing calculator technology first entered the classroom as a tool for learning, it was one of the few new and exciting mathematics tools for students. In the years since its introduction into the classroom, new technologies have increased exponentially providing increased graphics and capabilities. The graphing calculator is still a powerful tool for mathematics students to use during instruction. For achievement of the optimal instructional advantages, teachers must be users who understand the potential advantages and potential misconceptions of calculator capabilities. With this knowledge and understanding, the teacher can then guide the students through instruction with a tool that may assist in increased student learning.

Student learning is the ultimate outcome of instruction. Teachers have a variety of tools available to use during instruction. The available technology tools are developing at a very rapid pace and keeping up with the changes presents a challenge. As one of the earlier technologies in mathematics, the graphing calculator is still a viable

tool for algebra instruction. Research is available that focuses on the use of graphing calculators to teach specific algebraic skills; however, there is limited research available that focuses on the use of graphing calculator technology paired with discovery learning. The Final Report of the National Mathematics Advisory Panel calls for high quality research on particular uses of the graphing calculator targeting computation, problem solving, and conceptual understanding (National Mathematics Advisory Panel, 2008).

Chapter 3: Research Design and Methodology

To assess the effectiveness of two different instructional approaches to teaching slope to average/struggling and advanced learners, a research project was undertaken during the 2011–2012 school year. The study was an investigation of the instructional strategy of discovery learning with the graphing calculator used as the learning tool incorporated into the learning activity. The study was an investigation of whether a discovery learning activity completed before direct instruction takes place resulted in higher student achievement on a common final assessment than those students receiving direct instruction followed by graphing calculator reinforcement. The algebraic concept of slope provided an opportunity for discovery learning by using the graphing calculator to explore the relationship between the equation of the line and the graph of the line. Using the discovery lesson, students described the graph of the line when given the equation of the line. Additionally, students explained how a change in one component of the equation affected the graph of the line.

This research study was completed in three high schools and the two middle schools that are feeder schools to the three high schools. One Algebra I teacher from each school was selected and two of their classes were the focus of the study, one using each approach. This study was an investigation of the following research questions:

1. Is there is a difference in achievement of average/struggling learners in high school Algebra I when taught by development of the concept of slope through graphing calculator technology and discovery learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing

activities using graphing calculator technology?

2. Is there is a difference in achievement of advanced learners in middle school Algebra I when taught by development of the concept of slope through graphing calculator technology and discovery learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing activities using graphing calculator technology?
3. Is there a difference in achievement between the middle school Algebra I students, the advanced learners, and high school Algebra I students, the average/struggling learners, when taught by development of the concept of slope through graphing calculator technology and discovery learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing activities using graphing calculator technology?

Population and Sample

The target population of this study was average/struggling students enrolled in high school Algebra I and advanced students enrolled in middle school Algebra I. The average/struggling learners are students who take Algebra I for the first time in high school. These students have completed Math 7 and math eight in middle school. The majority of these students were enrolled in grade nine. The advanced learners studied in this research project were middle school students enrolled in Algebra I. These students have completed math six and/or Math 7. All students were enrolled in Algebra I classes that contained the same curricular content taught from the same pacing guide sequence.

The sample for this study was drawn from three high schools and two middle schools from a school division located in the southeastern part of Virginia. The three high schools and two middle schools within the division were fully accredited based on Virginia requirements. Although all of the schools were accredited, there was still a gap in achievement between minority students and majority students. Additionally, students with disabilities were also part of the gap in achievement. To narrow the gap in achievement, the school division was incorporating professional development to support instructional strategies that have the potential to increase student achievement. The sample classrooms were similar in demographics to the school division demographics. The school division's enrollment was approximately 15,000 total students in grades K-12. The demographics of the division included 68% African American students and 22% Caucasian students (VDOE, 2011). Approximately 16% of the city population lived below the poverty level at the time of the study (infoplease, 2012).

The 10 sample classes for this study were taught by 5 Algebra I teachers. One teacher from each of the two middle schools and one teacher from each of the three high schools were selected. Each of the teachers successfully completed the school division's Calculator Competency Assessment. This division-created assessment tool indicated that the teacher had at least a basic level of competency using the graphing calculator. The school division required each middle school teacher and high school teacher to complete the calculator competencies assessment. This assessment included the skills used to teach the secondary mathematics curriculum, including basic calculator set-up, operations and functions, and graphing capabilities. The complete list of skills contained in the Graphing Calculator Competencies Assessment is located in Appendix A.

A professional development session was held and the purpose of the study was shared with the teachers. Each teacher was provided with all materials needed for the treatment activity and for the final assessment. The treatment activity was discussed including how the activity should be facilitated. This activity was self-directed in nature as step-by-step procedures were provided for students. The teachers assumed the role of facilitator as the students completed the activity. Specifically, the teacher should not tell students the answers but rather redirect the students to the activity procedures. Teachers were also instructed to conduct direct instruction in the same manner for both the control group and the experimental group. The professional development took place approximately a week before the teachers began instruction on slope of a line.

Two classes from each teacher's schedule participated in the study. The student sample was a cluster sampling as the classes were defined by the school's master schedule. Random sampling was used to determine which classes were the experimental groups. The number of students enrolled in each class was between 20 and 25 in both the middle schools and the high schools. All Algebra I classes involved in the study were similar in racial and gender make-up (See Table 1).

The coursework prior to enrolling in Algebra I differed between the groups. The middle school students involved in the study moved from grade six mathematics to Algebra I or moved from grade seven mathematics to Algebra I. Historically, this group had reported a pass rate in the upper nineties on the Virginia Standards of Learning End-of-Course Assessment. The students enrolled in Algebra I at the high school level completed both grade seven mathematics and grade eight mathematics in middle school.

Table 1

Student Demographics

High School Demographics					
	African American	White	Other	Male	Female
Experimental	33	4	8	25	24
Control	33	7	1	22	19
Middle School Demographics					
	African American	White	Other	Male	Female
Experimental	31	8	1	20	20
Control	34	14	1	26	23

The study was conducted during the normal course of instruction during the school day and in alignment with the division's pacing guide for instruction. To determine the level of achievement before the treatment, the results from the middle school Benchmark One assessment and the high school midterm exam were analyzed. Both assessments included the same objectives and were created at the division level. The results of these assessments were used to determine if the level of content achievement of the control groups and the experimental groups were equivalent at the beginning of this study (see Appendices B and C).

Data Collection and Instrumentation

Teachers were selected randomly from a relatively small pool of teachers based upon the sections of Algebra I contained in their teaching schedule. The teachers who were eligible as participants in the study, had to teach at least two sections of Algebra I. The experimental group from each teacher's schedule was randomly assigned. The

treatment was completed during the unit on slope, SOL Standard A.6 (VDOE, 2012).

This standard focuses on recognizing that equations of the form $y = mx + b$ and $Ax + By = C$ are equations of lines. Students are also expected to write an equation of a line when given the graph of a line or given points that lie on the line. The control group received direct instruction followed by graphing calculator use to reinforce the instruction, which was the most common method of instructional delivery in this school division. The teachers involved in the study had similar teaching styles. Direct instruction was the main form of content delivery. The researcher observed these similarities during classroom visits prior to the study. It is noted that all mathematics classrooms were equipped with classroom sets of graphing calculators, so students had access to the graphing calculator on a daily basis.

The experimental group completed a discovery-based activity from the VDOE Enhanced Scope and Sequence (2001) that utilized the graphing calculator as part of the discovery learning lesson, which covered the same learning objectives as the control group. This *Transformation Investigation* activity directed students to graph the equation $y = x$ using the graphing calculator. Then students sketched the graph on a coordinate plane. Students graphed $y = x$, $y = x + 2$, $y = x + 4$, and $x + 6$ and sketched them on the same coordinate plane. Next, the students analyzed how the lines were alike, where they crossed the y-axis, and analyzed what happened to the parent graph $y = x$ when the constant was added to the equation. Students next graphed and analyzed equations such as $y = 2(x+2)$ and $y = 2(x + 4)$, where the slope of the line was changed.

The researcher provided professional development for the participating

teachers focusing on incorporating the discovery learning activity into instruction. The discovery learning activity was provided to the teachers during the professional development meeting (see Appendix E). After the students completed the discovery learning activity, direct instruction followed. Students in the experimental group received the same instruction the students in the control group received, only after the discovery activity was completed. The control groups and experimental groups in the middle schools and the high schools utilized 90-minute block schedules so all classes received 90 minutes of mathematics instruction daily. Following instruction, the control groups and the experimental groups were administered the same assessment (see Appendix F).

The assessment administered after instruction was similar to the format of the Virginia Department of Education Released Test Items (VDOE, 2010). In this assessment, students graphed and sketched linear equations, completed short answer questions, and completed questions that required students to create generalizations. The assessment included the following objectives:

- Find the slope of the line, given the equation of a linear function in slope-intercept form.
- Find the slope of the line, given the equation of a linear function in standard form.
- Find the slope of a line, given the graph of a line.
- Find the slope of a line, given the x-intercept and y-intercept
- Write an equation of a line when given two points on the line whose coordinates are integers.
- Write an equation of a line when given the slope and a point on the line whose

coordinates are integers.

- Write an equation of a line when given the graph of the line.

The Table of Specifications details the design of the assessment (see Appendix G). The assessment answers that students provided were numerical, so the answers were scored as either correct or incorrect. The answers did not require any subjective decisions on the part of the assessment scorer. The study assessment questions were modeled after state assessment questions thus maintaining alignment of the treatment activity to the tested objectives.

Data Analysis

The data obtained from this study were compared using an independent samples *t*-test. This test allowed comparison of the means of the two separate groups where the groups had approximately equal variances. Equivalence in prior achievement of the middle school groups and high school groups was determined by analyzing results of common assessments using an independent samples *t*-test. The data were obtained from the middle school Benchmark One assessment and the high school midterm examination. Both assessments were created at the school division level and included the same instructional objectives.

During the course of this study, the experimental group used a discovery learning activity, while the control group received direct instruction. Both groups used the graphing calculator during instruction. Experimental groups and control groups completed a common assessment and the assessment scores were analyzed using a two group post-test design. An independent samples *t*-test was used to compare the assessment scores of both groups. The first analysis compared the

assessment results of the high school control groups and the high school experimental groups. The second analysis compared the assessment scores of the middle school control groups and the middle school experimental groups. The third analysis compared the assessment scores from all control groups to all experimental groups. Box-and-Whisker Plots were used in conjunction with the independent samples *t*-tests to analyze the dispersion of the data for each of the focus groups.

A final analysis was conducted on individual items from the common assessment that was administered as the final part of the study. A pairwise comparison was conducted to obtain data on each assessment item. The pairwise comparison allowed comparisons between both treatment groups and both experimental groups for each assessment question.

Limitations of the Study

As the results of this study were analyzed, several limitations were determined.

1. Because of the master scheduling of Algebra I classes, the number of classes available for inclusion in the study was limited. Limitations for teacher selection were also noted, as it was necessary for teachers to have two Algebra I classes included in their teaching schedule.
2. Because of student absences when the treatment activity was completed in class, the treatment activity was not completed by a small number of students in both the middle school groups and the high school groups. The assessment data for these students could not be used in the analysis. A small number of assessment scores from both groups could not be used in the analysis. Because of absences on the day the assessment was administered. It is noted that high absences occurred

with the high school groups. Absences could also have occurred during the instruction that followed, thus affecting achievement on the assessment.

3. This study consisted of one discovery learning activity that was used to introduce the concept of slope. The use of one activity may not have provided ample experiences for the students.
4. The discovery learning professional development for teachers was a one-time session. The professional development discussed the discovery learning procedures, the lesson plan for the treatment activity was shared, and the facilitation of discovery learning lesson was discussed. A multi-part ongoing professional development may have strengthened the facilitation of the treatment activity.
5. The discovery learning experience may have been a new process for the students. Experiences in mathematics classes tend to include finding the solution to a mathematics problem after receiving instruction involving similar problems. Drawing individual conclusions may not be a common practice for students in mathematics classes.

Chapter 4: Findings

This study was an investigation of two approaches for teaching slope of a line in Algebra I classes. Interest in this topic grew from the Algebra for All initiative that has resulted in more students completing Algebra I in middle school. The students who complete Algebra I in middle school are learners who are more advanced while the students who enroll in Algebra I in high school are average/struggling learners. This study investigated the use of a discovery learning approach in addition to direct instruction to teach slope of a line. The same instructional approaches were used with advanced learners as well as with average/struggling learners.

Population and Sample

This study took place within an urban public school setting situated in a city where approximately 16% of the city population lived below the poverty level at the time of the study (infoplease, 2011). The school division had approximately 15,000 students enrolled in grades K-12. The demographic composition of the division included 68% African-American and 22% Caucasian (VDOE, 2011). At the secondary level, the school division had three fully accredited middle schools and three fully accredited high schools. One Algebra I teacher was selected from two of the middle schools and three of the high schools. Two classes from each teacher's schedule were selected to be the focus of the study through cluster sampling. The experimental group and control group were selected randomly from the two selected classes. The teachers involved in the study had teaching experience in mathematics that ranged between 5 and 15 years. As part of their teaching experience, each teacher demonstrated graphing calculator competency as determined by the school division's Graphing Calculator Competencies Assessment. The teachers who

participated in this study utilized these graphing calculator skills during instruction and students used the graphing calculator as a routine part of instruction. These graphing calculator practices were common in each high school and each middle school.

The sample size for the middle school groups consisted of 89 students taught by 2 teachers. The sample size for the high school groups consisted of 78 students taught by 3 teachers. The student participant information is shown in Table 2.

Table 2

Student Participants

School	Experimental n	Control n
Middle School A	24	24
Middle School B	16	25
High School A	18	14
High School B	11	14
High School C	8	13

After determining the initial equivalence of groups, this study is focused on two instructional approaches for teaching slope of a line. One approach consisted of completing a discovery-based activity utilizing the graphing calculator before students received formal instruction on slope of a line. The second approach consisted of formal instruction that was delivered through direct instruction alone. The students in the experimental classes completed the discovery-based activity before direct instruction took place. Both groups completed the final assessment when the instruction on slope of a line was completed. These assessment results were compiled and analyzed.

Tests and Data Collection

The academic achievement of the experimental and control groups were compared prior to the study by conducting an independent samples *t*-test using benchmark assessment scores for the middle school students and midterm exam scores for the high school students. A frequency table reflecting individual scores is located in Appendix D. These two assessments covered the same Algebra I objectives since the high schools utilized a 4-by-4 block schedule and completed instruction in 18 weeks and the middle schools utilized a traditional 36-week instructional schedule. The benchmark assessment data for the middle school participants was calculated ($M = 70.79$, $SD = 11.28$) and the midterm exam data for the high school participants was calculated ($M = 53.94$, $SD = 11.18$) $p < .001$. There were no significant differences in the scores of both groups $t(165) = -1.296$, $p = .197$. The benchmark assessment data and the midterm exam data are shown in Table 3 and Table 4. Appendix D is a frequency table of the middle school benchmark scores and high school midterm exam scores.

Table 3

High School Pre-treatment Assessment

	N	Mean	Std. Deviation	Std. Error Mean
Exam high school	69	53.94	11.183	1.346

High School Pre-treatment Assessment

Test Value = 0						
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Exam High school	40.068	68	.000	53.942	51.26	56.63

Table 4

Middle School Pre-treatment Assessment

	N	Mean	Std. Deviation	Std. Error Mean
Benchmark	89	70.79	11.279	1.196
Middle School				

Middle School Pre-treatment Assessment						
Test Value = 0						
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Benchmark	59.207	88	.000	70.787	68.41	73.16

analysis of the high school pre-study assessments was conducted $t(78) = 0.150$, $p = 0.180$. The high school experimental group had one outlier score of 25% and the high school control group had one outlier score of 24%. The mean for the high school experimental group, which consisted of 37 students, was 54.35% with a standard deviation of 13.265. The mean for the high school treatment group, which consisted of 41 students, was 53.95% with a standard deviation of 10.159. The independent samples t -test analysis indicated that there were no significant differences between the two groups. Table 5 is an illustration of the data.

Table 5

Midterm Exam Scores for High School Experimental and Control Groups

		N	Mean	Std. Deviation	Std. Error Mean
Exam high	Experimental	37	54.35	13.265	2.181
school	Control	41	53.95	10.159	1.587

The high school midterm exam data was also analyzed in terms of score dispersion. The range of the experimental group was 62 with one outlier of 24% included. The range calculated excluding the outlier was 50. The range of the control group was 51, which included one outlier of 24%. The range of the control group calculated with the outlier excluded was 35. The experimental group had wider dispersion of data than the control group. The median scores for each group differed by two. The experimental group had a median of 50 and the control group had a median of 49.5. The top half of the scores, the values above the median, of the experimental group were more widely dispersed than the scores of the control group. The top 25% of the data for the experimental group ranged from 64% to 86% while the top 25% of the control group ranged from 63% to 75%. The data for each group is shown using box-and-whisker plots in Figure 5.

An analysis of the pre-study midterm assessment was conducted for the middle school participants $t(89) = -0.121, p = 0.759$. The middle school experimental group from both middle schools consisted of 40 students ($M=70.62, SD = 12.055$) and the control groups from both middle schools consisted of 49 students ($M = 70.92, SD = 10.729$). The analysis of the groups showed no significant differences in the study assessment. It is noted that the experimental group did have one outlier score of 24%.

Table 6 summarizes the analysis of the data for the middle school groups.

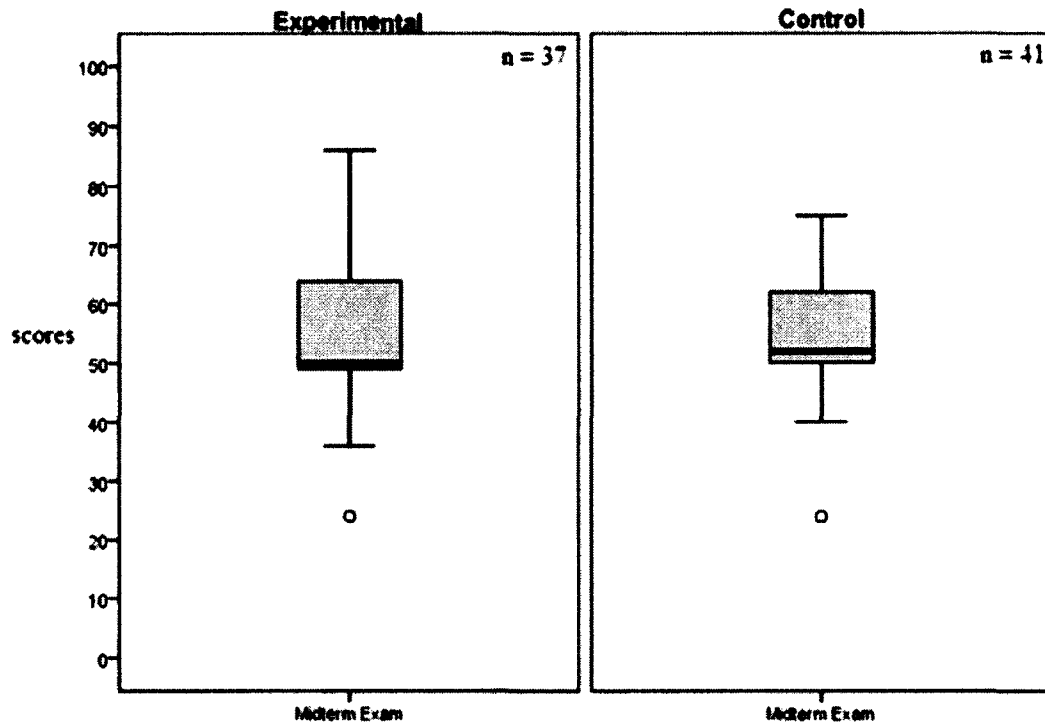


Figure 5. High School Midterm Exam Scores – Pre-treatment Assessment

Table 6

Benchmark Assessment Scores for Middle School Experimental and Control Groups

		N	Mean	Std. Deviation	Std. Error Mean
Benchmark	Experimental	40	70.62	12.055	1.906
middle school	Control	49	70.92	10.729	1.533

The analysis of the dispersion of the data indicated that there was little difference between the experimental and control groups. The experimental group had one outlier of 24%. The range of the data for the experimental group was 67 when determined including the outlier and 39 when calculated without the outlier. The range of the data

for the control group was 42. The data for the experimental and control groups were similar for the top 50% of the data for both groups. The median for the experimental group was 70.5% and 72% for the control group. The lower half of the data for the control group showed more of a spread than the experimental group. The minimum value for the control group was 48% as compared to 52% in the experimental group. The data for each group is shown in the box-and-whisker plot in Figure 6.

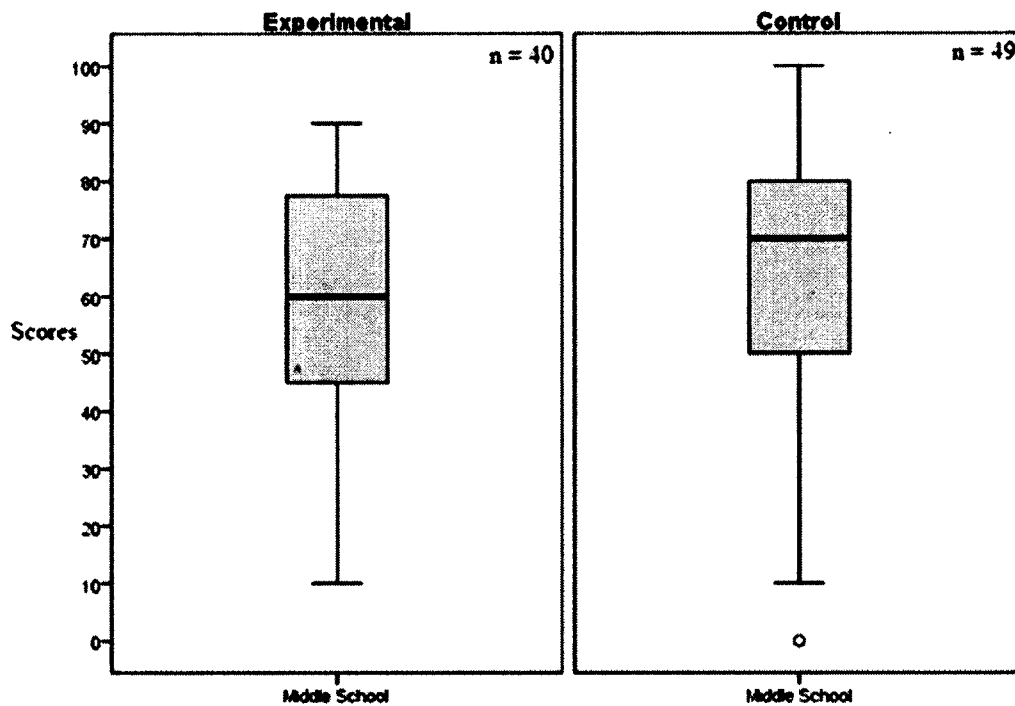


Figure 6. Middle School Benchmark Assessment 1 Pre-treatment Assessment

Each of the experimental groups completed the discovery learning activity prior to receiving formal instruction on slope of a line (see Appendix E). The activity was obtained from Virginia Department of Education created lesson plans from the Enhanced Scope and Sequence (VDOE, 2011). This activity allowed students to investigate slope of a line content using the graphing calculator as an exploration device. Students graphed the given equation of a line, the parent function, and then graphed several transformations

of the line. Throughout the activity, students made observations based on patterning they observed while completing the activity. Upon completion of the treatment activity, students received instruction in the same manner as the control groups. The content was delivered through direct instruction following the Madeline Hunter direct instruction format (Burns, 2005). The division lesson plan format requires all components of the Madeline Hunter format be included in daily plans. The instructional delivery began with the presentation of a warm-up problem followed by several problems that were presented to the whole class. The teacher modeled the problems and similar problems were assigned as guided practice. The lesson concluded with independent practice of additional problems. Graphing calculators were used as a tool during the instructional process.

Instruction on slope of a line lasted approximately four weeks and all students completed an assessment based on slope of a line objectives at the end of this four-week period. This assessment was developed by modeling questions from released test items from the Algebra I state assessment (see Appendix F). Released test items are questions that were used on End-of-Course Assessments from previous years' test administrations. The assessment items are available on the Virginia Department of Education website; however, no released items were used during class instruction (VDOE, 2012). Graphing calculator use was permitted during the assessment. The data from the assessment was analyzed in terms of the research questions posed in this study. The individual assessment scores are presented in Appendix H

Research Questions

The study was undertaken with the focus on the research questions that follow.

1. Is there is a difference in achievement of average/struggling learners in high school Algebra I when taught by development of the concept of slope through graphing calculator technology and discovery learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing activities using graphing calculator technology?

The assessment scores for the high school experimental groups and control groups were analyzed using an independent samples *t*-test. The analysis indicated no significant differences between the treatment group ($M = 44.59$, $SD = 32.795$) and the control group ($M = 45.85$, $SD = 26.644$) conditions $t(76) = -0.187$, $p = 0.852$. The results of the statistical data for the groups are presented in Table 7. The Levene's Test for Equality of Variances (See Table 8) was conducted to insure the assumptions of the *t*-test. The small sample size necessitated the Levene's Test. There were no significant differences between the control group and the experimental group indicating that the inclusion of the discovery learning activity did not increase the achievement of understanding slope of a line for the average/struggling learner.

Table 7

Statistics for Assessment Results for High School Experimental and Treatment Groups

		N	Mean	Std. Deviation	Std. Error Mean
High	Experimental	37	44.59	32.795	5.391
school	Control	41	45.85	26.644	4.161

Table 8

Independent Samples Test High School

		Levene's Test for Equality of Variances		<i>t</i> -test for Equality of Means						
		F	Sig.	t	df	Sig. (2- tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
High	Equal									
school	variances assumed	2.802	.098	-.187	76	.852	-1.259	6.738	-14.680	12.162
	Equal variances not assumed			-.185	69.474	.854	-1.259	6.810	-14.844	12.326

The assessment scores for the high school experimental group and control group were compared based on the spread of the data. The experimental group had a range of 100 and the control group had a range of 90. The top half of the data for the experimental group fell between 40% and 100%. Of the scores, 25% fell between 40% and 70% and 25% were between 70% and 100%. The top half of the data for the control group fell between 50% and 90%. Of the data, 25% fell between 50% and 60% and 25% fell between 60% and 90%. The middle 50% of the data was more closely clustered in the control group with scores falling between 26% and 50% while the scores for the experimental group fell between 20% and 50%. While the experimental group showed a wider range of scores, the control group contained higher scores. The control group

contained half of the scores above 50%. The scores in the control group were clustered more closely together while the scores in the experimental group contained a wider spread. The comparison of the data is shown in the box-and-whisker plot in Figure 7.

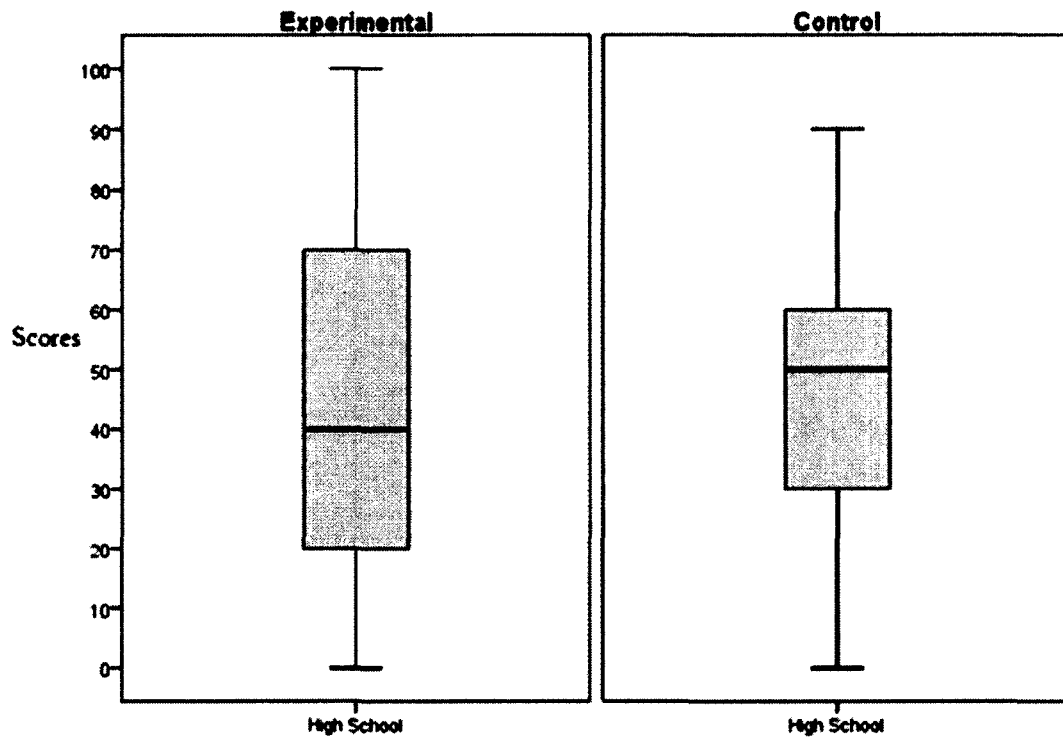


Figure 7. High School Common Assessment Results

Based on the analysis of the data by the independent samples *t*-test and box-and-whisker analysis, the inclusion of a discovery learning activity did not increase the achievement levels of average/struggling learners at the high school level.

2. Is there a difference in achievement of advanced learners in middle school Algebra I when taught by development of the concept of slope through graphing calculator technology and inquiry learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing activities using graphing calculator technology?

The second analysis involved the assessment scores for the middle school experimental and control groups using an independent samples *t*-test. The analysis indicated no significant differences between the treatment group ($M = 57.62$, $SD = 22.043$) and the control group ($M = 66.12$, $SD = 23.235$) conditions $t(87) = -1.756$, $p = 0.083$. The results of the statistical data for the groups are presented in Table 9. The Levene's Test for Equality of Variances (Table 10) was conducted to insure the assumptions of the *t*-test. The small sample size necessitated the Levene's Test.

Table 9

Statistics for Assessment Results for Middle School Experimental and Treatment Groups

		N	Mean	Std. Deviation	Std. Error Mean
Middle	Experimental	40	57.62	22.043	3.485
school	Control	49	66.12	23.235	3.319

The middle school experimental group and control group were compared in terms of dispersion of the data using a box-and-whisker plot. The control group had a larger spread of the data with a range of 100 including an outlier of a score of 0%. The range with the outlier not included is 90. The top 50% of the data in the control group fell between 70% and 100% while the top 50% of the data in the experimental group fell between 60% and 90%. It is noted that 75% of the students in the experimental group scored 45% or higher on the assessment while the students in the control group scored 50% or higher on the assessment. The box-and-whisker plot shows the dispersion of the data for each group (Figure 8).

Table 10

Independent Samples Test Middle School

		Levene's Test		<i>t</i> -test for Equality of Means						
		for Equality of								
		Variances								
				95% Confidence Interval of the						
				Difference						
				Sig. (2-tailed)						
				Mean Difference						
				Std. Error Difference						
				Lower Upper						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Middle school	Equal variances assumed	.246	.621	-1.756	87	.083	-8.497	4.839	-18.115	1.120
	Equal variances not assumed			-1.766	85.009	.081	-8.497	4.813	-18.067	1.072

Based on the analysis of the data by the independent samples *t*-test and box-and-whisker analysis, the inclusion of a discovery learning activity did not increase the achievement levels of advanced learners at the high school level.

1. Is there a difference in achievement between the middle school and high school Algebra students when taught by development of the concept of slope through graphing calculator technology and inquiry learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing activities using graphing calculator technology?

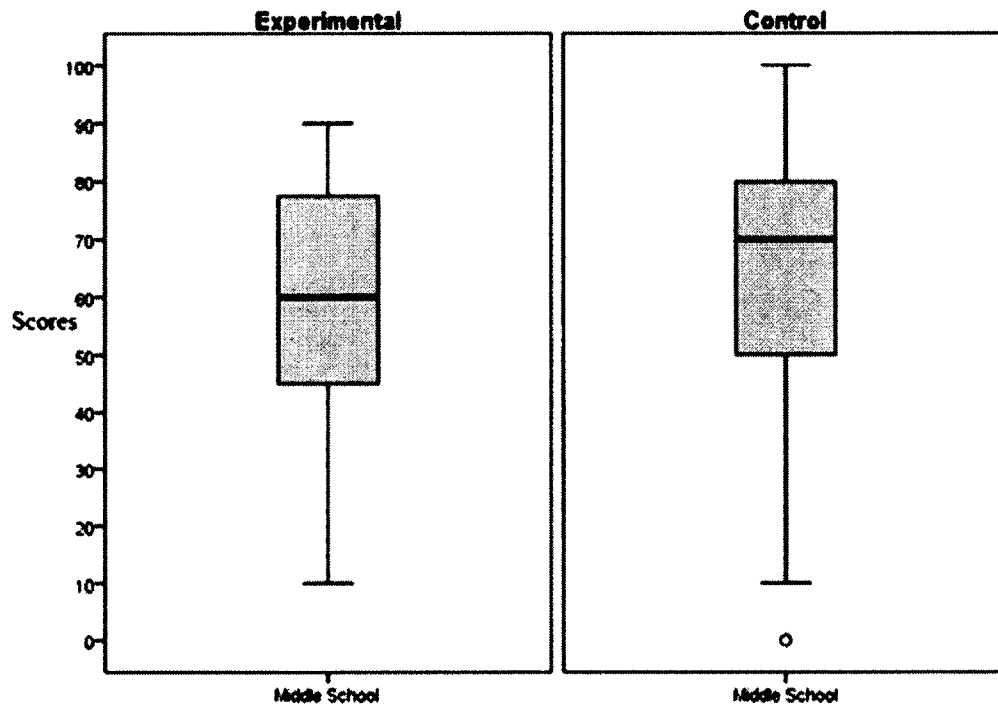


Figure 8. Middle School Common Assessment Results

The final independent samples *t*-test analyzed the control groups from the high schools and middle schools and the experimental groups from the high schools and middle schools. This analysis compared all students in the experimental groups to all students in the control groups. The analysis indicated no significant differences between the experimental group ($M = 51.36$, $SD = 26.707$) and the control group ($M = 56.89$, $SD = 28.315$) conditions $t(165) = -1.296$, $p = 0.197$. The results of the statistical data for the groups are presented in Table 11. The Levene's Test for Equality of Variances (Table 12) was conducted to ensure the assumptions of the *t*-test. The small sample size necessitated the Levene's Test.

Table 11

Statistics for Assessment Results for Experimental and Treatment Groups

		N	Mean	Std. Deviation	Std. Error Mean
High school and	Experimental	77	51.36	28.315	3.227
middle school	Control	90	56.89	26.707	2.815

Table 12

Independent Samples Test Experimental Group and Control Group

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2- tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower Upper	
High school and middle school	Equal variances assumed	.597	.441	-1.296	165	.197	-5.525	4.263	-13.942	2.891
	Equal variances not assumed			-1.290	157.704	.199	-5.525	4.282	-13.983	2.933

The assessment scores for all students in the experimental groups were compared to the assessment scores for all students in the control groups in terms of the dispersion of

the scores. Both groups had scores ranging from 0% to 100% resulting in the same range of 100. The middle 50% of the scores for the experimental groups fell between 25% and 72.5% while the middle 50% of the scores for the control group fell between 40% and 80%. The box-and-whisker plot in Figure 9 shows the dispersion of the assessment scores.

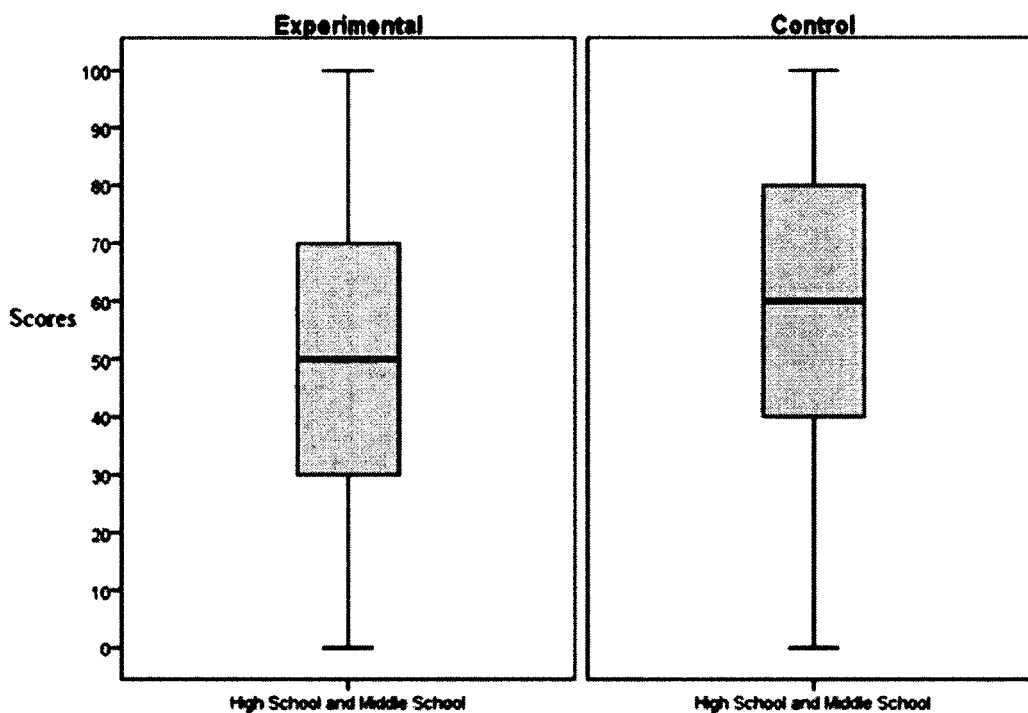


Figure 9. High School and Middle School Assessment Results

Based on the analysis of the data by the independent samples *t*-test and box-and-whisker analysis, the inclusion of a discovery learning activity did not increase the achievement levels of average/struggling learners at the high school level or advanced learners at the middle school level.

An analysis of variance was conducted and the resulting pairwise comparison was analyzed to determine if any significant differences were observed for specific items on the common assessment. Item responses were analyzed by assigning a value of one to

each correct response and assigning a value of two to each incorrect response. The assessment items were analyzed in terms of four groups: high school control, high school experimental, middle school control, and middle school experimental. Significant differences were observed for 6 of the 10 assessment items. Assessment items are identified as Question 1, Question 2, etcetera. Analysis of variance showed a main effect for Question 1, which is an assessment of the ability to determine slope of a line given an equation in slope-intercept form $F(3,163) = 7.89$, $p < .001$, $\eta^2 = .127$. There was a significant difference ($p < .001$) between the high school control group ($M = 1.56$, $SD = .502$) and the middle school control group ($M = 1.14$, $SD = .354$). There was also a significant difference ($p = .013$) between the high school control group ($M = 1.56$, $SD = .502$) and the middle school experimental group ($M = 1.25$, $SD = .439$). The high school experimental group ($M = 1.46$, $SD = .505$) and the middle school control group ($M = 1.14$, $SD = .354$) showed significant differences ($p = .009$). Table 13 is a display of the results for Question 1.

Analysis of variance showed a main effect for Question 2, which was an assessment of the ability to determine slope of a line given an equation in standard form. $F(3,163) = 7.02$, $p < .001$, $\eta^2 = .114$. There was a significant difference ($p < .001$) between the high school control group ($M = 1.88$, $SD = .331$) and the middle school control group ($M = 1.45$, $SD = .503$). There was also a significant difference ($p = .010$) between the high school control group ($M = 1.88$, $SD = .331$) and the middle school experimental group ($M = 1.53$, $SD = .504$).

Table 14 is a display of the results for Question 2.

Table 13

Pairwise Comparison Question 1

Dependent Variable	(I) ID	(J) ID	Mean Difference			95% Confidence Interval for Difference*	
			(I-J)	Std. Error	Sig.*	Lower bound	Upper Bound
Question 1	High School Control	High School Experimental	.102	.102	1.000	-.170	.373
		Middle School Control	.418*	.095	.000	.165	.672
		Middle School Experimental	.311*	.100	.013	.045	.577
	High School Experimental	High School Control	*.102	.102	1.000	-.373	.170
		Middle School Control	.317*	.098	.009	.056	.577
		Middle School Experimental	.209	.102	.253	-.064	.483
	Middle School Control	High School Control	-.418*	.095	.000	-.672	-.165
		High School Experimental	-.317*	.098	.009	-.577	-.056
		Middle School Experimental	-.107	.096	1.000	-.362	.148
Middle School Experimental	High School Control	-.311*	.100	.013	-.577	-.045	
	High School Experimental	-.209	.102	.253	-.483	.064	

Based on estimated marginal means

a. Adjustment for multiple comparisons: Bonferroni.

*. The mean difference is significant at the .05 level.

Table 14

Pairwise Comparison Question 2

Dependent Variable	(I) ID	(J) ID	Mean Difference			95% Confidence Interval for Difference*	
			(I-J)	Std. Error	Sig.*	Lower bound	Upper Bound
Question 1	High School Control	High School Experimental	.202	.104	.325	-.076	.481
		Middle School Control	.429*	.097	.000	.169	.689
		Middle School Experimental	.328*	.102	.010	.055	.601
	High School Experimental	High School Control	-.202	.104	.325	-.481	.076
		Middle School Control	.227	.100	.150	-.041	.494
		Middle School Experimental	.126	.106	1.000	-.155	.406
	Middle School Control	High School Control	-.409*	.097	.000	-.689	-.169
		High School Experimental	-.227	.100	.150	-.494	.041
		Middle School Experimental	-.101	.098	1.000	-.363	.161
Middle School Experimental	High School Control	-.328*	.102	.010	-.601	-.055	
	High School Experimental	-.126	.105	1.000	-.406	.155	
	Middle School Control	.101	.098	1.000	-.161	.363	

Based on estimated marginal means

a. Adjustment for multiple comparisons: Bonferroni.

*. The mean difference is significant at the .05 level.

Analysis of variance showed a main effect for Question 5, which was an assessment of the ability to determine the equation of a line given two points on the line

$F(3,163) = 7.58, p < .001, \eta p^2 = .122$. There was a significant difference ($p < .001$) between the high school control group ($M = 1.68, SD = .471$) and the middle school control group ($M = 1.22, SD .422$). There was also a significant difference ($p = .013$) between the high school control group ($M = 1.68, SD = .471$) and the high school experimental group ($M = 1.35, SD = .484$). Table 15 is a display of the results for Question 5.

Table 15

Pairwise Comparison Question 5

Dependent Variable	(I) ID	(J) ID	Mean Difference			95% Confidence Interval for Difference*	
			(I-J)	Std. Error	Sig.*	Lower bound	Upper Bound
Question 1	High School Control	High School Experimental	.332*	.106	.013	.048	.615
		Middle School Control	.458*	.099	.000	.193	.723
		Middle School Experimental	.208	.104	.286	-.070	.486
	High School Experimental	High School Control	-.332*	.106	.013	-.615	-.048
		Middle School Control	.127	.102	1.000	-.146	.400
		Middle School Experimental	-.124	.107	1.000	-.409	.162
	Middle School Control	High School Control	-.458*	.099	.000	-.723	-.193
		High School Experimental	-.127	.102	1.000	-.400	.146
		Middle School Experimental	-.251	.100	.079	-.517	.016
Middle School Experimental	High School Control	-.206	.104	.286	-.486	.070	
	High School Experimental	.124	.107	1.000	-.162	.409	
	Middle School Control	.251	.100	.079	-.016	.517	

Based on estimated marginal means

a. Adjustment for multiple comparisons: Bonferroni.

*. The mean difference is significant at the .05 level.

Analysis of variance showed a main effect for Question 8, which was an assessment of the ability to determine the equation of a line given the graph of the line $F(3,163) = 8.19, p < .001, \eta p^2 = .131$. There was a significant difference ($p < .001$) between the high school control group ($M = 1.63, SD = .488$) and the middle school control group ($M = 1.18, SD .391$). There was also a significant difference ($p = .008$) between the high school control group ($M = 1.63, SD = .488$) and the middle school

experimental group ($M = 1.30$, $SD = .464$). The high school experimental group ($M = 1.49$, $SD = .507$) and the middle school control group ($M = 1.18$, $SD = .391$) showed significant differences ($p = .018$). Table 16 is a display of the results for Question 8.

Table 16

Pairwise Comparison Question 8

Dependent Variable	(I) ID	(J) ID	Mean Difference			95% Confidence Interval for Difference*	
			(I-J)	Std. Error	Sig.*	Lower bound	Upper Bound
Question 1	High School Control	High School Experimental	.148	.104	.953	-.131	.426
		Middle School Control	.450*	.097	.000	.190	.711
		Middle School Experimental	.334*	.102	.008	.061	.607
	High School Experimental	High School Control	-.148	.104	.953	-.426	.131
		Middle School Control	.303*	.100	.018	.035	.570
		Middle School Experimental	-.116	.098	1.000	-.378	.146
	Middle School Control	High School Control	-.334*	.102	.006	-.607	-.061
		High School Experimental	-.186	.105	.165	-.467	.094
		Middle School Experimental	-.116	.098	1.000	-.378	.146
Middle School Experimental	High School Control	-.334	.102	.006	-.607	-.061	
	High School Experimental	-.186	.105	.465	-.467	.094	
	Middle School Control	.116	.098	1.000	-.146	.378	

Based on estimated marginal means

a. Adjustment for multiple comparisons: Bonferroni.

*. The mean difference is significant at the .05 level.

Analysis of variance showed a main effect for Question 9, which was an assessment of the ability to determine the slope of a line given the x-intercept and the y-intercept $F(3,163) = 3.38$, $p = .067$, $\eta^2 = .067$. There was a significant difference ($p = .006$) between the high school control group ($M = 1.59$, $SD = .499$) and the middle school control group ($M = 1.22$, $SD = .423$). Table 17 is a display of the results for Question 9.

Analysis of variance showed a main effect for Question 10, which was an assessment of the ability to determine the equation of a line given the graph of the line $F(3,163) = 3.71$, $p = .013$, $\eta^2 = .064$. There was a significant difference ($p = .015$) between the high school control group ($M = 1.51$, $SD = .506$) and the middle school

control group (M = 1.20, SD .407). Table 18 is a display of the results for Question 10.

Table 17

Pairwise Comparison Question 9

Dependent Variable	(I) ID	(J) ID	Mean Difference			95% Confidence Interval for Difference*	
			(I-J)	Std. Error	Sig.*	Lower bound	Upper Bound
Question 1	High School Control	High School Experimental	.126	.109	1.000	-.166	.418
		Middle School Control	.177	.102	.508	-.096	.450
		Middle School Experimental	.360*	.107	.006	.074	.647
	High School Experimental	High School Control	-.126	.109	1.000	-.418	.166
		Middle School Control	.051	.106	1.000	-.229	.332
		Middle School Experimental	.234	.110	.208	-.059	.528
	Middle School Control	High School Control	-.177	.102	.506	-.450	.096
		High School Experimental	-.051	.105	1.000	-.332	.229
		Middle School Experimental	.183	.103	.460	-.091	.458
Middle School Experimental	High School Control	-.360*	.107	.006	-.647	-.074	
	High School Experimental	-.234	.110	.206	-.528	.059	
	Middle School Control	-.183	.103	.450	-.458	.091	

Based on estimated marginal means

a. Adjustment for multiple comparisons: Bonferroni.

*. The mean difference is significant at the .05 level.

Table 18

Pairwise Comparison Question 10

Dependent Variable	(I) ID	(J) ID	Mean Difference			95% Confidence Interval for Difference*	
			(I-J)	Std. Error	Sig.*	Lower bound	Upper Bound
Question 1	High School Control	High School Experimental	.053	.107	1.000	-.234	.339
		Middle School Control	.308*	.100	.015	.041	.576
		Middle School Experimental	.162	.106	.749	-.119	.443
	High School Experimental	High School Control	-.053	.107	1.000	-.339	.234
		Middle School Control	.255	.103	.085	-.020	.531
		Middle School Experimental	.109	.108	1.000	-.179	.386
	Middle School Control	High School Control	-.308*	.100	.015	-.579	-.041
		High School Experimental	-.255	.103	.085	-.531	.020
		Middle School Experimental	-.146	.101	.899	-.415	.123
Middle School Experimental	High School Control	-.162	.105	.749	.443	.119	
	High School Experimental	-.109	.108	1.000	-.398	.179	
	Middle School Control	.146	.101	.899	-.123	.415	

Based on estimated marginal means

a. Adjustment for multiple comparisons: Bonferroni.

*. The mean difference is significant at the .05 level.

The analysis of the assessment items suggests that the average/struggling learners achieved lower assessment scores than the advanced learners on the items where significant differences were identified.

Data analysis of the common assessment overall scores did not identify significant differences between the high school control group and high school experimental group. Likewise, analysis of the middle school control group and middle school experimental group did not identify any significant differences in overall assessment scores. The final analysis of overall assessment scores between the high school and middle school control groups and the high school and middle school experimental groups did not identify any significant differences. An analysis of variance was conducted to determine if there were any significant differences between individual items on the common assessment. Significant differences were found in 6 of the 10 assessment items. The conclusions based on the analysis of the data indicate potential recommendations for further study on the topic of discovery learning using handheld graphing technology. The assessment items where significant differences were found indicated that the high school control group obtained the highest mean for the six questions that were analyzed. The mean reflects the value assigned to each question. A correct answer was assigned a value of 1 and an incorrect answer was assigned a value of 2. A higher mean indicated that the score is closer to 2 (incorrect response) than to 1 (correct response). Table 19 is a display of the results of the analysis of the assessment items.

Table 19

Common Assessment Item Analysis

	High School Control	High School Experiment	Middle School Control	Middle School Experiment
Question 1				
Sig.			.000	.013
Mean	1.56		1.14	1.25
Std Dev.	.502		.354	.439
Sig.			.009	
Mean		1.46	1.14	
Std Dev.		.505	.351	
Question 2				
Sig.			.000	.010
Mean	1.88		1.45	1.55
Std Dev.	.331		.503	.504
Question 5				
Sig.		.013	.000	
Mean	1.68	1.35	1.22	
Std Dev.	.471	.181	.422	
Question 8				
Sig.			.000	.008
Mean	1.63		1.18	1.30
Std. Dev.	.188		.391	.464
Sig.		1.49	.018	
Mean		.507	1.18	
Std Dev.			.391	
Question 9				
Sig.				.006
Mean	1.59			1.22
Std. Dev.	.499			.423
Question 10				
Sig.			.015	
Mean	1.51		1.20	
Std. Dev.	.506		.407	

A significant difference was observed between the high school control group and the high school experimental group on Question 5. The high school experimental group showed higher achievement than the high school control group. Significant differences also were observed between the high school control group and the middle school control group for Questions 1, 2, 5, 8, 9, and 10. The middle school control group showed higher achievement than the high school control group. Significant differences were also observed between the high school control group and the middle school experimental group for Questions 1, 2, and 8. The middle school experimental group showed higher achievement. Table 20 is a summary of these results.

Table 20

Summary of Questions with Significant Differences

	High School Control Group	Group with Higher Achievement
High School Experimental	Question 5	High School Experimental
Middle School Control	Question 1	Middle School Control
	Question 2	Middle School Control
	Question 5	Middle School Control
	Question 8	Middle School Control
	Question 9	Middle School Control
	Question 10	Middle School Control
Middle School Experimental	Question 1	Middle School Experimental
	Question 2	Middle School Experimental
	Question 8	Middle School Experimental

Question 1 and Question 2 were assessments determining slope given the equation of the line. Question 5 was an assessment writing the equation of a line given two points on the line. Question 8 and question 10 were assessments writing the equation of a line given the graph of the line. Question 9 was an assessment finding the slope of a line given the x-intercept and the y-intercept.

The inclusion of a discovery learning activity did not result in any significant differences in scores on the post-treatment assessment for advanced learners or average/struggling learners. However, significant differences were noted on particular assessment items.

Chapter 5: Conclusions and Recommendations

The results of this study indicated that no significant differences were observed when incorporating a discovery learning activity prior to direct instruction when teaching the concept of slope of a line to middle school and high school Algebra I students. This chapter is a summary and discussion of the purpose of the study. The research questions are restated and related back to the literature base. Data collections and findings are explained and, finally, recommendations for further research are posed.

Summary

Graduation and college or career readiness are the ultimate intended outcomes of K-12 instruction. Algebra I is a graduation requirement in the state of Virginia and this is a gatekeeper course for some students. Specifically, it is a course that may hinder enrollment in higher-level mathematics courses for some at-risk or struggling learners

Algebra I curriculum content includes the concept of slope of a line taught using paper and pencil and technology tools. Technology is an approach recommended for all students and has been found to be helpful with concept development with at-risk or struggling students (Lapp et al., 2000). The instruction of slope of a line includes paper-and-pencil graphing and computations with the graphing calculator technology used for verifying solutions computed by hand. Additionally, graphing calculator technology can facilitate the exploration of a variety of functions (NCTM, 2000). Assessment of these explorations is aligned with instruction when the use of paper-and-pencil methods and graphing calculator technology are utilized when a topic is taught and when that topic is tested. The calculator technology can also serve as informal assessment for students as they work with graphing calculator technology.

Purpose of the Study

This study was an investigation of the sequencing of instruction involving discovery learning and direct instruction of slope of a line in Algebra I classrooms. The experimental group completed a discovery learning activity before receiving direct instruction on slope of a line, while the control group received direct instruction only. The use of graphing calculator technology was infused during instruction and assessment. The purpose of the study was to determine if the inclusion of a discovery learning activity would improve student achievement of the topic of slope of a line with average/struggling learners and/or advanced learners.

Research Questions

Three research questions were investigated during the course of this study.

1. Is there a difference in achievement of average/struggling learners in high school Algebra I when taught by development of the concept of slope through graphing calculator technology and discovery learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing activities using graphing calculator technology?
2. Is there a difference in achievement of advanced learners in middle school Algebra I when taught by development of the concept of slope through graphing calculator technology and discovery learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing activities using graphing calculator technology?

3. Is there a difference in achievement between the middle school Algebra I students, the advanced learners, and high school Algebra I students, the average/struggling learners, when taught by development of the concept of slope through graphing calculator technology and discovery learning followed by reinforcing exercises using graphing calculator technology compared to the direct instruction approach to teaching slope of a line followed by reinforcing activities using graphing calculator technology?

Literature Related to the Problem Investigated

The discovery learning activity in this study incorporated the use of graphing calculator technology to develop the concept of slope of a line. The literature reviewed indicated advantages and disadvantages regarding the use of graphing calculator technology during instruction. The advantages of the integration of graphing calculator technology are discussed below. Students used the graphing calculator as a tool during the discovery learning investigation, which allowed students the ability to explore the concept of slope. The graphing calculator technology permitted the students the opportunity to test ideas and conjectures about slope of a line as they graphed the equations (Martin, 2008). All learners were provided the opportunity to explore and generate new ideas as they graphed the equations as the slope of the line changed. This process is in alignment with strategies that should be made available to advanced or gifted learners as recommended by the VDOE (2012). The VDOE guidelines for gifted learners call for students to be exposed to advanced content while the pacing of instruction is varied. While the content in study activity was not advanced, the discovery approach required students to work with unfamiliar content as they worked through the

activity. As students progressed through the discovery learning activity, they worked with multiple representations of the same function. Students used the graphing calculator technology to view functions in equation format, graphical format, and table format. The blending of graphing calculator technology, tables, and representations allowed students to understand abstract topics (Lapp et al., 2000). Hennessy et al. (2001) determined that student reasoning skills and graphical interpretations are enhanced using graphing calculator technology. In this study, the graphing calculator technology was blended with the discovery learning activity. Students investigated several equations where the slope of a parent function was varied using the graphing calculator technology. This technology afforded all students the opportunity to view the equations and then make conjectures based on the graphs. This exploration-based discovery learning activity was selected in hopes of permitting students of varied ability levels to become central to the learning environment and experience discoveries of content related to slope of a line (Chamblee et. al, 2008).

Two areas in the study reflected some of the potential disadvantages found in the review of the literature. The first area, curriculum integration, may have influenced the outcome since discovery learning is not a strategy integrated into this school division's curriculum. Lopez (2001) found that graphing calculator technology must be integrated into instruction in an effective manner. The integration of the calculator-based discovery learning activity was included in the unit on slope of a line for the purpose of this study and may not have been integrated as effectively as possible since this was the only discovery learning lesson used.

The second area, professional development, focused on the specifics of the lesson

but, did not specifically address the integration of the activity in detail. During the professional development session, the teachers were instructed to assign the discovery learning activity before they began the direct instruction portion of the lesson. The North Central Regional Educational Laboratory policy brief on professional development states “teachers learn as a result of training, practice, and feedback, as well as individual reflection and group inquiry into their practice” (1994). The professional development provided in this study was a one-time session and may not have provided the necessary depth and reflection for optimal implementation of this discovery learning activity. The implementation of the discovery learning activity and the subsequent post-intervention assessment provided the data for this study and perhaps lack of sustained professional development may have affected the final data.

The review of the literature found advantages and disadvantages to graphing calculator use; however, the literature base was limited regarding the use of discovery learning incorporating graphing calculator technology. As described by Castronova (2002), the three components of discovery learning are student exploration, ownership in their own learning, and new knowledge constructed on prior learning. These components were evident in the discovery learning lesson. Students explored slope of a line and created generalizations based on graphing calculator outputs. The students worked independently and at their own pace during completion of the activity. In the final step, students developed ideas based on prior learning experiences. Students had prior experiences related to the discovery learning activity, which included using the graphing calculator, interpreting output, and graphing equations of lines in paper-and-pencil format.

Methods for Data Collection

Pre/post data were used for this study. To determine if the groups were similar in achievement levels prior to the study, assessment results for all of the groups involved were compared using an independent samples *t*-test. The middle school Benchmark 1 assessment was the source of the data for the middle school groups. The midterm exam from the Algebra I course provided the data for the high school groups. Both of these assessments covered the same Algebra I content. After the discovery learning activity was completed and direct instruction was delivered, all groups completed a common assessment. The data from this assessment was gathered from all groups and compared using an independent samples *t*-test. The final analysis was a pairwise comparison of each question from the common assessment.

Findings

The analysis of the pre-activity assessments revealed that the experimental group and control group for the average/struggling learners indicated that there were no significant differences in terms of achievement between the groups. However, a larger dispersion of pre-activity assessment scores was observed in the experimental group. The finding of the pre-activity assessment for the advanced learners indicated that there were no significant differences between the experimental group and the control group. These two groups showed a similar dispersion of pre-activity assessment scores.

The findings from the analysis of the independent samples *t*-test of the post intervention common assessment indicated no significant differences in the achievement of average/struggling learners in high school Algebra I groups when the discovery learning activity was incorporated into instruction before direct instruction was delivered.

It is noted that the assessment scores were clustered more closely in the control group. These scores were clustered closer to the median value of 50. Of the data, 25% fell between 50 and 60 and 25% fell between 30 and 50. The experimental group had a median value of 40 where 25% of the data fell between 40 and 70 and 25% fell between 20 and 40. The wider clustering of the scores in the experimental group may indicate that some potential misconceptions developed during the discovery learning activity.

The findings from the analysis of the independent samples *t*-test of the post-intervention common assessment for the advanced learners indicated no significant differences in student achievement in the topic of study. The dispersion of the assessment scores was similar between the two groups.

The final analysis of the post-intervention common assessment was conducted using the data from the advanced learners and the average/struggling learners. The independent samples *t*-test indicated that there were no significant differences between all experimental groups and all control groups involved in the study. The dispersion of the data was similar for both groups; however, the middle 50% of the data for the control groups was slightly higher than for the experimental groups.

While no significant differences were observed from the analysis of the common assessment scores, differences were observed in six of the items on the post-intervention common assessment. Two questions involved determining slope of a line given the equation of the line. One question involved determining slope of a line given the *x*-intercept and *y*-intercept. One question involved writing the equation of the line given two points on the line and the final two questions involved writing the equation of the line given the graph of the line. The middle school control group scores showed higher

achievement on all six questions than the achievement of the high school control group. The middle school experimental group scores were higher than the high school control group. The final difference was noted between the high school experimental group and the high school control group. The high school experimental group reported higher achievement than the high school control group. Where differences were observed in the six questions, the high school control group reported lower achievement.

Conclusions

The conclusions drawn from the analysis of the data are listed below and will be discussed in the next section.

The findings based on the data related to research question 1 yielded the following results:

1. The mean for the control group was 45.85% with a standard deviation was 26.644.
2. The mean for the experimental group was 44.59% with a standard deviation of 32.795.

The independent samples *t*-test indicated no significant differences were found. The conclusion drawn indicated that the inclusion of one discovery learning activity did not improve the understanding of the concept of slope of a line for average/struggling learners.

The findings based on the data for research question 2 yielded the following results:

1. The mean for the control group was 66.12% with a standard deviation of 23.235.
2. The mean for the experimental group was 57.62% with a standard deviation of

22.043.

The independent samples *t*-test indicated that no significant differences were observed. The conclusion drawn is that the inclusion of one discovery learning activity did not improve the understanding of the concept of slope of a line for advanced learners.

The third research question focused on comparing all control groups to all experimental groups. The findings based on the data are listed below:

1. The mean for the control groups was 56.89% with a standard deviation of 26.707.
2. The mean for the experimental groups was 51.36% with a standard deviation of 28.315.

The independent samples *t*-test indicated that no significant differences were observed. The conclusions drawn indicate that the inclusion of one discovery learning activity did not increase achievement of the concept of slope of a line.

The final analysis focused on individual assessment questions on the post treatment common assessment. A pairwise comparison yielded a main effect for questions 1, 2, 5, 7, 8, 9, and 10. Conclusions based on the analysis of the data determined that achievement on all specified questions was higher for the middle school control group than for the high school control group. The middle school experimental group showed a main effect where achievement was higher than the high school control group for questions 1, 2, and 8. The high school experimental group showed a main effect for question 5 where the achievement was higher than the high school control group. In questions where effects were noted, the high school control group achieved a lower achievement level than the other groups. The conclusion drawn indicated that the direct instruction model for delivering content on slope of a line was not effective for this

group of learners.

Discussion

The active hands-on learning process of discovery learning in this study consisted of three main attributes. These attributes include student exploration, student ownership of learning, and student creation of knowledge built on prior experience. Students explored problem-solving situations using graphing calculator technology and generalized about the content to create understanding. Through this process, students should have taken ownership of their learning since they set their own pace of learning within the class period. This learning took place by students building new understandings on prior knowledge. Students learn to graph an equation from a table of values in Math 8 courses and this knowledge is built upon in Algebra I courses. This knowledge was assessed through a post-intervention common assessment.

The analysis of the common assessment data indicated no significant difference between the experimental and control groups for advanced learners or average/struggling learners. The experimental groups did not show higher achievement in conceptual understanding because of the discovery learning activity. The discussion of the results can be categorized into three main topics: discovery learning, teacher training, and technology integration.

Discovery Learning

The discovery learning activity took initial steps toward conceptual understanding by providing opportunities to explore the changes in graphs based on changes made to the y-intercept of the equation and the slope of the line using the graphing calculator technology during the discovery learning activity. The discovery learning activity

provided guiding questions that students answered after manipulating a graph.

Castronova (2002) stated that students must build upon prior knowledge to build understanding. The students in the present study may not have possessed an adequate prior knowledge base that would have led to increased understanding. Perhaps if students experienced discovery learning activities with graphing calculators in previous Algebra lessons or simple discovery learning activities in previous math courses the outcomes may have indicated some significant differences (Kersaint, 2007). The activity referred to $y = A(x \pm B)$ where $A \neq 0$ and $B \geq 0$. Students often see the slope-intercept form as $y = Ax + B$ or $y = Ax - B$, which may have caused some confusion as they did not perceive the equations as equivalent. Students manipulated these equations, specifically the value of the y-intercept to positive and negative values while the slope remained constant. Perhaps graphing all equations on the same coordinate plane may have provided a clearer view that the slope did not change. One activity alone may not have provided students enough exposure to the functions to see the patterns (White-Clark et al., 2008). In a discovery learning environment, students set the pace for their learning. Lopez (2001) indicated that calculator use aids in the pace of learning. The pace of the discovery learning activity may have required additional time for students to analyze and interpret the graphs (Hennessy et al., 2001). Inclusion of additional math talk and discussion between students may have provided added time for students to build upon prior knowledge (Hallagan et al., ND). Time could have been built into the lesson for students to discuss the results of the discovery learning activity. Student discussion could focus on similarities and differences in the results achieved from the activity. This discussion may have focused the results achieved in the discovery learning activity since

the discovery learning activity experience may have been the first experience for students. A final observation from the assessment question analysis indicated that where significant differences were observed, the middle school experimental group showed higher achievement than the high school experimental group (see Table 20). This may indicate that advanced learners may build upon prior knowledge more effectively than the average/struggling learner may and may benefit from the inclusion of discovery learning activities. Likewise, the middle school control group also showed higher achievement. This may indicate that advanced learners also reach higher achievement through direct instruction. Perhaps these students learn more effectively when receiving content delivered in small, structured increments followed by practice of newly acquired skills (NIDI, 2011). It is possible that advanced learners can master the content of slope of a line regardless of the method of instructional delivery. Another possible reason could be that the advanced learners mastered the content of slope of a line at a higher level than the level of average/struggling learners. Additionally, any potential misconceptions developed during the discovery learning activity were not applicable to this group of students since they did not experience the discovery learning lesson.

Teacher Training

Discovery learning was also a new concept for teachers. There was a onetime professional development session held for the teachers. This session introduced discovery learning, provided a walkthrough of the discovery learning activity, and reviewed the common assessment. There may have been a need for more sustained professional development in the use of discovery learning. Incorporation of a best practice recommended by NCREL (1994) where teachers learn, practice, receive

feedback on, and have time for personal reflection on new instructional strategies may have enriched the professional development experience. This suggests that ongoing professional development may have been needed as discovery learning was introduced into instruction. Chamblee et al. (2008) found that teachers who experience sustained professional development with the graphing calculator began to look for mathematics applications involving graphing calculator use. This may indicate that ongoing professional development might include discovery learning coupled with graphing calculator technology. The placement of graphing calculator technology into classroom instruction is driven by the written curriculum. Kersaint (2007) recommended that teachers and curriculum leaders determine where graphing calculator technology best fits into the curriculum. This fit could be incorporated with discovery learning. Teacher training is critical when new initiatives are undertaken. Sustained professional development provides the opportunity for teachers to learn about new instructional strategies and receive support as they implement the new strategies.

Technology Integration

The NCTM Technology Principle (2000) indicated that deeper mathematical understanding might be achieved through responsible use of appropriate technology. The graphing calculator is considered appropriate technology for Algebra I students. Students in the present study completed a discovery learning activity where the slope was varied and the y-intercept remained constant. Perhaps including a lesson where the y-intercept remained constant and the slope was changed could have enriched the lesson and made the changes in the line more apparent. White-Clark et al. (2008) noted that when students view representations of functions before instruction, students are better able to see the

representations on a continuing basis. This might suggest that additional time may be needed to develop student understanding by viewing equations over a longer period before engaging in a discovery learning lesson. According to the Virginia curriculum, students in the state graph equations in math eight classes using paper-and-pencil methods. This is the sequencing Waites and Demana (2000) recommend. The graphing calculator is not used in Math 8; rather, students graph with paper-and-pencil methods. Students are first introduced to graphing calculator technology in Algebra I classes. The transition to the graphing calculator output may have created a graph students were not used to seeing and interpreting. Perhaps if students completed the activity in paper-and-pencil mode first and then used the calculator technology, the results may have differed.

The discovery learning activity in the present study incorporated the use of the graphing calculator. The graphing calculator allowed all students the ability to begin at the same level when graphing an equation of a line. The graphing calculator technology permits all students the ability to graph the line (Lee, 2007). Students are taught how to graph a line given the equation of a line in the Virginia Math 8 course. However, some students involved in the study may not have remembered how to graph an equation using paper-and-pencil methods, but with graphing calculator technology, all students were able to complete the discovery learning activity. The graphing calculator technology permitted all students the ability to experience the discovery learning activity. This raises the question if a student does not recall the paper-and-pencil process, does this affect understanding when using the graphing calculator technology. This could have potentially affected the level of conceptual understanding that was the intended outcome of the combination of discovery learning and direct instruction to teach the curriculum

content of slope of a line.

Curriculum integration of graphing calculator technology was in place in the curriculum guide of the school division in which the present study occurred. This was in line with the findings of Lapp and Cyrus (2000) who stated that graphing calculator use needs to be integrated into the curriculum to obtain understanding of important concepts.

Recommendations for Further Research

The findings from this study generated several topics for additional study.

1. The present study is limited by the small size of the sample; therefore, further research in this area may need to include a larger sample. Perhaps a study conducted in a larger school division may provide additional Algebra I classes and additional teachers, which could be included in the study. A larger sample size would also lessen the effects of student absences on the study.
2. The creation of instructional modules may expand the use of discovery learning activities that incorporate graphing calculator technology. Further studies in this area may also investigate student achievement when graphing calculator technology coupled with discovery learning is used through instructional modules integrated throughout the unit of study. The one-time use of the discovery learning activity did not produce a significant effect on student achievement. Additional focus on discovery learning and graphing calculator technology as an embedded part of the curriculum is suggested.
3. The expansion to multiple professional development training sessions may standardize the use of discovery learning activities. The incorporation of multiple discovery learning experiences into the curriculum will require ongoing

professional development for teachers. The use of one professional development session in this study may not have provided sufficient training for the teachers.

Additionally, further research may include scheduled classroom observations of the discovery learning activity and/or direct instruction lessons for ensuring teacher fidelity in using discovery learning strategies as intended.

4. The consideration of curriculum sequencing may have implications for the infusion of discovery learning into the teaching and learning of some math concepts. Thus, a connection between the Math 8 objectives and the Algebra I objectives could be completed as a discovery learning activity before the slope unit. Students could graph functions in paper-and-pencil mode in table format and check the graphs with the graphing calculator. This may provide a bridge for students between the paper-and-pencil method and graphing calculator technology method.
5. A final recommendation for further research would be providing a series of student experiences in discovery learning. The additional experiences may result in students becoming more comfortable with the discovery learning process and may lead to greater student achievement. A series of lessons integrated into the curriculum and used over the course of a school year or over the course of several mathematics topics may provide results that significantly influence student achievement.

As researchers continue to seek strategies for teaching algebraic concepts that are more effective, student experiences should be kept in the forefront. Alfred North Whitehead, an English mathematician and philosopher, is quoted “from the very

beginning of his education, the child should experience the joy of discovery” (1916).

Too often, mathematics education is not an exciting experience for students but instead, students are provided information and content rather than experiencing the joy of discovering mathematics through investigations and rich experiences.

Appendix A

Secondary Mathematics Graphing Calculator Competencies

Teacher Assessment

Level I – Middle School Teachers	Level II – Algebra I and Geometry Teachers
Set up calculator	Work with “y=” graphs
Order of Operations	Input and evaluate algebraic expressions
Evaluate nth roots	LIST(s)
Covert fractions to decimals and vice versa	Graph linear equations of the form “y=mx+b”
Evaluate expressions with exponents	Factor using the graphing calculator
Linear and exponential functions; line of best fit	Evaluate radical expressions
Graph linear equations	STO function
Statistics – input data – Create scatterplots, histograms, box and whisker plots	TEST function
Scientific notation	Graph quadratic functions
TEST functions	Locate the zeros of a function using the CALC function
STO functions	Solve right triangle problems (trigonometry function)
Use Table and Tableset function	LINK
	Simultaneous equations

Appendix B

Algebra 1 Benchmark 1 – Middle School

Name: _____

Class: _____

Date: _____

1. Write an algebraic expression for the verbal expression.

35 less the product of 4 and x

- A. $35 - 4x$
- B. $35 + 4x$
- C. $4x - 35$
- D. $-35 - 4x$

2. Translate the following statement into an algebraic expression.

Six decreased by the difference of s and t

A. $6 + (s - t)$

B. $6 + (s + t)$

C. $6 - (s - t)$

D. $(s - t) - 6$

3. Which variable expression represents the phrase "twice the sum of a number and 7"?

A. $2x + 7$

B. $2x \cdot 7$

C. $2(x - 7)$

D. $2(x + 7)$

4. Evaluate.

$$\frac{a-b^2}{c^3}$$

when $a = 2.25$, $b = 0.5$, and $c = 0.2$

- A. -50
- B. -25
- C. 25
- D. 250

5. Evaluate.

$$f - (fg + gh)$$

when $f = 8$, $g = -1$, and $h = 2$

- A. -2
- B. 2
- C. 6
- D. 18

6. Simplify. $\left(\frac{2a^3b}{16b^6}\right)^3$

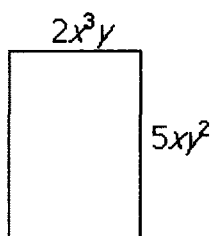
A. $\frac{a^9}{8b^3}$

B. $\frac{a^6}{2b^3}$

C. $\frac{a^9b^3}{2}$

D. $\frac{a^9}{512b^{15}}$

7. Which expression is equivalent to the area of the rectangle below?



A. $7x^3y^2$

B. $7x^4y^3$

C. $10x^3y^2$

D. $10x^4y^3$

8. Simplify: $(3x^2y^4)^4$

A. $8x^6y^8$

B. $8x^6y^{16}$

C. $81x^6y^8$

D. $81x^8y^{16}$

9. Simplify $(x^9)^0(x^7)^2$

A. x^{126}

B. x^{18}

C. 1

D. x^{14}

10. Simplify:

$$(5a^2b)(-6a^3b^4)$$

A. $-a^5b^5$

B. $-a^6b^4$

C. $-30a^5b^5$

D. $-30a^6b^4$

11. Simplify:

$$\frac{35x^6y^5}{7x^2y^3}$$

A. $5x^3y^2$

B. $5x^4y^2$

C. $5x^8y^9$

D. $28x^4y^3$

12. Simplify.

$$(-3ab)(7a^2b)$$

A. $-21a^2b$

B. $-21a^3b^2$

C. $4a^3b^2$

D. $42a^2b$

13. Simplify.

$$(2p^5q^3)(8pq^5)(-p^2q)$$

A. $-16p^8q^9$

B. $-16p^7q^8$

C. $10p^8q^9$

D. $10p^7q^8$

14. Simplify: $2y^4(3y^3 - 4y^2 + 2)$

A. $5y^7 - 2y^6 + 4y^4$

B. $5y^{12} - 2y^8 + 4$

C. $6y^7 - 8y^6 + 4y^4$

D. $6y^{12} - 8y^8 + 4y^4$

15. Simplify: $(x + y)(x - y)$

A. $x^2 - y^2$

B. $x^2 - 2xy + y^2$

C. $x^2 - 2xy - y^2$

D. $x^2 + 2xy - y^2$

16. Simplify:

$$(5st - 2)(5st + 2)$$

A. $10s^2t^2 - 20st - 4$

B. $25s^2t^2 + 20st + 4$

C. $25s^2t^2 + 4$

D. $25s^2t^2 - 4$

17. Simplify: $(m + 4)^2$

A. $m^2 - 8m + 16$

B. $m^2 - 4m + 8$

C. $m^2 + 8m + 16$

D. $m^2 + 4m + 16$

18. Simplify: $(4y - 9)^2$

A. $6y^2 + 81$

B. $16y^2 - 72y + 81$

C. $16y^2 - 36y + 81$

D. $8y^2 + 18$

19. Simplify: $\frac{24c^5d^3 + 36c^6d^6}{12c^3d^2}$

A. $2c^2d + 3c^3d^4$

B. $2c^2d + 36c^3d^4$

C. $2c^8d^5 + 3c^9d^8$

D. $12c^2d + 24c^3d^4$

20. Simplify: $(28y^4 - 14y^3 + 7y^3) \div (7y^3)$

A. $4y-1$

B. $21y - 7 + \frac{1}{y}$

C. $4y^3 + 7y^3 - 3$

D. $4y^3 - 2y^2 + 1$

21. Simplify

$$(-3p^2 + 9p - 7) + (-5p^2 - 3p - 10)$$

A. $(-8p^2 + 6p - 3)$

B. $(-8p^2 + 6p - 17)$

C. $(-2p^2 + 6p - 3)$

D. $(-2p^2 - 12p - 3)$

22. Simplify.

$$(3a^2b - 6ab^2) - (5 + 7ab^2 - 3a^2b)$$

A. $(-13ab^2 - 5)$

B. $(ab^2 - 5)$

C. $(3ab^2 - 5)$

D. $(6ab^2 - 13ab^2 - 5)$

23. $(4a - 3b^2 - a) + (b - 3 + 6a^2)$

A. $6a^2 - 3b^2 + 3a + b - 3$

B. $6a^2 - 3b^2 + 3a + b + 3$

C. $6a^2 - 3b^2 + 4a + b - 3$

D. $3a^2 + 3a + b - 3$

24. Factor. $16j^3k - 8j^6k^5 + 60j^3$

A. $4j^3(4k - 2j^3k^5 + 15)$

B. $4(4j^3k - 2j^6k^5 - 15j^3)$

C. $4j^3k(4 - 2j^3k^4 - 15)$

D. $2j^3(4k - 2j^3k^5 - 15)$

25. Simplify. $\sqrt{27}$

A. $3\sqrt{2}$

B. $3\sqrt{3}$

C. $9\sqrt{3}$

D. $3\sqrt{6}$

26. Evaluate. $\sqrt{10} \cdot 3\sqrt{2}$

A. $6\sqrt{5}$

B. $12\sqrt{5}$

C. $2\sqrt{7}$

D. 180

27. Simplify. $\sqrt{48}$

A. $4\sqrt{3}$

B. $16\sqrt{3}$

C. $2\sqrt{12}$

D. $4\sqrt{12}$

28. Find the slope of the line whose equation is $3x - 5y = 15$.

A. -5

B. $-\frac{3}{5}$

C. $\frac{3}{5}$

D. 3

29. Find the slope of the line containing the points $(-1, -4)$ and $(0, -4)$.

A. $\frac{4}{5}$

B. $\frac{5}{4}$

C. 1

D. zero slope

30. What is the slope of the line that passes through (2,4), (-3,5)?

A. $-\frac{1}{5}$

B. $\frac{1}{5}$

C. 5

D. -5

31. Find the slope of the line containing the points (-3,8) and (-1,0).

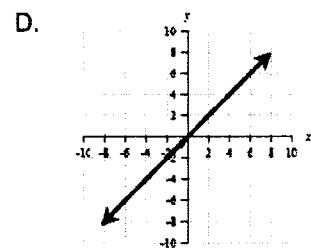
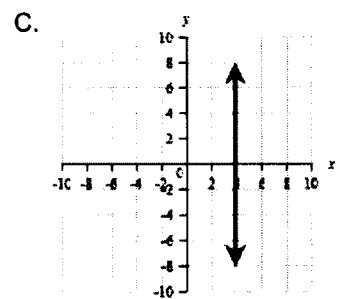
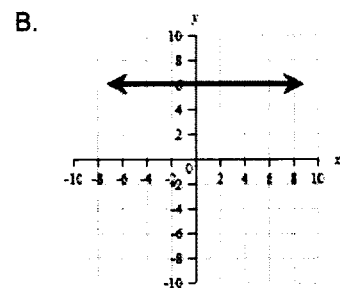
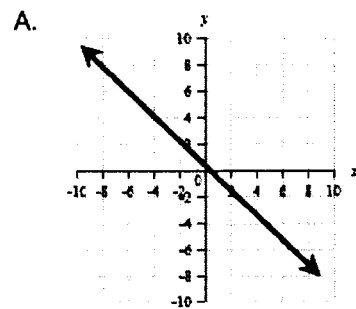
A. -4

B. -3

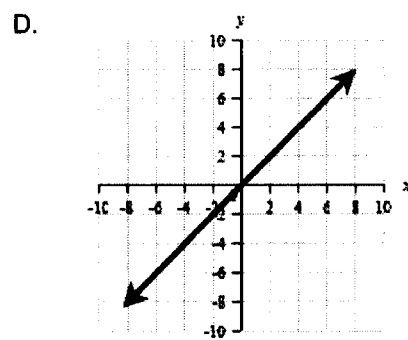
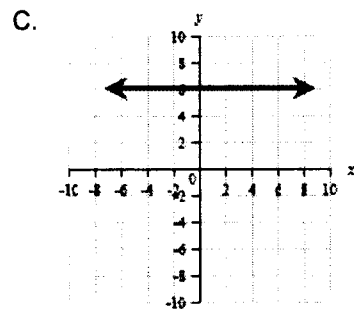
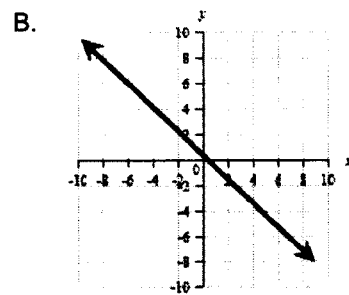
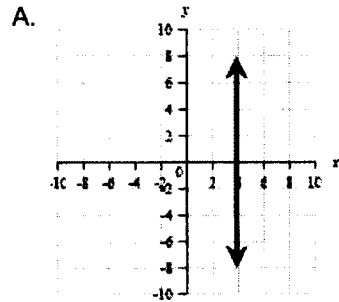
C. zero slope

D. undefined slope

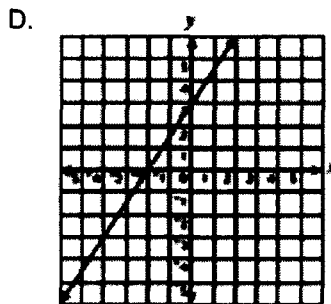
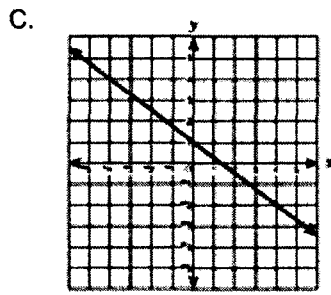
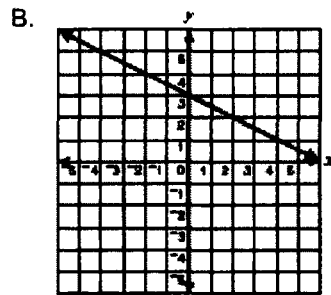
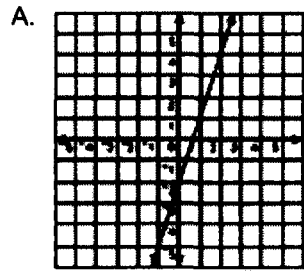
32. Which of the following lines has an undefined slope?



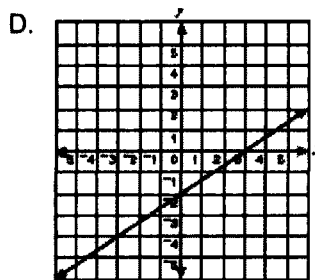
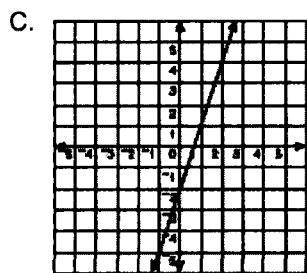
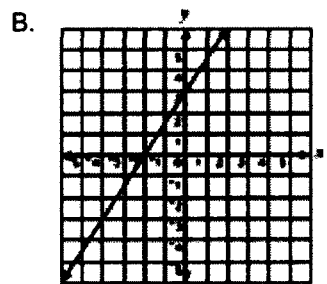
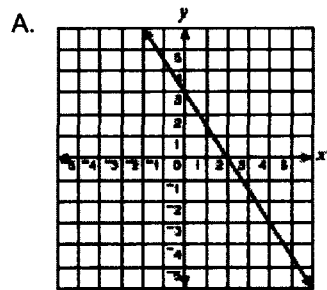
33. Which of the following lines has a negative slope?



34. Which is the graph of the line whose slope is $-\frac{1}{2}$ and whose y-intercept is 1?



35. Which of the following is the graph of $y = 3x - 2$?



Appendix C

Algebra 1 Midterm Exam – High School

Name: _____

Class: _____

Date: _____

1. Write an algebraic expression for the verbal expression.

35 less the product of 4 and x

- A. $35 - 4x$
- B. $35 + 4x$
- C. $4x - 35$
- D. $-35 - 4x$

2. Translate the following statement into an algebraic expression.

Six decreased by the difference of s and t

A. $6 + (s - t)$

B. $6 + (s + t)$

C. $6 - (s - t)$

D. $(s - t) - 6$

3. Which variable expression represents the phrase "twice the sum of a number and 7"?

A. $2x + 7$

B. $2x \cdot 7$

C. $2(x - 7)$

D. $2(x + 7)$

4. Evaluate.

$$\frac{a-b^2}{c^3}$$

when $a = 2.25$, $b = 0.5$, and $c = 0.2$

- A. -50
- B. -25
- C. 25
- D. 250

5. Evaluate.

$$f - (fg + gh)$$

when $f = 8$, $g = -1$, and $h = 2$

- A. -2
- B. 2
- C. 6
- D. 18

6. Simplify. $\left(\frac{2a^3b}{16b^6}\right)^3$

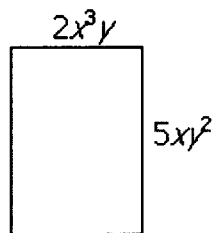
A. $\frac{a^9}{8b^3}$

B. $\frac{a^6}{2b^3}$

C. $\frac{a^9b^3}{2}$

D. $\frac{a^9}{512b^{15}}$

7. Which expression is equivalent to the area of the rectangle below?



A. $7x^3y^2$

B. $7x^4y^3$

C. $10x^3y^2$

D. $10x^4y^3$

8. Simplify: $(3x^2y^4)^4$

A. $8x^6y^8$

B. $8x^6y^{16}$

C. $81x^6y^8$

D. $81x^8y^{16}$

9. Simplify $(x^9)^0(x^7)^2$

A. x^{126}

B. x^{18}

C. 1

D. x^{14}

10. Simplify:

$$(5a^2b)(-6a^3b^4)$$

A. $-a^5b^5$.

B. $-a^6b^4$

C. $-30a^5b^5$

D. $-30a^6b^4$

11. Simplify:

$$\frac{35x^6y^5}{7x^2y^3}$$

A. $5x^3y^2$

B. $5x^4y^2$

C. $5x^8y^9$

D. $28x^4y^3$

12. Simplify.

$$(-3ab)(7a^2b)$$

A. $-21a^2b$

B. $-21a^3b^2$

C. $4a^3b^2$

D. $42a^2b$

13. Simplify.

$$(2p^5q^3)(8pq^5)(-p^2q)$$

A. $-16p^8q^9$

B. $-16p^7q^8$

C. $10p^8q^9$

D. $10p^7q^8$

14. Simplify: $2y^4(3y^3 - 4y^2 + 2)$

A. $5y^7 - 2y^6 + 4y^4$

B. $5y^{12} - 2y^8 + 4$

C. $6y^7 - 8y^6 + 4y^4$

D. $6y^{12} - 8y^8 + 4y^4$

15. Simplify: $(x + y)(x - y)$

A. $x^2 - y^2$

B. $x^2 - 2xy + y^2$

C. $x^2 - 2xy - y^2$

D. $x^2 + 2xy - y^2$

16. Simplify:

$$(5st - 2)(5st + 2)$$

A. $10s^2t^2 - 20st - 4$

B. $25s^2t^2 + 20st + 4$

C. $25s^2t^2 + 4$

D. $25s^2t^2 - 4$

17. Simplify: $(m + 4)^2$

A. $m^2 - 8m + 16$

B. $m^2 - 4m + 8$

C. $m^2 + 8m + 16$

D. $m^2 + 4m + 16$

18. Simplify: $(4y - 9)^2$

A. $6y^2 + 81$

B. $16y^2 - 72y + 81$

C. $16y^2 - 36y + 81$

D. $8y^2 + 18$

19. Simplify: $\frac{24c^5d^3 + 36c^6d^6}{12c^3d^2}$

A. $2c^2d + 3c^3d^4$

B. $2c^2d + 36c^3d^4$

C. $2c^8d^5 + 3c^9d^8$

D. $12c^2d + 24c^3d^4$

20. Simplify: $(28y^4 - 14y^3 + 7y^3) \div (7y^3)$

A. $4y-1$

B. $21y - 7 + \frac{1}{y}$

C. $4y^3 + 7y^3 - 3$

D. $4y^3 - 2y^2 + 1$

21. Simplify

$$(-3p^2 + 9p - 7) + (-5p^2 - 3p - 10)$$

A. $(-8p^2 + 6p - 3)$

B. $(-8p^2 + 6p - 17)$

C. $(-2p^2 + 6p - 3)$

D. $(-2p^2 - 12p - 3)$

22. Simplify.

$$(3a^2b - 6ab^2) - (5 + 7ab^2 - 3a^2b)$$

A. $(-13ab^2 - 5)$

B. $(ab^2 - 5)$

C. $(3ab^2 - 5)$

D. $(6ab^2 - 13ab^2 - 5)$

23. $(4a - 3b^2 - a) + (b - 3 + 6a^2)$

A. $6a^2 - 3b^2 + 3a + b - 3$

B. $6a^2 - 3b^2 + 3a + b + 3$

C. $6a^2 - 3b^2 + 4a + b - 3$

D. $3a^2 + 3a + b - 3$

24. Factor. $16j^3k - 8j^6k^5 + 60j^3$

A. $4j^3(4k - 2j^3k^5 + 15)$

B. $4(4j^3k - 2j^6k^5 - 15j^3)$

C. $4j^3k(4 - 2j^3k^4 - 15)$

D. $2j^3(4k - 2j^3k^5 - 15)$

25. Simplify. $\sqrt{27}$

A. $3\sqrt{2}$

B. $3\sqrt{3}$

C. $9\sqrt{3}$

D. $3\sqrt{6}$

26. Evaluate. $\sqrt{10} \cdot 3\sqrt{2}$

A. $6\sqrt{5}$

B. $12\sqrt{5}$

C. $2\sqrt{7}$

D. 180

27. Simplify. $\sqrt{48}$

A. $4\sqrt{3}$

B. $16\sqrt{3}$

C. $2\sqrt{12}$

D. $4\sqrt{12}$

28. Find the slope of the line whose equation is $3x - 5y = 15$.

A. -5

B. $-\frac{3}{5}$

C. $\frac{3}{5}$

D. 3

29. Find the slope of the line containing the points $(-1, -4)$ and $(0, -4)$.

A. $\frac{4}{5}$

B. $\frac{5}{4}$

C. 1

D. zero slope

30. What is the slope of the line that passes through (2,4), (-3,5)?

A. $-\frac{1}{5}$

B. $\frac{1}{5}$

C. 5

D. -5

31. Find the slope of the line containing the points (-3,8) and (-1,0).

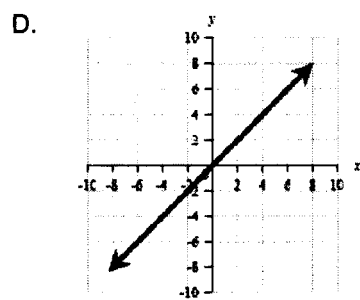
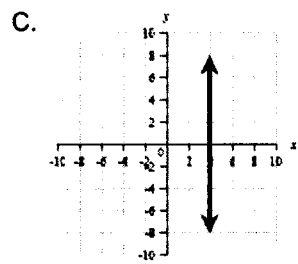
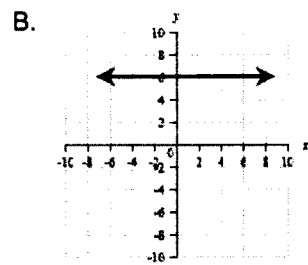
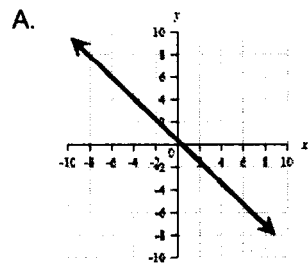
A. -4

B. -3

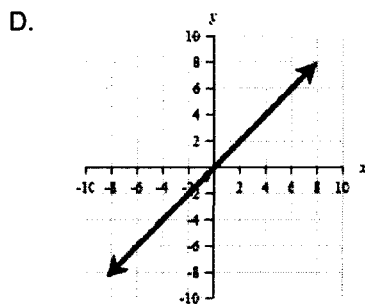
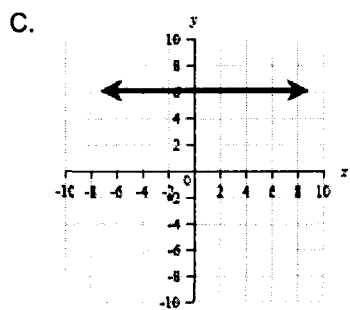
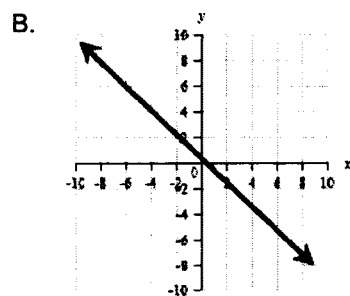
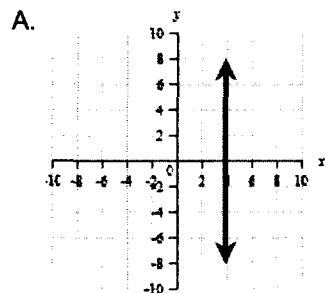
C. zero slope

D. undefined slope

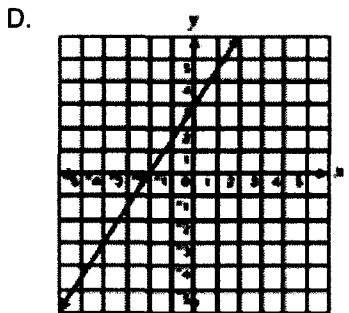
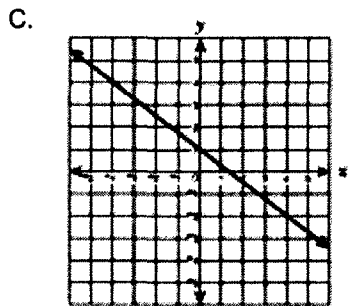
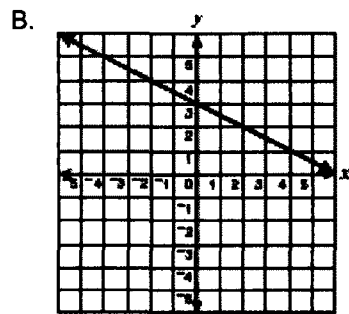
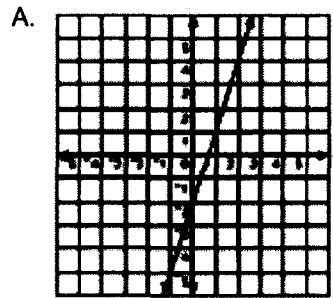
32. Which of the following lines has an undefined slope?



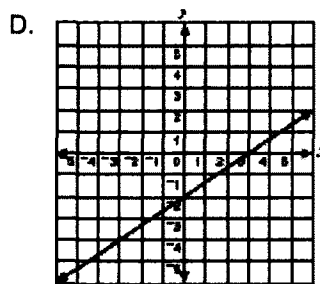
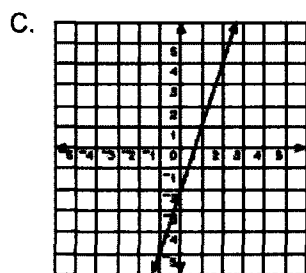
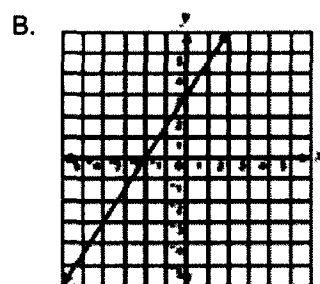
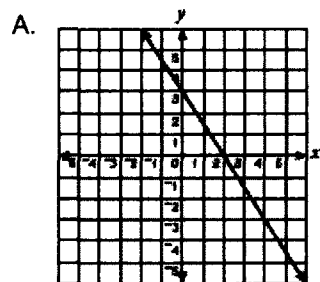
33. Which of the following lines has a negative slope?



34. Which is the graph of the line whose slope is $-\frac{1}{2}$ and whose y-intercept is 1?



35. Which of the following is the graph of $y = 3x - 2$?



Appendix D

Frequency Table - High School Exam Scores and Middle School Benchmark One Scores

High School Exam Scores				Middle School Benchmark Scores			
Score	Frequency	Score	Frequency	Score	Frequency	Score	Frequency
24	2	55	1	24	1	70	4
36	2	56	1	48	1	71	2
37	1	60	6	50	3	72	8
38	1	62	3	52	3	73	2
40	4	64	7	54	1	74	6
42	1	65	1	56	1	76	1
43	2	66	2	57	1	77	5
45	3	67	2	59	1	79	1
48	1	69	1	61	6	80	9
49	4	70	1	62	1	82	6
50	14	71	1	63	2	84	4
51	5	72	1	64	1	85	3
52	2	75	2	65	1	88	1
53	4	82	1	66	4	91	2
54	1	86	1	67	2		
				69	6		
		Total	78			Total	89

Appendix E

Transformation Investigation**Organizing topic**

Relations and Functions: Linear

Overview

Students investigate the significance of the components of the equation of a line.

Related Standards of Learning A.6, A.7, A.8**Objectives**

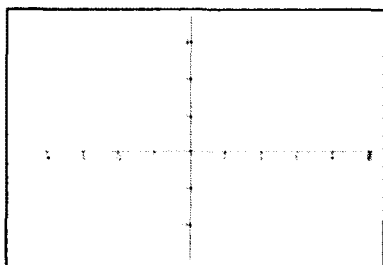
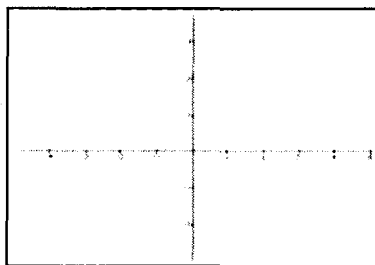
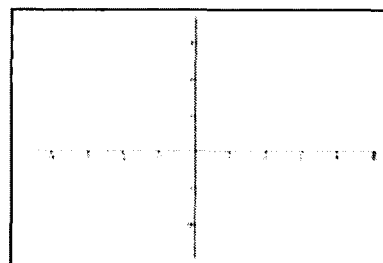
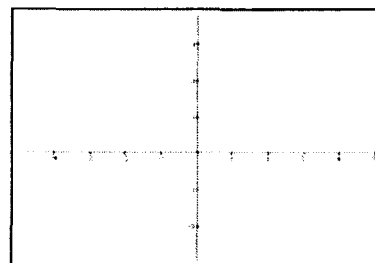
- The student will use the line $y = x$ as a reference and generalize the effect of changes in the equation on the graph of the line.
- The student will characterize the changes in the graph of the line as translations, reflections, and dilations.

Materials needed

- Graphing calculators

Instructional activity

1. Have students graph linear equations of the form $Y = A(X \pm B)$, when $A \neq 0$ and $B \geq 0$.
2. Have students set their calculator window to
 $X_{\min} = -10$
 $X_{\max} = 10$ $X_{\text{sc1}} = 1$
 $Y_{\min} = -6$
 $Y_{\max} = 6$

Part IBasic function: $Y = 1(X + 0)$  $Y_2 = 1X + 2$  $Y_3 = 1X + 4$  $Y_4 = 1X + 6$ 

Critical Statistics

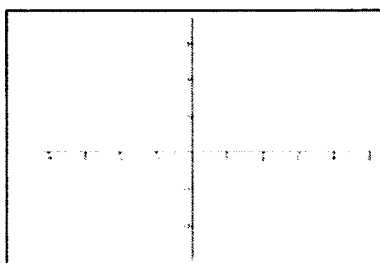
	Y	Y_1	Y_2	Y_3	Y_4
y -intercept					
x -intercept					
Slope					

1. What effect does "changing" B have on the basic function?
2. What generalization can you make about the change in the y -intercept if $B \geq 0$?
3. What generalization can you make about the change in the x -intercept if $B \geq 0$?

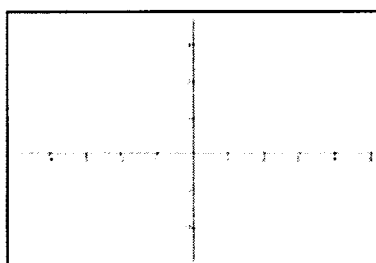
Part II

Basic function: $Y = 1(X + 0)$

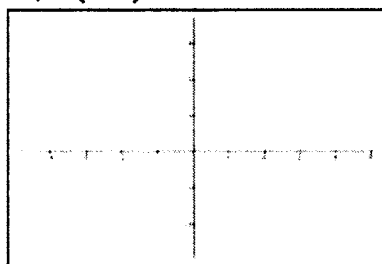
$$Y_2 = 1(X - 2)$$



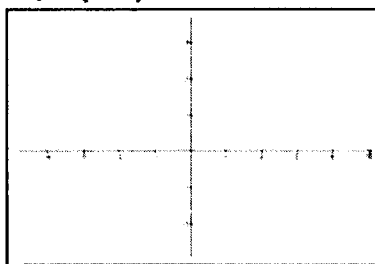
$$Y_3 = 1(X - 4)$$



$$Y_4 = 1(X - 6)$$



$$Y_5 = 1(X - 8)$$



Critical Statistics

	Y	Y_2	Y_3	Y_4	Y_5
y -intercept					
x -intercept					
Slope					

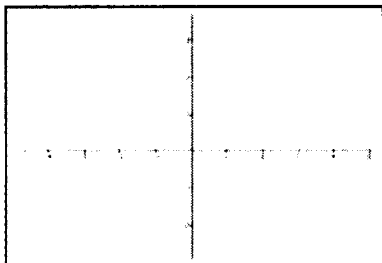
1. What effect does "changing" B have on the basic function?
2. What generalization can you make about the change in the y -intercept if $B \geq 0$?
3. What generalization can you make about the change in the x -intercept if $B \geq 0$?

4. Does the slope have an effect on the way the graph changes?

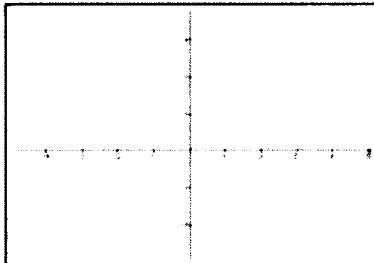
Part III

Sketch a graph for each of the following equations with a basic function: $Y = 2(X + B)$

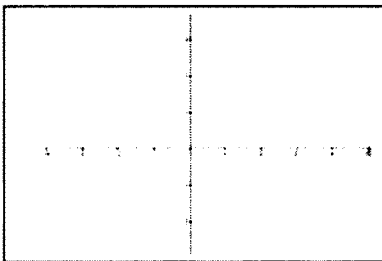
$$Y_1 = 2(X + 0)$$



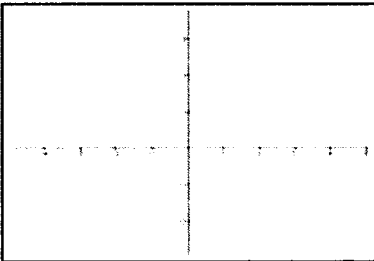
$$Y_2 = 2(X + 2)$$



$$Y_3 = 2(X + 4)$$



$$Y_4 = 2(X + 3)$$



Critical Statistics

	Y	Y_1	Y_2	Y_3	Y_4
y-intercept					
x-intercept					
Slope					

Compare the critical statistics of Y_1, Y_2, Y_3, Y_4 to the critical statistics of Y . What effect(s) does "changing" B have on the basic (parent) function?

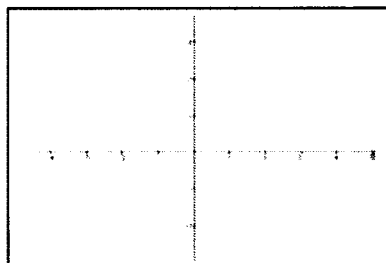
Generalizing: (if $B \geq 0$)

1. What was the slope in each of the problems above? Do you think that the slope has any effect on the graph?
2. Adding a value of B to the X in the previous problems resulted in a transformation of the x-intercept to the _____.
3. Adding a value of B to the X in the previous problems resulted in a transformation of the y-intercept to the _____.

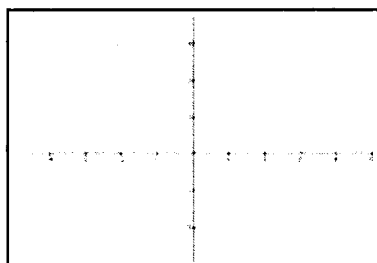
Part IV

Sketch a graph for each of the following equations with a basic function: $Y = 2(X - B)$

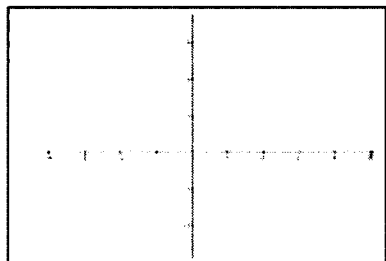
$$Y = 2(X - 0)$$



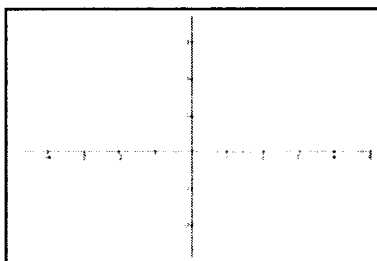
$$Y_1 = 2(X - 2)$$



$$Y_2 = 2(X - 3)$$



$$Y_3 = 2(X - 4)$$



Critical Statistics

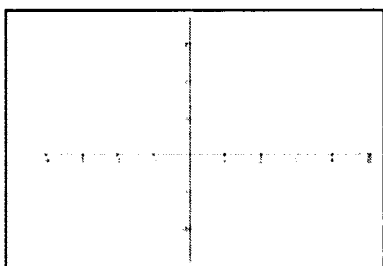
	Y	Y_1	Y_2	Y_3	Y_4
y-intercept					
x-intercept					
Slope					

1. What is the slope of each graph above?
2. What effect does subtracting a value of B have on the graph?
3. What is the effect of subtracting a value of B on the x -intercept? y -intercept?

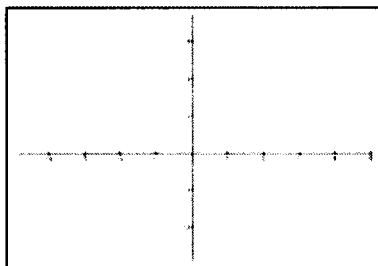
Part V

Sketch a graph for each of the following equations for the basic function

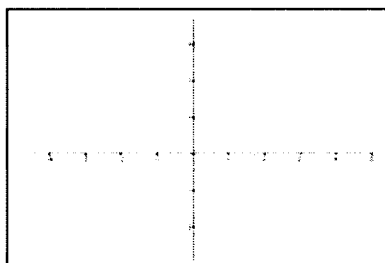
$$Y = -1(X + B)$$



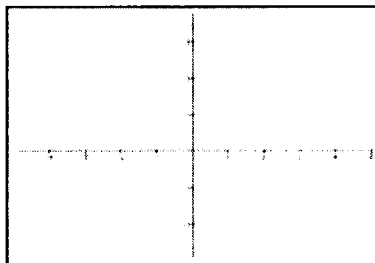
$$Y_1 = -1(X + 2)$$



$$Y_3 = -1(X + 4)$$



$$Y_4 = -1(X - 4)$$

**Critical Statistics**

	Y	Y_1	Y_2	Y_3	Y_4
y-intercept					
x-intercept					
Slope					

1. What is the slope of each graph above?
2. What effect does adding/subtracting a value of B have on changing the graph?
3. What is the effect of changing the slope on the x-intercept? y-intercept?

Generalizations for Parts I – V

1. When the slope of a line is a positive 1, adding a value of B to the X results in _____.
2. When the slope of a line is a positive 1, subtracting a value of B from the X results in _____.
3. When the slope of a line is A where $A > 0$, adding a value of B to the X results in _____.
4. When the slope (A) of a line is a positive number, subtracting a value of B from the X results in _____.
5. When the slope of a line is a negative 1, adding a value of B to the X results in _____.
6. When the slope of a line is negative 1, subtracting a value of B from the X results in _____.

7. When the slope of a line is negative ($A < 0$), adding a value of B to the X results in _____.
8. When the slope of a line is negative ($A < 0$), subtracting a value of B to the X results in _____.
9. Given that the slope of a line is 2 and the x -intercept is 5, what is the y -intercept?
_____ What would be an equation of this line? _____
10. Given that the slope of a line is 2, the x -intercept is R , and $R > 0$, what is the y -intercept?
_____ What would be an equation of this line? _____
11. Given that the slope of a line is m , the x -intercept is R , and $R > 0$, what is the y -intercept? _____ What would be an equation of this line? _____
12. Given that the slope of a line is 2 and the y -intercept is 6, then the x -intercept is _____.
What would be an equation of this line? _____
13. Given that the slope of a line is A and the y -intercept is B , then the x -intercept is _____.
An equation of the line is _____.




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DEPARTMENT OF EDUCATION

P. O. Box 2120
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May 30, 2012

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FROM:  Linda Wallinger
Assistant Superintendent for Instruction
SUBJECT: Copyright Request

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Appendix F

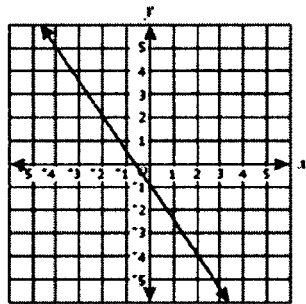
Name:

Date:

Teacher:

Block:

1. What is the slope of the line $y = 6x - 2$?
2. What is the slope of the line $8x - 2y + 6 = 0$?
- 3.

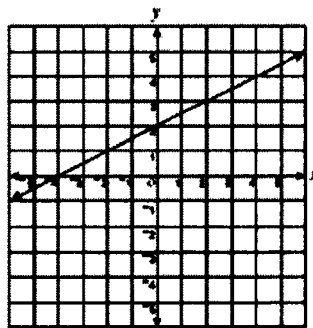


Which is closest to the slope of the line graphed above?

- A. $-\frac{6}{4}$
 - B. $-\frac{4}{6}$
 - C. $\frac{4}{6}$
 - D. $\frac{6}{4}$
4. What is the slope of the line that passes through $(-6, -10)$ and $(8, -4)$?

5. What is the slope of the line through (6, 4) and (-2, -8) ?
6. Write an equation for the line with slope = $-\frac{1}{2}$ and y-intercept of 5 ?
7. Write an equation for the line that passes through the origin and has a slope of $\frac{2}{3}$?

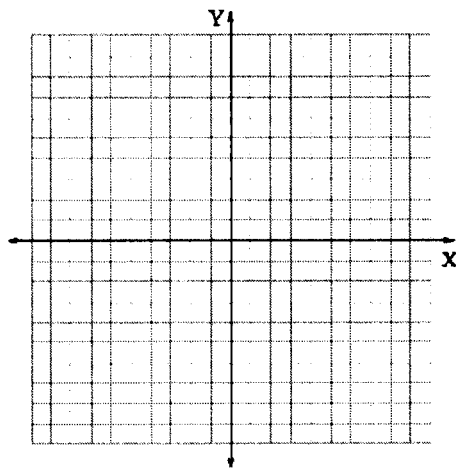
B.



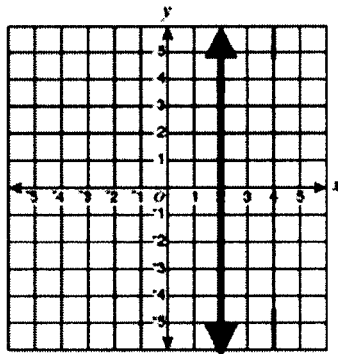
Which equation best represents the line shown?

- A. $y = 2x + 2$
- B. $y = 2x + 1$
- C. $y = \frac{1}{2}x + 2$
- D. $y = x + 2$

9. Graph the line with an x – intercept of 2 and a y – intercept of -3 ?



- 10.



Which equation best represents the line shown on the grid?

- A. $y = x - 4$
- B. $y = 4x$
- C. $x = 4$
- D. $y = 4$

Appendix G

Table of Specifications

Standard of Learning	Objective	Short Answer	Multiple Choice Without Graph	Multiple Choice With Graph	Graph a Line	Total Points
The student will graph linear equations and linear inequalities in two variables, including a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line. (VDOE, 2012)	Find the slope of the line, given the equation of a linear function in slope-intercept form.	1				10
	Find the slope of the line, given the equation of a linear function in standard form.	2				10
	Find the slope of a line, given the graph of a line.			3		10
	Find the slope of a line, given the x-intercept and y-intercept.				9	10
	Write an equation of a line when given two points on the line whose coordinates are integers.	4, 5				20
	Write an equation of a line when given the slope and a point on the line whose coordinates are integers.	6, 7				20
	Write an equation of a line when given the graph of the line.				8, 10	20
	Total					100

Appendix H

Final Assessment Scores

High School		High School	
Experimental	Control	Experimental	Control
70	0	100	40
50	40	20	90
40	90	90	90
40	10	20	90
0	70	20	40
70	60	0	50
40	20	40	50
70	60	90	60
50	60	100	50
80	70	20	20
40	50	20	20
20	60	0	0
30	30	10	40
60	50	10	60
50	20	10	30
100	30	10	50
20	80	10	10
90	70		10
100	90		30
60	60		0
			30

Middle School		Middle School	
Experimental	Control	Experimental	Control
90	40	90	90
50	90	50	100
70	60	50	80
30	100	40	40
30	70	50	70
80	100	60	20
60	80	65	80
80	50	70	50
10	80	75	80
80	80	55	40
40	70	40	50
70	70	80	40
60	60	80	50
80	80	10	0
60	70	60	50
50	90	40	40
60	80	60	80
60	80	20	10
70	90	10	40
90	60	80	100
			60

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